# Chiral plasma instabilities in non-Abelian gauge theories

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  [Tashiro, Vachaspati & Vilenkin 12, A. Boyarsky, O. Ruchayskiy, M. Shaposhnikov, 12]
- Crucial for understanding topological properties of non-Abelian gauge theories like QCD far away from equilibrium.

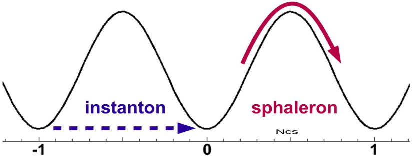
  [Y. Akamatsu & N. Yamamoto 13, Y. Hirono, D. Kharzeev, Y. Yin 15, K. Tuchin 17]

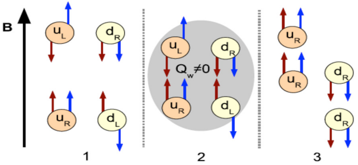
# Chiral Magnetic effect and the role of instabilities

- Early stages of heavy-ion collisions provides a unique laboratory to study topological properties of QCD.
- Local regions of CP odd domains created due to sphaleron transitions in QCD $\rightarrow$ leads to a net axial charge $j_0^a$.
- In presence of an external $U(1)$ magnetic field $\vec{B}$, a vector current generated

[ Kharzeev, McLerran, Warringa 07, Kharzeev, Fukushima, Warringa, 08]

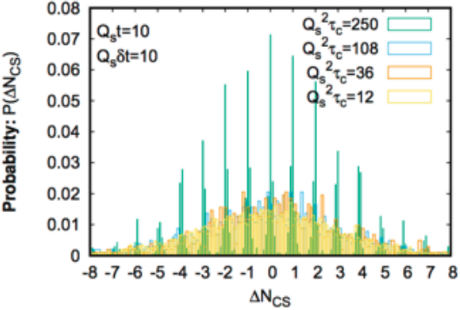
$$\vec{j}^v \propto j_0^a \; \vec{B} \qquad \text{Chiral Magnetic effect}$$





# Chiral Magnetic effect and the role of instabilities

- Early stages of heavy-ion collisions provides a unique laboratory to study topological properties of QCD.
- Strong $SU(3)$ color fields at early times lead to enhancement of sphaleron rates in pre-equilibrium stages. [Mace, Schlichting and Venugopalan, 16].



# Chiral Magnetic effect and the role of instabilities

- Early stages of heavy-ion collisions provides a unique laboratory to study topological properties of QCD.
- Crucial ingredients for understanding anomalous transport → sphaleron rates out of equilibrium [Mace, Schlichting, Venugopalan, 16] + anomalous charge production in the strong fields.
- Since axial charge is not conserved, just the knowledge of initial conditios not sufficient to describe it subsequent evolution.
- It is important to accurately account for chiral charge some of which may get absorbed in the gauge fields.

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- Initial gauge field occupation numbers large → amenable to classical statistical lattice gauge theory techniques.
- Specifically study the effects of strong back-coupling between fermion and gauge fields on the efficiency of helicity transfer.
- We also want to study the entire time evolution of this process which is not possible using kinetic theory.

# CLT for gauge theories on the lattice

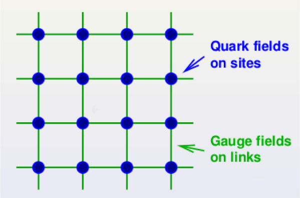
- Gauge theories are Constrained systems.
- The Hamiltonian in $A_0 = 0$ gauge is defined as

$$H_{YM} = \sum_{x,i} \frac{a_i^2}{g_c^2 a^3} \frac{E^i_{a,x} E^i_{a,x}}{2} + \sum_{x,i,j} \frac{a^3}{g_c^2 a_i^2 a_j^2} \ \mathrm{ReTr} \left[ 1 - U^{\square}_{ij}(x) \right] \ ,$$

