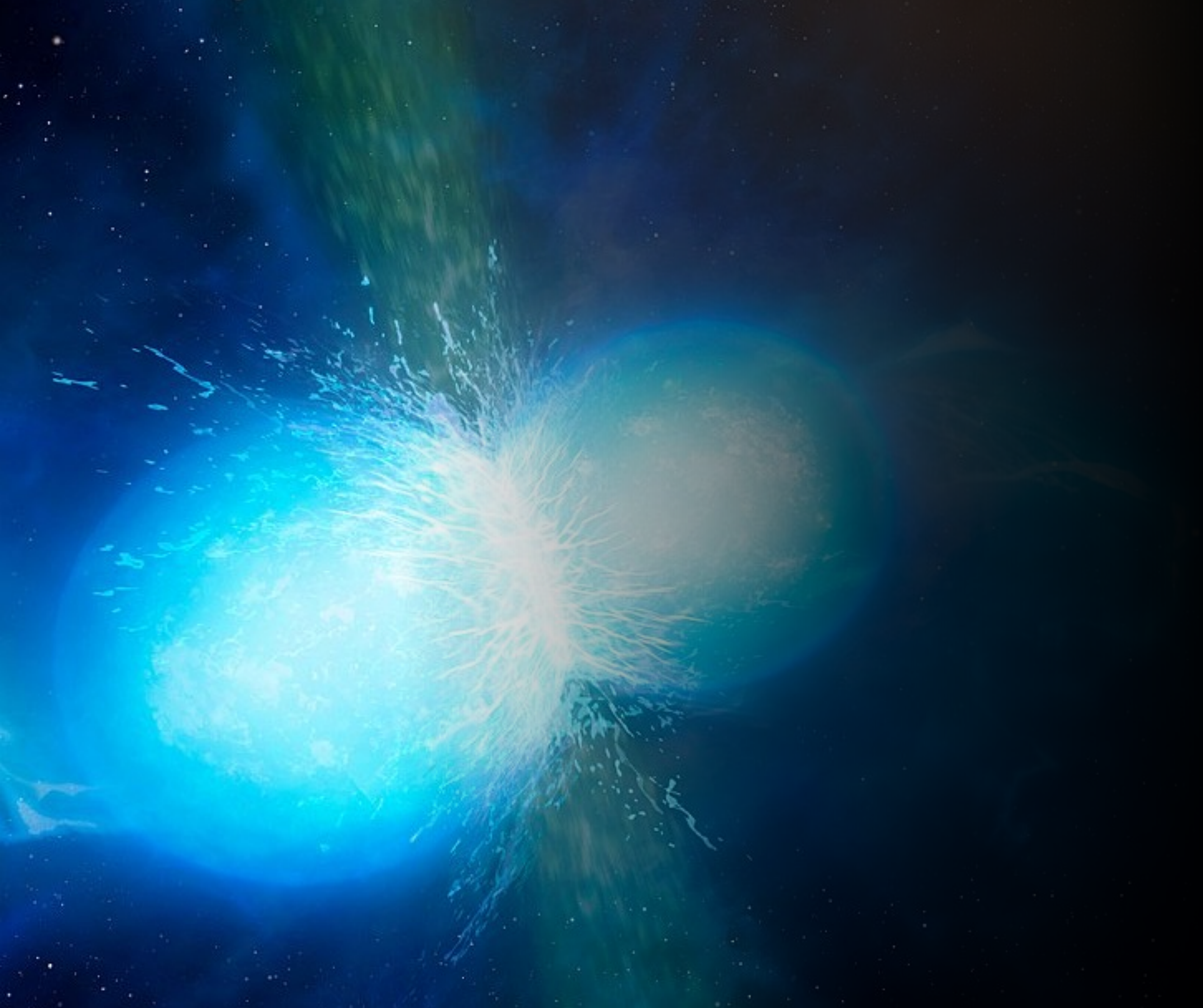

Axion condensates in a time dependent magnetic field

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Based on *Phys.Rev.D* 98 (2018) 10
and work with Lars Sivertsen,
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Dark Matter in Compact Objects, Stars, and in Low Energy Experiments, INT, UW 2022

Axions and axion like particles

Pseudoscalar particles originally postulated to solve the strong CP problem: QCD axions.

