# Coulomb Gauge LQCD Static Quark Potentials

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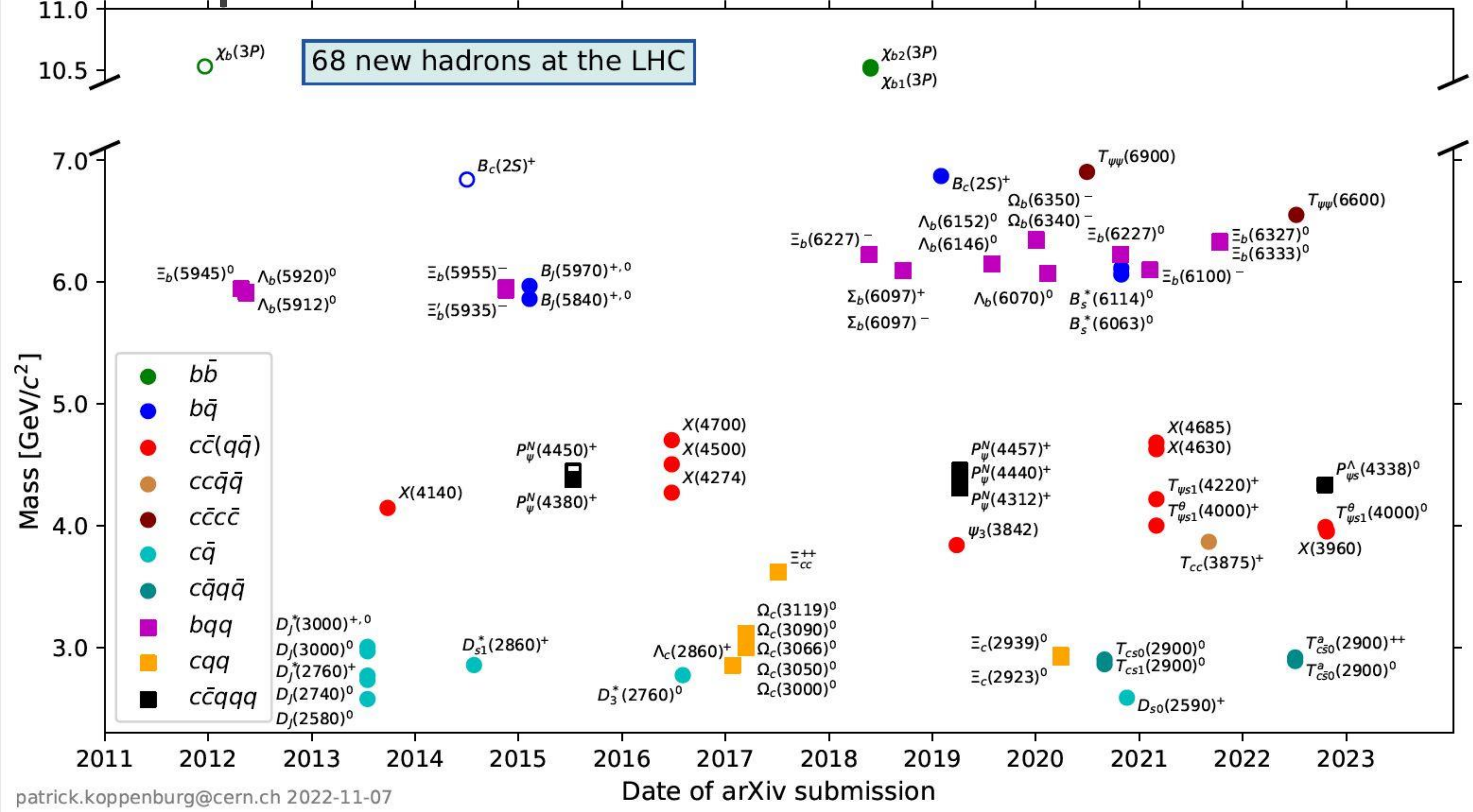
# Hadron spectrum




Plot from https://home.cern/news/news/physics/59-new-hadrons-and-counting

# Confinement

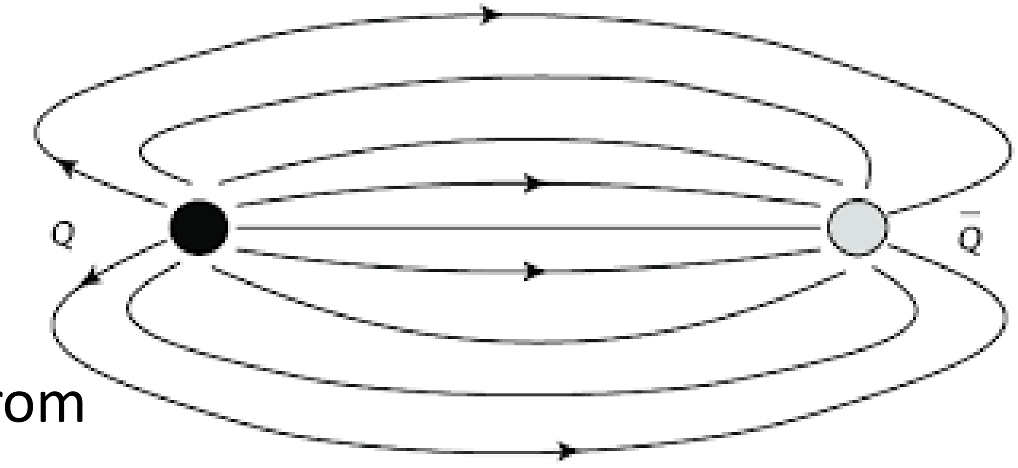
- The potential between static quark-antiquark pairs must be linear at large distances (before string-breaking)

- Wilson potential (from LQCD)

$$V(r) = A + \frac{B}{r} + \sigma r$$

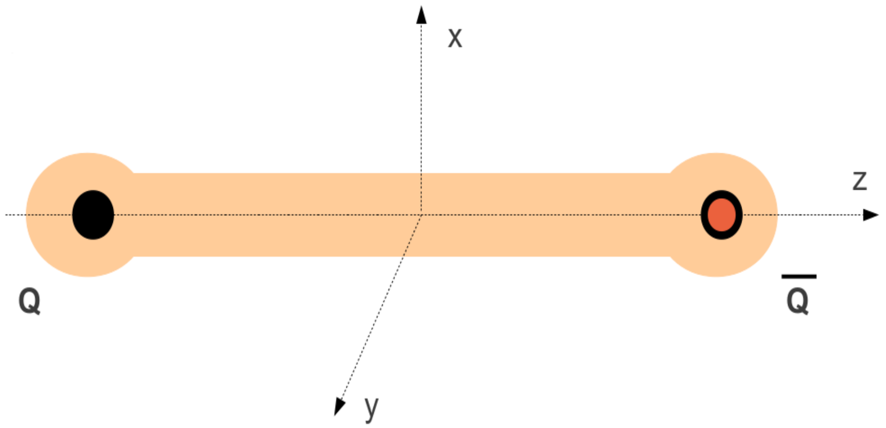
- Many models for charmonium, bottomonium come from this potential