- Evolution eq. for links and electric field variables are then derived from the lattice Hamiltonian yielding the following set of update rules

$$\partial_{x^0} U_{i,x} = -i \frac{a_i^2}{a^3} \ E^i_{a,x} t^a \ U_{i,x}$$

$$\partial_{x^0} E^i_{a,x} = \text{staples} + \left\langle \hat{j}^i_{a,x} \right\rangle$$



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- Fermion operator $\hat{\psi}$ written in terms of mode-functions [Aarts, Smit, 97, 98]

$$\hat{\psi}_{\mathrm{x}}(t) = \frac{1}{\sqrt{V}} \sum_{\lambda} \left( \hat{b}_{\lambda}(0)\phi^{u}_{\lambda}(t,\mathrm{x}) + \hat{d}^{\dagger}_{\lambda}(0)\phi^{v}_{\lambda}(t,\mathrm{x}) \right) \ ,$$

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- On a $N^3$ lattice $\rightarrow 4N_cN^3$ modes, prohibitively expensive on realistic volumes. In small volumes lattice cut-off effects are more severe!

# Implementation

- We start with non-interacting almost massless fermions with a large chiral imbalance $n_{u,v} = \frac{1}{\mathrm{e}^{\beta(\mathrm{E_p} \pm \mu_\mathrm{h})+1}}$ , $\beta\mu_h = 8$ .

- At $t = 0$ the electric field and links are a random superposition of plane waves to describe their initial vacuum fluctuations.

- The helicity transferred from the fermion to gauge sector during the evolution of electric fields

$$\partial_{x^0} E^i_{a,\mathrm{x}} = -\left(\frac{g_c a^3}{a_i}\right)\frac{\delta H}{\delta A^a_{i,\mathrm{x}}} + e^2 N_f J^i_a .$$

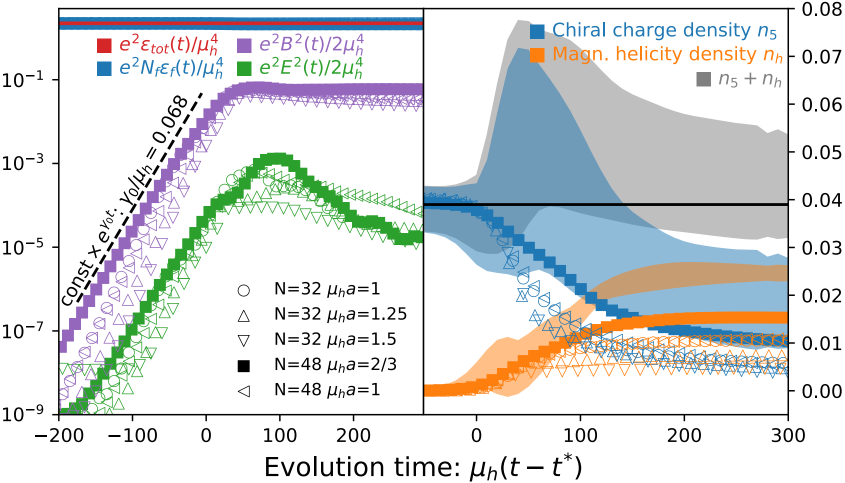
- We choose $e^2 N_f = 64$ to mimic the strong interactions.

[Mace, Mueller, Schlichting, SS, 19]

# Chiral plasma instabilities in Abelian gauge theories

[Mace, Mueller, Schlichting, SS, PRL 124, 191604 (2020)]

Strong chirality transfer but not much energy lost in the process!



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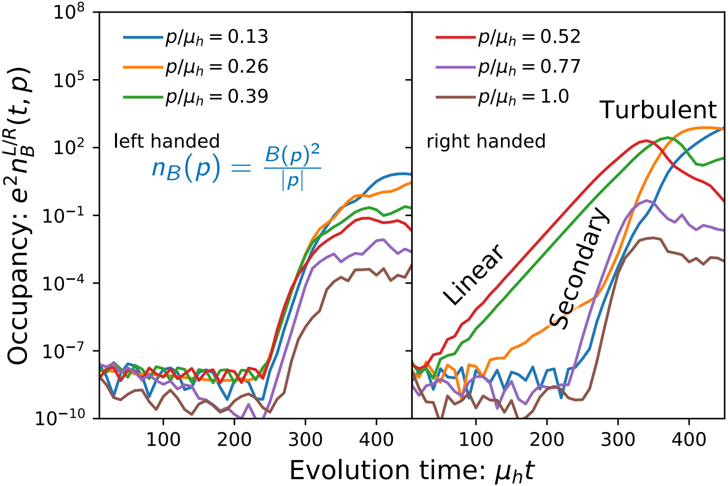
Linear growth of R-H modes starts immediately!



