Bayesian Inference of (3+1)D Relativistic Nuclear Dynamics from the RHIC Beam Energy Scan Data

Chun Shen (Wayne State University)

In collaboration with Syed Afrid Jahan, Hendrik Roch, Björn Schenke and Wenbin Zhao

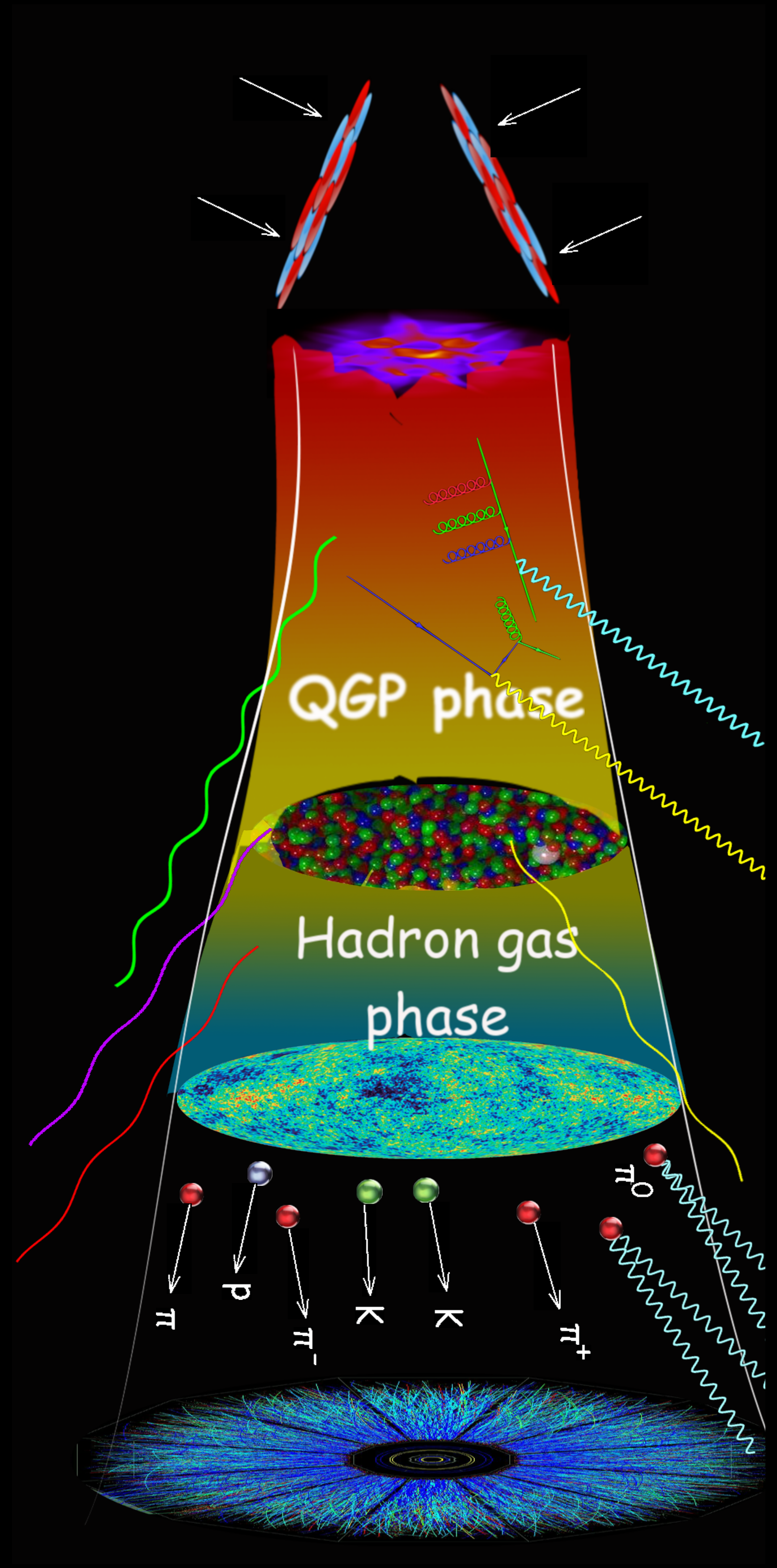
July 8, 2024

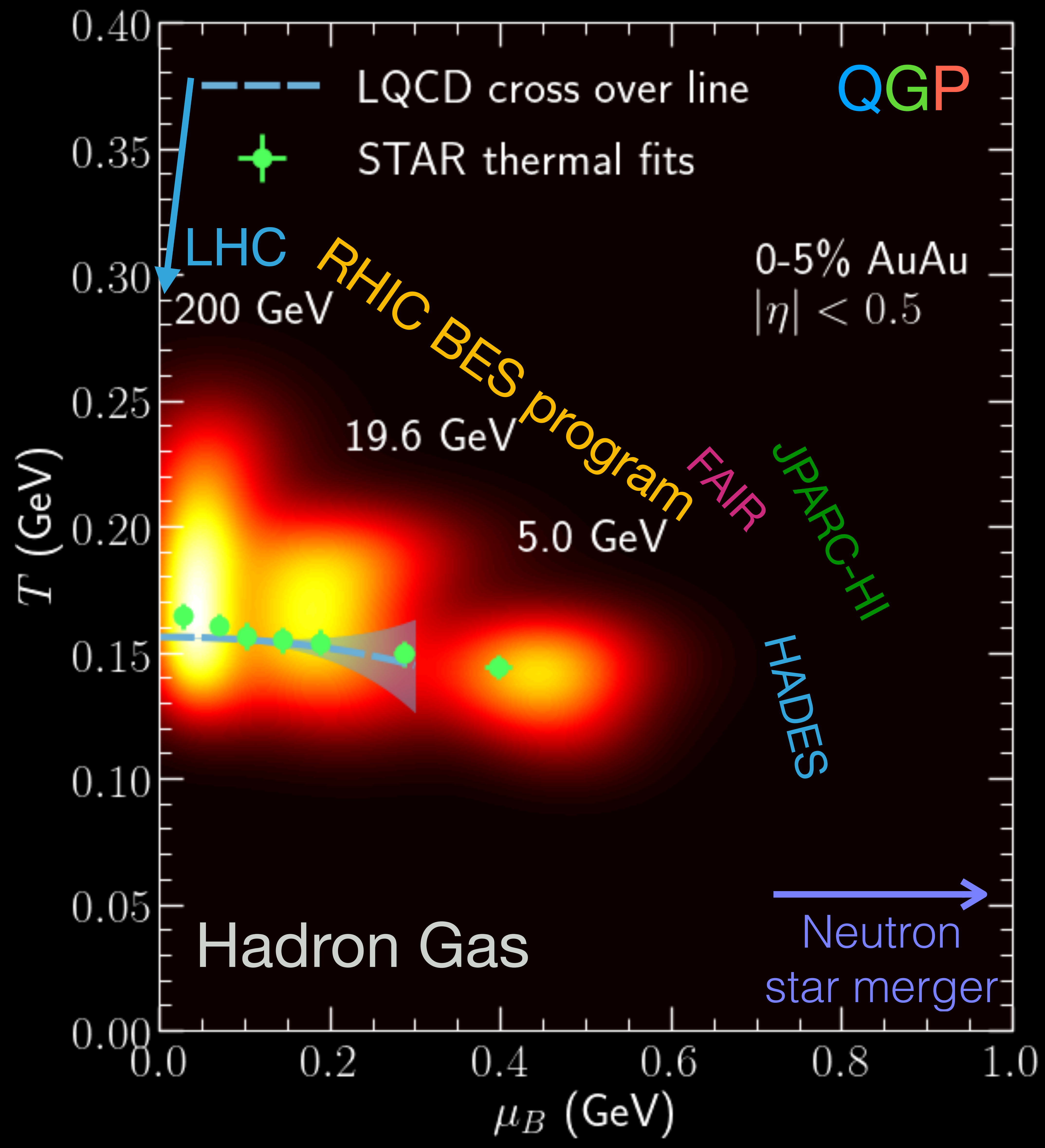
C. Shen, B. Schenke and W. Zhao, Phys.Rev.Lett. 132 (2024) 072301

H. Roch, S. A. Jahan and C. Shen, arXiv:2405.12019 [nucl-th]

Workshop on Inverse Problems and Uncertainty Quantification in Nuclear Physics

PROBING THE NUCLEAR MATTER PHASE DIAGRAM





- Search for a critical point & 1st order phase transition

  $c_s^2(T, \{\mu_q\})$

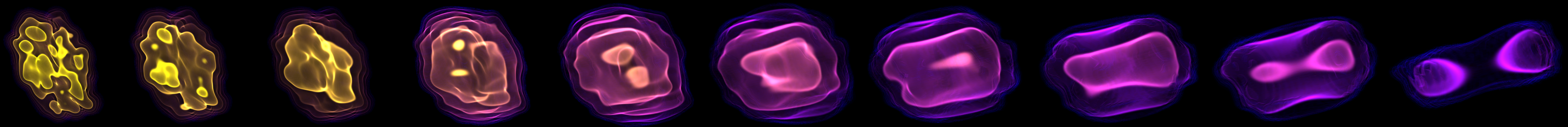
- How do the QGP transport properties change with baryon doping?

  $(\eta/s)(T, \{\mu_q\})$, $(\zeta/s)(T, \{\mu_q\})$

- Access to new transport phenomena

  Charge diffusion

THE MULTI-STAGE THEORETICAL FRAMEWORK



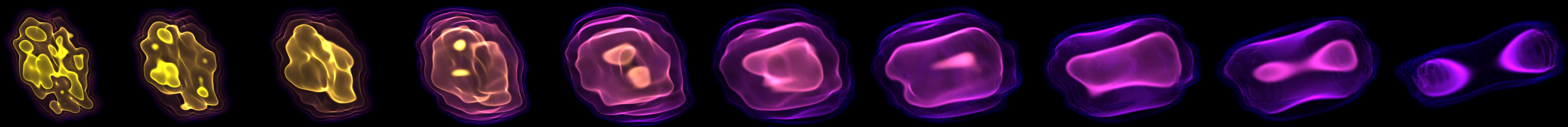
$T^{\mu\nu}$ 0^+ 0.5-1 τ (fm)