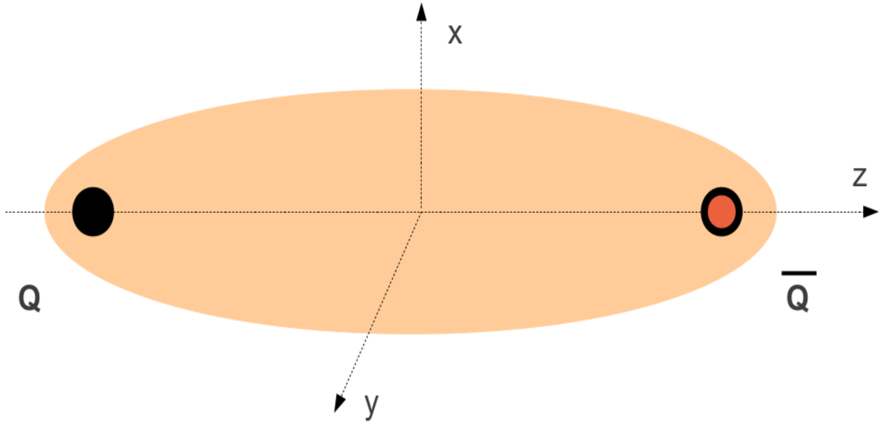
- Problem: This gives no information about *why* quarks are confined!



# Why Coulomb Gauge Lattice QCD?

- LQCD is the only way to probe quark-level interactions currently
- Need to fix the gauge to employ physical intuition/can understand QCD through analogy to QED in Coulomb Gauge
- Some questions remain about specifics of origin of Cornell potential, and flux tubes [1 2 3] on the Lattice
- Remarkable feature of Coulomb Gauge: $gA_0$ in Coulomb gauge is a renormalization-group invariant [4]

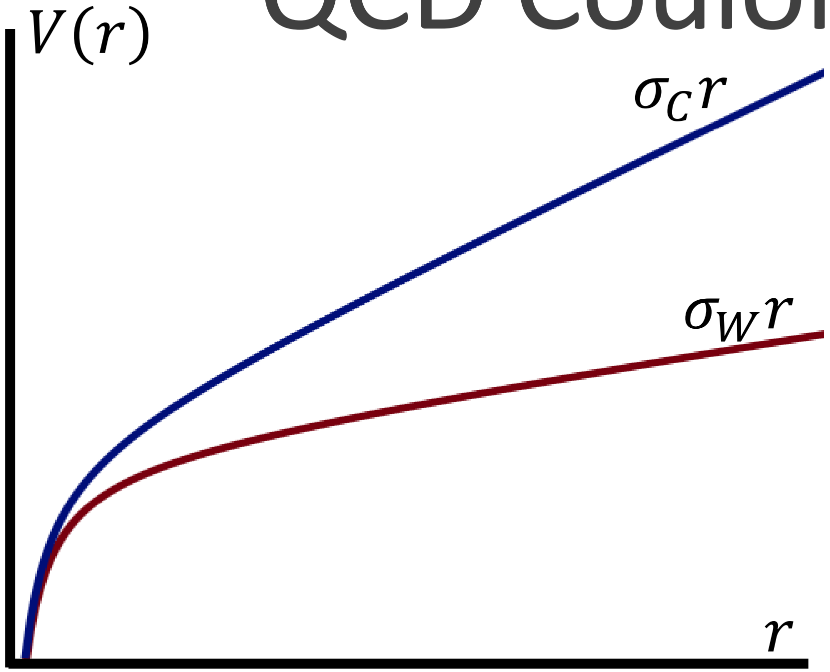


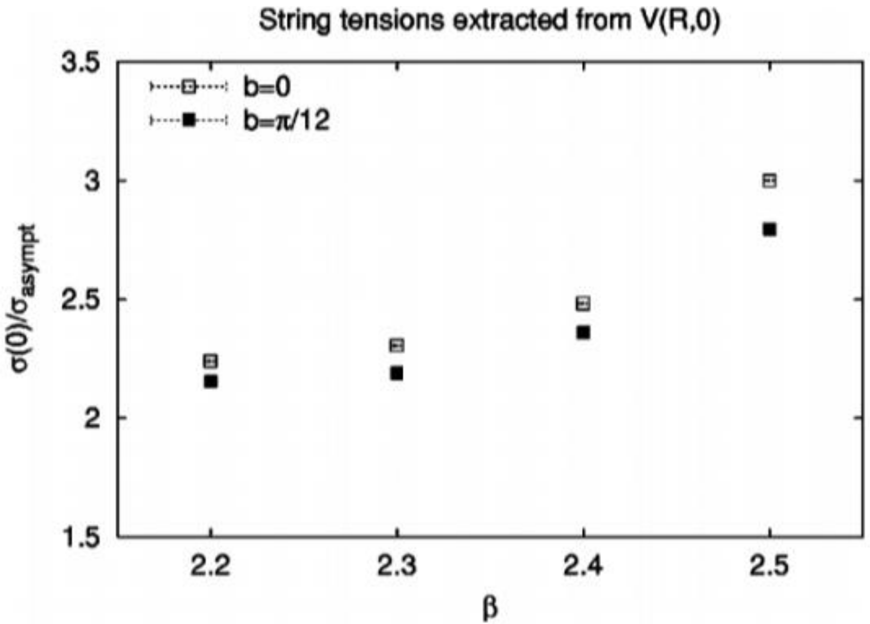





[1] P. O. Bowman and A. P. Szczepaniak, Phys. Rev.D70, 016002 (2004), arXiv:hep-ph/0403075[hep-ph].
[2] K. Chung and J. Greensite, Phys. Rev.D96, 034512 (2017), arXiv:1704.08995 [hep-lat].
[3] S. Dawid and A. P. Szczepaniak , Phys. Rev.D100, 074508 (2019)
[4] D Zwanziger, Nucl.Phys.B 518 (1998) 237-272

# QCD Coulomb Potential vs Wilson Potential




- Wilson potential = potential of static quark antiquark pair in ground state
- Coulomb potential = potential of static quark antiquark pair interacting *instantaneously* in Coulomb gauge
- Both potentials parameterized by Cornell potential

