

Charm Fluctuations in $(2+1)$ -flavor QCD at finite Temperature

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Motivation

- ▶ Strong interaction matter undergoes a chiral crossover at $T_{pc} = 156.5 \pm 1.5$ MeV. [HotQCD Collaboration, 2019]
- ▶ In heavy-ion collisions, relevant degrees of freedom change from partonic to hadronic in going from high temperature phase to temperatures below T_{pc} .

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- ▶ Existence of not-yet-discovered open charm states can be predicted by comparing Lattice results with HRG calculations.
- ▶ Signals of exotic charm states such as tetraquarks can shed light on how quarks arrange inside the bound states.

Hadron Resonance Gas (HRG) model

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- ▶ Charmed baryons and mesons separately contribute to the partition function of HRG, which in turn reflects in contributions to the pressure. $P_C(T, \vec{\mu})/T^4 = M_C(T, \vec{\mu}) + B_C(T, \vec{\mu})$.

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- ▶ $\hat{\mu}_X = \mu/T$, $X \in \{B, Q, S, C\}$.

Pressure calculation using HRG

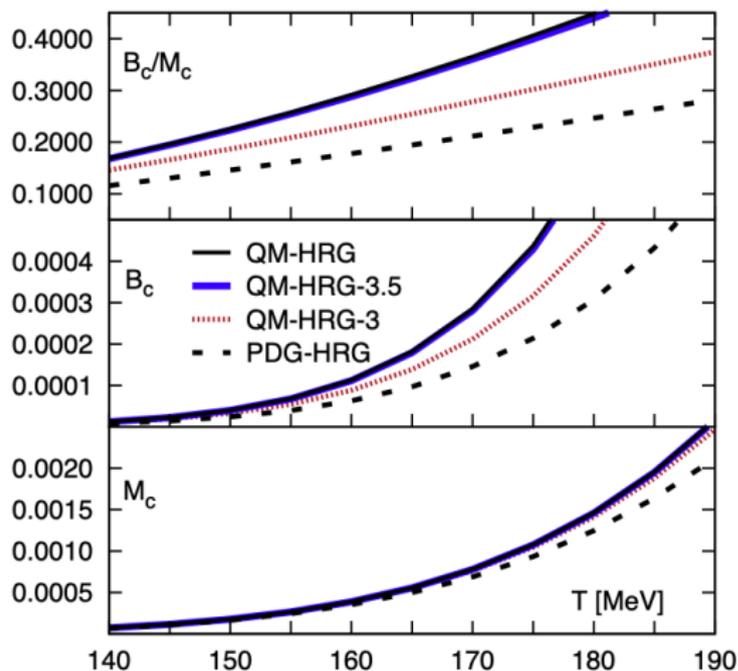


Figure: [HotQCD Collaboration, 2014]

Generalized susceptibilities of conserved charges

$$M_C(T, \vec{\mu}) = \frac{1}{2\pi^2} \sum_i g_i \left(\frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C) .$$

- ▶ $K_2(x) \sim \sqrt{\pi/2x} e^{-x} [1 + \mathcal{O}(x^{-1})]$. If $m_i \gg T$, then contribution to P_C will be exponentially suppressed.
- ▶ Λ_c^+ mass ~ 2286 MeV, Ξ_{cc}^{++} mass ~ 3621 MeV. At T_{pc} , contribution to B_C from Ξ_{cc}^{++} will be suppressed by a factor of 10^{-3} in relation to Λ_c^+ .

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$$\chi_{klmn}^{BQSC} = \frac{\partial^{(k+l+m+n)} [P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C) / T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \partial \hat{\mu}_S^m \partial \hat{\mu}_C^n} \Bigg|_{\vec{\mu}=0} .$$

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- ▶ At present, we have gone upto fourth order in calculating various cumulants.

Simulation details

- ▶ Partition function of QCD with 2 light, 1 strange and 1 charm quark flavors is :

$$\mathcal{Z} = \int \mathcal{D}[U] \{\det D(m_l)\}^{2/4} \{\det D(m_s)\}^{1/4} \{\det D(m_c)\}^{1/4} e^{-S_g}.$$

This can be used to calculate susceptibilities in the BQSC basis.

- ▶ We used (2+1)-flavor HISQ configurations generated by HotQCD collaboration for $m_s/m_l = 27$ and $N_\tau = 8$.
- ▶ We treated charm sector quark in the quenched approximation.
- ▶ We made use of 500 random vectors to calculate various traces per configuration, expect for $\text{Tr} \left(D^{-1} \frac{\partial D}{\partial \mu} \right)$, for which we used 2000 random vectors.

