Description of doubly-charmed tetraquark T_{cc}^{+} within a constituent quark model

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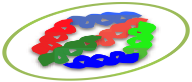
Exotic matter

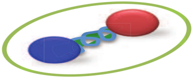
☞ QCD describes the interactions of coloured quarks and gluons and the formation of hadronic matter.

☞ QCD makes precise predictions at high energies; however, the theory cannot be used directly for describing the interactions of quarks inside hadrons.

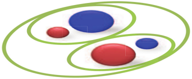
Conventional hadrons consists of baryons and mesons, hadrons with an alternative quark and gluon content are considered exotics.



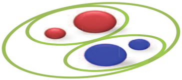
☞ Glueballs (only gluons)
An hypothetical composite particle which consists solely of gluon particles, without valence quarks.



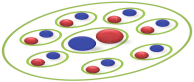
☞ Hybrids ($Q\bar{Q}g$)
Exotic properties are due to gluonic excitations.



☞ Molecules ($Q\bar{q} - \bar{Q}q$)
Shallow bound states of heavy mesons analogous to the deuteron.



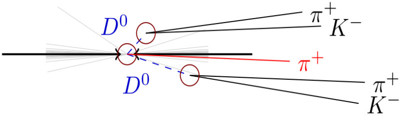
☞ Diquarkonium ($Qq - \bar{Q}\bar{q}$)
The constituent quarks are assumed to be clustered into color triplet diquarks.

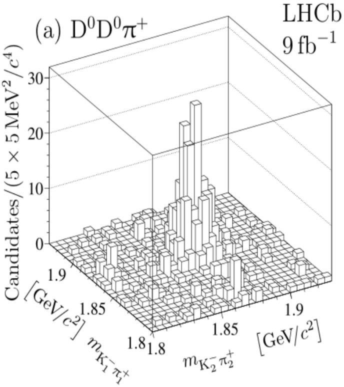


☞ Hadroquarkonium ($Q\bar{Q} - q\bar{q}$)
A compact core that is a color-singlet $Q\bar{Q}$ surrounded by light mesons.

The landmark of 2021: A signal in $D^0 D^0 \pi^+$

Event selection

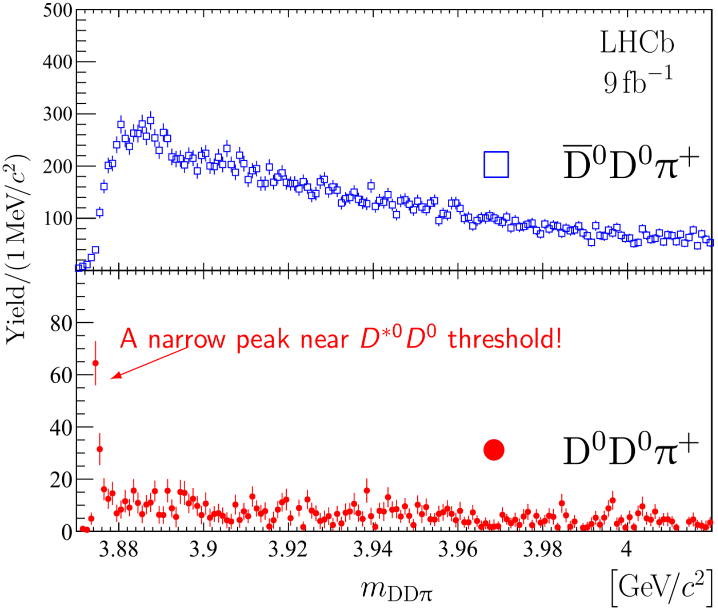




LHCb Coll., Nature Phys. 18 (2022) 751.
LHCb Coll., Nature Commun. 13 (2022) 3351.

☞ Observations:

- Select $D^0 D^0 \pi^+$ candidates from primary pp-vertex with detached $D^0 \to K^- \pi^+$.
- Require detached $K^- \pi^+$ with high p_T.
- Require good quality of tracks, vertices, and particle IDs.
- Ensure no K/π candidates belong to one track (clones).
- Ensure no reflections via miss-ID.
- Remove fake-D background using 2d fit to $(m_{K\pi} \times m_{K\pi})$.