String theory predicts axion like particles (ALP).

Dark matter candidate.

Can we find them?

What would be the observable signatures?

Ultralight axions

Huge mass range: 10^{-10} eV to 10^{-33} eV.

The entire range of masses is being explored for observational signatures.

Ultralight axions

Huge mass range: 10^{-10} eV to 10^{-33} eV.

Focusing around here

Why? Because this scale coincides with several other astrophysical energy scales.

Interplay of these scales → interesting theoretical and observational implications.

Condensates

Axions are bosons.

Can form Bose Einstein condensates: coherently oscillating spatial lump.

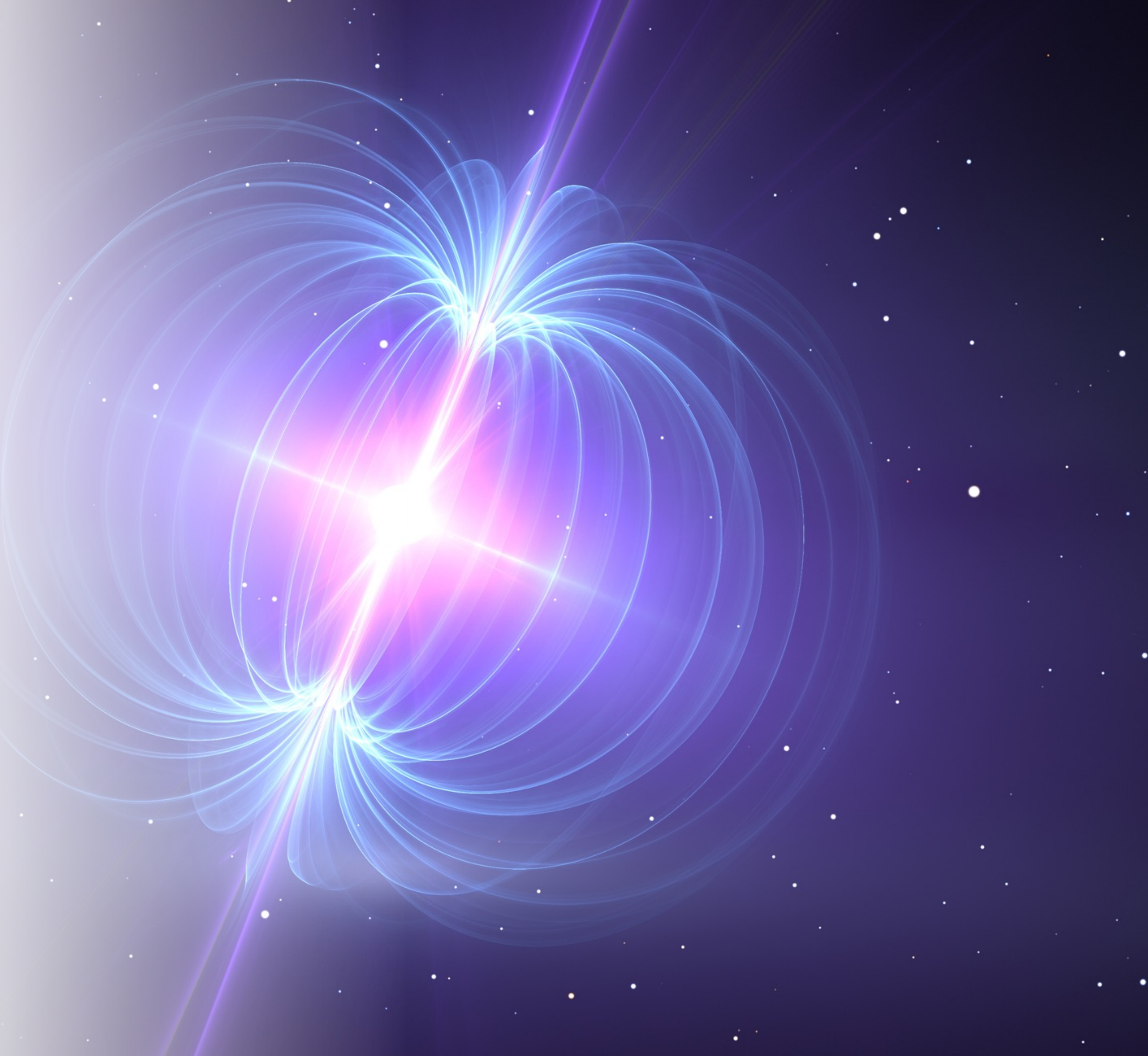
Condensates: Axion stars, superradiant condensates around black holes.

