non-linear Chiral Magnetic Waves in Schwinger model

Shuzhe Shi（施舒哲）

Tsinghua University

in collaboration with

arXiv: 2305.05685

Kazuki Ikeda, Dmitri Kharzeev

outline

- introduction
- model set up for quantum simulation
- quantum, non-linear CMW
- summary and outlook

introduction: CMW

PHYSICAL REVIEW D **83,** 085007 (2011)

Chiral magnetic wave

Dmitri E. Kharzeev[1,2,*] and Ho-Ung Yee[1,†]

$$\boldsymbol{J}_V^{\mathrm{CME}} = \frac{N_c e}{2\pi^2}\mu_A \boldsymbol{B}$$

$$\boldsymbol{J}_A^{\mathrm{CSE}} = \frac{N_c e}{2\pi^2}\mu_V \boldsymbol{B}$$

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strong B field limit

→

1+1 D

introduction: CMW

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⟶

1+1 D

$$\partial_t J_V^t + \partial_x J_V^x = 0\,,$$

$$\partial_t J_A^t + \partial_x J_A^x = 0\,.$$

(if $m = 0$, $E = 0$)

introduction: CMW

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bosonization ↓

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$\longrightarrow$

1+1 D

$$\partial_t J_V^t + \partial_x J_V^x = 0\,,$$

(if $m \neq 0$)

bosonization $\downarrow$

$$(\partial_t^2 - \partial_z^2 + \frac{g^2}{\pi})\phi + 2c\, m\, g \sin(2\sqrt{\pi}\phi) = 0.$$

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quantum effect?

1+1D massive Schwinger model

$$L = \int \left(-\frac{F^{\mu\nu}F_{\mu\nu}}{4} + \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - g\gamma^{\mu}A_{\mu} - m)\psi \right) \mathrm{d}x \, .$$

Hamiltonian

1+1D massive Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x \, .$$

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E: electric field

A: electric potential

$\psi, \bar{\psi}$: fermion field

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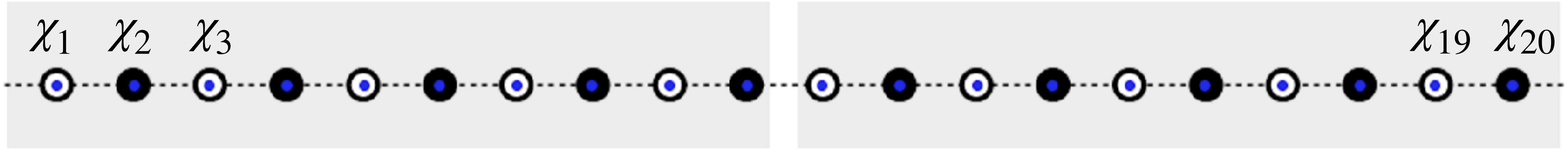
discretize and matrix(gate) representation:

Hamiltonian

1+1D massive Schwinger model



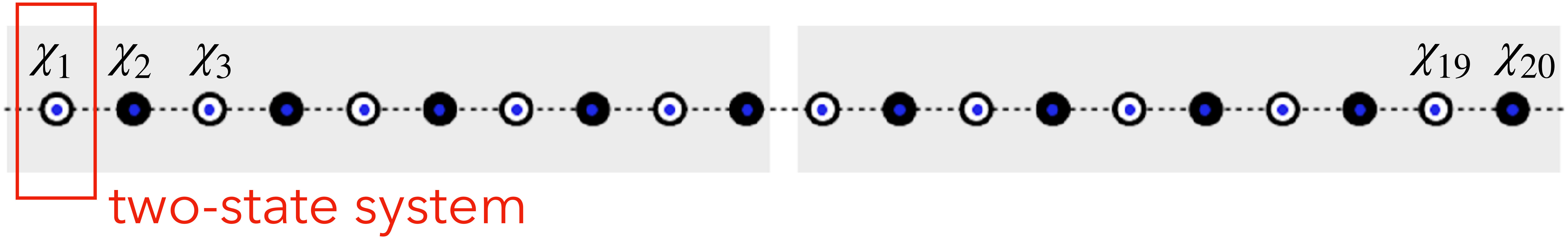
discretize and matrix(gate) representation:

staggered fermion that satisfied anti-commutation: $\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$

$$\psi(x = a\ n) \quad \leftrightarrow \quad \frac{1}{\sqrt{a}}\begin{pmatrix} \chi_{2n} \\ \chi_{2n-1} \end{pmatrix}$$

Kogut-Susskind

1+1D massive Schwinger model



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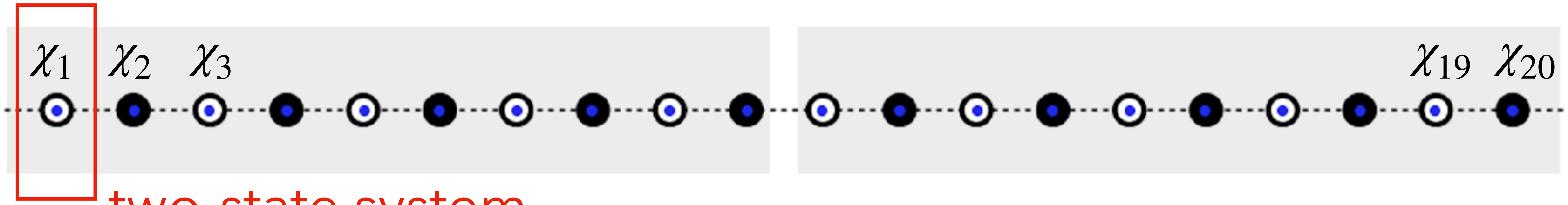

two-state system

discretize and matrix(gate) representation:

Pauli matrices: X, Y, Z

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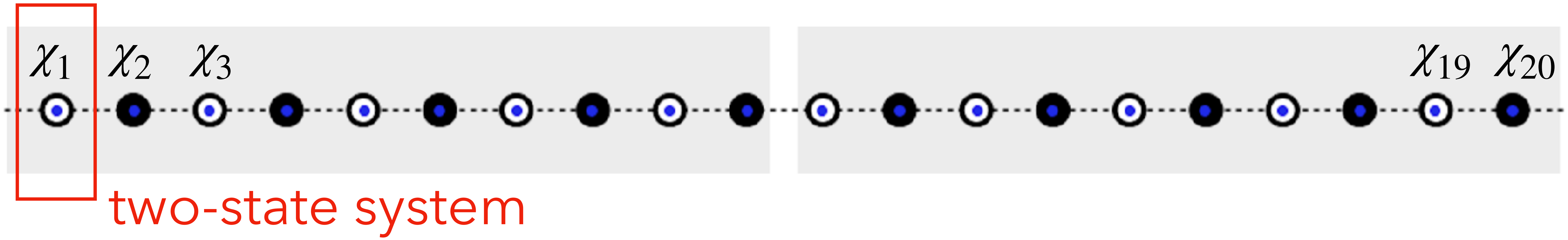
Kogut-Susskind

$$\chi_n = \frac{X_n - iY_n}{2}\prod_{m=1}^{n-1}(-iZ_m)$$

$$X_n \equiv \underset{1^{\text{st}}}{I} \otimes \cdots \otimes \underset{(n-1)^{\text{th}}}{I} \otimes \underset{n^{\text{th}}}{X} \otimes \underset{(n+1)^{\text{th}}}{I} \otimes \cdots \otimes I$$

1+1D massive Schwinger model



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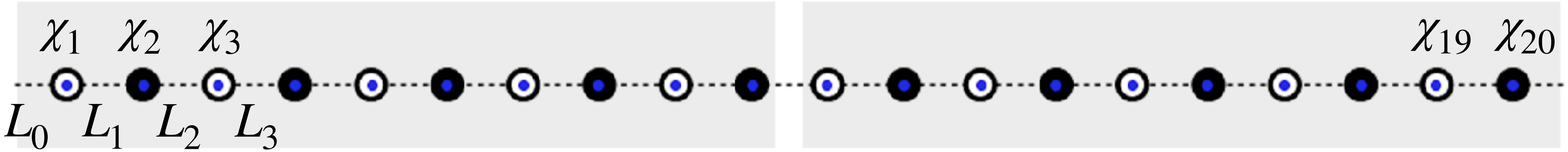
Jordan-Wigner

$$\{\chi_n^\dagger, \chi_m\} = \delta_{nm}\,, \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0\,.$$

Hamiltonian

1+1D massive Schwinger model



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gauge field fixed by Gauss' law: $\partial_1 E - g\,\bar{\psi}\gamma^0\psi = 0$