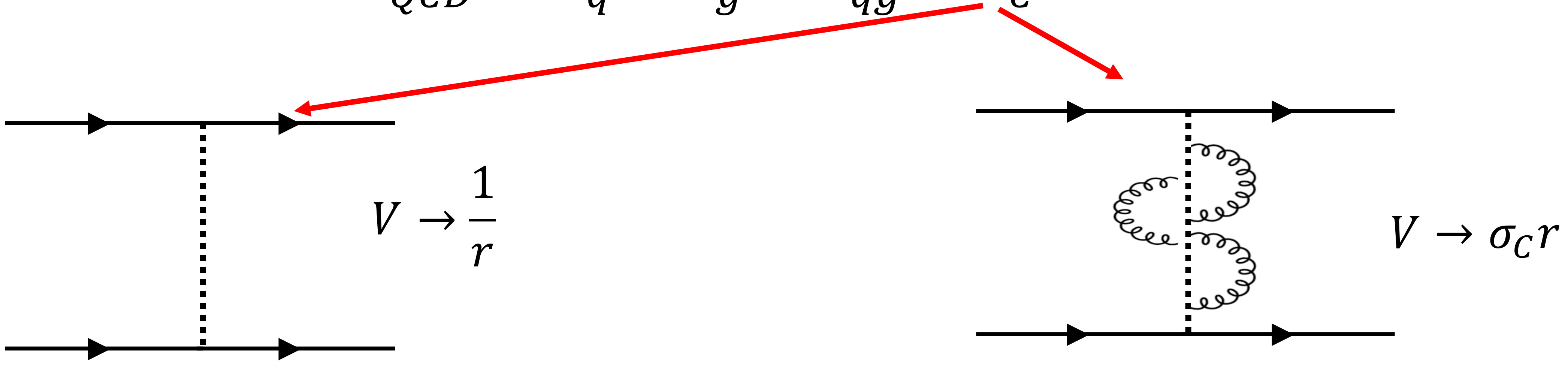
$$V(r) = A + \frac{B}{r} + \sigma r$$

- Confining behavior of Coulomb potential is *necessary* for Wilson confinement[5]


[5] D. Zwanziger, Phys. Rev. Lett.90, 102001 (2003), arXiv:hep-lat/0209105 [hep-lat].
Plot from: J. Greensite and A. P. Szczepaniak,Phys. Rev. D91, 034503(2015).

# Coulomb Gauge Hamiltonian:

$$H_{QCD} = H_q + H_g + H_{qg} + H_C$$




$$\langle q\bar{q}|H_{QCD}|q\bar{q}\rangle \approx V_C(R)$$

- The static quark-antiquark state which produces the coulomb potential is *not* the ground state!

$$H_{QCD}|q\bar{q}_{true}\rangle = \sigma_W r|q\bar{q}_{true}\rangle$$

$$|q\bar{q}_{true}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}gg\rangle + \cdots$$

$\Delta_{FP} = -(\partial \cdot D)$

# Coulomb Potential (Continuum)

$$V_C(\vec{p}) = g^2 \mathrm{Tr}\langle (\Delta_{FP})^{-1}(-\nabla^2)(\Delta_{FP})^{-1}\rangle$$

- Definition based off of IR behavior of ghost propagator
- Linearly rising potential would diverge like $\frac{1}{p^4}$
- Coulomb gauge not completely specified by $\partial_i A_i = 0$
- Enhancement of Coulomb potential near boundary of FMR

