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NAWA
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Spin polarization, pseudogauge transformations, and fluctuations

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Stony Brook University

24 Aug 2023

Chirality and Criticality: Novel Phenomena in Heavy-Ion Collisions

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Arpan Das
Avdhesh Kumar
Masoud Shokri
Gabriel Sophys
Ali Tabatabaee

Motivation

Phys.Rev.Lett. 94 (2005) 102301, Phys. Rev. C 77, 024906

→ Non-central ultra-relativistic HIC, due to spatial inhomogeneity, create large OAM, $\boldsymbol{L}_{\text{initial}} \approx 10^5\,\hbar$.

→ This OAM is along y-axis (orthogonal to the reaction $(x - z)$ plane) and may polarize spin of the QGP constituents.

Phys. Rev. Lett. 94 (2005) 102301

Predicted in 2005

→ This spin polarization is expected to be transferred to the hadrons leading to their global spin polarization along y -axis.

→ Among various spin-polarizable hadrons, Lambda $(\Lambda(\bar{\Lambda}))$ hyperons are special as they are self-analyzing.

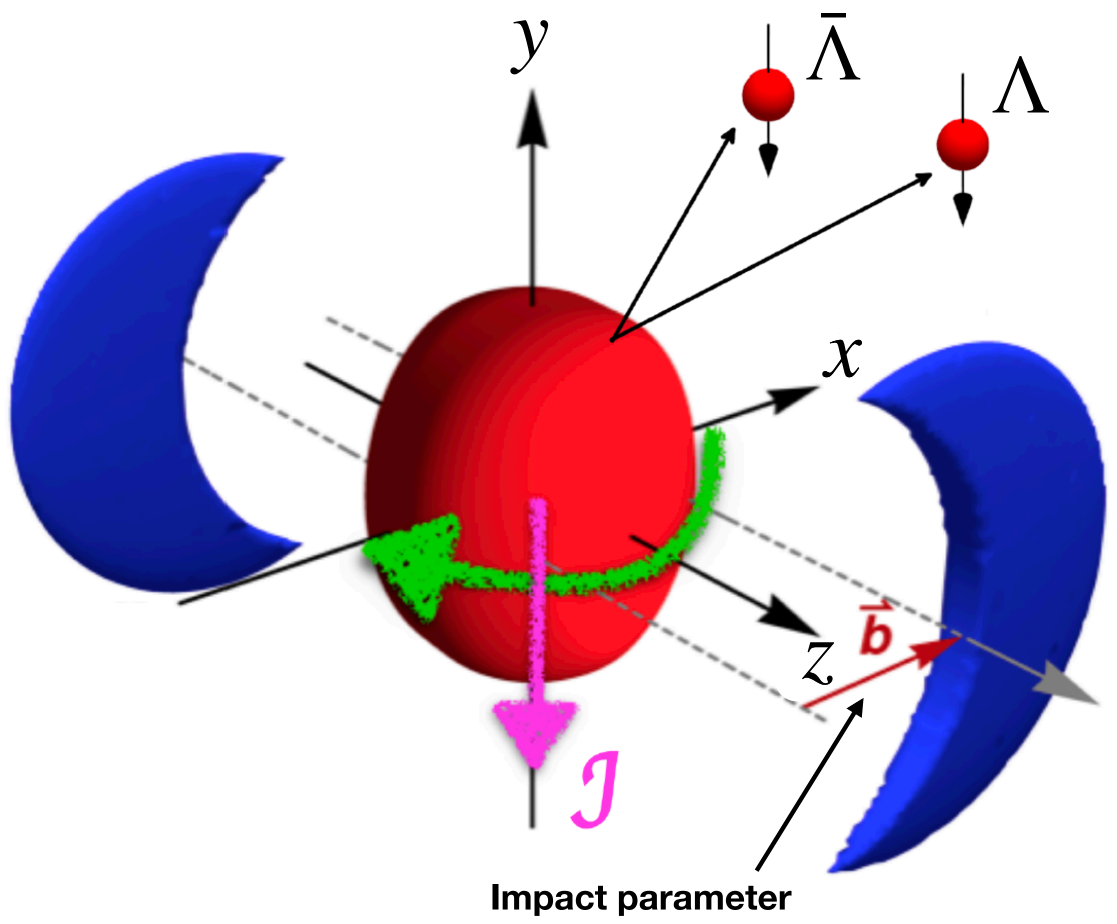

Figure: Schematic diagram of the initial angular momentum orientation in non-central heavy-ion collision.
(Prog.Part.Nucl.Phys. 108 (2019) 103709)

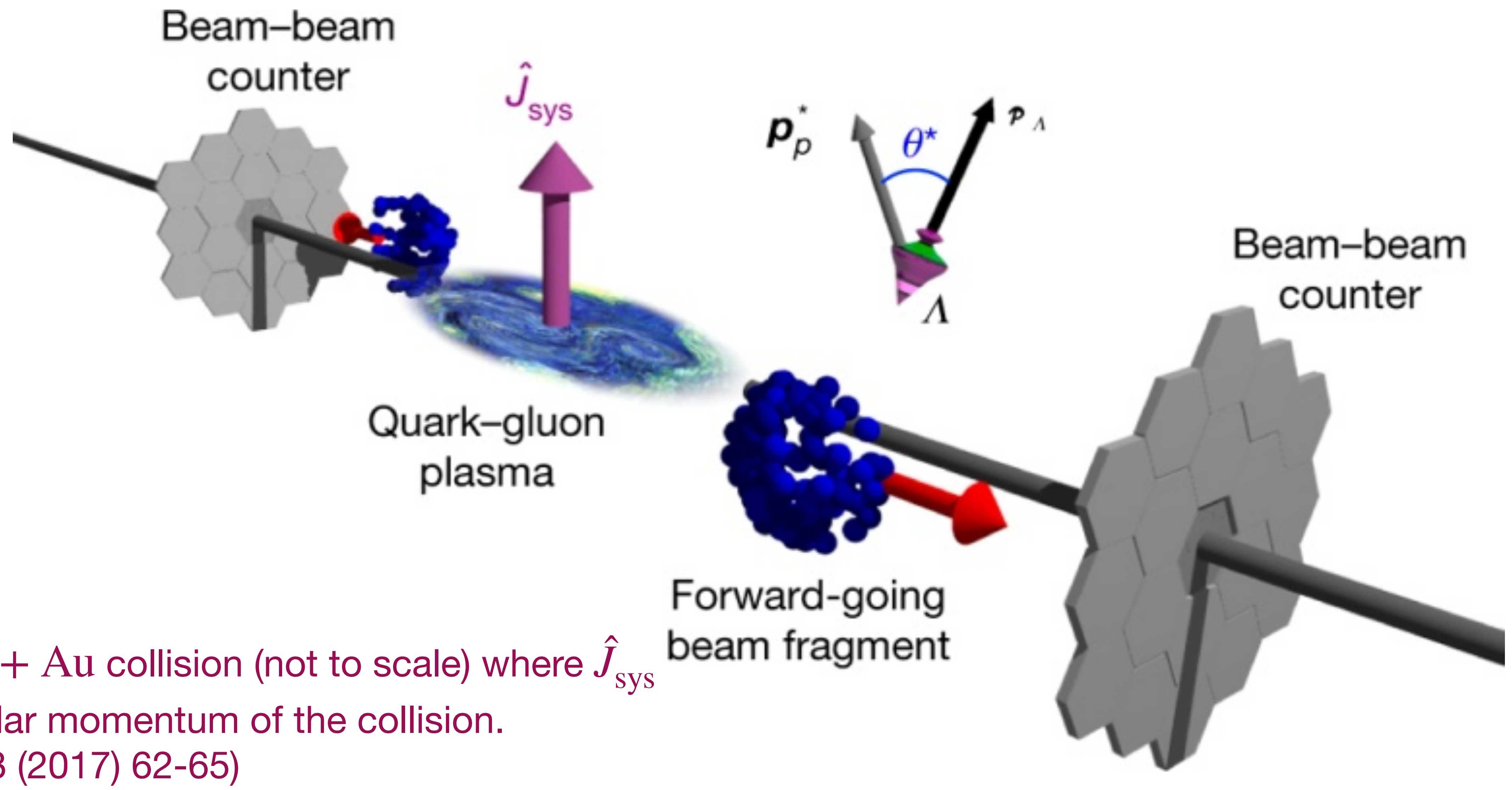

Figure: Schematic diagram of a $\mathrm{Au}+\mathrm{Au}$ collision (not to scale) where $\hat{J}_{\text{sys}}$ is the direction of the angular momentum of the collision.
(Nature 548 (2017) 62-65)

Motivation

- First observation of global spin polarization of $\Lambda(\bar{\Lambda})$ was by STAR collaboration, providing evidence of the vortical structure of QGP.
- It shows decreasing behavior with increase in collision energy.
- Differences between Λ and $\bar{\Lambda}$ polarization may be due to initial EM fields caused during the collisions, however, we do not have clear explanation yet.

Finite differences

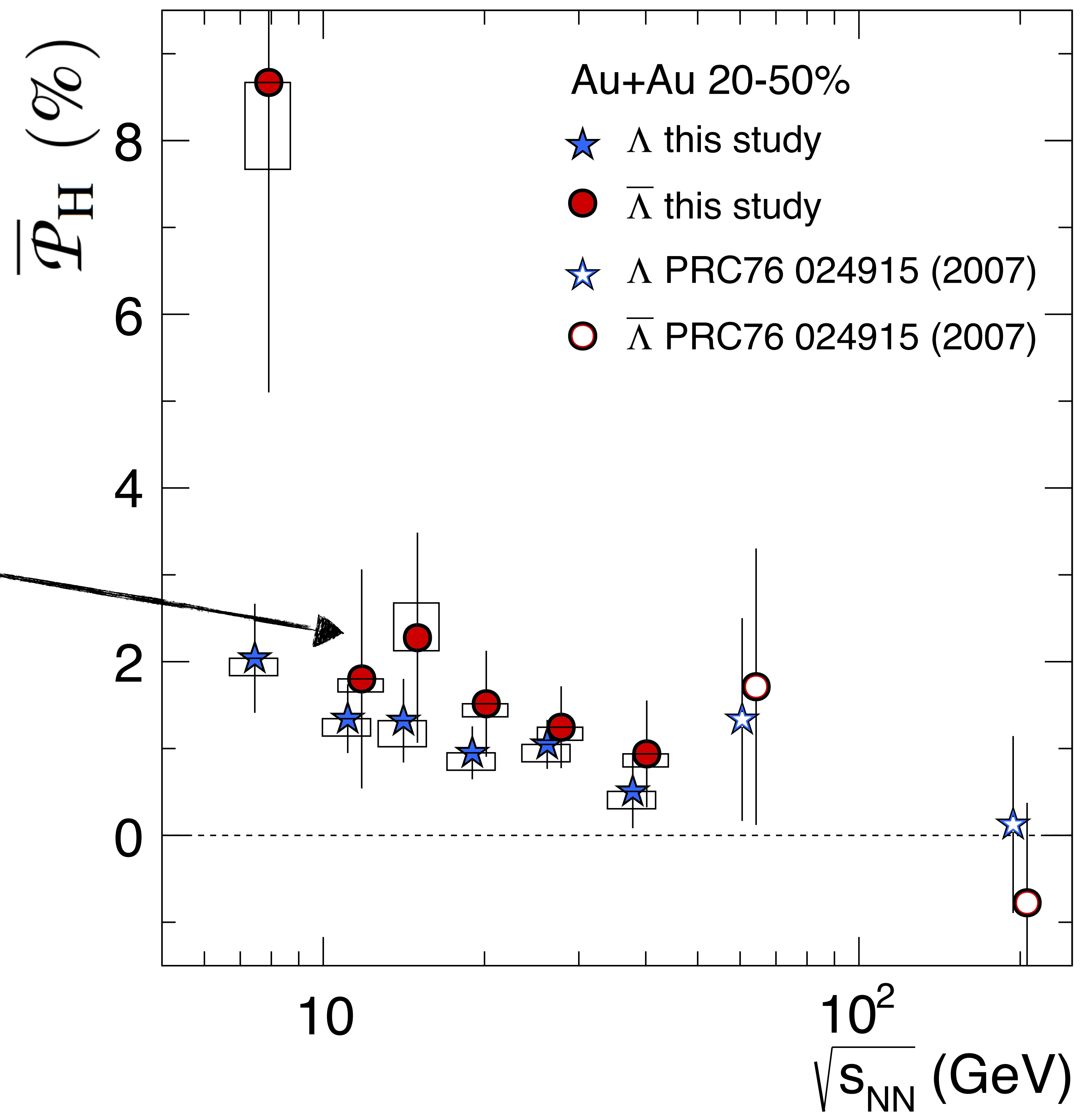

Figure: Average global spin polarization for $\Lambda(\bar{\Lambda})$ hyperons in 20-50% centrality $\mathrm{Au} + \mathrm{Au}$ collisions as a function of collision energy. (Nature 548 (2017) 62-65)

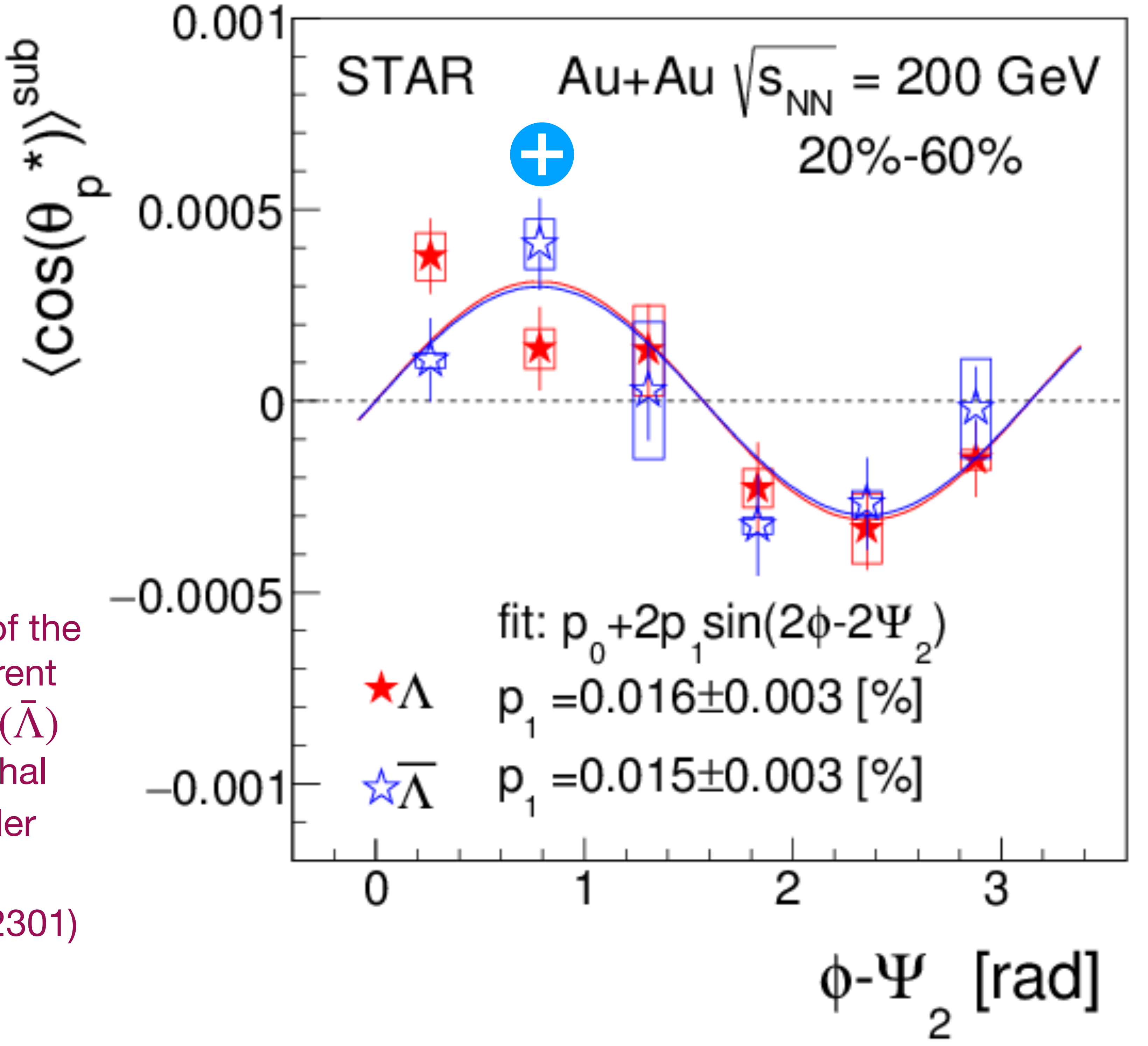

Figure: Cosine of the polar angle of the proton in the rest frame of its parent $\Lambda(\bar{\Lambda})$ that is averaged over all $\Lambda(\bar{\Lambda})$ particles as a function of azimuthal angle (ϕ) relative to second-order event plane (ψ). (Phys.Rev.Lett. 123 (2019) 13, 132301)

- Besides the global spin polarization along y-axis, STAR also observed spin polarization along the beam direction (z) which may result from the flow structure in the transverse plane.

