Probing QCD at high energy and density with Jets, INT 2023

Medium Modifications to Jet Angularities and semi-inclusive jet function

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Outline

- Jet substructure observable: Angularities
- Angularities in pp collision
- Medium modifications on jet angularities
- Groomed angularities
- Semi-inclusive jet function
- Summary and outlook

Jet Angularities

- Angularities are a class of jet substructure observables that characterize the angular and momentum distribution of partons inside a jet through a continuous free parameter a

- In e^+e^- collisions, the jet angularity is defined as

$$\tau_a^{e^+e^-} = \frac{1}{2E_J} \sum_{i\epsilon J} |\vec{p}_T^{\,i}| e^{-|\eta_i|(1-a)} \qquad \tau_a = \left(\frac{2E_J}{p_T}\right)^{2-a} \tau_a^{e^+e^-}$$

- Special cases

Thrust: $$\tau = \frac{1}{2E_J} \sum_{i\epsilon \text{jet}} |\vec{p}_T^{\,i}| e^{-|\eta_i|} \qquad (a = 0)$$

Broadening: $$\tau = \frac{1}{2E_J} \sum_{i\epsilon \text{jet}} |\vec{p}_T^{\,i}| \qquad (a = 1)$$

- Varying the exponent 'a' changes the sensitivity of the observable to collinear and soft radiation in the jet.

- For $a < 1$, the observable weighs collinear radiation more strongly while for a close to 1, the effect of soft radiation to the measured value of the observable cannot be ignored.

Soft Collinear Effective Theory

- SCET describes the interactions of soft and collinear degrees of freedom

Collinear $\longrightarrow$ Inhomogeneous momenta $\longrightarrow$ $Q(1,\lambda^2,\lambda)$ $\quad Q \longrightarrow$ Hard scale

Soft $\longrightarrow$ homogeneous momenta $\longrightarrow$ $Q(\lambda,\lambda,\lambda)$ $\quad \lambda \longrightarrow$ Power counting parameter

$$\mathscr{L}(\xi_n, \xi_{\bar{n}}, A_n, A_{\bar{n}}, A_s) = \mathscr{L}_n(\xi_n, A_n) + \mathscr{L}_{\bar{n}}(\xi_{\bar{n}}, A_{\bar{n}}) + \mathscr{L}_s(A_s)$$

$$\chi_n(x) = U_n^\dagger(x)\xi_n(x) \qquad \mathscr{B}^\mu_{n,\perp}(x) = \frac{1}{g}U_n^\dagger(x) iD^\mu_{n,\perp}U_n(x)$$

Gauge invariant fields

- Scaling with angularity exponent

$$p_n \sim Q(\lambda^2,1,\lambda) \qquad p_s \sim Q(\lambda^{2-a},\lambda^{2-a},\lambda^{2-a}) \qquad \lambda \sim \tau_a^{\frac{1}{2-a}}$$

- For the case $a < 1$ of angularities, this soft mode always scales smaller than λ

$$p_s^2 \ll p_c^2 \Rightarrow \text{SCET}_\text{I}$$

Factorization Theorem for Angularities

- For $a < 1$ angularities and $\tau_a^{\frac{1}{2-a}} \ll R$, the factorized differential distribution is

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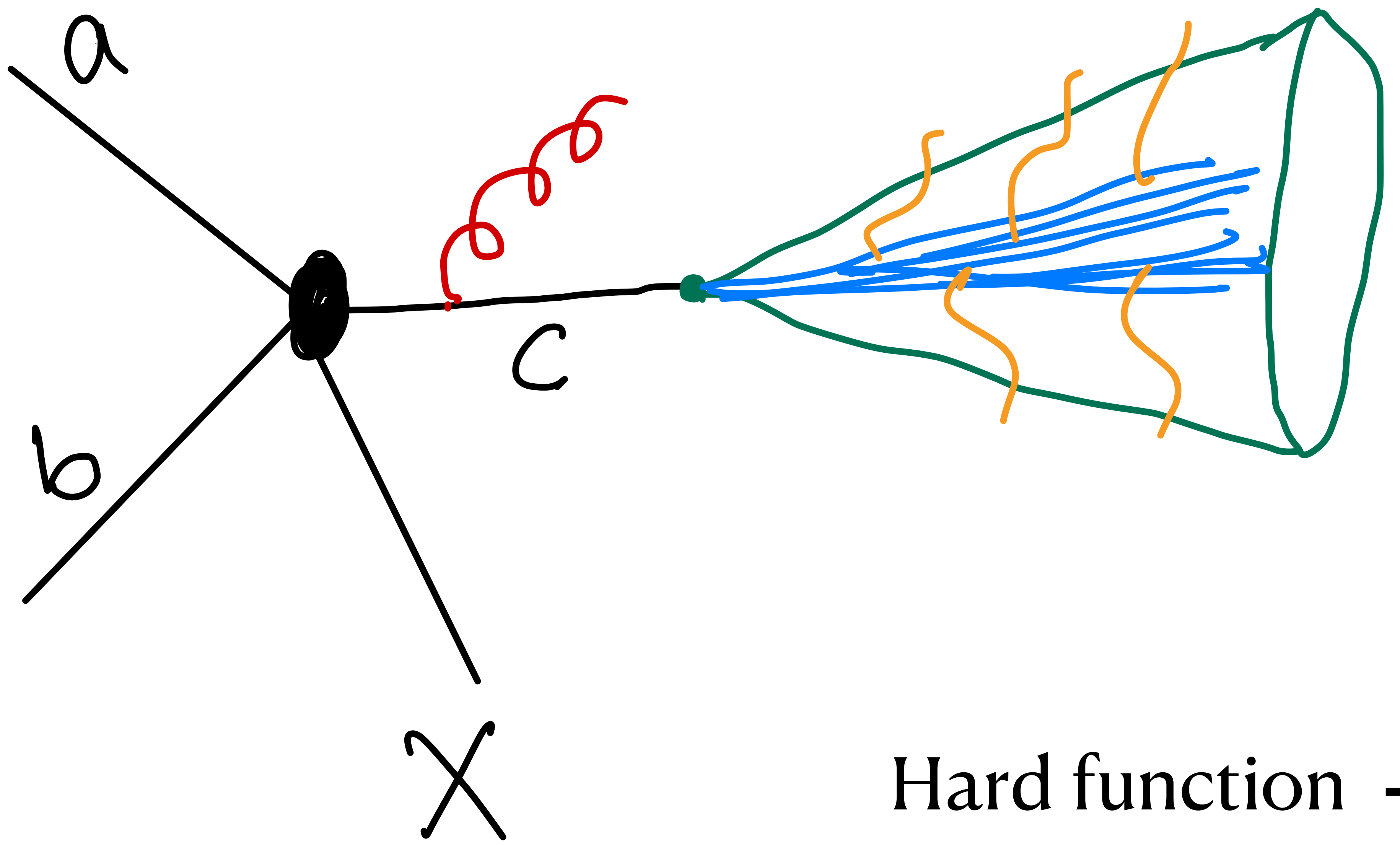
$$\frac{d\sigma^{pp\to(\mathrm{jet}\,\tau_a)X}}{d\tau_a\, dp_T\, d\eta} = \frac{2p_T}{s}\sum_{abc}\sum_i \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a,\mu)\int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b,\mu)\int_{z^{\min}}^1 \frac{dz}{z^2} H_{ab}^c(x_a,x_b,\hat{s},\hat{\eta},p_T/z,\mu)\,\mathscr{H}_{c\to i}(z,p_T R,\mu) J(\tau_a^c,p_T,\mu)\otimes S(\tau_a^s,p_T,R,\mu)$$

