Aspects of the gluonic structure of the proton from (lattice) QCD

Phiala Shanahan, MIT

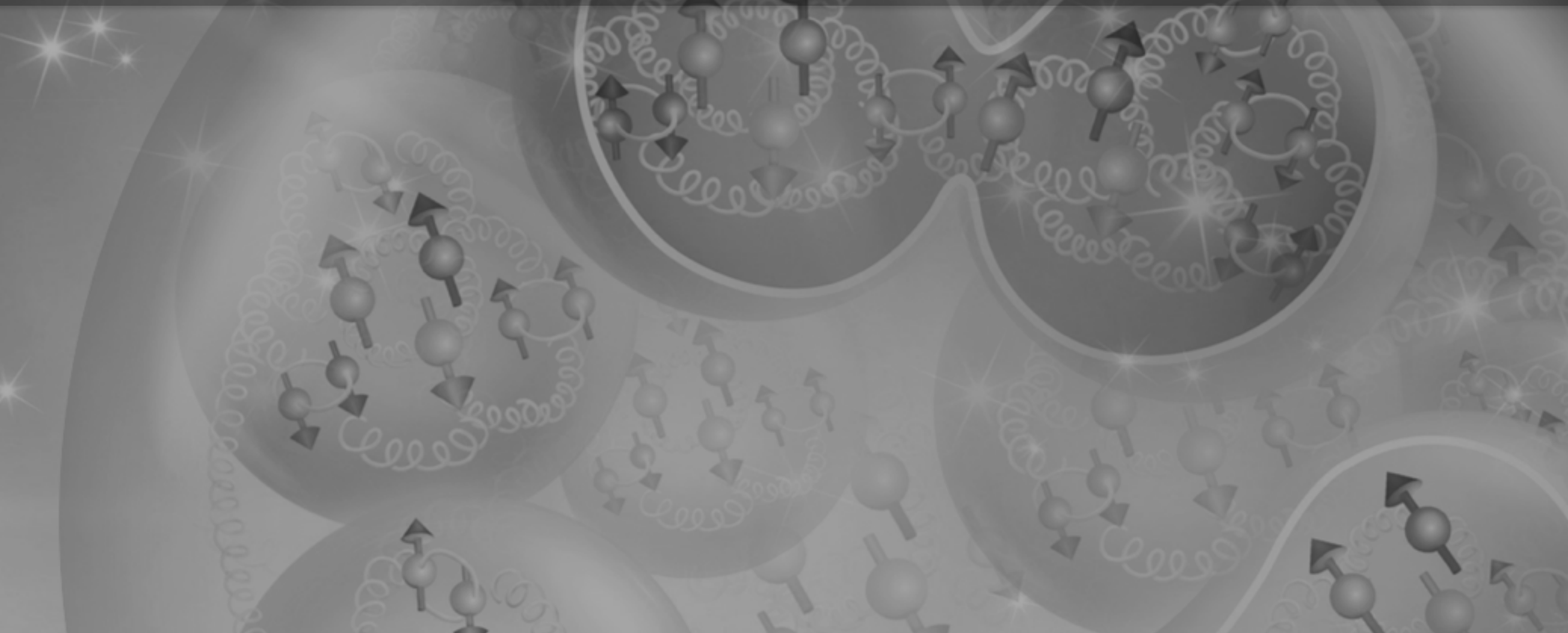


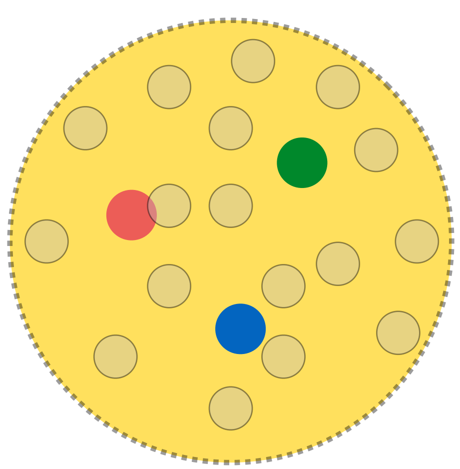
Image Credit: 2018 EIC User's Group Meeting

Massachusetts Institute of Technology

The gluon structure of the proton

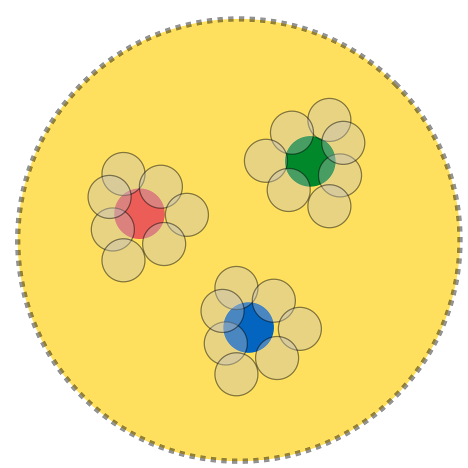
How does the gluon radius of a proton compare to the quark/charge radius?

MIT Bag Model



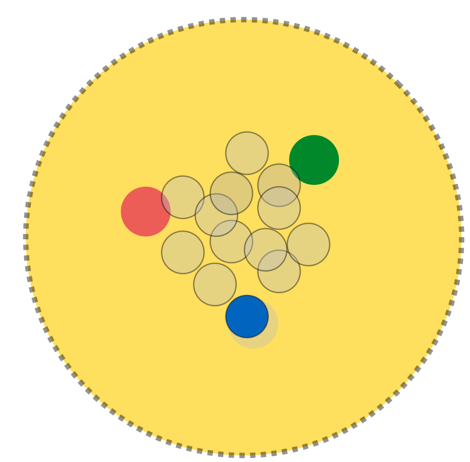
gluon radius > charge radius

Constituent Quark Model



gluon radius ~ charge radius

Lattice QCD with heavy quarks



gluon radius < charge radius

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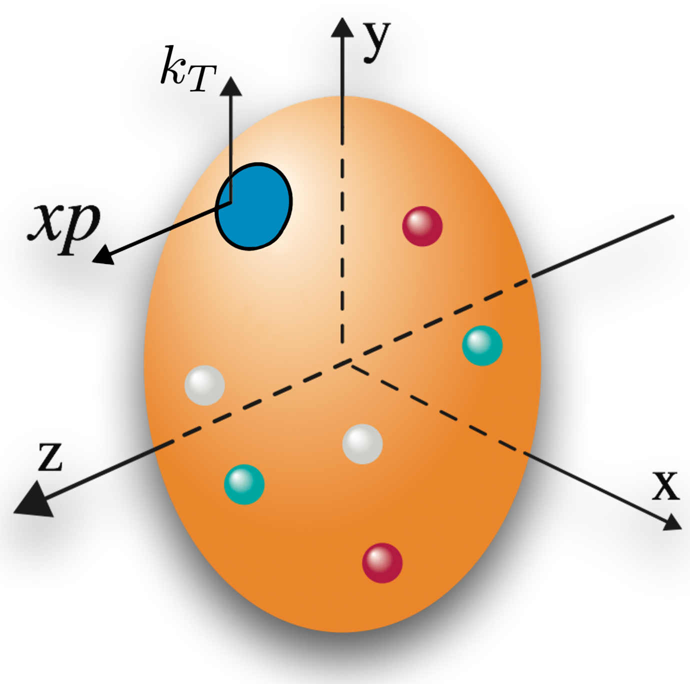
Partonic structure of the proton

Understanding the quark and gluon structure of matter



Three-dimensional partonic structure of the proton

PDF
$f_{q/H}(x)$
longitudinal

TMD
$f_{q/H}(x, k_T)$
+ transverse

$f_{q/H}(x, b_T)$
+ impact parameter

Fourier transform
$\xi = 0$

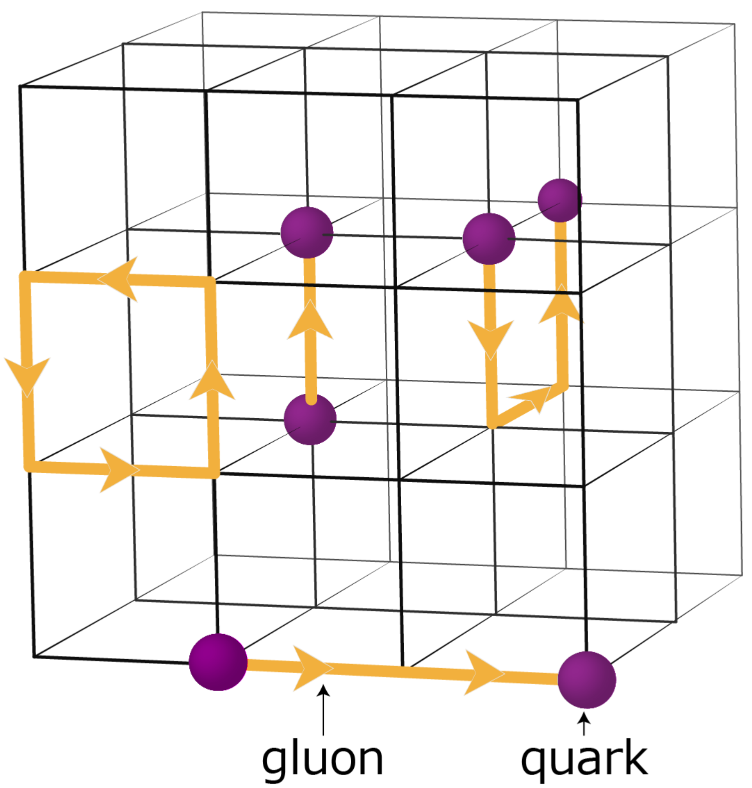
GPD
$H_{q/H}(x, \xi, \Delta_T^2)$

Community overview:
Constantinou et al.,
"2020 PDFLattice Report"
arXiv:2006.08636

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Discretise QCD onto 4D space-time lattice
- Approximate QCD path integral using Monte-Carlo methods and importance sampling
- Run on supercomputers and dedicated clusters
- Take limit of vanishing discretisation, infinite volume, physical quark masses



Lattice QCD

Numerical first-principles approach to non-perturbative QCD

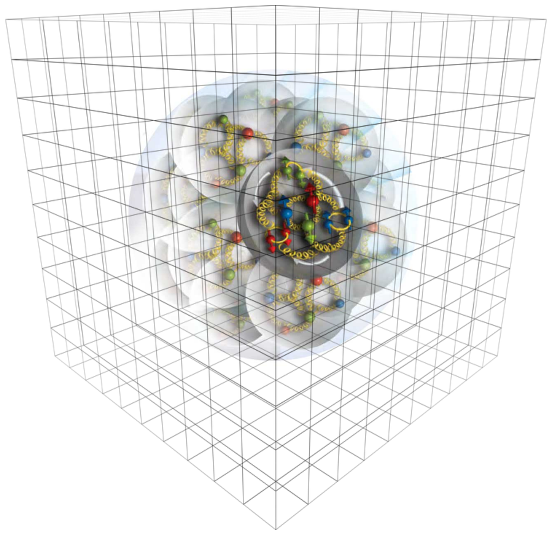
INPUT

Lattice QCD action has same free parameters as QCD: quark masses, α_S

- Fix quark masses by matching to measured hadron masses, e.g., π, K, D_s, B_s for u, d, s, c, b
- One experimental input to fix lattice spacing in GeV (and also α_S), e.g., $2S - 1S$ splitting in Υ, or f_π or Ω mass

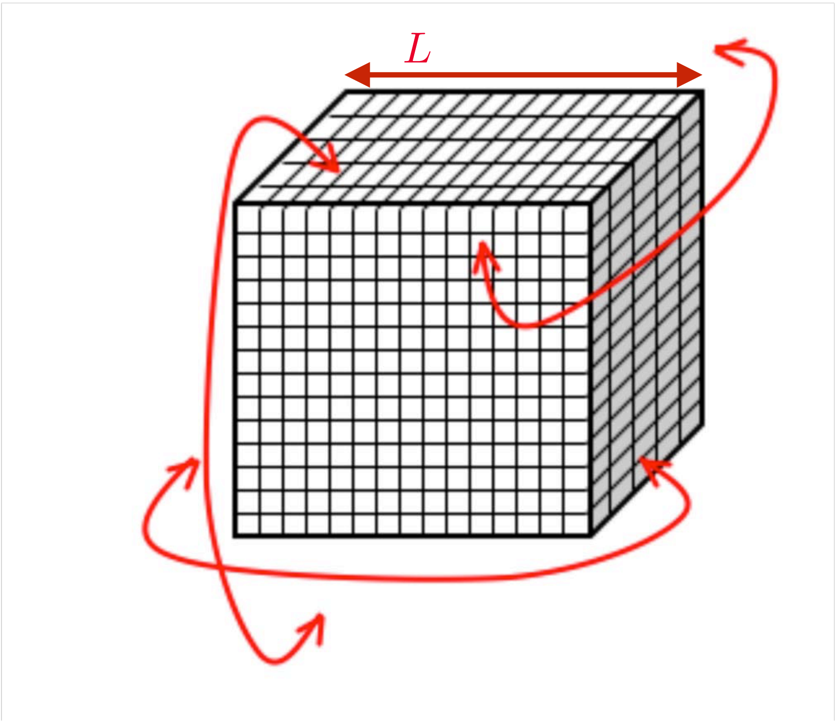
OUTPUT

Calculations of all other quantities are QCD predictions



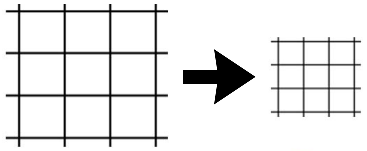
Uncertainties in lattice QCD

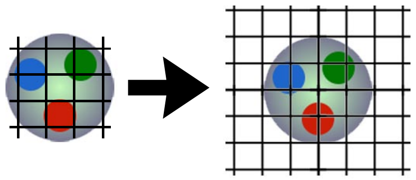
Differences between calculations



- **Uncertainties on a single "ensemble"**
  - Data volume (statistical sampling)
  - Fitting methodology (e.g., treatment of "excited-state contamination")
  - Renormalisation procedure (i.e., matching from lattice quantities to $\overline{\mathrm{MS}}$)

