Energy hierarchies governing quarkonium dynamics in Heavy Ion Collisions

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[Work in progress] with Balbeer Singh (TIFR)

Motivation

Separation of scales in quarkonia

- Quarkonia are non-relativistic bound states of heavy quark (Q) and anti-quarks ($\bar{Q}$)
- The large mass M provides a high energy scale
- They are characterized by the energy scales $M \gg \frac{1}{r} \gg E_b$ where r is the bound state size and E_b is the binding energy
- $M \gg \frac{1}{r}$ ensures that the heavy quarks in the bound states are non-relativistic
- $1/r \gg E_b$ means that at leading order in M, the $\bar{Q}Q$ interaction is a potential

Separation of scales

- For example, using potential models one finds for Bottomonia (Υ states):
  - $M \sim 4.7\text{GeV}$
  - $1/r \sim 1\text{GeV}$
  - $E_b \sim 0.5\text{GeV}$
- In QGP, they should be compared with $T \sim [0.2, 0.5]\text{GeV}$

Motivation

- Assume that $1/r \gg E_b, T$
- Depending on whether $E_b \gg T$, $T \gg E_b$ different processes dominate the dynamics of quarkonia in the QGP *[Brambilla, Ghiglieri, Vairo, Petreczky, Escobedo, Soto. (2008, 2010, 2011, 2013), Thermal pNRQCD] [Brambilla, Ghiglieri, Rapp, Riek, Du, Emerick, He (2010, 2011, 2012, 2017), scattering dynamics]*
- Our goal in this work is to numerically compare the relative contributions of the various processes in a simple setting

Formalism

The lagrangian

- The lagrangian in terms of the singlet and the octet wavefunctions is (pNRQCD)

$$\mathcal{L}_{pNRQCD} = \int d^3 r \mathrm{tr}\{S^\dagger[i\partial_0 - h_s]S + O^\dagger[iD_0 - h_o]O \\ + (O^\dagger \mathrm{r}\cdot g\mathrm{E}S) + \frac{1}{2}O^\dagger\{\mathrm{r}\cdot g\mathrm{E}, O\} + \ldots\}$$

- r is the relative separation between the Q and $\bar{Q}$
- E is the chromo-electric field
- $h_{o,s} = -\frac{\nabla^2}{M} + V_{s/o}(\mathrm{r})$

The potentials in the static limit

- V_s, V_o in the static limit is well understood. In this limit the kinetic energy of the heavy quarks is ignored. The relevant energy scale is much less than T
- We know that V_s, V_o are complex (thermal decay and dissociation). Real and imaginary parts have been calculated in weak coupling for both singlet *[Laine et. al. (2007); Brambilla et. al. (2008)]*, and octet *[Akamatsu (2013); Brambilla et. al. (2017)]* channels
- The real and imaginary parts for V_s also now well studied on the lattice although the imaginary part is challenging to compute *[Petreczky, Rothkopf, Weber (2018); Burnier, Kaczmarek, Rothkopf (2015); Burnier, Rothkopf (2017); Bala, Datta (2019); HOTQCD (2021)]*
- Less is known about V_o. The real and imaginary parts of the V_o have been computed in quenched QCD *[Bala, Datta (2021)]*
- Key finding, at large r, the real parts of V_s and V_o approach each other. This is what happens in weak coupling also

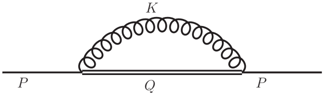
V_s, V_o at E_b scale

- The static approximation misses an important dynamic associated with energy transfer to and from the bound state
- Our goal is to explore this phenomenon
- Model assumption: V_s, V_o at this scale are dominantly real. Assume that thermal losses arise from dynamics $\sim E_b$
- Can be rigorously justified only in the limit $T \ll E_b$
- For $T \sim E_b$ it is just a model assumption
- In our model we take V_s to be the real part of the potential validated on the lattice *[Krouppa, Rothkopf, Strickland (2017)]*
- For V_o a well motivated choice is

$$V_o = \frac{g^2}{2N_c} \frac{e^{-m_D r}}{4\pi r} + V_s(\infty)$$

[Bala, Datta (2021)]. Today's results are without the screening

The self-energy correction to S



- The diagram in pNRQCD was first evaluated in *[Brambilla, Ghiglieri, Vairo, Petreczky (2008)]*
- The gluon is a "dressed" (in medium) gluon