$E(x = an) \quad \leftrightarrow \quad L_n$

$$L_n - L_{n-1} - \frac{Z_n + (-1)^n}{2} = 0\ ,$$

Hamiltonian

1+1D massive Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x \, .$$

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$$H = \frac{1}{4a} \sum_{n=1}^{N-1} (X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2} \sum_{n=1}^{N} (-1)^n Z_n + \frac{a\,g^2}{2} \sum_{n=1}^{N-1} L_n^2 \, .$$

$$Q_n \equiv \langle \bar{\psi}(a\,n)\gamma^0\psi(a\,n)\rangle = \frac{\langle Z_n\rangle + (-1)^n}{2a},$$

$$Q_{5,n} \equiv \langle \bar{\psi}(a\,n)\gamma^5\gamma^0\psi(a\,n)\rangle = \frac{\langle X_nY_{n+1} - Y_nX_{n+1}\rangle}{4a}$$

real-time evolution

initial state: vacuum + dipole

$$H|0\rangle = E_0|0\rangle$$

$$\langle\psi|Q_n|\psi\rangle_{t=0} = \langle 0|Q_n|0\rangle + D\,(\delta_{n,\frac{N}{2}} - \delta_{n,\frac{N}{2}+1}),$$
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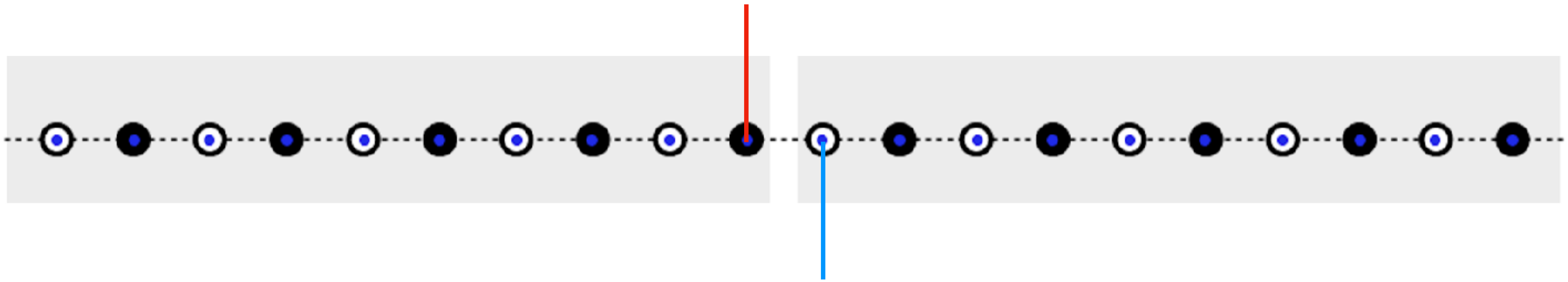

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time-dependent Schroedinger equation:

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\,i\,H|\psi(t)\rangle$$

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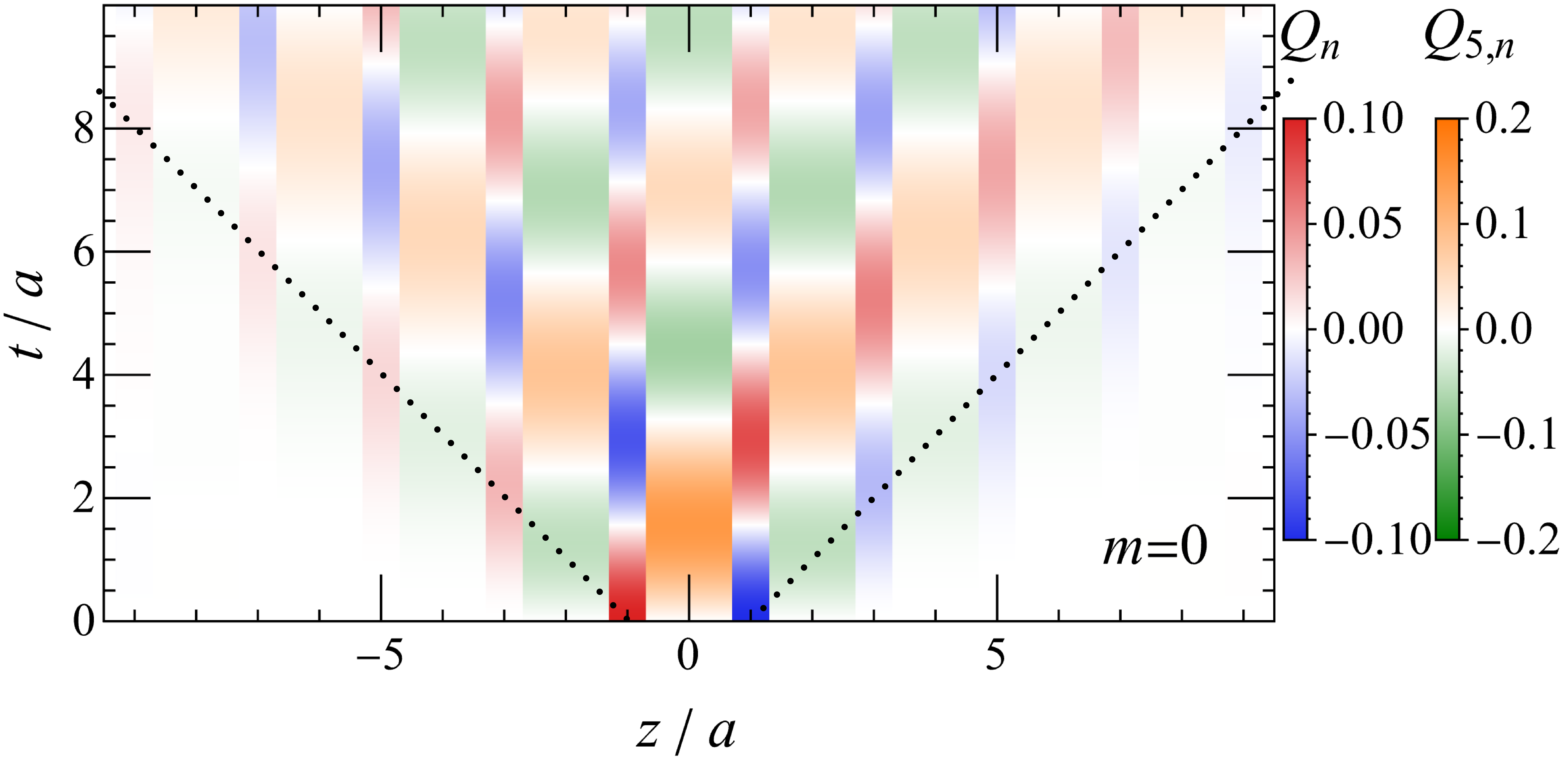
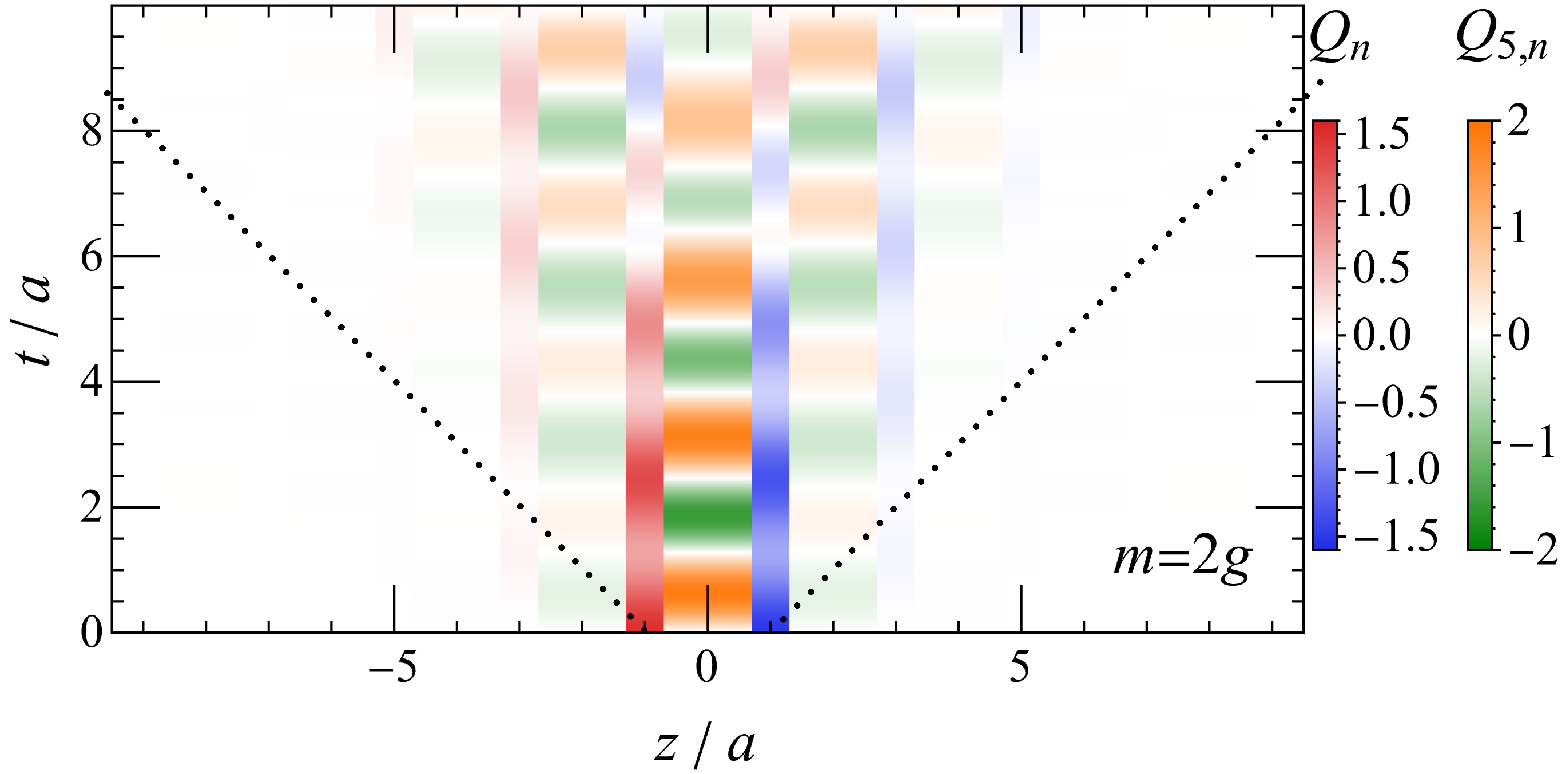
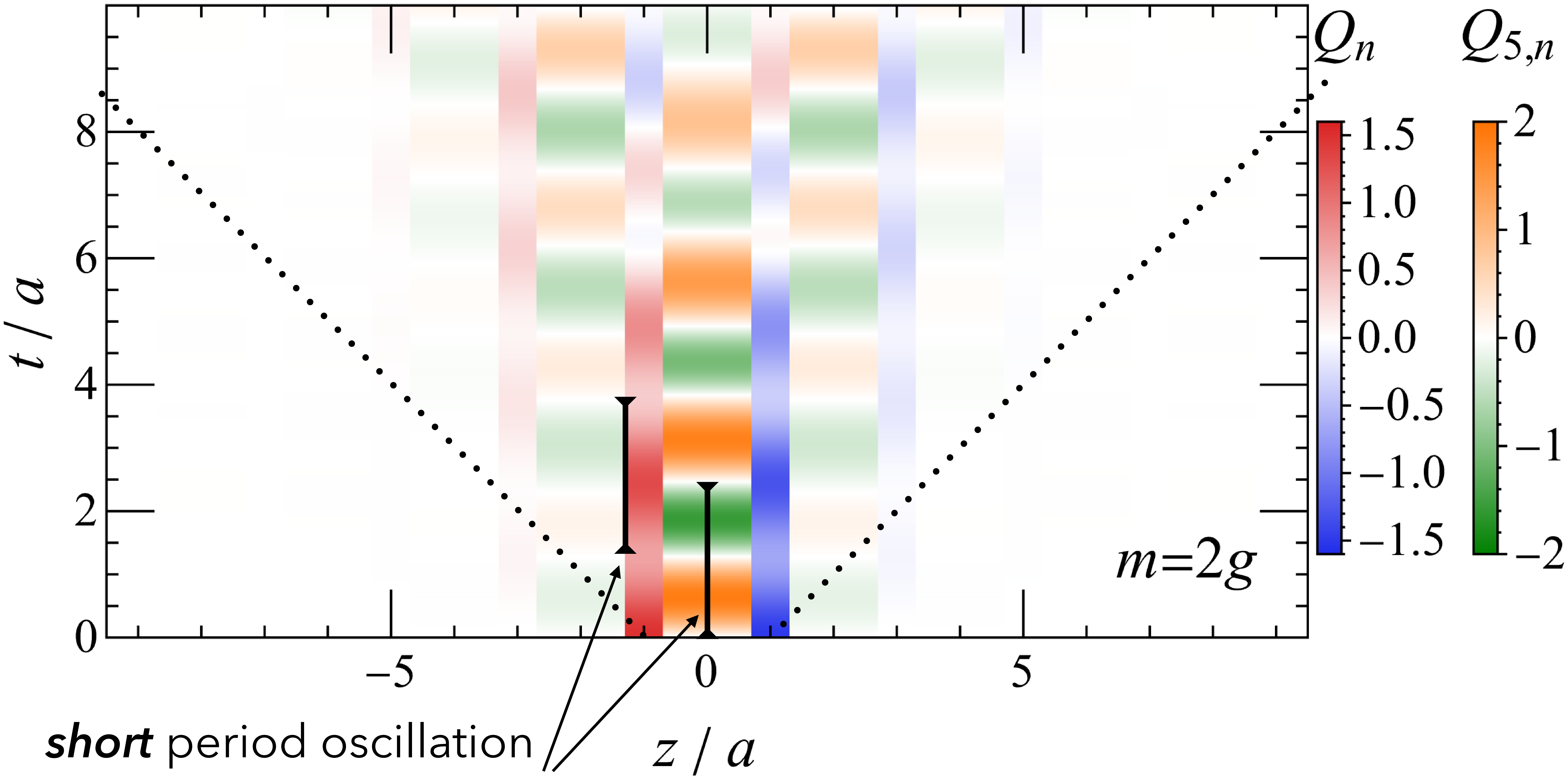
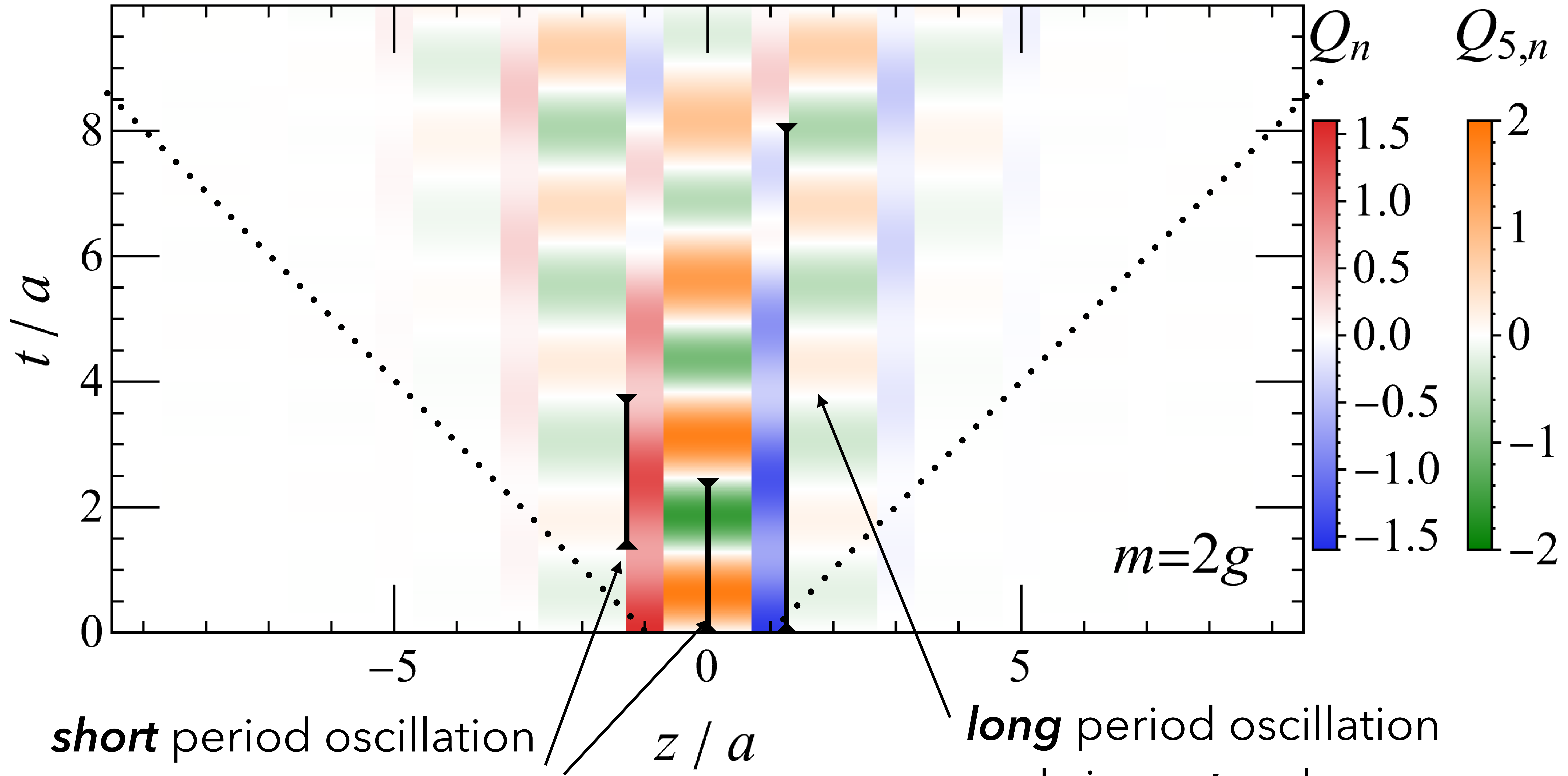