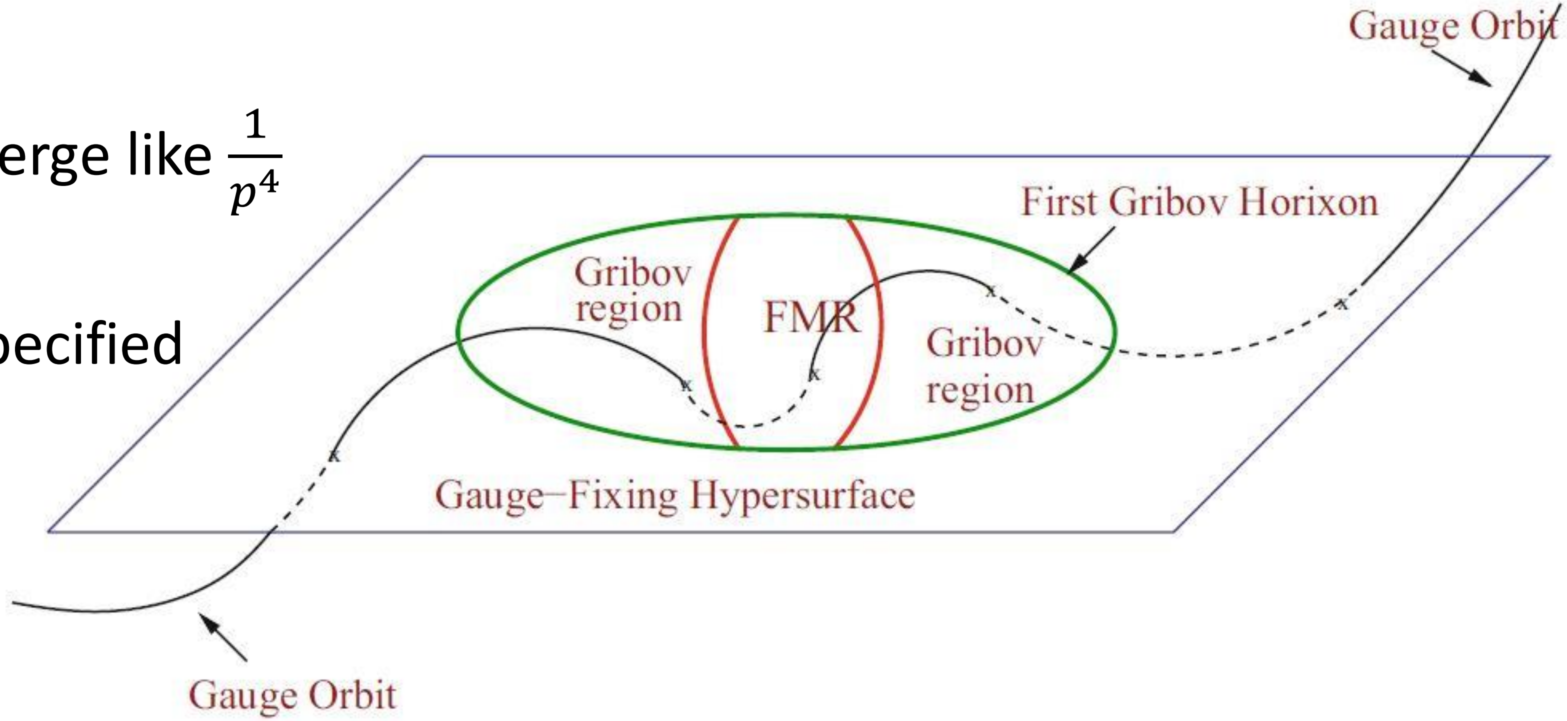
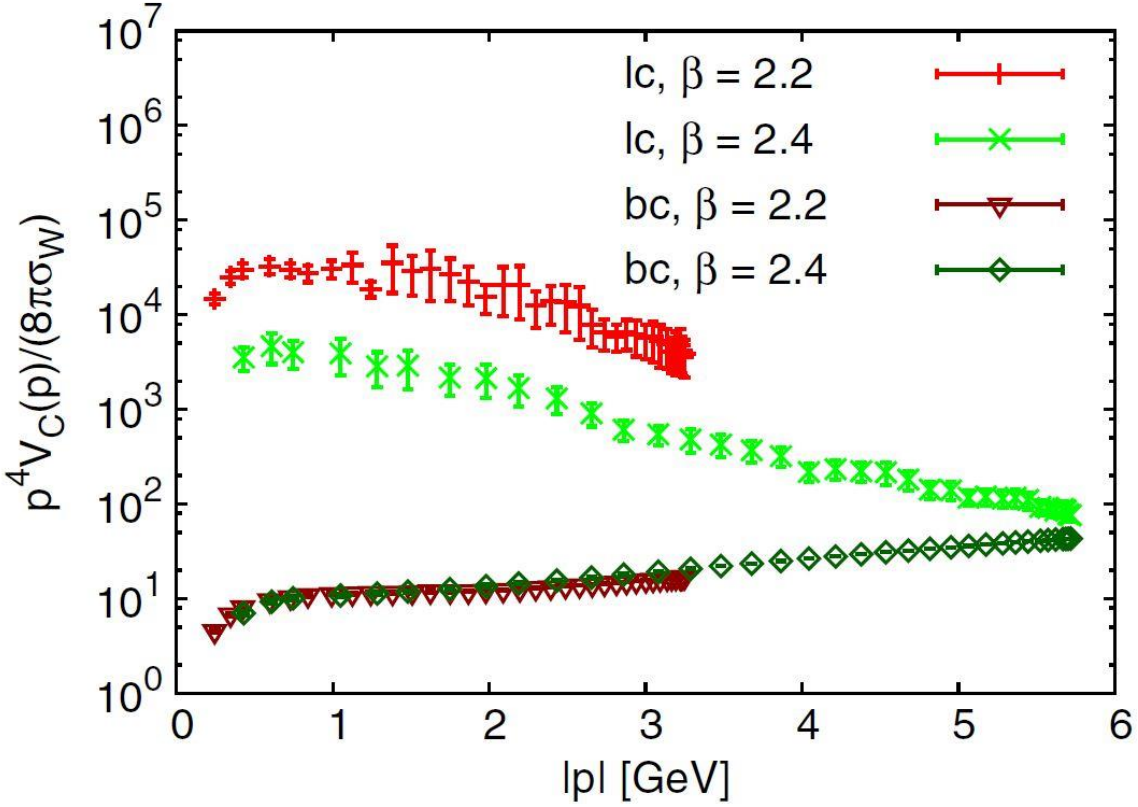



Figure from "An Introduction to the Confinement Problem", J. Greensite

# Coulomb Potential (Continuum)

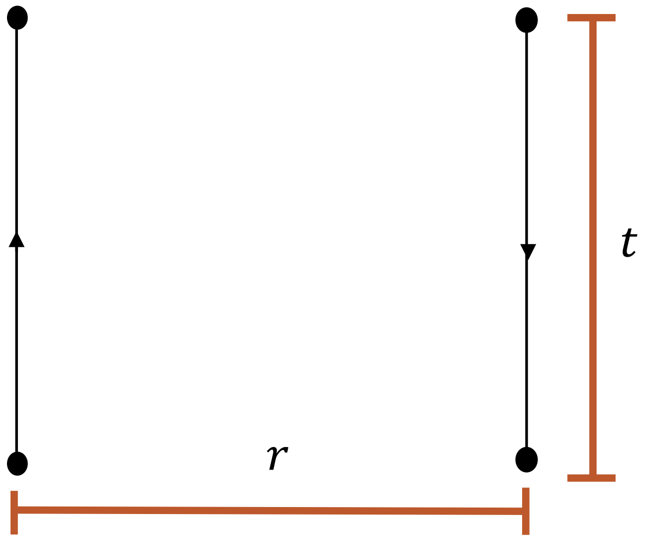



- This definition suffers from Gribov copies!
- Unclear where the issue comes from (Lattice discretization artifacts, insufficient constraints, etc.)

Plot from H. Reinhardt, G. Burgio, D. Campagnari, E. Ebadati, J.Heffner, M. Quandt, P. Vastag, and H. Vogt, Adv. High Energy Phys. 2018, 1 (2018).

$L(\vec{x},t) = P\, e^{i\int_0^t A_0(\vec{x},t')dt'}$

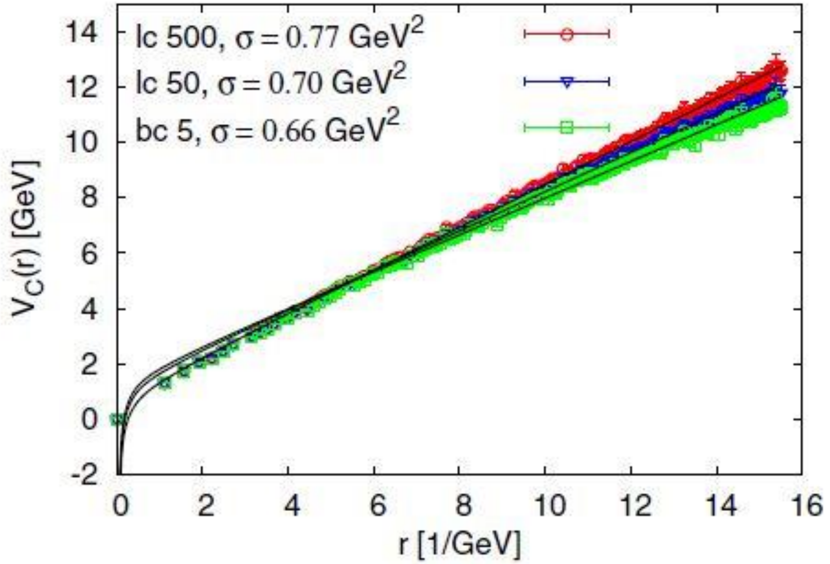
# Coulomb Potential (Continuum)




$$G(r,t) = \langle \mathrm{Tr}[L^\dagger(0,t)L(\vec{r},t)] \rangle$$

$$= \sum_n |c_n|^2 e^{-E_n T}$$

$$V(r,t) = -\frac{d}{dt}\log G(r,t)$$




$$t \to \infty \Rightarrow V(r,t) \to E_{ground}$$

$$t \to 0 \Rightarrow V(r,t) \to V_C(r) = \frac{\sum_n E_n |c_n|^2}{\sum_n |c_n|^2}$$