Decay and conversion to photons

Axions couple to photons.

One axion to two photons coupling: stimulated emission, spontaneous emission from condensates.

Conversion of one axion to a photon in a background magnetic field: neutron stars.



Highlights: what is exciting?

What are the different contexts in which the scales of around $\sim 10^{-11}$ eV arises.

How these scales conspire to facilitate electromagnetic radiation from axion condensates.

Neutron stars with strong magnetic fields can play an important role in facilitating this decay.

Binary neutron star mergers with time dependent magnetic fields can further facilitate such decays.

Axion condensates decaying to photons

Lagrangian

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{C\,\beta}{4\pi f_a}\underbrace{\phi\,\epsilon_{\mu\nu\sigma\lambda}F^{\mu\nu}F^{\sigma\lambda}}_{\text{Axion-photon coupling}} + \frac{1}{2}\left(\partial_\mu\phi\right)^2 - \underbrace{V(\phi)}_{\text{Axion potential: condensate physics}}$$

To get to axion condensate profile:

Set $C = 0$.

Solve classical EOM, to find localized lumps.

To get to EM fields, radiation:

Write equations of motion for electromagnetic fields.

Plug in the condensate profile.

Serves as a source term for EM fields.

Superradiant Condensates

Axion condensates around Black holes, take into account gravity in equation of motion.

Mass of the axion that condenses is set by the black hole mass.

For a few solar mass black holes, the axion mass is around 10^{-11}eV.

For larger mass black holes, the axion mass is even smaller.

Since the minimum mass of a Black hole is of the order of a few solar mass, the maximum mass of superradiant condensate axions is 10^{-11}eV.

String Axiverse 2010, Arvanitaki, Dimopolous, Dubovsky, Kaloper, March-Russell

Phys. Rev. D **91**, 084011, Arvanitaki, Baryakhtar, Huang

Phys. Rev. D96, 035019 (2017), Baryakhtar, Lasenby, Teo

Other types of axion condensates

Solve axion EOM for a fixed axion number.

The axion potential (with or without gravity) forces the axions to form clumps.

Axion mass is not constrained as in the case of superradiant condensates (constrained by Black hole mass).

Condensate size set by the equation of motion for a fixed axion mass and a fixed number of axions*.

A useful approximation for the condensate

Coherently oscillating uniform condensate

$$\phi \sim \phi_0 \cos(m_a t)$$

Axion mass.

Better approximation $\phi \sim \phi_0(r) \cos(\omega\, t)$

spatial dependence contained here

ω close to m_a

Typically $\phi_0(r)$ approximated by $\operatorname{sech}\left(\frac{r}{R}\right)$ profile where the radius of the condensate is R.

JCAP 01 (2018) 037, Schiappacasse, Hertzberg

Background magnetic field

Axion condensate acts like a transmitting antenna

Plug in $B = B_0\hat{z}$ here

$$\dot{\mathbf{E}} = \nabla \times \mathbf{B} + \left(\frac{C\beta}{\pi f_a}\right)(\dot{\phi}\mathbf{B} + \nabla\phi \times \mathbf{E}),$$