Quadruple structure in flow

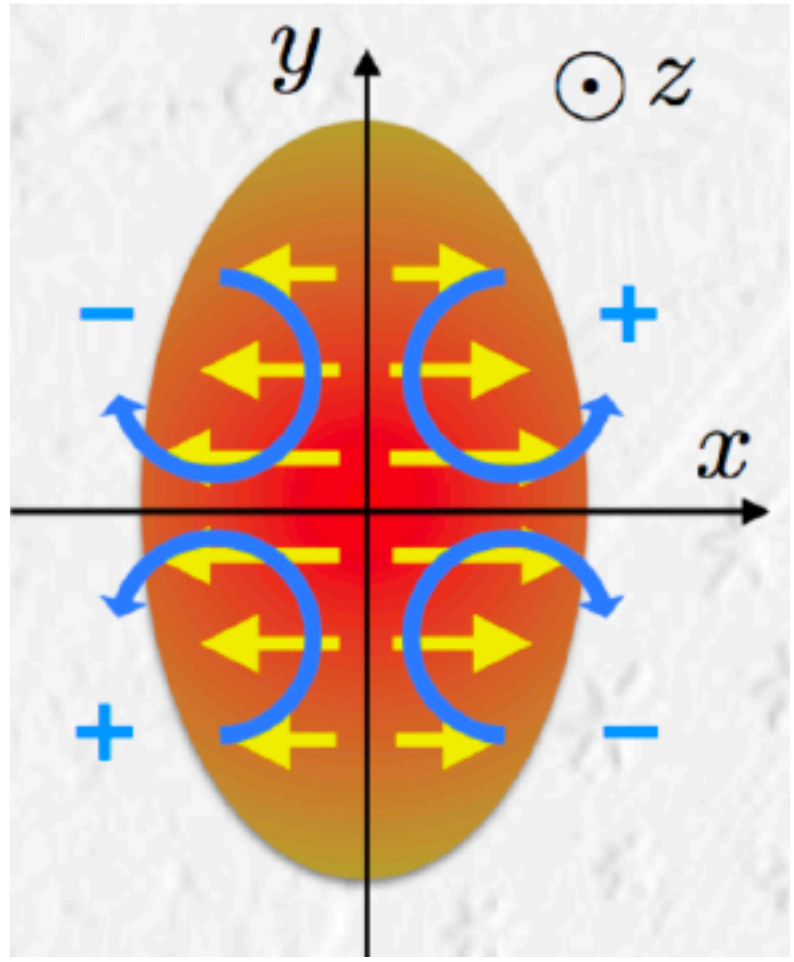

Figure: Schematic view of the flow structure in the transverse plane which may generate longitudinal polarization. (Phys.Rev.Lett. 120 (2018) 1, 012302) (EPJ Web Conf. 171 (2018) 07002)

Developments towards hydrodynamics with spin

Lagrangian effective field theory approach

D. Montenegro, G. Torrieri, Phys.Rev. D94 (2016) no.6, 065042
D. Montenegro, L. Tinti, G. Torrieri, Phys. Rev. D 96(5) (2017) 056012; Phys. Rev. D 96(7) (2017) 076016
D. Montenegro, G. Torrieri, Phys. Rev. D 100, 056011 (2019)

Hydrodynamics with spin based on entropy-current analysis

K. Hattori, M. Hongo, X-G Huang, M. Matsuo, H. Taya, PLB 795 (2019) 100-106
K. Fukushima, S. Pu, PLB 817 (2021) 136346

Hydrodynamics of spin currents using presence of torsion

D. Gallegos, U. Gursoy, A. Yarom arXiv:2101.04759

Relativistic viscous hydrodynamics with spin using Navier-Stokes type gradient expansion analysis

D. She, A. Huang, D. Hou, J. Liao, arXiv:2105.04060

Relativistic viscous spin hydrodynamics from chiral kinetic theory

S. Shi, C. Gale, and S. Jeon, Phys. Rev. C 103, 044906 (2021)

Spin polarization generation from vorticity through nonlocal collisions

N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke, arXiv:2005.01506, arXiv:2103.04896

Spin polarisation due to thermal shear

F. Becattini, M. Buzzegoli, and A. Palermo, arXiv:2103.10917
S. Y. F. Liu and Y. Yin, arXiv:2103.09200

Not enough space to include all papers!

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	Citeable	Published
Papers	360	360
Citations	10,087	10,087
h-index	57	57
Citations/paper (avg)	28	28

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Outline

What do I mean?

→ Why we need spin hydrodynamics?

→ Pseudogauge transformations

→ GLW pseudogauge based spin hydro

→ 1st part

→ Pseudogauge ambiguity

→ Can we resolve this ambiguity?

→ 2nd part

Why we need spin hydro?

Models that assume LTE of spin degrees of freedom are able to explain global spin polarization measurement.

What does it mean?

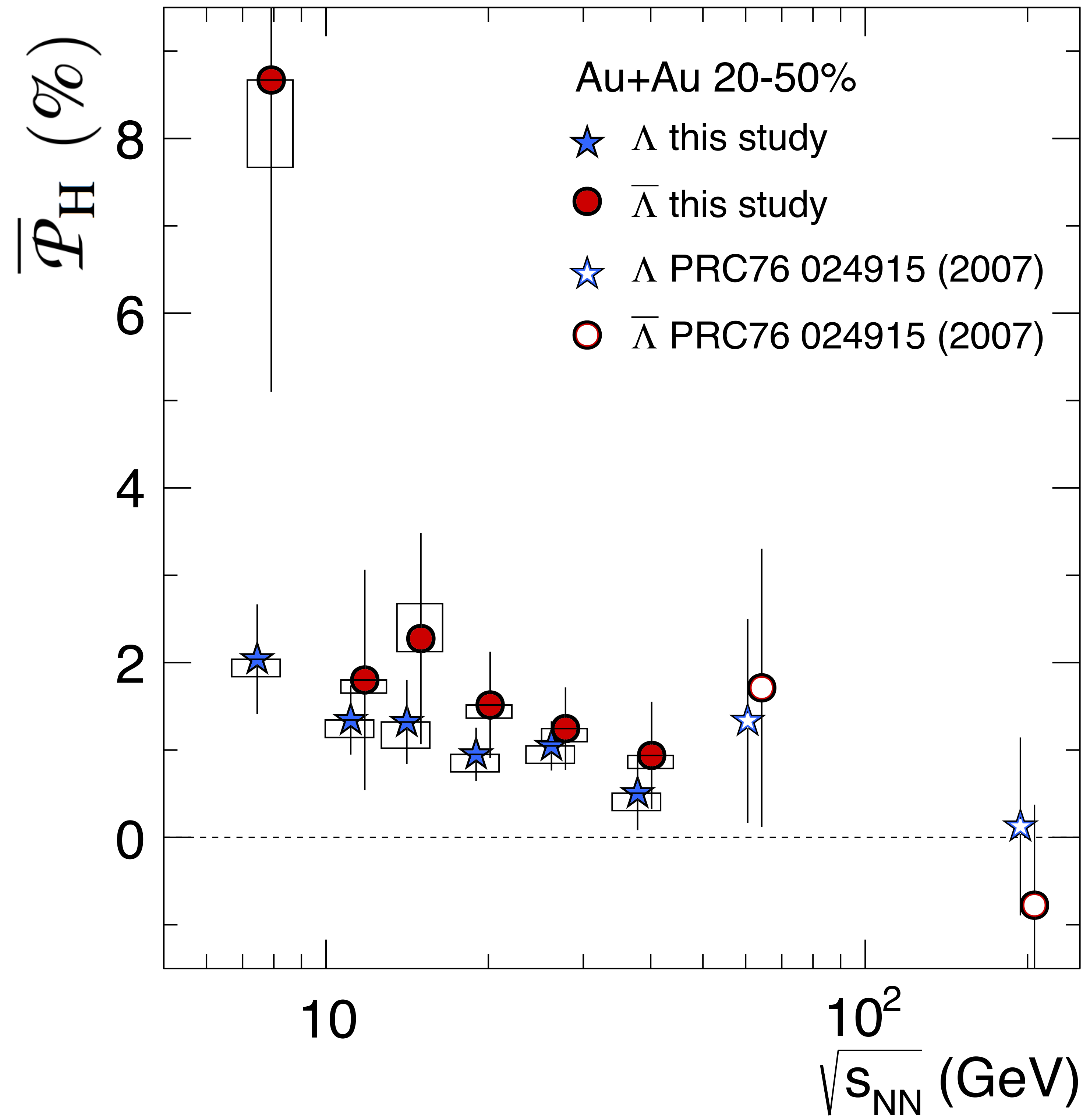


Figure: Average global spin polarization for $\Lambda(\bar{\Lambda})$ hyperons in 20-50% centrality $\mathrm{Au}+\mathrm{Au}$ collisions as a function of collision energy. (Nature 548 (2017) 62-65)

Why we need spin hydro?

In local thermodynamic equilibrium, one can establish a link between spin and thermal vorticity

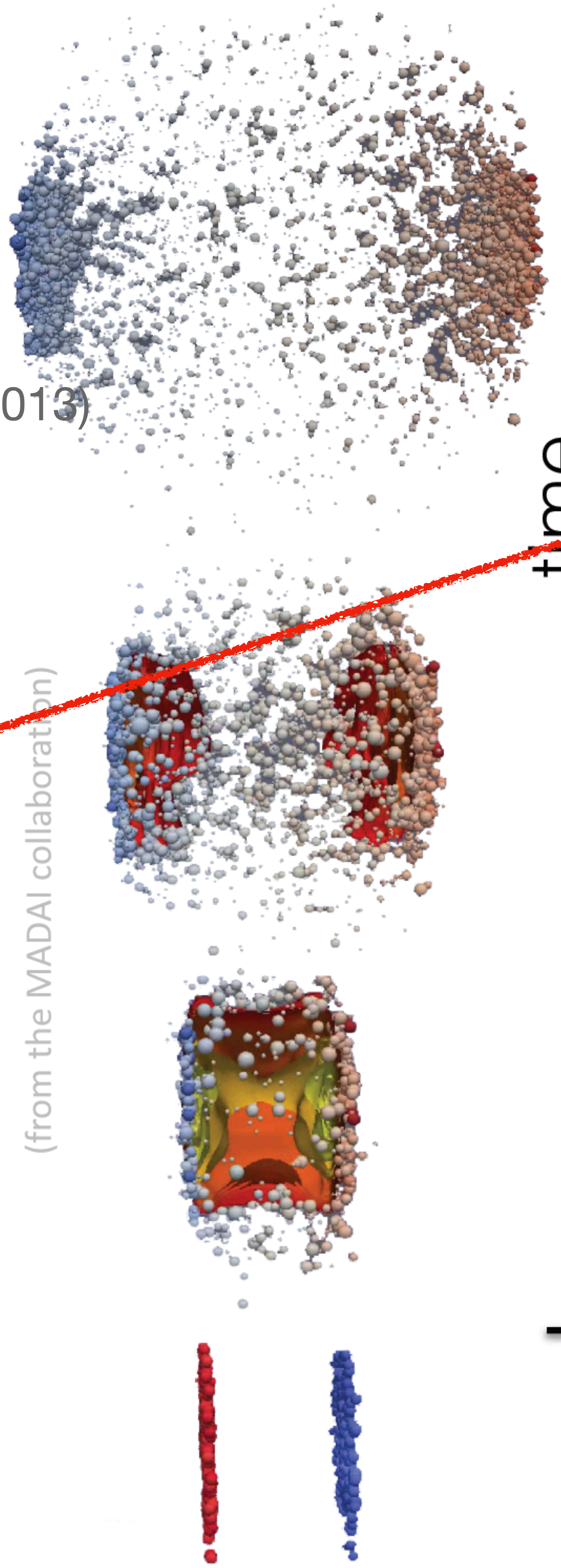
Becattini F, Piccinini F. Ann. Phys. 323:2452 (2008)
Becattini F, Chandra V, Del Zanna L, Grossi E. Ann. Phys. 338:32 (2013)
Fang R, Pang L,Wang Q,Wang X. Phys. Rev. C 94:024904 (2016)

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int \mathrm{d}\Sigma_\lambda p^\lambda n_F(1-n_F)\boxed{\varpi_{\rho\sigma}}}{\int \mathrm{d}\Sigma_\lambda p^\lambda n_F}$$

$$\boxed{\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) \qquad \beta^\mu = \frac{u^\mu}{T}}$$

$$n_F = (1+\exp[\beta\cdot p - \mu Q/T])^{-1}$$

Allows to extract polarization at the freeze-out hypersurface in any model which provides u^μ, T and μ