Charm Mass Tuning on Lattice

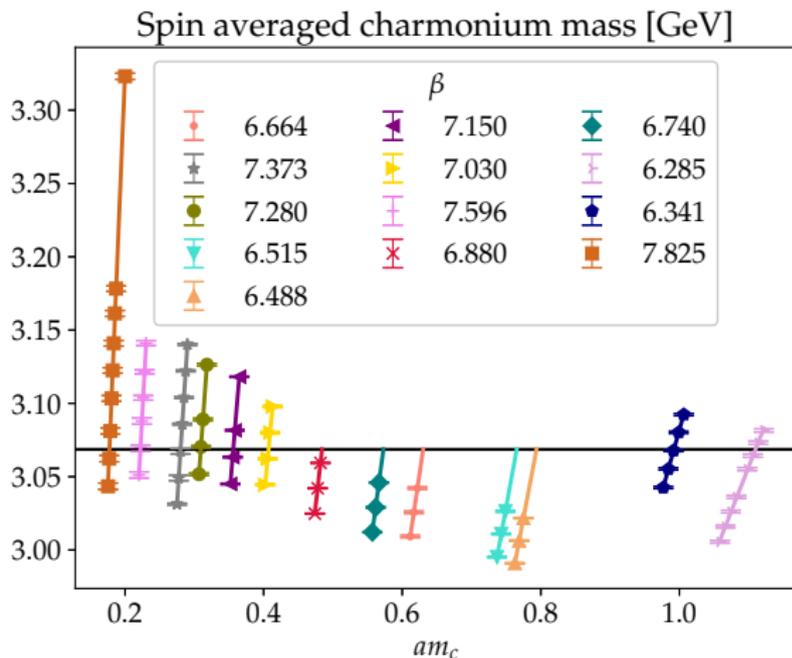
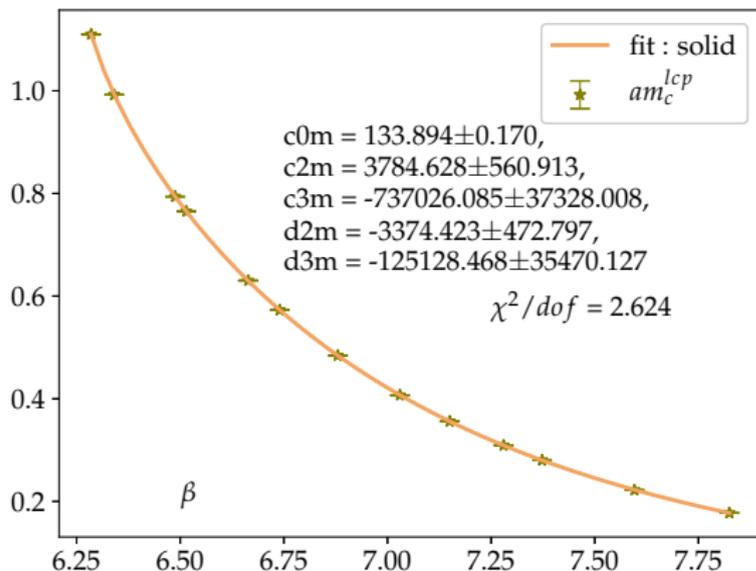


Figure: f_K scale setting from [D.Bollweg et al., 2021] has been used. Black horizontal line corresponds to mass $[(3m_{J/\psi} + m_{\eta_{c\bar{c}}})/4] = 3.06865$ GeV.