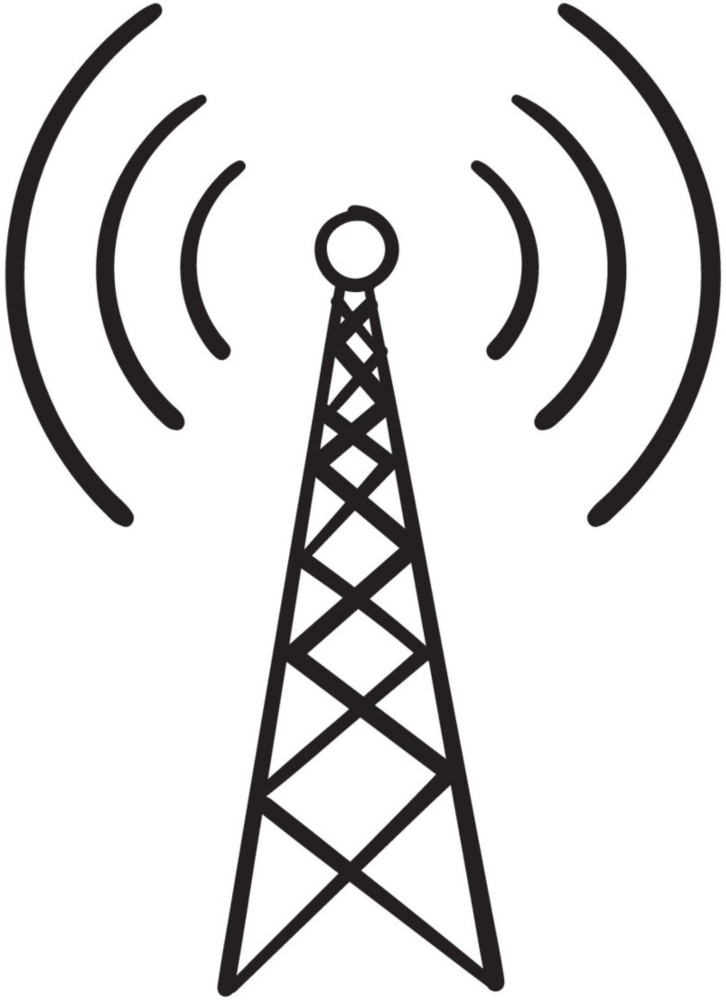
$$\dot{\mathbf{B}} = -\nabla \times \mathbf{E},$$

$$\nabla \cdot \mathbf{E} = \left(\frac{C\beta}{\pi f_a}\right)\nabla\phi \cdot \mathbf{B},$$

Plug in $\phi \sim \phi_0 \cos(m_a t)$ here

$$\nabla \cdot \mathbf{B} = 0,$$

$$\partial_t^2\phi - \nabla^2\phi + \partial_\phi V(\phi) = \left(\frac{C\beta}{\pi f_a}\right)\mathbf{E} \cdot \mathbf{B}.$$