- **Uncertainties of result "extrapolated to physical point"**
  - Continuum extrapolation: range of lattice spacings
  - Infinite-volume extrapolation: range of volumes
  - Tuning of the bare quark masses: values of pion, kaon masses, extrapolation/interpolation thereof, isospin-breaking, …

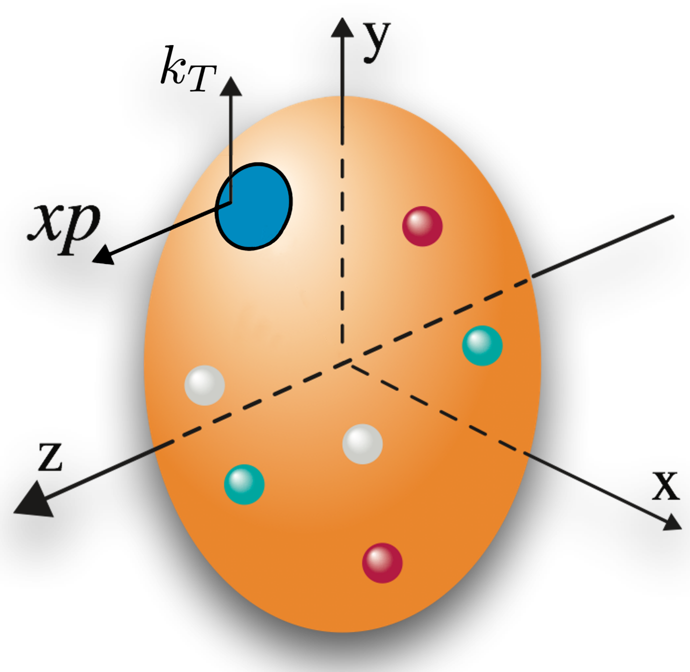




$m_\pi \to 140\mathrm{MeV}$

Partonic structure of the proton

Understanding the quark and gluon structure of matter



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Generalised Form Factors

Generalised form factors encode moments of Generalised Parton Distribution functions (GPDs)

e.g., Gravitational form factors

- Encode "graviton scattering" from the nucleon
- Related to leading-twist chiral-even GPDs

Gluon field-strength tensor

$$\int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p', s' | G_a^{\{\mu\alpha}(-\frac{\lambda}{2}n) \left[\mathcal{U}^{(A)}_{[-\frac{\lambda}{2}n, \frac{\lambda}{2}n]}\right]_{ab} G_{b\alpha}^{\ \ \nu\}}(\frac{\lambda}{2}n) | p, s\rangle$$

$$= \frac{1}{2}\left(H_g(x,\xi,t)\bar{U}(p',s')P^{\{\mu}\gamma^{\nu\}}U(p,s) + E_g(x,\xi,t)\bar{U}(p',s')\frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{2M}U(p,s)\right) + \dots ,$$

GPDs(Bjorken x, skewness, mom transfer)

$$\int_0^1 \mathrm{d}x\ H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t)\,, \qquad \int_0^1 \mathrm{d}x\ E_g(x,\xi,t) = B_g(t) - \xi^2 D_g(t)$$

Gluon gravitational form factors

$$\Delta_\mu = p'_\mu - p_\mu$$
$$P_\mu = (p_\mu + p'_\mu)/2$$
$$t = \Delta^2$$
$$n^2 = 0$$

Gravitational FFs encode EMT

- **Gravitational form factors** describe matrix elements of Energy-Momentum Tensor
  e.g., traceless gluon EMT for nucleon:

$$\langle p', s' | G^a_{\{\mu\alpha} G^{a\alpha}{}_{\nu\}} | p, s\rangle = \bar{U}(p', s') \left(A_g(t)\, \gamma_{\{\mu} P_{\nu\}} + B_g(t) \frac{i\, P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D_g(t) \frac{\Delta_{\{\mu}\Delta_{\nu\}}}{4M_N} \right) U(p, s)$$

Gluon field-strength tensor

Gluon gravitational form factors

$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2 \quad t = \Delta^2$

- Sum rules of gluon and quark GFFs in forward limit
  - Momentum fraction. $A_a(0) = \langle x \rangle_a$ → $\sum_{a=q,g} A_a(0) = 1$
  - Spin $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t))$ → $\sum_{a=q,g} J_a(0) = \frac{1}{2}$
  - D-terms $D_a(0)$ less known but equally fundamental!
    - $D_a(t)$ GFFs encodes pressure and shear distributions

D-term from JLab DVCS

Experimental determination of DVCS D-term and extraction of proton pressure distribution

[Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)]

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r)$$

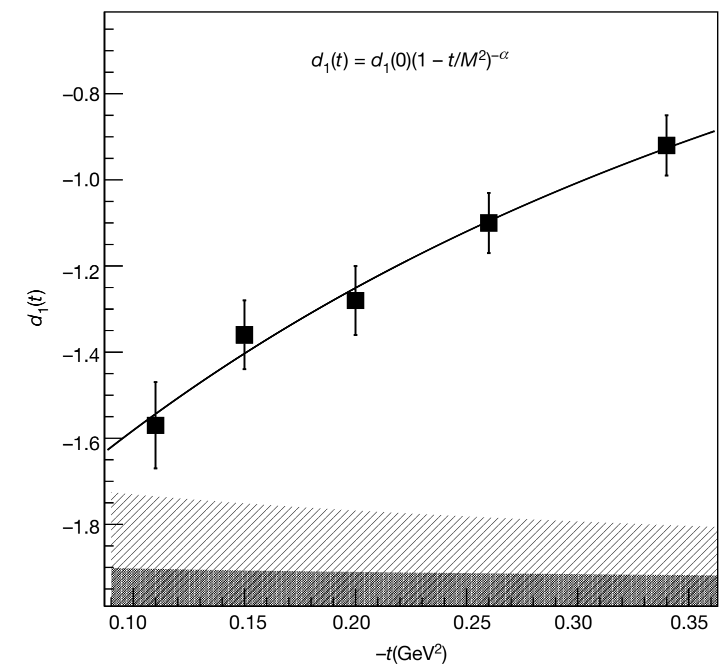
- Peak pressure near centre ~10^{35} Pascal, greater than pressure estimated for neutron stars
- Key assumptions: gluon D-term same as quark term, tripole form factor model, $D_u(t,\mu) = D_d(t,\mu)$

EXP + LQCD

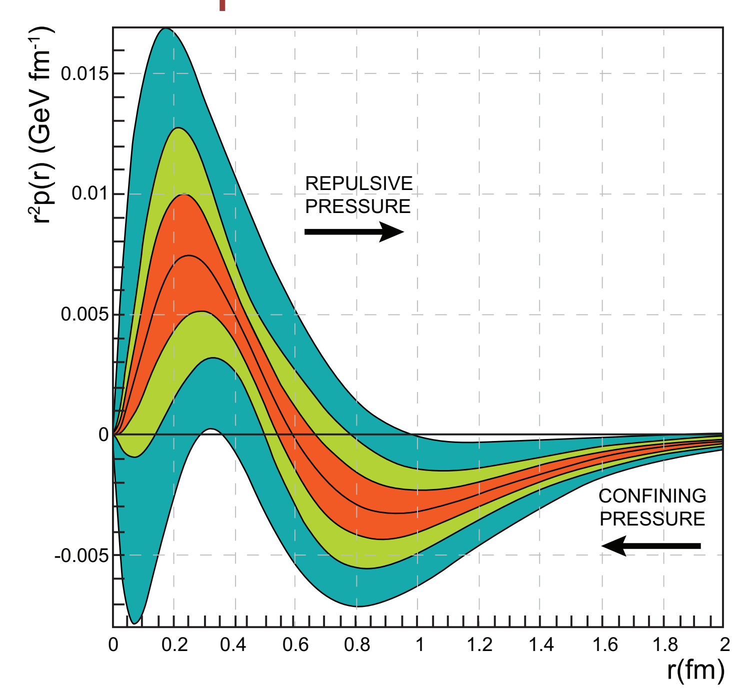
complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]