The self-energy correction

- The self energy correction is given formally by

$$\Sigma_{11}(P, \mathrm{r}) = -\, i g^2 \frac{C_F}{3} \mathrm{r}^i \int \frac{d^4 k}{(2\pi)^4} [\langle \mathrm{EE} \rangle]_{11}(k^0, \mathrm{k}) \frac{i}{q^0 - \frac{\mathrm{q}^2}{2M} - h_s + i\epsilon} \mathrm{r}^i$$

- $P = (p^0, \mathrm{p})$ is the center of mass momentum of the initial state (assumed at rest with $\mathrm{p} = 0$), K is the gluon momentum, and $Q = P - K$
- $\mathrm{q}^2/(2M)$ is the centre of mass kinetic energy (recoil) of the octet. $|\mathrm{q}| \sim T$, $q^0, h_s \sim E_b$. Thus the recoil is suppressed by T/M and can be ignored
- $[\langle \mathrm{EE} \rangle]_{11}(k^0, \mathrm{k}) = \int e^{ik^0 \cdot t' - i\mathrm{k} \cdot \mathrm{r}'} \langle \mathrm{E}^a(\mathrm{r}', t') \mathrm{E}^a(0, 0) \rangle_{11}$ (not shown the adjoint connections that we will not need in leading order)

$\Im m[\Sigma]$

- Σ_{11} has both a real and an imaginary piece. The imaginary piece is directly related to decay and we focus on it here
- In weak coupling at leading order

$$
\begin{aligned}
\Im m\Sigma(P,r) = {} & \frac{C_F g^2}{3} \times \\
& \mathrm{r}_k \int \frac{d^4K}{(2\pi)^3} \delta(q^0 - h_o)[k_0^2 \rho_{ii}(K) + k_i^2 \rho_{00}(K)] \\
& [\theta(q_0)(\theta(-k_0) + f(k_0)) + \theta(-q_0)(\theta(k_0) + f(k_0))]\mathrm{r}_k
\end{aligned}
$$

- Here $\rho_{00}(K)$ and $\rho_{ii}(K)$ are components of the gluon spectral functions which are related to the longitudinal and the transverse spectral densities,

$$\rho_{\mu\nu}(K) = \mathcal{P}^L_{\mu\nu}\rho_L(K) + \mathcal{P}^T_{\mu\nu}\rho_T(K)\ .$$

$\Im m[\Sigma]$

- For gluon absorption, $k^0 < 0$ and we get,

$$\Im m[\Sigma(p^0, 0, r)] = \frac{C_F g^2}{3} \mathrm{r}^k \tilde{\kappa}(k^0) \mathrm{r}^k$$

-

$$\tilde{\kappa}(k^0) = \frac{C_F g^2}{3} \int \frac{d^3k}{(2\pi)^3} [k_0^2 \rho_{ii}(K) + k_i^2 \rho_{00}(K)] \theta(q_0) f(k_0)|_{k^0 = p^0 - q^0}$$

- A related quantity $\kappa = \langle EE \rangle_{\mathrm{symm}} =$
$\frac{C_F g^2}{6} \int \frac{d^3k}{(2\pi)^2} \frac{dk_0}{2\pi} \delta(k_0)(1 + 2f(k_0)) \left(k^2 \rho_{00}(k_0, k) + k_0^2 \rho_{ii}(k_0, k) \right)$
- $\lim_{k^0 \to} \tilde{\kappa}(k^0) = \kappa$ which is the momentum diffusion constant

The EE correlator

- In the $k^0 = 0$ (static) limit, or more generally if $k^0 \ll T$ (for example if the binding energy is much smaller than T), only the longitudinal mode contributes and $\tilde{\kappa}$ is the same as the momentum diffusion constant *[Brambilla et. al. (2019); previous talk]*

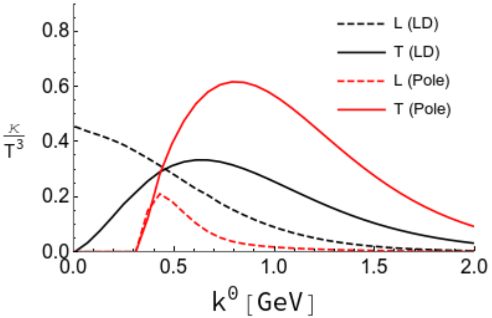
The EE correlator

- To compute EE we just need ρ_T and ρ_L
- To get some intuition, for the longitudinal components, the Landau Damping (LD) contribution has the structure

$$\rho_L(|k^0| < \mathrm{k}) = \frac{2[\Im m \Pi_L]}{[\mathrm{k}^2 + \Re e \Pi_L]^2 + [\Im m \Pi_L]^2}$$

- The pole contribution $\rho_L(|k^0| > \mathrm{k})$ is activated in the time like regime (pole contribution). Similarly one computes the tansverse contribution in the space-like and the time like regime

The EE spectral function



- $g = 2$ *[Kaczmarek, Zantow (2004); Burnier, Rothkopf (2017)]*, $T = 250$MeV.