$$x_a^{\min} = W \qquad x_b^{\min} = \frac{1-V}{1-VW/x_a} \qquad z^{\min} = \frac{1-V}{x_b} + \frac{VW}{x_a}$$



Step 1: Factorized jet production

$f_a \otimes f_b \otimes H_{ab}^c \otimes \mathcal{G}_c(z,\tau_a,R)$

Step 2: $\tau^{1/(2-a)} << R$

$\mathcal{G}_c(z,\tau_a,R) = \mathcal{H}_{c\to i}(z) \times J(\tau_a^c) \otimes S(\tau_a^s)$

Hard function $\longrightarrow$ Observable independent

Hard function

- For H^c_{ab} : we compute all processes up-to next-to-leading-order.

- $\mathscr{H}_{c\to i}$ satisfies a DGLAP-type evolution equation

$$\mu\frac{d}{d\mu}\mathscr{H}_{c\to i}(z,p_TR,\mu)=\sum_k\int_z^1\frac{dz'}{z'}\gamma_{ck}\left(\frac{z}{z'},p_TR,\mu\right)\mathscr{H}_{k\to i}(z',p_TR,\mu)$$

H^c_{ab} — $\mu_H\sim p_T$

$\mathscr{H}_{c\to i}$ — $\mu_{\mathscr{H}}\sim p_TR$

J_i — $\mu_J\sim p_T(\tau_a)^{\frac{1}{(2-a)}}$

S_i — $\mu_S\sim\frac{p_T\tau_a}{R^{1-a}}$

For $c=q$:

H^c_{ab}:
$qq\to qX$
$q\bar{q}\to q/\bar{q}X$
$qg\to q/\bar{q}X$
$gg\to qX$

For $c=g$

H^c_{ab}:
$gg\to gX$
$qg\to gX$
$q\bar{q}\to gX$
$qq\to gX$

Hard matching function encodes radiation at virtuality p_TR.

Angularity jet function

- Jet function is related to spin averaged QCD splitting function with appropriate phase factors

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$$J_i(\ldots) = \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon} 2g_s^2 \sum_{i^1 i^2 \ldots i^n} \int \prod_{n=1}^{m} \frac{d^d k_n}{(2\pi)^{d-1}} P_{i \to i^1 i^2 \ldots i^n} \delta(\ldots)\delta(k_n^2)$$

$$\hat{\tau} = \left(\frac{2E_J}{p_T}\right)^{2-a} \frac{1}{2E_J}\left(|\vec{k}_T| e^{-|\eta|(1-a)} + |\vec{q}_T| e^{-|\eta|(1-a)}\right)$$

- Measurement function is angularity

$$\delta(\tau_a - \hat{\tau}_a) \qquad \hat{\tau}_a = p_T^{a-2} k_\perp^{2-a}(x^{a-1} + (1-x)^{a-1})$$

$$P_{i \to jk}(x, k_\perp) = \frac{1}{k_\perp^2} P_{i \to jk}(x) \quad \longrightarrow \quad \text{Vacuum splitting functions}$$

- Vacuum jet function

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$$J(\tau_a, p_T, \mu) = \delta(\tau_a) + \frac{\alpha_s(\mu)}{\pi \ (2-a)} \left(\delta(\tau_a) f_1 + \left[\frac{1}{\tau_a}\right]_+ f_2 + \left[\frac{\ln \tau_a}{\tau_a}\right]_+ f_3 \right)$$

Angularity soft function

- RGE for vacuum jet function

$$\mu\frac{d}{d\mu}J_i^{\rm vac}(\tau_a,p_T,R,\mu)=\int d\tau_a'\,\gamma_{J_i}(\tau_a-\tau_a',p_T,R,\mu)\,J_i^{\rm vac}(\tau_a',p_T,R,\mu)$$

$$\gamma_{J_i}=\frac{\alpha_s(\mu)}{\pi}\left[\delta(\tau_a)\left(2b_i+\frac{(2-a)}{(1-a)}\ln\left(\frac{\mu^2}{p_T^2}\right)C_i\right)-\frac{2}{(1-a)}C_i\left[\frac{1}{\tau_a}\right]_+\right]$$

- Jet algorithm function

$$\Theta_{\rm alg}\equiv\Theta\left(\omega x\,(1-x)\tan\frac{R}{2}-k_\perp\right)$$

- Soft function $S_g(\ldots)=\dfrac{1}{N^2-1}\langle 0|\,\bar{Y}_n\delta(\ldots)Y_{\bar{n}}|X\rangle\langle X|\,\bar{Y}_{\bar{n}}Y_n|\,0\rangle$

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- RGE for soft function

$$\gamma_{S_i}=\frac{2\alpha_s(\mu)C_i}{\pi(1-a)}\left\{\left[\frac{1}{\tau_a}\right]_+-\ln\left(\frac{\mu R^{1-a}}{p_T}\right)\delta(\tau_a)\right\}$$

$$\mu\frac{d}{d\mu}S_i^{\rm vac}(\tau_a,p_T,R,\mu)=\int d\tau_a'\,\gamma_{S_i}(\tau_a-\tau_a',p_T,R,\mu)\,S_i^{\rm vac}(\tau_a',p_T,R,\mu)$$

Non-perturbative effects via Shape function

- As $\mu_S \sim \Lambda_{\text{QCD}}$, non-perturbative effects become relevant → incorporate through a shape function