Charm Mass Parametrization

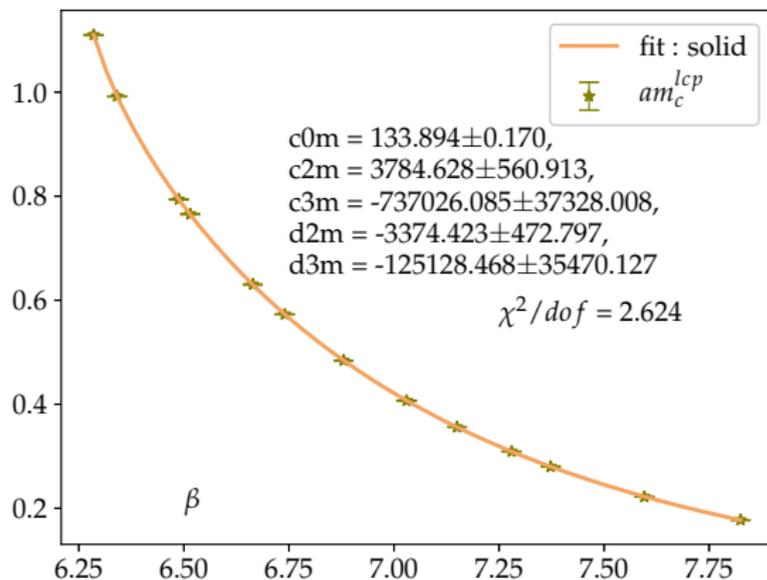
$$am_c^{lcp} = \left(\frac{20b_0}{\beta} \right)^{\frac{4}{9}} c_{0m} f(\beta) \left[\frac{1 + c_{2m} \left(\frac{10}{\beta} \right) f^2(\beta) + c_{3m} \left(\frac{10}{\beta} \right) f^3(\beta)}{1 + d_{2m} \left(\frac{10}{\beta} \right) f^2(\beta) + d_{3m} \left(\frac{10}{\beta} \right) f^3(\beta)} \right].$$

$f(\beta)$ is the 2-loop QCD beta function.



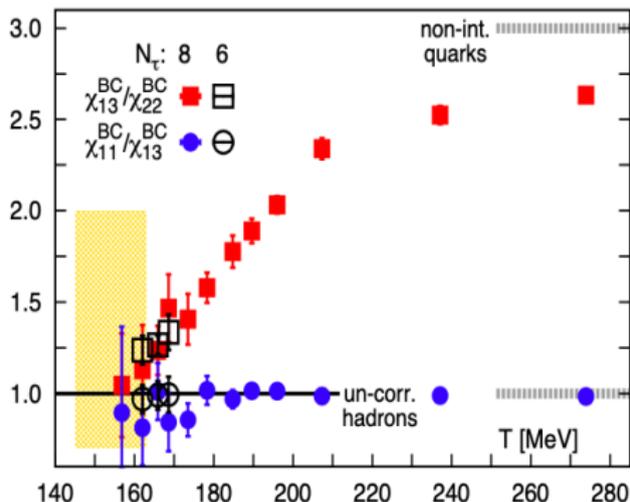
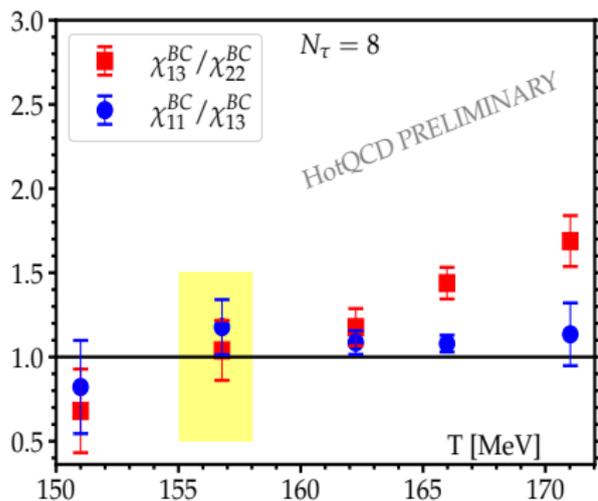
Charm Mass Parametrization

m_c/m_s quoted by PDG is $11.76^{+0.05}_{-0.10}$. In the continuum we obtain, $m_c/m_s = 11.64$.
 am_s parametrization from [HotQCD Collaboration, 2014] is used for the comparison.



