Anomalies and θ terms on the lattice

Hersh Singh

InQubator for Quantum Simulation (IQuS)
& Institute for Nuclear Theory
Department of Physics, University of Washington

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Stephan Caspar, HS [Phys.Rev.Lett. 129 (2022) 2, 022003]
Mendel Nguyen, HS [Phys.Rev.D 107 (2023) 1, 014507]

Why study lattice regularizations?

- rigorous definition of a QFT beyond perturbation theory
- novel insights into continuum subtleties
- compute nonperturbative observables in a controlled way

Why study **new** lattice regularizations?

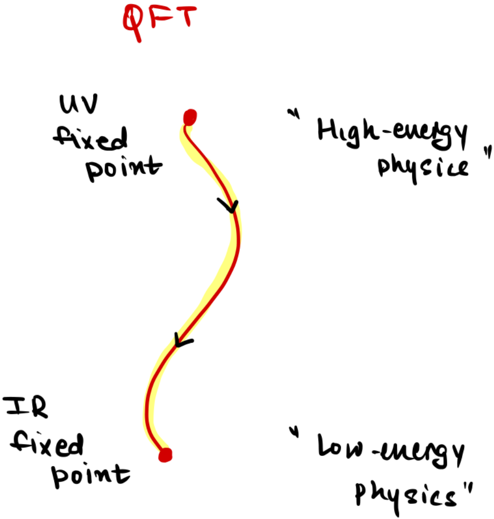
- solve sign problems
- theoretical insights
- use new techniques and platforms (tensor networks, quantum computers)

“Lattice regularization”

- I will use the word “lattice regularization” in a strict manner
- In particular, for a lattice model to be a “lattice regularization of a QFT”, it needs to have a well-defined continuum limit.
- Wilson’s insight: Continuum limit of a QFT emerges near a second-order critical point.

Lattice regularization

A classical-statistical lattice model or a quantum lattice Hamiltonian with a parameter which can be tuned to a second-order critical point.



- A lattice regularization must reproduce the physics of all scales
- Otherwise, it is just a “low-energy EFT”

A toy model of QCD

- **$O(3)$ nonlinear sigma model in 1+1 dimensions**
- Continuum action

$$S[\vec{n}(x)] = \frac{1}{2g^2} \int d^2x \, \partial_\mu \vec{n} \cdot \partial^\mu \vec{n} \tag{1}$$

  with $\vec{n} \in \mathbb{R}^3$ and $|\vec{n}| = 1$.

- g is classically dimensionless coupling
- toy model for QCD: asymptotic freedom, dynamical mass generation, dimensional transmutation, θ-vacua

(3+1)d SU(N) Yang-Mills vs. (1+1)d $O(3)$

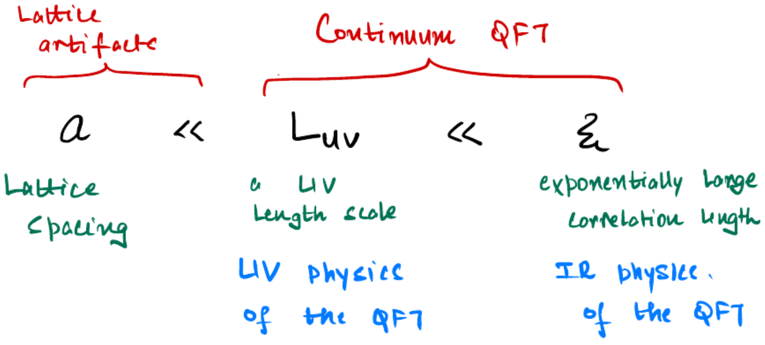
$SU(N)$ YM

- $3+1$-dimensional
- Local gauge symmetry
- Asymptotically free
- Dimensional transmutation
- Nonperturbative mass gap
- Nontrivial topology, θ-term

$O(3)$ NLσM

- $1+1$-dimensional
- Global $O(3)$ symmetry
- Asymptotically free
- Dimensional transmutation
- Nonperturbative mass gap
- Nontrivial topology, θ-term

The challenge of asymptotic freedom



- To get the continuum limit, we need to recover both the IR physics and the UV physics