Initial State +
Pre-equilibrium dynamics

Hydrodynamics

Hadronic Transport

THE MULTI-STAGE THEORETICAL FRAMEWORK



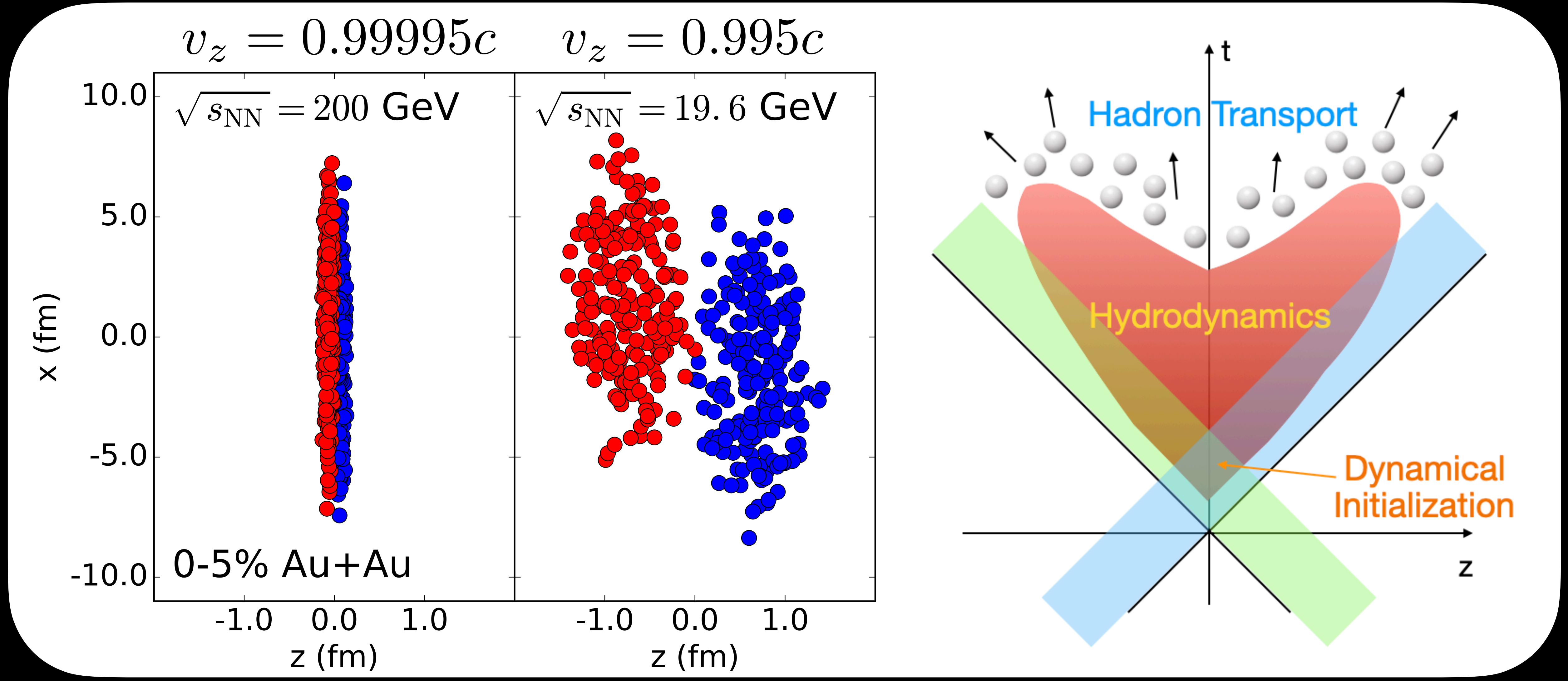
$T^{\mu\nu}$ 0^+ 0.5-1 τ (fm)

Initial State +
Pre-equilibrium dynamics

Hydrodynamics

Hadronic Transport

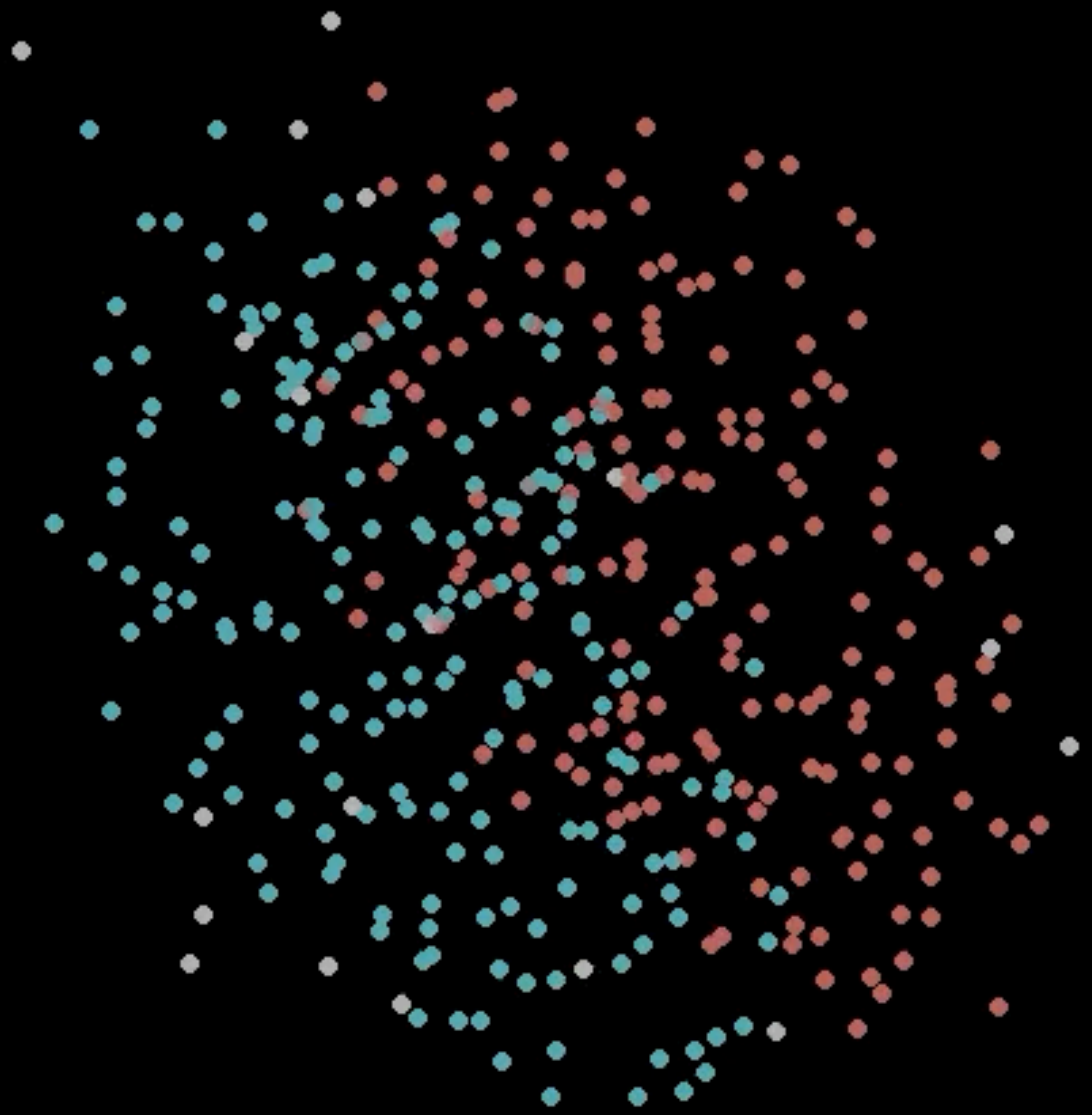
RHIC BES energies



$$\partial_\mu T^{\mu\nu} = J^\nu_{\mathrm{source}}$$

$$\partial_\mu J^\mu = \rho_{\mathrm{source}}$$

0-5% Au+Au @ 19.6 GeV



THE MODEL PARAMETERS

TABLE I. The 20 model parameters and their prior ranges.

Parameter	Prior	Parameter	Prior
B_G (GeV^{-2})	[1, 25]	$\alpha_{\rm string\,tilt}$	[0, 1]
$\alpha_{\rm shadowing}$	[0, 1]	$\alpha_{\rm preFlow}$	[0, 2]
$y_{{\rm loss},2}$	[0, 2]	η_0	[0.001, 0.3]
$y_{{\rm loss},4}$	[1, 3]	η_2	[0.001, 0.3]
$y_{{\rm loss},6}$	[1, 4]	η_4	[0.001, 0.3]
$\sigma_{y_{\rm loss}}$	[0.1, 0.8]	$\zeta_{\max}$	[0, 0.2]
$\alpha_{\rm Rem}$	[0, 1]	$T_{\zeta,0}$ (GeV)	[0.15, 0.25]
λ_B	[0, 1]	$\sigma_{\zeta,+}$ (GeV)	[0.01, 0.15]
$\sigma_x^{\rm string}$ (fm)	[0.1, 0.8]	$\sigma_{\zeta,-}$ (GeV)	[0.005, 0.1]
$\sigma_\eta^{\rm string}$	[0.1, 1]	$e_{\rm sw}$ (GeV/fm^3)	[0.15, 0.5]

MODEL TRAINING & OBSERVABLE SELECTION

A 20-dimensional model parameter space with 1,000 training points

Au+Au	Hydro events per design	Avg. hadronic events per hydro
200 GeV	1,000	1,000
19.6 GeV	2,000	4,000
7.7 GeV	2,000	8,000