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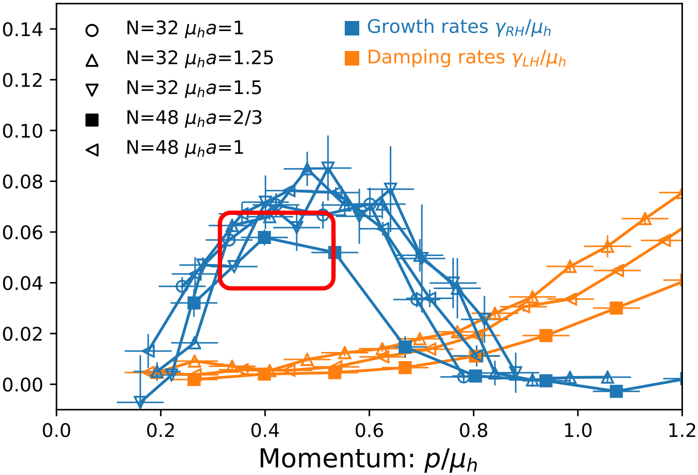
Intermediate R-H modes grows at the fastest rate!



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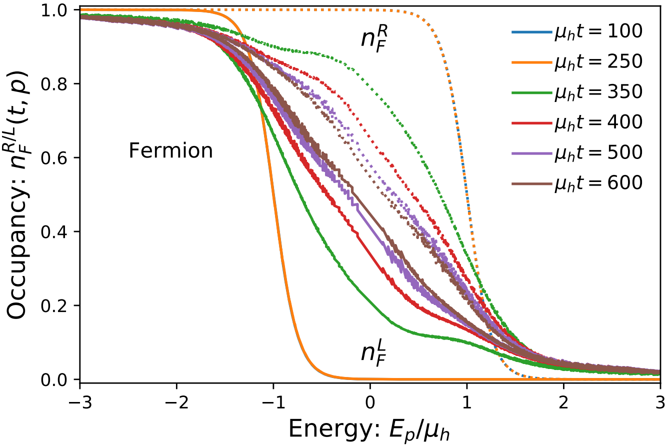
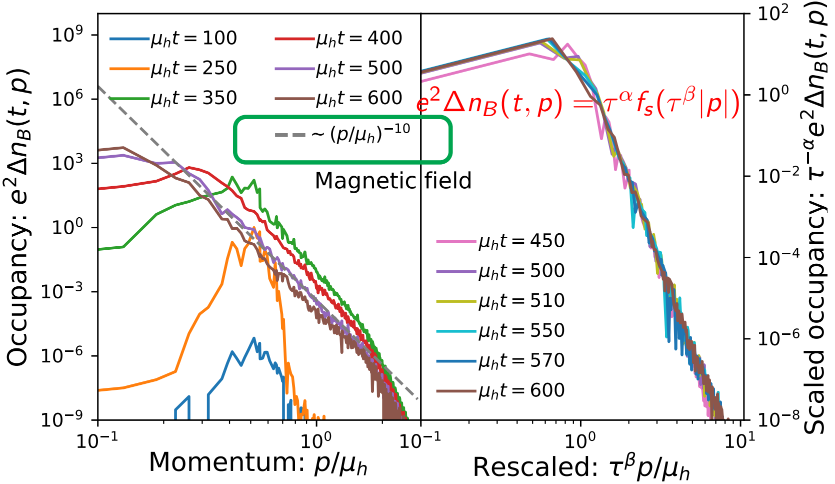
Fermions heated up significantly during growth of secondary instabilities.