Traditional lattice regularization

- **$O(3)$ nonlinear sigma model in 1+1 dimensions**
- Lattice regulated action:

$$S = \frac{1}{2g^2} \int d^2x \, \partial_\mu \vec{n} \cdot \partial^\mu \vec{n} \tag{2}$$

$$\Big\downarrow \text{ Naïve discretization}$$

$$S = -\frac{1}{g^2} \sum_{\langle xy \rangle} \vec{n}_x . \vec{n}_y \tag{3}$$

- 2d $O(3)$ NLSM is the continuum QFT which emerges in the $g \to 0$ limit of the lattice model

$O(3)$ NLSM at arbitrary θ

- So far, we have talked about the $O(3)$ NLSM at $\theta = 0$.
- Just like QCD, the $O(3)$ NLSM allows for a topological θ term

$$S_\theta[\vec{\phi}] = \frac{1}{g^2} \int d^2x (\partial_\mu \vec{\phi})^2 + i\theta Q[\vec{\phi}] \tag{4}$$

where

$$Q[\vec{\phi}] = \frac{1}{8\pi} \int d^2x \, \varepsilon_{\mu\nu} \, \vec{\phi} \cdot (\partial^\mu \vec{\phi}) \times (\partial^\nu \vec{\phi}) \tag{5}$$

is the *topological* theta term.

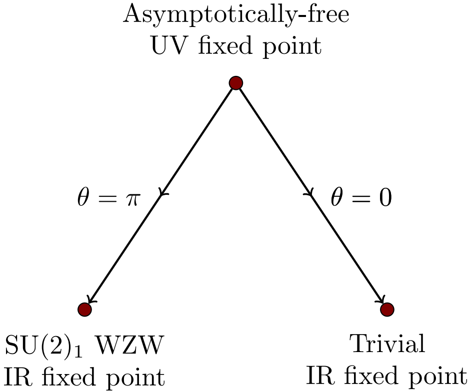
In nature, $\theta < 10^{-10} \implies$ **Strong CP problem**

Physics of θ

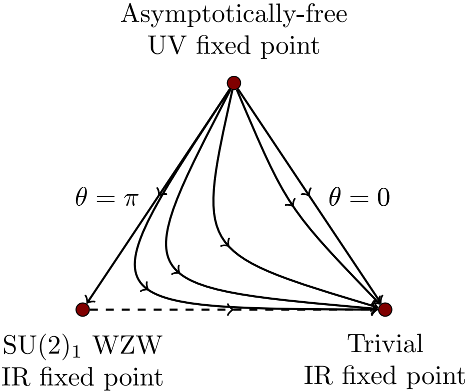
$$S_\theta[\vec{\phi}] = S_0 + i\theta Q[\vec{\phi}]$$

- The physics of θ is totally non-pertubative
- θ does not show up in perturbation theory $\implies$ UV physics unchanged.
  - S_θ is an *asymptotically free* theory for all θ with a non-pertubatively generated energy scale.
- What about the IR physics?
  - θ non-perturbatively changes IR physics
  - At $\theta = \pi$, the low-energy physics is completely different from $\theta = 0$!
  - It is, in fact, massless in the IR $\implies$ flows to the SU$(2)_1$ WZW CFT.
- What happens at arbitrary θ?

RG flow


Asymptotically-free
UV fixed point
$\theta = \pi$
$\theta = 0$
$SU(2)_1$ WZW
IR fixed point
Trivial
IR fixed point

RG flow



Asymptotically-free
UV fixed point
$\theta = \pi$
$\theta = 0$
SU$(2)_1$ WZW
IR fixed point
Trivial
IR fixed point

The theory at $\theta \neq 0, \pi$

- $\theta = 0, \pi$ points are very special due to their integrability [Zamolodchikov et al, 1978; 1992] and we have a good understanding.
- At $\theta \neq 0, \pi$ the situation is not so clear.
- Many attempts to understand the behavior of the theory, but questions still remain...

The problem with θ

- It has been argued that the topological charge has ultraviolet divergences for the $O(3)$ model and is not a physical quantity [Schwab, 1982; Luscher, 1982; Blatter et al, 1996].
- If so, it might happen that θ is an irrelevant parameter (for $\theta \neq \pi$) and simply renormalizes to zero.
- Other studies, by studying the theory about $\theta = \pi$ WZW point, have argued that that in fact there is a critical θ_c below which the theory renormalizes to zero, but is nontrivial for $\theta \geq \theta_c$ [Controzzi, Mussardo, 2003; Venuti et al 2005].
- **Question: Is there a continuum QFT S_θ for each value of θ?**
- We should sort this out clearly. Yang-Mills is even harder...