DVCS (quark) D-term



Radial pressure distribution

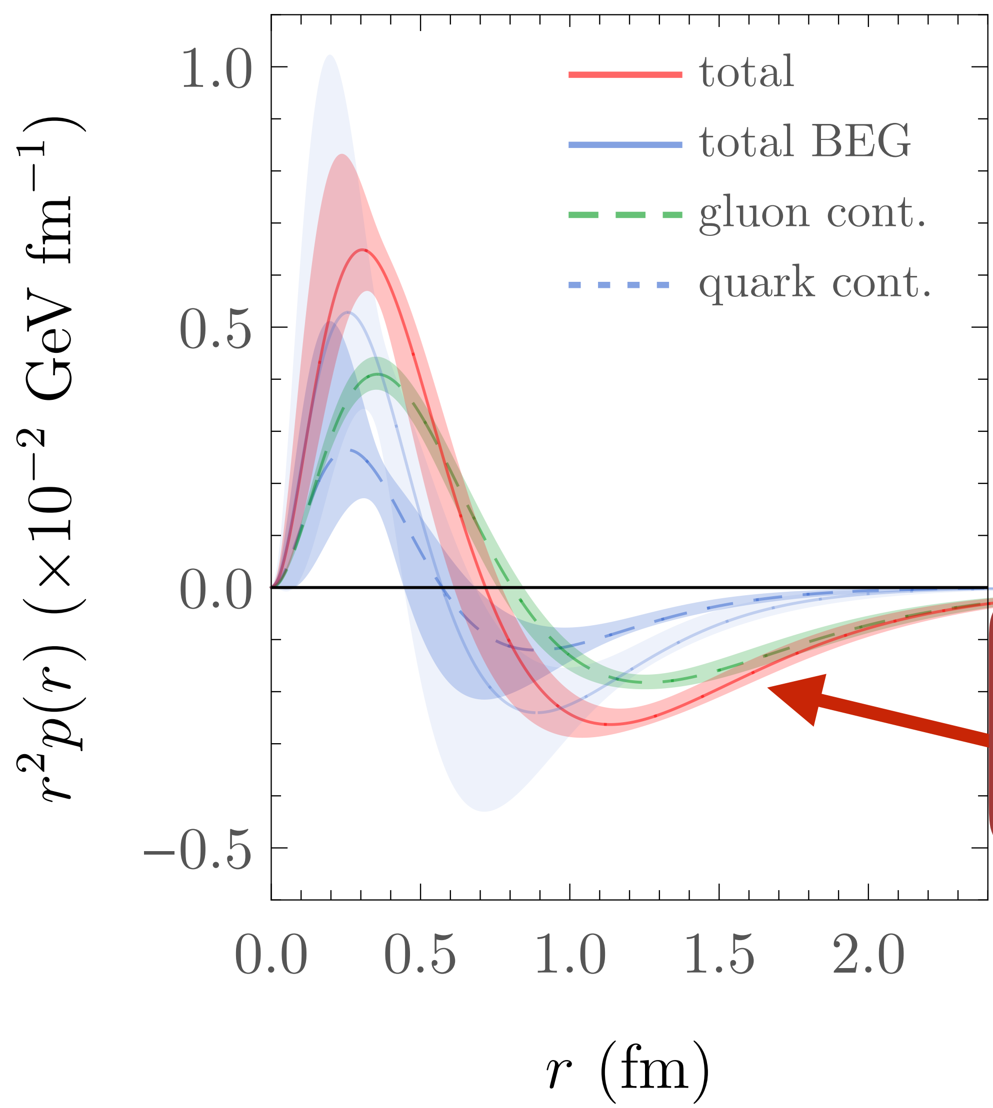


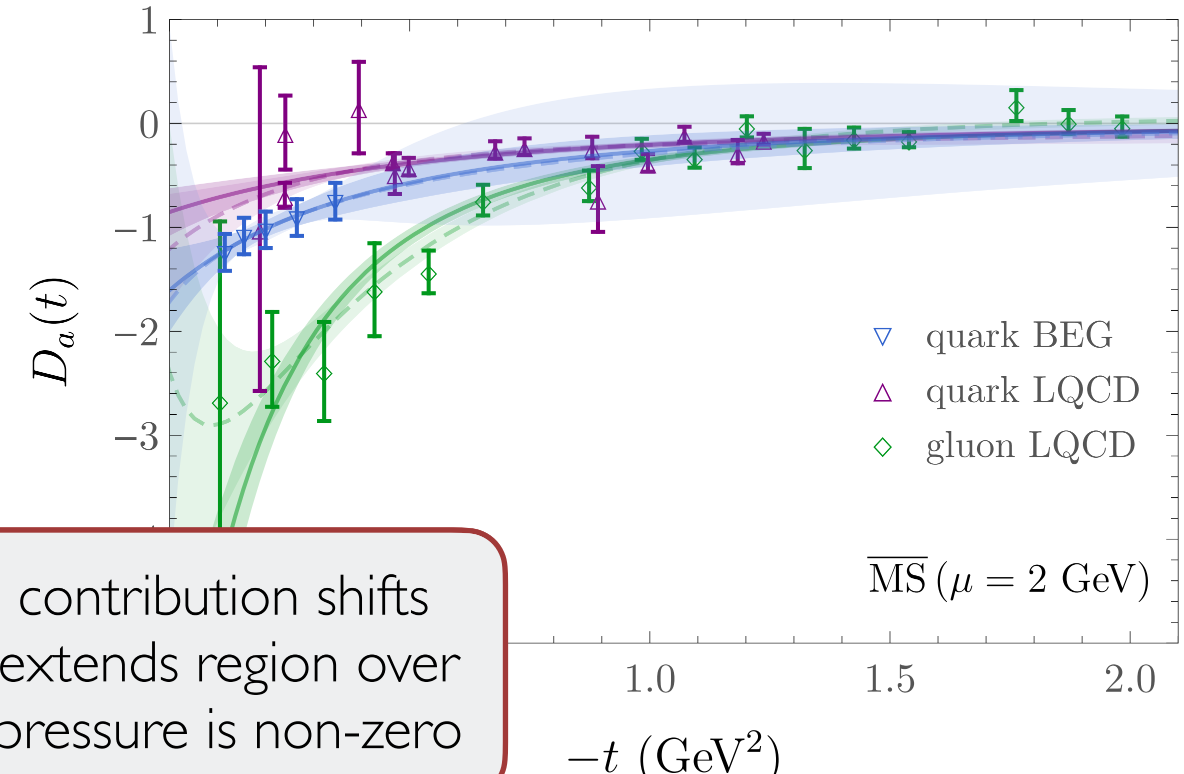
Nucleon D-term GFFs from LQCD

EXP + LQCD 2019

first complete pressure determination

[First lattice calculation of gluon GFFs, some uncontrolled systematics
Shanahan, Detmold PRL 122 072003 (2019)]





gluon contribution shifts peaks, extends region over which pressure is non-zero

Gluon GFFs: Shanahan, Detmold, PRD 99, 014511 (2019)
Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)
Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

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EXP + LQCD 2019

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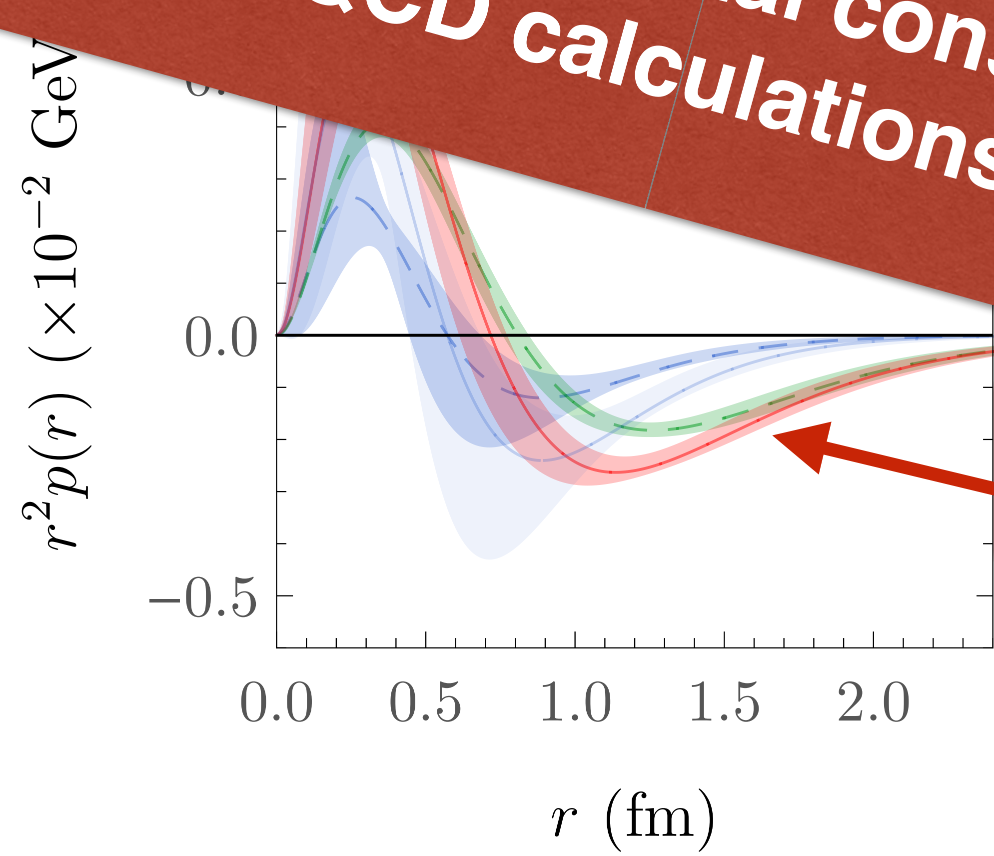
Rapid progress since 2019!

- **First experimental constraints of gluon GFFs**
- **First LQCD calculations with physical pion mass**



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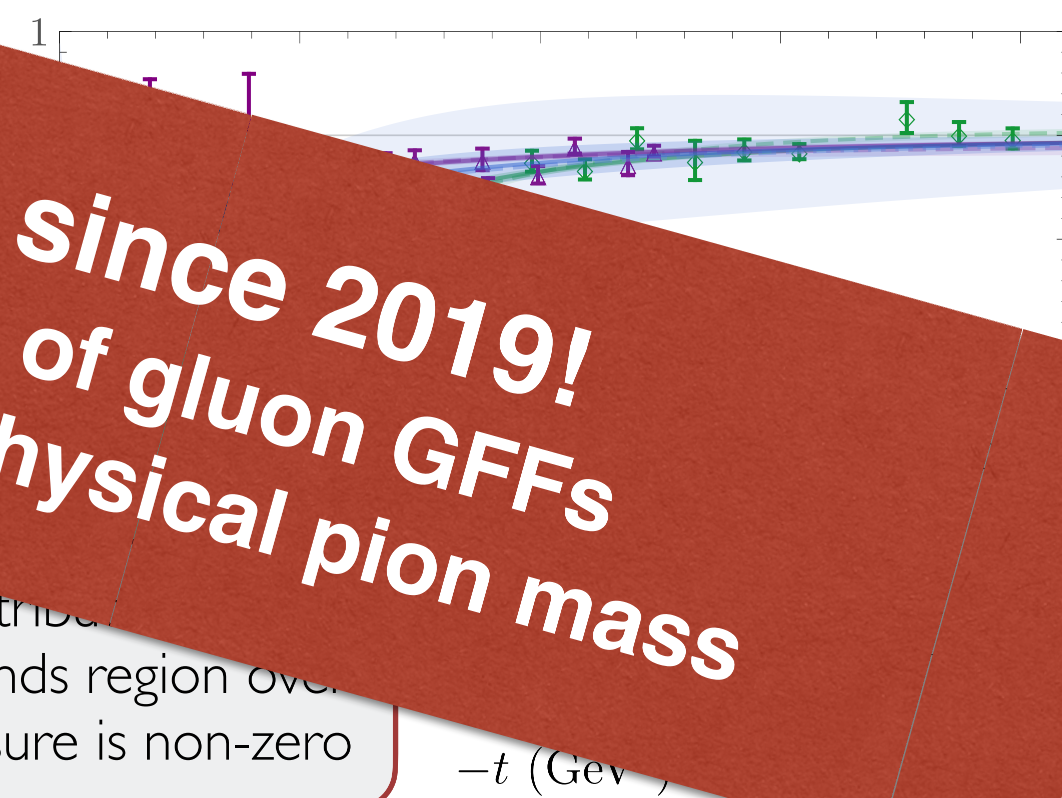

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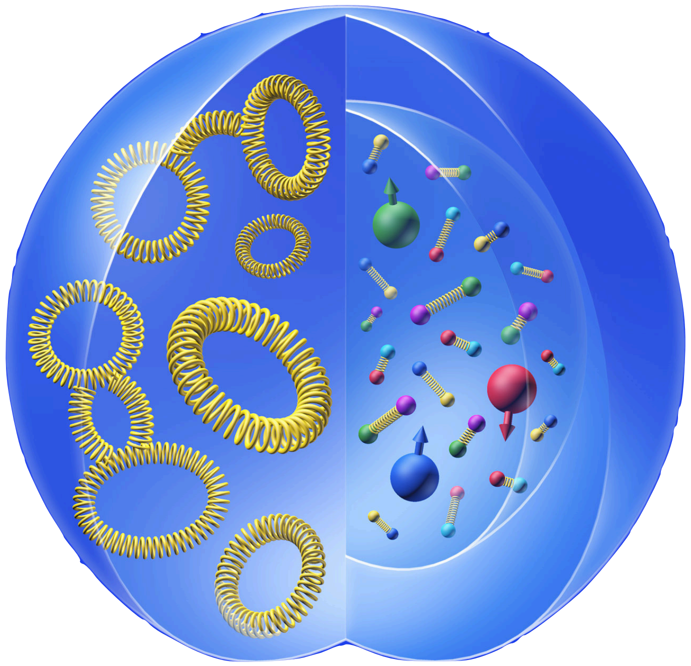
New proton+pion GFF calculations

New calculations in 2022-2024!

- Quark masses corresponding to a close-to-physical pion mass
- All contributions from light quarks and gluons separately computed
- Non-pert. renormalisation incl. mixing
- [Still a single ensemble, no control of discretisation effects]



Pefkou, Hackett, Shanahan, Phys.Rev.Lett. 132 (2024) 25, 251904
Pefkou, Hackett, Shanahan, Phys.Rev.D 108 (2023) 11, 114504
Pefkou, Hackett, Shanahan, Phys.Rev.D 105 (2022) 5, 054509

Proton GFFs from lattice QCD

Solving system yields flavour-decomposition of all three GFFs

- **First complete decomposition** of proton gravitational form factors into u, d, s, g contributions from lattice QCD in 2023/2024
- Physical pion mass
- Non-pert. renormalisation incl. mixing
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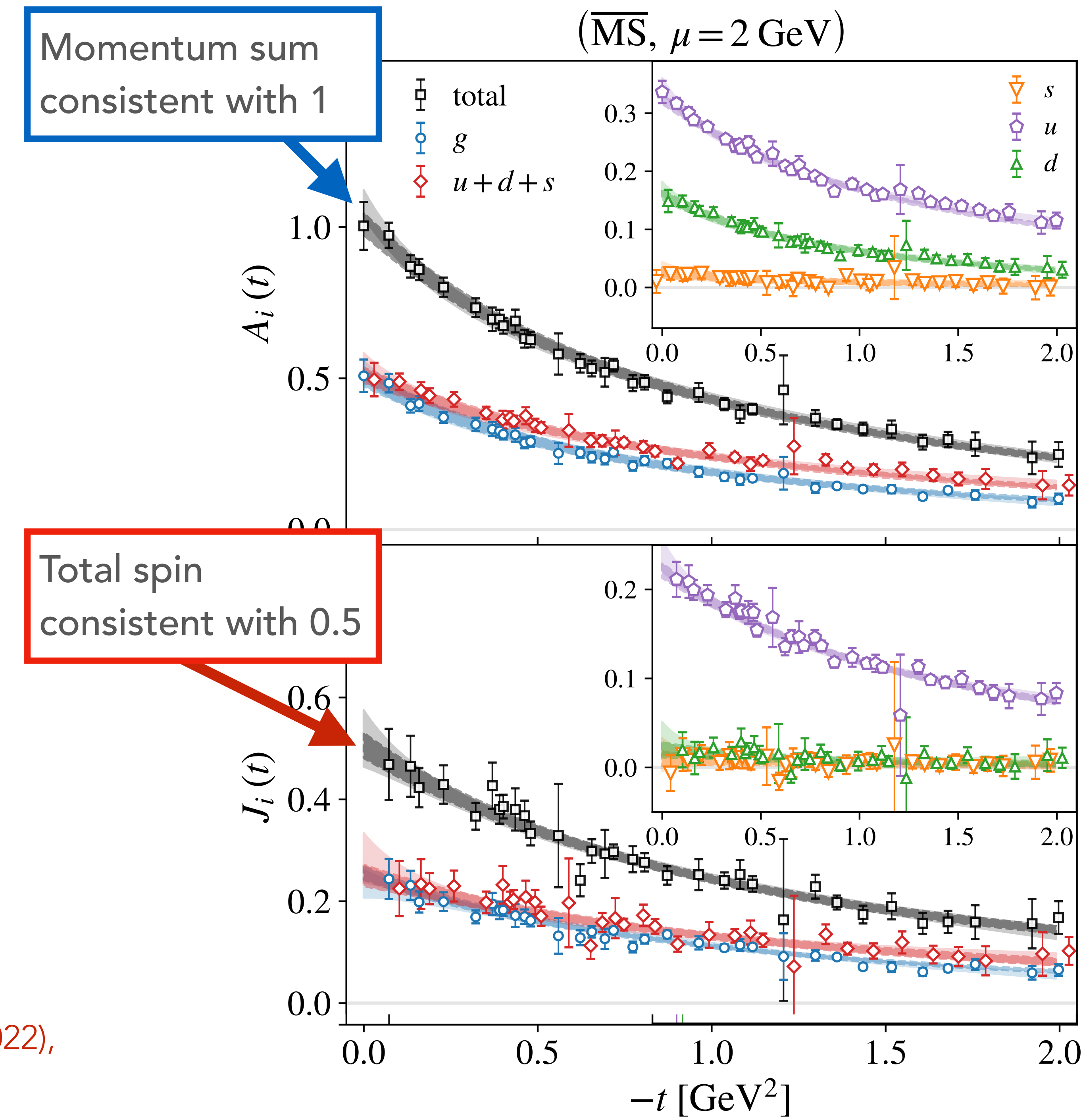

