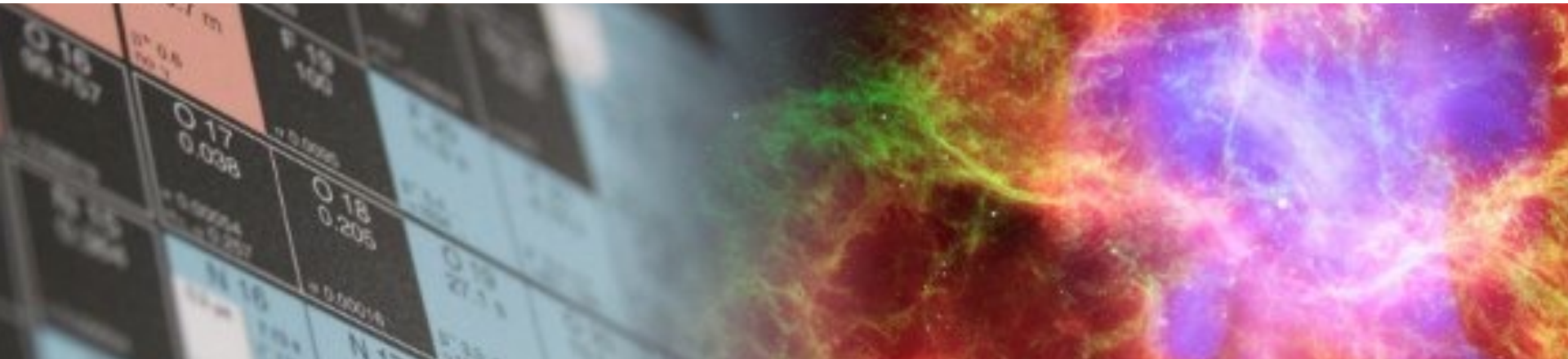


# Equation of state developments for nuclear matter

Achim Schwenk



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT



INT Program “Neutron Rich Matter on Heaven and Earth”, July 22, 2022



European Research Council  
Established by the European Commission

ERC AdG EUSTRONG



Bundesministerium  
für Bildung  
und Forschung

# Outline

Chiral effective field theory for nuclear forces




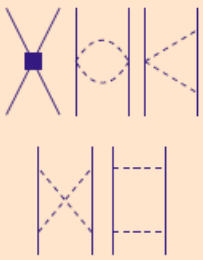


Ab initio calculations of nuclear matter:  
cold, finite temperature and arbitrary proton fraction

Constraints at intermediate densities:  
astro, heavy-ion collisions, functional RG

Nuclear experiments: dipole response, CREX, PREX

# Chiral effective field theory for nuclear forces

Systematic expansion (power counting) in low momenta  $(Q/\Lambda_b)^n$

		NN	3N	4N	
LO	$\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$				based on symmetries of strong interaction (QCD)
NLO	$\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$				long-range interactions governed by pion exchanges



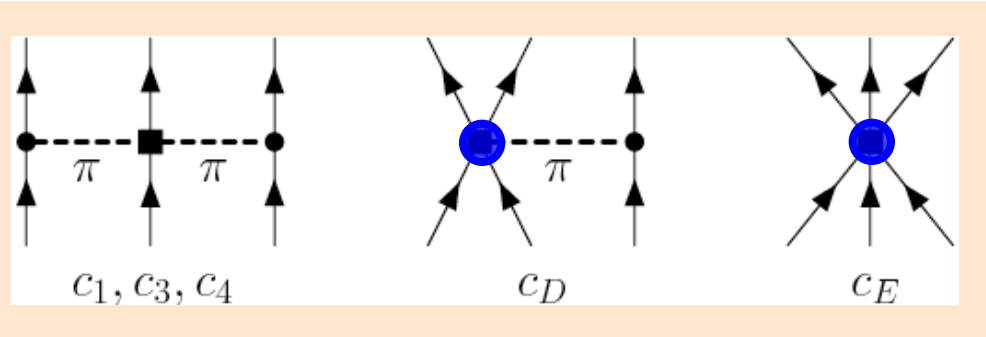
Weinberg (1990,91)

# Chiral effective field theory for nuclear forces

Systematic expansion (power counting) in low momenta  $(Q/\Lambda_b)^n$

	NN	3N	4N
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$			
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$			
N <sup>2</sup> LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			
N <sup>3</sup> LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$			

powerful approach for many-body interactions



only 2 new couplings at N<sup>2</sup>LO

all 3- and 4-neutron forces

predicted to N<sup>3</sup>LO

Hebeler, AS (2010), Tews, Krüger et al. (2013)

derived in (1994/2002)

+ ... (2011) ... (2006) ...