Lattice formulation

- In the conventional approach, θ introduces a severe sign problem in the naive formulation (imaginary coefficient in Euclidean spacetime)

$$S_\theta[\vec{\phi}] = \frac{1}{g^2}\int d^2x(\partial_\mu\vec{\phi})^2 + i\theta Q[\vec{\phi}] \quad (6)$$

- Actually, the $\theta = \pi$ sign problem can in fact be solved using a meron cluster algorithm [Bietenholz, A. Pochinsky, U.-J. Wiese 1996]
- Bögli, Niedermayer, Pepe, Wiese (2011) studied the θ-vacua using non-standard ("topological") actions:
  - In their approach the sign problem is "mild" for smaller lattices.
  - Concluded that S_θ is unique for each θ.
- It would be good to have a completely sign-problem free way of studying θ vacua.

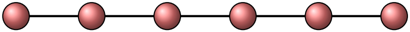
Qubit regularization

Is there another lattice regularization of this model which solves some of these problems?

- As a model of qubits? "qubit regularization"
- We need to obtain
  - UV physics (asymptotic freedom)
  - IR physics (θ-vacua)

Haldane Conjecture

- In 1981, Haldane surprised both condensed matter and high-energy communities
- Consider the antiferromagnetic spin-S Heisenberg chain



$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1} \tag{7}$$

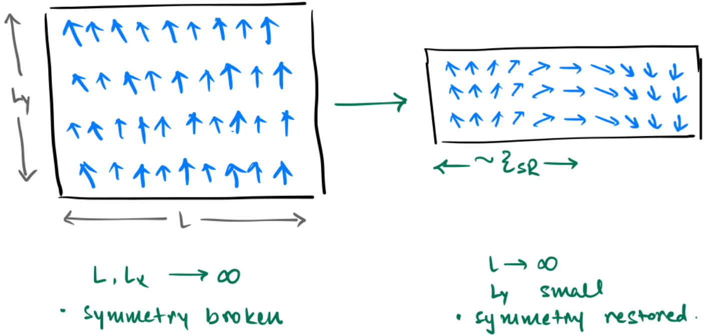
- Haldane Conjecture: at **low energies**

$$\text{Spin-}S \text{ chain} \leftrightarrow \text{O(3) sigma model at } \theta = 2\pi S \tag{8}$$

S=1/2 chain	$\theta = \pi$ NLSM	massless
S=1 chain	$\theta = 0$ NLSM	massive

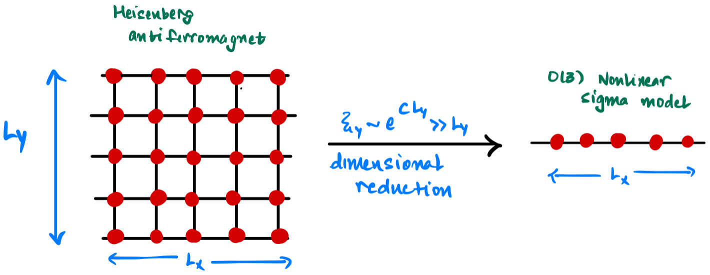
- How do we take the continuum limit (asymptotic freedom)?

UV: asymptotic freedom from dimensional reduction



- Start with a 2+1d lattice. Make L, L_y large $\implies$ Symmetry breaking $SO(3) \to SO(2)$, massless goldstone modes
- What happens as make L_y small? $SO(3)$ symmetry cannot be broken. System orders at length scales ξ_{SR} (*symmetry restoration scale*). Goldstone modes pick up a mass $\sim \xi_{\text{SR}}^{-1}$
- Asymptotic freedom in 1+1d theory ensures that $\xi_{\text{SR}} \sim e^{\# L_y} \gg L_y$. Therefore, the system is effectively (1+1)d.

UV: asymptotic freedom from dimensional reduction



$$H = J \sum_{ij} \vec{S}_i \cdot \vec{S}_j$$

- The continuous fields $\vec{n}$ arise from collective Goldstone mode excitations of the spin-1/2 variables $\vec{S}_i$
- Dimensional reduction back to (1+1)-d theory! [Chandrasekharan, Wiese, 1997]
- Also has been generalized to QCD using quantum link models [Brower et al, 1999]

UV and IR

- This provides a recipe to get the UV physics of asymptotically free theories
- But what about IR? Can we generate a θ term in the IR?

