Reformulation of the pionless effective field theory for practical few- and many-body application

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Motivation

Universal corrections to the superfluid gap in a cold Fermi gas

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Gapless neutron superfluidity in the crust of the accreting neutron stars KS 1731−260 and MXB 1659−29

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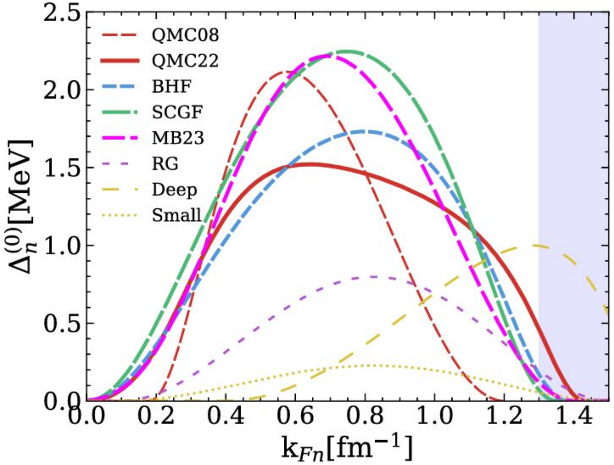

Variational and parquet-diagram calculations for neutron matter. III. *S*-wave pairing

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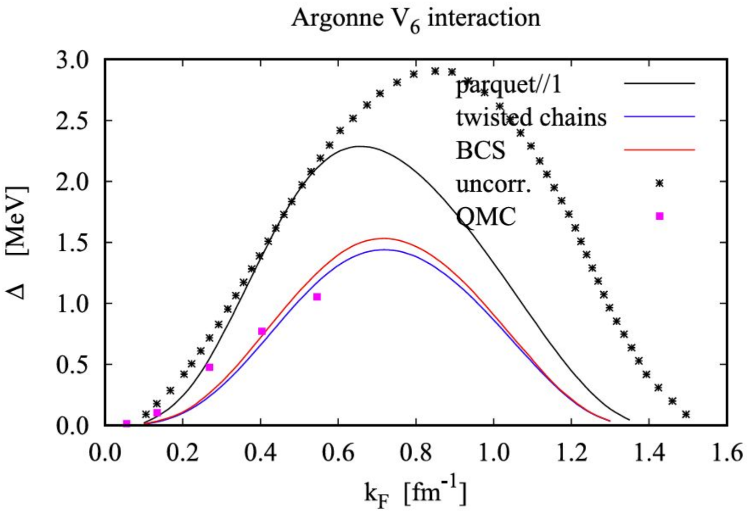
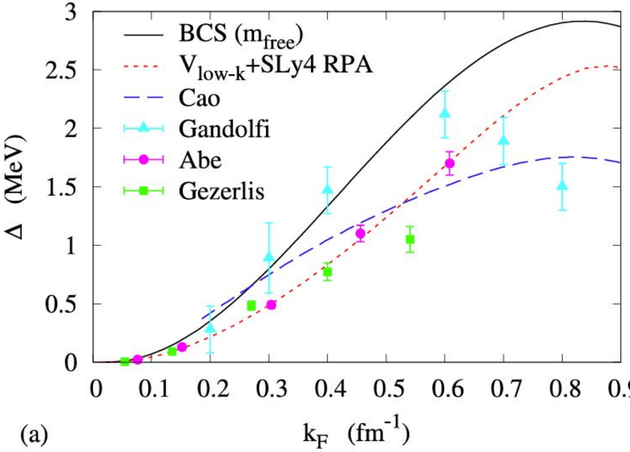
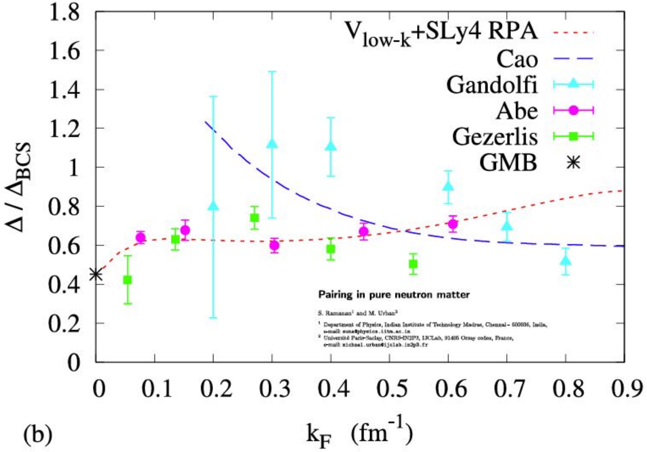