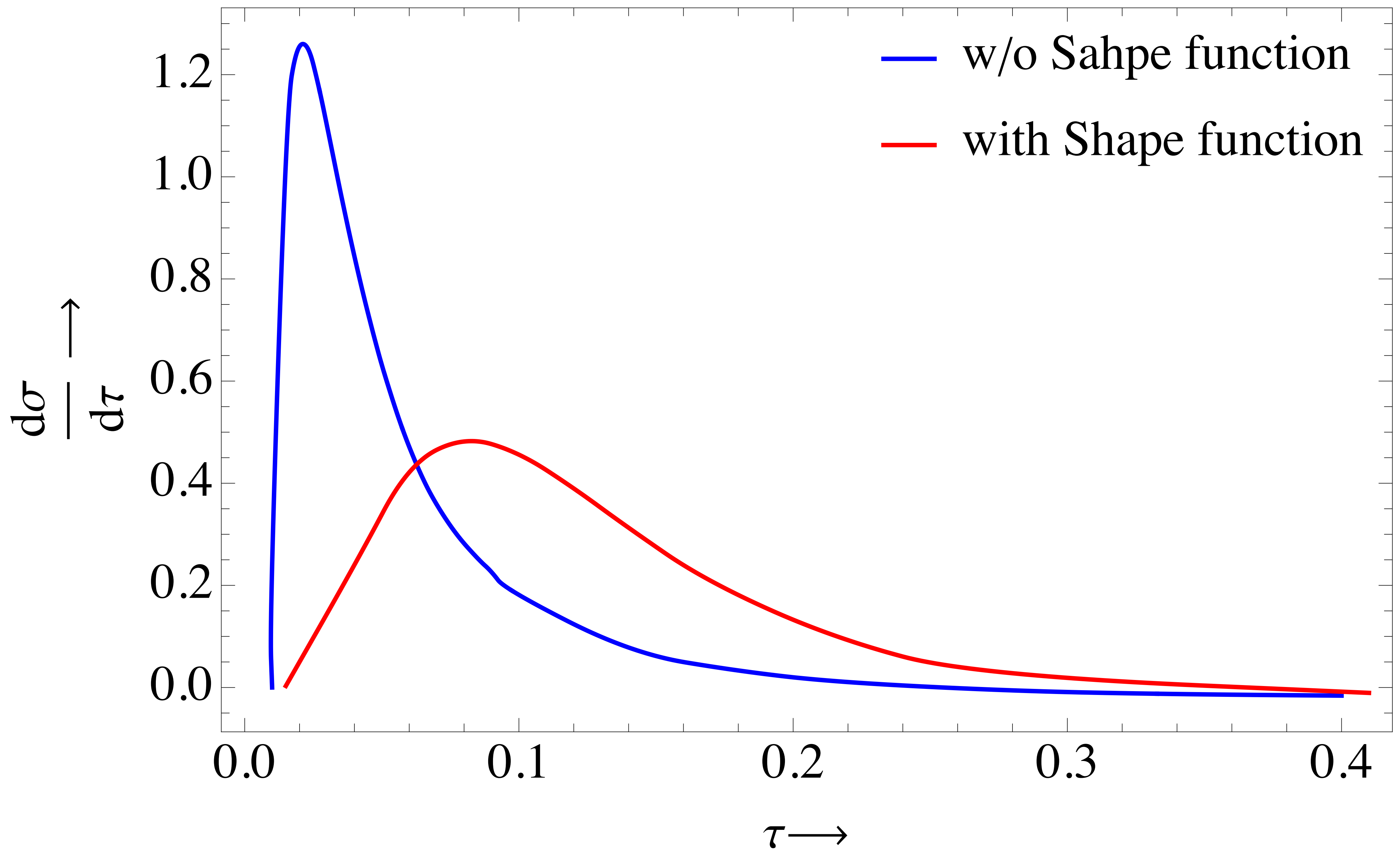
$$S_{NP}(k) = \frac{4k}{\Omega_a^2} exp(-2k/\Omega_a) \qquad \text{with} \qquad \Omega_a = \frac{\Omega_0}{1-a} \qquad \Omega_0 \sim 1\,\text{GeV}$$

- Ω_0 can be universally obtained by a global fit

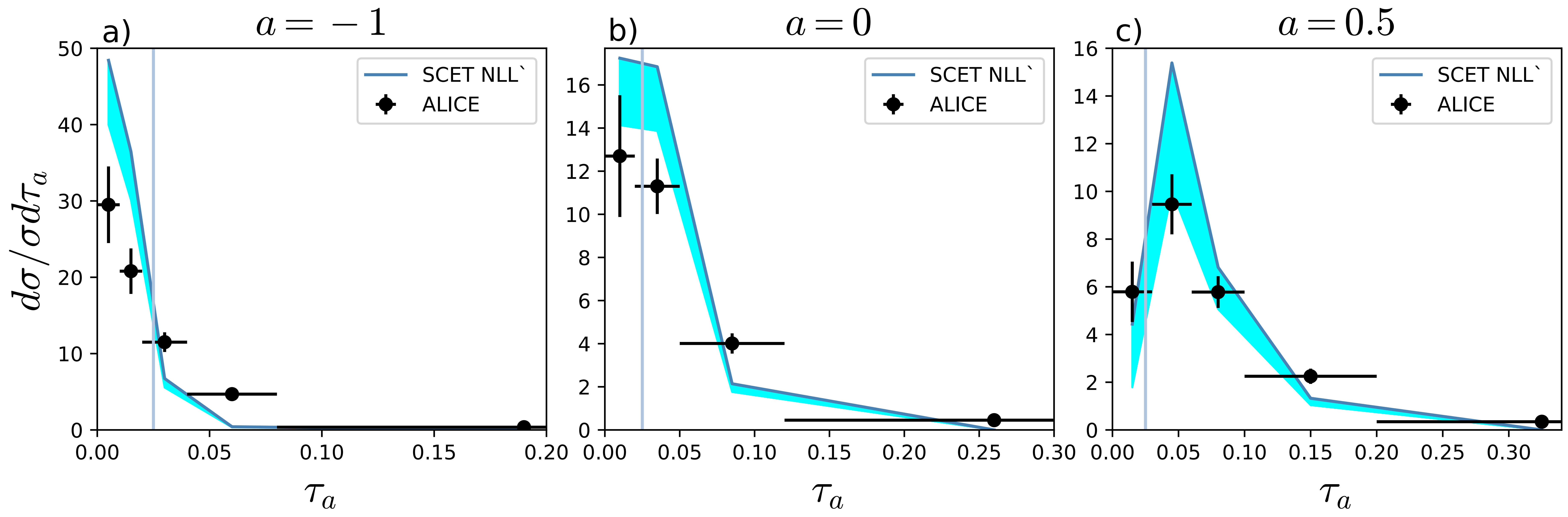
- The final angularity cross-section is then