# Self-similar cascade at late times

Scaling exponents $\alpha = 1.1(5)$, $\beta = 0.4(1)$ , $\tau = \mu_h.t - 375$ , Helicity consv.

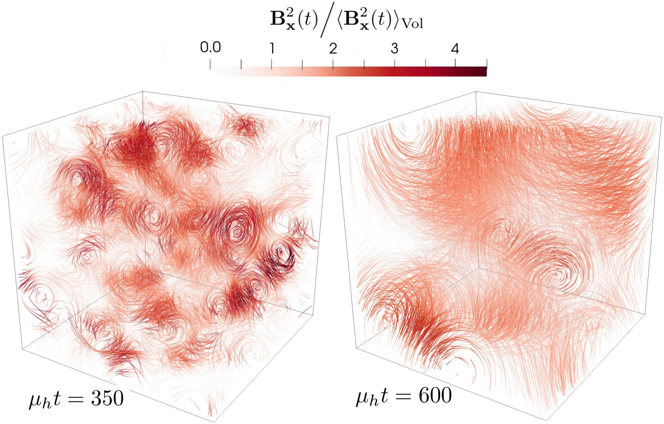


$e^2 \Delta n_B(t,p) = \tau^{\alpha} f_s(\tau^{\beta}|p|)$

Magnetic field

# Formation of large-scale magnetic fields

Inverse cascade from $l = \mu_h^{-1}$ to larger scales $l = \tau^{\beta}\mu_h^{-1}$.



# Our results in the context of recent studies

- Within chiral kinetic theory the growth of unstable modes $0 \leq k \leq \alpha\mu_5/\pi$ are characterized by the exponent $\gamma = \frac{4\alpha\mu_5}{\pi^2 m_D^2} k^2 (1 - \frac{\pi k}{\alpha\mu_5})$ [Y. Akamatsu & N. Yamamoto 13].

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- Our study suggests that under strong back-coupling the onset of instabilities and turbulence is quicker.
- An earlier study observed that backreaction of fermions on the electromagnetic field prevents the system from acquiring chirality imbalance[P. V. Buividovich & M. V. Ulybyshev 16] → we observe an efficient transfer of chirality. This may be due to better control over chiral properties of fermions on a finite lattice.

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- For finite mass, time scale for chirality flipping may exceed the one needed to create helical magnetic fields. Cannot sustain the imbalance without an energy source, for example, via a turbulent mechanism [D. B. Kaplan, S. Reddy & S. Sen, 17].

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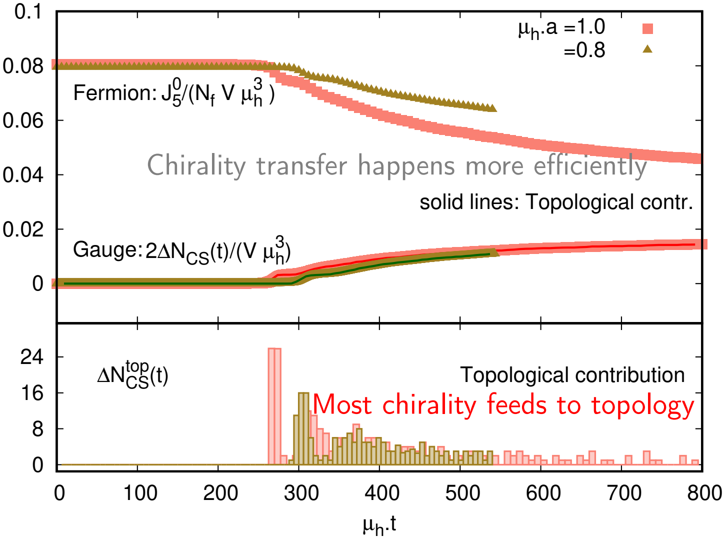
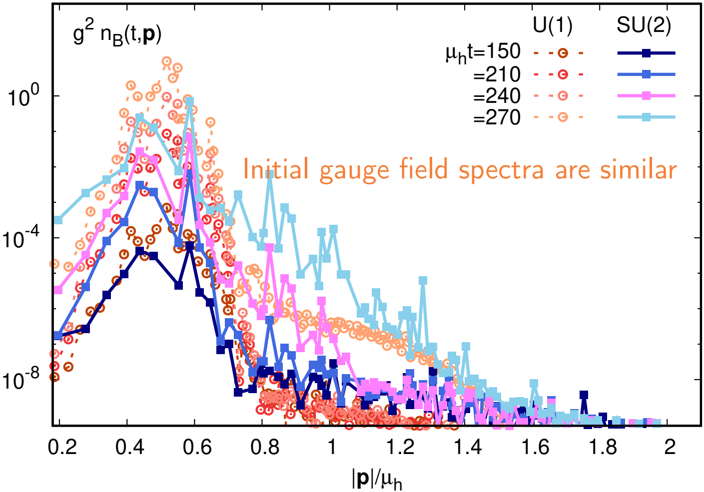
- Our prelim. study shows this could be a possibility but we need to perform runs with heavier quark mass.
- For Abelian plasma, collisions of fermions do not affect the instability when $\mu_5 \gg T$.
- In non-Abelian plasma the time scale of the onset of instabilities is expected to be comparatively faster [Y. Akamatsu & N. Yamamoto 13] → Need to perform a similar long-time evolution of chirally imbalance non-Abelian plasma with strong back-coupling.