IR: θ term in spin chains

- $\theta \neq 0, \pi$ breaks charge conjugation symmetry $C : \vec{n} \to -\vec{n}$ since $C : i\theta Q \to -i\theta Q$.
- In terms of the spin variables, it can be shown using bosonization [Affleck, 1988]

$$a^{-1}\vec{S}_n = \vec{J}_L + \vec{J}_R + i(-1)^n c(\mathrm{Tr}\, g)\vec{\sigma}. \tag{9}$$

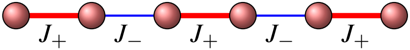
- Note that “charge conjugation” $g \mapsto -g$ maps to translation by one unit $S_n \mapsto S_{n+1}$.
- Manifestation of the antiferromagnetic nature of the spin chain

- Therefore, to generate a θ term in the spin system, we must break this translation-by-one symmetry.

IR: θ term in spin chains

- Therefore, to generate a θ term in the spin system, we must break this translation-by-one symmetry.
- For example, we can stagger the couplings on even and odd bonds

$$J_{\pm} = J(1 \pm \gamma). \tag{10}$$



- For this case, [Haldane, Affleck]

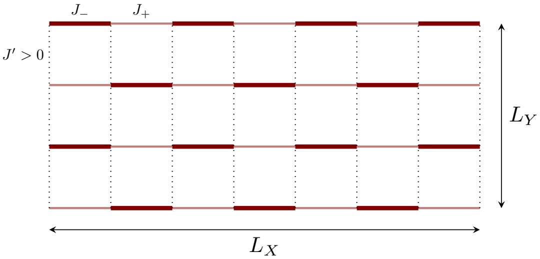
$$\theta = 2\pi S(1 + \gamma). \tag{11}$$

- Can be generalized to spin ladders [Sierra, 1996; Sierra et al, 1997]

Taking the continuum limit with θ term

- We can finally put the two pieces of the puzzle together
  - UV = Asymptotic freedom $\implies$ Dimensional reduction
  - IR = topological θ term $\implies$ C breaking using staggered couplings
- Therefore, we can now take the *continuum limit* of these models at non-trivial θ!

θ-term with D-theory



- *Proposal: Continuum limit of the $O(3)$ NLSM with θ term obtained in the $L_X \gg L_Y \gg 1$ limit*
- Analysis of spin ladders [1] suggests, for $J_\pm = J(1 \pm \gamma)$,

$$\theta \approx 2\pi S L_Y (1 + c\gamma) \implies |\theta - \pi| = c\pi\gamma L_Y \text{ (odd } L_y\text{)} \tag{12}$$

- **A gift:** *no sign problem!* So we can actually numerically check this.

[1] Sierra 1995; Martin-Delgado, Shankar, Sierra 1996

Probing the continuum limit for asymptotically free theories

- To probe the universal behavior of the continuum limit, we can use the **step scaling function** as a convenient tool [Luscher, Weisz, Wolff, 1991]
- Put the asymptotically free theory in a box of size L (natural length scale)
- Define a dimensionless renormalized coupling $\bar{g}^2(L)$
    - For example, we can choose $\bar{g}^2(L) = M(L)L$, where $M(L)$ is the finite-volume mass gap
- All dimensionless observables depend only on the renormalized coupling $\bar{g}^2(L)$.

Step scaling function

- We will look at the universal function $F(z)$ defined by

$$\frac{\xi(\beta,2L)}{\xi(\beta,L)} \equiv F\left(\xi(\beta,L)/L\right) \tag{13}$$

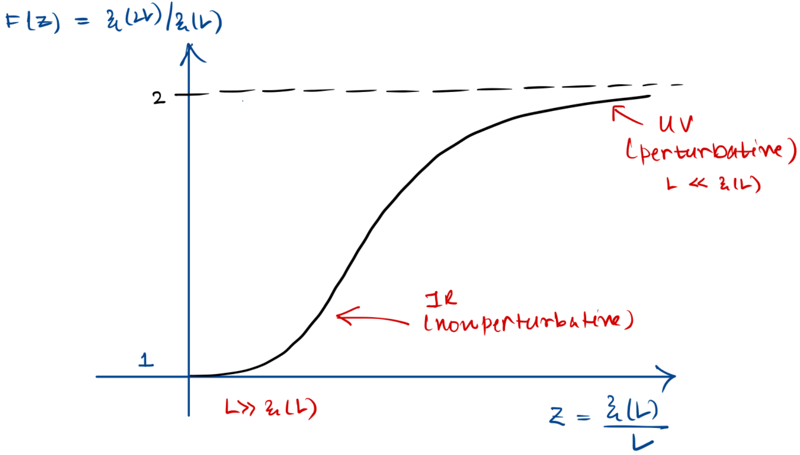
  where β is a bare coupling and $z = \xi(\beta,L)/L$ is the renormalized coupling