LHCb
$9\,\mathrm{fb}^{-1}$
$\overline{\mathrm{D}}^0\mathrm{D}^0\pi^+$
A narrow peak near $D^{*0} D^0$ threshold!
$\mathrm{D}^0\mathrm{D}^0\pi^+$
Yield/(1 MeV/c^2)
$m_{\mathrm{DD}\pi}$ [GeV/c^2]
500
400
300
200
100
80
60
40
20
0
3.88
3.9
3.92
3.94
3.96
3.98
4

☞ Modeling procedure:

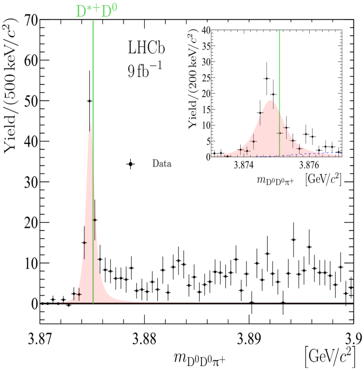
- BW signal $[(DD)_S\ \pi\ P\text{-wave}]$ + background of 2-body phase-space + polynomial.
- Convolved with the detector resolution, root-mean-square of 400 keV.
- The statistical significance of the peak is overwhelming, 22σ.
- Significance of 4.3σ for having the peak below threshold.

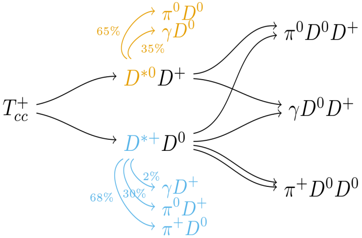
☞ Model results:

Parameter	Value
N	117 ± 16
$\delta m_{\rm BW}$	-273 ± 61 keV/c^2
$\Gamma_{\rm BW}$	410 ± 165 keV

☞ Model assumptions:

- $J^P = 1^+$ state that decays to DD^* in a S-wave.
- T_{cc}^+ is an isoscalar because absence of a signal in the D^0D^+ and $D^+D^0\pi^+$ mass distributions.
- No isospin violation in couplings to $D^{*+}D^0$ and $D^{*0}D^+$.

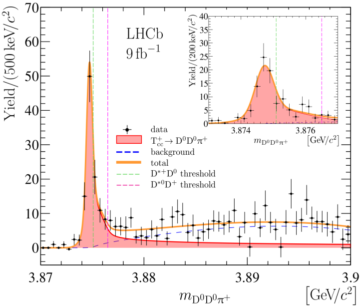


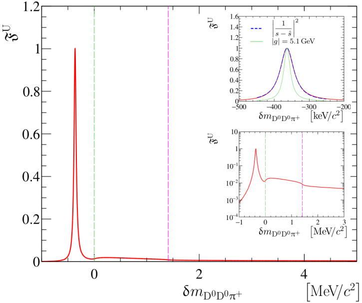


☞ Observations:

- Nearly-isolated resonance below the $D^{*+}D^0$ threshold.
- Long tail with cusps at the $D^{*+}D^0$ and $D^{*0}D^+$ thresholds.
- Peak position: (-360 ± 40) keV.
  $\rightarrow$ The most precise ever with respect to threshold.
- Full width at half maximum (FWHM): (48 ± 2) keV.
- Lifetime: $\tau \approx 10^{-20}$ s.
  $\rightarrow$ Unprecedentedly large for exotic hadrons.

An extremely narrow state, very close to threshold, which is a strong candidate for a pure molecular state.



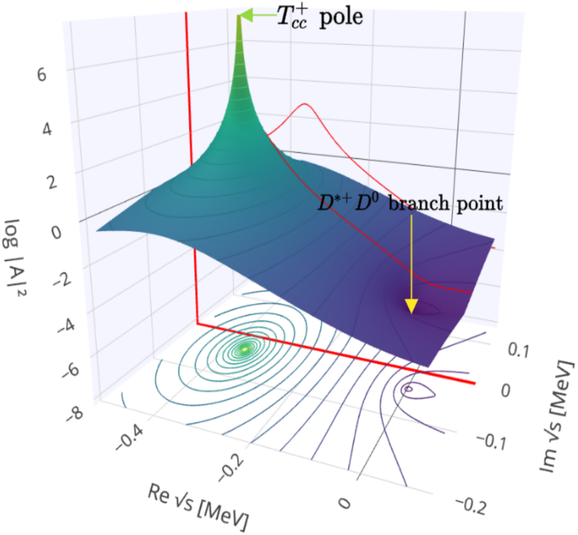


☞ Observations:

- Dynamic amplitude of $DD^* \to DD^*$ scattering.
- The construction is guided by unitarity and analyticity.
- Analytic continuation is not trivial due to three-body decays.
- The pole parameters:

$$\delta m_{\text{pole}} = (-360 \pm 40^{+4}_{-0})\,\text{keV}\,,$$
$$\Gamma_{\text{pole}} = (48 \pm 2^{+0}_{-14})\,\text{keV}\,.$$

M. Mikhasenko et al., Phys. Rev D98 (2018) 096021; JHEP 08 (2019) 080.



Molecular nature of the T_{cc}^{+} state as a DD^{*} system using a **constituent quark model**

- Spectra.
  Segovia:2009zz, Segovia:2015dia, Yang:2019lsg, Ortega:2020uvc.
- Electromagnetic, weak and strong decays and reactions.
  Segovia:2011zza, Segovia:2011dg, Segovia:2014mca, Martin-Gonzalez:2022qwd.
- Coupling with meson-meson thresholds.
  Ortega:2016pgg, Ortega:2018cnm, Ortega:2021xst, Ortega:2021fem.

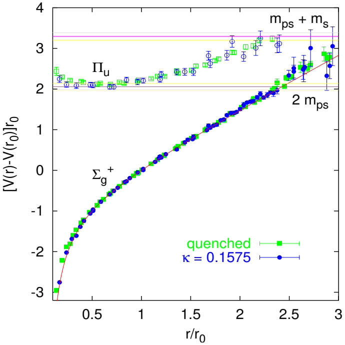
Advantage of using a model with relatively large history: all the parameters of the model have already been constrained before → we present a parameter-free calculation.

Main pieces of the theoretical approach

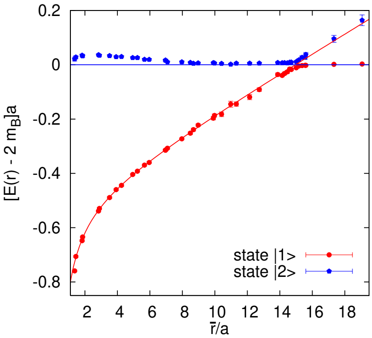
☞ The interaction between quarks is given by a **Constituent Quark Model (CQM)** whose phenomenological potential involves a confining interaction + Goldstone-boson exchanges between light quarks + the perturbative one-gluon exchange term.

☞ The interaction between mesons is deduced using the **Resonating Group Method (RGM)** which derives the Hamiltonian for meson-meson states from the quark-antiquark interaction.

G.S. Bali *et al.* Phys. Rep. 343 (2001) 1.



G.S. Bali *et al.* Phys. Rev. D71 (2005) 114513.



LINEAR SCREENED POTENTIAL

$$V_{CON}(r) = \left[-a_c(1-e^{-\mu_c r}) + \Delta\right](\vec{\lambda}_i \cdot \vec{\lambda}_j)$$

- *Flavor independent*
- $r \to 0 \quad \Rightarrow \quad V_{CON}(r) \to (-a_c\mu_c r + \Delta)(\vec{\lambda}_i \cdot \vec{\lambda}_j) \quad \Rightarrow$ *Linear.*
- $r \to \infty \quad \Rightarrow \quad V_{CON}(r) \to (-a_c + \Delta)(\vec{\lambda}_i \cdot \vec{\lambda}_j) \quad \Rightarrow$ *Threshold.*

- QCD Lagrangian invariant under the chiral transformation

  Chiral symmetry is spontanously broken

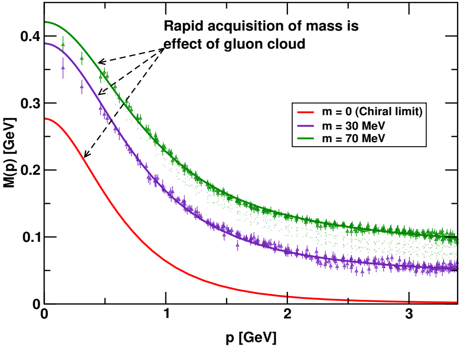
$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - M(q^2)U^{\gamma_5}\right)\psi$$