# Lattice Framework

- Anisotropic Wilson Action

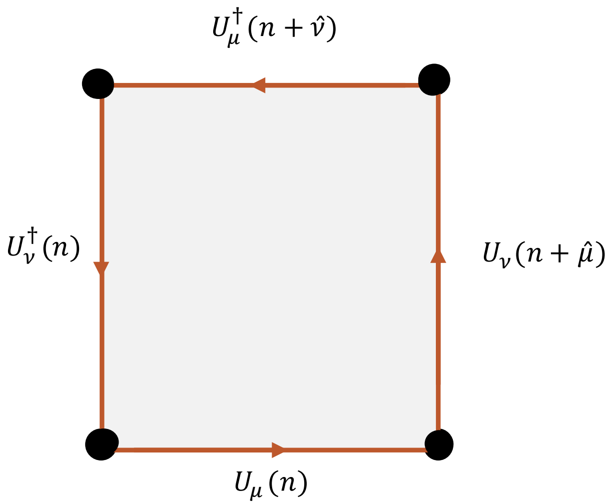
$$S = \sum_n \left[ \beta_s \sum_{j>i=1}^{3} \left(1 - \frac{1}{2} \mathrm{Tr}\, U_{ij}(n)\right) + \beta_t \sum_{i=1}^{3} \left(1 - \frac{1}{2} \mathrm{Tr}\, U_{0i}(n)\right)\right]$$