- $\xi(\beta,L)$ is a definition of finite-volume correlation length: the "second-moment" correlation length

$$\xi(L) = \frac{\sqrt{\tilde{G}(0)/\tilde{G}(2\pi/L) - 1}}{2\sin(\pi/L)} \tag{14}$$

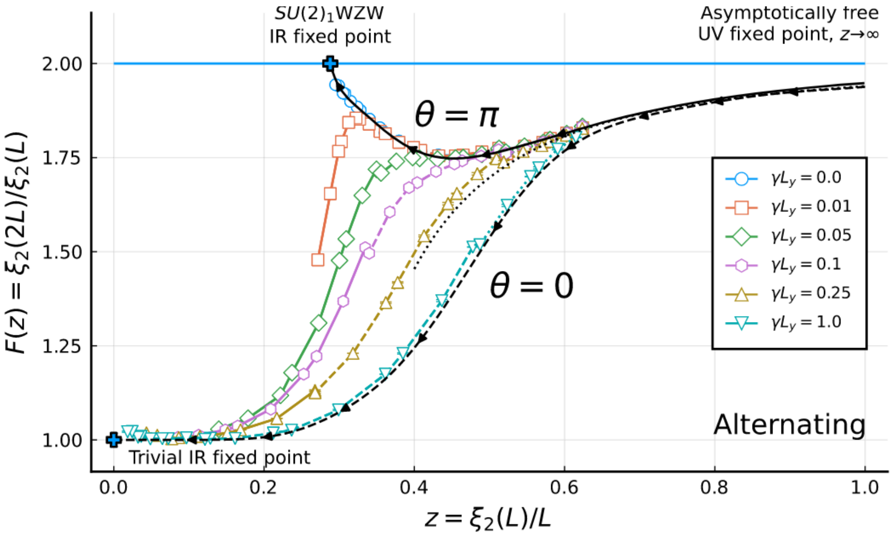
- Easy to measure

Step scaling function: qualitative behavior

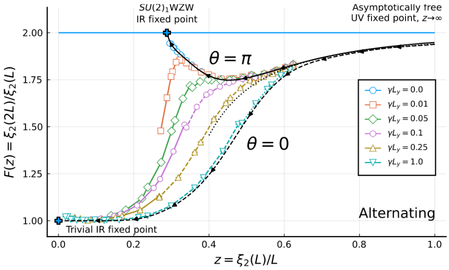


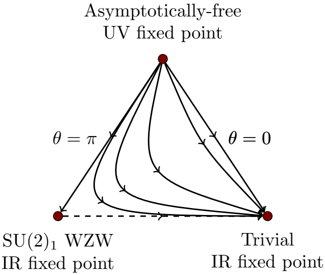
$$z = \xi(L)/L, \quad F(z) = \xi(2L)/\xi(L) \tag{15}$$

Step-scaling function and the RG flow



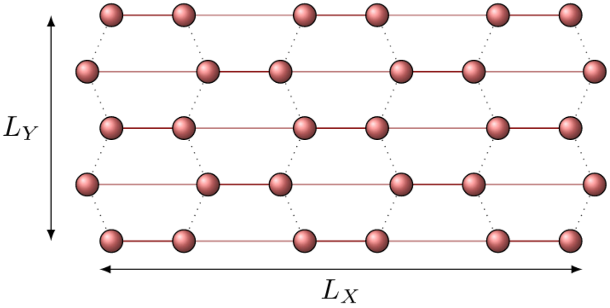
Step-scaling function and the RG flow





The step-scaling curves mimic the expected RG flow diagram beautifully!

On quantum simulators



- On Rydberg systems with native Ising-type interactions, we can use Floquet engineering techniques to implement Heisenberg interactions [2]

[2] arXiv: 2207.09438 [Ciavarella, Caspar, HS, Savage, Lougovski, 2022]

Summary (so far)

- The 2d $O(3)$ NLSM allows for a θ term, just like QCD.
- However, physics of θ is non-perturbative and therefore hard to study – both analytically and on the lattice (sign problem)
- We constructed a lattice regularization using "qubits" for the $O(3)$ NLSM with a θ term
    - Completely **solves the sign problem** present in conventional approaches for the θ term, for the first time.
    - Allowed us to take the **continuum limit** and demonstrate **asymptotic freedom** for various θ
    - Step-scaling curves give a quantitative instantiation of the RG flow
    - Very natural for quantum simulators with qubit degrees of freedom
- Opens up many paths forward...
    - systematic understanding of the RG flow as a function of θ, comparison with analytical results from instanton calculations, ...