Lattice QCD: Pefkou, Hackett, Shanahan, PRD 105, 054509 (2022), PRD 108, 114504 (2023), PRL 132, 251904 (2024)

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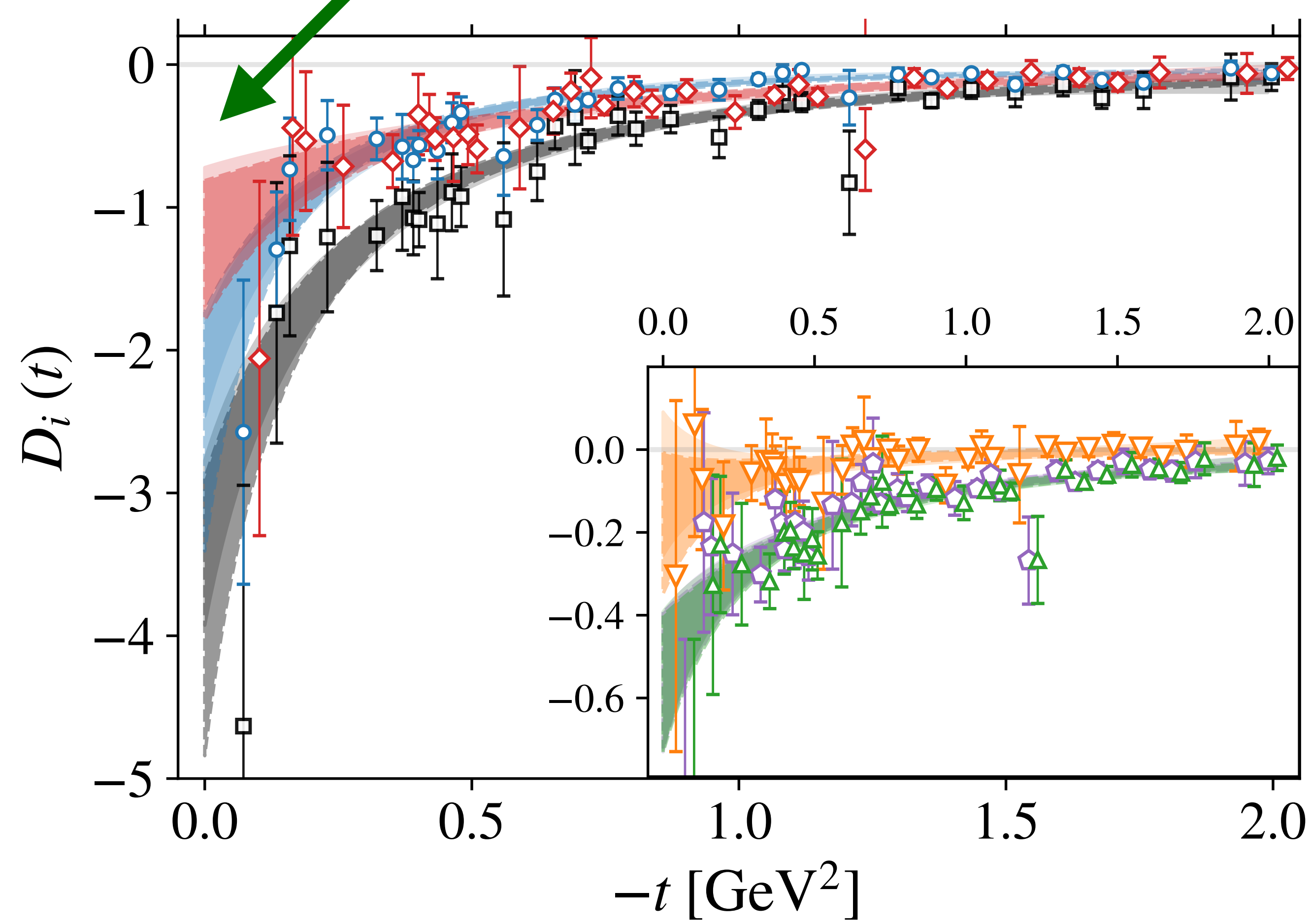
D-term decomposition previously unknown

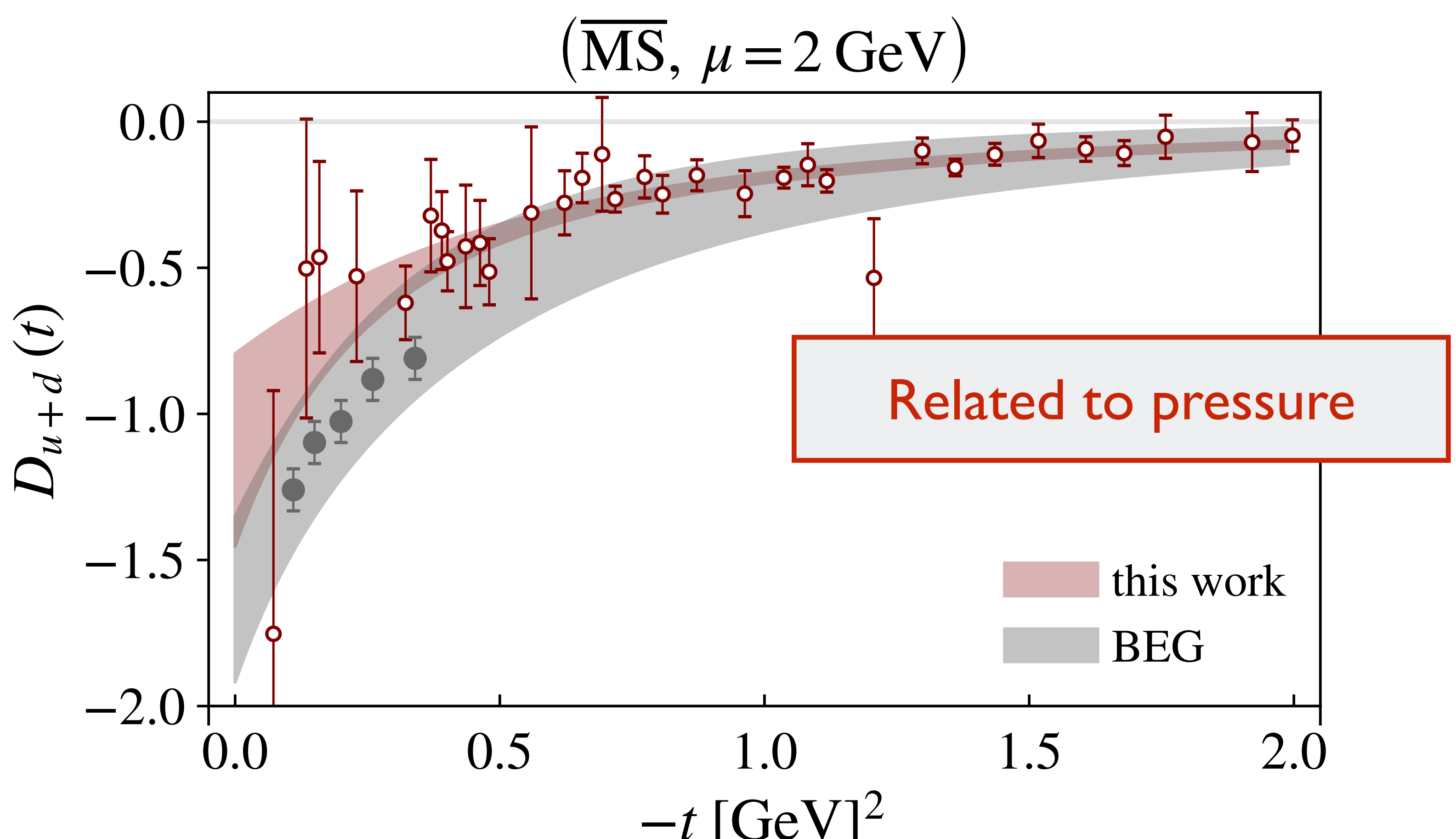


Lattice QCD: Pefkou, Hackett, Shanahan, PRD 105, 054509 (2022), PRD 108, 114504 (2023), PRL 132, 251904 (2024)

Gravitational FFs c.f. experiment

Compare quark D GFF with 2018 results from DVCS
[Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)]

- Consistency and complementarity: lattice result more precise at large t, experimental constraints are at small t



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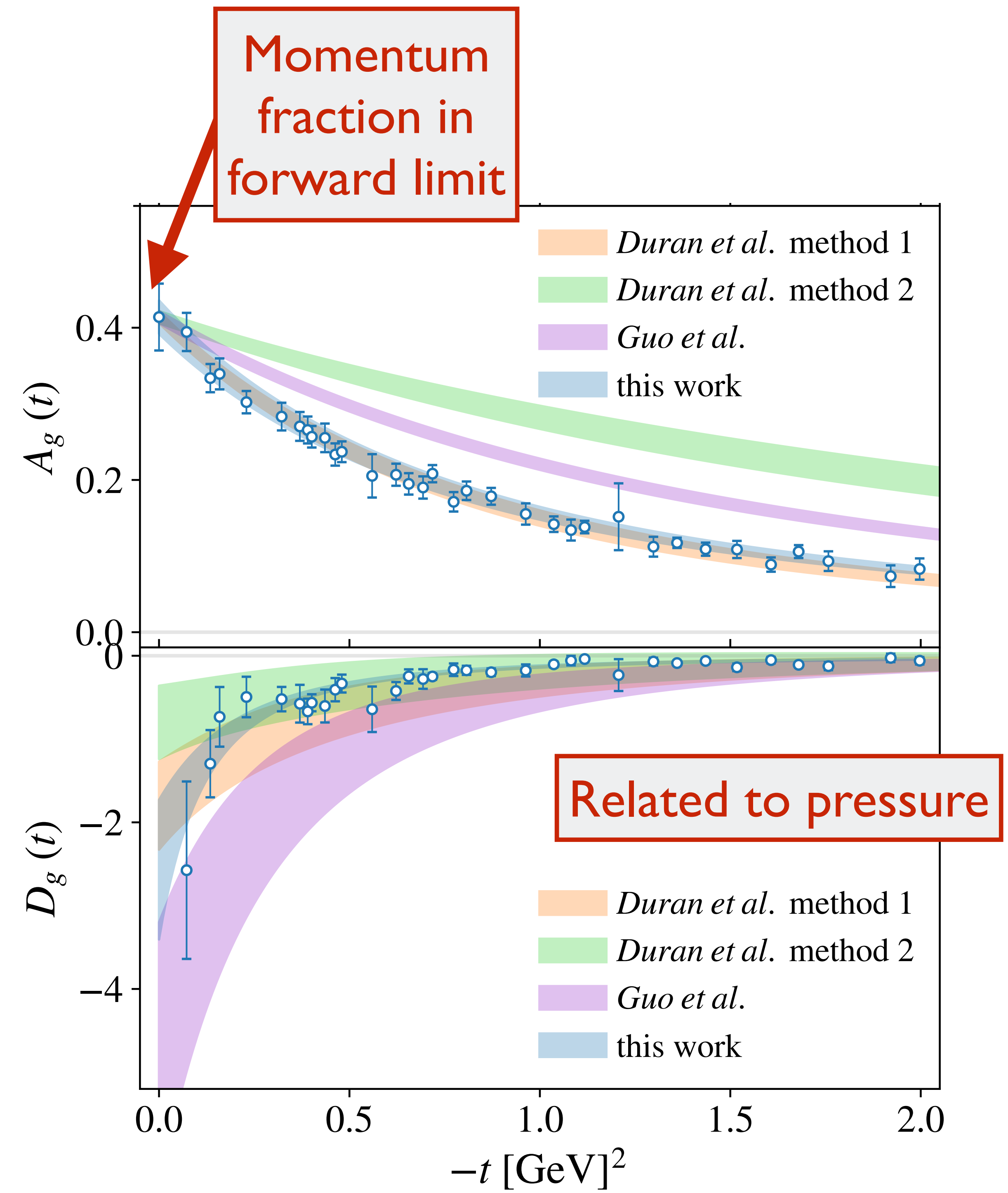
Gravitational FFs c.f. experiment

Synergy between lattice QCD and experiment continues!