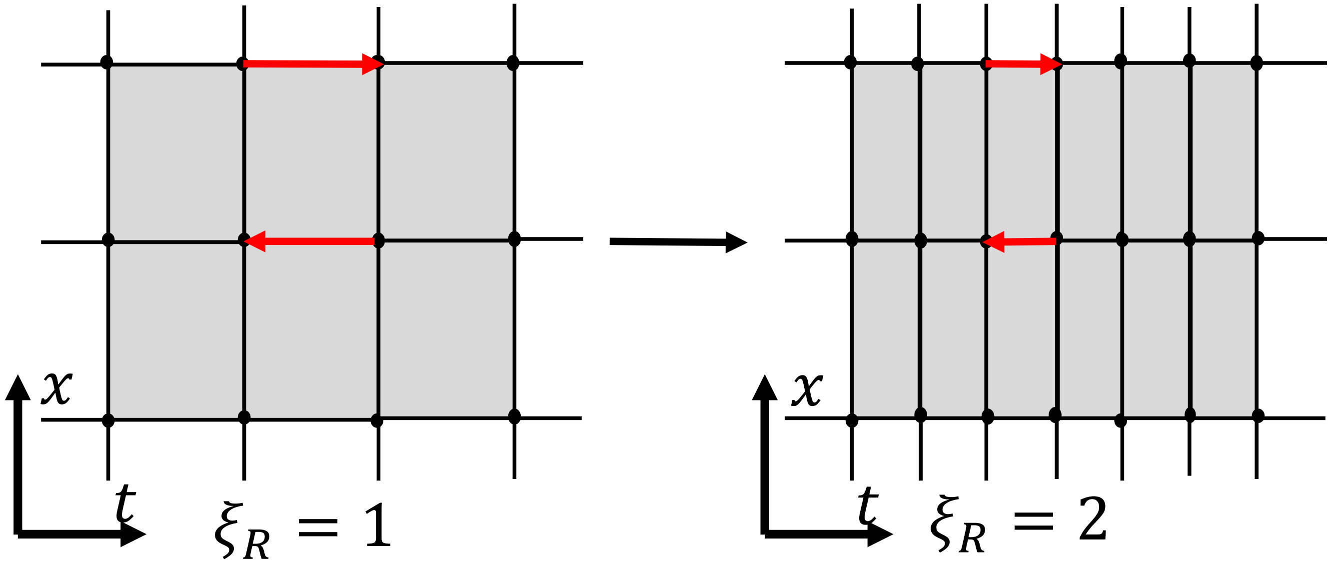
$$\beta = 2N_C/g^2$$

$$\beta_s = \frac{\beta}{\xi_0}$$

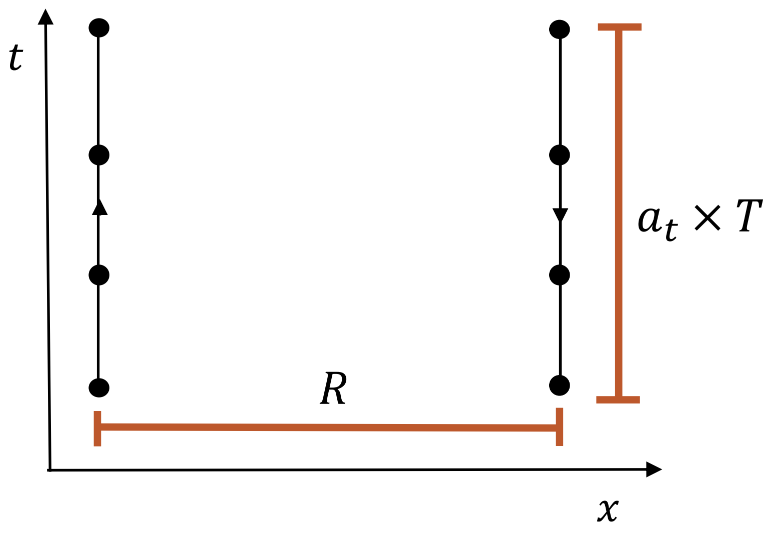
$$\beta_t = \xi_0\, \beta$$

$$\xi_R = \frac{a_s}{a_t}$$



# Coulomb Potential (Lattice)




$$L(\vec{x},T) = \prod_{m}^{T/m} U_0(\vec{x}, m a_t)$$

$$G(R,T) = \langle \mathrm{Tr}\left[L^\dagger(0,T)L(R,T)\right]\rangle$$

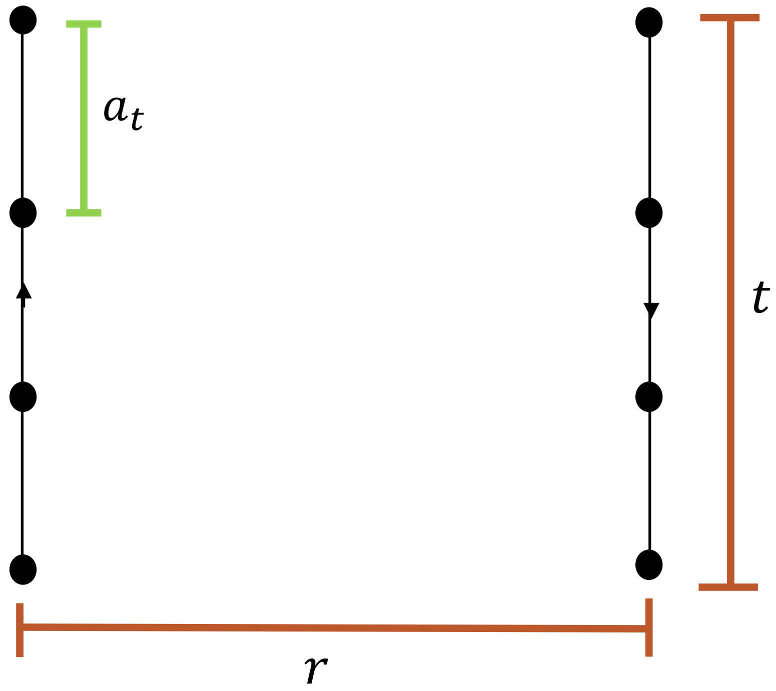
$$V(R,T) = \frac{1}{a_t}\log\frac{G(R,T)}{G(R,T+a_t)}$$

$$V_C(R) \equiv V(R,0) = -\frac{1}{a_t}\log G(R,a_t)$$

# Which Limit?

$$V_C(r) = \lim_{t \to 0} V(r,t)$$

$$t = T \times a_t$$




a) $\lim_{a_t \to 0} \lim_{T \to 0}$

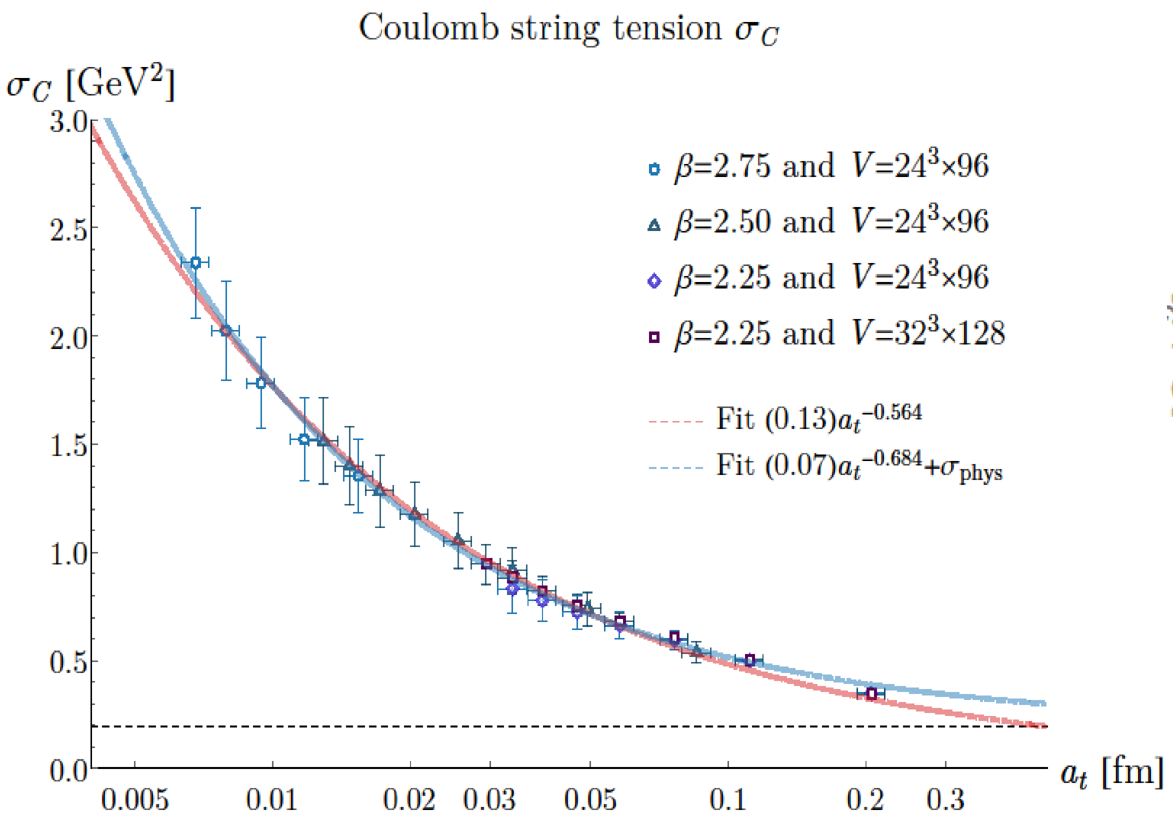
b) $\lim_{a_t \to 0}$ (finite $T \neq 0$)