$$\frac{d\sigma}{dp_T d\tau} = \int \frac{d\sigma^{\text{pert}}}{dp_T d\tau}(\tau - \tau_{\text{shift}}(k)) S_{NP}(k) dk$$

$$\tau_{\text{shift}}(k) = \frac{k}{p_T R}$$



Resummed Distributions for pp-collisions



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- Bands are obtained by varying μ from $p_T - 2p_T$

$$R = 0.4 \qquad 80 < p_T < 100 \qquad \Omega_0 = 0.35 \text{ GeV}$$

Soft Collinear Effective Theory with Glaubers

- Glaubers are off-shell modes
- Scaling of the Glaubers depends on the source that emits these modes

$$p_G \sim Q(\tilde{\lambda}^2, \tilde{\lambda}^2, \tilde{\lambda}) \quad p_S \sim Q(1,1,\tilde{\lambda}) \quad p_S \sim Q(\tilde{\lambda}^2,1,\tilde{\lambda})$$

$$p_G \sim Q(\tilde{\lambda}, \tilde{\lambda}^2, \tilde{\lambda}) \quad p_S \sim Q(\tilde{\lambda}, \tilde{\lambda}, \tilde{\lambda})$$

$$\tilde{\lambda} \sim \frac{T}{Q}$$

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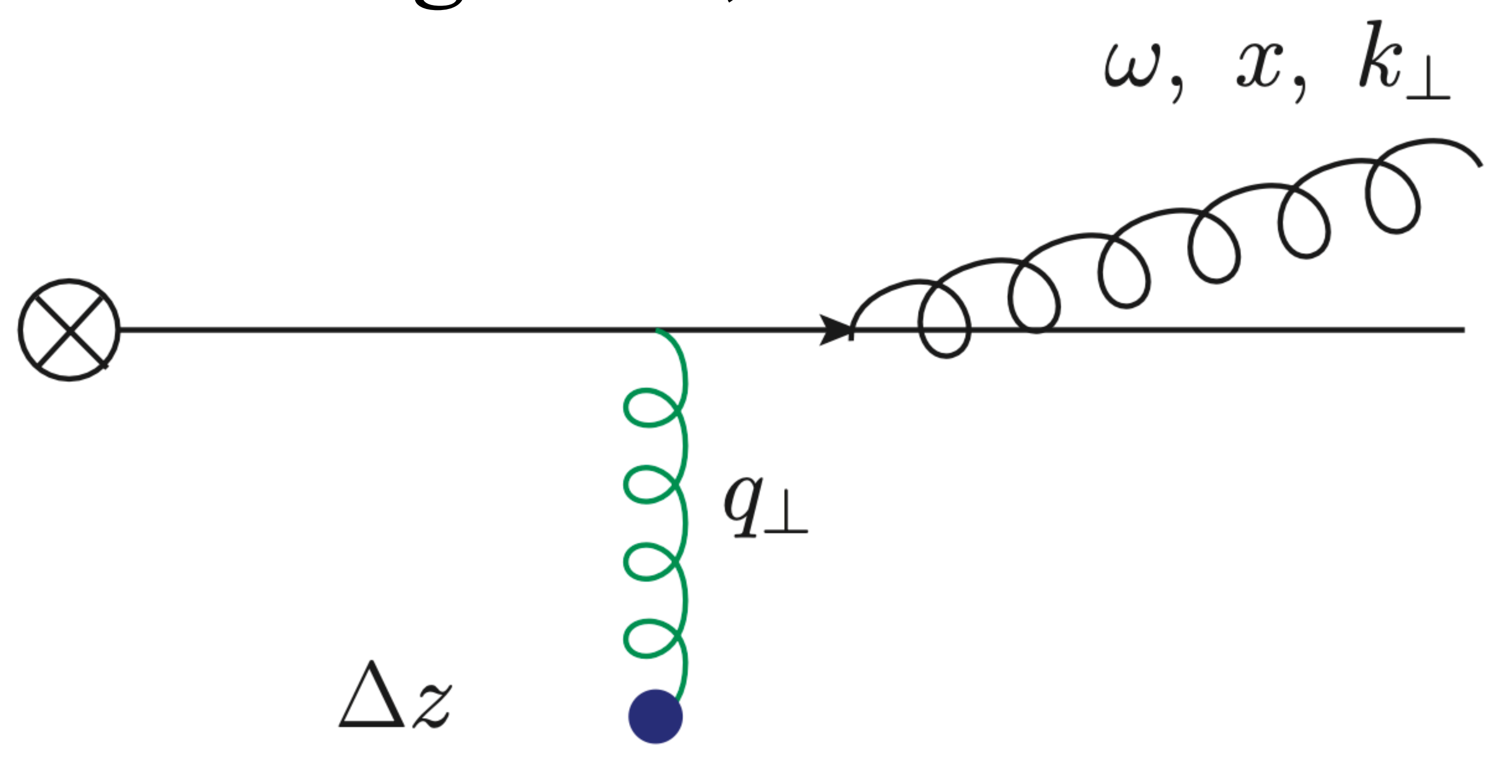
- Glauber Lagrangian in addition to SCET Lagrangian

$$\mathscr{L}_G(\chi_n, \mathscr{B}_n, \eta) = \sum_{q,p,p'} e^{-i(q+p-p')\cdot x}\left(\frac{1}{2}\bar{\chi}_{n,p'}\Gamma^{\nu,a_{qqG}}\gamma_\mu \bar{n}^\mu \chi_{n,p} - i\Gamma^{\nu\mu\lambda,abc}_{ggG}(\mathscr{B}^c_{n,p'})_\lambda(\mathscr{B}^b_{n,p})_\nu\right)\bar{\eta}\Gamma^{\delta,a}_s \eta \Delta_{\nu\delta}(q)$$

- To see the LPM effect, we require minimum two scattering sources. To leading order, in the small x-limit

$$x\frac{dN_{q\to q\,g}}{dx d^2k_\perp} = \tilde{\alpha}\int_0^{\bar{L}} \frac{d\Delta z}{\lambda} d^2q_\perp \frac{1}{\sigma}\frac{d\sigma}{d^2q_\perp}\frac{2k_\perp\cdot q_\perp}{k_\perp^2(q_\perp-k_\perp)^2}\left[1-\cos\left(\frac{(q_\perp-k_\perp)^2\Delta z}{x\,\omega}\right)\right]$$



with $$\tau_f = \frac{x\,\omega}{(q_\perp - k_\perp)^2}$$

Medium modified jet function

- Medium modification to the jet function are incorporated via splitting functions

$$J_i^{\text{med}}(\tau_a, p_T, \mu) = \int d\Phi_2^c\, \sigma_2^c \delta(\tau_a - p_T^{2-a}\left\{x^{a-1}k_\perp^{2-a} + (1-x)^{a-1}q_\perp^{2-a}\right\})$$

$$d\Phi_2^c = 2(2\pi)^{3-2\epsilon}(2E_J)\int \frac{d^dk}{(2\pi)^{d-1}}\delta(k^2)\theta(k^0)\int \frac{d^dq}{(2\pi)^{d-1}}\delta(q^2)\theta(q^0)\delta(2E_J - k^+ - q^+)\delta^{d-2}(k_\perp + q_\perp)$$