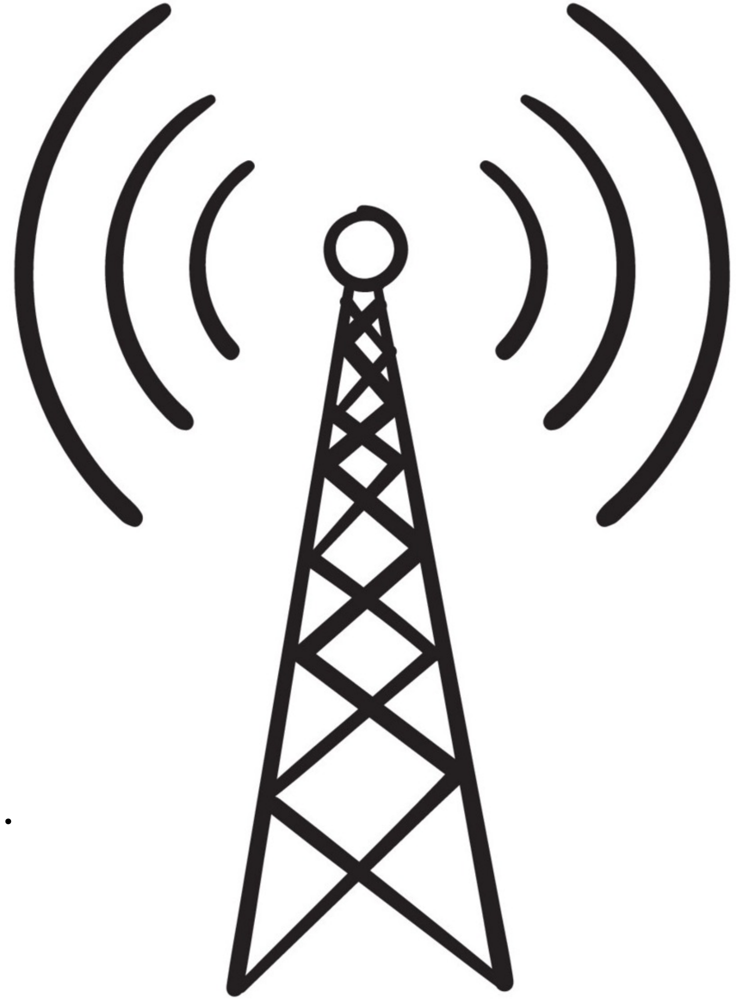


Background magnetic field

Axion condensate acts like a transmitting antenna

$$\dot{\boldsymbol{E}} = \boldsymbol{\nabla}\times\boldsymbol{B} - \frac{c\beta}{\pi f_a}\phi_0 \underbrace{B_0 m_a \sin(m_a t)\hat{z}}_{\text{Oscillating current source}}\,.$$

EM Radiation!



To capture propagating EM waves, consider finite size condensates.

Waves propagate out from the source, i.e. location of the condensate.

With a sech$\left(\frac{r}{R}\right)$ condensate profile

Using $\phi = \tilde{\phi}\,\mathrm{sech}\left(\frac{r}{R}\right)\cos(m_a t)$,

Note these spatial profiles

$$\mathbf{E}_{\rm r}(\mathbf{x},t) = -\partial_t \mathbf{A}(\mathbf{x},t) - \nabla A^0(\mathbf{x},t)$$

$$\approx \frac{C\beta}{\pi f_a}\frac{\tilde{\phi} B_0 m_a \pi^2 R^2}{4|\mathbf{x}|}\left[\frac{\tanh(\pi k_{m_a} R/2)}{\cosh(\pi k_{m_a} R/2)}\cos(m_a t - k_{m_a}|\mathbf{x}|)\right]\hat{\mathbf{x}}_z$$

$$+\left(\frac{C\beta}{\pi f_a}\right)\frac{\tilde{\phi} B_0 k_{m_a} \pi^2 R^2}{4|\mathbf{x}|}\left[\frac{\tanh(\pi k_{m_a} R/2)}{\cosh(\pi k_{m_a} R/2)}\cos(m_a t - k_{m_a}|\mathbf{x}|)\right]\hat{\mathbf{x}},$$

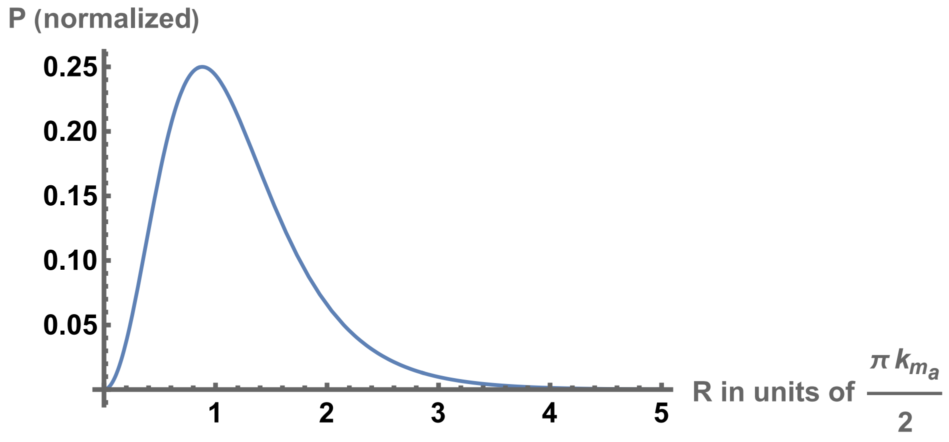
$$\mathbf{B}_{\rm r}(\mathbf{x},t) = \nabla \times \mathbf{A}(\mathbf{x},t)$$