- Pseudo-Goldstone Bosons ($\vec{\pi}$, K_i and η_8)

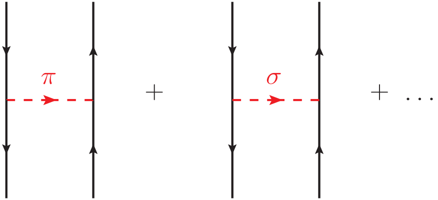
$$U^{\gamma_5} = \exp\left(i\pi^a\lambda^a\gamma_5/f_\pi\right)$$
$$\sim 1 + \frac{i}{f_\pi}\gamma^5\lambda^a\pi^a - \frac{1}{2f_\pi^2}\pi^a\pi^a + \ldots$$

- Constituent quark mass

$$M(q^2) = m_q F\left(q^2\right) = m_q\left[\frac{\Lambda^2}{\Lambda^2+q^2}\right]^{1/2}$$



C.D. Roberts, arXiv:1109.6325v1 [nucl-th]



Beyond the chiral symmetry breaking scale

$\Downarrow$

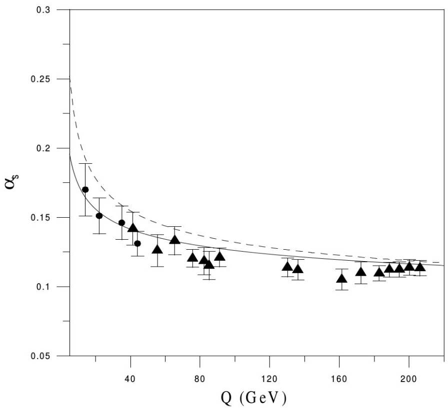
Dynamics to be governed by QCD perturbative effects

- We take them into account through the one-gluon exchange (OGE) potential.
- The OGE is a standard color Fermi-Breit interaction obtained from the vertex:

$$\mathcal{L}_{qqg} = i\sqrt{4\pi\alpha_s}\,\bar{\psi}\gamma_\mu G_c^\mu \lambda^c \psi$$

- Effective scale dependent strong coupling constant:

$$\alpha_s(\mu) = \frac{\alpha_0}{\ln\left(\frac{\mu^2+\mu_0^2}{\Lambda_0^2}\right)}$$



J. Vijande *et al.* J. Phys. G31 (2005) 481.

- The 1-hadron wave function:

$$\psi_A = \phi_A(\vec{p}_A)\,\chi_{SF}\,\xi_c$$

- The 2-hadron wave function:

$$\begin{aligned}\psi_{AB} &= \mathcal{A}\left[\chi(\vec{P})\,\psi_{AB}^{SF}\right] \\ &= \mathcal{A}\left[\phi_A(\vec{p}_A)\,\phi_B(\vec{p}_B)\,\chi(\vec{P})\,\chi_{AB}^{SF}\,\xi_c\right] \\ &= \int \mathcal{A}\left[\phi_A(\vec{p}_A)\,\phi_B(\vec{p}_B)\,\delta^{(3)}(\vec{P}-\vec{P}_i)\,Z(\vec{P}_{\rm CM})\right]\chi(\vec{P}_i)d\vec{P}_i\end{aligned}$$

- Dynamics of the bound state governed by the Schrödinger equation:

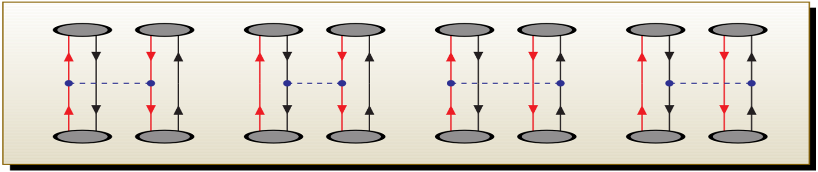
$$(\mathcal{H}-E_T)\,|\,\psi\,\rangle = 0 \quad \Leftrightarrow \quad \mathcal{H} = \sum_{i=1}^{N}\frac{\vec{p}_i^{\,2}}{2m_i} + \sum_{i<j} V_{ij} - T_{\rm CM}$$

$$\left(\frac{\vec{P}'^{\,2}}{2\mu} - E\right)\chi(\vec{P}') + \int \left({}^{\rm RGM}V_D(\vec{P}',\vec{P}_i) + {}^{\rm RGM}K_E(\vec{P}',\vec{P}_i)\right)\chi(\vec{P}_i)d\vec{P}_i = 0$$