Q_n
$Q_{5,n}$
0.10
0.05
0.00
−0.05
−0.10
0.2
0.1
0.0
−0.1
−0.2
m=0
t / a
z / a
0
2
4
6
8
−5
0
5

real-time evolution





short period oscillation
in both ***vector*** and ***axial*** charge



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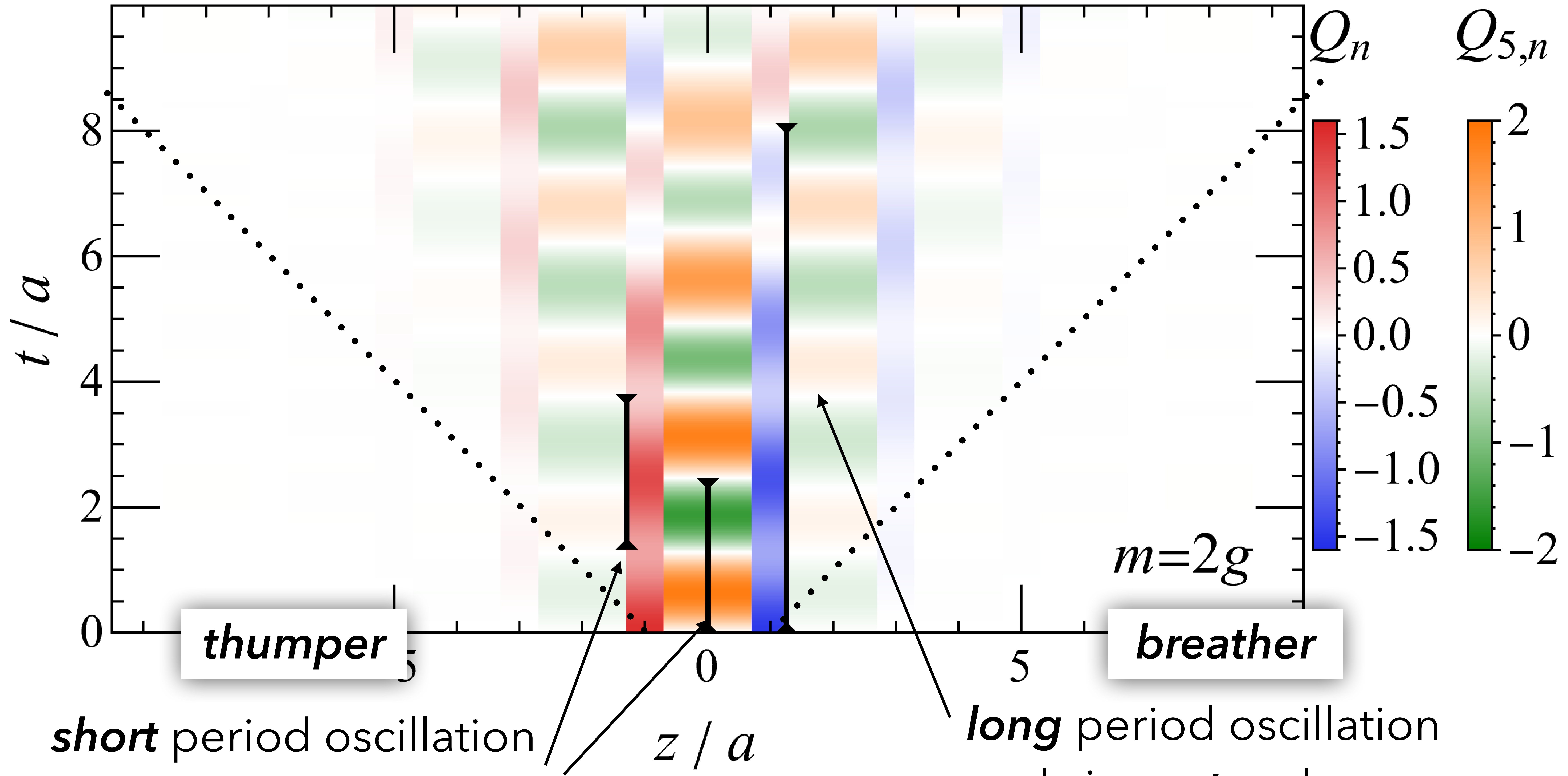
long period oscillation
only in ***vector*** charge



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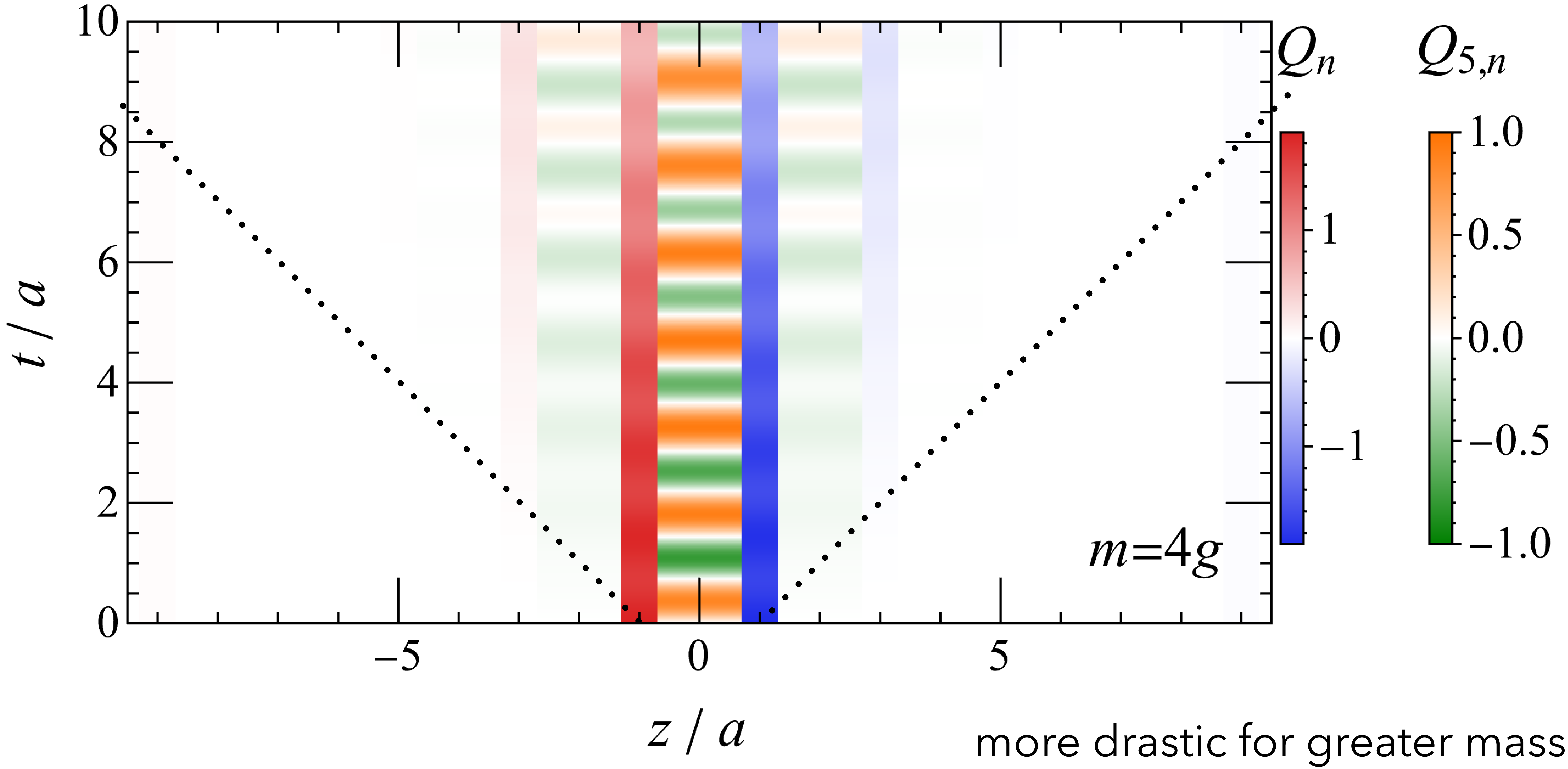

more drastic for greater mass

origin of fast and slow oscillating modes

$$H|k\rangle = E_k|k\rangle$$

$$|\Psi(t=0)\rangle = \sum_k c_k|k\rangle$$

$$O(t) \equiv \langle\Psi(t)|O|\Psi(t)\rangle = \sum_{k,l} c_k c_l^* e^{i(E_l - E_k)t}\langle l|O|k\rangle$$