# Chiral effective field theory for nuclear forces

Systematic expansion (power counting) in low momenta  $(Q/\Lambda_b)^n$

IOP Publishing

Journal of Physics G: Nuclear and Particle Physics

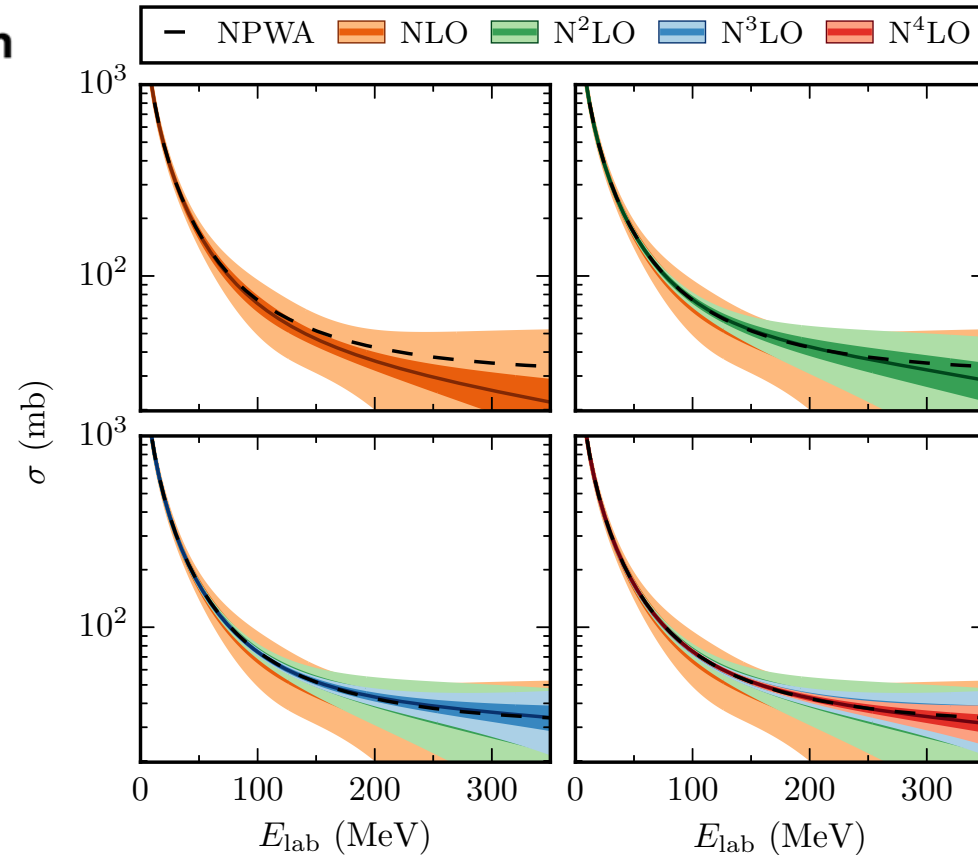
J. Phys. G: Nucl. Part. Phys. 42 (2015) 034028 (20pp)