- Dynamics of the scattering state governed by the Lippmann-Schwinger equation:

$$T_\alpha^{\alpha'}(z;p',p) = V_\alpha^{\alpha'}(p',p) + \sum_{\alpha''}\int dp''\,p''^2\,V_{\alpha''}^{\alpha'}(p',p'')\frac{1}{z-E_{\alpha''}(p'')}T_\alpha^{\alpha''}(z;p'',p)$$

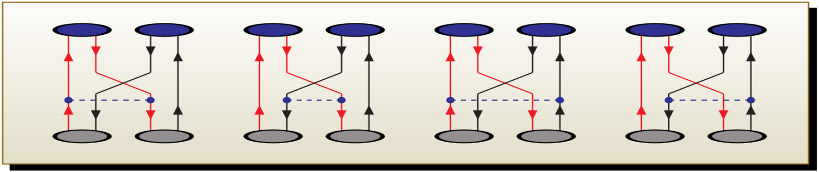
Direct terms



$$ {}^{\text{RGM}}V_D(\vec{P}', \vec{P}) = \sum_{i\in A, j\in B} \int d\vec{p}_{A'}\, d\vec{p}_{B'}\, d\vec{p}_A d\vec{p}_B \times $$
$$ \times\; \phi^*_{A'}(\vec{p}_{A'})\phi^*_{B'}(\vec{p}_{B'}) V_{ij}(\vec{P}', \vec{P}) \phi_A(\vec{p}_A)\phi_B(\vec{p}_B) $$

- V_{ij} the interaction at quark level given by the CQM.
- $i(j)$ the indices that run inside the constituents of the $A(B)$ meson.
- $p_{A(B)}$ is the relative internal momentum of the $A(B)$ meson.
- The wave functions of the mesons act as natural cut offs for the potentials.

Exchange terms



The non-local and energy-dependent exchange kernel ${}^{\mathrm{RGM}}K_E$, that models the quark rearrangement between mesons, can be written as

$$ {}^{\mathrm{RGM}}K_E(\vec{P}',\vec{P}_i) = {}^{\mathrm{RGM}}V_E(\vec{P}',\vec{P}_i) - E_T\, {}^{\mathrm{RGM}}N_E(\vec{P}',\vec{P}_i) $$

i.e. it can be separated in a potential term plus a normalization term where E_T is the total energy of the system and $\vec{P}_i$ is a continuous parameter:

$$ \begin{aligned} {}^{\mathrm{RGM}}V_E(\vec{P}',\vec{P}_i) = \int & d\vec{p}_{A'}\, d\vec{p}_{B'}\, d\vec{p}_A d\vec{p}_B d\vec{P} \phi^*_{A'}(\vec{p}_{A'}) \times \\ & \times \phi^*_{B'}(\vec{p}_{B'}) \mathcal{H}(\vec{P}',\vec{P}) P_{12} \left[\phi_A(\vec{p}_A)\phi_B(\vec{p}_B)\delta^{(3)}(\vec{P}-\vec{P}_i) \right] \\ {}^{\mathrm{RGM}}N_E(\vec{P}',\vec{P}_i) = \int & d\vec{p}_{A'}\, d\vec{p}_{B'}\, d\vec{p}_A d\vec{p}_B d\vec{P} \phi^*_{A'}(\vec{p}_{A'}) \times \\ & \times \phi^*_{B'}(\vec{p}_{B'}) P_{12} \left[\phi_A(\vec{p}_A)\phi_B(\vec{p}_B)\delta^{(3)}(\vec{P}-\vec{P}_i) \right] \end{aligned} $$

where $\mathcal{H}$ is the Hamiltonian at quark level.

With the aim of evaluating the molecular nature of the T_{cc}^{+}:

- Coupled-channels calculation of the $J^P = 1^+$ $cc\bar{q}\bar{q}'$ sector including the meson-meson thresholds: $D^0 D^{*+}$ (3875.10), $D^+ D^{*0}$ (3876.51) and $D^{*0} D^{*+}$ (4017.11). The DD^*-pair can be in relative $^3S_1$ or $^3D_1$ partial waves.