# What happens in a non-Abelian SU(2) plasma?



[Schlichting, SS, arxiv:2211.11365]

# SU(2) vs U(1) plasma



Initial gauge field spectra are similar

[Schlichting, SS, arxiv:2211.11365]

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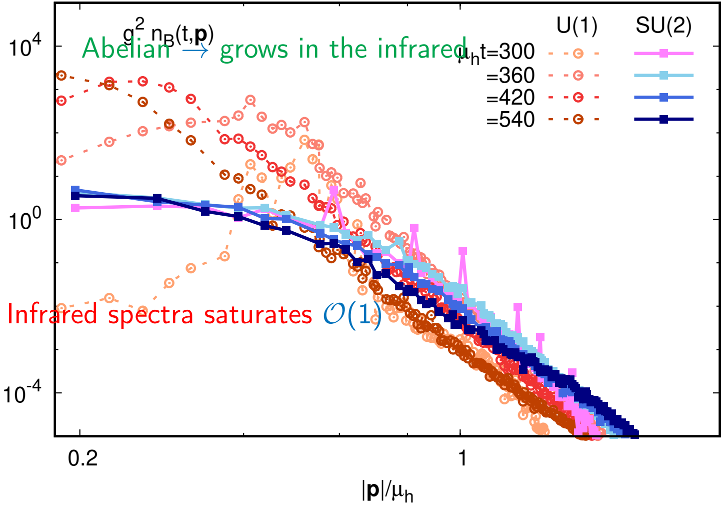

Abelian → grows in the infrared

Infrared spectra saturates $\mathcal{O}(1)$

[Schlichting, SS, arxiv:2211.11365]

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- The initial chirality washout due to thermal sphaleron transitions occurs at a time-scale $\tau_{\mathrm{sph}} = \frac{\chi_t}{2\Gamma_{\mathrm{sph}} T}$ [ L. D. McLerran, E. Mottola, and M. E. Shaposhnikov, 91].

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- Hence the chirality washout is expected to be on $\tau_{\text{sph}} \sim 5/T$ which is not larger than the lifetime of the fireball.
- Erasure of the initial pockets of chirality imbalance will be compensated by creation of tiny regions of net-chirality due to thermal sphaleron transitions → for a realistic study one needs to disentagle the modes $|\vec{p}| \sim g^2 T$ from the modes at scale $\sim T$ [D. Bodeker, 98].

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- Unlike the Abelian case, the non-Abelian modes do not show an infrared enhancement. Most of the initial chirality absorbed by the topology of the gauge fields.
- Including all orders quantum fluctuations in gauge sector is extremely challenging!