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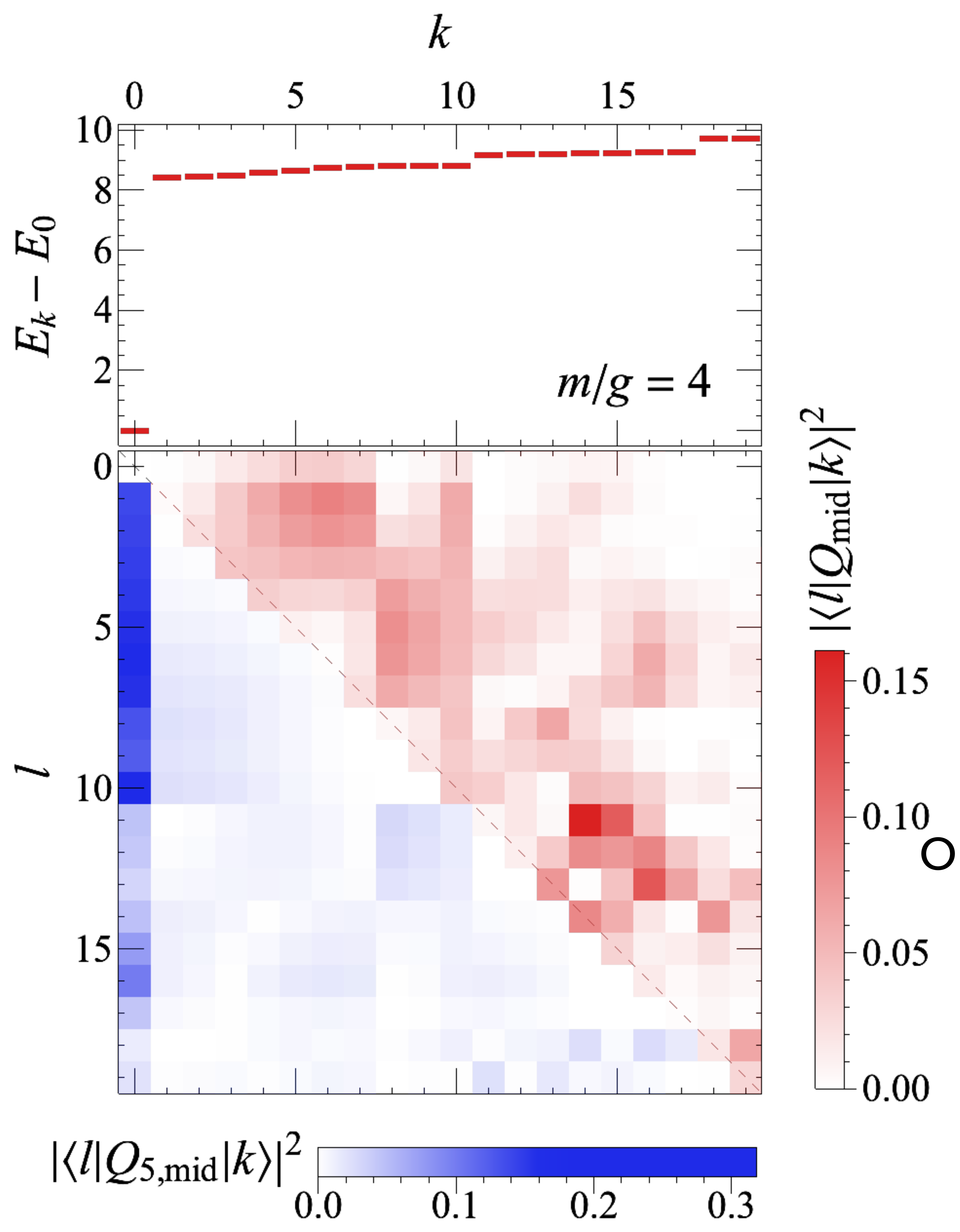
oscillation *frequency* ← energy difference

oscillation *strength* ← matrix element

origin of fast and slow oscillating modes



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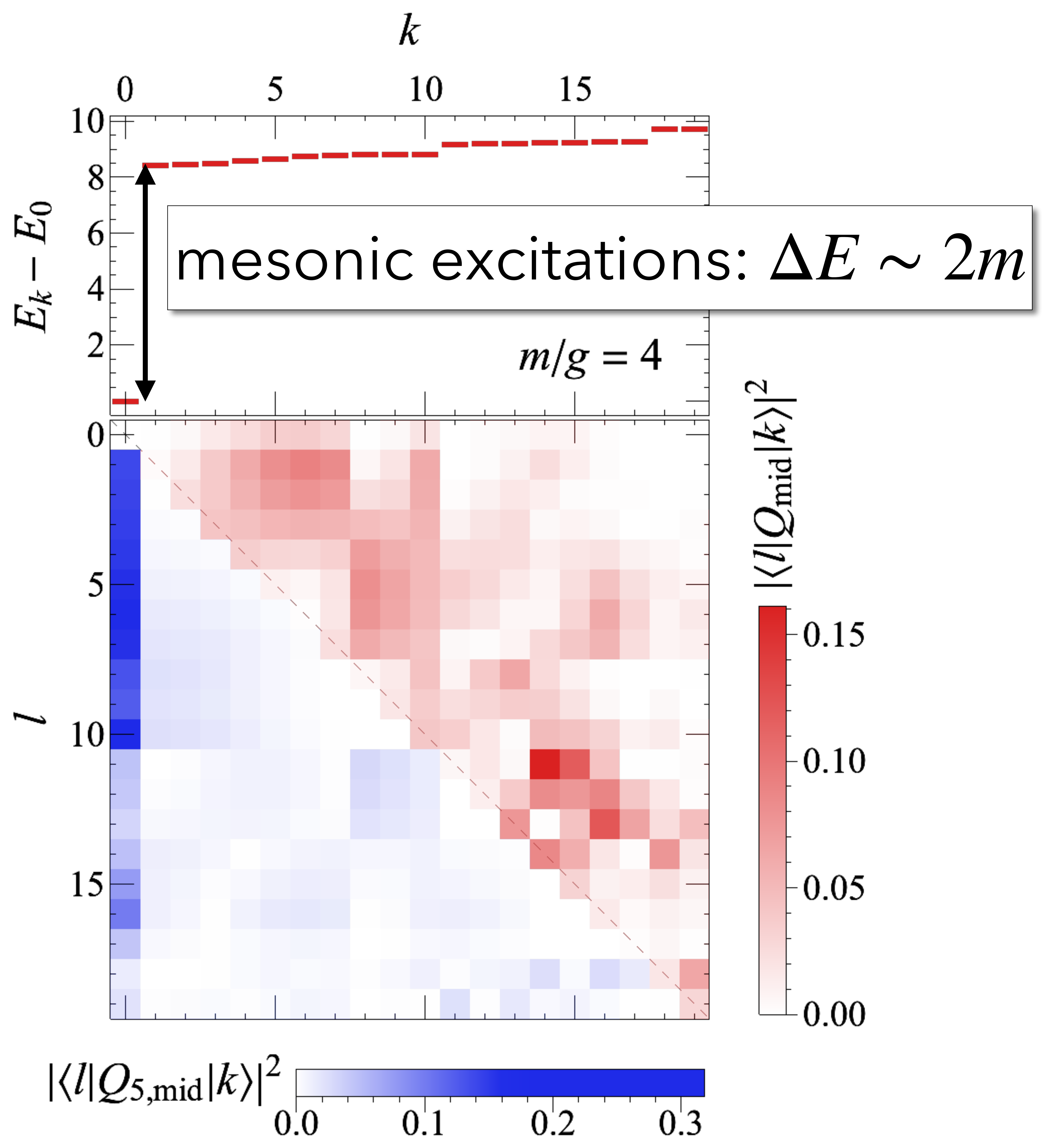