relativistic heavy-ion collision

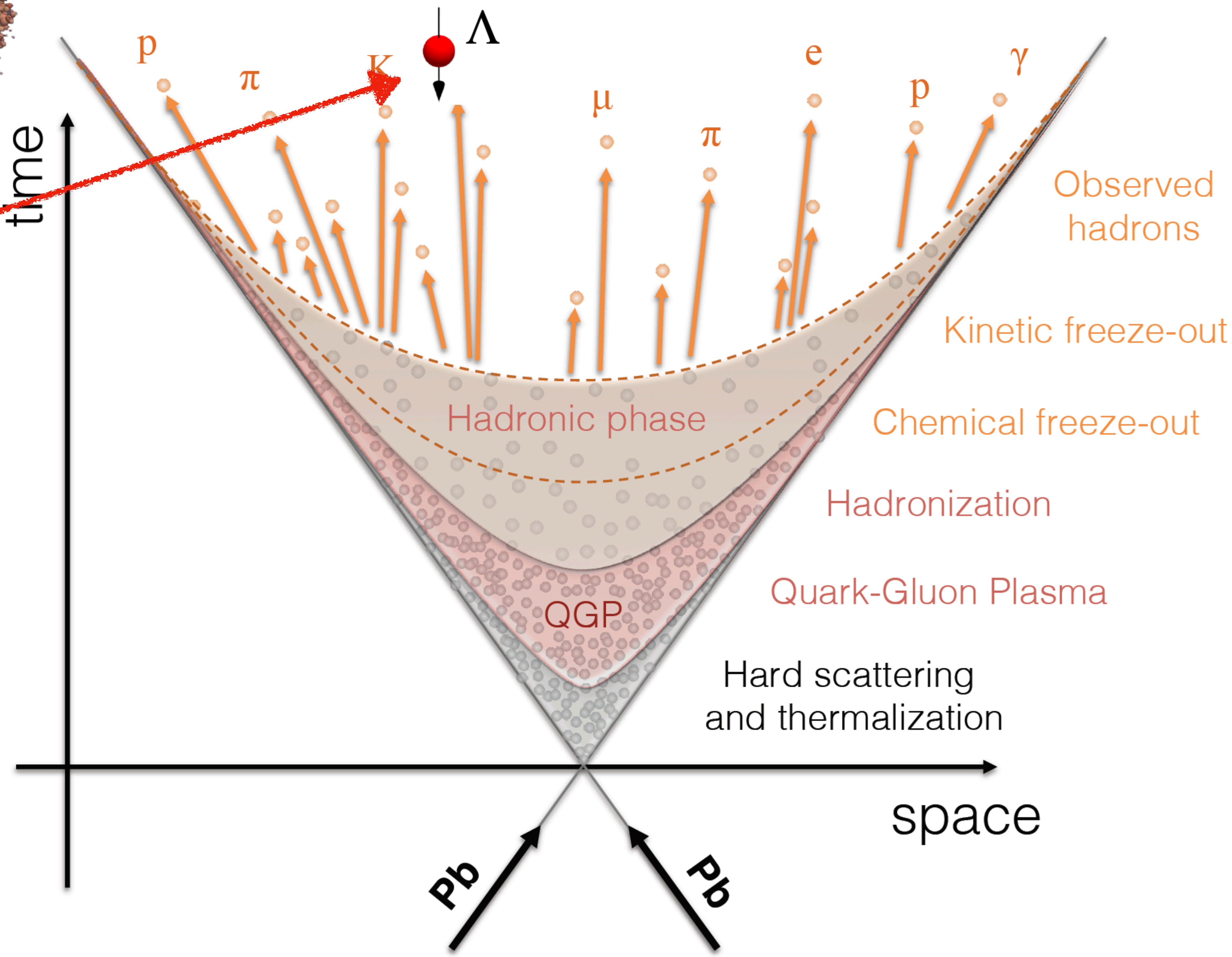


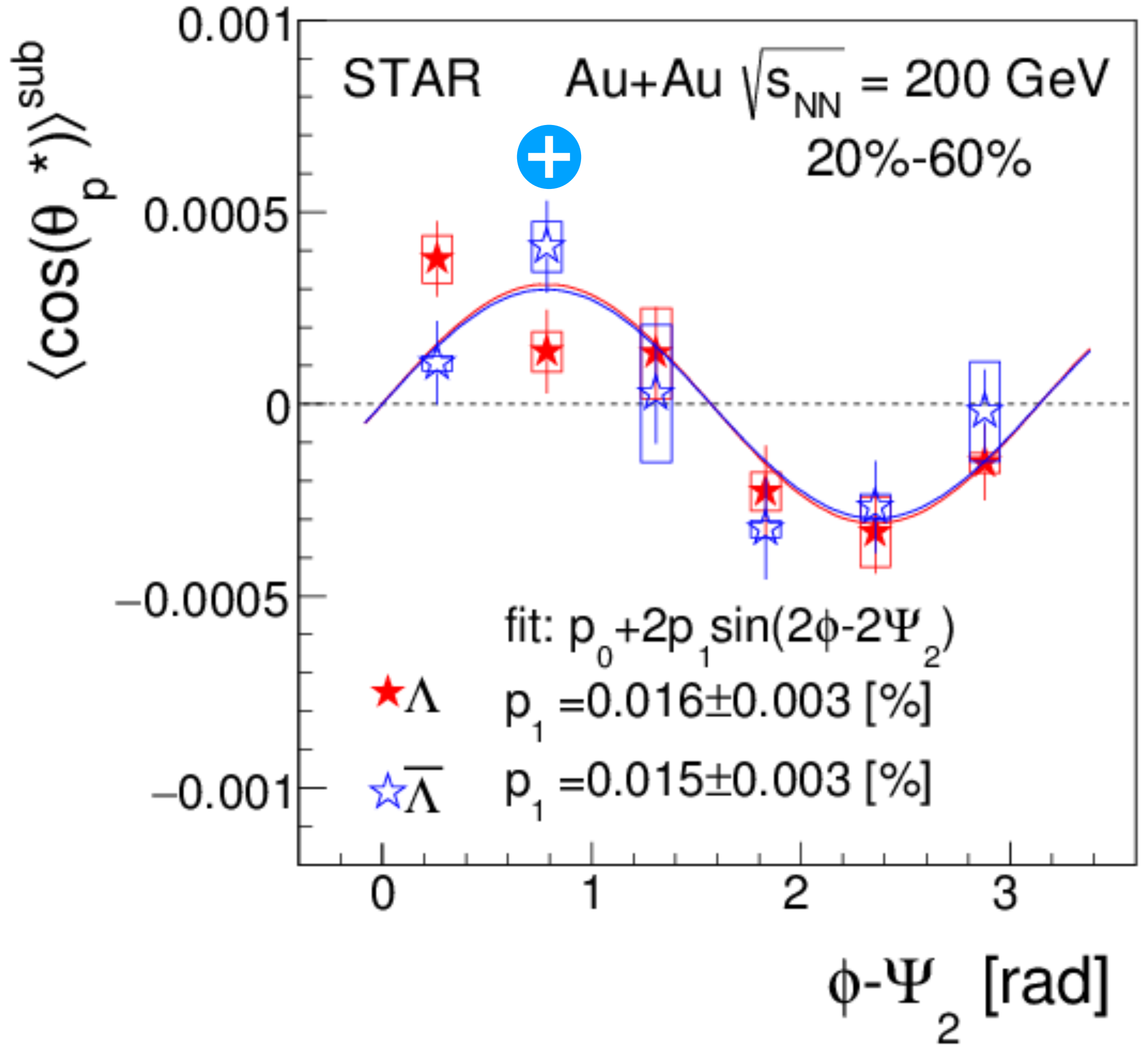
Figure: D.D. Chinellato

Why we need spin hydro?

→ Models that assume LTE of spin degrees of freedom are able to explain global spin polarization measurement.

→ They were unsuccessful to provide clear explanation for the azimuthal angle dependence of longitudinal polarization. Recent progress with thermal shear have some agreement.

Phys.Rev.Lett. 127 (2021) 27, 272302, Phys.Rev.Lett. 127 (2021) 14, 142301

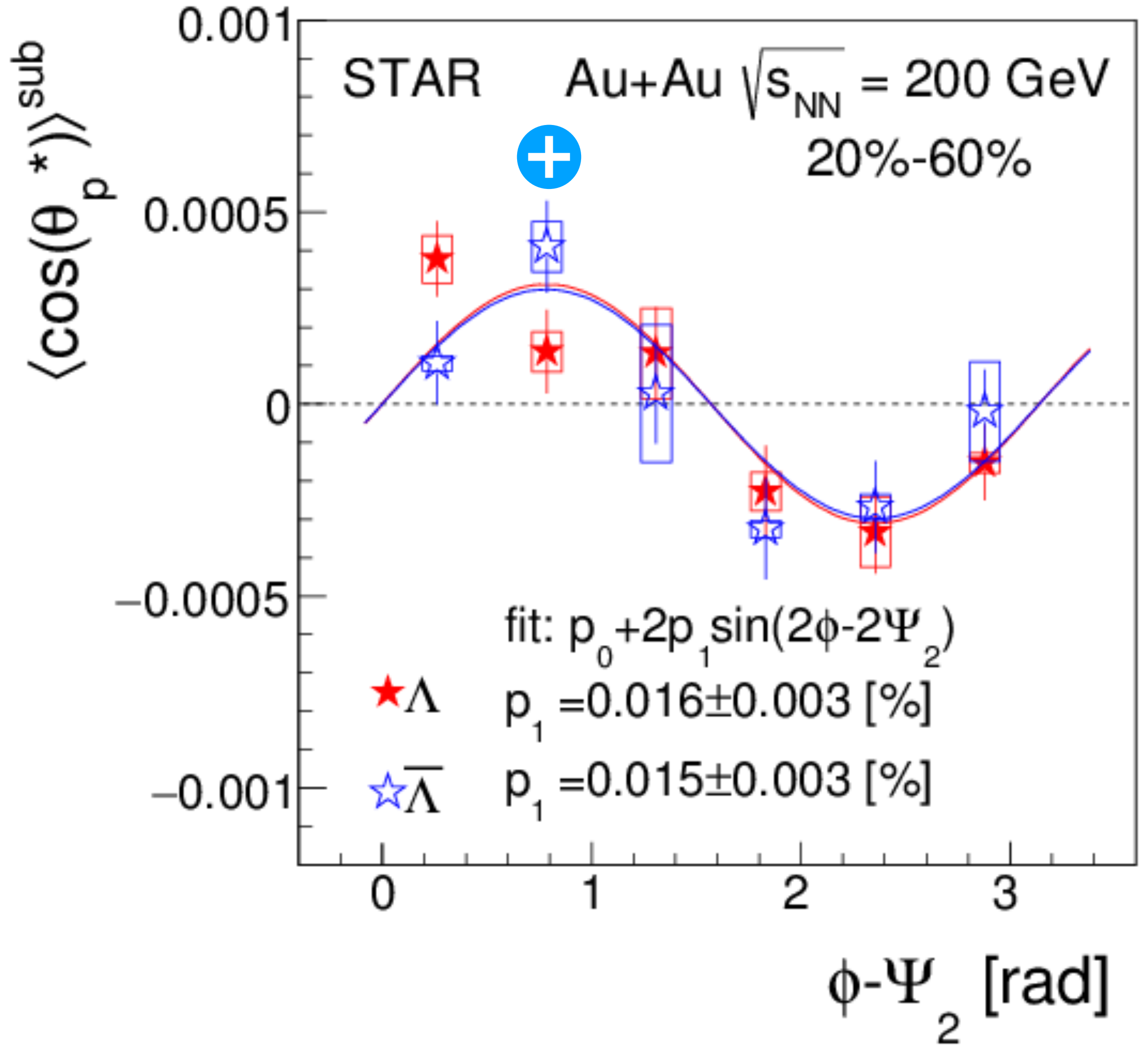


Why we need spin hydro?

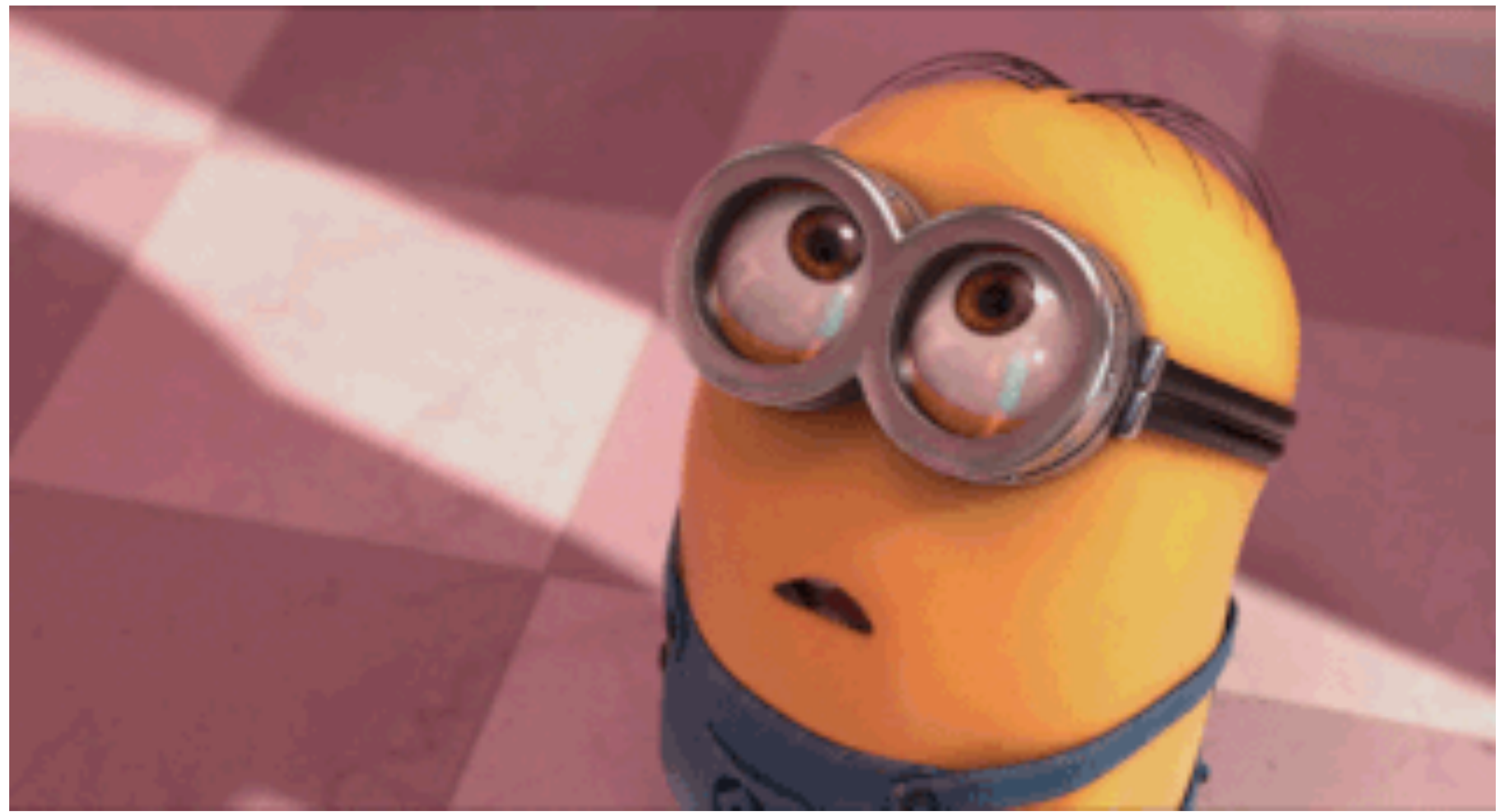
→ Models that assume LTE of spin degrees of freedom are able to explain global spin polarization measurement.

→ However, they were unsuccessful to provide clear explanation for the azimuthal angle dependence of longitudinal polarization. But there are recent progress which have some agreement.

Phys.Rev.Lett. 127 (2021) 27, 272302, Phys.Rev.Lett. 127 (2021) 14, 142301



These discrepancies raise some questions



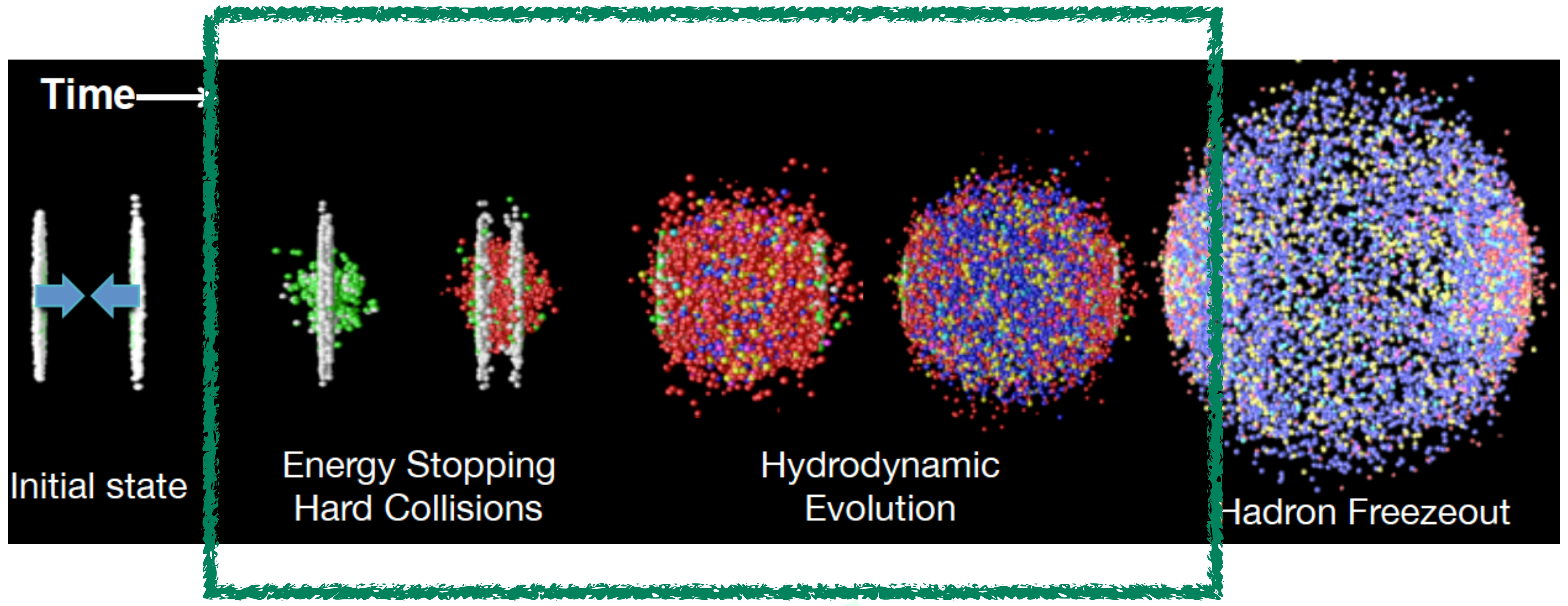
Why we need spin hydro?