$$\approx \frac{C\beta}{\pi f_a}\frac{\tilde{\phi} B_0 m_a \pi^2 R^2}{4|\mathbf{x}|}\left[\frac{\tanh(\pi k_{m_a} R/2)}{\cosh(\pi k_{m_a} R/2)}\cos(m_a t - |\mathbf{x}| k_{m_a})\right](\hat{\mathbf{x}} \times \hat{\mathbf{x}}_z),$$

Here, $k_{m_a} = m_a$

With a sech$\left(\frac{r}{R}\right)$ condensate profile

$$P = \left(\frac{C\beta}{\pi f_a}\right)^2 \frac{\tilde{\phi}^2 B_0^2 m_a^2 R^4 \pi^5}{12} \left(\frac{\tanh(\pi k_{m_a} R/2)}{\cosh(\pi k_{m_a} R/2)}\right)^2 .$$



Note this spatial profile.

For $R \gg \frac{1}{m_a}$, radiation is suppressed. So, large condensates don't radiate efficiently.

Neither do very small ones.

For efficient radiation we need $R \sim \frac{1}{m_a}$

But this is in free space.

What if you had a plasma medium

If the medium has a plasma frequency of ω_P.

All formulae remain the same, except $k_{m_a} = \sqrt{m_a^2 - \omega_P^2}$

So, condensates with large radius can radiate efficiently as long as $k_{m_a} \ll m_a$ and $R \sim 1/k_{m_a}$. This happens when $m_a \sim \omega_P$.

$$P = \left(\frac{C\beta}{\pi f_a}\right)^2 \frac{\tilde{\phi}^2 B_0^2 m_a^2 R^4 \pi^5}{12} \left(\frac{\tanh(\pi k_{m_a} R/2)}{\cosh(\pi k_{m_a} R/2)}\right)^2 .$$

A resonance in radiation of large condensates!
First conspiracy.

Where can such a conspiracy happen: Interstellar medium?

Model the interstellar medium with ionized Hydrogen with density 1/cc and 0.001/cc at $T = 10^4 - 10^6$K.

One can use the Drude model to describe the plasma.

Involves augmenting Maxwells equations with constitutive relation for the current in linear response $J = \sigma E$

where σ can be frequency dependent,

$$\sigma(\omega) = \frac{4\pi n_e e^2 \tau_{\rm coll}}{m_e(1 - i\tau_{\rm coll}\omega)}$$

Interstellar plasma

$$\sigma(\omega) = \frac{4\pi n_e e^2 \tau_{\rm coll}}{m_e(1 - i\tau_{\rm coll}\omega)}$$

n_e = electron density $\quad$ τ_{coll} = collision time

m_e = electron mass

In the limit of $\tau_{\text{coll}} \gg \frac{1}{\omega}$, collision-less regime.

$$\sigma(\omega) = i\,\frac{4\pi n_e\, e^2}{m_e\omega}$$

Interstellar plasma

Plug this conductivity in Maxwell's equation

$$\underset{\sim k^2}{\underset{\downarrow}{\nabla \times B}} - \underset{\sim \omega^2}{\underset{\downarrow}{\dot{E}}} - \underset{\sim \omega_P^2}{\underset{\downarrow}{\sigma E}} = \text{source}$$

(In the collision-less limit)

Wave equation with a dispersion relation $\omega = \sqrt{k^2 + \omega_P^2}$

$$\text{where } \omega_P = \frac{4\pi n_e\, e^2}{m_e}$$

Interstellar plasma

Is in the collision-less limit for frequencies $\sim 10^{-10} - 10^{-11}$ eV.

Plasma frequency is $\omega_P \sim 10^{-10}$ eV plugging in a density of 1/cc!

For smaller density, the frequency can be even smaller.

Right in the range of the mass scale of interest 10^{-11} eV.