Open Science Grid delivered 5 million CPU hours for the data generation

604 experimental data points

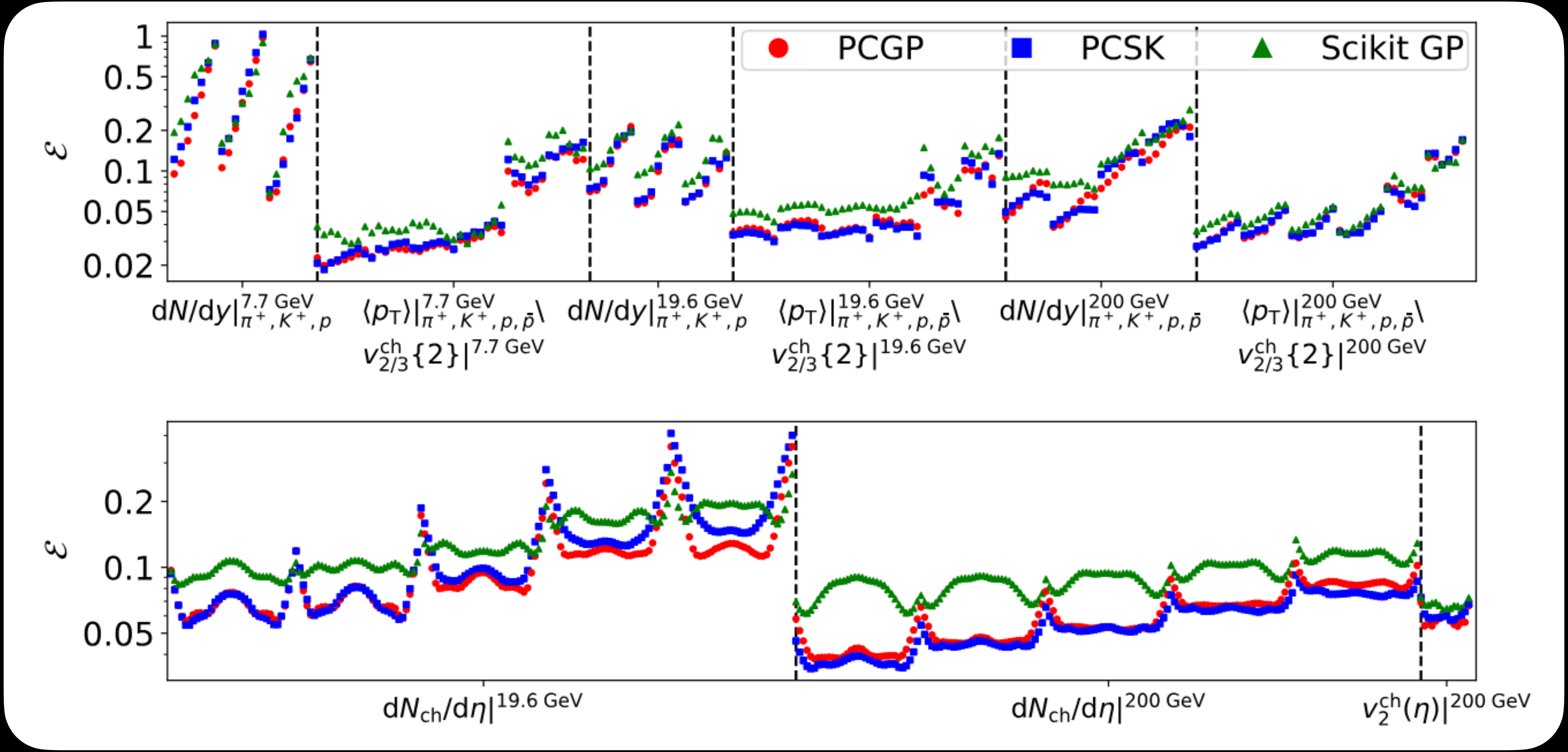
Au+Au	STAR midrapdity data vs. centrality	PHOBOS rapidity distribtion
200 GeV	$dN/dy(\pi^+, K^+, p, \bar{p})$ $\langle p_T \rangle(\pi^+, K^+, p, \bar{p})$ $v_2^{ch}\{2\}, v_3^{ch}\{2\}$	$dN^{ch}/d\eta$ $v_2(\eta)$
19.6 GeV	$dN/dy(\pi^+, K^+, p)$ $\langle p_T \rangle(\pi^+, K^+, p, \bar{p})$ $v_2^{ch}\{2\}, v_3^{ch}\{2\}$	$dN^{ch}/d\eta$
7.7 GeV	$dN/dy(\pi^+, K^+, p)$ $\langle p_T \rangle(\pi^+, K^+, p, \bar{p})$ $v_2^{ch}\{2\}, v_3^{ch}\{2\}$	

Phys. Rev. C79, 034909 (2009) Phys. Rev. C98, 034918 (2018)
Phys. Rev. C96, 044904 (2017) Phys. Rev. C74, 021901 (2006)

EVALUATE THE QUALITY OF MODEL EMULATION

H. Roch, S. A. Jahan and C. Shen, arXiv:2405.12019 [nucl-th]

$$\varepsilon \equiv \sqrt{\left\langle \left(\frac{\text{prediction} - \text{truth}}{\text{truth}} \right)^2 \right\rangle}$$



- The PCGP and PCSK emulators are more accurate than the standard GP from the scikit-learn package

EVALUATE THE QUALITY OF MODEL EMULATION

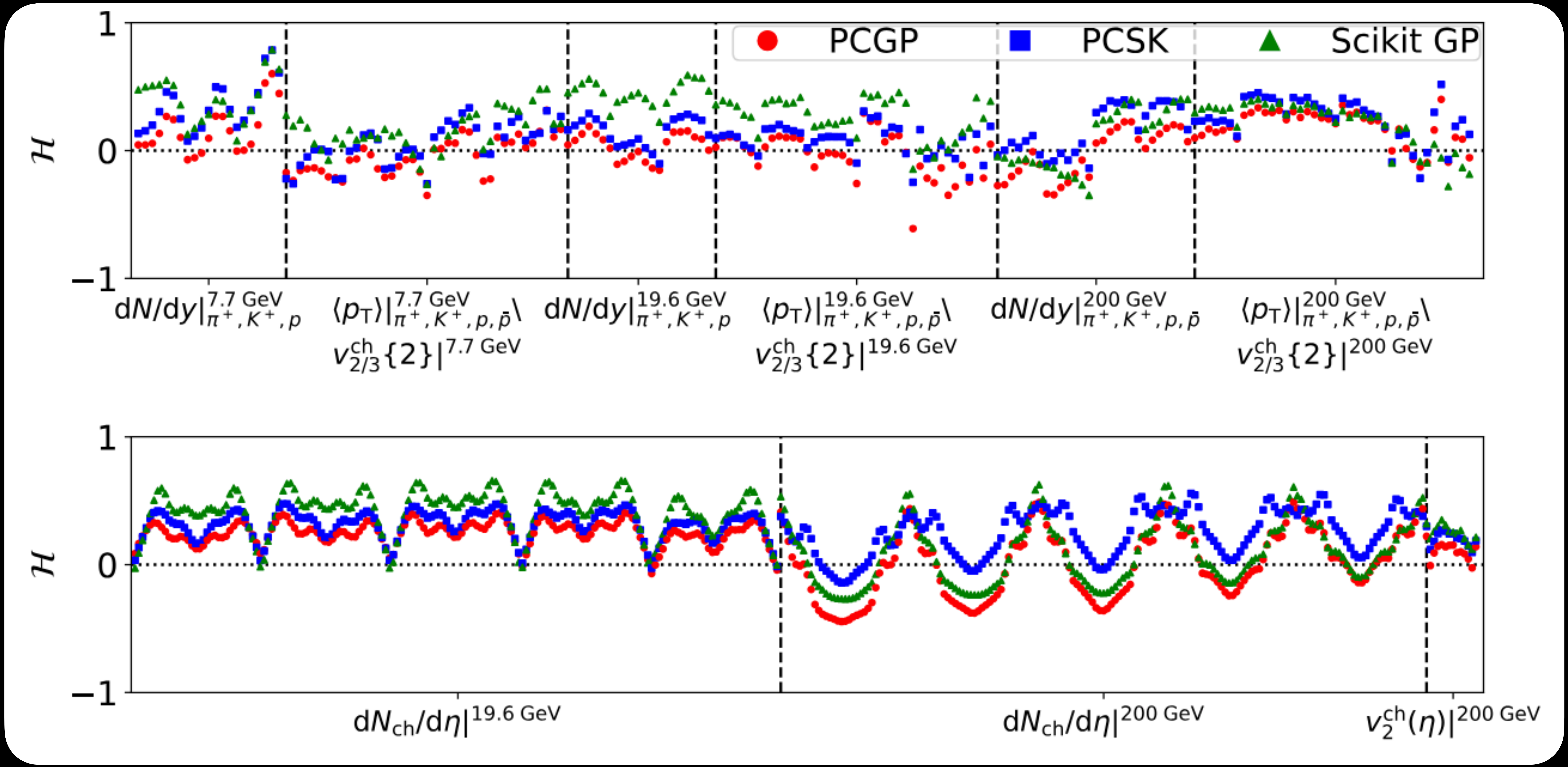
H. Roch, S. A. Jahan and C. Shen, arXiv:2405.12019 [nucl-th]

$$\mathcal{H} \equiv \ln\left(\sqrt{\left\langle\left(\frac{\text{prediction} - \text{truth}}{\text{pred . uncertainty}}\right)^2\right\rangle}\right)$$

$\mathcal{H} = 0$: Best

$\mathcal{H} > 0$: Predicted uncertainty is too small

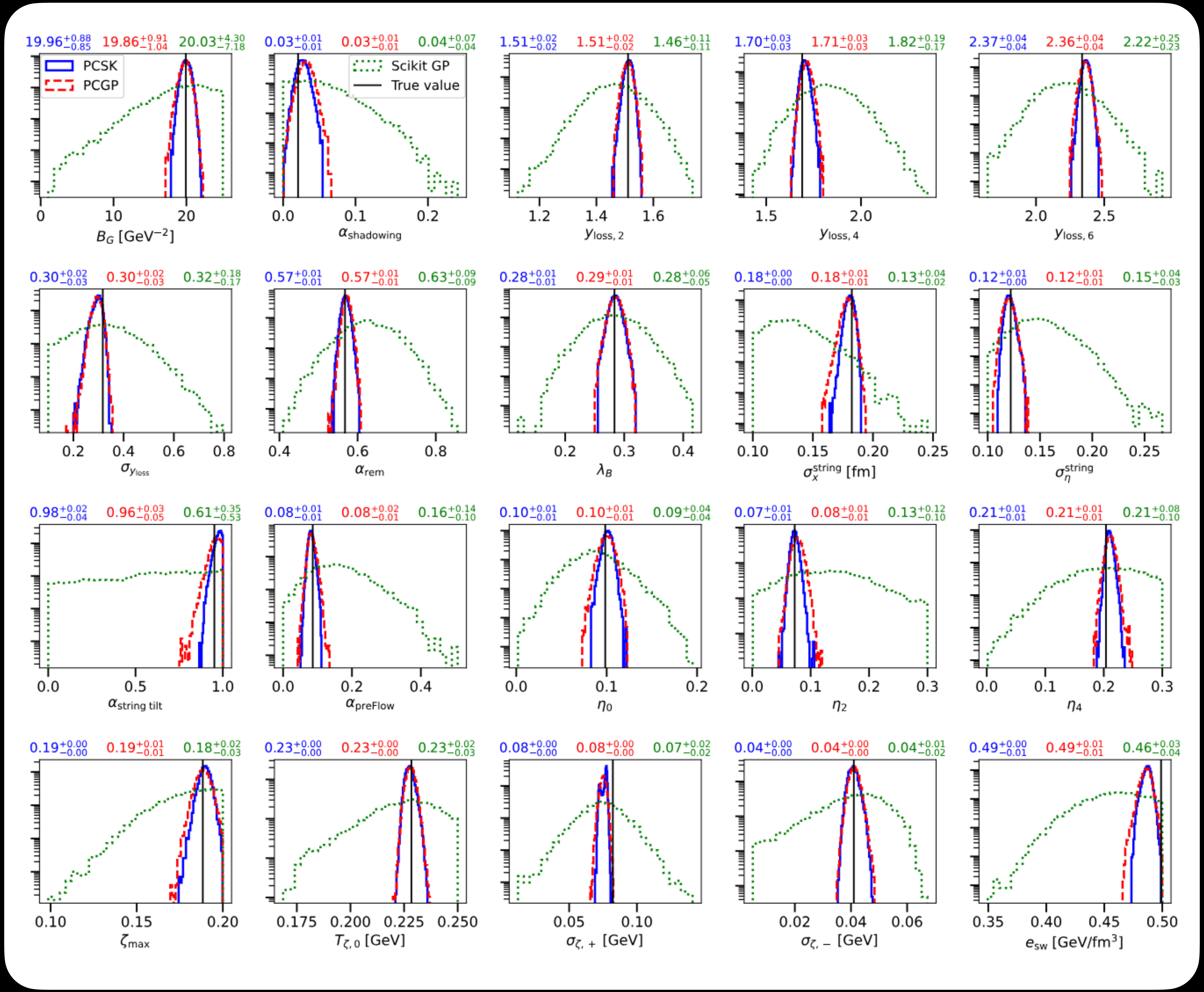
$\mathcal{H} < 0$: Predicted uncertainty is too large