What do we know about κ?

- At $T \approx 1.5 T_c$ lattice calculations give *[Brambilla et. al. (2020)]*
$$2 < \frac{\kappa}{T^3} < 2.7$$
- *[Banerjee et. al. (2011, 2022); Francis et. al. (2012); Brambilla et. al. (2020), Ding et. al. (2011, 2021)]*
- Well known that leading order weak coupling calculations underestimate κ by a factor of roughly 5
- Do not know enough about finite frequency form of $\tilde{\kappa}$ from non-perturbative calculations *[Ilgenfritz et. al. (2018)]*

Γ

- From $\Im m[\Sigma]$ one can calculate the decay width of singlet states
- For eg., longitudinal Landau Damping (LD) gives,

$$
\begin{aligned}
\Gamma_L &= 2\langle\phi|\Im m\Sigma^L_{11}|\phi\rangle \\
&= \frac{C_F g^4 N}{6\pi}\int \frac{f(k_0)d^3p}{(2\pi)^3}\int_0^\infty \frac{d^3k}{(2\pi)^3}\frac{k\theta(k-k_0)}{(k^2-\Re\Pi_{00})^2+\Im\Pi_{00}^2} \\
&\times \int_{\frac{k+k_0}{2}}^\infty dqq^2\left(2+\frac{k^4}{4q^4}-\frac{k^2}{q^2}\right)(f(q-k_0)-f(q)) \\
&\times |\langle\phi|r|o\rangle|^2.
\end{aligned}
$$

- Similar expressions for longitudinal pole, transverse LD, and transverse pole contributions

Γ

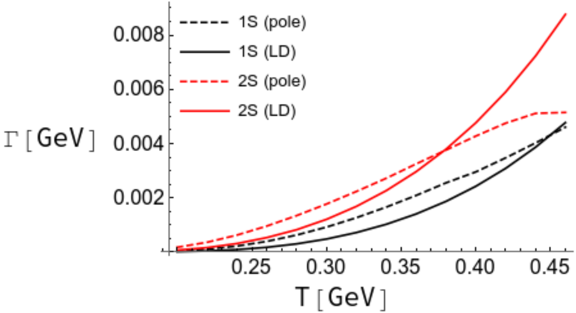
- In $T \gg E_b$, $E_b \gg T$ *[Brambilla, Ghiglieri, Vairo, Petreczky, Escobedo, Soto. (2008, 2010, 2011, 2013)]* the results for the width are well known. However, for $T \sim E_b$ one needs to evaluate the spectral function, and the states $|s\rangle$ and $|o\rangle$ numerically.
- Calculating decay widths in this regime was one of our goals

S and O states

- Finally, we need the states $|s\rangle$ and $|o\rangle$
- Actually, the states are not pure states but we need to follow the evolution of the density matrix. *See previous talk*
- Our goal in this work is to understand the relative contributions between the various processes in a simple setting. We model $|s\rangle$, $|o\rangle$ states as instantaneous eigenstates of h_s and h_o (adiabatic approximation)
- The model is now completely specified and we can now calculate the decay widths

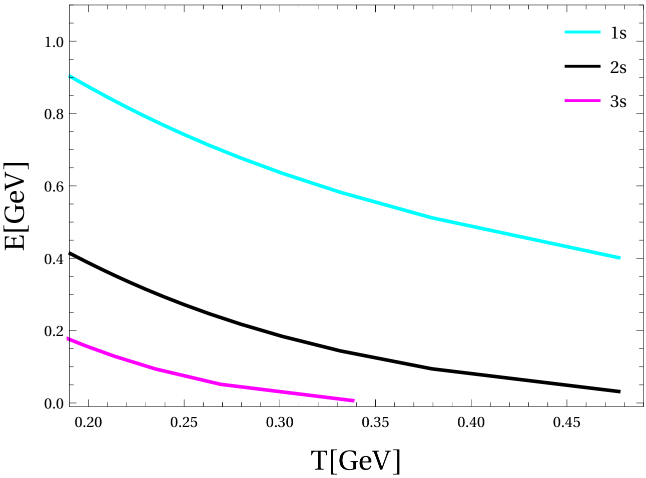
Results (preliminary)