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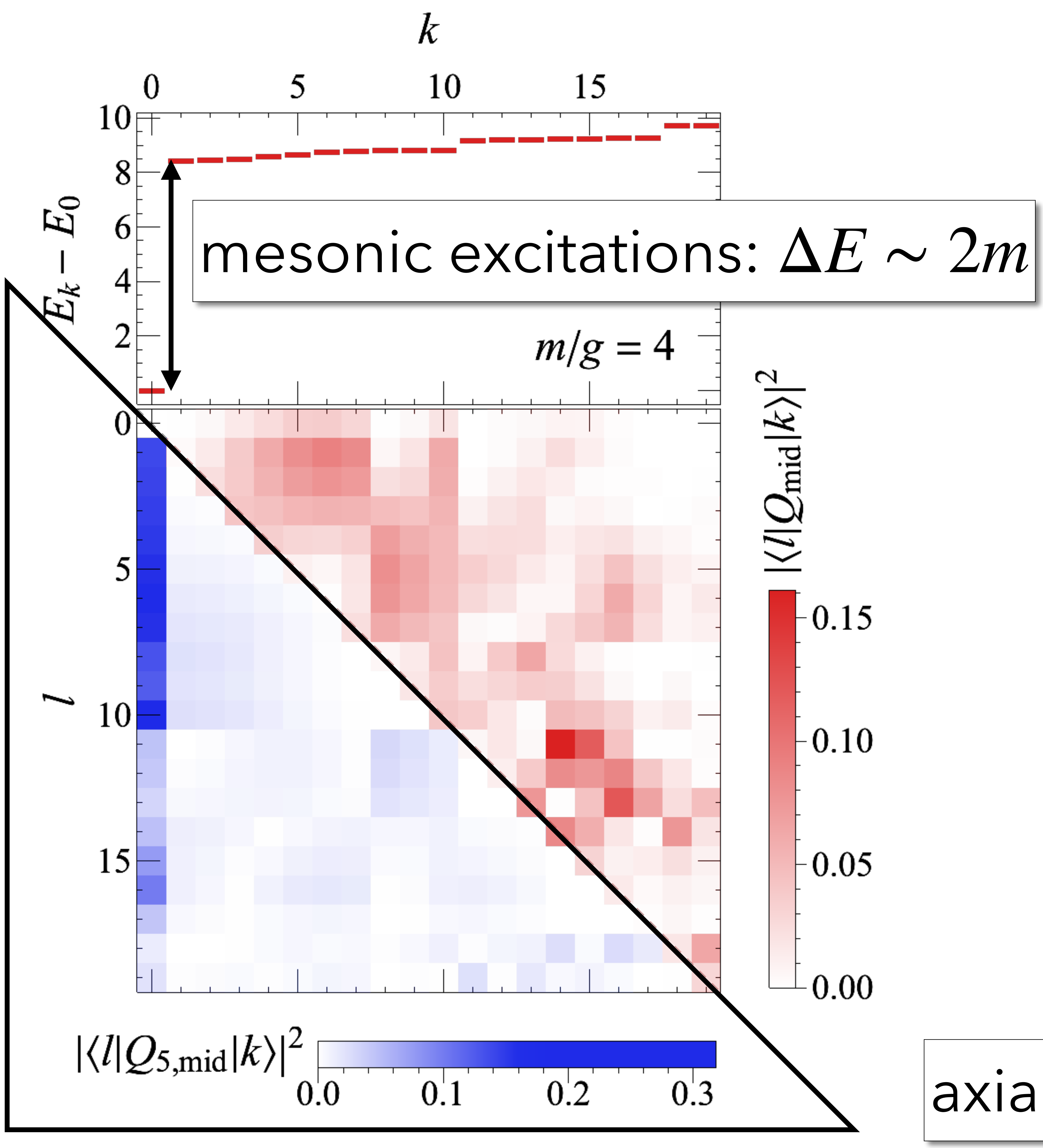

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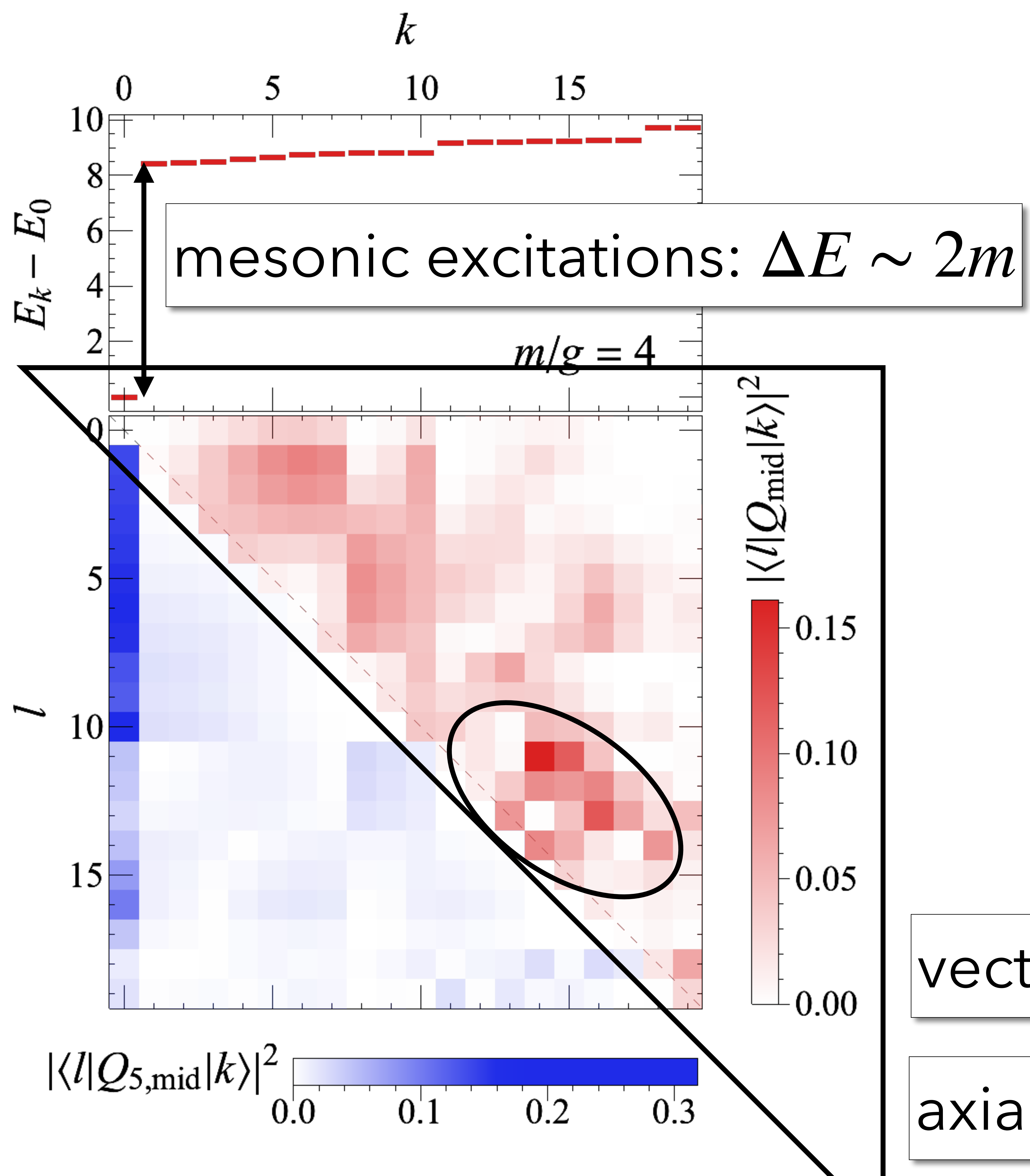
axial charge: ground state ↔ excitation