- We saw that there is a lattice regularization of the θ term where θ appears as the staggering of couplings

$$\text{Staggering } \gamma \longleftrightarrow \theta \text{ term} \tag{16}$$

- But: why does such a regularization exist? Did we simply get lucky?
- Is there a way to systematically explore this space of lattice regularizations?

- An interesting perspective comes from symmetries and **anomalies**

Lattice regularizations of θ vacua: Anomalies and qubit models

Mendel Nguyen*
Department of Physics, North Carolina State University, Raleigh, North Carolina 27607, USA

Hersh Singh†
InQubator for Quantum Simulation (IQuS), Department of Physics,
University of Washington, Seattle, Washington 98195-1550, USA and
Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195-1550, USA

[Phys.Rev.D 107 (2023) 1, 014507]

Symmetries and Anomalies

- It is clear that symmetries play a huge role in constructing lattice regulators.
  - It is ideal if the lattice regulator explicitly preserves a symmetry of the continuum theory
- However, some symmetries have a subtle structure, which we call an **anomaly**

’t Hooft Anomalies

- The word *anomaly* has many meanings..
- For us, anomaly = **’t Hooft anomaly**

“’t Hooft anomaly”

G is a genuine global symmetry of the theory, but it cannot be gauged.

“Mixed ’t Hooft anomaly”

G_1, G_2 are genuine global symmetries. They can be gauged individually, but gauging one breaks the other.

Classic example of a mixed ’t Hooft anomaly

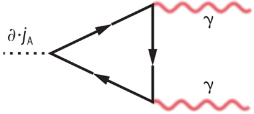
$U(1) \times U(1)_A$ for a free Dirac fermion

- A *free* Dirac fermion has exact global (vector) $U(1)$ and (chiral) $U(1)_A$ symmetries

$$\psi \xmapsto{U(1)} e^{i\theta}\psi \tag{17}$$

$$\psi \xmapsto{U(1)_A} e^{i\theta\gamma_5}\psi \tag{18}$$

- However, *if you gauge* $U(1)$*, then you lose* $U(1)_A$!



- In other words, $U(1)$ and $U(1)_A$ cannot be gauged simultaneously.

A mixed anomaly in the sigma models

- At $\theta = 0, \pi$, the $O(3)$ model has exact global SO(3) and charge conjugation C symmetries

$$\vec{\phi} \xmapsto{SO(3)} e^{i\theta \hat{n}\cdot\vec{J}}\vec{\phi} \tag{19}$$

$$\vec{\phi} \xmapsto{C} -\vec{\phi} \tag{20}$$

(21)

- This is a mixed 't Hooft anomaly at $\theta = \pi$ between SO(3) and C[3]
- If you gauge SO(3), then you lose C!

$$\langle \mathcal{O} \rangle \xmapsto{C} e^{i\,\text{Anomaly}} \langle \mathcal{O} \rangle \tag{22}$$

[3] [Gaiotto, Kapustin, Komargodski, Seiberg, 2017]

Anomalies and lattice regularizations?

- We are interested in constructing new lattice regularizations
- The presence of an ’t Hooft anomaly for G must be reflected in a lattice regularization.

Gauging a symmetry on the lattice

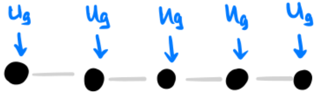
- Assume a lattice regulator for a QFT with some global symmetry G, with
  1. (1) same spacetime dimensionality
  2. (2) exact symmetry on the lattice
  3. (3) locality
  4. (4) symmetry implemented **"onsite"**

- Alternate possibilities (which we do not consider)
  - (1) can be violated: boundary of SPT bulk
    - example: domain wall fermions for chiral symmetry
  - (2) can be violated: emergent symmetry at low-energies

Onsite symmetries

A symmetry is onsite if its action factorizes over the local Hilbert spaces

$$
\begin{aligned}
|\psi\rangle &= |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_N\rangle \\
&\Big\downarrow g \in G \\
U|\psi\rangle &= U_g|\phi_1\rangle \otimes U_g|\phi_2\rangle \otimes \cdots \otimes U_g|\phi_N\rangle
\end{aligned}
$$



- Onsite symmetry $\implies$ we can gauge it on the lattice by introducing link variables

- But if there is an anomaly, there must be some obstruction to this procedure!