c) $\lim_{T \to 0} \lim_{a_t \to 0}$

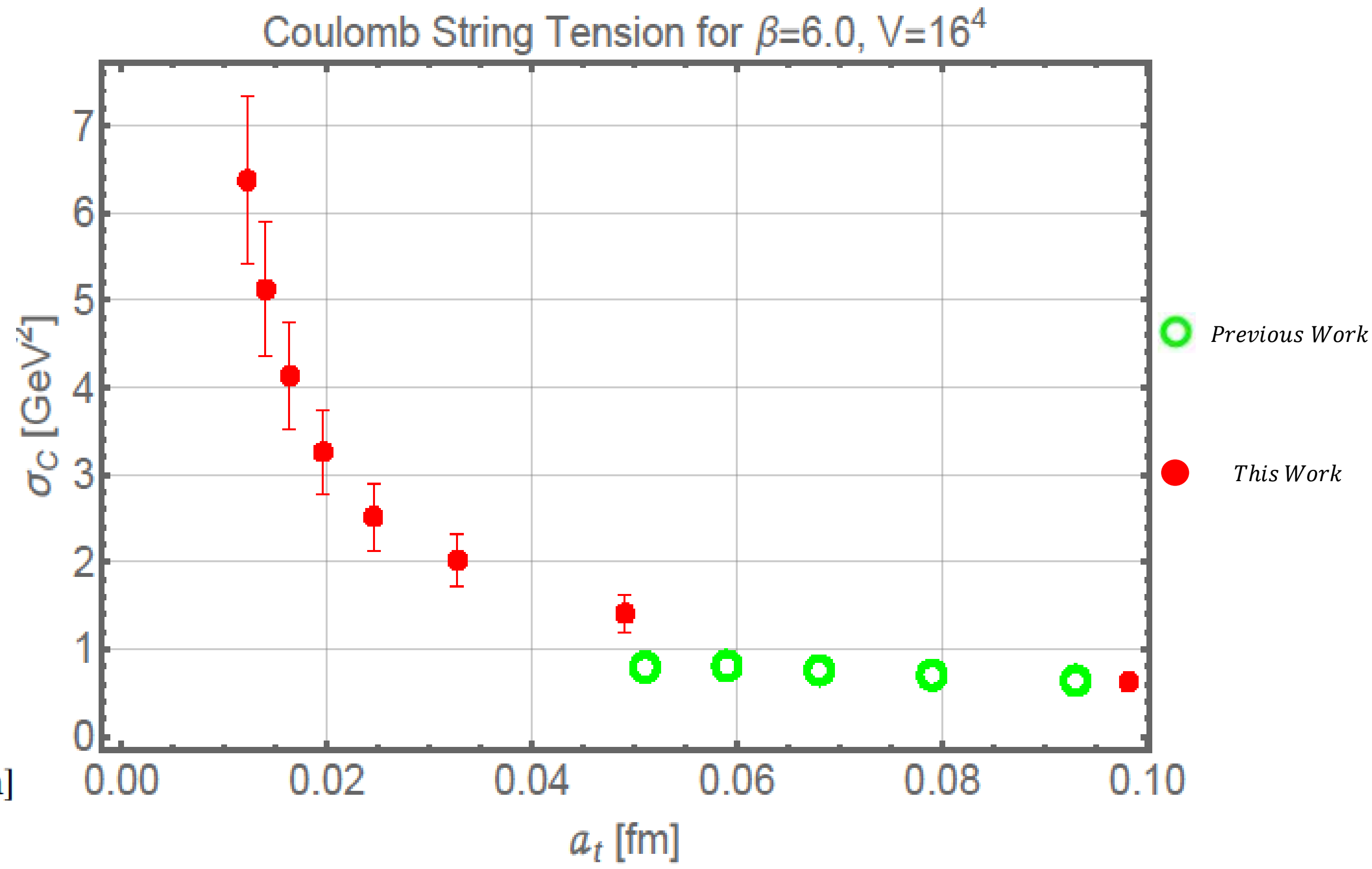
d) $\lim_{T \to 0}$ (finite $a_t \neq 0$)

# Definition $a) \lim_{a_t \to 0} \lim_{T \to 0} V(R,T) = V_C(R) = -\frac{1}{a_t} \log G(R, a_t)$

SU(2)

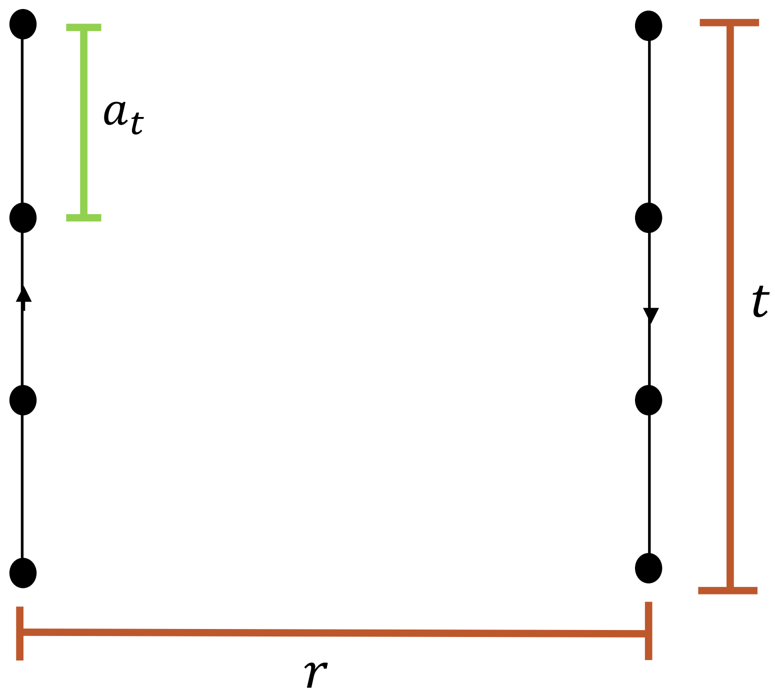



SU(3)



$$V(R,T) = \frac{1}{a_t} \log \frac{G(R,T)}{G(R,T+a_t)}$$

# Problem: $G(R,0) = 1$ *discontinuously*




*a)* $\lim_{a_t \to 0} \lim_{T \to 0}$

*b)* $\lim_{a_t \to 0}$ (finite $T \neq 0$)