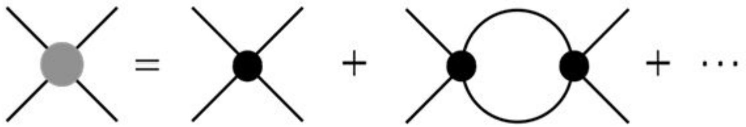


Free Space EFT

$$\mathcal{L} = \sum_{\sigma=\uparrow,\downarrow} \left[\psi_\sigma^\dagger \left(i\hbar\partial_t + \frac{\hbar^2 \vec{\nabla}^2}{2M} \right) \psi_\sigma - \frac{1}{2} g \left(\psi_\sigma^\dagger \psi_\sigma \right)^2 \right]$$

In free space, this generates a scattering amplitude

$$T(k) = -\frac{4\pi}{M} \left[-\frac{1}{a} - ik \right]^{-1} \qquad g \overset{\overline{MS}}{=} \frac{4\pi a}{M}$$

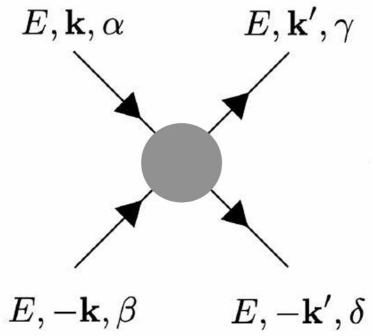


In pionless EFT, $g \sim \frac{4\pi}{M\aleph} \sim Q^{-1}$ and loops $\sim Q \rightarrow$ Loops must be summed to all orders!

In-Medium

Working at T=0, consider the in-medium four point function:

$$i\Gamma_{\alpha\beta,\gamma\delta}(\mathbf{k},\mathbf{k}';2E) =$$



An attractive interaction in the presence of the Fermi surface will pair

$$\boxed{\left[\Gamma\left(\mathbf{k},\mathbf{k}';\Re[2E]+i\Delta\right)\right]^{-1}=0}$$

Expansion for the Gap

$$\log\left(\frac{\Delta}{\omega_{k_F}}\right) = \frac{c_{-1}}{\lambda} + c_0 + c_1\lambda + \ldots \qquad \lambda = \frac{2k_F a}{\pi}$$

$$c_{-1} = 1 \longrightarrow \Delta_{\text{BCS}} = \frac{8}{e^2}\omega_{k_F}\exp\left(\frac{\pi}{2k_F a}\right)$$

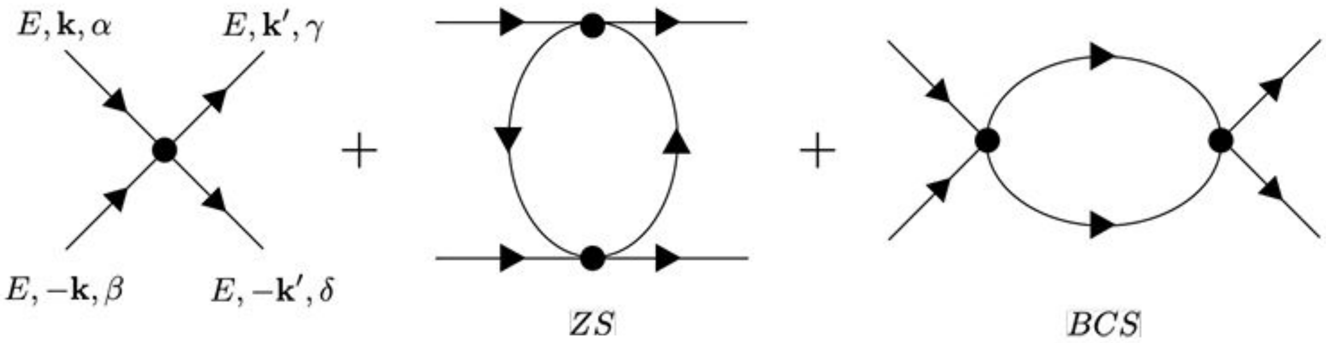
$$c_0 = \frac{7}{3}\left(\log 2 - 1\right) \longrightarrow \Delta_{\text{GM}} = \frac{1}{(4e)^{1/3}}\Delta_{\text{BCS}}$$

$$c_1 = 0.47619(20) \longrightarrow \Delta = \Delta_{\text{GM}}\left(1 + \frac{0.95238(40)}{\pi}k_F a\right)$$