doi:10.1088/0954-3899/42/3/034028

## A recipe for EFT uncertainty quantification in nuclear physics

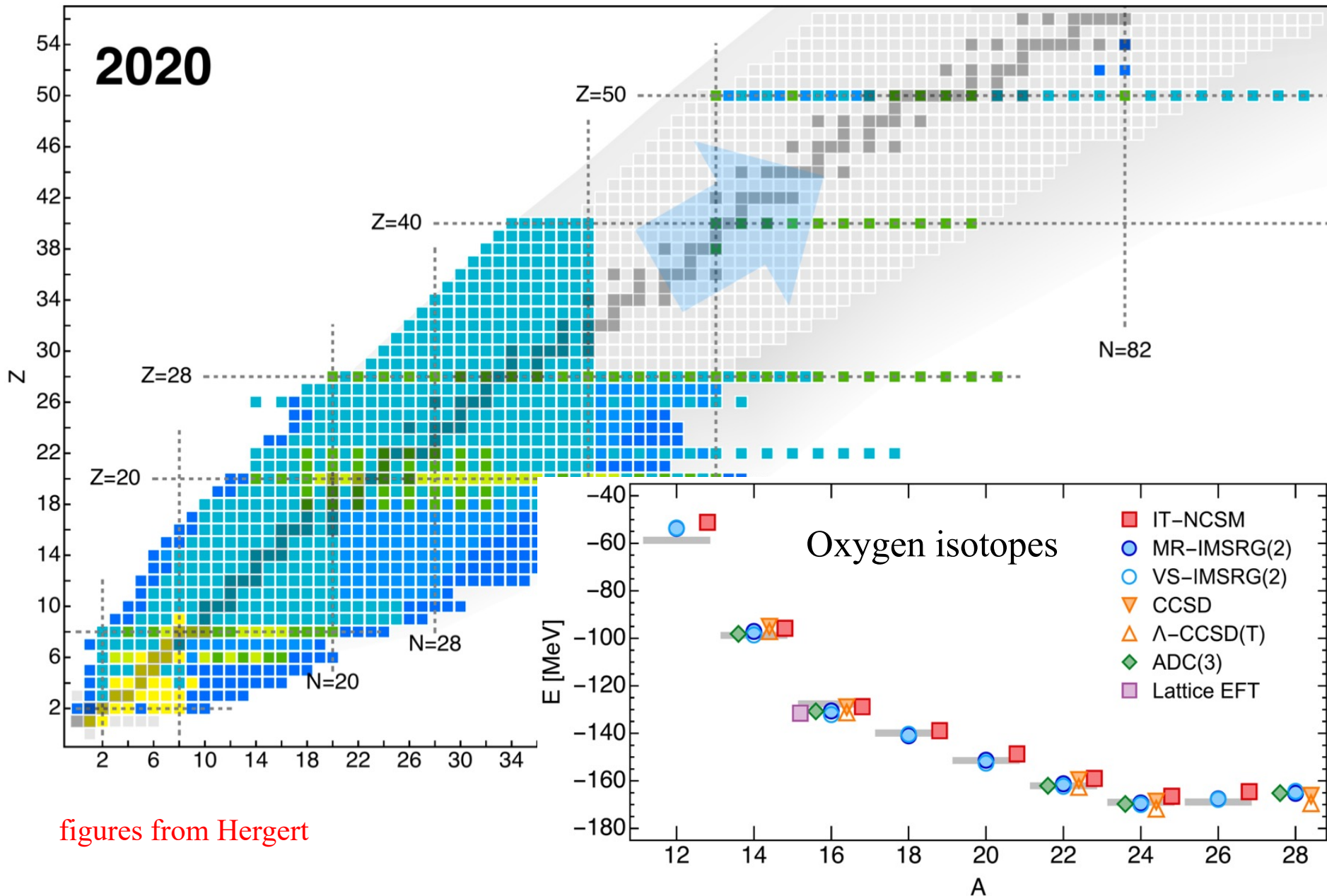
R J Furnstahl<sup>1</sup>, D R Phillips<sup>2</sup> and S Wesolowski<sup>1</sup>