- The PCGP and PCSK emulators give more reliable uncertainty estimation than that from the Scikit GP

IMPACTS OF EMULATOR PRECISION ON CLOSURE TEST

H. Roch, S. A. Jahan and C. Shen, arXiv:2405.12019 [nucl-th]

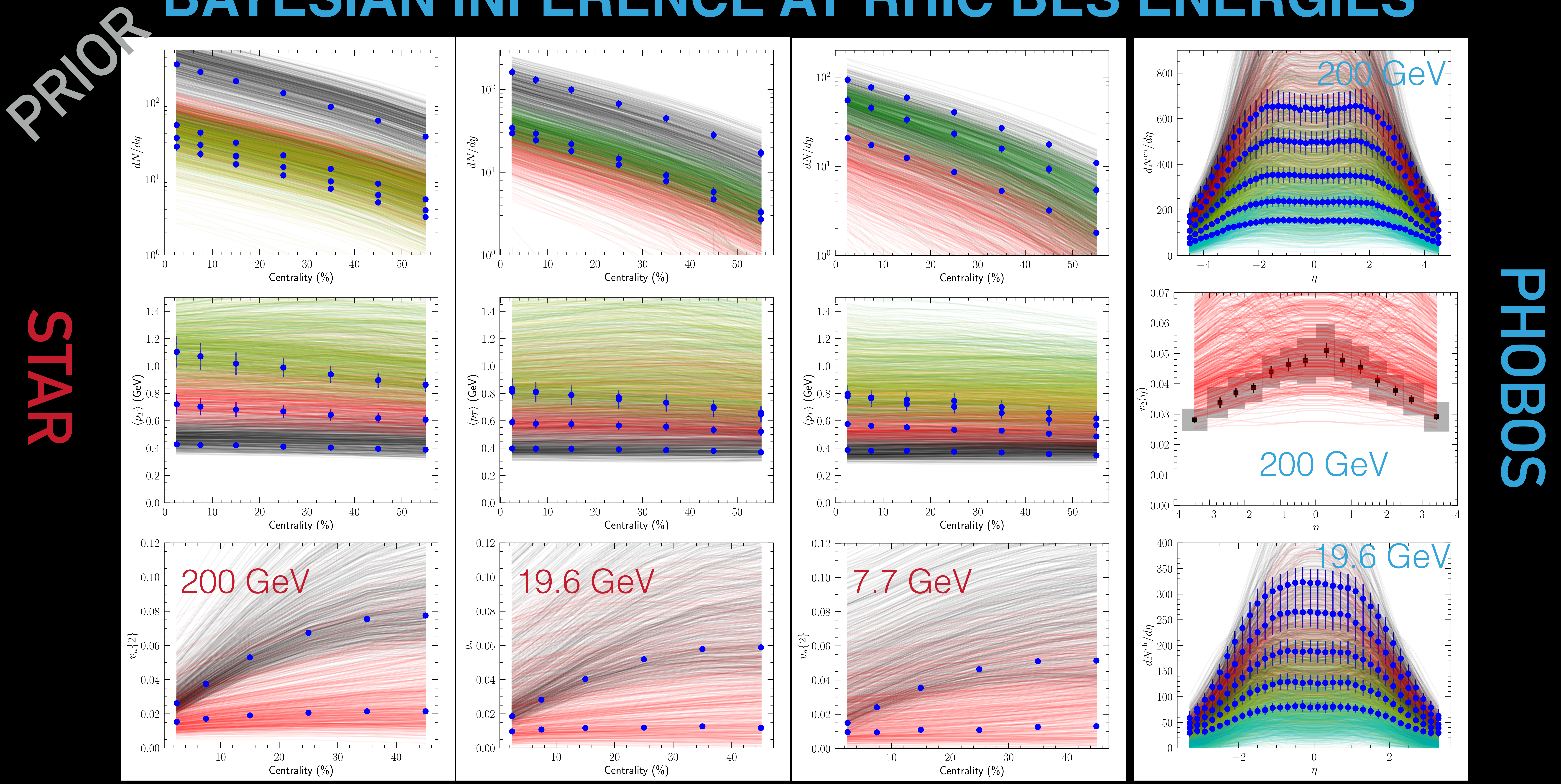


- The more accurate PCGP and PCSK emulators give tighter posterior on model parameters than that from the Scikit GP

$$\Delta \equiv \frac{1}{N_{\mathrm{param.}}} \int \left| \frac{\boldsymbol{\theta} - \boldsymbol{\theta}_{\mathrm{truth}}}{\boldsymbol{\theta}_{\mathrm{max}} - \boldsymbol{\theta}_{\mathrm{min}}} \right|^2 p(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta},$$

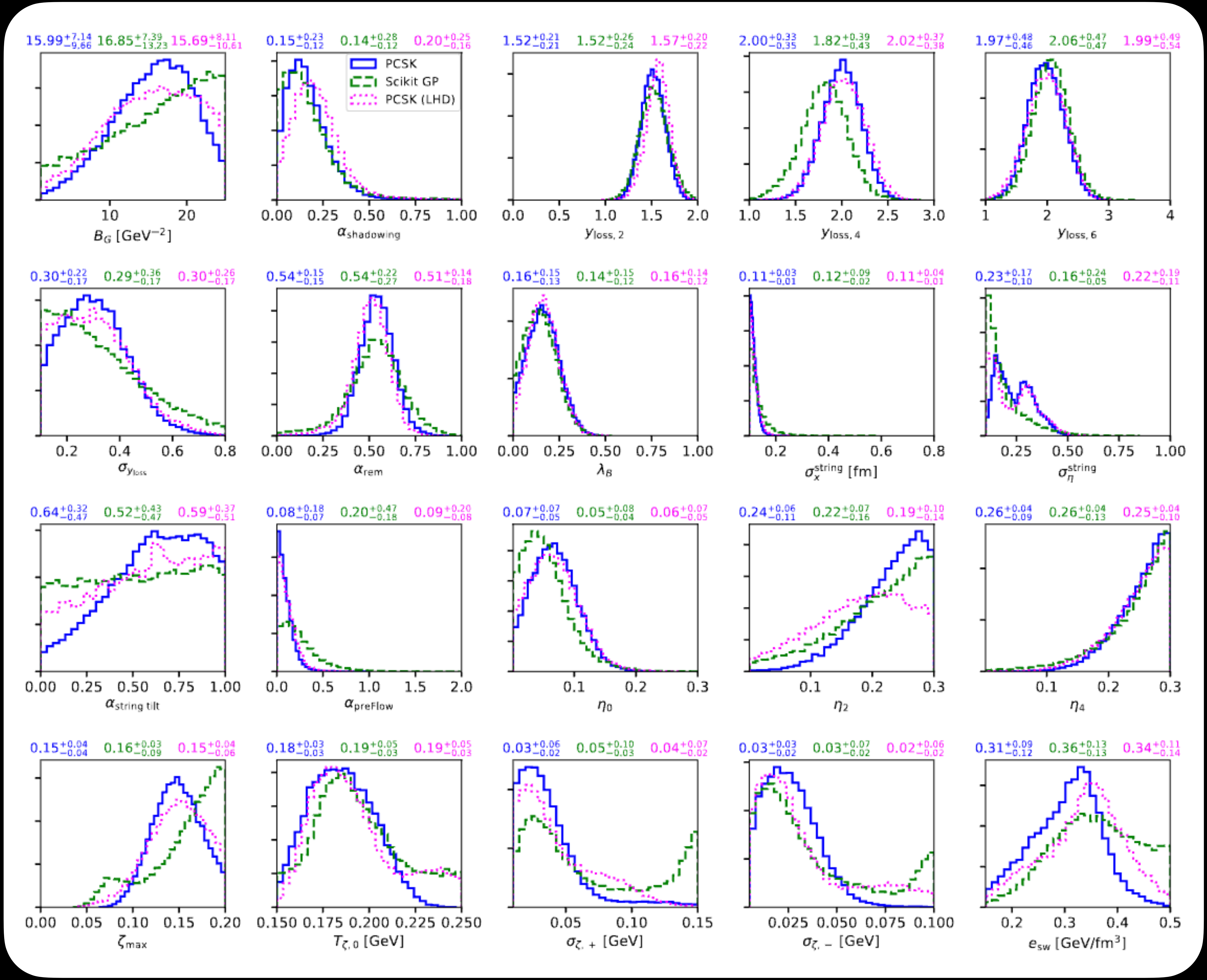
Emulator	Δ
PCGP	4.5×10^{-4}
PCSK	5.7×10^{-4}
Scikit GP	2.5×10^{-2}