With an axion frequency(mass) of $\omega \sim m_a \sim 10^{-11}$ eV, the wavelength for resonance $\mathrm{k}_{\mathrm{m_a}} = \sqrt{\omega^2 - \omega_P^2}$ is much smaller than m_a itself → efficient radiation for large condensates of size $R \sim 1/\sqrt{\omega^2 - \omega_P^2}$.

So, large condensates of axions of mass 10^{-11} eV can decay quickly, thanks to the interstellar medium

Time dependent B field: another way to enhancement

Consider a time dependent magnetic field, of frequency Ω.

Then Maxwells equations

$$\dot{\boldsymbol{E}} = \boldsymbol{\nabla}\times\boldsymbol{B} - \frac{C\beta}{\pi f_a}\tilde{\phi}\, B_0 \cos(\Omega t)\, m_a \sin(m_a t)\hat{z}\,.$$

The current source term separates into two frequencies
$(m_a - \Omega)$ and $(m_a + \Omega)$.

Power radiated

$$P_{\rm av} \approx \frac{4\pi}{3}\left(\frac{C\beta}{\pi f_a}\frac{B_0 m_a\tilde{\phi}}{8}\pi^2 R^2\right)^2\left[\frac{\tanh\left(\frac{\pi(m_a+\Omega)R}{2}\right)^2}{\cosh\left(\frac{\pi(m_a+\Omega)R}{2}\right)^2}+\frac{\tanh\left(\frac{\pi|m_a-\Omega|R}{2}\right)^2}{\cosh\left(\frac{\pi|m_a-\Omega|R}{2}\right)^2}\right]$$