## Bayesian uncertainty estimates and model checking



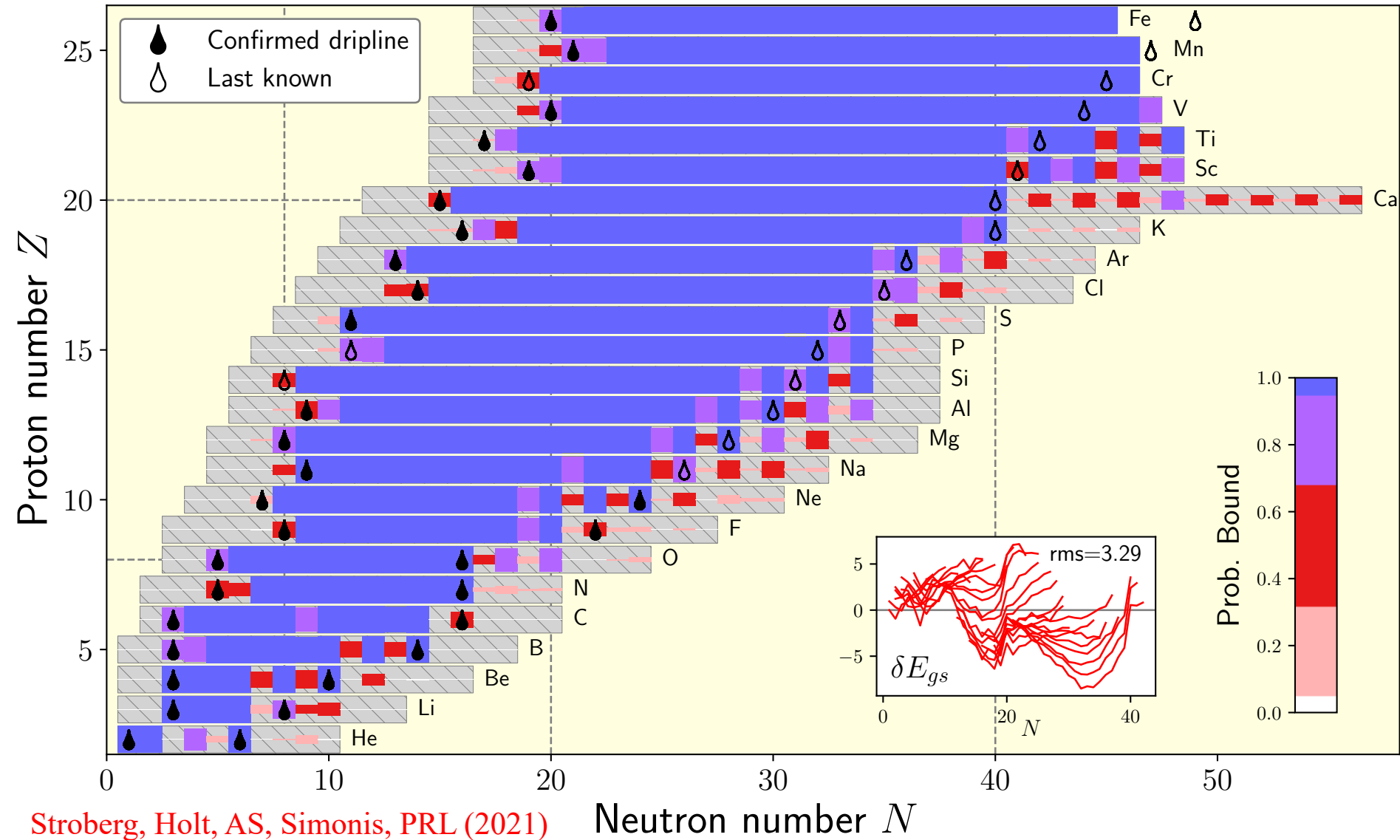
Furnstahl, Phillips, Klos, Wesolowski, Melendez (2015-)