- The system recalls the $X(3872)$ problem studied in Ortega:2009hj using the same formalism, but in this case there is explicit charm content.

- We cannot couple the molecular system to charmonium states when studying the T_{cc}^{+} structure; this would have provided additional attraction.

- Besides the direct interaction between $D^{(*)}D^{(*)}$ pairs, we have to consider exchange diagrams to deal with indistinguishable quarks from different mesons in the molecule.

Details of the calculation (II)

Indeed, the right mixture of mesons must be considered in order to have a well-defined isospin state. The quark content of D mesons is $c\bar{q}$. Using the isospin doublets $D = (D^+, -D^0)$ and $D^* = (D^{*+}, -D^{*0})$ the isospin basis is built as

$$|D^*D, I=0\rangle = -\frac{1}{\sqrt{2}}\left(|D^{*+}D^0\rangle - |D^{*0}D^+\rangle\right),$$
$$|D^*D, I=1\rangle = -\frac{1}{\sqrt{2}}\left(|D^{*+}D^0\rangle + |D^{*0}D^+\rangle\right).$$

The energy difference between D^0D^{*+} and D^+D^{*0} is roughly 1.4 MeV, larger than the binding energy of the T_{cc}^+, so to explore possible isospin breaking effects of this threshold energy difference we will perform a calculation in charged basis, defined as

$$|D^{*0}D^+\rangle = -\frac{1}{\sqrt{2}}\left(|D^*D, I=1\rangle - |D^*D, I=0\rangle\right),$$
$$|D^{*+}D^0\rangle = -\frac{1}{\sqrt{2}}\left(|D^*D, I=1\rangle + |D^*D, I=0\rangle\right).$$

Results

☞ The calculation shows one bound state below the lower D^0D^{*+} threshold, with a mass with respect to the latter threshold of -387 keV.

☞ Most of the attraction is due to pseudo-Goldstone boson exchanges, but the state is unbound unless the exchange kernel is considered.

☞ The state is basically a D^0D^{*+} molecule, with $\sim$87% probability due to its proximity to this threshold; the other 13% corresponds to the D^+D^{*0} channel.

☞ Essentially an $I = 0$ state ($\sim$81%), but the mass difference between D^0D^{*+} and D^+D^{*0} thresholds adds a sizable isospin breaking to the bound state, $\sim$19% of $I = 1$.

☞ The pole position and partial widths are

E_B (keV)	$M - i\frac{\Gamma}{2}$ (MeV)	$\Gamma_{D^0D^0\pi^+}$ (keV)	$\Gamma_{D^0D^+\pi^0}$ (keV)	$\Gamma_{D^0D^+\gamma}$ (keV)	Γ (keV)
-387	$3874.713 - i\,0$	49	26	6	84

The state is sensitive to three body effects, so we have performed a similar calculation introducing the energy dependent self-energy of the D^* meson. In this calculation the binding energy goes to 278 keV while the width drops to 42 keV, in better agreement with LHCb analysis.

Details on calculating partial decay widths

The T_{cc}^{+} state can only decay strongly if the meson D^* inside the DD^* system disintegrates. As the D^* width is small, the decay can be calculated perturbatively considering the D^* an unstable particle decaying into $D\pi$ or $D\gamma$:

$$\Gamma_{D^0 D^0 \pi^+} = \Gamma_{D^{*+} \to D^0 \pi^+} \int_0^{k_{\rm max}} k^2 dk |\chi_{D^0 D^{*+}}(k)|^2 \frac{(M_T - E_{D^0} - E_{D^{*+}})^2}{(M_T - E_{D^0} - E_{D^{*+}})^2 + \frac{\Gamma_{D^*}^2}{4}} \,,$$

where $\Gamma_{D^{*+} \to D^0 \pi^+}$ is the $D^{*\,+}$ experimental partial width to $D^0\pi^+$, $\chi_{D^0 D^{*+}}(k)$ is the wave function of the channel $D^0 D^{*\,+}$, E_D are the total energies of the mesons involved in the reaction and $k_{\rm max}$ is the maximum on-shell momentum of the $D^0 D^0 \pi^+$ system:

$$k_{\,\rm max} = \frac{1}{2M_T} \sqrt{\left[M_T^2 - (2m_{D^0} + m_{\pi^+})^2\right] \left[M_T^2 - m_{\pi^+}^2\right]} \,,$$