Why spin-thermal approach does not fully capture differential observables?

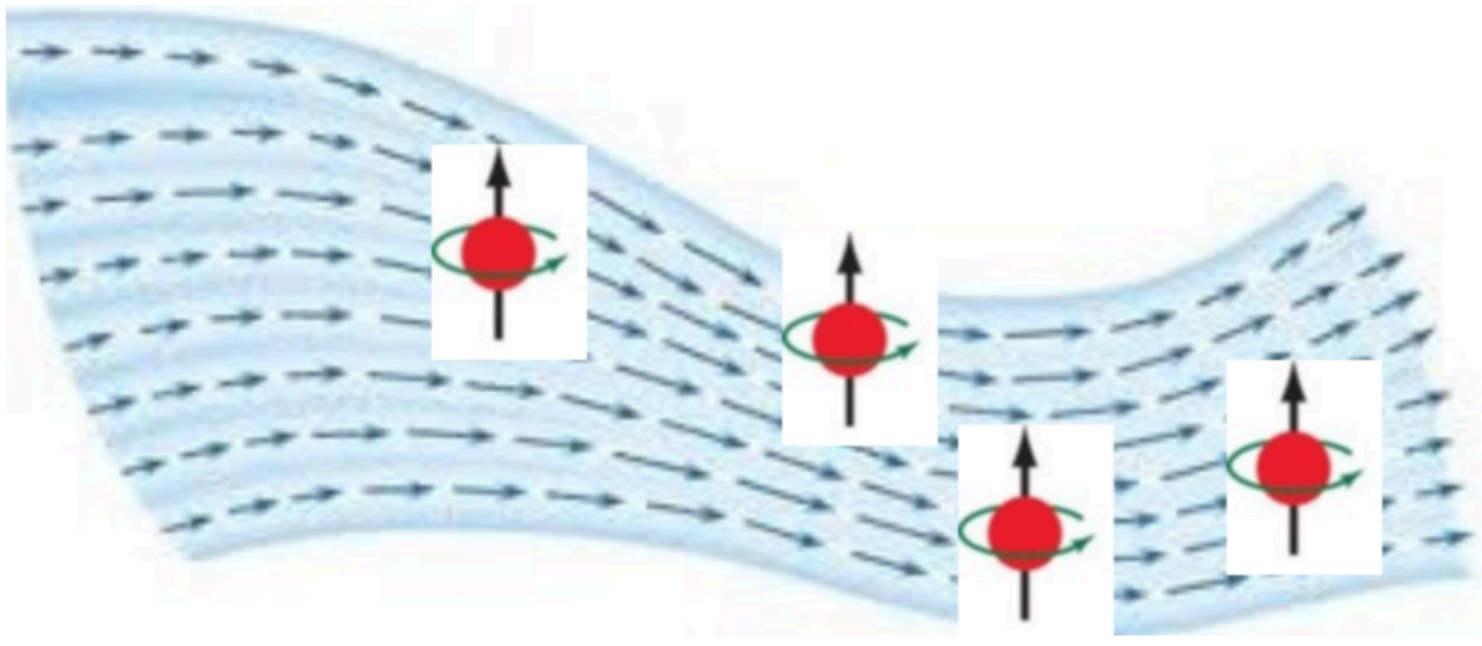
Is spin polarization always enslaved to thermal vorticity?

Is there non-trivial space-time dynamics of spin?

Relativistic fluid dynamics forms the basis of HIC models



Fluid dynamics with spin?



Most of the time close to equilibrium but the dissipation is also important

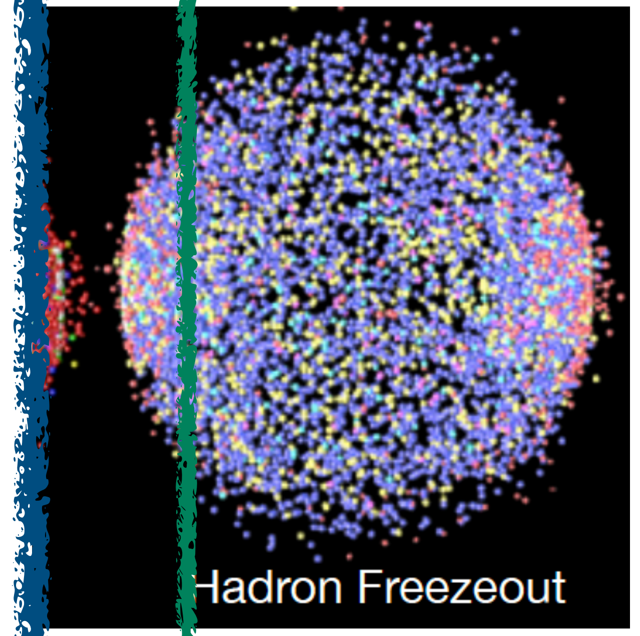
Why we need spin hydro?

Why spin-therm
d

Is spin polarizatio

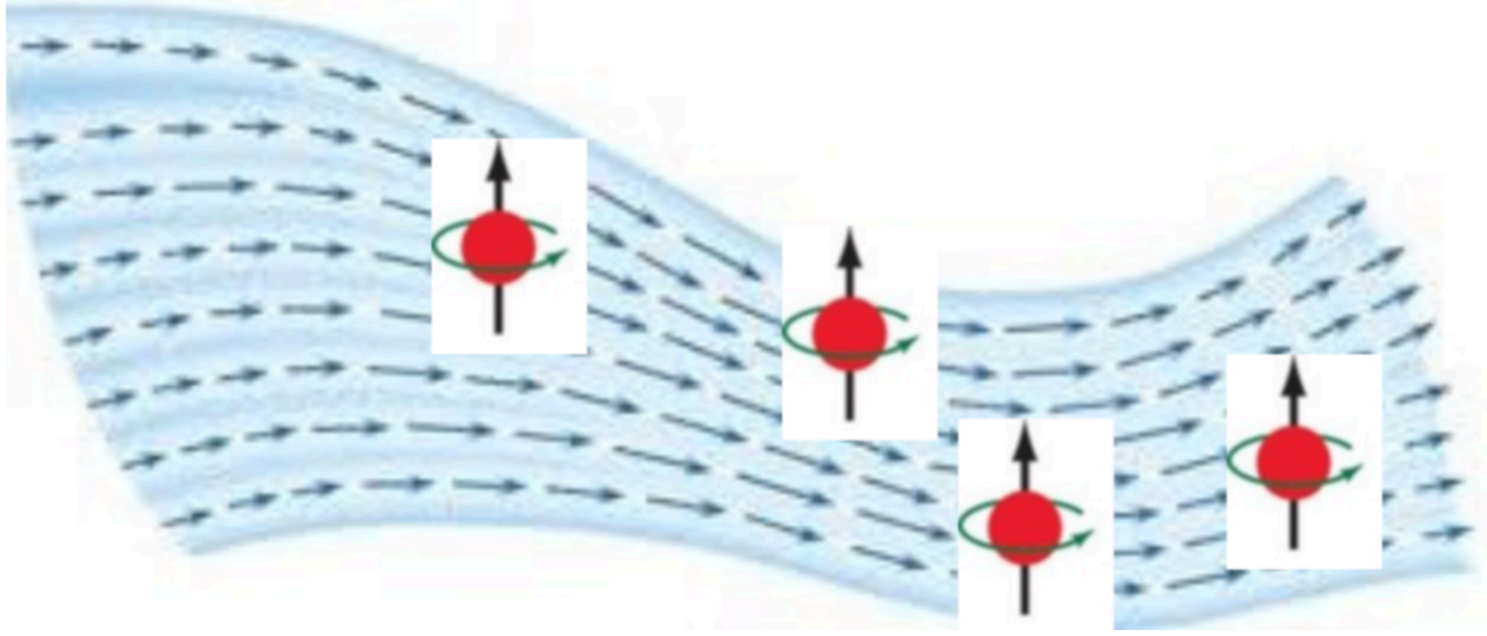
Is there non-tr

mics forms
lels



One way to answer is to have the theory and find out!

the time
close to equilibrium
but the dissipation
is also important



Wigner function and transport equation

$$W_{\alpha\beta}(x,k) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}k\cdot y} \langle : \bar{\psi}_\beta(x_+)\,\psi_\alpha(x_-) : \rangle$$

Wigner function for massive Dirac particles

Dirac equation

$$(i\,\hbar\,\gamma^\mu\,\partial_\mu - m)\,\psi(x) = \hbar\,\rho(x) = -\frac{\partial\,\mathscr{L}_I}{\partial\,\bar{\psi}}$$

Using the total Lagrangian density

$$\mathscr{L}(x) = \mathscr{L}_D(x) + \mathscr{L}_I(x)$$

Does not contain gauge-field interactions

$$\mathscr{L}_D(x) = \frac{i\hbar}{2}\bar{\psi}(x)\,\gamma^\mu \overleftrightarrow{\partial}_\mu\,\psi(x) - m\,\bar{\psi}(x)\,\psi(x)$$

Transport equation

$$\left(i\hbar\frac{\gamma^\mu\partial_\mu}{2} + \gamma^\mu k_\mu - m\right) W(x,k) = \hbar\, C_{\alpha\beta}[W(x,k)]$$

$$C_{\alpha\beta}[W(x,k)] \equiv \int \frac{d^4y}{\left(2\pi\hbar\right)^4} e^{-\frac{i}{\hbar}k\cdot y} \langle : \rho_\alpha(x_-)\,\bar{\psi}_\beta\,(x_+) : \rangle$$

Collisional kernel

arXiv:2203.15562
Phys.Rev.D 104 (2021) 1, 016022
GLW, Relativistic Kinetic Theory and its Applications

Wigner function and transport equation

$W(x,k) = W_{\mathrm{eq}}(x,k) + \delta W(x,k)$

- Neglect initial correlations
- Consider only binary collisions
- Wigner function varies slowly in space and time on the microscopic scale corresponding to the interaction range

$$C_{\alpha\beta} = \frac{(2\pi\hbar)^6}{(2m)^4} \sum_{r_1,r_2,s_1,s_2} \int d^4k_1 d^4k_2 d^4q_1 d^4q_2 \;\; {}_{\text{in}}\left\langle k_1 - \frac{q_1}{2}, k_2 - \frac{q_2}{2}; r_1, r_2 \middle| \Phi_{\alpha\beta}(k) \middle| k_1 + \frac{q_1}{2}, k_2 + \frac{q_2}{2}; s_1, s_2 \right\rangle_{\text{in}}$$

$$\times \prod_{j=1}^{2} \bar{u}_{s_j}\left(k_j + \frac{q_j}{2}\right) \left\{ \underbrace{W(x,k_j)\delta^{(4)}(q_j)}_{\text{Local}} - i\hbar \left[\partial^\mu_{q_j}\delta^{(4)}(q_j)\right] \underbrace{\partial_\mu W(x,k_j)}_{\text{Non-Local}} \right\} u_{r_j}\left(k_j - \frac{q_j}{2}\right)$$

$$W(x,k) = \frac{1}{4}\left[\mathbf{1}_{4\mathrm{x}4}\, F(x,k) + i\,\gamma^5\, P(x,k) + \gamma^\mu\, V_\mu(x,k) + \gamma^5\,\gamma^\mu\, A_\mu(x,k) + \Sigma^{\mu\nu}\, S_{\mu\nu}(x,k)\right]$$

Clifford algebra decomposition

arXiv:2203.15562
Phys.Rev.D 104 (2021) 1, 016022

GLW, Relativistic Kinetic Theory and its Applications

Wigner function and transport equation

$A^{\star\mu\nu} = (1/2)\,\epsilon^{\mu\nu\alpha\beta}\,A_{\alpha\beta}$

$X = \Sigma_n \hbar^n\, X^{(n)}\,,\, C_X = \Sigma_n \hbar^n\, C_X^{(n)}$

Real parts

$$k\cdot V - m\,F = \hbar\, D_F$$
$$-\frac{\hbar}{2}\partial\cdot A - m\,P = \hbar\, D_P$$
$$k_\mu\, F - \frac{\hbar}{2}\partial^\nu S_{\nu\mu} - m\,V_\mu = \hbar\, D_{V,\mu}$$
$$\frac{\hbar}{2}\partial_\mu\, P - k^\beta S^\star{}_{\mu\beta} - m\,A_\mu = \hbar\, D_{A,\mu}$$
$$\hbar\,\partial_{[\mu}V_{\nu]} - \epsilon_{\mu\nu\alpha\beta}\,k^\alpha\, A^\beta - m\,S_{\mu\nu} = \hbar\, D_{S,\mu\nu}$$

Imaginary parts

$$\frac{\hbar}{2}\partial\cdot V = \hbar\, C_F$$
$$k\cdot A = \hbar\, C_P$$
$$\frac{\hbar}{2}\partial_\mu\, F + k^\nu S_{\nu\mu} = \hbar\, C_{V,\mu}$$
$$-k_\mu\, P - \frac{\hbar}{2}\partial^\beta S^\star{}_{\mu\beta} = \hbar\, C_{A,\mu}$$
$$-2k_{[\mu}V_{\nu]} - \frac{\hbar}{2}\epsilon_{\mu\nu\alpha\beta}\,\partial^\alpha\, A^\beta = \hbar\, C_{S,\mu\nu}$$

Kinetic equations using semi-classical expansion

$$k\cdot\partial\, F^{(0)} = 2\,m\, C_F^{(0)}$$

$$k\cdot\partial\, F^{(1)} = 2\,m\, C_F^{(1)} + \partial\cdot D_V^{(0)}$$

For scalar component

$$k\cdot\partial\, A_\mu^{(0)} = 2\,m\, C_{A,\mu}^{(0)} - 2\,k_\mu\, D_P^{(0)}$$

$$k\cdot\partial\, A_\mu^{(1)} = 2\,m\, C_{A,\mu}^{(1)} - 2\,k_\mu\, D_P^{(1)} - \frac{1}{2}\epsilon_{\mu\beta\gamma\delta}\,\partial^\beta D_{S(0)}^{\gamma\delta}$$

For axial-vector component

arXiv:2203.15562

Phys.Rev.D 104 (2021) 1, 016022

GLW, Relativistic Kinetic Theory and its Applications

Spin effects ?

General Boltzmann-like spin kinetic equation

$$\int \mathrm{d}\Gamma = \int d^4k\,\delta\left(k^2 - m^2\right) \int \mathrm{dS}(k)$$

which can then be combined to have

$$k \cdot \partial\, \mathfrak{f}(x, k, \mathfrak{s}) = m\, \mathfrak{C}(\mathfrak{f}) = m\left(\tilde{C}_F - \mathfrak{s} \cdot \tilde{C}_A\right)$$

where

$$\mathfrak{f}(x, k, \mathfrak{s}) = \frac{1}{2}\left(\tilde{F}(x, k) - \mathfrak{s} \cdot \tilde{A}(x, k)\right)$$

$$\mathfrak{C}[\mathfrak{f}] = \mathfrak{C}_l[\mathfrak{f}] + \hbar \mathfrak{C}_{nl}^{(1)}[\mathfrak{f}] = \mathfrak{C}_l^{(0)}[\mathfrak{f}] + \hbar \mathfrak{C}_l^{(1)}[\mathfrak{f}] + \hbar \mathfrak{C}_{nl}^{(1)}[\mathfrak{f}]$$

arXiv:2203.15562

Phys.Rev.D 104 (2021) 1, 016022

GLW, Relativistic Kinetic Theory and its Applications

Canonical currents

→ Being an effective theory, hydrodynamics is defined at a length scale larger than the mean free path of microscopic particles but smaller than the system size.