# Great progress in ab initio calculations of nuclei



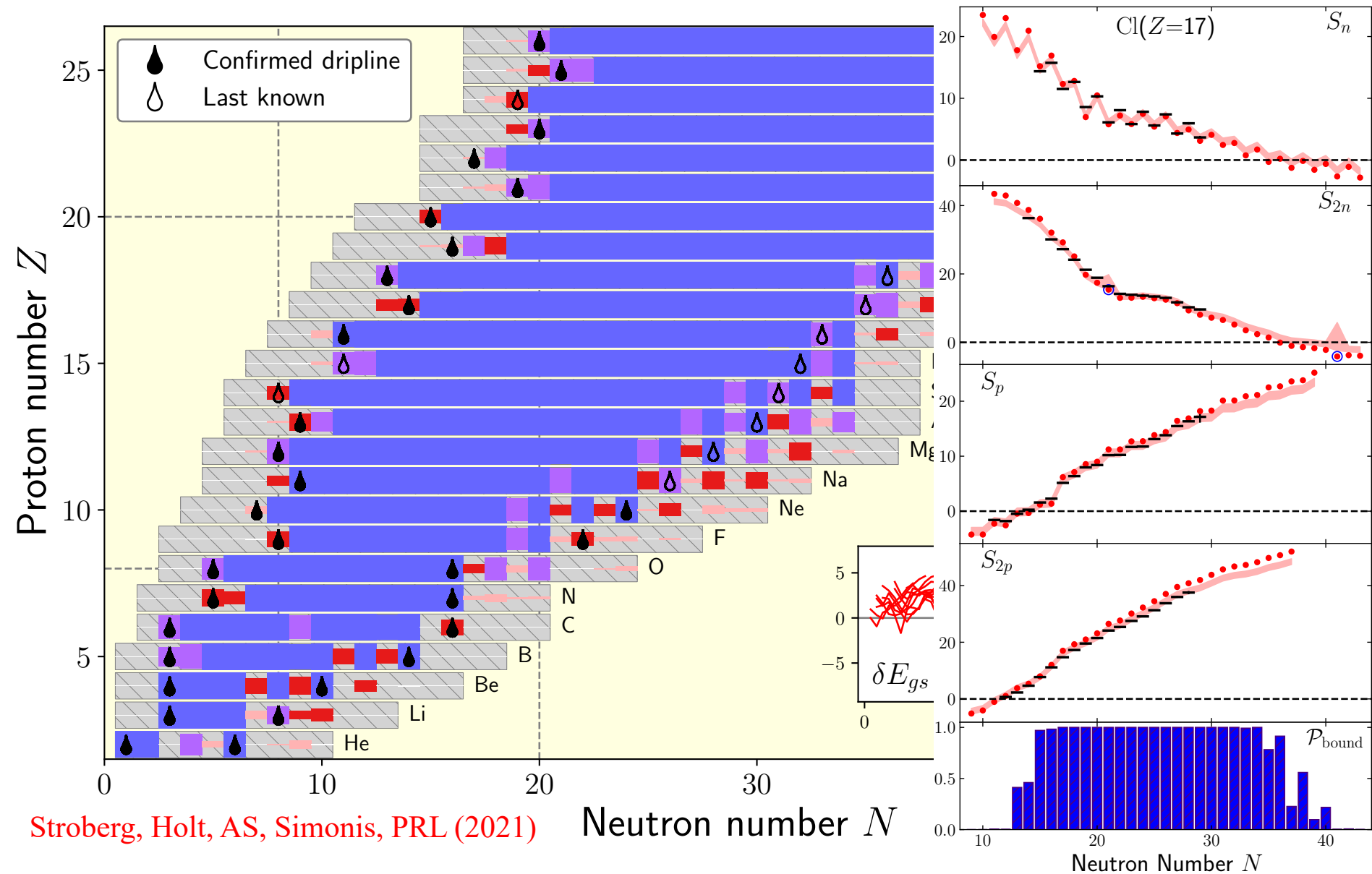
figures from Hergert

# Nuclear landscape based on a chiral NN+3N interaction



ab initio is advancing to global theories, limitations due to input NN+3N

# Nuclear landscape based on a chiral NN+3N interaction



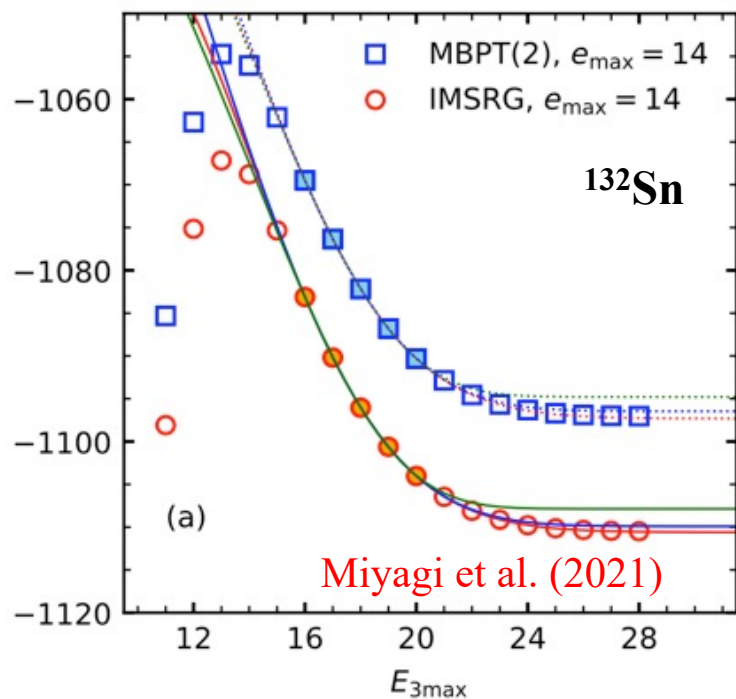
ab initio is advancing to global theories, limitations due to input NN+3N