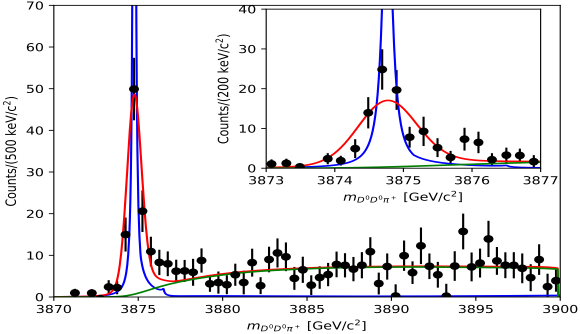
where M_T is the mass of the T_{cc}^{+}. The $D^0 D^0 \pi^+$ threshold is located at about 3869 MeV, *i.e.* there is not much phase space available, which explains the small partial width obtained.

Besides the T_{cc}^{+} below $D^0 D^{*+}$, we also find a molecular candidate slightly below the $D^+ D^{*\,0}$ threshold in the $J^P = 1^+$ $cc\bar{q}\bar{q}'$ sector.

State	$\mathcal{P}_{D^0 D^{*+}}$ (%)	$\mathcal{P}_{D^+ D^{*0}}$ (%)	$\mathcal{P}_{D^{*+} D^{*\,0}}$ (%)	$\mathcal{P}_{I=0}$ (%)	$\mathcal{P}_{I=1}$ (%)
T_{cc}	86.8	13.1	0.1	81.3	18.7
T'_{cc}	16.9	83.1	0.01	57.7	42.3

State	E_B (keV)	$M - i\,\frac{\Gamma}{2}$ (MeV)	$\Gamma_{D^0 D^0 \pi^+}$ (keV)	$\Gamma_{D^0 D^+ \pi^0}$ (keV)	$\Gamma_{D^0 D^+ \gamma}$ (keV)
T_{cc}	-387	$3874.713 - i\,0$	49	26	6
T'_{cc}	-3	$3876.507 - i\,0.129$	175	140	40

Experimental line-shape description



☞ Observations:

- The theoretical position of the pole and the width allows us to describe the experimental line shape with good accuracy.
- Although the predicted width is much smaller than the experimental data, one should consider the resolution function with a root mean square of 400 keV.
- The T_{cc} peak is clearly visible, while the T'_{cc} peak appears as a small bump smeared by the resolution.
- The normalization of the curve is the only free parameter, which has been calculated with a χ^2-minimization procedure.

Channel	a_{sc} (fm)	r_{eff} (fm)	g ($\text{GeV}^{-1/2}$)
$D^0 D^{*+}$	-7.14	-0.49	0.12
$D^+ D^{*0}$	$-8.98 + 8.57\,i$	$0.82 + 0.48\,i$	0.07
$D^{*0} D^{*+}$	$0.20 + 0.02\,i$	$-6.09 - 6.23\,i$	< 0.01

Observations:

- The scattering length of the lower threshold $D^0 D^{*+}$ is -7.14 fm, fully compatible with the experimental estimation, $a_{sc}^{\text{LHCb}} = -7.15(51)$ fm.
- The LHCb collaboration only gives an upper limit of $r_{\text{exp}} > -11.9(16.9)$ fm at 90(95)% CL. Our calculation throws a compatible value of -0.49 fm.
- The scattering lengths and effective ranges for $D^+ D^{*0}$ and $D^{*0} D^{*+}$ are complex numbers. The imaginary part indicates the existence of inelastic channels.
- The real part of the scattering length of the $D^+ D^{*0}$ is large and negative, compatible with the bound state $T'_{cc}(3876)$.
- Coupling to $D^0 D^{*+}$ and $D^+ D^{*0}$ channels are of the same order, which indicates that the isospin breaking is basically due to the mass difference of the thresholds.

T_{cc}^{+} partners and the bottom sector (I)

☞ We have searched for T_{cc}^{+} partners in alternative spin-parity sectors and thresholds, such as the DD system in $J^P = 0^+$ or D^*D^* with spin-parity $J^P = 0^+$, 1^+ and 2^+.

☞ We have not found additional bound states. We do, however, find a virtual state and a resonance in $J^P = 0^+$ in isospin 1 and 0, respectively, just below the D^*D^* threshold.