- No new divergence in $\epsilon \to 0$ limit
- Explicit divergence in $a \to 2$ limit

$$\sigma_2^c = \sum_{j,k}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon} 2g_s^2 P_{i\to jk}^{\text{med}}(x, k_\perp)$$

$$J_i^{\text{med}}(\tau_a, p_T, \mu) = \frac{\alpha_s}{\pi}\frac{}{2-a}\frac{\mu^{2\epsilon}e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\frac{1}{\tau_a^{\frac{2\epsilon-a}{2-a}}}\sum_{j,k}\int dx\,\left(x^{a-1}+(1-x)^{a-1}\right)^{\frac{2\epsilon-2}{2-a}} P_{i\to jk}^{\text{med}}\left(x, \frac{p_T\tau^{\frac{1}{2-a}}}{(x^{a-1}+(1-x)^{a-1})^{\frac{1}{2-a}}}\right)$$

Medium modifications

- Total jet function

$$J_i(\tau_a, p_T, R, \mu) = J_i^{med}(\tau_a, p_T, R, \mu) + J_i^{vac}(\tau_a, p_T, R, \mu)$$

- Jet energy loss

$$\epsilon_g = 2\pi \left(\int_0^{\frac{1}{2}} dx\, x + \int_{\frac{1}{2}}^{1} dx\,(1-x) \right) \int_{2p_T x(1-x)\tan\frac{R}{2}}^{2p_T x(1-x)\tan\frac{R_0}{2}} dk_\perp\, k_\perp \left[P^{\text{med}}_{g\to gg}(x, k_\perp) + 2N_f P^{\text{med}}_{g\to q\bar{q}}(x, k_\perp) \right]$$

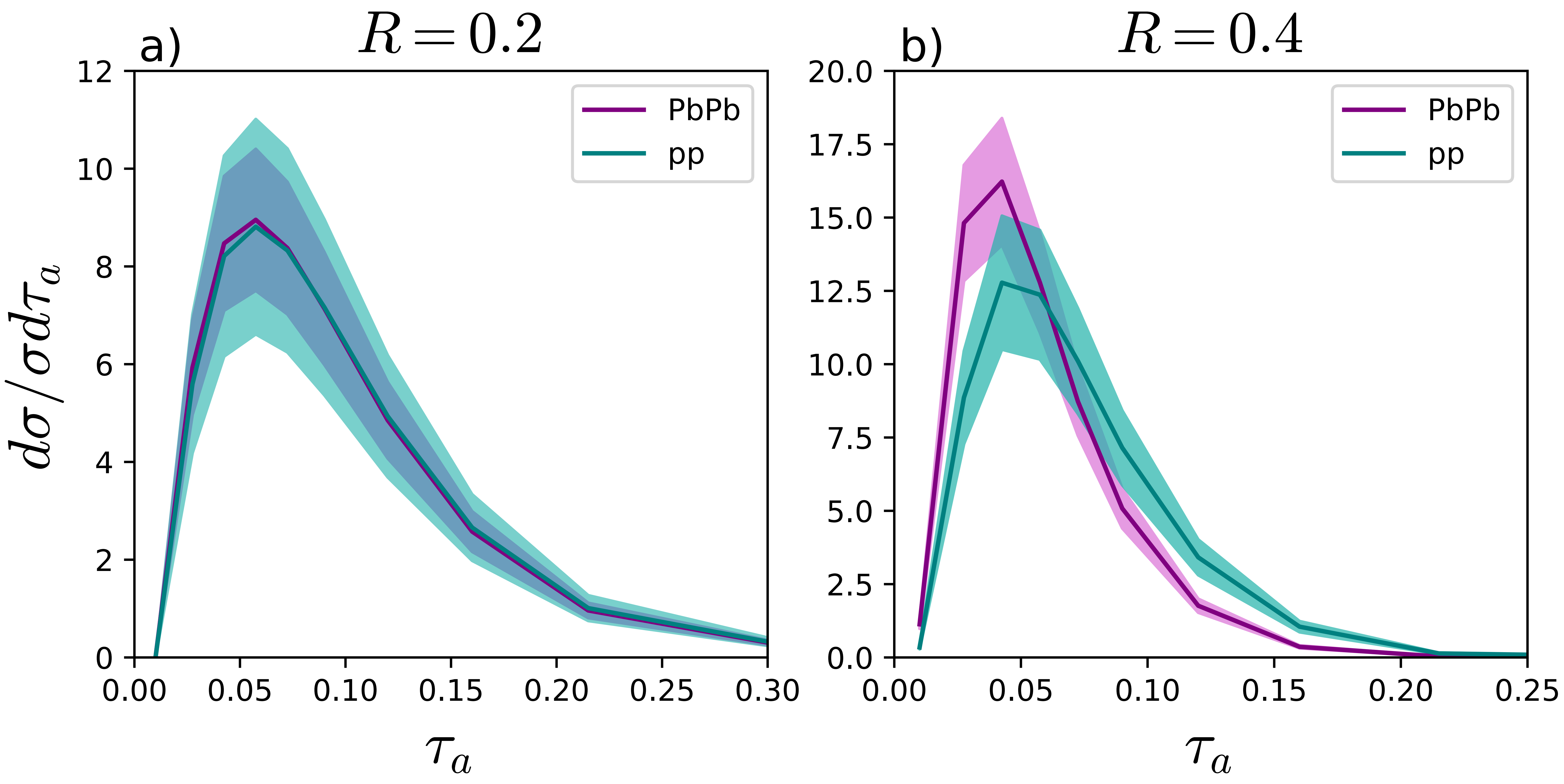
- Medium parameters are averaged quantities

$$L(b) = \frac{\int d^2r\, l(b,r,\theta)\, P(b,r,\theta)}{\int d^2r\, P(b,r,\theta)}$$