Results

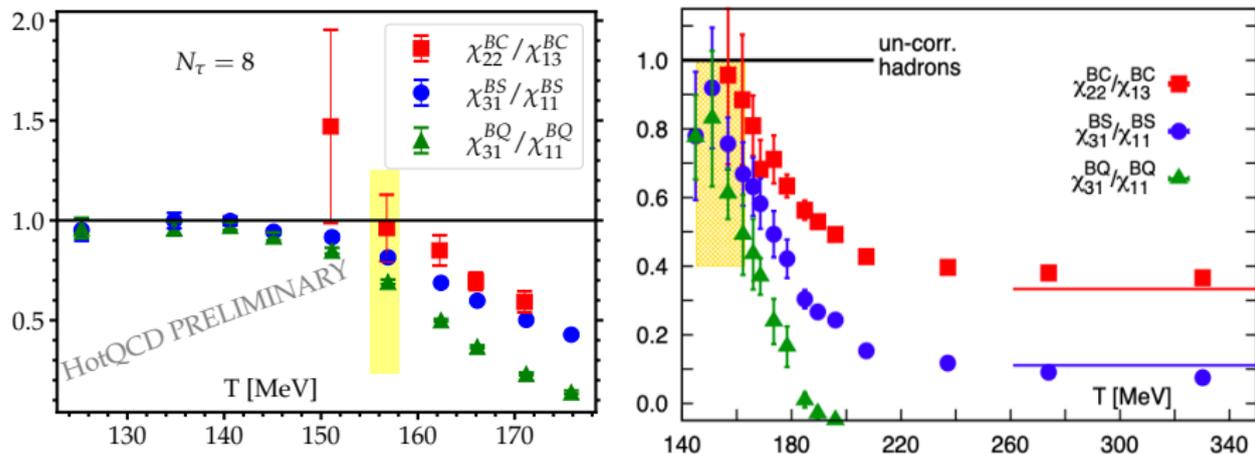
Near $T = 156$ MeV, left figure has on order of magnitude larger statistics. Right figure is the older HotQCD analysis [A. Bazavov et al., 2014].



Crossover region has shrunk and we have better control over the errors there. With further addition of statistics, results will look more precise.

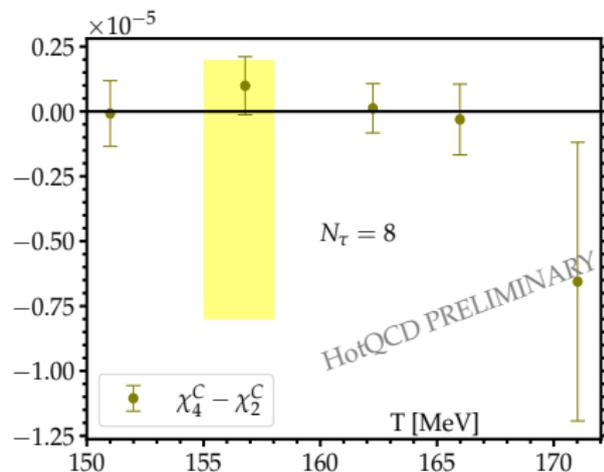
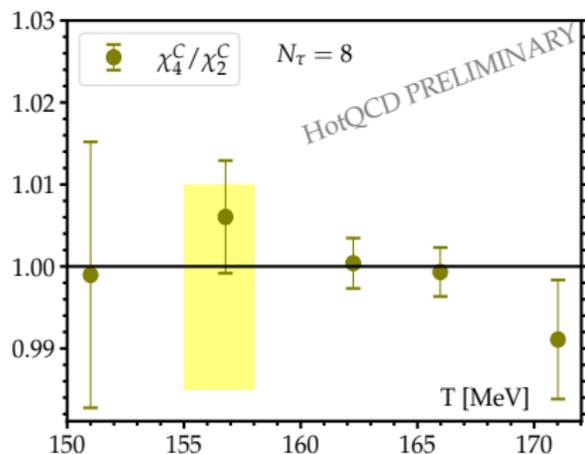
Results

Right figure is the older HotQCD analysis. BQ and BS ratios in the left figure were calculated by making use of all the available statistics. This is clearly reflected in the errors.



BQ and BS correlations separated by even $\hat{\mu}_B$ derivatives should be unity in the HRG phase.