☞ Additionally, in $J^P = 1^+$, besides the T_{cc} and T'_{cc}, we find a pole in the second Riemann sheet just below the D^*D^* threshold. Their properties are:

J^P	I	Mass (MeV)	Width (MeV)	E_B (MeV)	$\mathcal{P}_{D^*D^*}$ (%)	Type
0^+	0	4018.0	8.15	0.9	95.6	Resonance
	1	4016.9	0.6	-0.2	98.8	Virtual
1^+	–	4014.0	0	-3.1	38.5	Virtual

☞ We have analyzed the $J^P = 1^+$ $\bar{b}\bar{b}ud$ sector in a coupled-channels calculation analog to that of the T_{cc}^{+}; that is, in charged basis including the B^0B^{*+}, B^+B^{*0} and $B^{*+}B^{*\,0}$ thresholds.

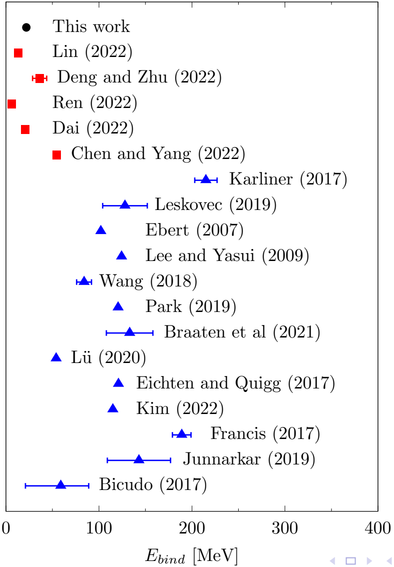
Mass (MeV)	E_B (MeV)	$\mathcal{P}_{B^0B^{*+}}$ (%)	$\mathcal{P}_{B^+B^{*0}}$ (%)	$\mathcal{P}_{B^{*+}B^{*\,0}}$ (%)	$\mathcal{P}_{I=0}$ (%)	$\mathcal{P}_{I=1}$ (%)
10582.2	-21.9	47.8	50.0	2.2	99.99	0.01
10593.5	-10.5	51.0	48.6	0.4	0.02	99.98

☞ We have searched for further T_{bb} states in the $J^P = 0^+$ and 2^+ sectors, including all the meson-meson channels in a relative S-wave; that is, $BB + B^*B^*$ for 0^+ and B^*B^* for 2^+.

J^P	I	Mass (MeV)	Width (MeV)	E_B (MeV)	$\mathcal{P}_{BB}$ (%)	$\mathcal{P}_{B^*B^*}$ (%)	Γ_{BB} (MeV)	$\Gamma_{B^*B^*}$ (MeV)
0^+	0	10553.0	0	6.0	92	8	0	0
		10640.7	2.8	8.7	76	24	2.8	0
	1	10545.9	0	13.1	93	7	0	0
		10672.6	72.0	-23.2	39	61	30.7	41.3
2^+	1	10642.3	0	7.1	-	100	-	0

These results show a populated spectroscopy in the bottom sector, which can be detected in future searches.

We compare our isoscalar T_{bb} molecule with the predictions from other theoretical studies. Our binding energy agrees with those reported by molecular descriptions, whereas the tetraquark studies usually show much deeper bound states.



Conclusions

- The T_{cc}^{+} is found as a D^0D^{*+} molecule (87%) with a binding energy of 387 keV/c^2 and a width of 81 keV, in agreement with the experimental measurements.
- The quark content of the state forces the inclusion of exchange diagrams to treat indistinguishable quarks between the D mesons, which are found to be essential to bind the molecule.
- The $D^0D^0\pi^+$ line shape, scattering lengths and effective ranges of the molecule are also analyzed, which are found to be in agreement with the LHCb analysis.
- We search for further partners of the T_{cc}^{+} in other charm and bottom sectors, finding different candidates. In particular, in the charm sector we found a shallow $J^P = 1^+$ D^+D^{*0} molecule (83%), dubbed T'_{cc}, just 1.8 MeV above the T_{cc}^{+} state.
- In the bottom sector, an isoscalar and an isovector $J^P = 1^+$ bottom partners were identified as BB^* molecules lying 21.9 MeV/c^2 ($I = 0$) and 10.5 MeV/c^2 ($I = 1$), respectively, below the B^0B^{*+} threshold.