Power Counting with the BCS Singularity

$$iG_0(k_0,\mathbf{k})\delta_{\gamma\alpha} = i\delta_{\gamma\alpha}\left(\frac{i}{k_0-\omega_k+i\epsilon} - 2\pi\delta(k_0-\omega_k)\theta(k_F-k)\right)$$

$$i\Gamma_{\alpha\beta,\gamma\delta}(\mathbf{k},\mathbf{k}';2E) \quad =$$

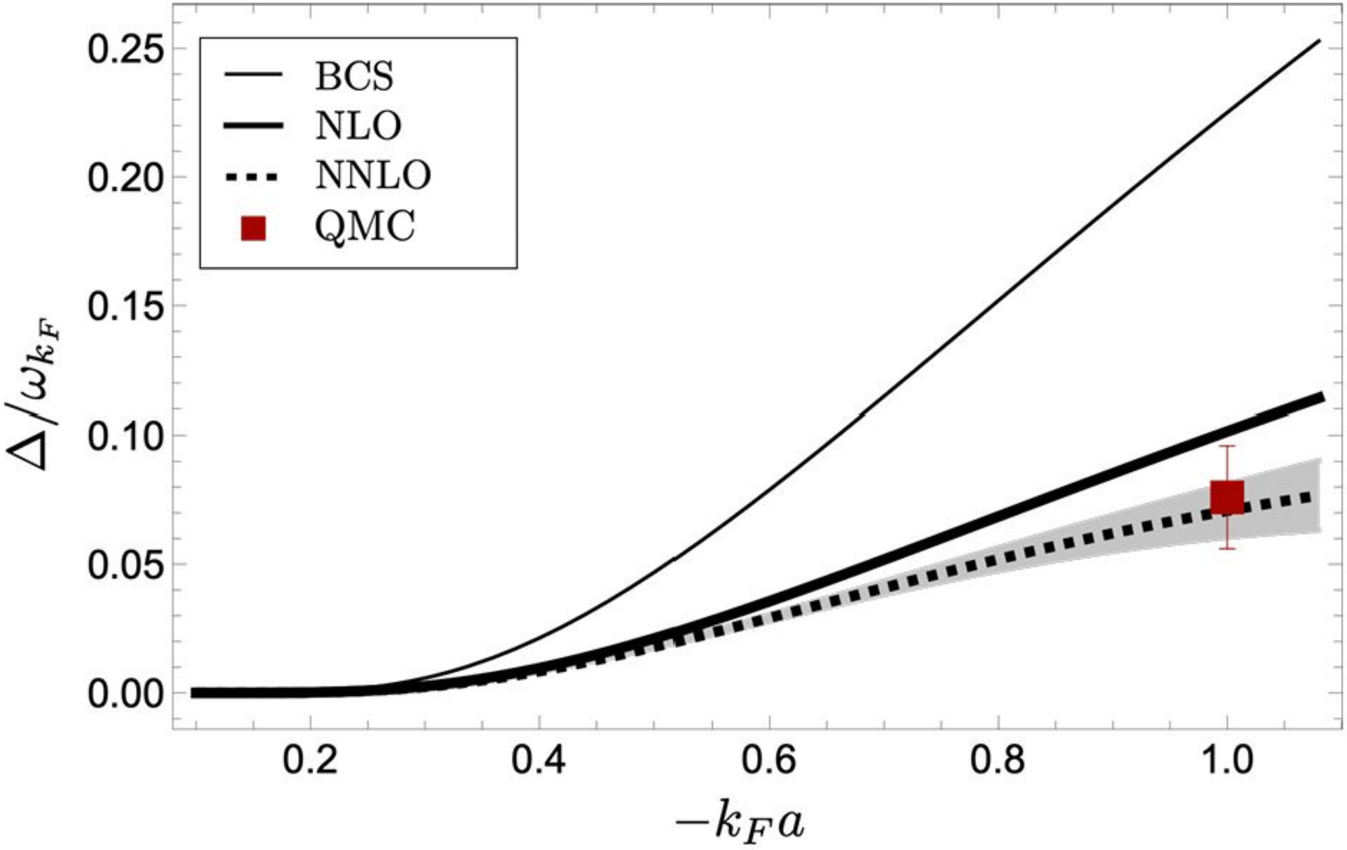


$g \qquad g^2\Pi_{ph} \qquad g^2\Pi_{pp}$

Π_{pp} is enhanced by the BCS kinematical singularity $\quad \Pi_{pp} \sim \dfrac{1}{g}$

Must sum loops to all orders for arbitrarily weak g. Only have control when $\quad k_F a < 1$

The Gap Results



Looking Forward

Want to compute gap at phenomenologically interesting densities.

- Current determination limited to $k_F \lesssim 10 \text{ MeV}$
- Nuclear saturation density $k_F \sim 300 \text{ MeV}$

We aim to extend the radius of convergence of this expansion using the pionless EFT. Two major obstacles:

- The unnaturally large a breaks the perturbative expansion.
- Adding effective range corrections will make the potential less perturbative.

Effective Range Corrections in Pionless EFT

$$V(p,p') = C_0 + \frac{C_2}{2}(p^2 + p'^2)$$

Add subleading terms to V to soften it.

$$V(p,p') = g\mathcal{G}(p)\mathcal{G}(p')$$

Pick V so that it generates the appropriate scattering amplitude up to subleading contributions. (Infinite number of choices.)

Is nonlocality ok?

$$C_0 \to C_0 \exp(-p^2/\Lambda^2) \exp(-p'^2/\Lambda^2)$$

Effective Range Corrections in Pionless EFT

$$V(p, p') = g\mathcal{G}(p)\mathcal{G}(p') \qquad \mathcal{G}(p) = \frac{1}{\sqrt{\gamma_1^2 + p^2}}$$

This potential generates the effective range to all orders:

$$T(k) = -\frac{4\pi}{M}\left[-\frac{1}{a} + \frac{1}{2}rk^2 - ik\right]^{-1}$$

In terms of the S-matrix poles $k = i\gamma_1, i\gamma_2$,

$$g = -\frac{4\pi}{M}(\gamma_1 + \gamma_2) \qquad a = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} \qquad r = \frac{2}{\gamma_1 + \gamma_2}$$