- First experimental constraint on gluon GFFs in 2023 from J/ψ photoproduction
[Duran et al., Nature 615, 813-816 (2023)]

- Lattice calculation important in distinguishing between analyses using different models



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Guo et al., 2308.13006 (2023); BEG 2310.11568 (2023)

Proton quark and gluon radii

Define quark and gluon radii from energy and longitudinal force densities

$$\langle r_i^2 \rangle^{\text{mass}} = \frac{\int d^3\mathbf{r}\, r^2 \varepsilon_i(r)}{\int d^3\mathbf{r}\, \varepsilon_i(r)}\,, \quad \langle r_i^2 \rangle^{\text{mech}} = \frac{\int d^3\mathbf{r}\, r^2 F_i^{||}(r)}{\int d^3\mathbf{r}\, F_i^{||}(r)}\,.$$

- Mass and mechanical radii of proton comparable to charge radius
- Gluons act to extend radius defined by quark contributions

$$\varepsilon_i(r) = m\left[A_i(t) - \frac{t(D_i(t)+A_i(t)-2J_i(t))}{4m^2}\right]_{\text{FT}}\,,$$

$$F_i^{||}(r) = p_i(r) + 2s_i(r)/3$$

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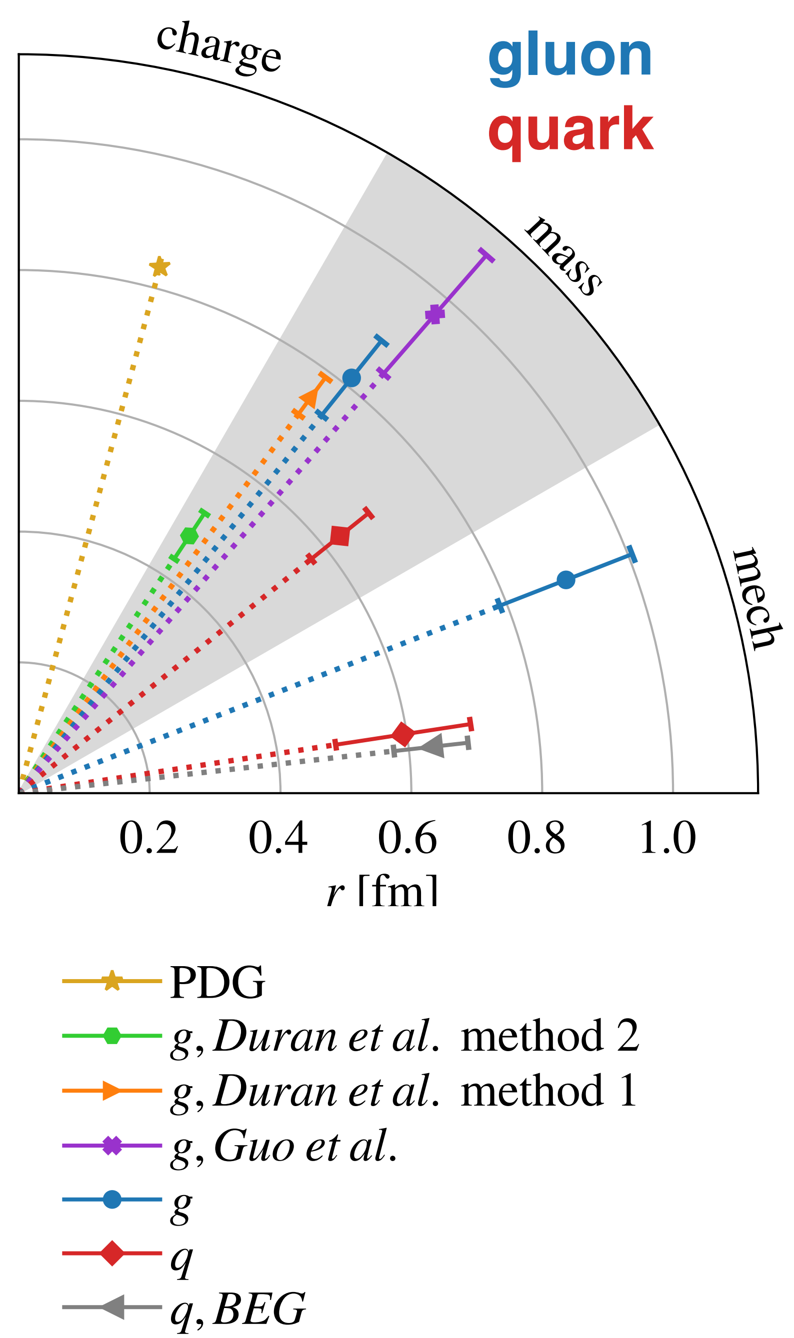

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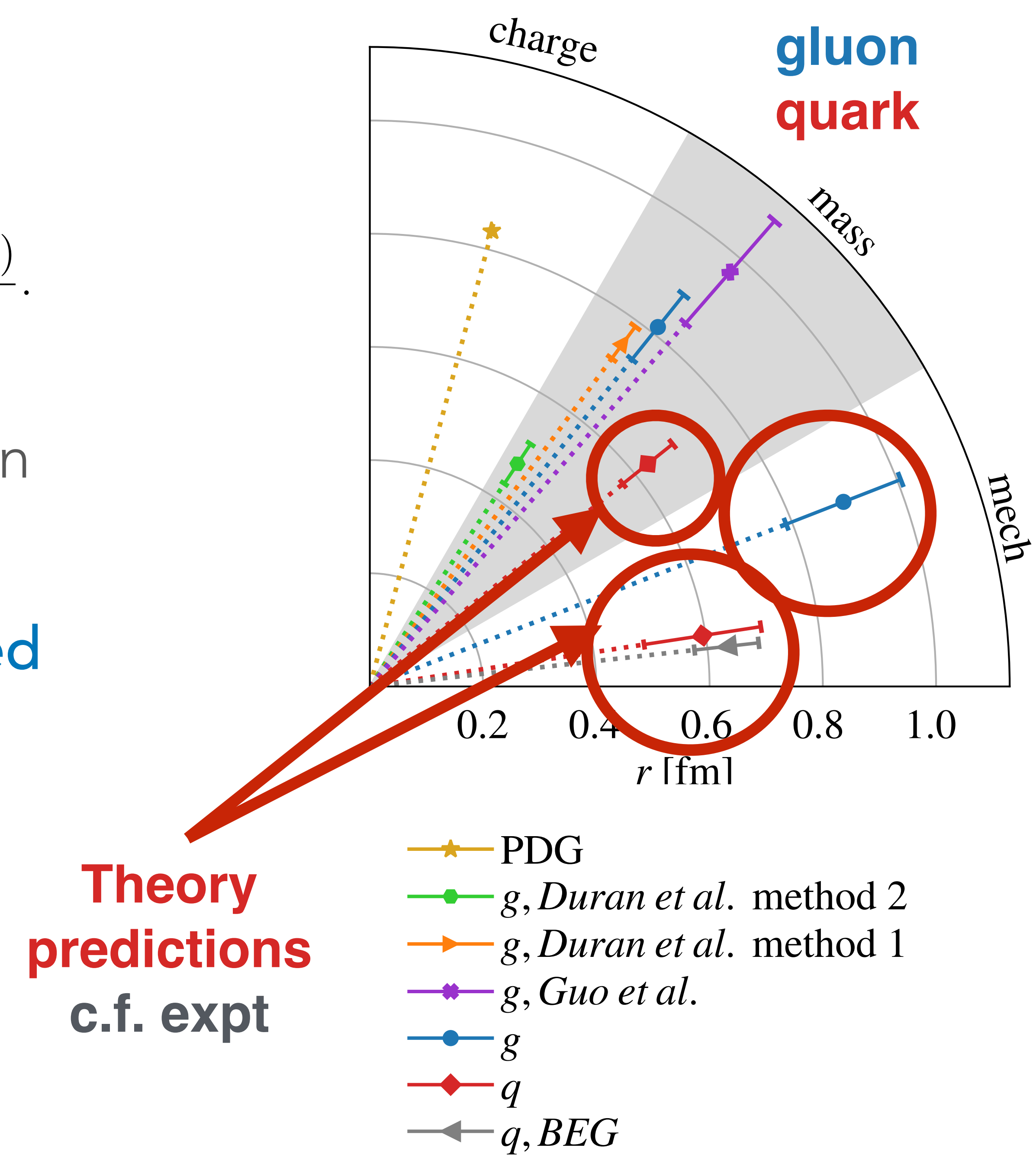

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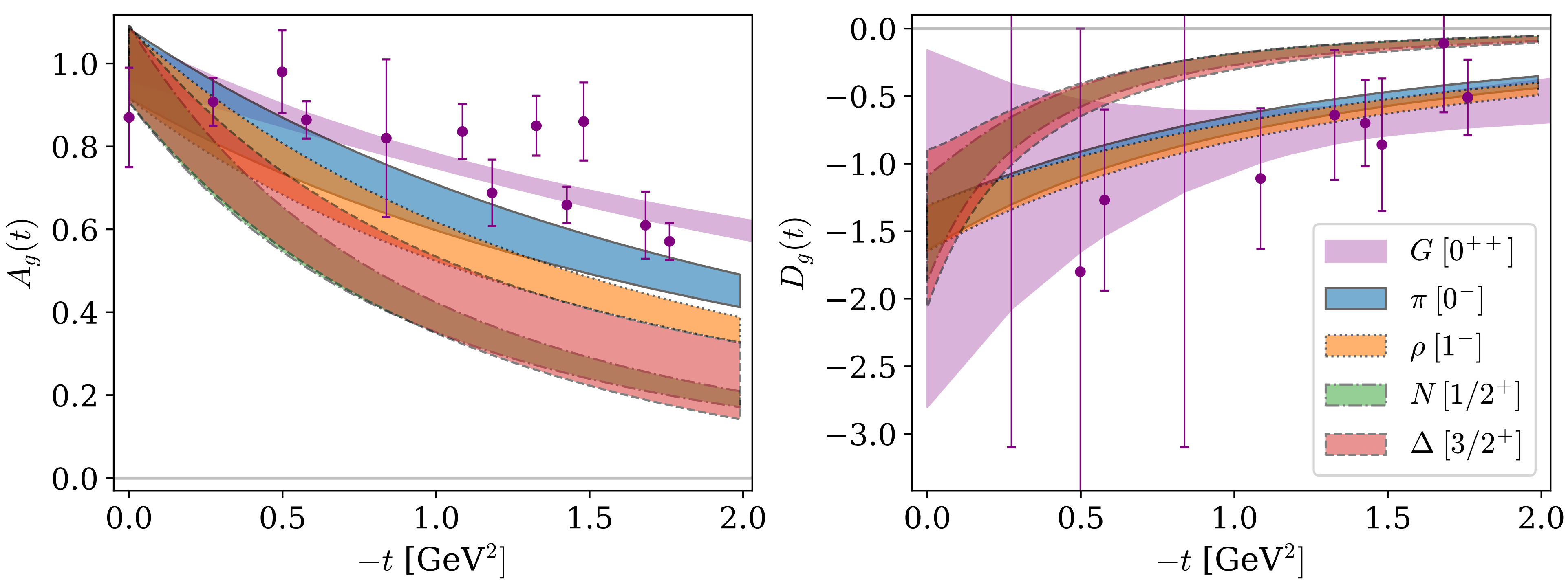
Gravitational FFs of Glueballs

There are multiple experimental candidates for glueball states with various quantum numbers; what is their structure?

- **QUESTION:** Can structure measurements act as ‘smoking gun’ for ID of Glueball or glue hybrid states?