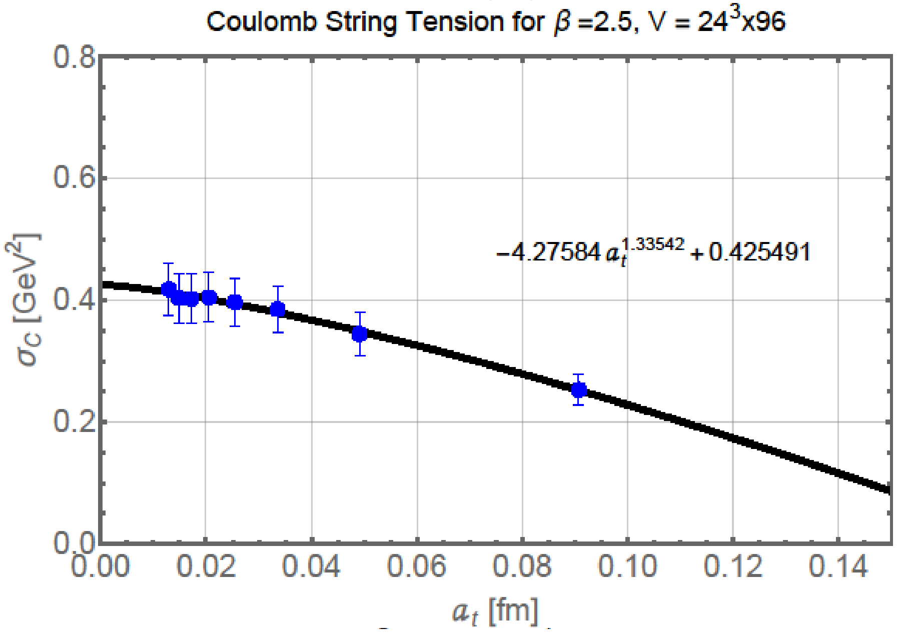
*c)* $\lim_{T \to 0} \lim_{a_t \to 0}$

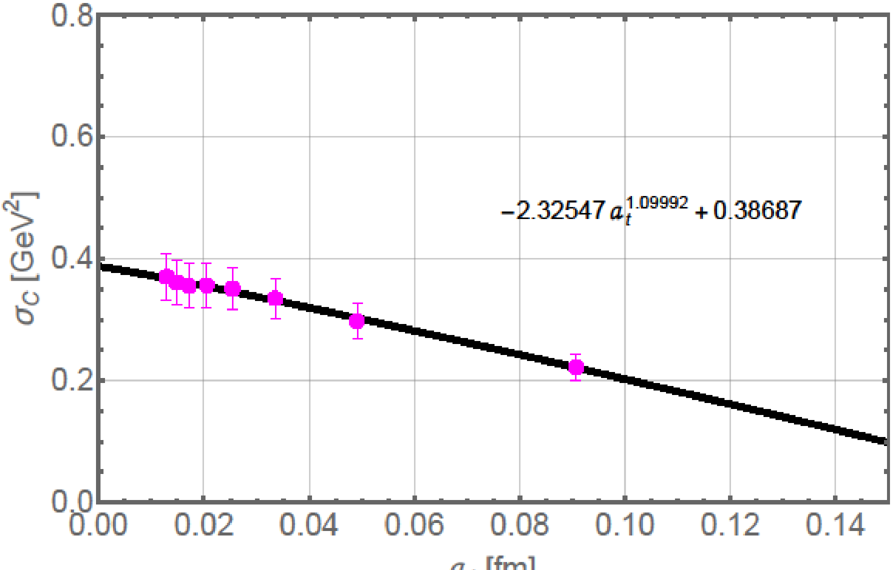
*d)* $\lim_{T \to 0}$ (finite $a_t \neq 0$)

$$\Rightarrow \text{Try} \quad \lim_{a_t \to 0} \frac{1}{a_t} \log \frac{G(R,a_t)}{G(R,2a_t)} \stackrel{?}{=} \lim_{a_t \to 0} \frac{1}{a_t} \log \frac{G(R,2a_t)}{G(R,3a_t)}$$

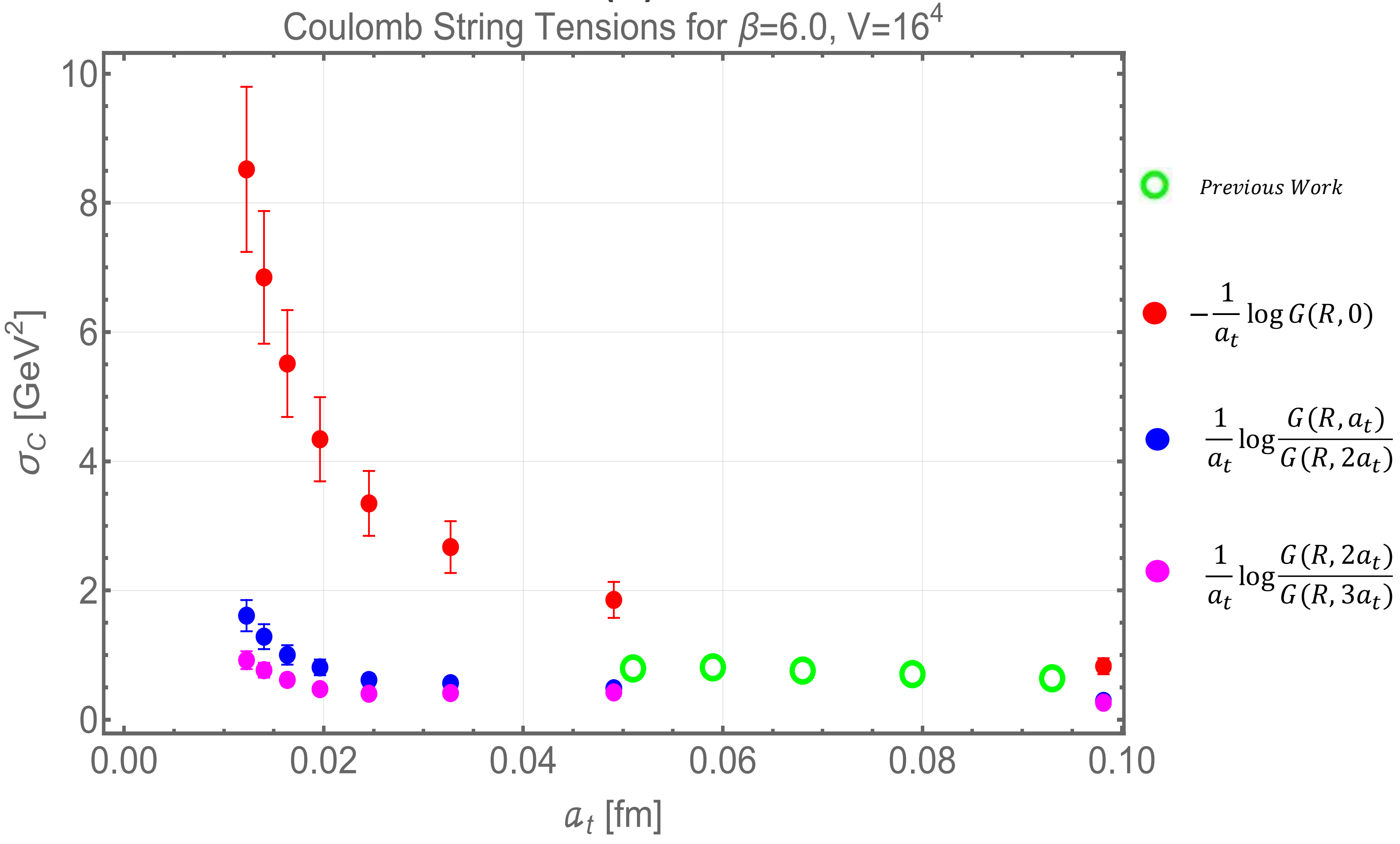
# Definition $b)\ \lim_{a_t \to 0}$ (finite $T \neq 0$) $= V_C(R)$

SU(2)




SU(3)



# Summary

- Coulomb Gauge Physics is important for understanding hadron spectrum, confinement

- Preliminary results show the Coulomb String tension is *closer* to the Wilsonian string tension than previous calculations

- Be careful when you take limits!