Parameter	$0-10\,\%$	$10-30\,\%$
b (fm)	3.34	7.01
L (fm)	4.96 ± 1.22	3.56 ± 0.99
T_0 (MeV)	456	437
T (MeV)	308 ± 84	248 ± 128

Results

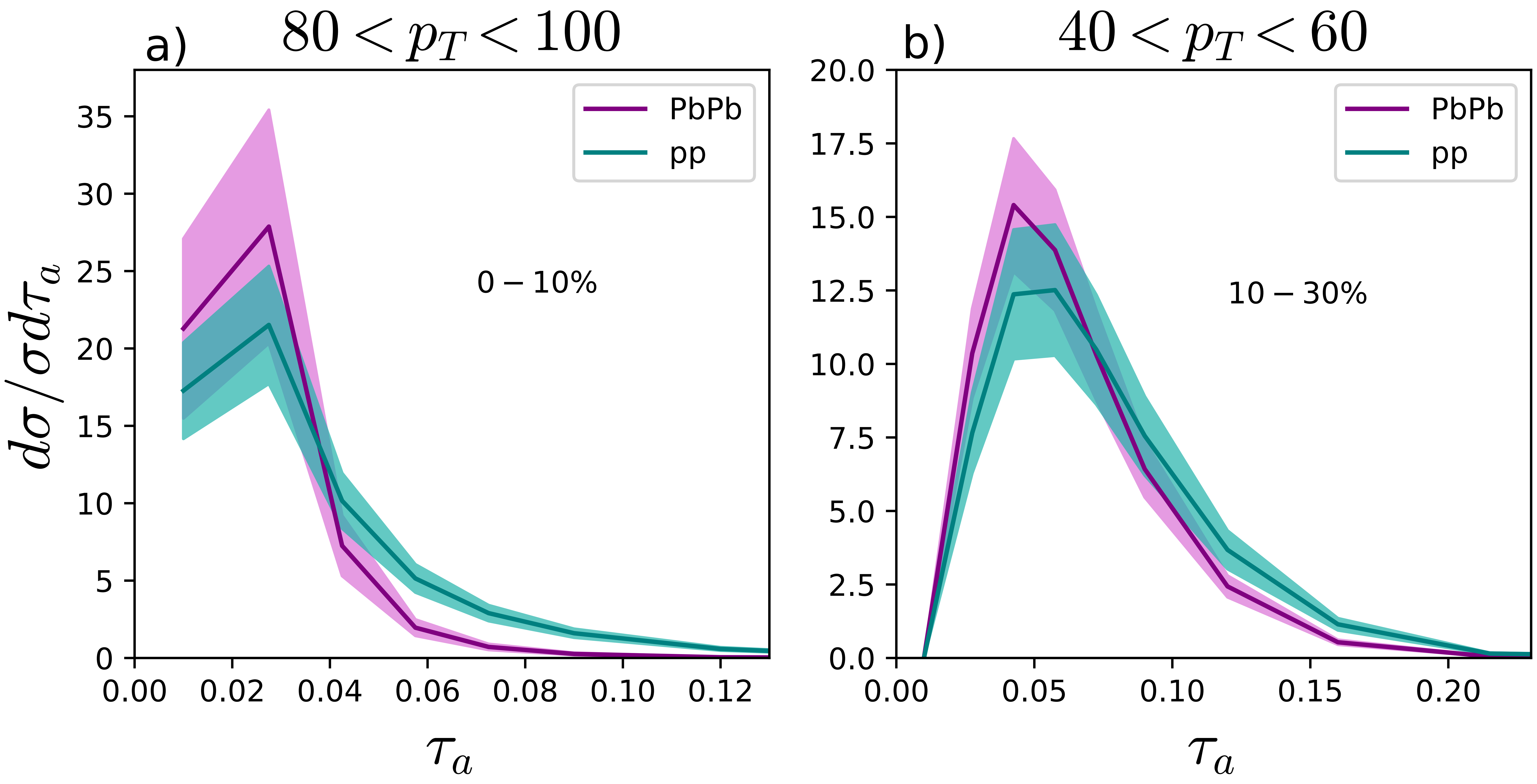
- pp vs AA distributions



Normalised differential distributions in PbPb (central) and pp for $a = 0$, jet parameters $40 < p_T < 60$ and (a) $R = 0.2$, (b) $R = 0.4$. The theoretical error bands correspond to variation in μ from p_T to $2p_T$.