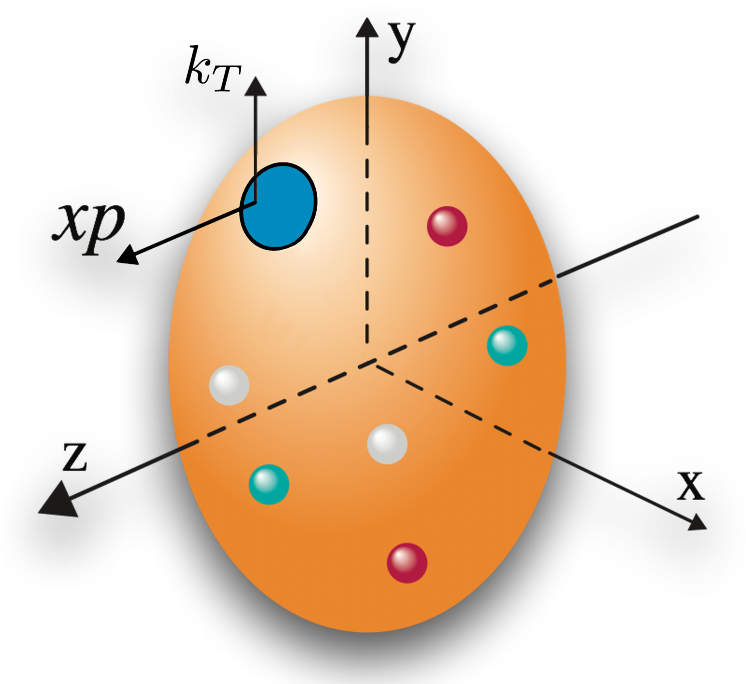
- **NEW (unpublished):** First calculations of aspects of the gluonic structure of Glueball states (c.f. structure of hadrons)



Mesons: Pefkou, Hackett, Shanahan, Phys.Rev.D 105 (2022); Glueball: Abbott et al. [2410.02706]

Partonic structure of the proton

Understanding the quark and gluon structure of matter



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Collins-Soper evolution kernel

PDF
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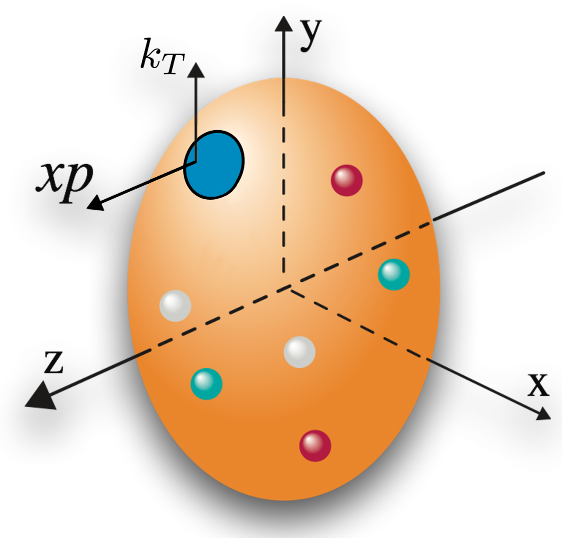


Collins-Soper Evolution Kernel

$$\gamma_\zeta^q(\mu, b_T) = \zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta)$$

- Governs TMD rapidity evolution
- Needed to match lattice QCD calculations of TMDs through "quasi-TMDs" to physical TMDs

- Perturbative at short distances $\mu, b_T^{-1} \gg \Lambda_{\mathrm{QCD}}$
- Non-perturbative for $b_T^{-1} \lesssim \Lambda_{\mathrm{QCD}}$ → lattice QCD?
- Independent of hadron

Phiala Shanahan, MIT

Collins-Soper evolution kernel

Estimates of size of nonperturbative contributions to CS kernel

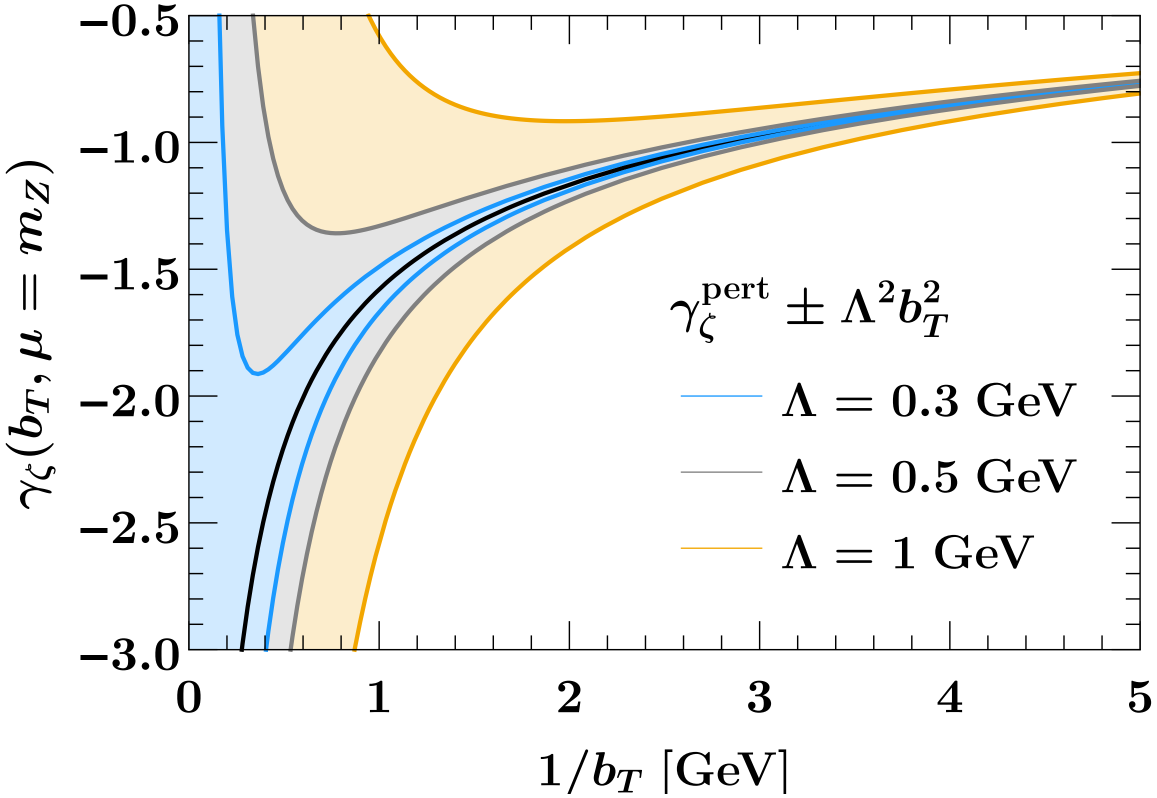


Figure: Iain Stewart

- Large uncertainties in estimates of the size of nonperturbative contributions to the CS kernel

CS kernel from phenomenological fits

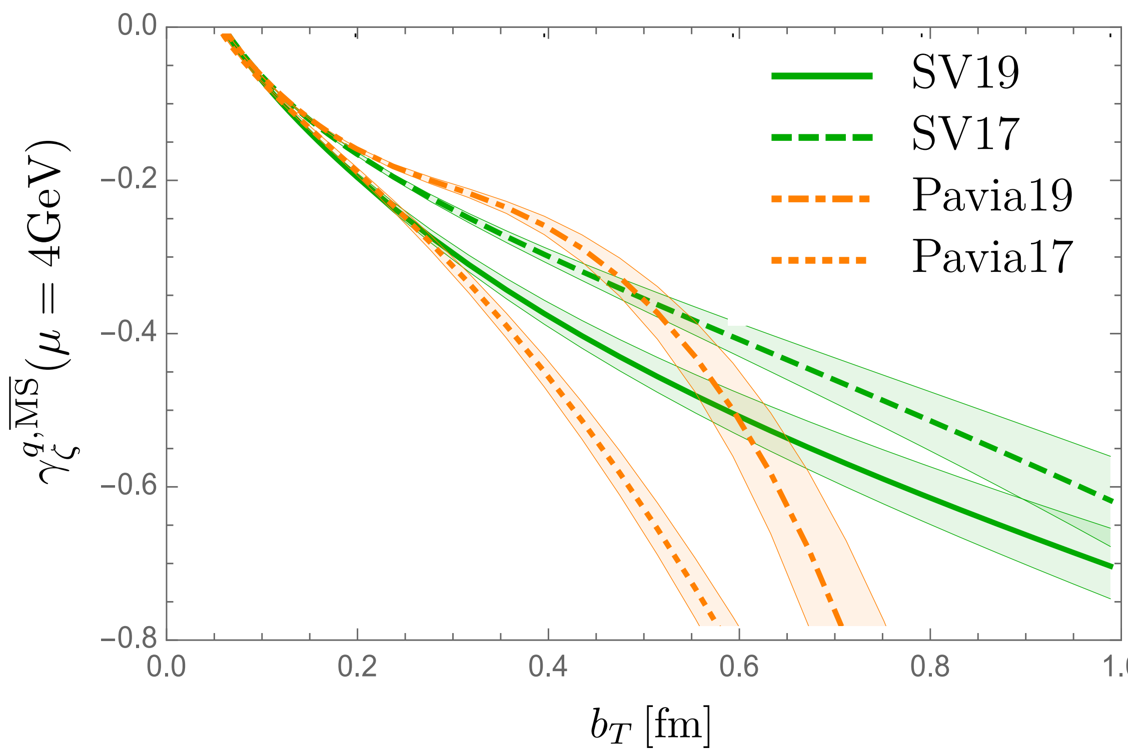


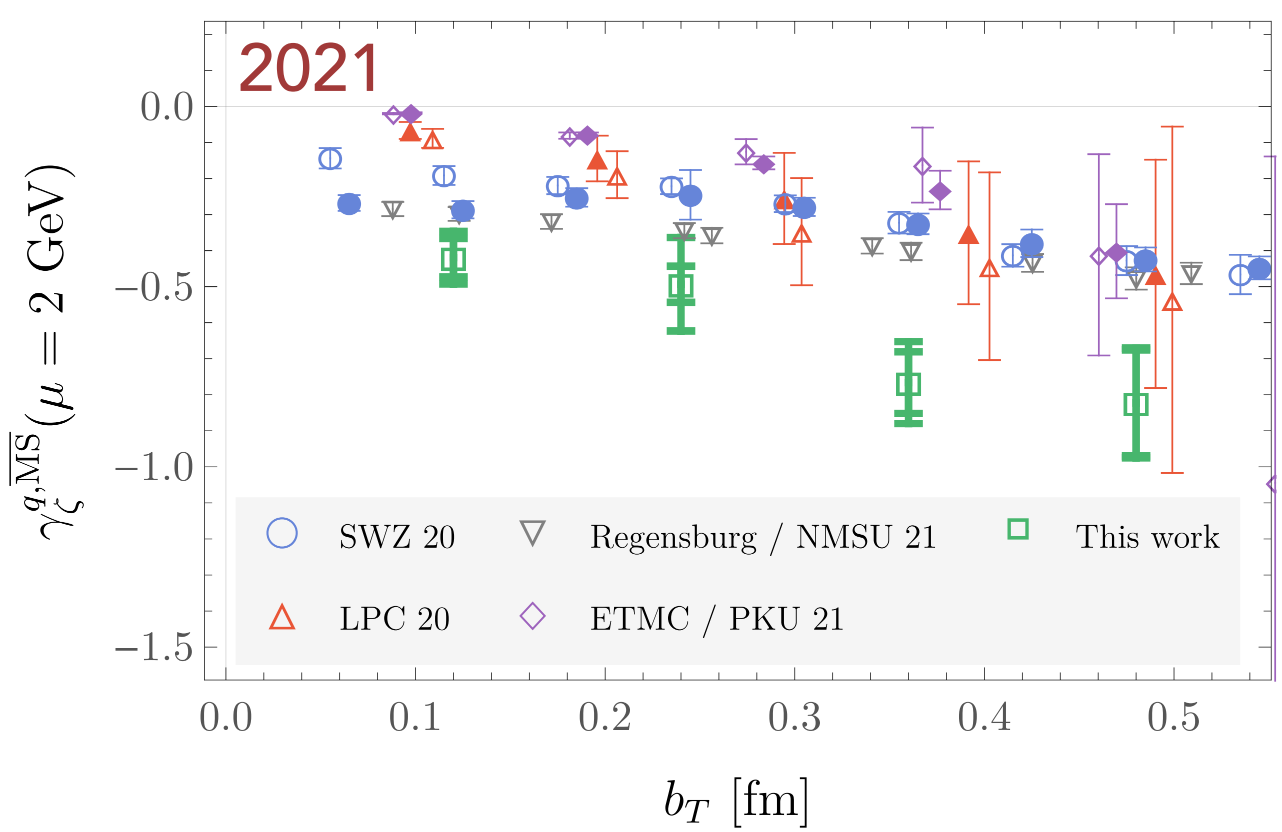
Figure adapted from Vladimirov [2003.02288]
[see also e.g., Kang, Prokudin, Sun,Yuan Phys.Rev.D 93 (2016), Collins, Rogers Phys.Rev.D 91 (2015), Sun, Yuan Phys.Rev.D 88 (2013)]

- LQCD constraints in nonperturbative region with ~10-20% uncertainties would allow model differentiation