BAYESIAN INFERENCE AT RHIC BES ENERGIES



BAYESIAN INFERENCE AT RHIC BES ENERGIES

POSTERIORS FROM DIFFERENT EMULATORS



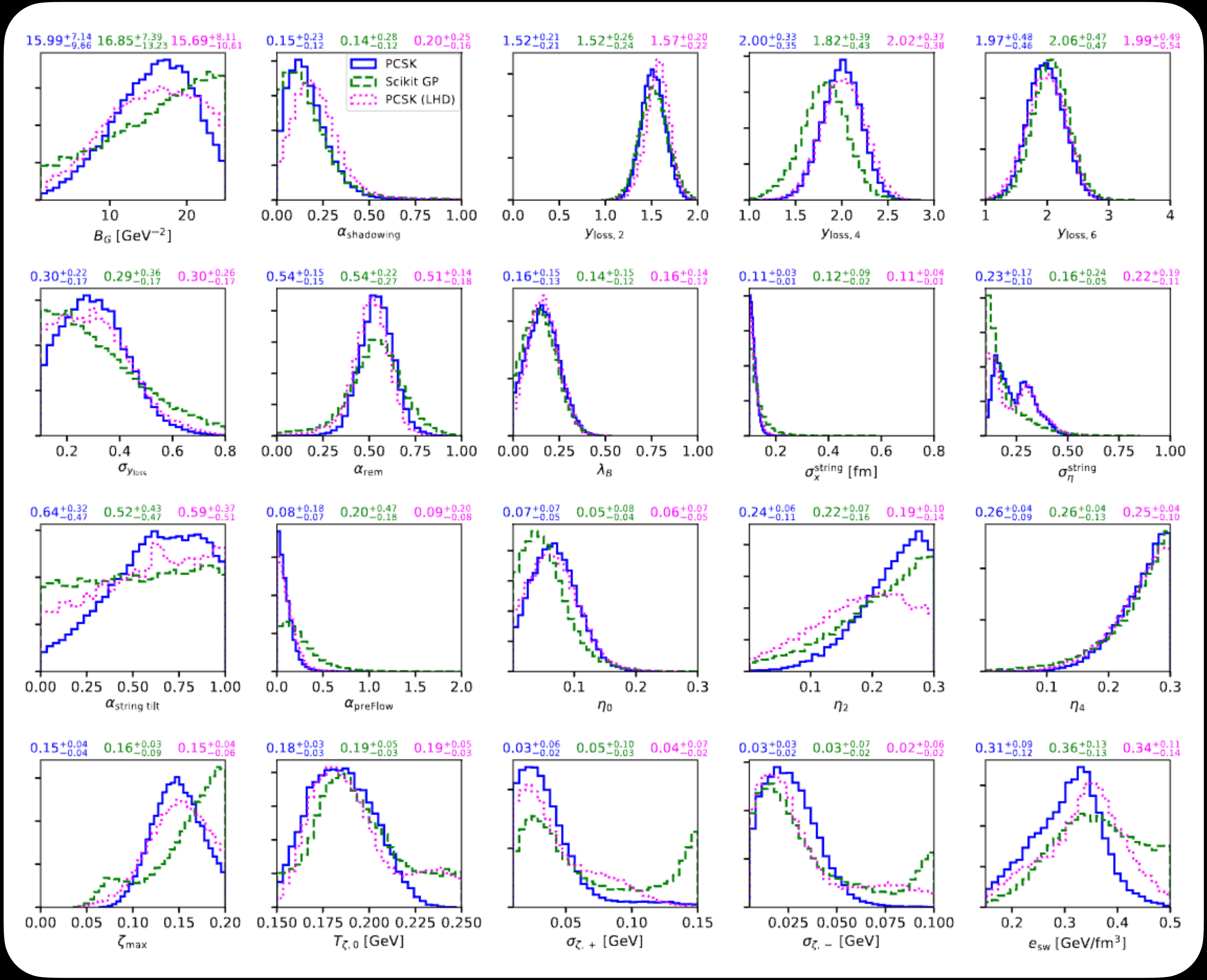
H. Roch, S. A. Jahan and C. Shen, in progress

- We compare different posteriors using the KL divergence:

$$D_{KL} = \int p(\theta) \ln \left(\frac{p(\theta)}{q(\theta)} \right) d\theta$$

Models	D_{KL}
PCSK	24.6
Scikit GP	20.9
PCSK (LHD)	22.4

POSTERIORS FROM DIFFERENT EMULATORS



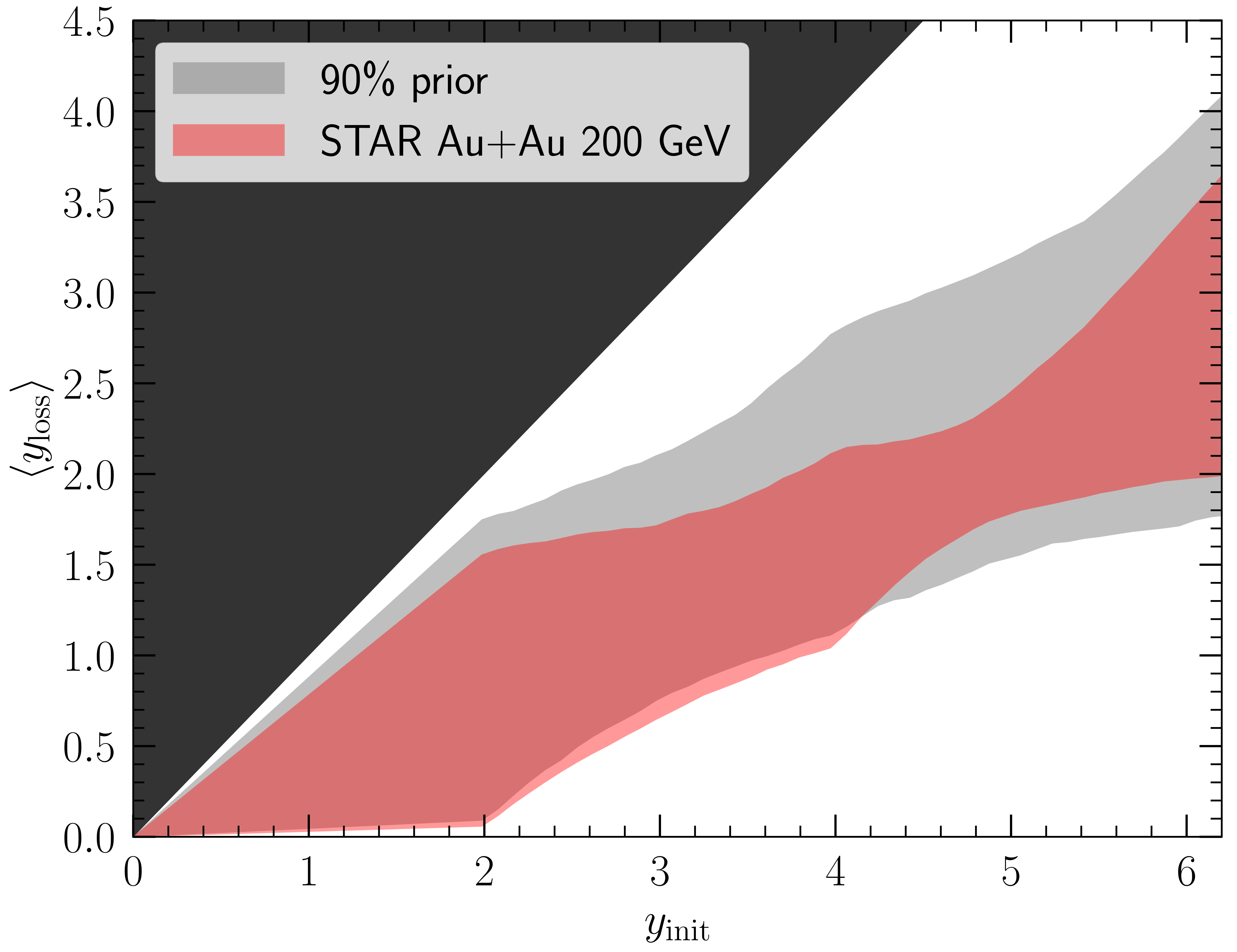
H. Roch, S. A. Jahan and C. Shen, in progress

- We compare different posteriors using Bayes factor:

$$\mathscr{B}_{A/B} = \frac{P(y_{\mathrm{exp}}\,|\,A)}{P(y_{\mathrm{exp}}\,|\,B)}$$

Models	$\mathscr{B}$
PCSK/Scikit GP	6.9
LHD + HPP/ LHD	-1.4

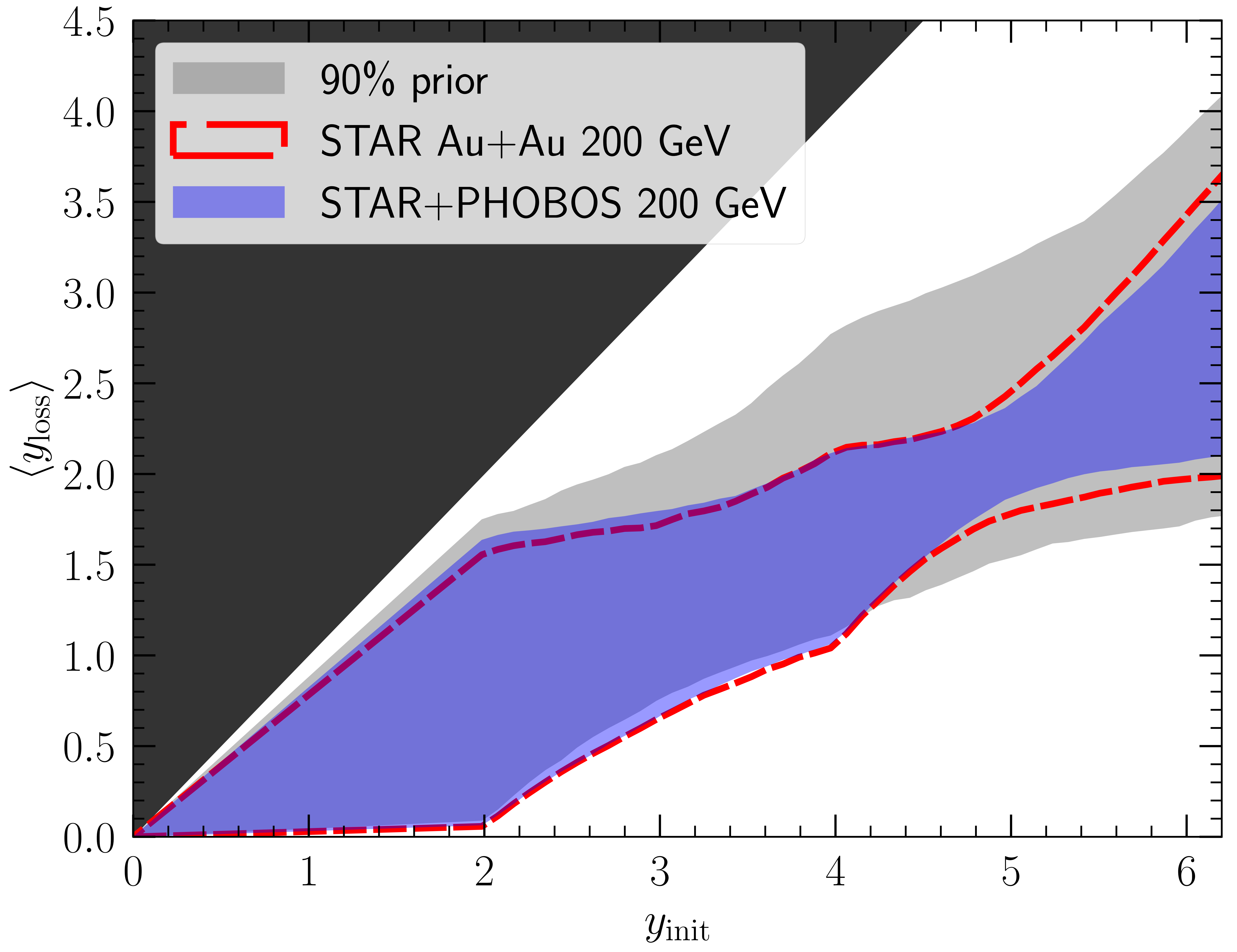
INITIAL-STATE STOPPING



- Mid-rapidity particle productions at 200 GeV yields $y_{\rm loss} \sim 2$ for $y_{\rm init} \sim 5$