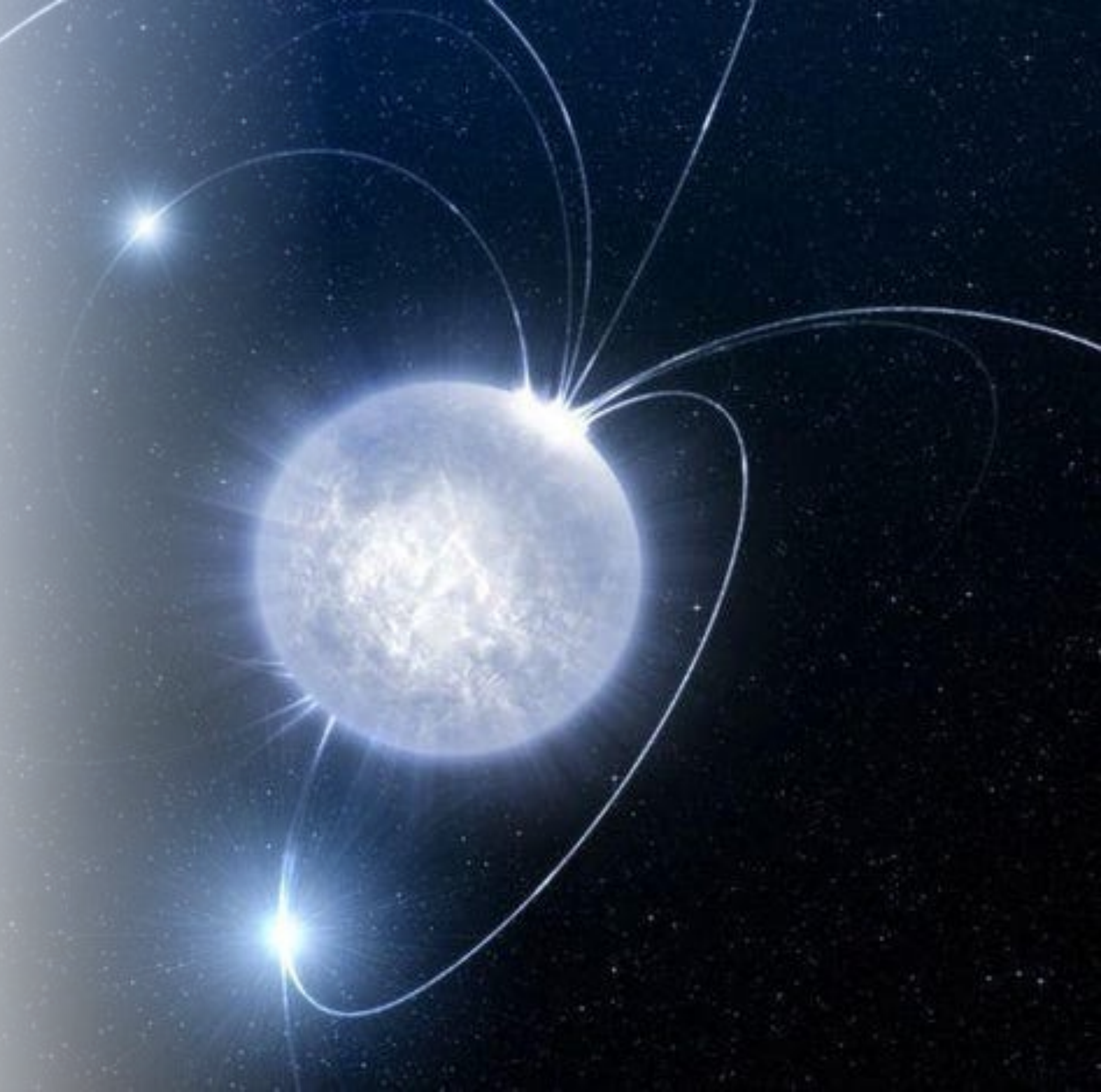
Time dependent B field

So, resonant enhancement (efficient radiation) can take place

for $R \gg \frac{1}{m_a}$, so long as Ω is close to m_a.

Where can we find time dependent magnetic fields?

Off axis magnetic field in neutron stars.



Frequency of magnetic field due to off axis rotation(NS)

Set by the orbital frequency of the neutron star.

What's the orbital frequency?

For millisecond pulsars, it's in the range of KHz, i.e. 10^{-12}eV!

Again in the mass scale range we are interested in.

$10^{-11} - 10^{-12}$ eV axion condensates in a time-dependent B field may decay through resonant EM radiation.

Another conspiracy at rescue.

List of scales converging

ISM plasma frequency

Orbital frequency of neutron stars

Axion mass scales of 10^{-11} eV.

If you put these together, you can get a resonant enhancement of large axion condensate EM radiation.

Interesting to explore how these scales interact with each other when their order of magnitude are the same.

Putting magnetic field and plasma frequency together

Compute radiation in a time dependent background B field and a plasma frequency.

Resonant condition becomes

$$R\sqrt{(m_a \mp \Omega)^2 - \omega_P^2} \sim 1$$

So, $m_a \pm \Omega$ have to be close to ω_P to get resonant enhancement.

For very large $R,\ R \gg \frac{1}{m_a}$, $m_a \pm \Omega$ and ω_P have to be within a few percent.

i.e. not enough to be of the same order. The cancellation has to be almost exact.

Putting magnetic field and plasma frequency together

This is in general not possible, even if the different scales are of the same order.

But if one of the scales Ω or ω_P varies slowly with time, even though $m_a - \Omega$ may not match ω_P exactly at one instant,

it can do so at a later time.

Where can we find time dependent magnetic field frequency?

Binary mergers?

Two neutron stars with strong magnetic fields orbiting.

Rotation frequency increases with time.

Possible source of time dependent Ω?

If so, can allow a range of axion condensates to decay as the stars orbit.

Outlook

What are the possible signatures of such decay?

Cannot be electromagnetic, wavelength is too long for detection through EM. Has to be gravitational.

These decays have implications for the stability of axion condensates in a magnetic field.

Lifetime of condensates is dependent on photon coupling.

Outlook 2

The scales considered in this talk converge. But neutron star environments are more complex than described here.

Magnetic field evolution is also more complicated than described.

More realistic description of these effects needed to accurately capture the decay of the condensates.

Several conceptual questions still remain.

1. Take into account back-reaction of the axion condensates in EM radiation?
2. Does the resonance survive when back-reaction is taken into account ?