# First ab initio calculations of $^{208}\text{Pb}$

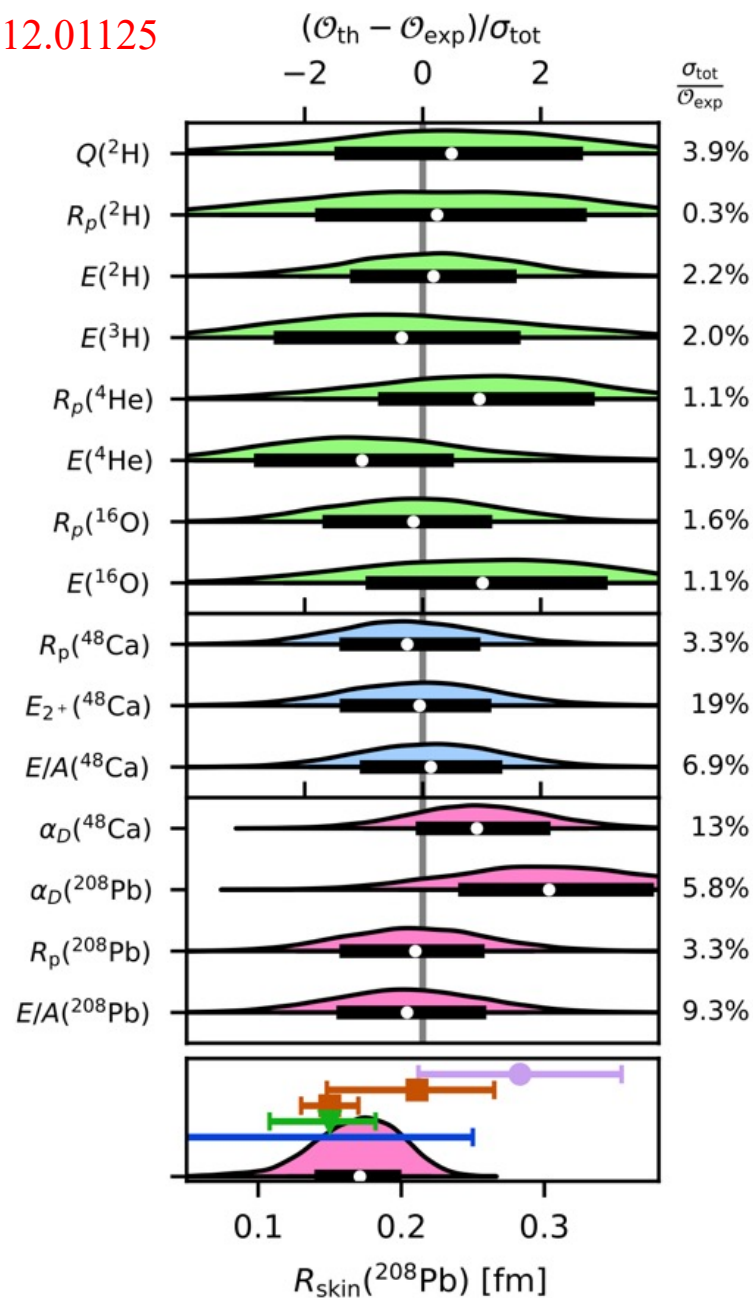
Hu, Jiang, Miyagi et al. [Chalmers, ORNL, TRIUMF], arXiv:2112.01125