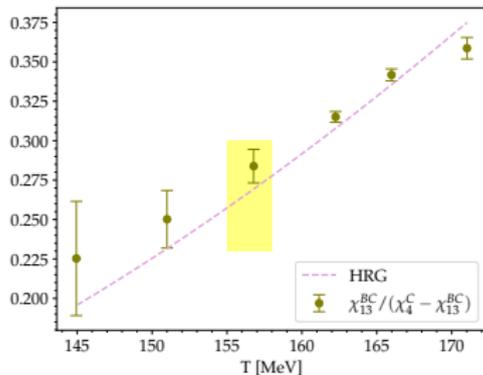
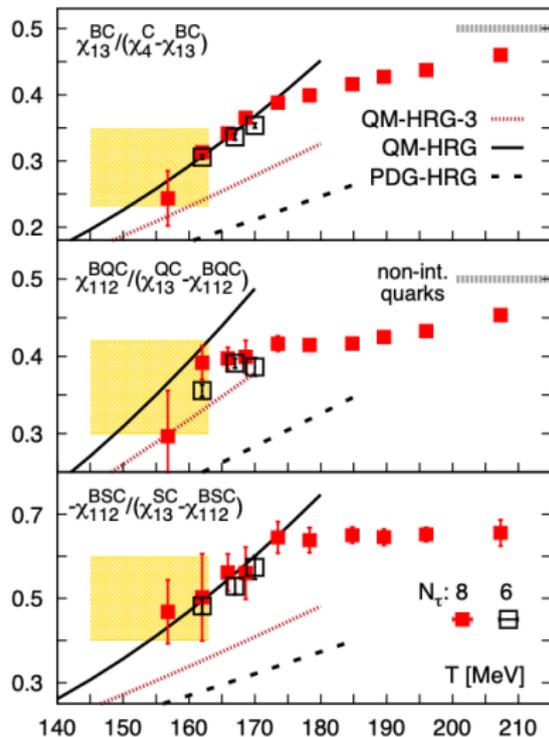
$|C| > 1$ states



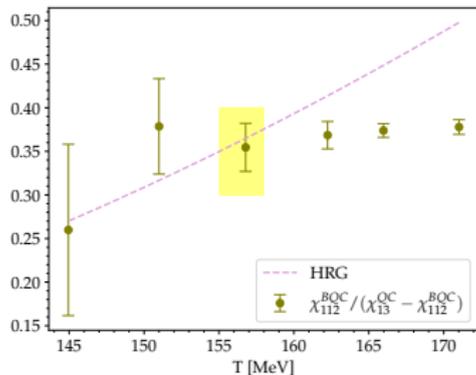
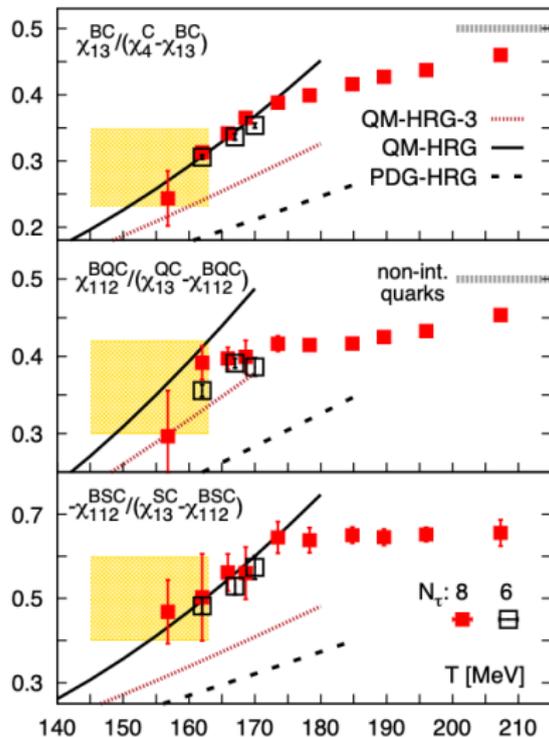
We also want to explore the multiple charm sector. Ratio on left indicates that the contribution to partial pressure from $|C| > 1$ sector is indeed very small and difference on right quantifies it. For HRG,