→ For formulating hydrodynamics with spin, we need to define energy-momentum ($T^{\mu\nu}$) and spin ($S^{\lambda,\mu\nu}$) currents as ensemble averages of their respective normal-ordered QFT operators

$$\boxed{T^{\mu\nu} = \langle : \hat{T}^{\mu\nu} : \rangle\,, \quad S^{\lambda,\mu\nu} = \langle : \hat{S}^{\lambda,\mu\nu} : \rangle}$$

Canonical currents

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$$\boxed{T^{\mu\nu} = \langle : \hat{T}^{\mu\nu} : \rangle \,, \quad S^{\lambda,\mu\nu} = \langle : \hat{S}^{\lambda,\mu\nu} : \rangle}$$

→ For a system with spin we have

$$\hat{J}^{\lambda,\mu\nu} = \hat{L}^{\lambda,\mu\nu} + \hat{S}^{\lambda,\mu\nu} = x^{\mu}\hat{T}^{\lambda\nu} - x^{\nu}\hat{T}^{\lambda\mu} + \hat{S}^{\lambda,\mu\nu}$$

Conservation of TAM

$$\partial_\lambda \hat{J}^{\lambda,\mu\nu} = \partial_\lambda \hat{L}^{\lambda,\mu\nu} + \partial_\lambda \hat{S}^{\lambda,\mu\nu} = \hat{T}^{\mu\nu} - \hat{T}^{\nu\mu} + \partial_\lambda \hat{S}^{\lambda,\mu\nu} = 0$$

gives

$$\partial_\lambda \hat{S}^{\lambda,\mu\nu} = \hat{T}^{\nu\mu} - \hat{T}^{\mu\nu}$$

Antisymmetric parts of $T^{\mu\nu}$

We also have $\partial_\mu \hat{T}^{\mu\nu} = 0$

ψ & $\bar{\psi}$ are Dirac field operators

$\mathscr{L}_D$ is Dirac Lagrangian

$g^{\mu\nu} = \{1, -1, -1, -1\}$

$\overleftrightarrow{\partial} \equiv \overrightarrow{\partial} - \overleftarrow{\partial}$

For massive Dirac particles:

$$\hat{T}^{\mu\nu}_{\rm Can} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}\psi - g^{\mu\nu}\mathscr{L}_D$$

$$\hat{S}^{\lambda,\mu\nu}_{\rm Can} = \frac{i}{8}\bar{\psi}\left\{\gamma^{\lambda}, \left[\gamma^{\mu}, \gamma^{\nu}\right]\right\}\psi$$

Do not lead to proper global spin

$$\hat{S}^{\mu\nu}_{C} \equiv \int_{\Sigma} d\Sigma_\lambda \, \hat{S}^{\lambda,\mu\nu}_{C}$$

QFT, Itzykson and Zuber (Saclay 1980)

de Groot–van Leeuwen–van Weert pseudo-gauge

→ However, one can obtain new pair of $\hat{T}^{\mu\nu}$ and $\hat{S}^{\lambda,\mu\nu}$ using $\hat{T}^{\mu\nu}_{\text{Can}}$ and $\hat{S}^{\lambda,\mu\nu}_{\text{Can}}$ through pseudo-gauge transformation

Rept.Math.Phys. 9 (1976) 55-82,

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_{\text{Can}} + \frac{1}{2}\partial_\lambda\left(\hat{\Pi}^{\lambda,\mu\nu} + \hat{\Pi}^{\nu,\mu\lambda} + \hat{\Pi}^{\mu,\nu\lambda}\right)$$

$$\hat{S}^{\lambda,\mu\nu} = \hat{S}^{\lambda,\mu\nu}_{\text{Can}} - \hat{\Pi}^{\lambda,\mu\nu} + \partial_\rho \hat{\Upsilon}^{\mu\nu,\lambda\rho}$$

$$\hat{\Pi}^{\lambda,\mu\nu} = -\hat{\Pi}^{\lambda,\nu\mu}$$

$$\hat{\Upsilon}^{\mu\nu,\lambda\rho} = -\hat{\Upsilon}^{\nu\mu,\lambda\rho} = -\hat{\Upsilon}^{\mu\nu,\rho\lambda}$$

$$\sigma^{\mu\nu} = (i/2)\left[\gamma^\mu, \gamma^\nu\right]$$

→ One may have several choices of $\hat{\Pi}^{\lambda,\mu\nu}$ & $\hat{\Upsilon}^{\mu\nu,\lambda\rho}$, however, we choose

$$\hat{\Pi}^{\lambda,\mu\nu} = \frac{i}{4m}\bar{\psi}\left(\sigma^{\lambda\mu}\overleftrightarrow{\partial}^{\nu} - \sigma^{\lambda\nu}\overleftrightarrow{\partial}^{\mu}\right)\psi$$

$$\hat{\Upsilon}^{\mu\nu,\lambda\rho} = 0$$

$$\boxed{\begin{aligned} \hat{T}^{\mu\nu}_{\text{GLW}} &= -\frac{1}{4m}\bar{\psi}\overleftrightarrow{\partial}^{\mu}\overleftrightarrow{\partial}^{\nu}\psi \\ \hat{S}^{\lambda,\mu\nu}_{\text{GLW}} &= \bar{\psi}\left[\frac{\sigma^{\mu\nu}}{4} - \frac{1}{8m}\left(\gamma^\mu\overleftrightarrow{\partial}^{\nu} - \gamma^\nu\overleftrightarrow{\partial}^{\mu}\right)\right]\gamma^\lambda\psi + \text{h.c} \end{aligned}}$$

S. De Groot, W. Van Leeuwen, and C. Van Weert, Relativistic Kinetic Theory. Principles and Applications. North Holland, 1, 1980

Prog.Part.Nucl.Phys. 108 (2019) 103709

Relativistic hydrodynamics with spin

Prog.Part.Nucl.Phys. 108 (2019) 103709

→ Using Wigner function (in equilibrium) $W_{\rm eq}(x,k) = W^+_{\rm eq}(x,k) + W^-_{\rm eq}(x,k)$

$$\left(i\hbar\frac{\gamma^\mu\partial_\mu}{2} + \gamma^\mu k_\mu - m\right) W_{\rm eq}(x,k) = \hbar\, C[W_{\rm eq}(x,k)]$$

Transport equation

$$X^\pm = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu\right]\left[1 \pm \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]$$

$$\Sigma^{\mu\nu} = (i/4)\left[\gamma^\mu,\gamma^\nu\right], \quad \xi(x) = \mu_B/T, \quad \beta_\mu(x) = U^\mu/T$$

$$z = m/T$$

"+" means particle contribution

"−" means antiparticle contribution

$$W^+_{\rm eq}(x,k) = \frac{1}{2}\sum_{r,s}\int {\rm dP}\,\delta^{(4)}(k-p)\,\mathcal{U}^r(p)\,\bar{\mathcal{U}}^s(p)\,f^+_{rs}(x,p)$$

$$W^-_{\rm eq}(x,k) = -\frac{1}{2}\sum_{r,s}\int {\rm dP}\,\delta^{(4)}(k+p)\,\mathcal{V}^s(p)\,\bar{\mathcal{V}}^r(p)\,f^-_{rs}(x,p)$$

and ansatz for local equilibrium distribution functions

$$f^+_{rs}(x,p) = \frac{1}{2\,m}\,\bar{\mathcal{U}}_r(p)\,X^+\,\mathcal{U}_s(p) = \frac{1}{2\,m}\,\bar{\mathcal{U}}_r(p)\,\exp\left[-\beta_\mu(x)p^\mu + \xi(x)\right]\left[1 + \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]\mathcal{U}_s(p)$$

$$f^-_{rs}(x,p) = -\frac{1}{2\,m}\,\bar{\mathcal{V}}_s(p)\,X^-\,\mathcal{V}_r(p) = -\frac{1}{2\,m}\,\bar{\mathcal{V}}_s(p)\,\exp\left[-\beta_\mu(x)p^\mu - \xi(x)\right]\left[1 - \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]\mathcal{V}_r(p)$$

Dirac spinors

Spin polarization tensor
(Spin chemical potential)

We obtain

$$W^\pm_{\rm eq}(x,k) = \frac{1}{4m}\int {\rm dP}\,e^{-\beta\cdot p\,\pm\,\xi}\,\delta^{(4)}(k\mp p)\left[2\,m(m\pm\gamma^\mu p_\mu) \pm \frac{1}{2}\,\omega_{\mu\nu}(\gamma^\mu p_\mu \pm m)\Sigma^{\mu\nu}(\gamma^\mu p_\mu \pm m)\right]$$

Relativistic hydrodynamics with spin

Prog.Part.Nucl.Phys. 108 (2019) 103709

$\longrightarrow$ Decomposing Wigner function using Clifford algebra expansion

$\Sigma^{\mu\nu} = (i/4)\left[\gamma^\mu, \gamma^\nu\right], \quad \xi(x) = \mu_B/T, \quad \beta_\mu(x) = U^\mu/T$

$\mathscr{C} = \cosh(\xi), \quad \Delta^{\mu\nu} = g^{\mu\nu} - (U^\mu U^\nu)/(U\cdot U)$

$\mathscr{B}_{(0)} = -\frac{2}{z^2}\frac{\mathscr{E}_{(0)} + \mathscr{P}_{(0)}}{T}, \quad \mathscr{A}_{(0)} = 2\mathscr{N}_{(0)} - 3\mathscr{B}_{(0)}$

$z = m/T$

One can derive the <u>constitutive relations</u> for

- Net baryon current

$$N^\alpha(x) = \langle : \bar{\psi}\gamma^\alpha \psi : \rangle$$
$$= \mathrm{tr}\int d^4k\, \gamma^\alpha \left(W^+_{\mathrm{eq}}(x,k) - W^-_{\mathrm{eq}}(x,k)\right)$$

$$N^\alpha(x) = \mathscr{N}\, U^\alpha$$

with

$$\mathscr{N} = 4\,\sinh(\xi)\,\mathscr{N}_{(0)}(T)$$

$$\mathscr{N}_{(0)}(T) = \frac{T^3}{2\pi^2} z^2 K_2(z)$$

- Energy-momentum tensor

$$T^{\mu\nu}_{\mathrm{GLW}}(x) = \langle : \hat{T}^{\mu\nu}_{\mathrm{GLW}} : \rangle$$
$$= \frac{1}{m}\,\mathrm{tr}\int d^4k\, k^\mu k^\nu \left(W^+_{\mathrm{eq}}(x,k) + W^-_{\mathrm{eq}}(x,k)\right)$$

$$T^{\mu\nu}_{\mathrm{GLW}}(x) = (\mathscr{E} + \mathscr{P})\, U^\mu U^\nu - \mathscr{P}\, g^{\mu\nu}$$

with

$$\mathscr{E} = 4\,\cosh(\xi)\,\mathscr{E}_{(0)}(T)$$
$$\mathscr{P} = 4\,\cosh(\xi)\,\mathscr{P}_{(0)}(T)$$

$$\mathscr{E}_{(0)}(T) = \frac{T^4}{2\pi^2} z^2 \left[zK_1(z) + 3K_2(z)\right]$$

$$\mathscr{P}_{(0)}(T) = T\mathscr{N}_{(0)}(T)$$

Relativistic hydrodynamics with spin

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- Spin tensor $$S^{\alpha,\beta\gamma}_{\rm GLW} = \langle : \hat{S}^{\alpha,\beta\gamma}_{\rm GLW} : \rangle = \frac{\hbar}{4}\int d^4k\, {\rm tr}\left[\left(\left\{\sigma^{\beta\gamma},\gamma^{\alpha}\right\} + \frac{2i}{m}\left(\gamma^{[\beta}k^{\gamma]}\gamma^{\alpha} - \gamma^{\alpha}\gamma^{[\beta}k^{\gamma]}\right)\right)\left(W^{+}_{\rm eq}(x,k) + W^{-}_{\rm eq}(x,k)\right)\right]$$

$$S^{\alpha,\beta\gamma}_{\rm GLW} = U^{\alpha}\left(\mathscr{A}_1\,\omega^{\beta\gamma} + \mathscr{A}_2\,U^{[\beta}\,\omega^{\gamma]}{}_{\delta}\,U^{\delta}\right) + \mathscr{A}_3\left(U^{[\beta}\,\omega^{\gamma]\alpha} + g^{\alpha[\beta}\,\omega^{\gamma]}{}_{\delta}\,U^{\delta}\right)$$

Fluid-flow four-velocity

Spin polarization tensor

with

$$\left.\begin{aligned}
\mathscr{A}_1 &= \mathscr{C}\left(\mathscr{N}_{(0)} - \mathscr{B}_{(0)}\right)\\
\mathscr{A}_2 &= \mathscr{C}\left(\mathscr{A}_{(0)} - 3\mathscr{B}_{(0)}\right)\\
\mathscr{A}_3 &= \mathscr{C}\,\mathscr{B}_{(0)}
\end{aligned}\right\}$$ Thermodynamic coefficients

$\Sigma^{\mu\nu} = (i/4)\left[\gamma^{\mu},\gamma^{\nu}\right], \quad \xi(x) = \mu_B/T, \quad \beta_{\mu}(x) = U^{\mu}/T$

$\mathscr{C} = \cosh(\xi), \quad \Delta^{\mu\nu} = g^{\mu\nu} - (U^{\mu}U^{\nu})/(U\cdot U)$

$\mathscr{B}_{(0)} = -\dfrac{2}{z^2}\dfrac{\mathscr{E}_{(0)} + \mathscr{P}_{(0)}}{T}, \quad \mathscr{A}_{(0)} = 2\mathscr{N}_{(0)} - 3\mathscr{B}_{(0)}$

$z = m/T$

Relativistic hydrodynamics with spin

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- Spin tensor $S^{\alpha,\beta\gamma}_{\rm GLW} = \langle : \hat{S}^{\alpha,\beta\gamma}_{\rm GLW} : \rangle = \frac{\hbar}{4} \int d^4k \, {\rm tr} \left[\left(\{\sigma^{\beta\gamma}, \gamma^\alpha\} + \frac{2i}{m} \left(\gamma^{[\beta}k^{\gamma]}\gamma^\alpha - \gamma^\alpha \gamma^{[\beta}k^{\gamma]}\right) \right) \left(W^+_{\rm eq}(x,k) + W^-_{\rm eq}(x,k) \right) \right]$

$$S^{\alpha,\beta\gamma}_{\rm GLW} = U^\alpha \left(\mathscr{A}_1 \, \omega^{\beta\gamma} + \mathscr{A}_2 \, U^{[\beta} \omega^{\gamma]}{}_\delta U^\delta \right) + \mathscr{A}_3 \left(U^{[\beta} \omega^{\gamma]\alpha} + g^{\alpha[\beta} \omega^{\gamma]}{}_\delta U^\delta \right)$$

Fluid-flow four-velocity

Spin polarization tensor

with

$$\left.\begin{aligned} \mathscr{A}_1 &= \mathscr{C}\left(\mathscr{N}_{(0)} - \mathscr{B}_{(0)}\right) \\ \mathscr{A}_2 &= \mathscr{C}\left(\mathscr{A}_{(0)} - 3\mathscr{B}_{(0)}\right) \\ \mathscr{A}_3 &= \mathscr{C}\,\mathscr{B}_{(0)} \end{aligned}\right\} \text{Thermodynamic coefficients}$$

$$\rightarrow \boxed{\partial_\alpha N^\alpha = 0 \,, \; \partial_\alpha T^{\alpha\beta}_{\rm GLW} = 0 \,, \; \partial_\alpha S^{\alpha,\beta\gamma}_{\rm GLW} = 0.}$$

Modeling of the spin polarization dynamics

→ Mean spin polarization per particle (momentum-dependent)

$$\langle \pi_\mu \rangle_p = \frac{E_p \dfrac{d\Pi_\mu(p)^*}{d^3p}}{E_p \dfrac{d\mathcal{N}(p)}{d^3p}}$$

Pauli Lubanski four-vector

Momentum density of all particles

$$E_p \frac{d\Pi_\mu(p)^*}{d^3p} = -\frac{1}{(2\pi)^3 m} \int \cosh(\xi)\, \Delta\Sigma_\lambda\, p^\lambda\, e^{-\beta\cdot p}\, (\omega^\star_{\mu\beta} p^\beta)^*$$

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4}{(2\pi)^3} \int \cosh(\xi)\, \Delta\Sigma_\lambda\, p^\lambda\, e^{-\beta\cdot p}$$