origin of fast and slow oscillating modes



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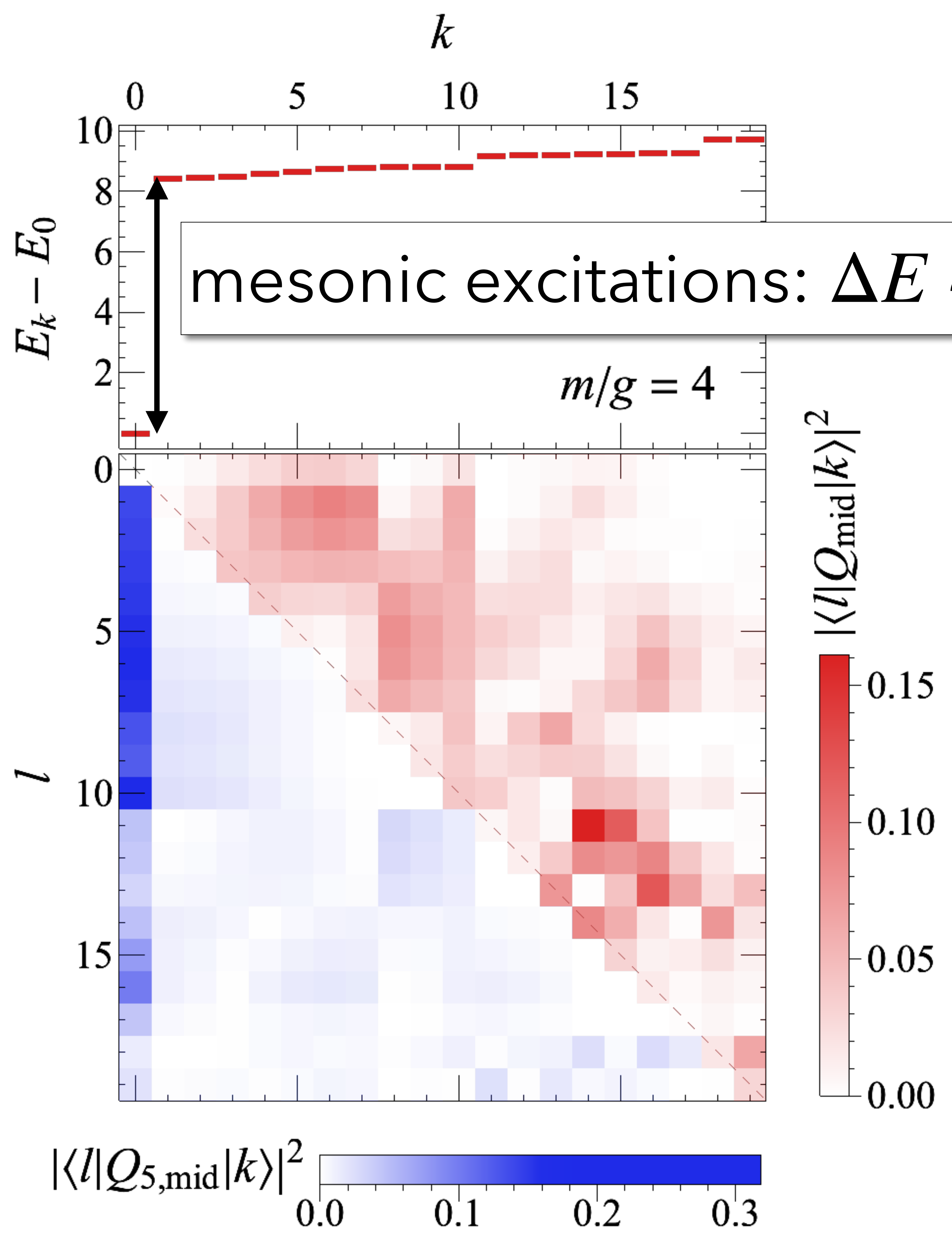
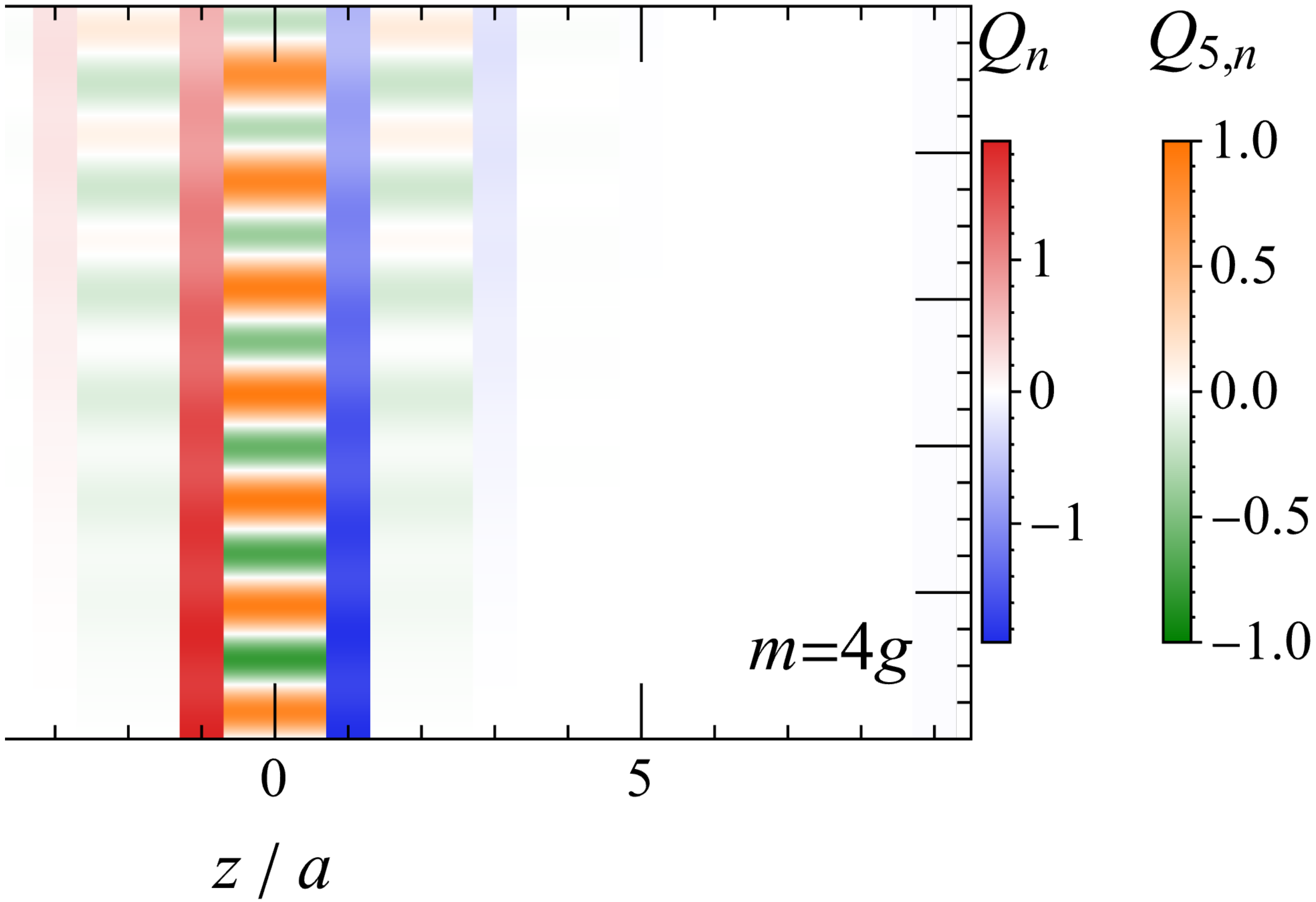
vector charge: excitation ↔ excitation

axial charge: ground state ↔ excitation

origin of fast and slow oscillating modes



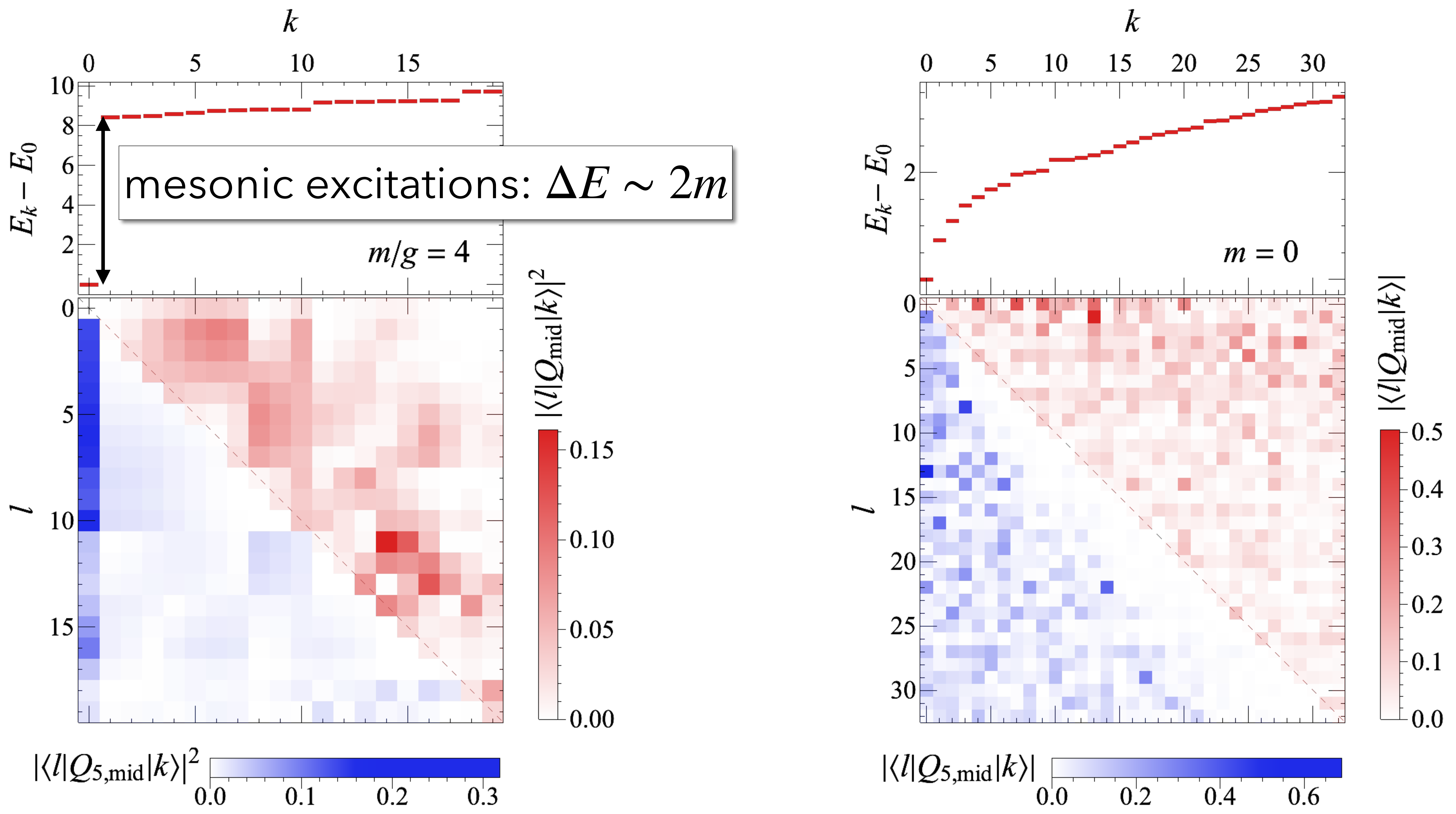
mesonic excitations: $\Delta E \sim 2m$



vector charge: excitation ↔ excitation

axial charge: ground state ↔ excitation

real-time evolution



summary

- real-time quantum evolution of Chiral Magnetic Wave
  - spread out of light-cone
  - fast oscillation in axial charge + slow oscillation in vector charge
- Need ***quantum computers*** to approach the continuum limit.

why quantum computer?

dimension of state vector $= 2^N$ N : number of lattice sides

dimension of Hamiltonian $=$ ~~$2^N \times 2^N$~~ sparse $\sim 2N \times 2^N$

N	dimension	memory of Hamiltonian	# of qubit (N)
8	256	~ 131 kB	8
12	4,096	~ 3.1 MB	12
16	65,536	~ 67 MB	16
20	1,048,576	~ 1.3 GB	20
24	16,777,216	~ 26 GB	24
28	268,435,456	~ 481 GB	28

unrealistic in a "classical" computer,
but plausible in the state-of-art quantum computer?