QFT Lore

"There are no anomalies on the lattice"

- Example: attempts to put chiral fermions on the lattice result in doublers [Nielsen, Ninomiya]

Obstructions to gauging on the lattice

- A way in which it is impossible to gauge a symmetry is that the symmetry is **not** onsite
  - Well appreciated in cond-mat [4]
- If the symmetry is offsite, there is no obvious way to gauge it on the lattice
- Indeed, in the spin-chain regularization, the charge conjugation symmetry was offsite

$$C : \vec{S}_i \mapsto \vec{S}_{i+1} \tag{23}$$

- The spin-1/2 chain naturally realizes the $\theta = \pi$ model, which has a SO$(3) \times$ C anomaly
- Now, we see that the offsite-ness of the symmetry was no accident – it is *almost* forced by the anomaly!

[4] For example: Jian, Bi, Xu (2018); Cho, Hsieh, Ruy (2017)

Obstructions to gauging on the lattice

- But what if we *insist* that an anomalous symmetry is onsite on the lattice?
- Consider the **standard lattice action** at $\theta = 0$

$$S_0 = -\frac{1}{g^2} \sum_{\langle xy \rangle} \vec{n}_x \cdot \vec{n}_y \tag{24}$$

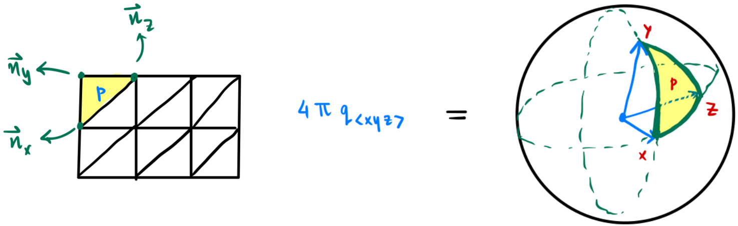
- Both SO(3) and C are **onsite**
- Indeed, a topological θ term on the lattice was defined by [Berg, Lüscher, 1981] which maintains this property

A topological definition of the θ term on the lattice

- Topological θ term on the lattice [Berg, Lüscher, 1981]

$$S[\vec{n}] = S_0[\vec{n}] + i\theta Q[\vec{n}] \quad (25)$$

$$Q[\vec{n}] = \sum_{\langle xyz \rangle} q_{\langle xyz \rangle} \quad (26)$$



- The local topological charge density $4\pi q_{\langle xyz \rangle}$ is just the area of the spherical triangle formed by $\vec{n}_x, \vec{n}_y, \vec{n}_z$.

Obstruction?

- So if the symmetry is both exact and onsite on the lattice, where is the obstruction? What prevents us from gauging it on the lattice?
- We find that this rather well-known lattice model **explicitly** reproduces the exact anomaly on the lattice[5]!

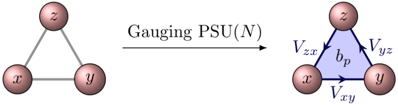
[5] [Nguyen, HS, Phys.Rev.D 107 (2023) 1, 014507]

How to detect the anomaly?

SO(3) $\times$ C anomaly

- We turn on a background gauge field (A, B) for the SO(3) symmetry and then perform a C transformation

$$Z \xmapsto{\text{Gauging PSU}(N)} \tilde{Z}[A, B] \tag{27}$$



- We then check whether Z is invariant under C.

$$\text{C}: \begin{cases} \tilde{Z}[A, B, \theta = 0] \mapsto \tilde{Z}[A, B, \theta = 0], \\ \tilde{Z}[A, B, \theta = \pi] \mapsto \tilde{Z}[A, B, \theta = \pi] \underbrace{e^{-ik\int b}}_{\text{anomaly}} \end{cases} \tag{28}$$

- Analogy: Chiral anomaly $\implies$ we turn on background field for $U(1)$ and find that the partition function is not invariant under $U(1)_\chi$

“~~No~~ anomalies on the lattice”