Freeze-out hyper-surface element

→ Mean spin polarization per particle (momentum-independent)

$$\langle \pi_\mu \rangle = \frac{\int dP\, \langle \pi_\mu \rangle_p\, E_p \dfrac{d\mathcal{N}(p)}{d^3p}}{\int dP\, E_p \dfrac{d\mathcal{N}(p)}{d^3p}}$$

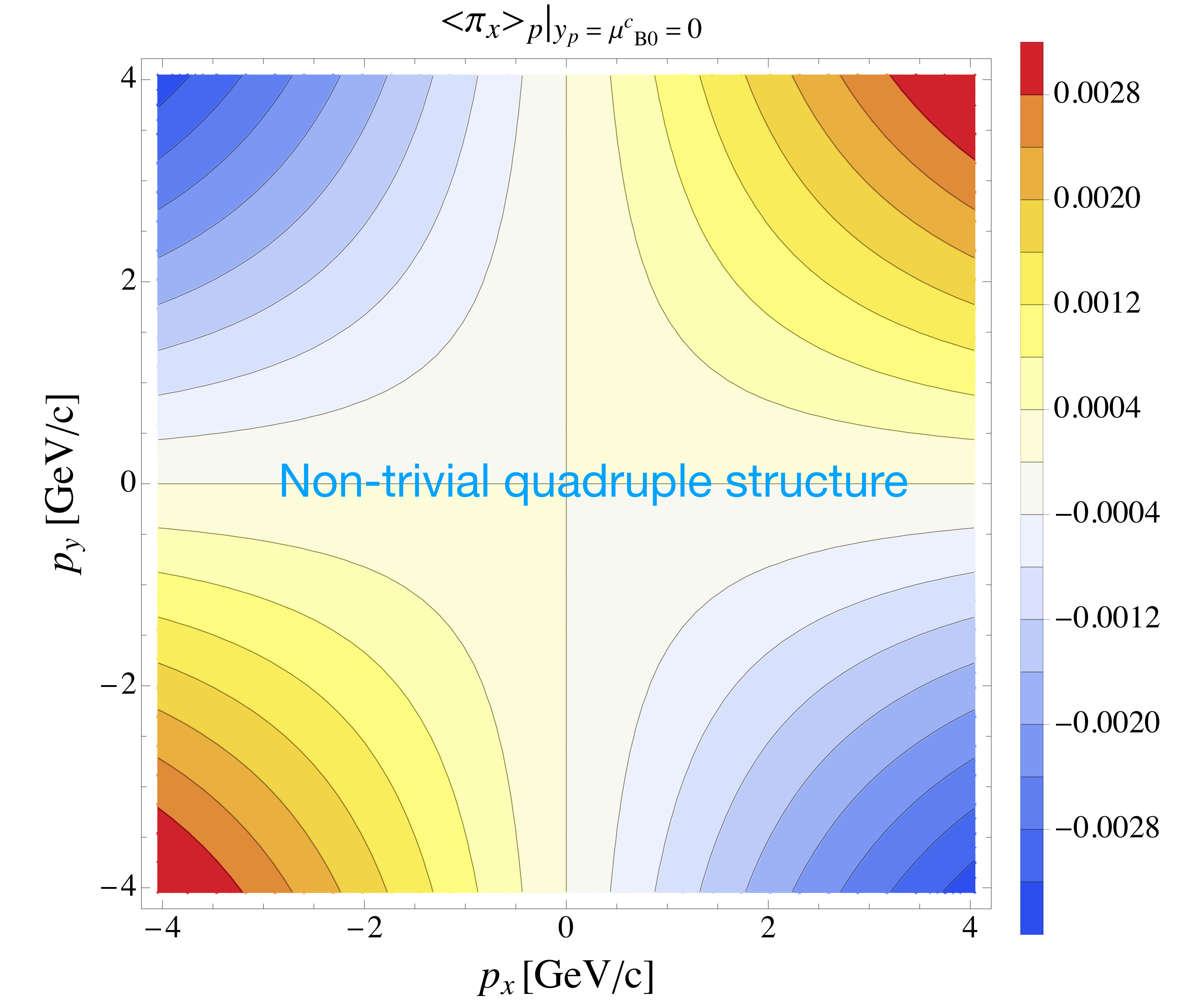
$$\langle \pi_\mu(p_T) \rangle = \frac{\frac{1}{2\,\pi} \int d\phi_p\, \sin(2\phi_p)\, E_p \dfrac{d\Pi^*_\mu(p)}{d^3p}}{\int d\phi_p\, E_p \dfrac{d\mathcal{N}(p)}{d^3p}}$$

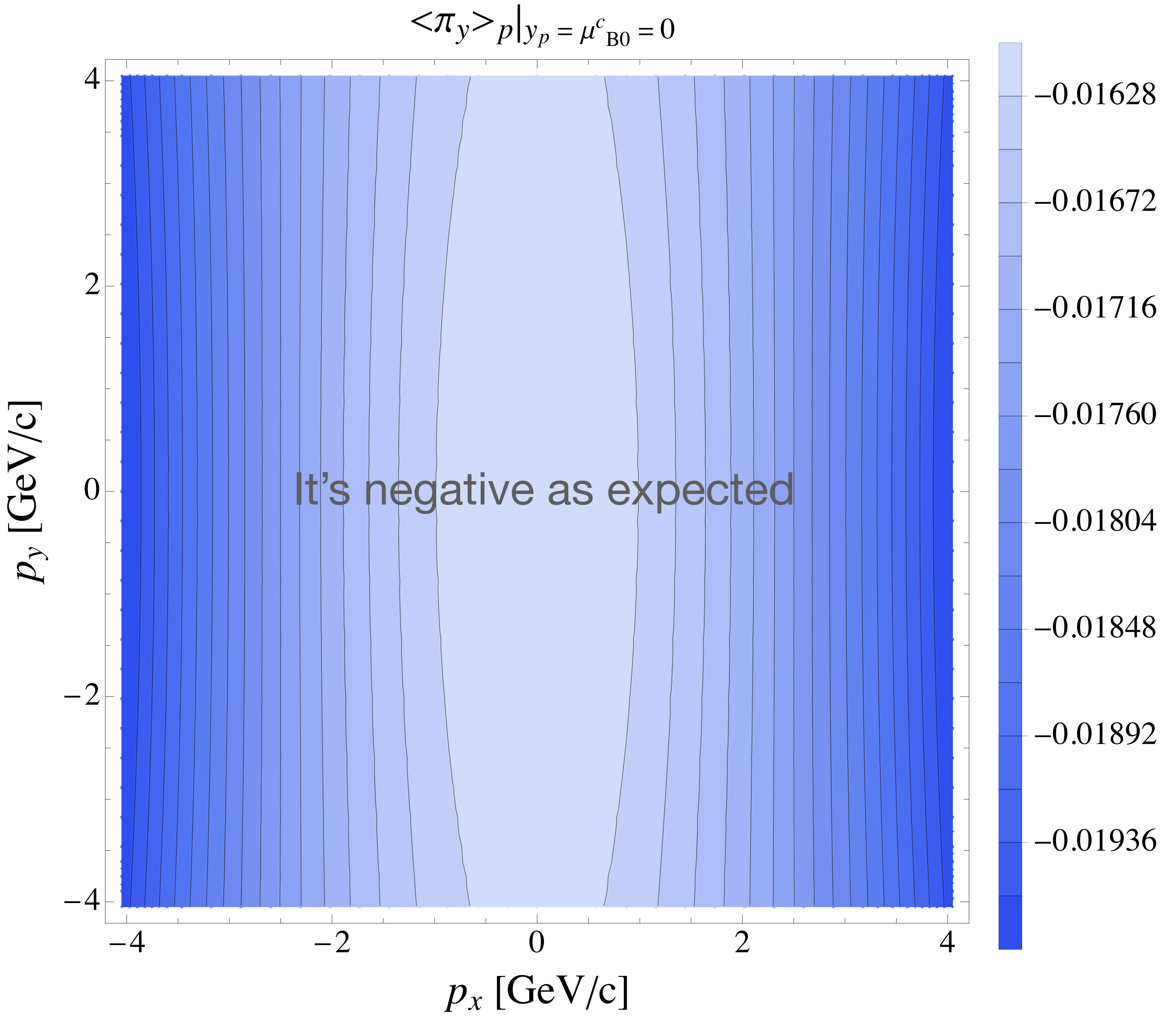
$$\langle \pi_\mu(\phi_p) \rangle = \frac{\int p_T\, dp_T\, E_p \dfrac{d\Pi^*_\mu(p)}{d^3p}}{\int d\phi_p\, p_T\, dp_T\, E_p \dfrac{d\mathcal{N}(p)}{d^3p}}$$

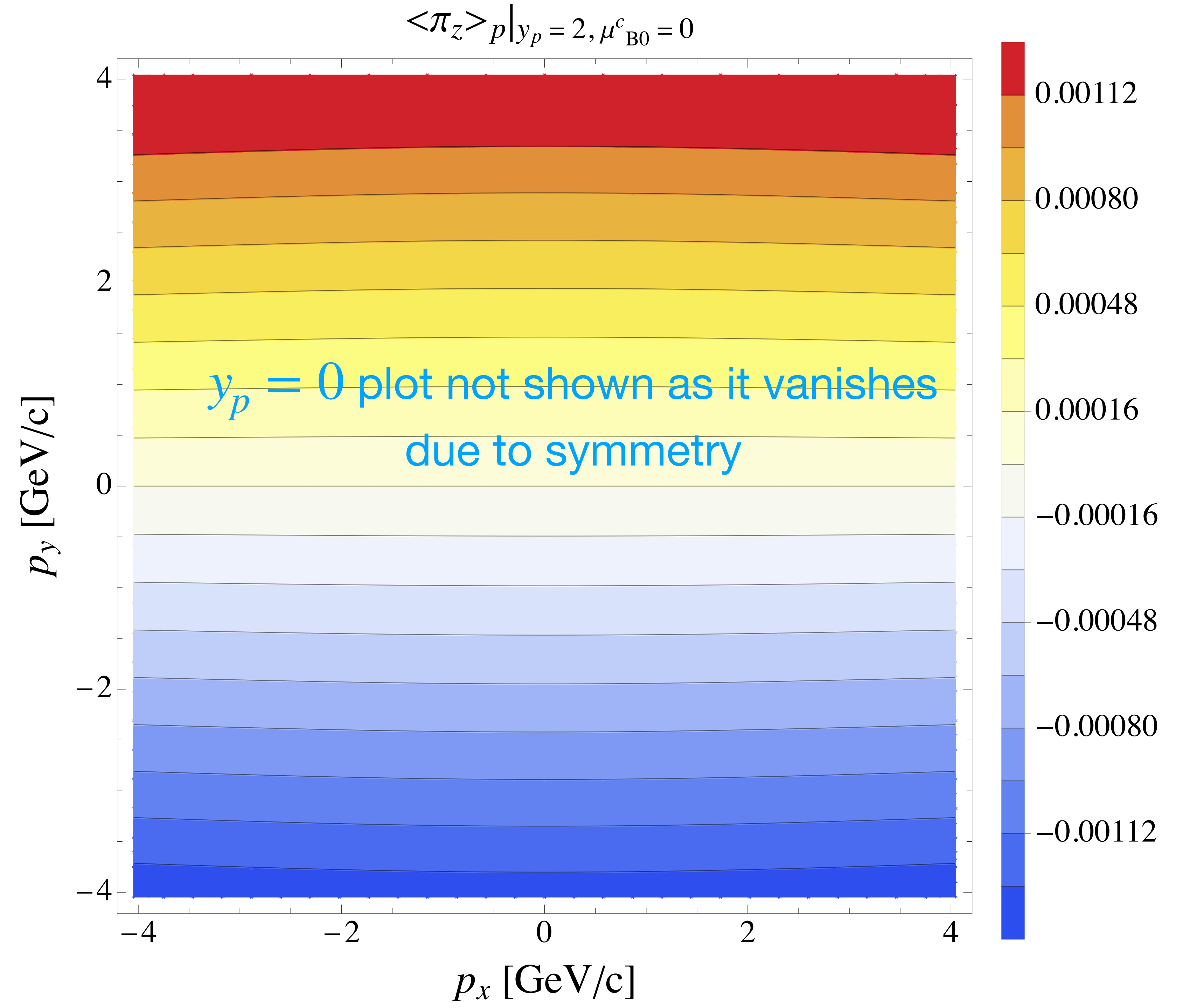
Modeling of the spin polarization dynamics

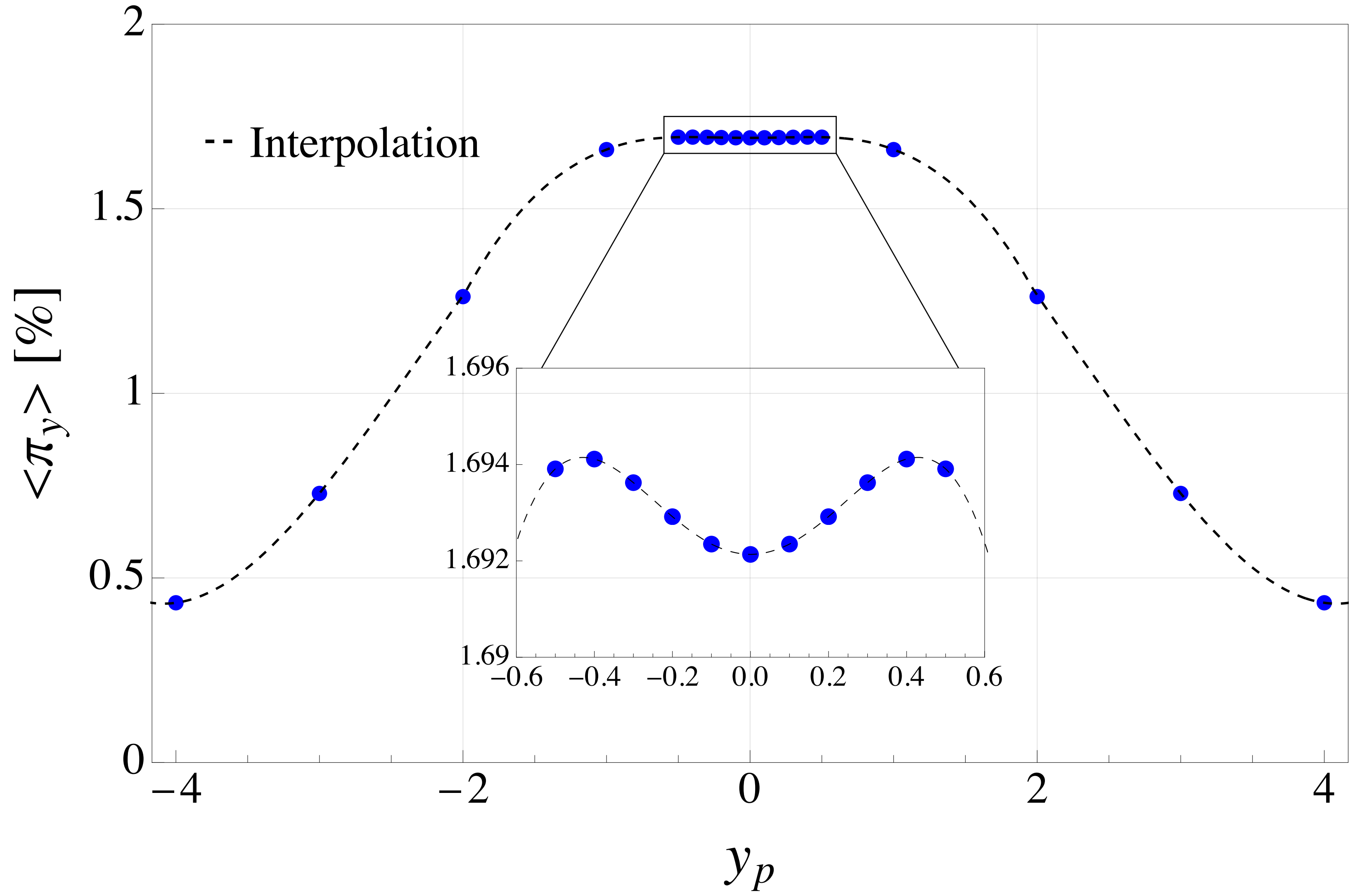
$\mu_B \neq 0$ plots not shown as they are qualitatively similar