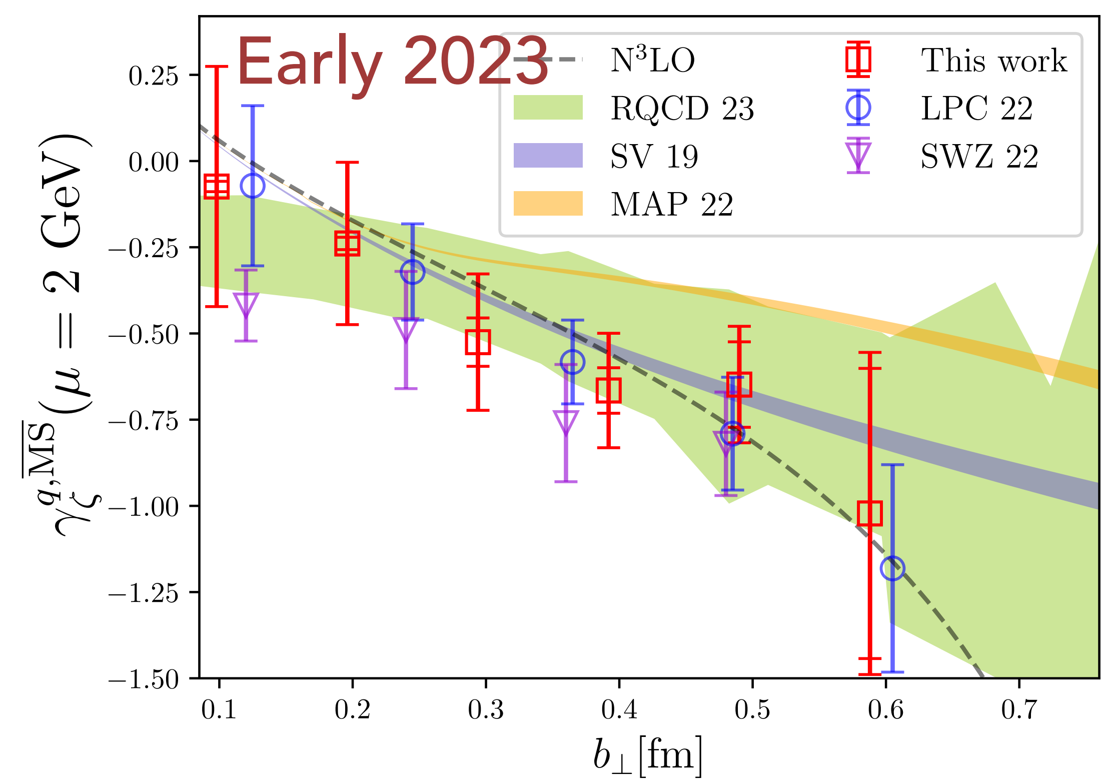
Collins Soper kernel from lattice QCD

Status of lattice QCD calculations of the CS kernel in early 2023

- Lattice QCD results broadly consistent with phenomenology
- Many calculations, not yet at the level of complete systematic control: consistency improving as systematics are addressed



[LPC 2005.14572; Regensburg/NMSU 2103.16991; ETMC/PKU 2106.13027; SWZ 2003.06063, 2107.11930]



[Figure from LPC 2306.06488; SWZ 22 2107.11930; LPC 22 2204.00200; RQCD 23 2103.16991]

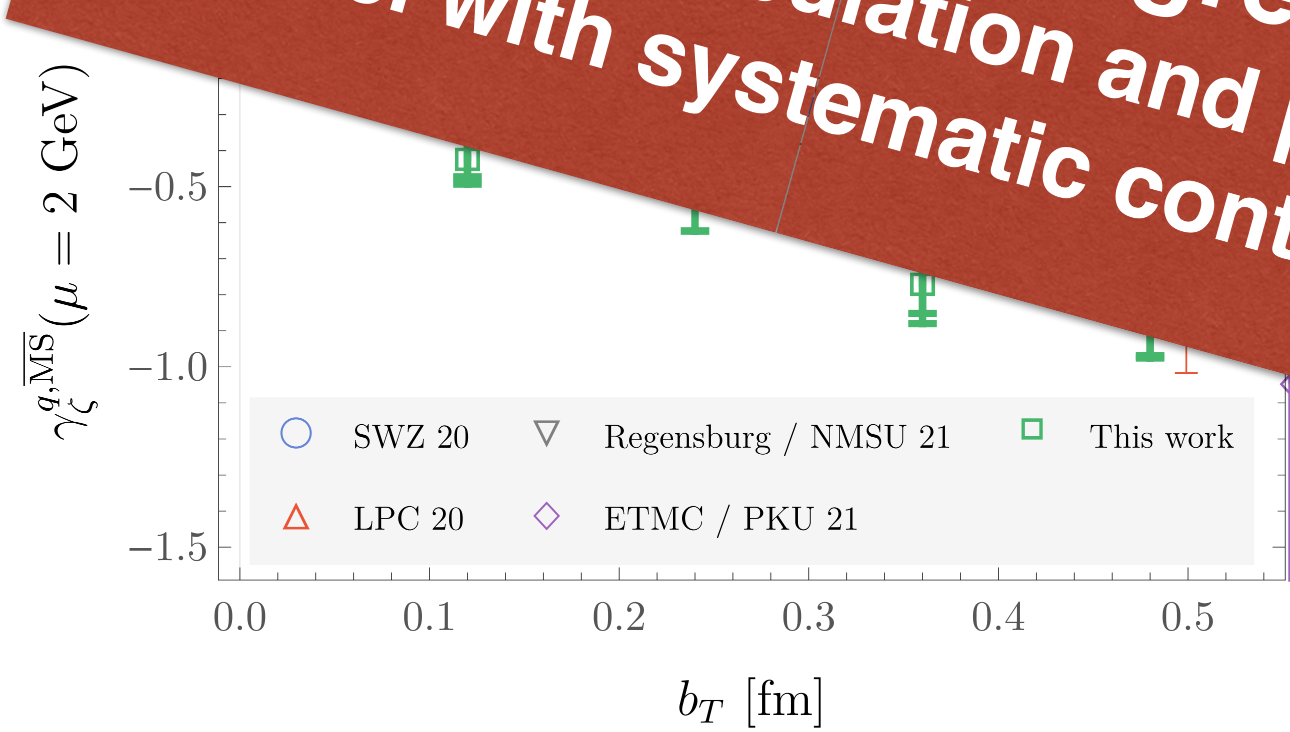
Collins Soper kernel from lattice QCD

Status of lattice QCD calculations of the CS kernel in early 2023

- ...ttice QCD results broadly consistent with phenomenology
- ... not yet at the level of complete systematic control:
  ...tematics are addressed

Major progress since 2023!

- First QCD calculation and parameterisation of the CS kernel with systematic control



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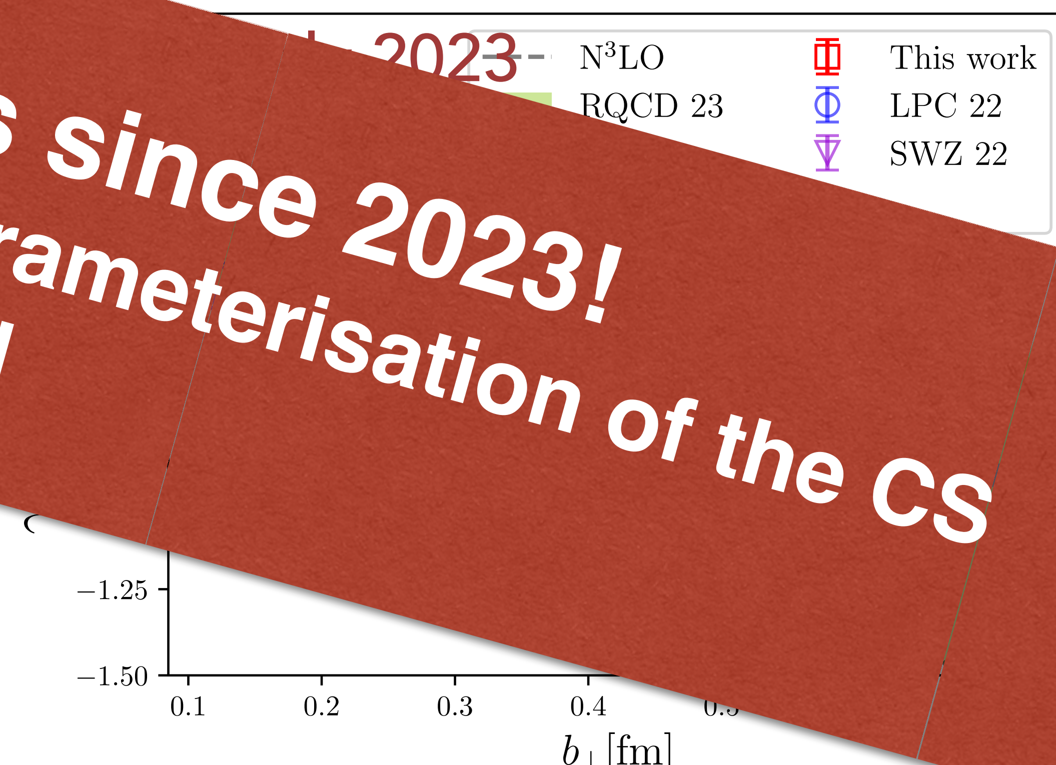

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TMDPDFs from lattice QCD

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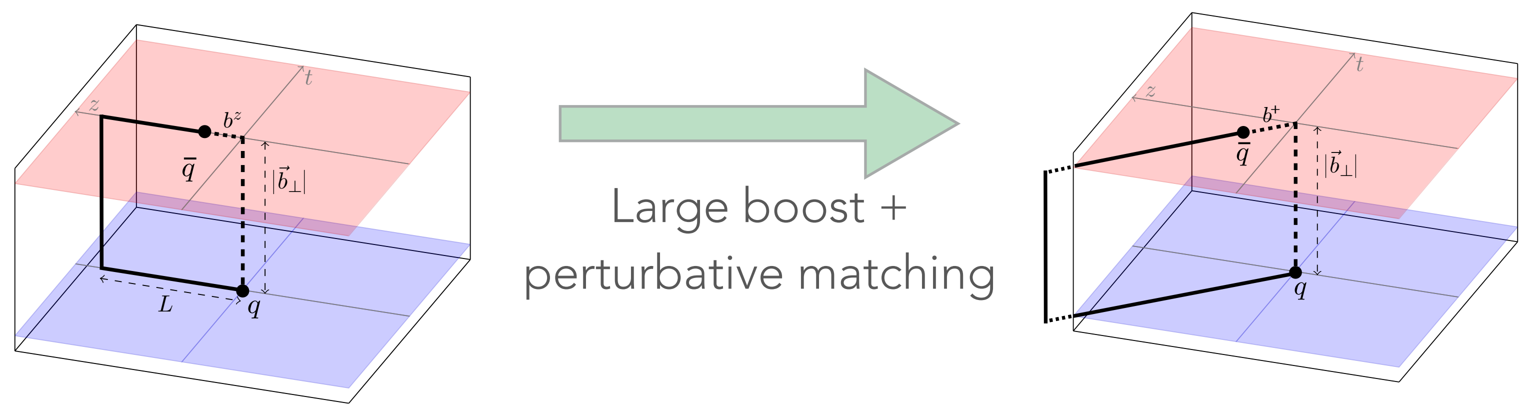
+ transverse

TMD Collaboration

- TMDPDF defined by matrix element of non-local light-cone quark bilinear operator
  - Calculate via quasi-TMD prescription

    [Ji, Sun, Xiong,Yuan '14, Ji, Jin, Yuan, Zhang, Zhao, PRD99 (2019), Ebert, Stewart, Zhao, PRD99 (2019), JHEP09 (2019), Ji, Liu, Liu, [1911.03840] (2019), Nucl.Phys.B (2020), Vladimirov, Schäfer, Phys.Rev.D 101 (2020)]

Spatial staple calculable in LQCD



$$\underbrace{\tilde{f}_{u\text{-}d}(x, \vec{b}_T, \mu, P^z)}_{\text{quasi-TMD from LQCD}} = \underbrace{C^{\mathrm{TMD}}_{u\text{-}d}(\mu, xP^z)}_{\text{perturbative matching}} \underbrace{g^S_q(b_T, \mu)}_{\text{non-pert. soft factor}} \exp\left[\frac{1}{2}\underbrace{\gamma^q_\zeta(\mu, b_T)}_{\text{Collins-Soper evolution kernel}} \ln\frac{(2xP^z)^2}{\zeta}\right] \underbrace{f_{u\text{-}d}(x, \vec{b}_T, \mu, \zeta)}_{\text{desired TMDPDF}}$$

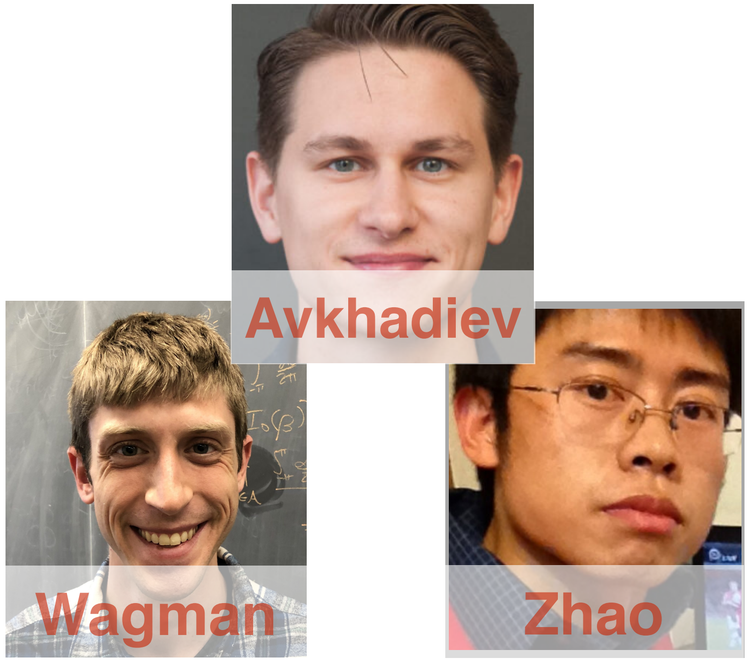
Collins Soper kernel from lattice QCD

Five steps to CS kernel from lattice QCD:

1. Calculation of bare quasi-wavefunctions
2. Renormalisation and matching to the $\overline{\text{MS}}$ scheme
3. Fourier transform to momentum space
4. Ratio, perturbative matching, power corrections
5. Continuum extrapolation

2023/2024: First calculation at quark masses yielding a close-to-physical pion mass

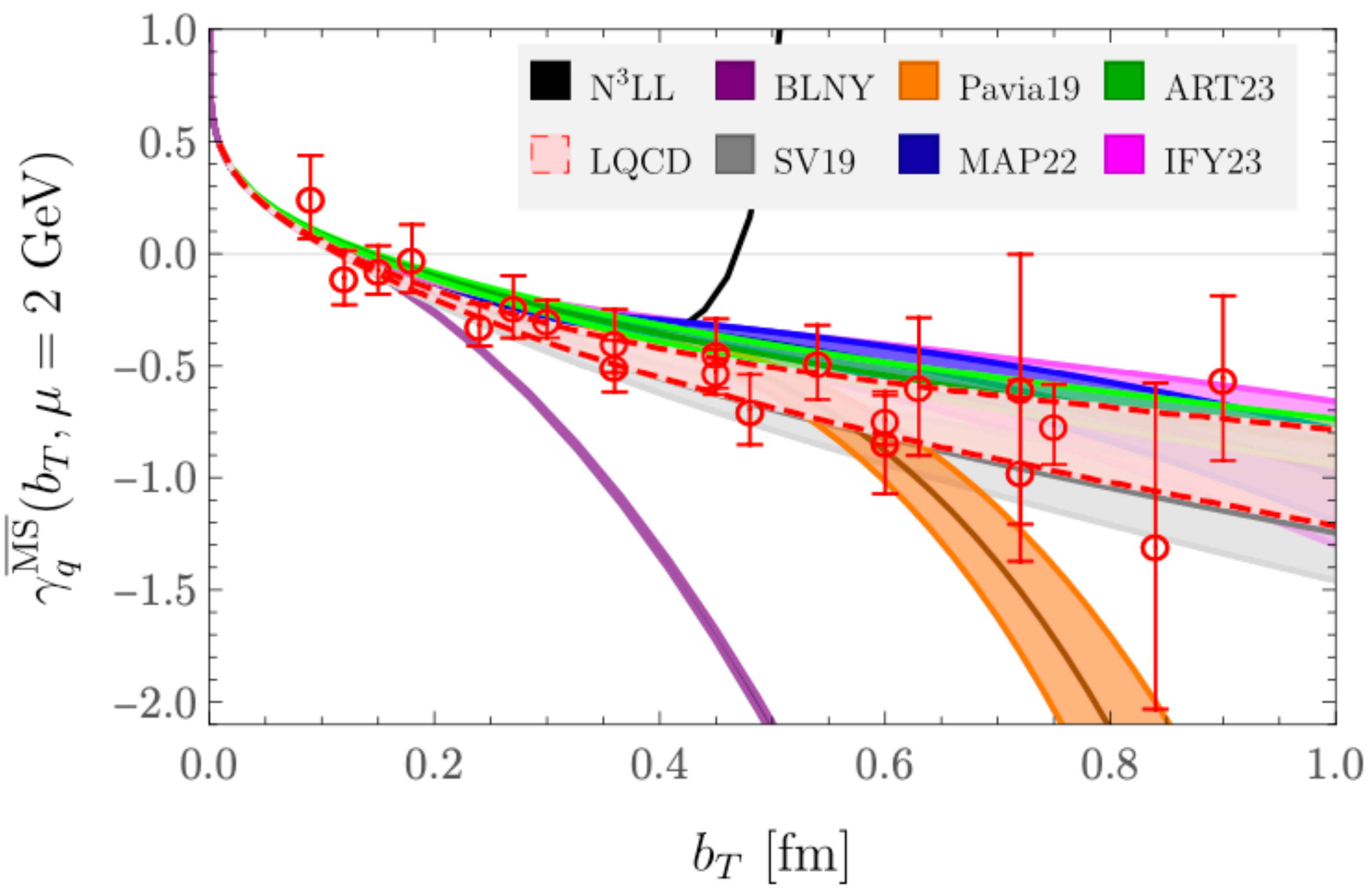
- Nonperturbative operator mixing
- Investigate effect of different perturbative orders of matching, resummation, etc



Avkhadiev, Shanahan, Wagman, Zhao; Phys.Rev.D 108 (2023) 11, 114505; Phys.Rev.Lett. 132 (2024) 23, 231901

Collins Soper kernel from lattice QCD

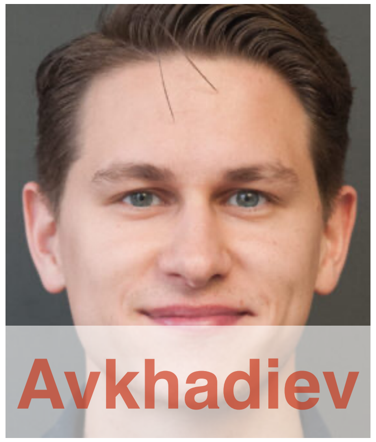
- First calculation at ~physical pion mass and NNLO + NNLL matching
- Allows first parameterisation of kernel constrained by pure theory results
- Begins to distinguish between phenomenological models in non-perturbative regime

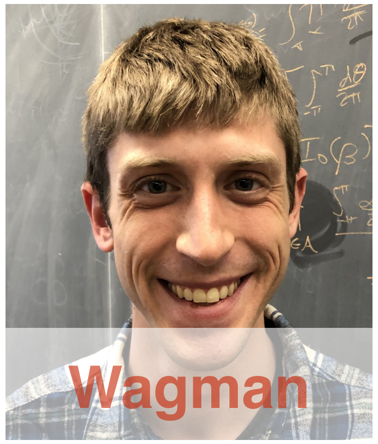


Avkhadiev, Shanahan, Wagman, Zhao; Phys.Rev.D 108 (2023) 11, 114505; Phys.Rev.Lett. 132 (2024) 23, 231901

Gluon Collins-Soper Kernel

- *Experiment:*
    - Can expect first constraints on gluon TMDs and CS kernel from future Electron-Ion Collider
- *Theory:*
    - **Perturbative region:** from 1-loop PT, quark and gluon CS kernels differ by only a group theory factor (C_A vs C_F)
    - **Non-perturbative region:** no information yet!
    - *First LQCD calculation in progress*

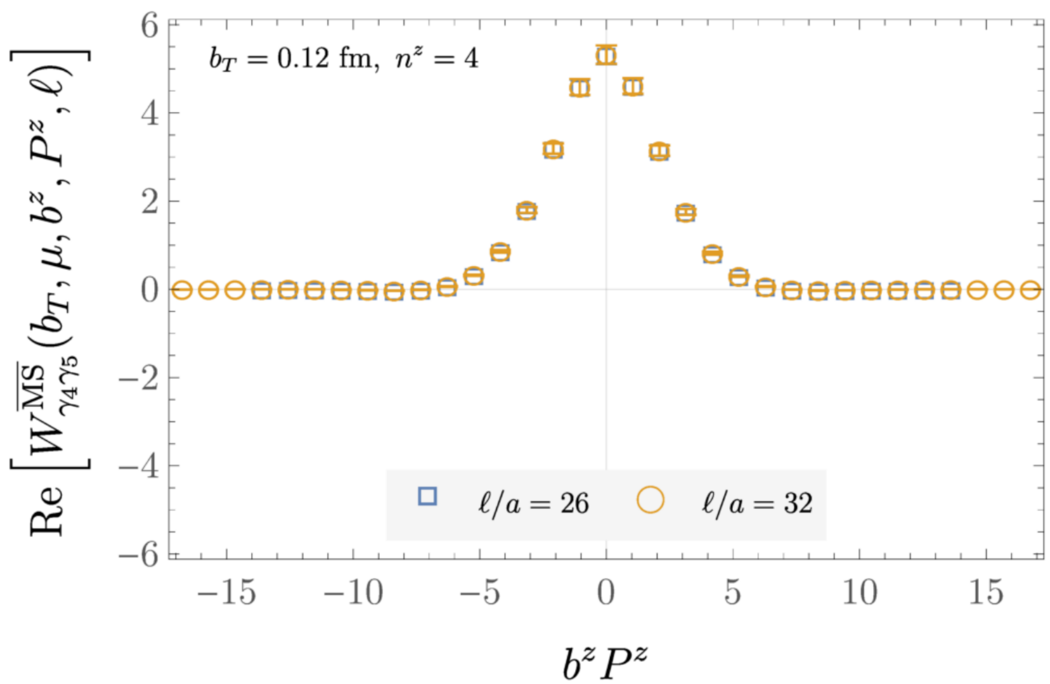




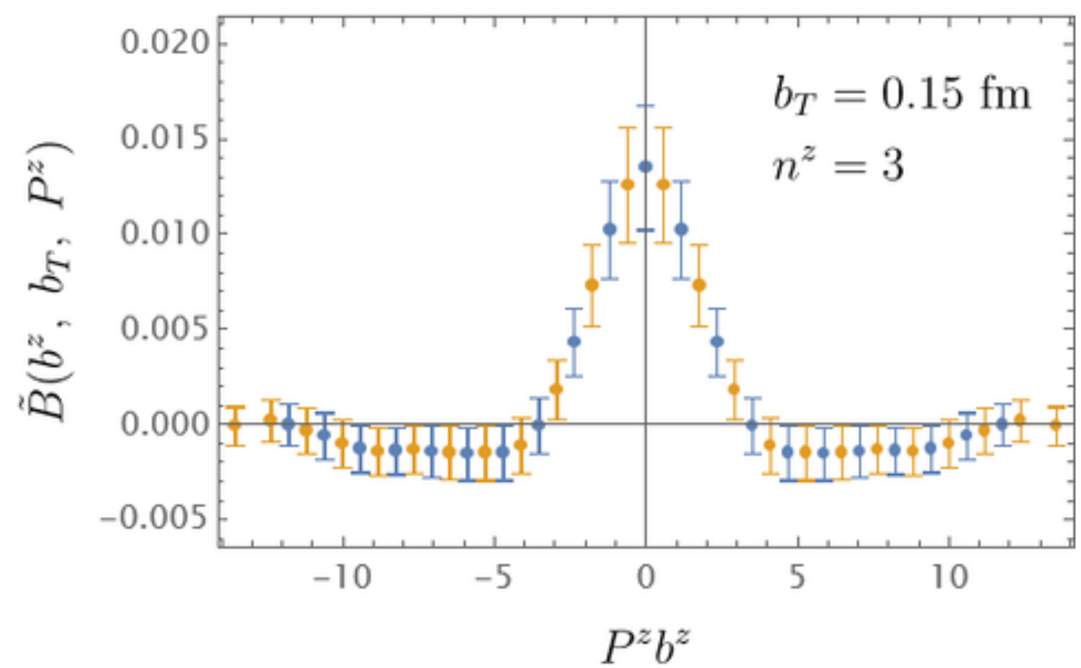
Gluon Collins-Soper Kernel

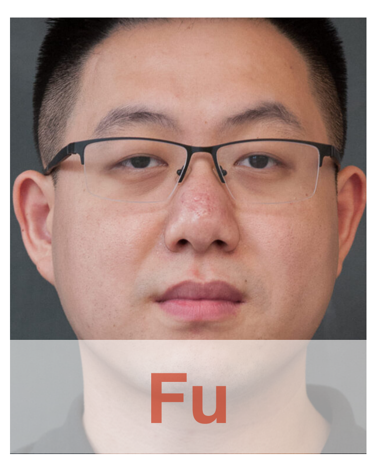
Work in progress (ETA 2025): High statistics required for gluon CS kernel!

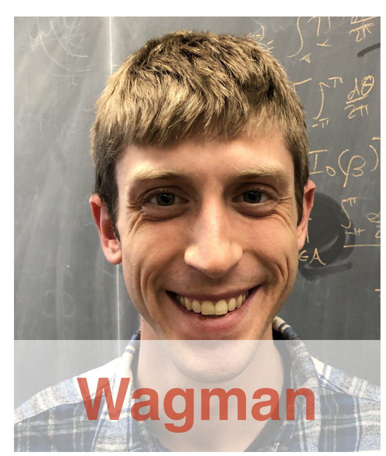
Quark quasi-beam function



Gluon analogue (same stats)







Parton structure from LQCD

Future colliders will dramatically alter our knowledge of the quark and gluon structure of hadrons and nuclei

- Work towards a complete 3D picture of parton structure (moments, x-dependence of PDFs, GPDs, TMDs)

- **New** lattice QCD calculations of the Collins-Soper kernel for TMD rapidity evolution

- **New** flavour-decomposition of generalised form factors of the proton and other hadrons, yielding decomposition of distributions including shear and pressure

Lattice QCD calculations in hadrons and light nuclei will complement and extend understanding of fundamental structure of nature

Image Credit: 2018 EIC User's Group Meeting