- The lore “no anomalies on the lattice” is incomplete
- Indeed there can be anomalies on the lattice, much like the continuum formulations
- Other examples:
  - Kahler-Dirac fermions [Catterall, 2022]
  - Ginsparg-Wilson fermions for chiral anomaly[6] [Lüscher, Neuberger, Narayanan, Kaplan, ...]
  - Modified Villain formulations [Fazza, Sulejmanpasic 2022; Gorantola, Lam, Seiberg, Shao 2021; Sulejmanpasic, Gattringer...] [see also Theo’s talk at this workshop]

[6] for a modified chiral symmetry

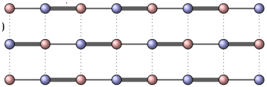
Anomalies and Lattice Regularizations of θ theta vacua

Anomaly

Lattice
symmetric, local, same d

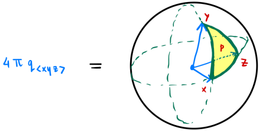
Offsite symmetry

- "qubit regularization"
- Staggered couplings
- No sign problem!
- Natural for quantum computers (finite-dimensional local Hilbert spaces)



Exact anomaly

- Berg–Lüscher θ term
- Manifestly topological
- Sign problems
- ∞-dimensional local Hilbert space



Guidance from anomalies

- These arguments seem general. Do all models with mixed 't Hooft anomalies have such a dichotomy of lattice regularizations?
- Can generalize the O(3) constructions to a wider class of 2d asymptotically free theories, called the Grassmannian nonlinear sigma model.
- Here, instead of S^2, the fields P live on

$$P_x \in \mathsf{Gr}_k(N) = \frac{U(N)}{U(N-k) \times U(k)} \tag{29}$$

with the action

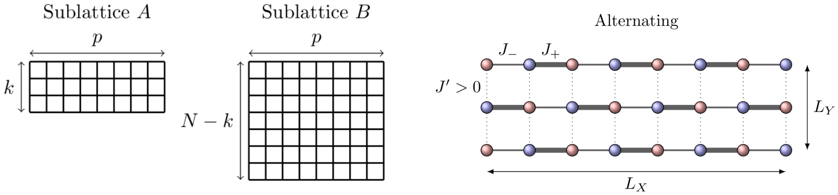
$$S = \frac{1}{g^2} \int d^2x \, \mathrm{Tr}(\partial_\mu P)^2 + \frac{\theta}{4\pi} \int d^2x \, \epsilon^{\mu\nu} \, \mathrm{Tr} \, P \, \partial_\mu P \, \partial_\nu P \tag{30}$$

- These $\mathsf{Gr}_k(N)$ models also have an anomaly at $\theta = \pi$ between PSU(N) and C for (N, k)=(even, odd)[7].

[7] for other cases, we have a more subtle scenario called "global inconsistency"

Lattice regularization for Grassmannian models

- Qubit regularization
  - Now, we have $SU(N)$ spins at each site in certain conjugate representations[8]



  - Again, we can argue that a continuum limit at a fixed θ arises in the $L_y \to \infty$ limit if you keep γL_y fixed.
- Conventional regularization
  - The geometric Berg–Lüscher construction can also be generalized

[8] [Read, Sachdev, 1989]

Summary

- In the quest to find new lattice regulators with unique advantages for quantum/classical simulation and theoretical insights, anomalies can be a guide.
- The importance of anomalies been long appreciated for chiral fermions on the lattice.
- For the $O(3)$ model (and Grassmannian $\mathrm{Gr}_k(N)$ models), we saw a dichotomy of regularizations: qubit and conventional, which reflect how the $\mathrm{SO}(3) \times \mathrm{C}$ anomaly manifests.
- They have quite different advantages! Which one is useful depends on the question.
- There are very suggestive parallels with 4d nonabelian gauge theories
  - Indeed, pure $SU(N)$ Yang-Mills has a very similar anomaly at $\theta = \pi$, between time reversal and $\mathbb{Z}_N$ center symmetry [9]
  - What does this say for lattice/qubit regularizations of QCD?

[9] [Gaiotto et al, 2017]

Outlook

- We have demonstrated that a lattice regularization of the $O(3)$ NLSM with arbitrary θ can be constructed using qubits
  - solved a sign problem along the way
- Lattice regularizations where anomalies are manifested differently seem to have quite different properties! What does this imply for lattice QCD?
  - Anomalies can indeed be present on the lattice, going against an old lore. Implications?
  - What about chiral fermions?
- The space of such non-traditional formulations of lattice QFTs is quite rich and important for near-term quantum computers

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