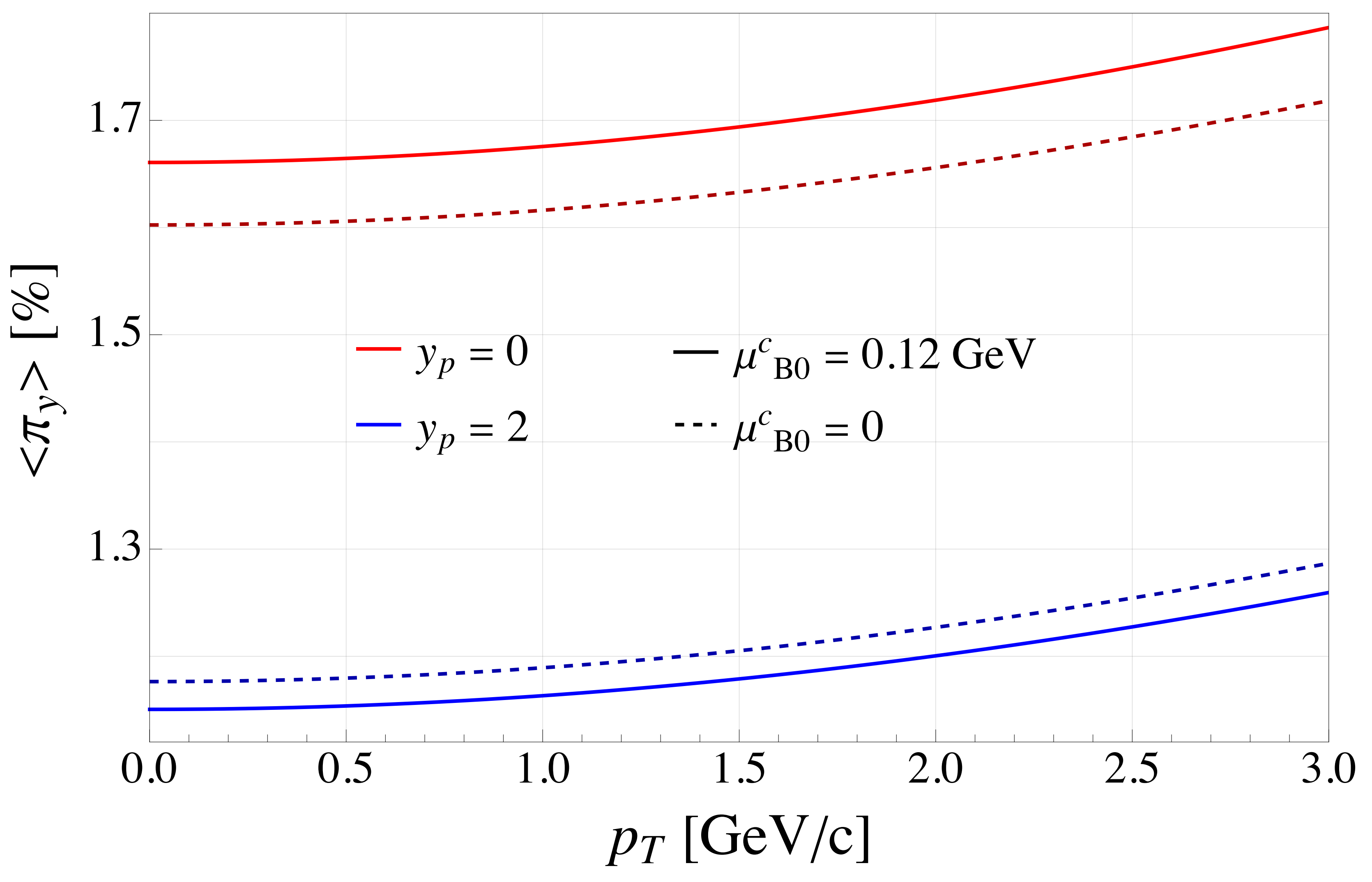
→ Non-boost-invariant and transversely homogeneous

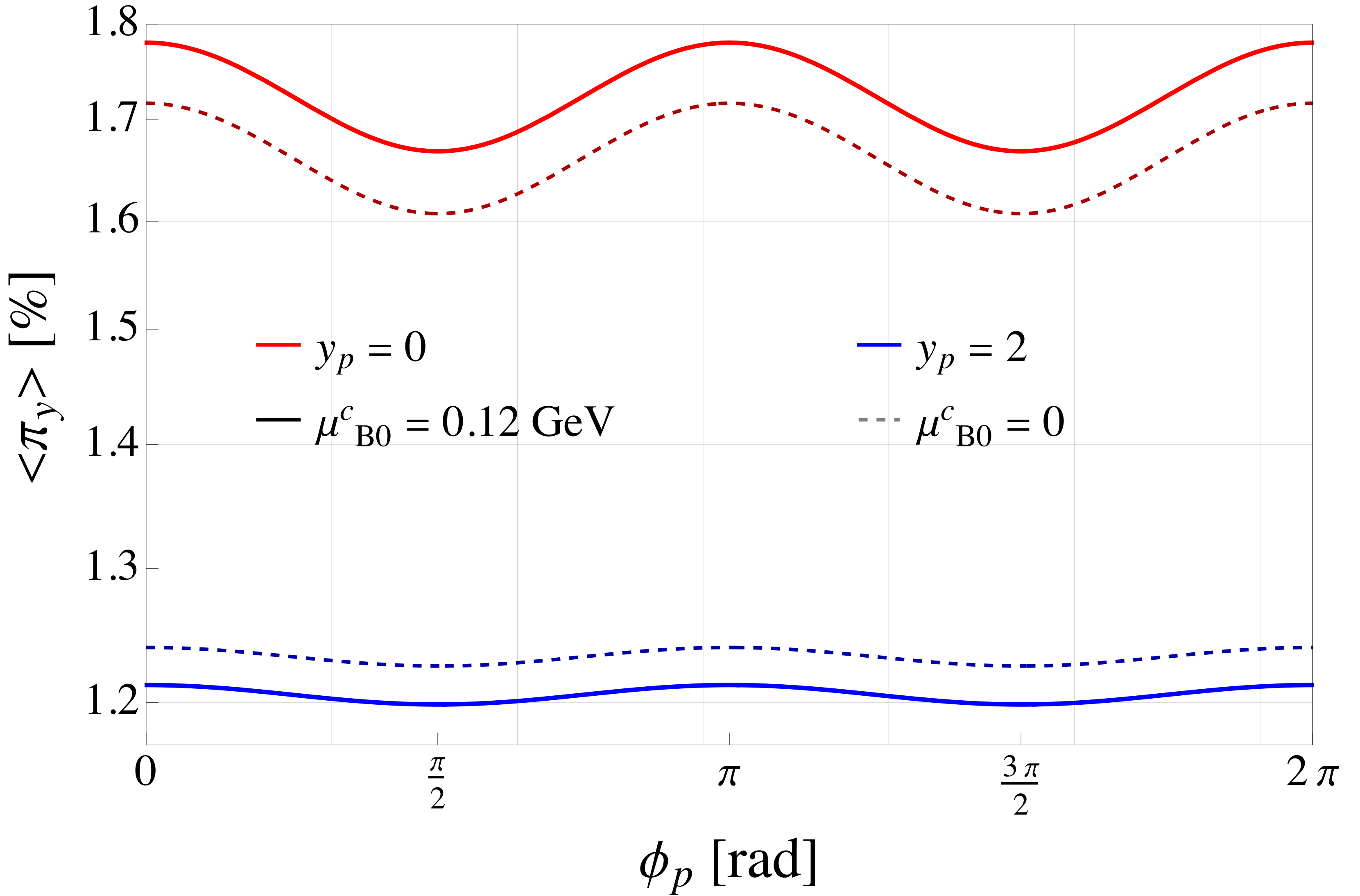












Part-I summary

The spin polarization provides a sensitive new probe of the QGP properties

The disagreements motivates developments of dynamical models

The fluid dynamics with spin seems to be a natural framework

Spin hydrodynamics depends on pseudo gauge

Pseudogauge ambiguity

Can: Canonical
BR: Belinfante–Rosenfeld
GLW: de Groot-van Leeuwen-van Weert
HW: Hilgevoord-Wouthuysen

⟶ Various pseudogauges

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_{\text{Can}} + \frac{1}{2}\partial_\lambda(\hat{\Pi}^{\lambda,\mu\nu} + \hat{\Pi}^{\nu,\mu\lambda} + \hat{\Pi}^{\mu,\nu\lambda})$$

$$\hat{S}^{\lambda,\mu\nu} = \hat{S}^{\lambda,\mu\nu}_{\text{Can}} - \hat{\Pi}^{\lambda,\mu\nu} + \partial_\rho \hat{\Upsilon}^{\mu\nu,\lambda\rho}$$

$$\hat{T}^{\mu\nu}_{\text{Can}} = \frac{i}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \psi - g^{\mu\nu}\mathscr{L}_D$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{Can}} = \frac{i}{8}\bar{\psi}\left\{\gamma^\lambda, \left[\gamma^\mu, \gamma^\nu\right]\right\}\psi$$

$$\hat{T}^{\mu\nu}_{\text{BR}} = \frac{i}{4}\bar{\psi}\left(\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu\right)\psi - g^{\mu\nu}\mathscr{L}_D$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{BR}} = 0$$

$$\hat{T}^{\mu\nu}_{\text{GLW}} = -\frac{1}{4m}\bar{\psi}\overleftrightarrow{\partial}^\mu \overleftrightarrow{\partial}^\nu \psi$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{GLW}} = \bar{\psi}\left[\frac{\sigma^{\mu\nu}}{4} - \frac{1}{8m}\left(\gamma^\mu \overleftrightarrow{\partial}^\nu - \gamma^\nu \overleftrightarrow{\partial}^\mu\right)\right]\gamma^\lambda \psi + \text{h.c}$$

$$\hat{T}^{\mu\nu}_{\text{HW}} = \hat{T}^{\mu\nu}_{\text{Can}} + \frac{i}{2m}\left(\partial^\nu \bar{\psi}\sigma^{\mu\beta}\partial_\beta \psi + \partial_\alpha \bar{\psi}\sigma^{\alpha\mu}\partial^\nu \psi\right) - \frac{i}{4m}g^{\mu\nu}\partial_\lambda\left(\bar{\psi}\sigma^{\lambda\alpha}\overleftrightarrow{\partial}_\alpha \psi\right)$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{HW}} = \hat{S}^{\lambda,\mu\nu}_{\text{Can}} - \frac{1}{4m}\left(\bar{\psi}\sigma^{\mu\nu}\sigma^{\lambda\rho}\partial_\rho \psi + \partial_\rho \bar{\psi}\sigma^{\lambda\rho}\sigma^{\mu\nu}\psi\right)$$

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Pseudogauge ambiguity

Can: Canonical
BR: Belinfante–Rosenfeld
GLW: de Groot-van Leeuwen-van Weert
HW: Hilgevoord-Wouthuysen

→ Various pseudogauges

Which one is physical?

Which one describes data?

Is there any general pseudogauge?

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_{\text{Can}} + \frac{1}{2}\partial_\lambda(\hat{\Pi}^{\lambda,\mu\nu} + \hat{\Pi}^{\nu,\mu\lambda} + \hat{\Pi}^{\mu,\nu\lambda})$$

$$\hat{S}^{\lambda,\mu\nu} = \hat{S}^{\lambda,\mu\nu}_{\text{Can}} - \hat{\Pi}^{\lambda,\mu\nu} + \partial_\rho\hat{\Upsilon}^{\mu\nu,\lambda\rho}$$

$$\hat{T}^{\mu\nu}_{\text{Can}} = \frac{i}{2}\bar{\psi}\gamma^\mu\overleftrightarrow{\partial}^\nu\psi - g^{\mu\nu}\mathscr{L}_D$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{Can}} = \frac{i}{8}\bar{\psi}\left\{\gamma^\lambda, \left[\gamma^\mu, \gamma^\nu\right]\right\}\psi$$

$$\hat{T}^{\mu\nu}_{\text{BR}} = \frac{i}{4}\bar{\psi}\left(\gamma^\mu\overleftrightarrow{\partial}^\nu + \gamma^\nu\overleftrightarrow{\partial}^\mu\right)\psi - g^{\mu\nu}\mathscr{L}_D$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{BR}} = 0$$

$$\hat{T}^{\mu\nu}_{\text{GLW}} = -\frac{1}{4m}\bar{\psi}\overleftrightarrow{\partial}^\mu\overleftrightarrow{\partial}^\nu\psi$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{GLW}} = \bar{\psi}\left[\frac{\sigma^{\mu\nu}}{4} - \frac{1}{8m}\left(\gamma^\mu\overleftrightarrow{\partial}^\nu - \gamma^\nu\overleftrightarrow{\partial}^\mu\right)\right]\gamma^\lambda\psi + \text{h.c}$$

$$\hat{T}^{\mu\nu}_{\text{HW}} = \hat{T}^{\mu\nu}_{\text{Can}} + \frac{i}{2m}\left(\partial^\nu\bar{\psi}\sigma^{\mu\beta}\partial_\beta\psi + \partial_\alpha\bar{\psi}\sigma^{\alpha\mu}\partial^\nu\psi\right) - \frac{i}{4m}g^{\mu\nu}\partial_\lambda\left(\bar{\psi}\sigma^{\lambda\alpha}\overleftrightarrow{\partial}_\alpha\psi\right)$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{HW}} = \hat{S}^{\lambda,\mu\nu}_{\text{Can}} - \frac{1}{4m}\left(\bar{\psi}\sigma^{\mu\nu}\sigma^{\lambda\rho}\partial_\rho\psi + \partial_\rho\bar{\psi}\sigma^{\lambda\rho}\sigma^{\mu\nu}\psi\right)$$

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Pseudogauge dependence of quantum fluctuations

$$\hat{T}^{00}_a = \frac{1}{(a\sqrt{\pi})^3}\int d^3\boldsymbol{x}\ \hat{T}^{00}(x)\ \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right)$$

→ Quantum fluctuations of energy in subsystems of a hot relativistic gas of spin-1/2 particles

$$\sigma^2(a,m,T) = \langle :\hat{T}^{00}_a::\hat{T}^{00}_a:\rangle - \langle :\hat{T}^{00}_a:\rangle^2$$

Variance

$$\sigma_n(a,m,T) = \frac{\left(\langle :\hat{T}^{00}_a::\hat{T}^{00}_a:\rangle - \langle :\hat{T}^{00}_a:\rangle^2\right)^{1/2}}{\langle :\hat{T}^{00}_a:\rangle}$$

Normalized standard deviation

$$\sigma^2_{\rm Can}(a,m,T) = 2\int dKdK'f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}'}))\left[(\omega_{\boldsymbol{k}}+\omega_{\boldsymbol{k}'})^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}+\boldsymbol{k}\cdot\boldsymbol{k}'+m^2)e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} - (\omega_{\boldsymbol{k}}-\omega_{\boldsymbol{k}'})^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}+\boldsymbol{k}\cdot\boldsymbol{k}'-m^2)e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\right]$$

$$\sigma^2_{\rm GLW}(a,m,T) = \frac{1}{2m^2}\int dKdK'f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}'}))\left[(\omega_{\boldsymbol{k}}+\omega_{\boldsymbol{k}'})^4\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}-\boldsymbol{k}\cdot\boldsymbol{k}'+m^2\right)e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} - (\omega_{\boldsymbol{k}}-\omega_{\boldsymbol{k}'})^4\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}-\boldsymbol{k}\cdot\boldsymbol{k}'-m^2\right)e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\right]$$

$$\sigma^2_{\rm HW}(a,m,T) = \frac{2}{m^2}\int dKdK'f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}'}))\left[\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}+\boldsymbol{k}\cdot\boldsymbol{k}'+m^2\right)^2\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}-\boldsymbol{k}\cdot\boldsymbol{k}'+m^2\right)e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} - (\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}+\boldsymbol{k}\cdot\boldsymbol{k}'-m^2)^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}-\boldsymbol{k}\cdot\boldsymbol{k}'-m^2)e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\right]$$