enabled by 3N advances



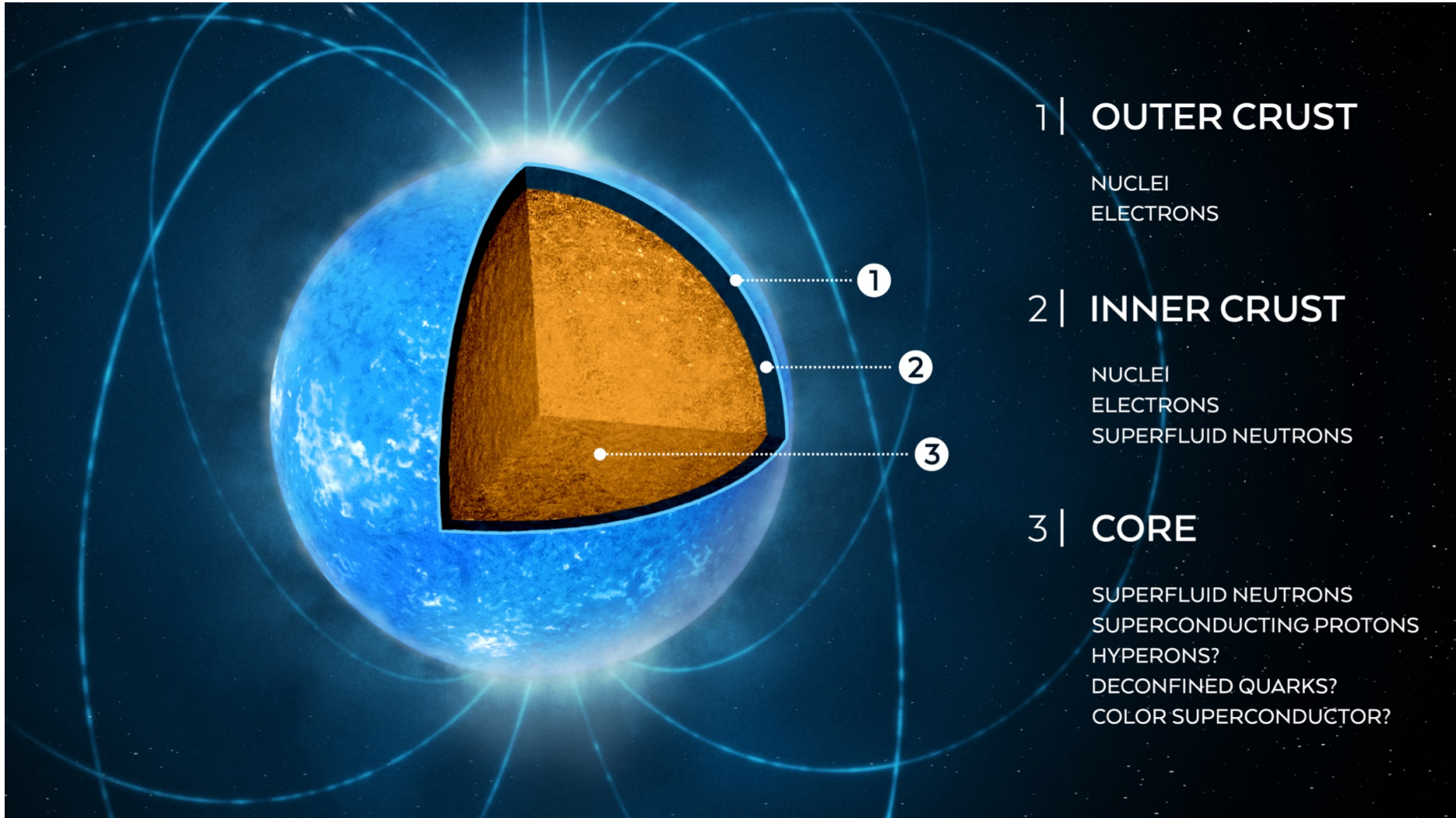
history matching to explore  
uncertainties in NN+3N interactions

predicted **neutron skin of  $^{208}\text{Pb}$**



# Extreme matter in neutron stars

governed by the same strong interactions



# Outline

Chiral effective field theory for nuclear forces

**Ab initio calculations of nuclear matter:**

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Nuclear experiments: dipole response, CREX, PREX