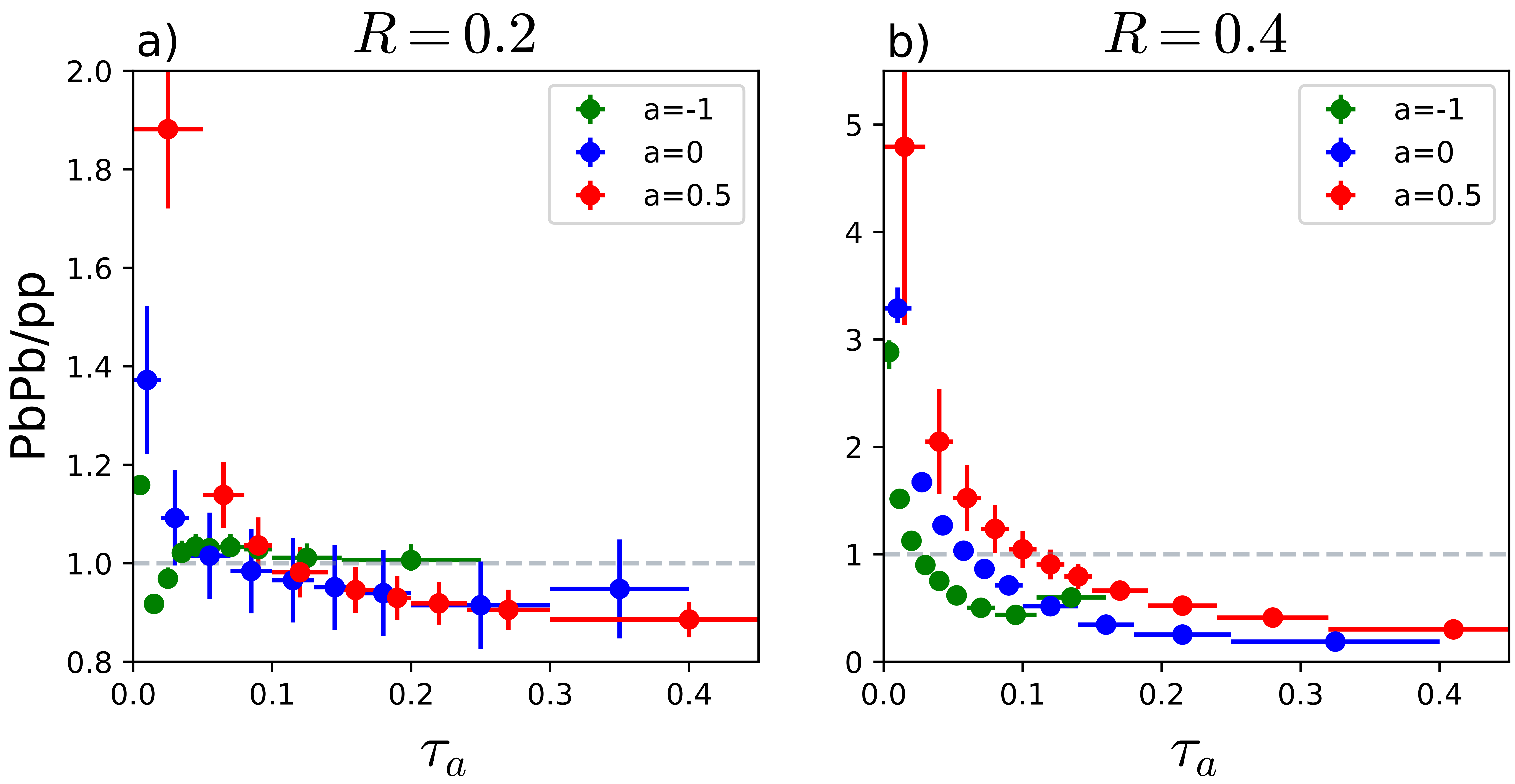
Results

- pp vs AA distributions



Normalised angularity distributions for $a = 0$ with (a) $80 < p_T < 100$, $0 - 10\,\%$ centrality and (b) $40 < p_T < 60$, $10 - 30\,\%$ centrality, for a jet with $R = 0.4$

Results



- For both the figures $40 < p_T < 60$ and centrality is 0-10%.
- Initial state energy loss is not included

Groomed angularities

- Less sensitive to soft radiation

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- Factorization

$$\frac{d\sigma}{d\eta\, dp_T\, d\tau_a} = \sum_{ijk} \int \frac{dx_i}{x_i} f_i(x_i, \mu) \int \frac{dx_j}{x_j} f_j(x_j, \mu) \int \frac{dz}{z} H_{ijk}(x_i, x_j, \eta, p_T/z, \mu) G_k(z, \tau_a, p_T R, z_{\rm cut}, \beta, \mu)$$

$$G_i(z, \tau_a, p_T R, z_{\rm cut}, \beta, \mu) = J_i(z, p_T R, \mu)\delta(\tau_a) + \delta(1-z)\Delta G_i(z, \tau_a, p_T R, z_{\rm cut}, \beta, \alpha_s(\mu))$$

- Measurement function

$$\Delta G_i(z, \tau_a, p_T R, z_{\rm cut}, \beta, \alpha_s(\mu)) = \int d\Phi_2\, \sigma^c_{2,i}\, \Theta\left(\omega x(1-x)\tan\frac{R}{2} - k_\perp\right)\left[\theta_1 \delta\left(\tau_a - p_T^{a-2} k_\perp^{2-a}(x^{a-1} + (1-x)^{a-1})\right) + ..\right]$$

$$\theta_1 = \Theta\left(x > z_{\rm cut}\left(\frac{\Delta R}{R}\right)^\beta\right)\Theta\left(1 - x > z_{\rm cut}\left(\frac{\Delta R}{R}\right)^\beta\right)$$

Semi-inclusive jet function

- Scales in the problem

Medium
$T \sim (0.5\text{-}1)\,\text{GeV}$

$p_T \longrightarrow$ Jet energy

$p_T R \longrightarrow$ Jet Virtuality

$\Lambda_{QCD} \longrightarrow$ Non-perturbative

$\lambda \sim R$

$\tilde{\lambda} \sim \dfrac{T}{Q}$

$P_c \sim p_T(1, \lambda^2, \lambda)$

$P_M \sim (T, T, T)$

$p_T R$ — Perturbative

- For a virtual photon probing the nuclear medium

$$x = \frac{Q^2}{2P \cdot q} \qquad x_{HIC} \sim \frac{(p_T R)^2}{2 p_T T}$$

- Two regimes for x

Glauber

$P_c \sim p_T(\lambda, \lambda^2, \lambda)$

$x << 1$	$x \sim 1$
$T \gg p_T R^2$	$T \sim p_T R^2$
$P_M \sim p_T(\tilde{\lambda}, \tilde{\lambda}, \tilde{\lambda})$	$P_M \sim p_T(R^2, R^2, R^2)$
$\lambda \sim \tilde{\lambda}$	

SCET1

Factorization

- SCET1 subleading operators

$$\mathcal{L}^{(1)} = g\bar{\chi}_n \mathcal{B}_{us(n)} \cdot P_\perp \frac{\bar{n}}{\bar{\mathcal{P}}} \chi_n - 2g \left[P_\perp^\mu \mathcal{B}_n^\perp \right] \left[\mathcal{B}_{n\nu}^\perp, \mathcal{B}_{us\mu}^\perp \right] \qquad \mathcal{L}^{(2)} = \bar{\chi}_n \left(t^a \gamma_\perp^\mu \frac{1}{\bar{P}} t^b \gamma_\perp^\nu \right) \frac{\bar{n}}{2} \chi_n \mathcal{B}_{us(n)}^{a\mu} \mathcal{B}_{us(n)}^{b\nu}$$

$$\mathcal{B}_{us}^\mu = \left[\frac{1}{in \cdot \partial_{us}} in_\nu G_{us}^{b\nu\mu} \mathcal{Y}_n^b \right]$$

At leading order contribution comes from Wilson lines.