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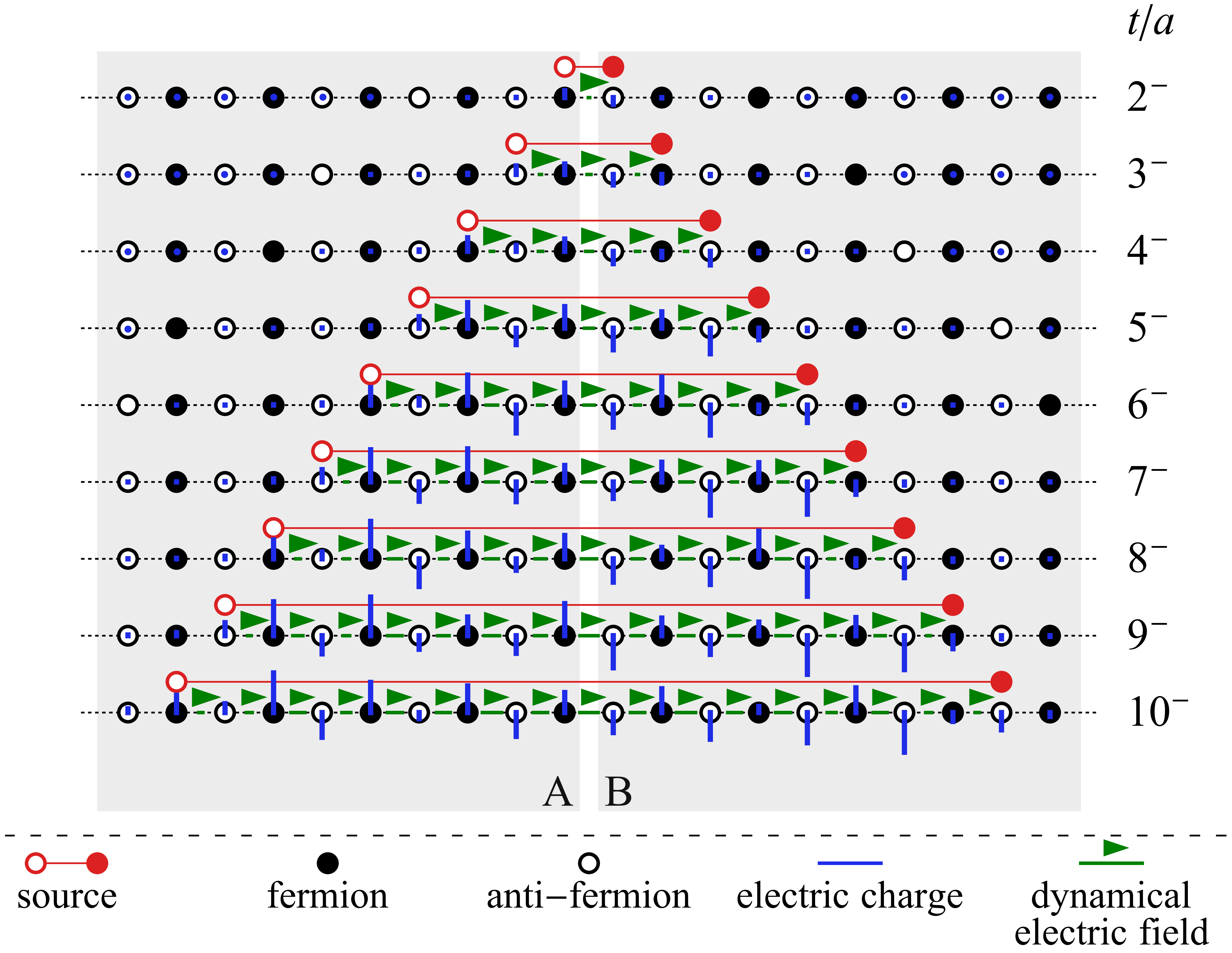
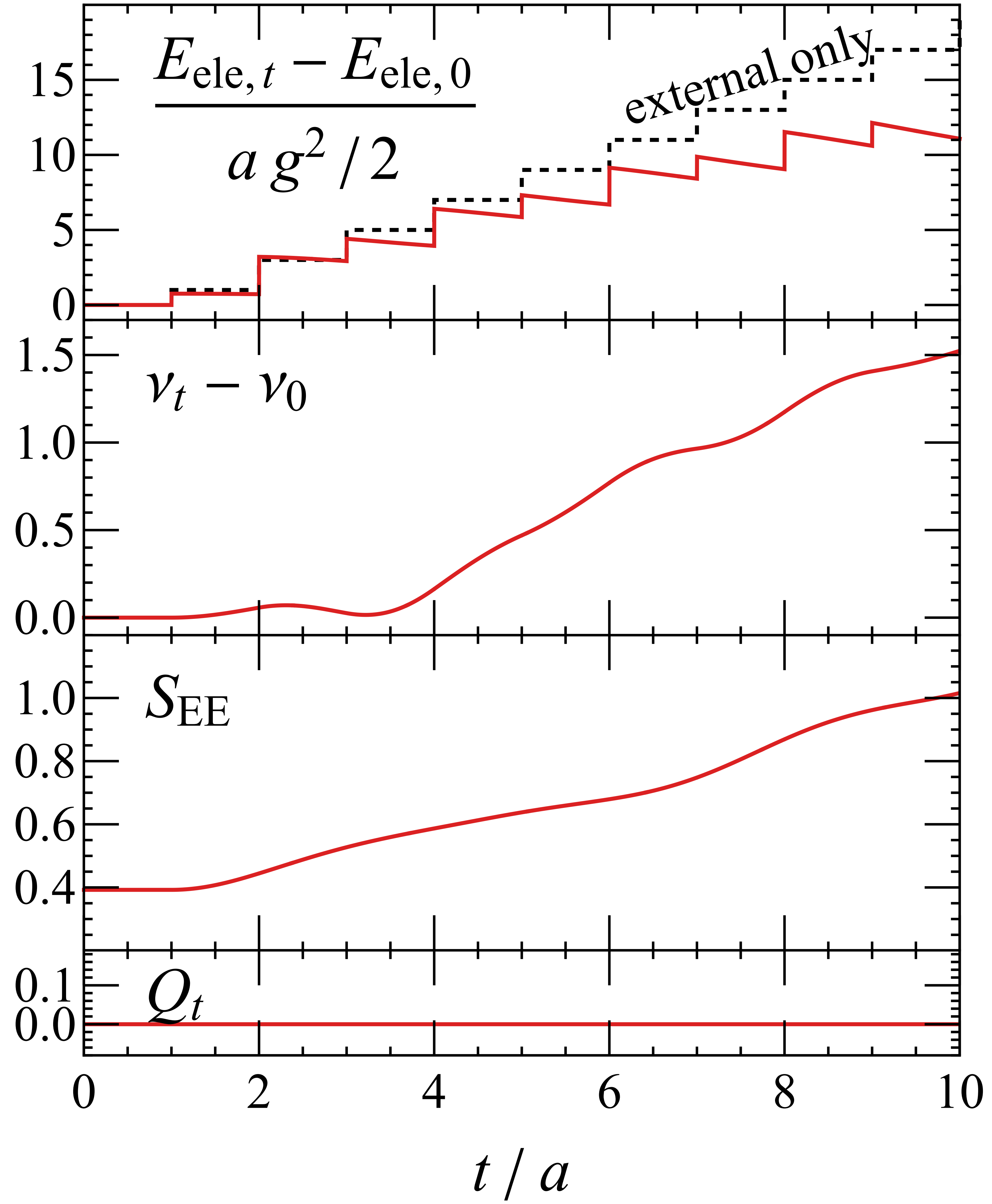
classical hardware in this work

performance not satisfying…

more properties of the massive Schwinger model

- entanglement in jet production





A. Florio, D. Frenklakh, K. Ikeda, D. Kharzeev, V. Korepin, *SS*, K. Yu

PhysRevLett.131.021902 (arXiv: 2305.05685)

more properties of the massive Schwinger model

- finite temperature, finite chemical potential:

$$\langle O \rangle_{\rm th} \equiv {\rm Tr}(\rho_{\rm th}\, O) \qquad \rho_{\rm th} \equiv \frac{e^{-(H-\mu Q)/T}}{{\rm Tr}(e^{-(H-\mu Q)/T})}$$

K. Ikeda, D. Kharzeev, R. Meyer, *SS*, arXiv: 2305.00996