color bands indicate 90% credible interval in the posterior

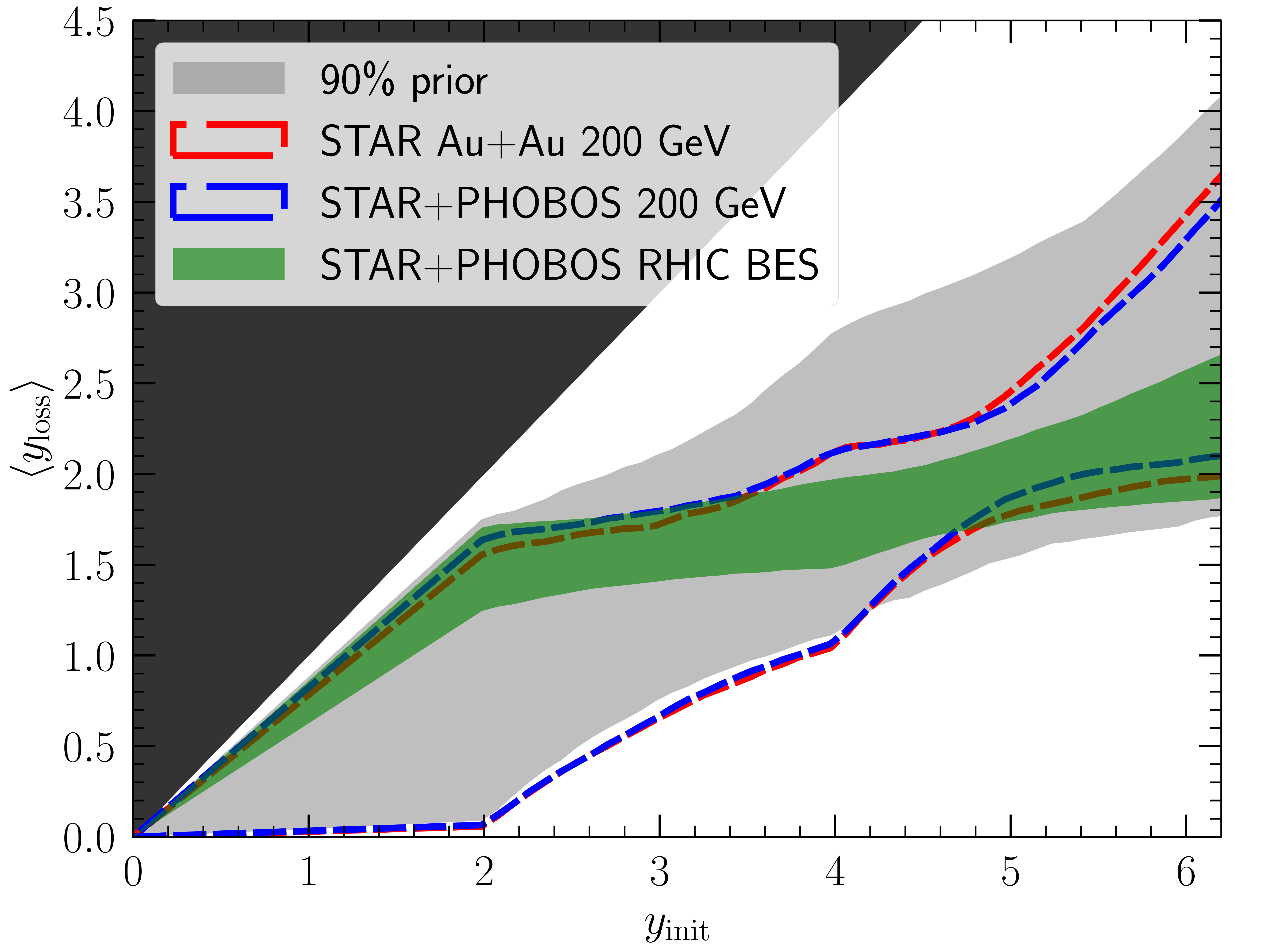
INITIAL-STATE STOPPING



- Mid-rapidity particle productions at 200 GeV yields $y_{\mathrm{loss}} \sim 2$ for $y_{\mathrm{init}} \sim 5$
- The rapidity distributions from PHOBOS give small improvements to the constraint

color bands indicate 90% credible interval in the posterior

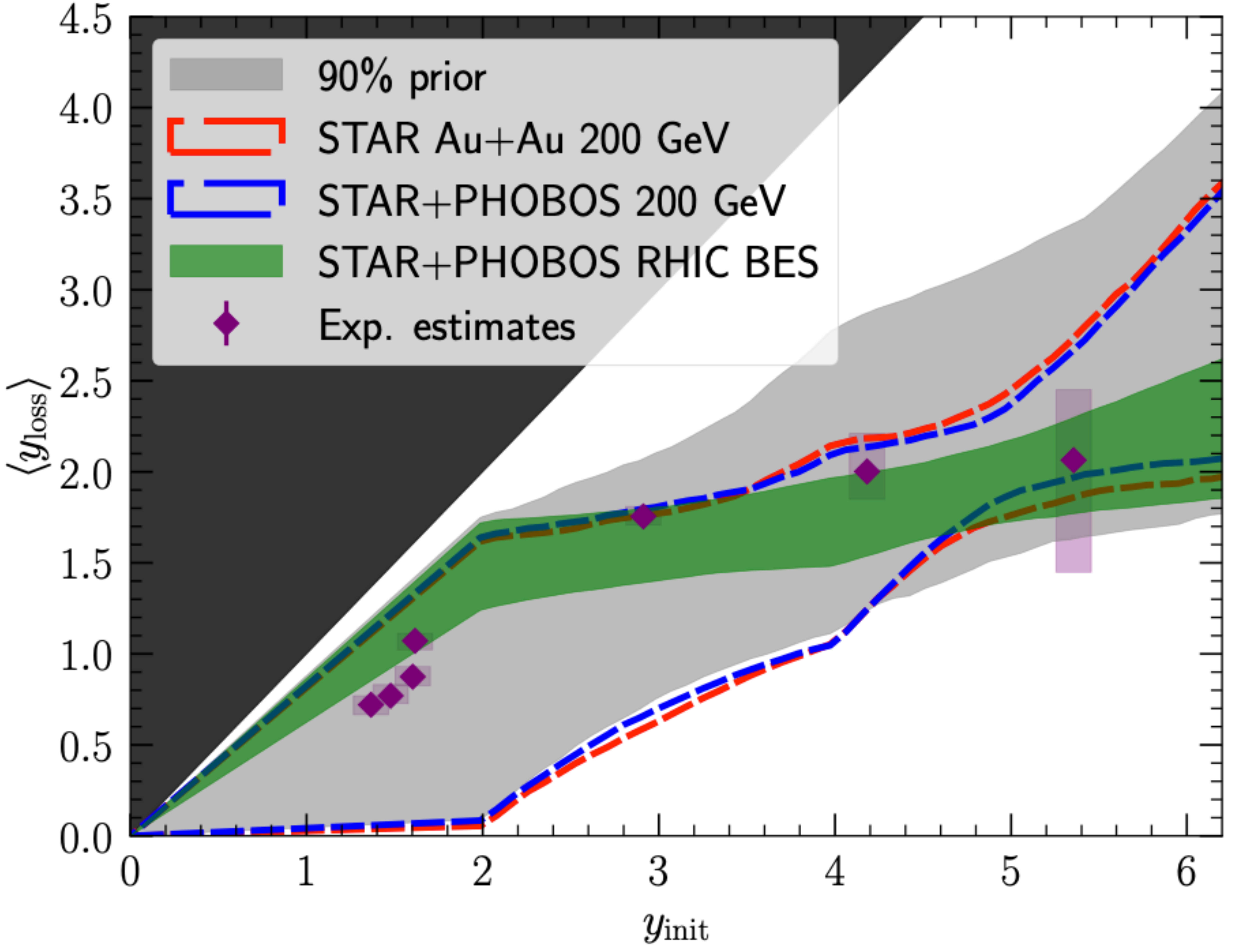
INITIAL-STATE STOPPING



- Mid-rapidity particle productions at 200 GeV yields $y_{\rm loss} \sim 2$ for $y_{\rm init} \sim 5$
- The rapidity distributions from PHOBOS give small improvements to the constraint
- Particle production from 7.7, 19.6, and 200 GeV sets strong constrain on $y_{\rm loss}(y_{\rm init})$ for $y_{\rm init} \in [0,6]$

color bands indicate 90% credible interval in the posterior

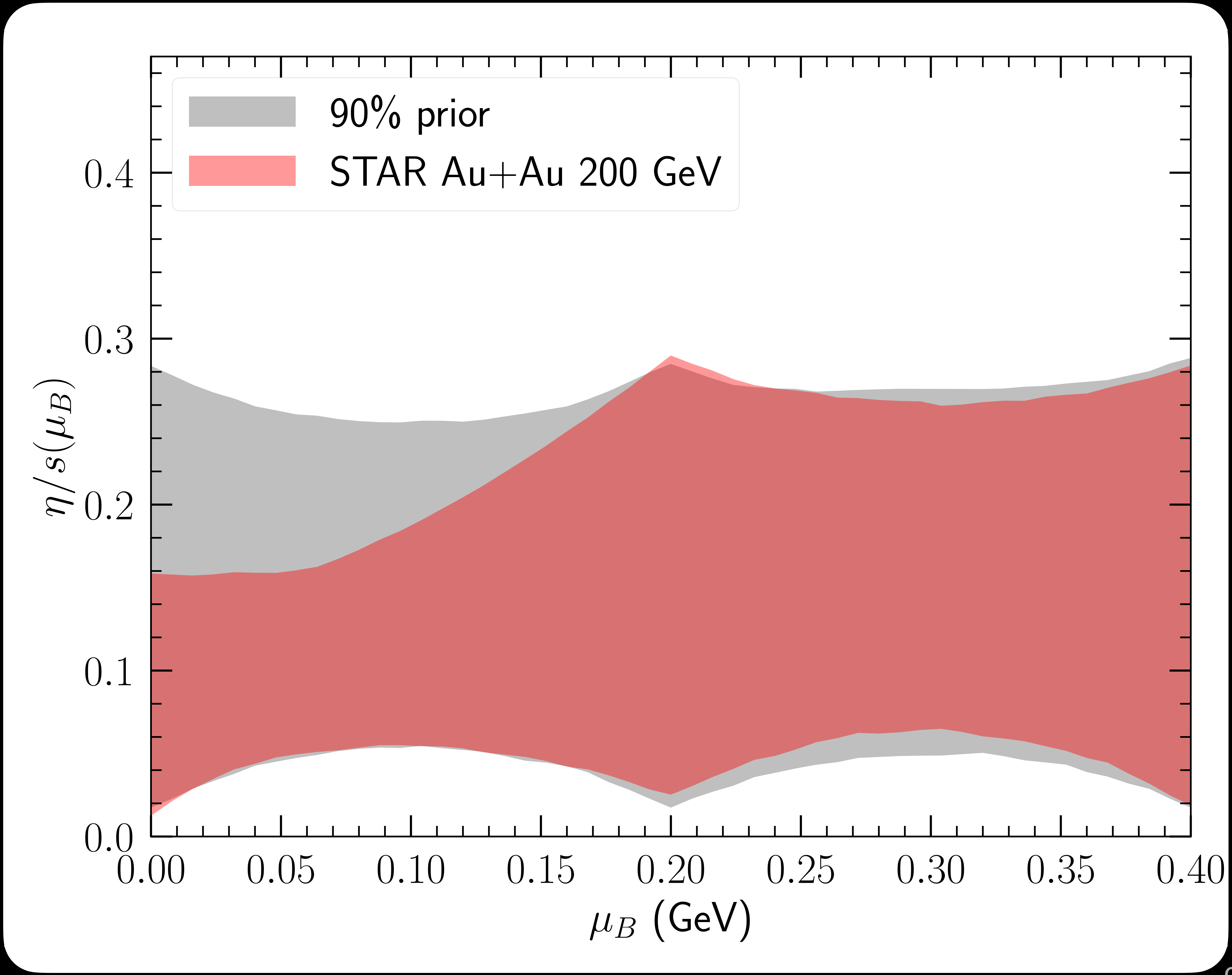
INITIAL-STATE STOPPING



- Mid-rapidity particle productions at 200 GeV yields $y_{\rm loss} \sim 2$ for $y_{\rm init} \sim 5$
- The rapidity distributions from PHOBOS give small improvements to the constraint
- Particle production from 7.7, 19.6, and 200 GeV sets strong constrain on $y_{\rm loss}(y_{\rm init})$ for $y_{\rm init} \in [0,6]$