$$\chi_4^C - \chi_2^C = 12B_{C,2} + 72B_{C,3}.$$

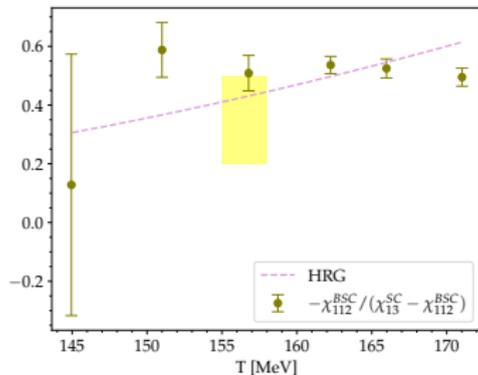
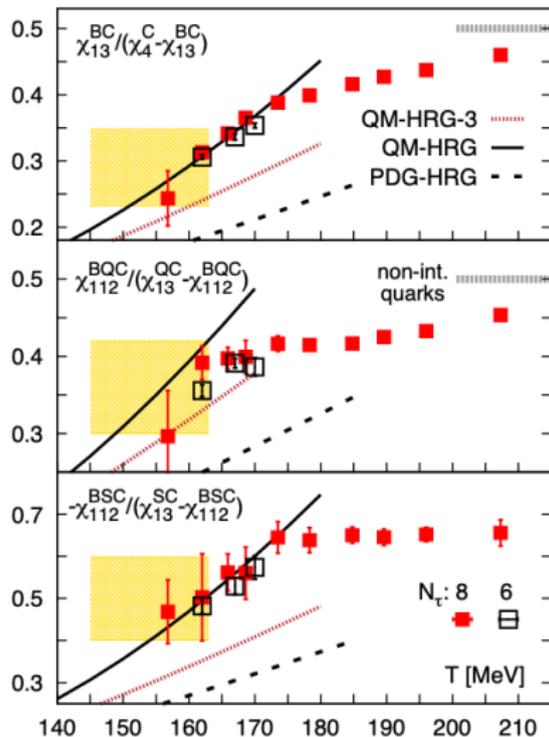
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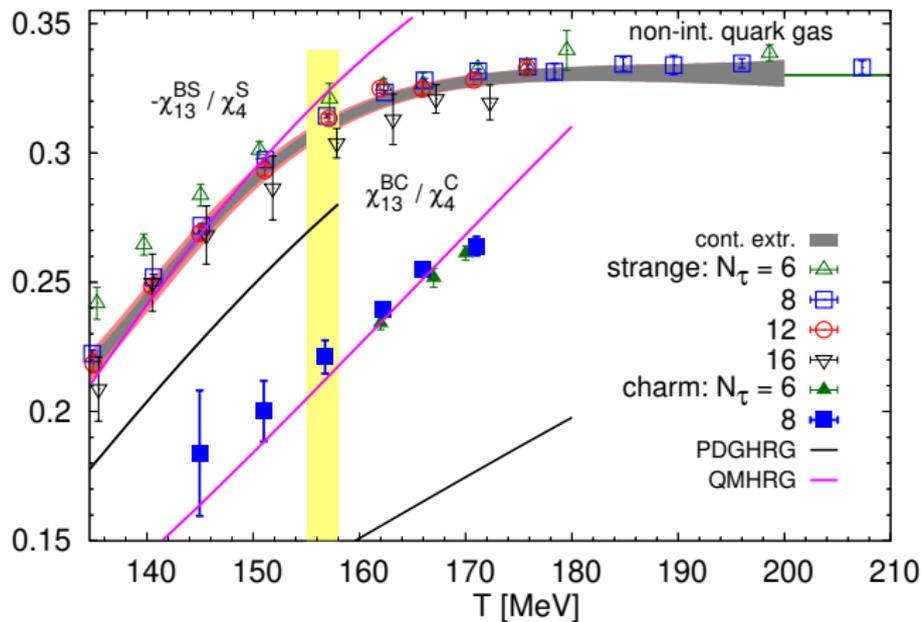
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Conclusions & Outlook

- ▶ Until now only one-third of the available statistics was used. Our aim is to make use of the available high statistics.

$N_\tau = 8$				$N_\tau = 12$			
β	m_l	T[MeV]	#conf.	β	m_l	T[MeV]	#conf.
6.175	0.003307	125.28	1,471,861				
6.245	0.00307	134.84	1,275,380	6.640	0.00196	135.24	330,447
6.285	0.00293	140.62	1,598,555	6.680	0.00187	140.80	441,115
6.315	0.00281	145.11	1,559,003	6.712	0.00181	145.40	416,703
6.354	0.00270	151.14	1,286,603	6.754	0.00173	151.62	323,738
6.390	0.00257	156.92	1,602,684	6.794	0.00167	157.75	299,029
6.423	0.00248	162.39	1,437,436	6.825	0.00161	162.65	214,671
6.445	0.00241	166.14	1,186,523	6.850	0.00157	166.69	156,111
6.474	0.00234	171.19	373,644	6.880	0.00153	171.65	144,633
6.500	0.00228	175.84	294,311	6.910	0.00148	176.73	131,248

Conclusions & Outlook

- ▶ We see deviations from HRG model in the open charm sector near $T_{pc} = 156.5 \pm 1.5$ MeV. This is consistent with the older HotQCD analysis.
- ▶ Our analysis shows that there are missing states in the PDG record.
- ▶ We want to go to $N_\tau = 12, 16$ to quantify the cut-off effects.