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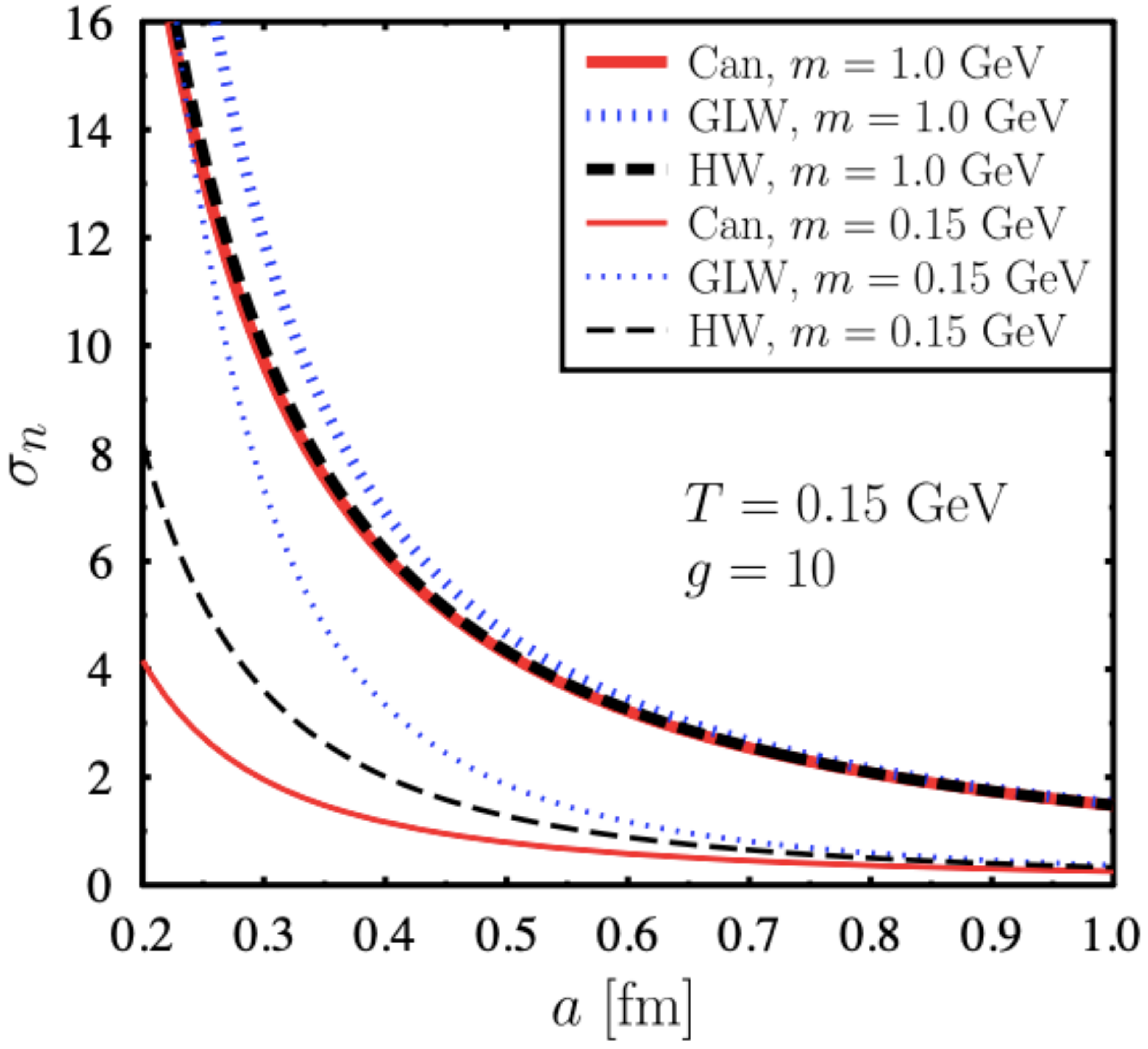
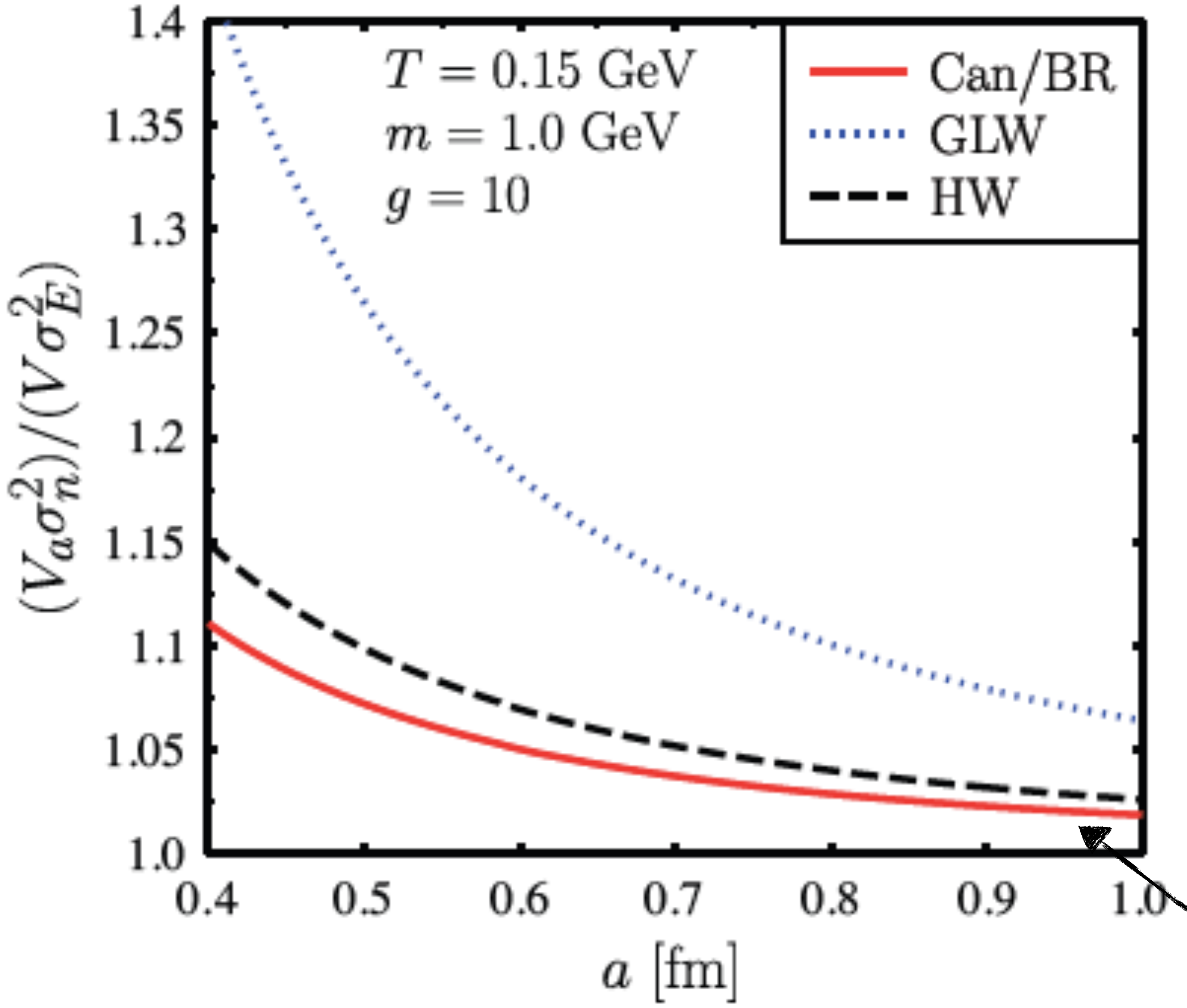
Pseudogauge dependence of quantum fluctuations





Agree with the known canonical ensemble formula

Quantum fluctuations of energy in very small thermodynamic systems is pseudogauge dependent

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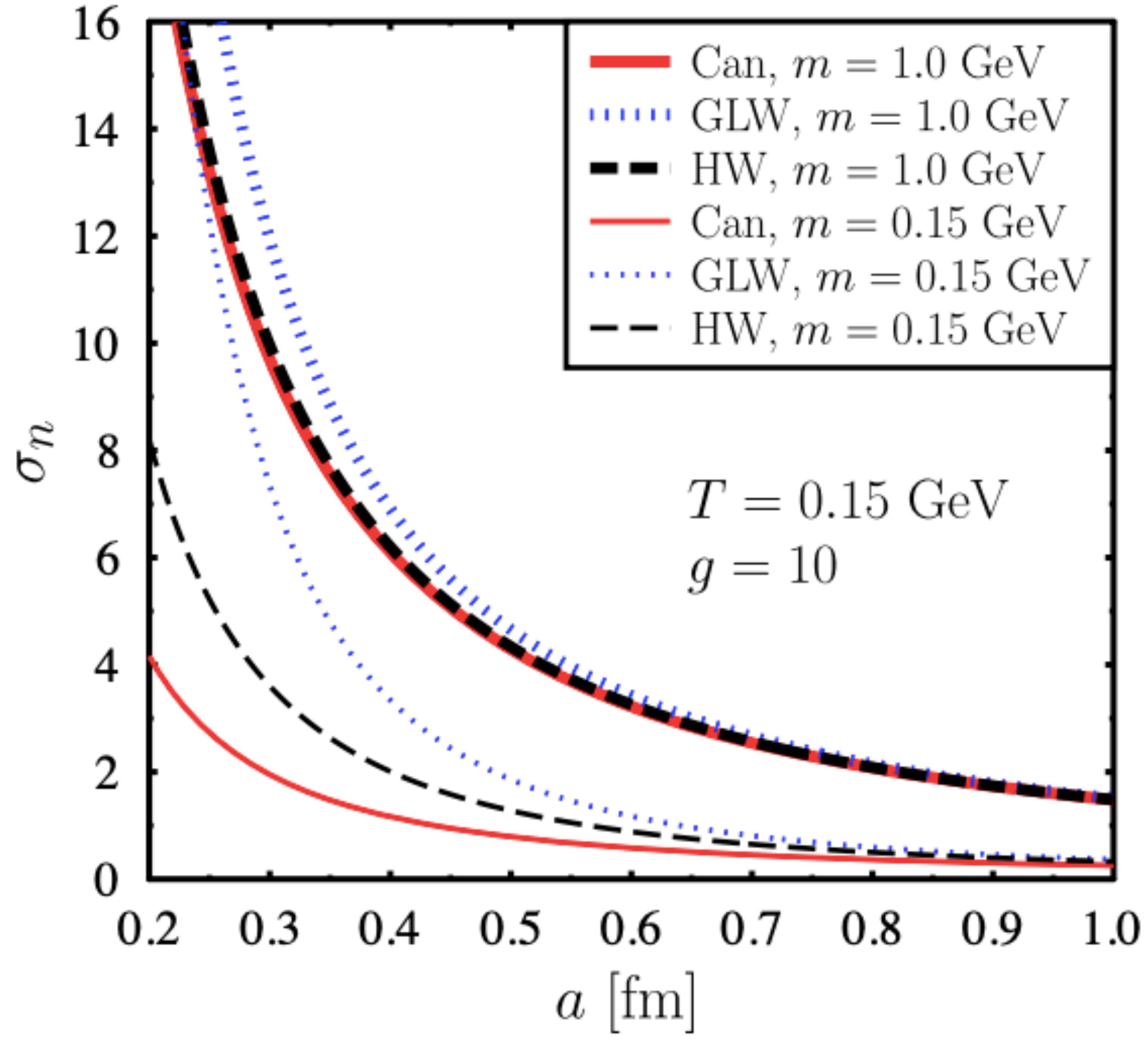
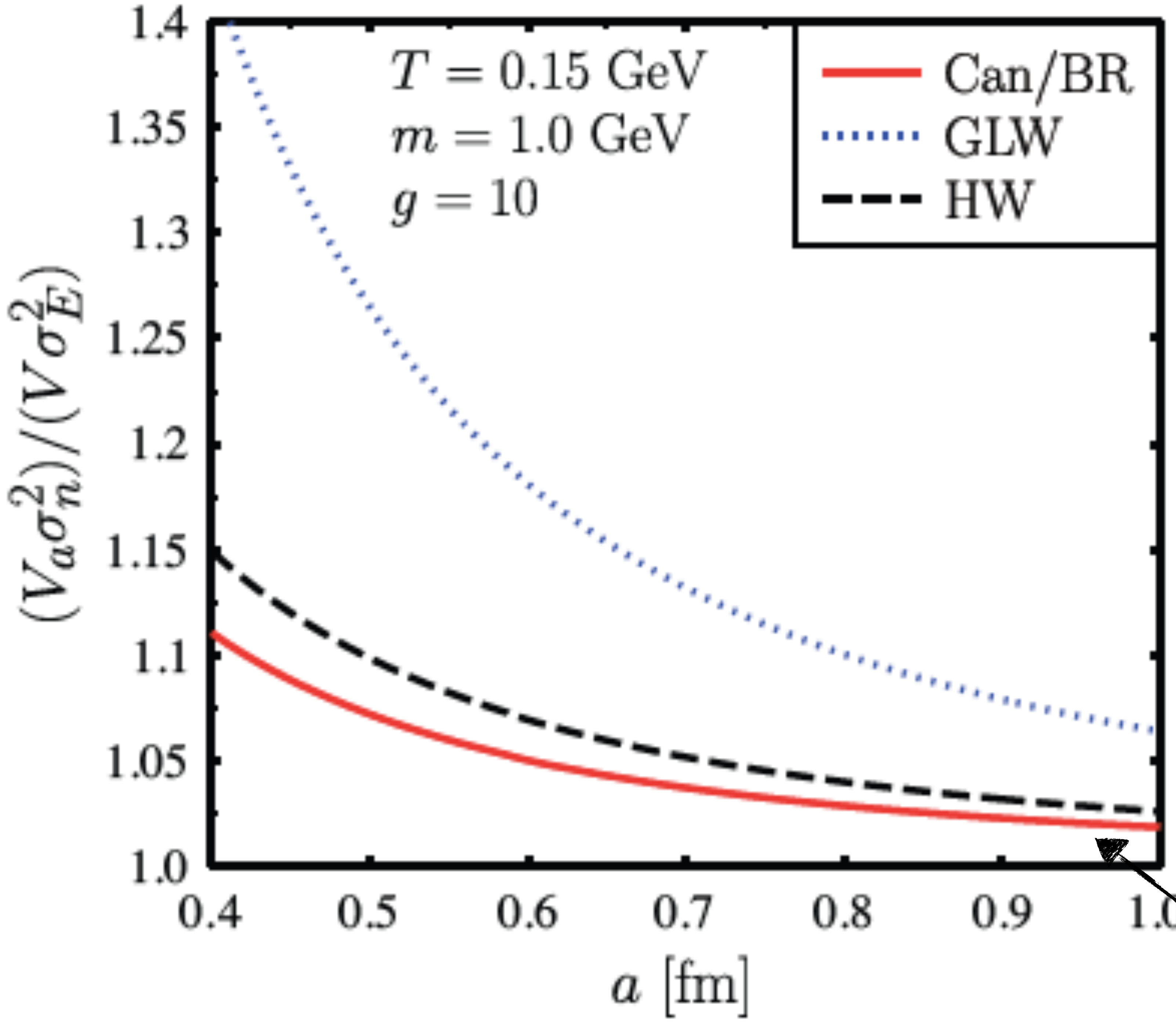
Pseudogauge dependence of quantum fluctuations





Agree with the known canonical ensemble formula

Not all spin operators follow SO(3) angular momentum algebra except canonical spin

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Can we remove the ambiguity?

→ Canonical $\hat{T}^{\mu\nu}$

- Not gauge invariant in theories with gauge fields
- Asymmetric in the presence of spin even there are no interactions
- Not unique in linearized gravity
- Fails to be traceless in theories which are manifestly scale invariant

→ Use pseudo gauge, but?

Emmy Noether, “Invariante Variationsprobleme”, Gottinger Nachrichten (1918), 235-257

Annals Phys. 309 (2004) 306-389

Physics Letters B 843 (2023) 137994

Coming soon (arXiv:2308.xxxxx)

Thank you for listening!