color bands indicate 90% credible interval in the posterior

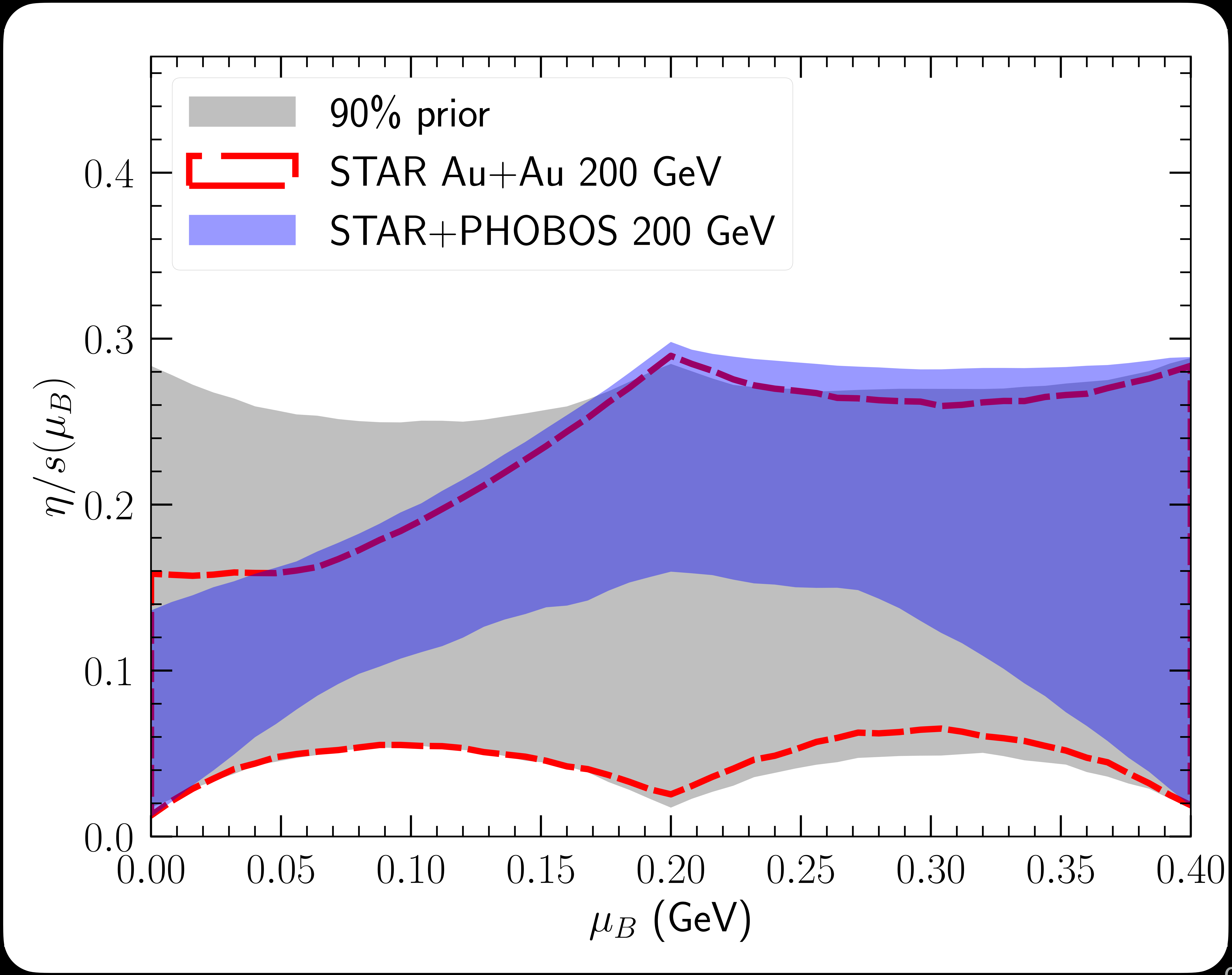
SHEAR VISCOSITY $\eta/s(\mu_B)$



- Mid-rapidity data at 200 GeV can constrain η/s around $\mu_B = 0$

color bands indicate 90% credible intervals of the posterior

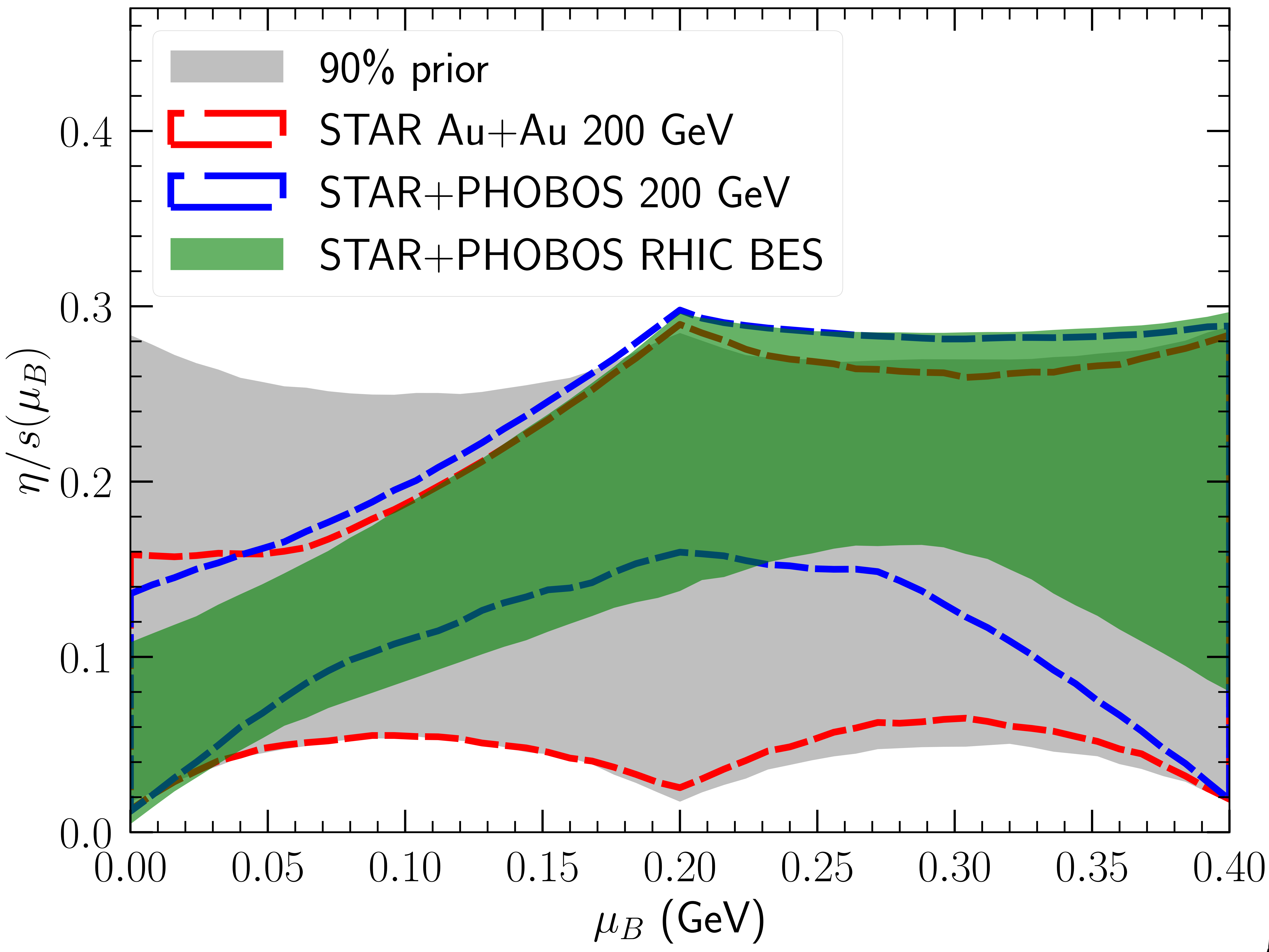
SHEAR VISCOSITY $\eta/s(\mu_B)$



- Mid-rapidity data at 200 GeV can constrain η/s around $\mu_B = 0$
- The $dN^{\mathrm{ch}}/d\eta$ and $v_2(\eta)$ at 200 GeV *significantly* improve the η/s constraint at $\mu_B \sim 0.2$ GeV