$$\gamma_1 = 157.93 \text{ MeV} \quad , \quad \gamma_2 = -7.93 \text{ MeV}$$

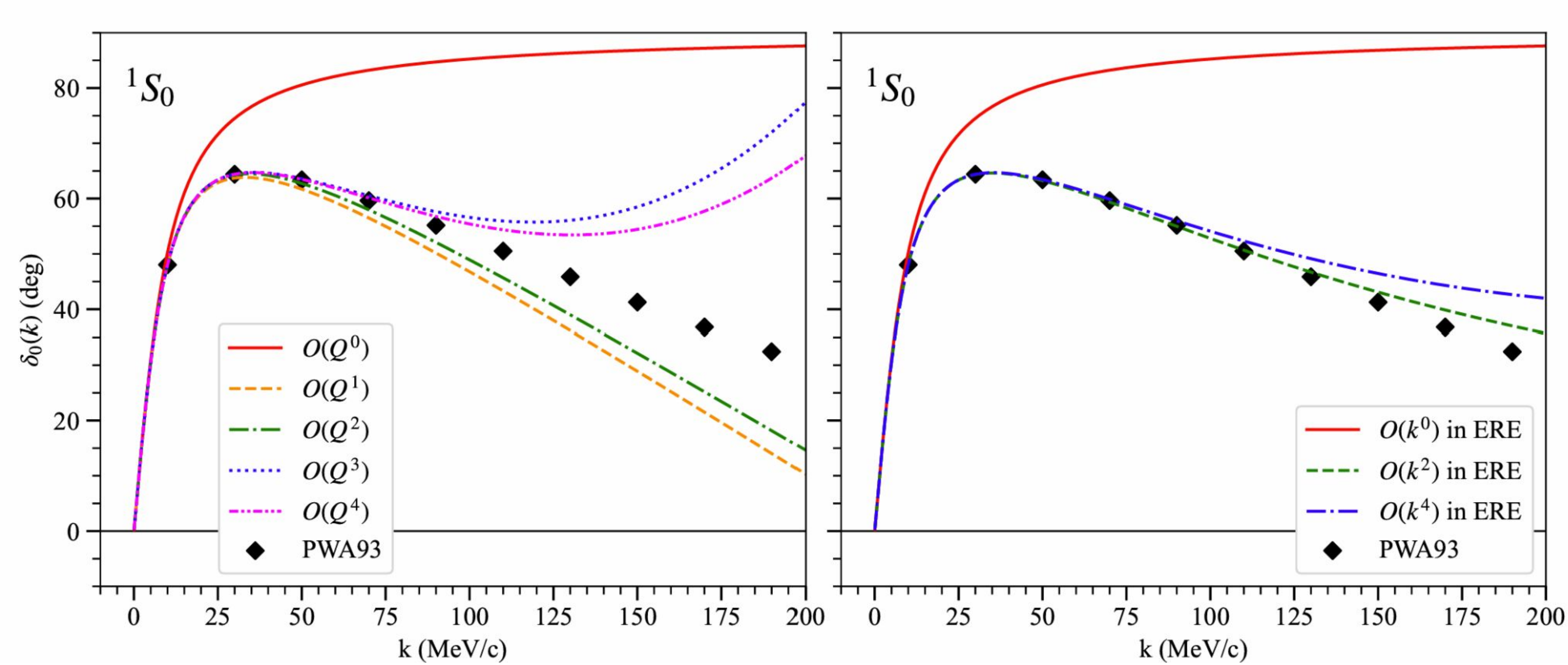
Shape Parameter Corrections in Pionless EFT