- Cross-section

$$\Sigma = \lim_{t->\infty} \mathrm{Tr}[e^{iHt} \rho(0) e^{-iHt} \mathcal{M}] \qquad \rho(0) = | e^+e^- \rangle \langle e^+e^- | \otimes \rho_M(0)$$

- Total Hamiltonian

$$H = H_n + H_{us} + H_{int} \theta(\tau - t) + CL_\mu J^\mu$$

$H_{SCET} = H_n + H_{us} + H_{int}$

$$\Sigma = C^2 L_{\mu\nu} \int d\omega^- H^{\mu\nu}_{e^+e^- \to q} \lim_{t \to \infty} \int d^4s d^4r e^{-i\omega r} \mathrm{Tr} \left[e^{-iH_{HSET}t} (\bar{\chi}_{n,I} Y_n^\dagger)(0,s) \rho_M(0) \frac{\bar{\not{n}}}{2} (Y_n \chi_{n,I})(r+s) e^{iH_{HSET}t} \theta_{alg} \right]$$

Factorization

- Factorization at $\mathcal{O}(H^0_{int})$ reproduces the vacuum one

$$\Sigma = V\ C^{\,2} L_{\mu\nu} \int d\omega^- H^{\mu\nu}_{e^+e^-\to q} \int dr^+ \int d^2 r_\perp e^{-i\omega_- r_+} \mathrm{Tr}\left[\bar\chi_{n,I}(0)\frac{\bar{n}\!\!\!/}{2}\chi_{n,I}(r^+, r^\perp)\theta_{alg}\right]$$

- Factorization at $\mathcal{O}(H^2_{int})$

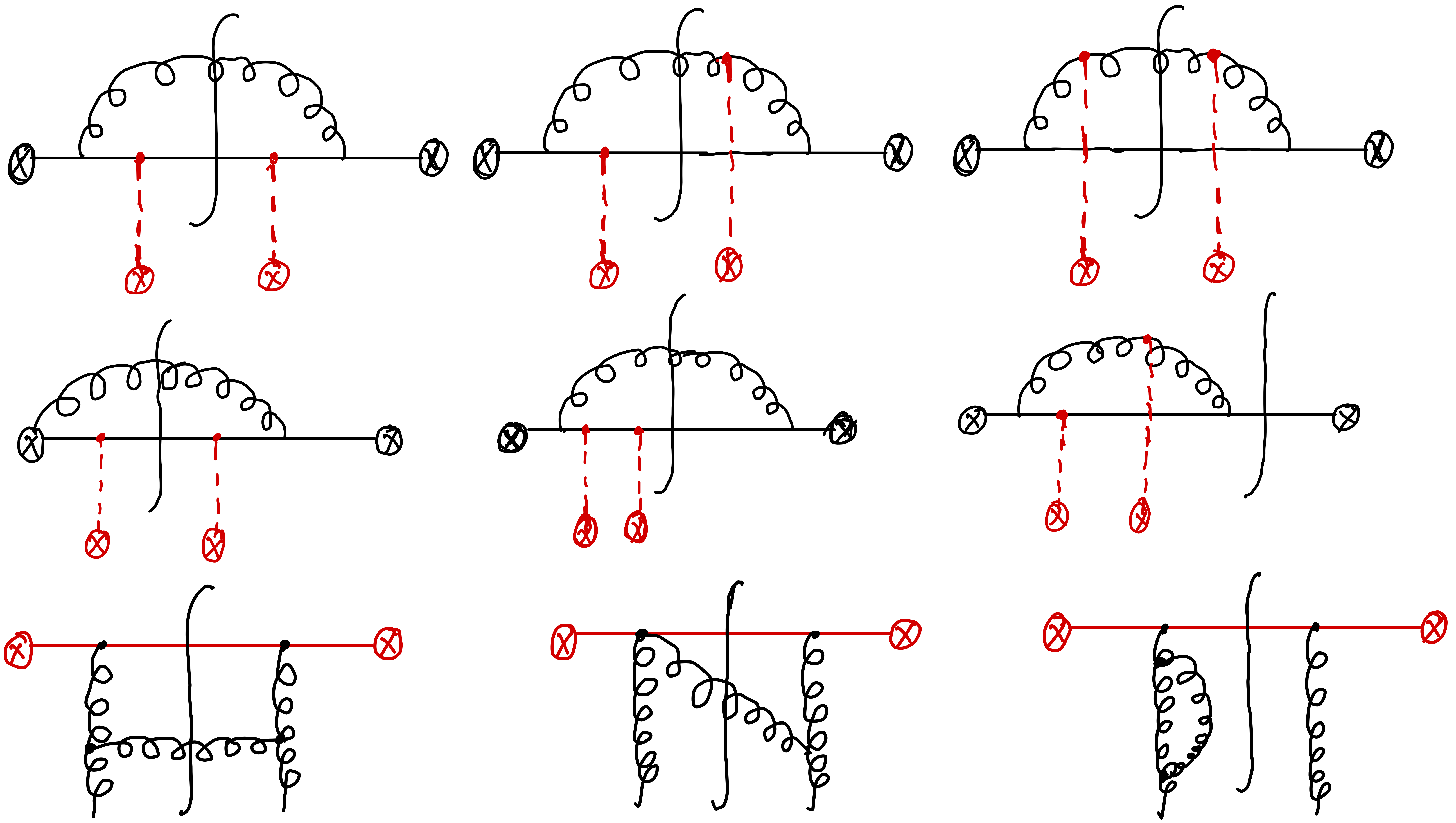
$$\Sigma^2_R = V\ C^{\,2} L_{\mu\nu} \int d\omega^- H^{\mu\nu}_{e^+e^-\to q} \int dr^+ \int d^2 r_\perp e^{-i\omega_- r_+} \int d^4u\, d^4v\, J^{(2)}_{\mu\nu}(u,v) U^{\mu\nu}(u,v)$$

$$U_{\mu\nu}(u,v) = \langle X_s | \, T\left[e^{-i\int dt_l H^{int}_{us}(t_l)} g\mathcal{B}^{\perp,\mu}_{us(n),I}(u^- + v^-, v^+, v^\perp)\right]\rho_M(0)\bar{T}\left[e^{-i\int dt_r H^{int}_{us}(t_r)} g\mathcal{B}^{\perp,\nu}_{us(n),I}(u^- - v^-, v^+, v^\perp)\right] | X_s\rangle$$

$$J^{(2)}_R(u,v) = \mathrm{Tr}\left[T\left\{\left(\bar\chi_n P_{\mu\perp}\frac{\bar{n}\!\!\!/}{\bar{P}}\chi_n\right)(v-u)\bar\chi_n(s)\right\}\frac{\bar{n}\!\!\!/}{2}\bar{T}\left\{\left(\bar\chi_n P_{\nu\perp}\frac{\bar{n}\!\!\!/}{\bar{P}}\chi_n\right)(v+u)\chi_n(r^+ + s^+, s^-, s^\perp + r^\perp)\theta_{alg}\right\}\right]$$

Order by order factorization

Diagrams



Summary and outlook

- Jet angularities allow to study a class of substructure observables with sensitivity to collinear emissions controlled by a continuous parameter, a.

- Jet-medium interactions modelled through off-shell Glauber gluons generated by color gauge fields of the medium.

- For $a < 1$, all medium modifications consistently incorporated in the medium modified jet function through the medium splittings.

- For a cleaner understanding of medium effects on the jet core, one needs to look at groomed angularities $\rightarrow$ less sensitive to hadronization and jet selection effects