color bands indicate 90% credible intervals the posterior

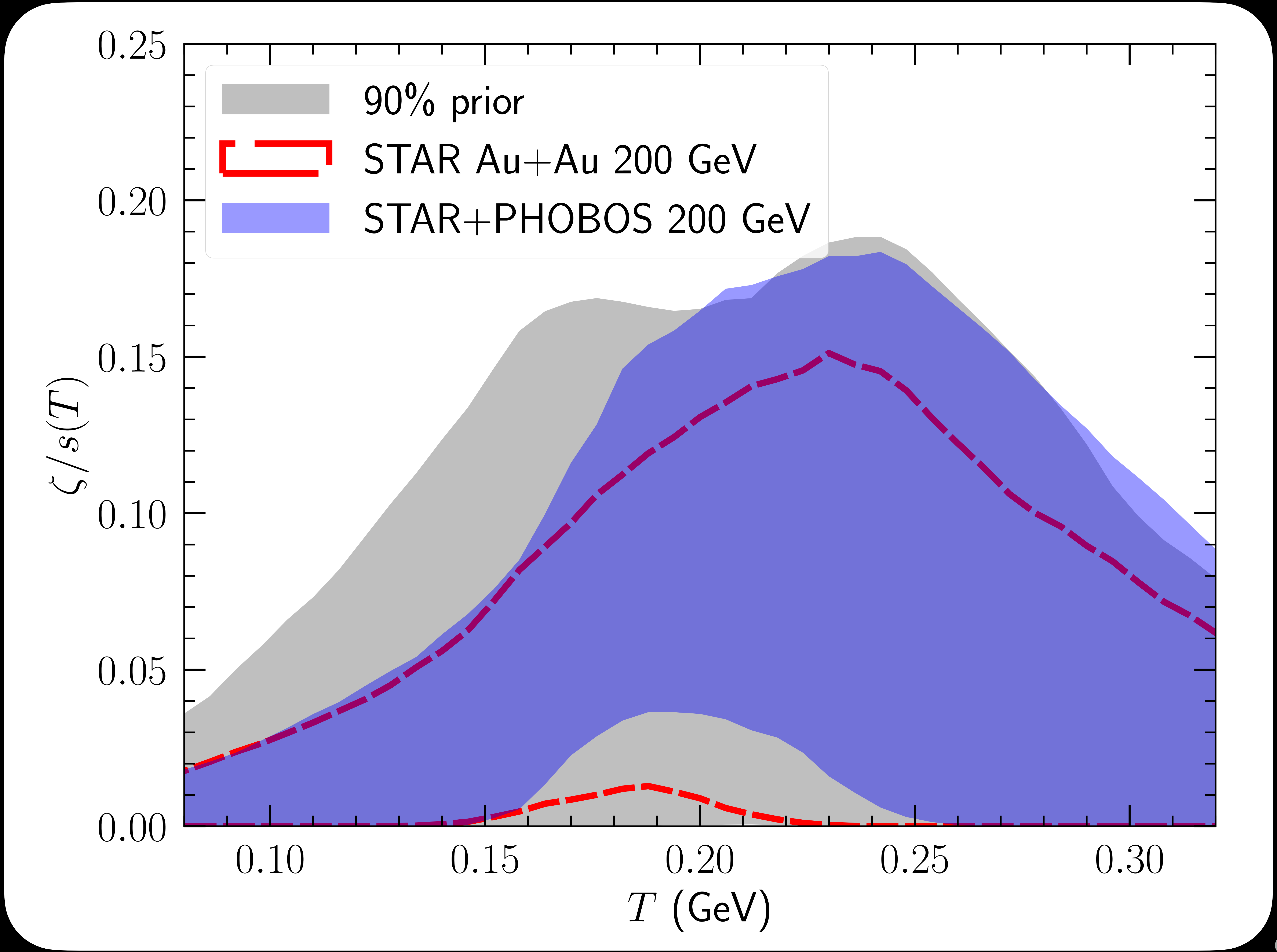
SHEAR VISCOSITY $\eta/s(\mu_B)$



- Mid-rapidity data at 200 GeV can constrain η/s around $\mu_B = 0$
- The $dN^{\rm ch}/d\eta$ and $v_2(\eta)$ at 200 GeV *significantly* improve the η/s constraint at $\mu_B \sim 0.2$ GeV
- The full RHIC BES data (STAR+PHOBOS) shows that the QGP η/s is **larger** at finite μ_B than that at $\mu_B = 0$

color bands indicate 90% credible intervals in the posterior

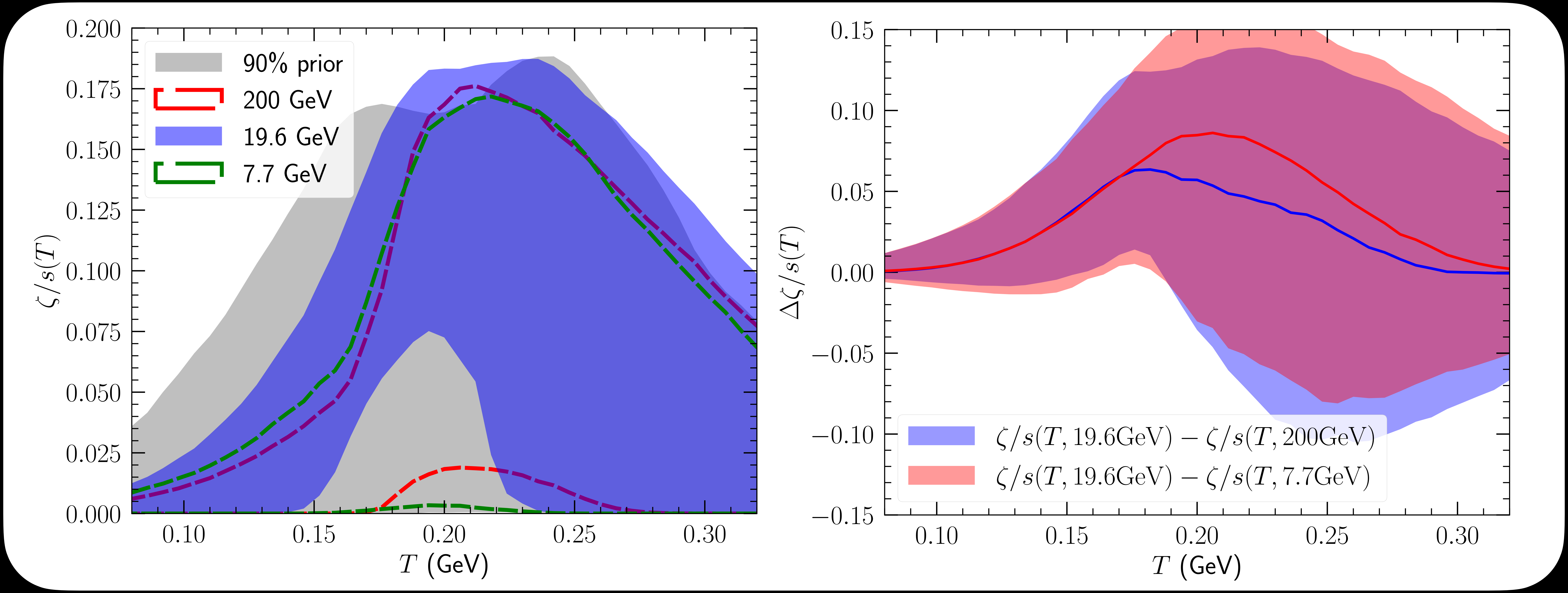
BULK VISCOSITY $\zeta/s(T)$



- Mid-rapidity identified particle yields and their $\langle p_T \rangle$ at 200 GeV set constraints on the temperature dependence of the QGP bulk viscosity
- The additional PHOBOS data shifts the posterior $\zeta/s(T)$ to larger values

color bands indicate 90% credible interval in the posterior

BULK VISCOSITY $\zeta/s(T, \sqrt{s})$



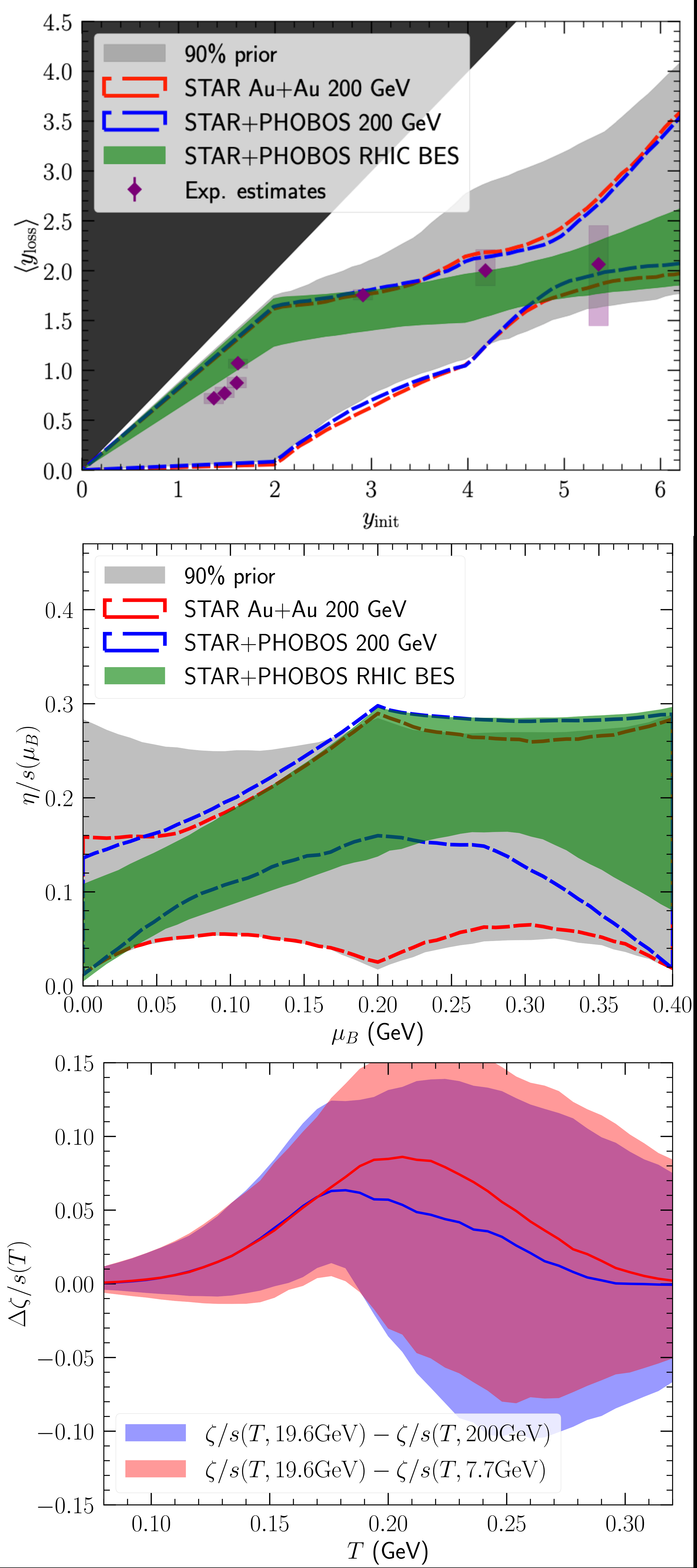
color bands indicate 90% credible interval in the posterior

- Allowing $\zeta/s(T)$ to be an independent function for the three collision energies, our calibration suggests a larger $\zeta/s(T)$ at 19.6 GeV than those at 200 and 7.7 GeV for $T \in [0.15, 0.2]$ GeV

Hint for softening EoS at $\mu_B = 0.2$ GeV?

CONCLUSIONS

- We performed a comprehensive Bayesian Inference study at multiple RHIC BES energies with a state-of-the-art event-by-event (3+1)D hybrid framework
- With the RHIC BES phase I data, robust constraints are obtained for initial state nuclear stopping, QGP $\eta/s(\mu_B)$, and $\zeta/s(T, \sqrt{s})$ for the first time
- The QGP effective η/s is larger at finite μ_B, while $\zeta/s(T)$ shows a hint for non-monotonic energy dependence around $\sqrt{s} = 19.6$ GeV
- Our work marks an important step towards quantitative characterization of the QCD phase structure with the RHIC BES and future FAIR programs



Color bands indicate 90% credible interval in the posterior