$$\mathcal{G}(p) = \prod_{a=1}^{n} \frac{1}{\sqrt{\gamma_a^2 + p^2}}$$

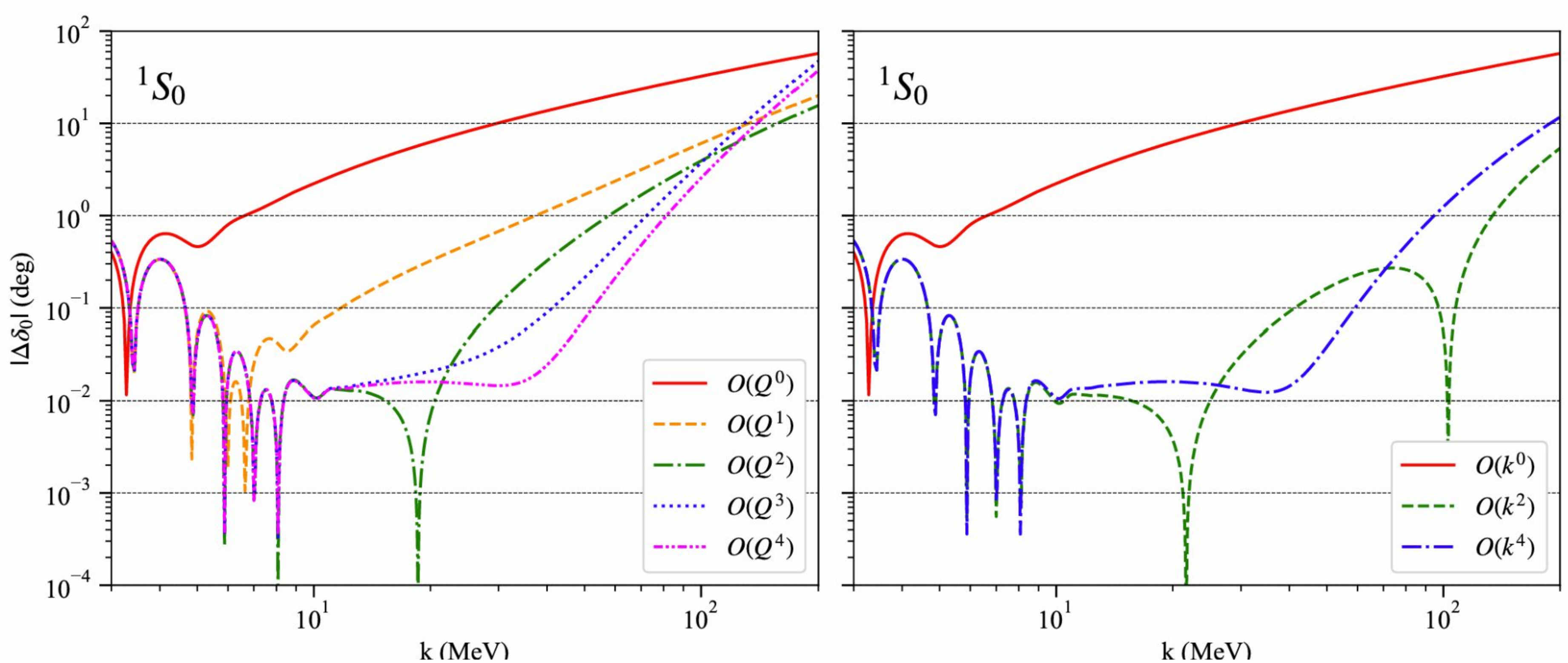
$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + \ldots + v_n k^{2n}$$