Comparison of the contributions



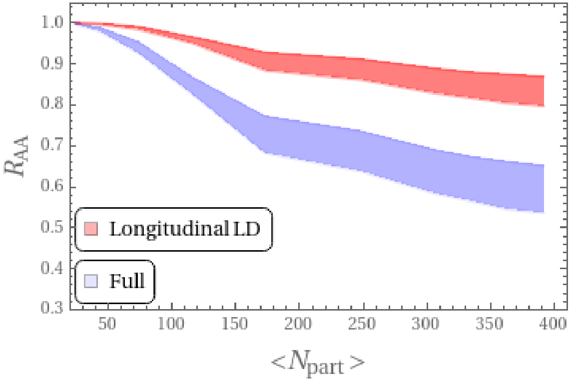
- For intuition, taking $E_b(1S) = 0.7\text{GeV}$ and $E_b(2S) = 0.25\text{GeV}$ as illustrative numbers and keeping the wavefunction T independent
- To see the overall effect during the evolution, we follow the thermal evolution in a Bjorken expanding medium

Binding energies



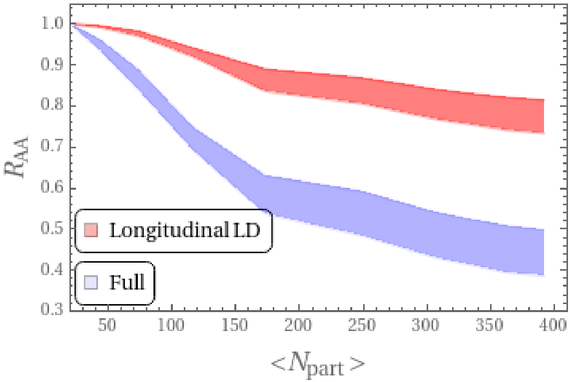
- Binding energy of the states with T

$R_{AA}(1S)$



- The uncertainty band shows the uncertainty associated with the Bjorken evolution and its connection with N_{part}

$R_{AA}(2S)$



- Surprisingly similar fractional contribution from Landau damping for $1S$ and $2S$

Implication for open quantum system approaches

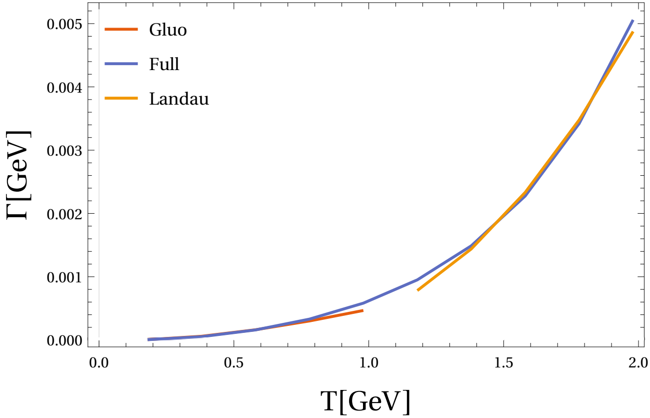
- $\langle EE \rangle$ at finite k^0 contributes substantially, for the $1s$, $2s$ states
- For open quantum system approaches, this has an important implication for the Markovian approximation
- If $\langle EE \rangle[k^0 = 0]$ dominates losses, then in the master equation for the system density matrix, one can assume that the system evolution is slow and hence the evolution evolution has no memory. This gives rise to the Lindblad equation where the evolution operator is local in time *[Akamatsu (2014, 2017, 2020), Brambilla et. al. (2016, 2017, 2021)]*
- If finite frequency effects are relevant *[eg. Sharma, Tiwari (2017)]*, the evolution equation involves a convolution in time and the evolution is not Markovian

Summary

- In a simple setting we see that $\langle EE \rangle$ at finite k^0 contributes substantially for the $1s$ states and (somewhat surprisingly) the $2s$ states
- The higher excited states are close enough to the continuum and $\langle EE \rangle$ may be governed dominantly by Landau damping
- Non-perturbative effects could make the relative contribution of finite frequency correlations weaker, as we know that κ is substantially under-predicted in weak coupling but the spectral function at high frequencies should be perturbative
- Higher order corrections in E_b/T to Lindblad maybe helpful in capturing some of these effects *[previous talk]*
- Finally, formation dynamics not well captured in this classical model and one needs to do a OQS study to pin these effects

Backup slides

Limiting cases



- In $T \gg E_b$, $E_b \gg T$ and taking g small (0.2), approach results from *[Brambilla et. al. (2008, 2013)]*