This can be extended to higher partial waves, the mixed channel, etc.

Performance in np-scattering



Performance in np-scattering



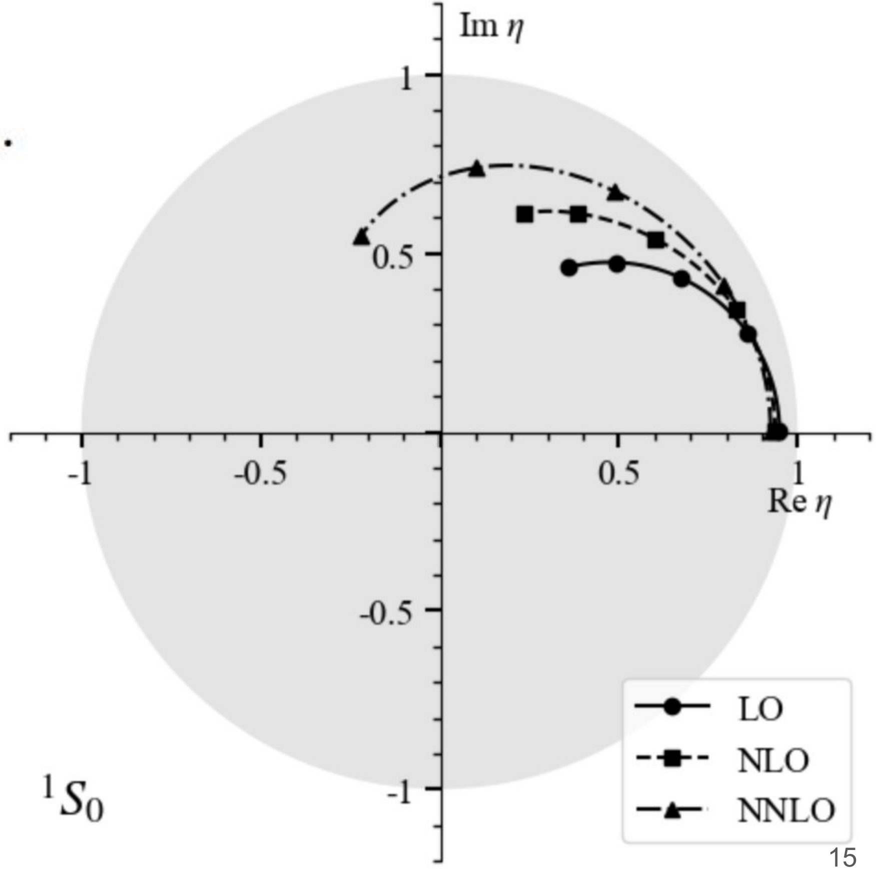
Perturbativeness

$$T = V + VG_0V + VG_0VG_0V + \dots$$

The convergence of the Born series can be quantified with Weinberg eigenvalues

$$\hat{V}\hat{G}_0(z)\,|\Psi_\nu(z)\rangle \;=\; \eta_\nu(z)\,|\Psi_\nu(z)\rangle$$

For the contact interaction $\eta = -ika$



$^1S_0$

The Path Forward

$$g \longrightarrow \; V(\mathbf{k}, \mathbf{k}')$$