# N<sup>3</sup>LO calculation of neutron matter and symmetric matter

Monte-Carlo evaluation  
of energy diagrams  
up to 4th order in MBPT

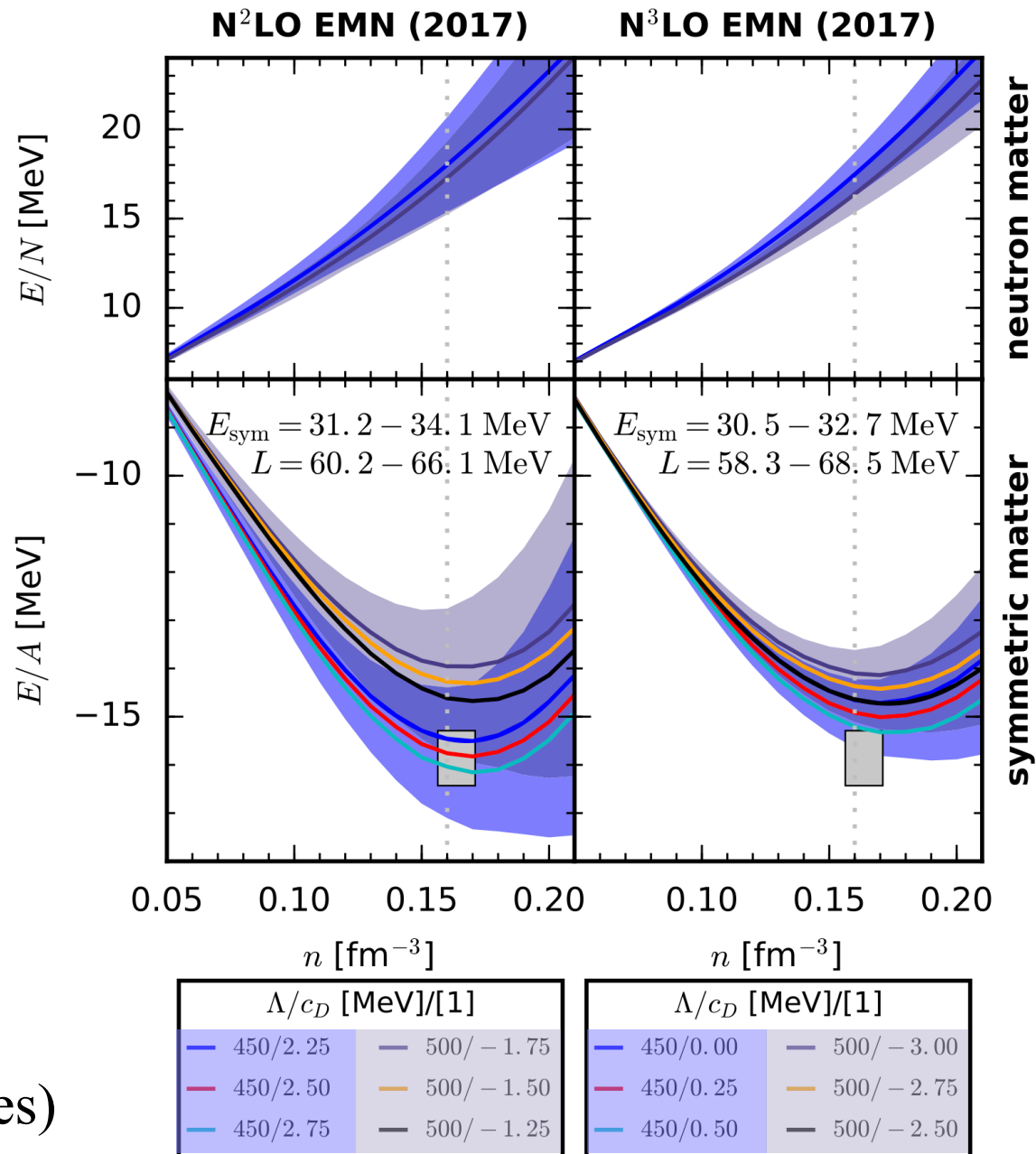
Drischler, Hebeler, AS, PRL (2019)

including NN, 3N, 4N  
3N fit to saturation region

all many-body forces  
to N<sup>3</sup>LO predicted for  
neutron matter

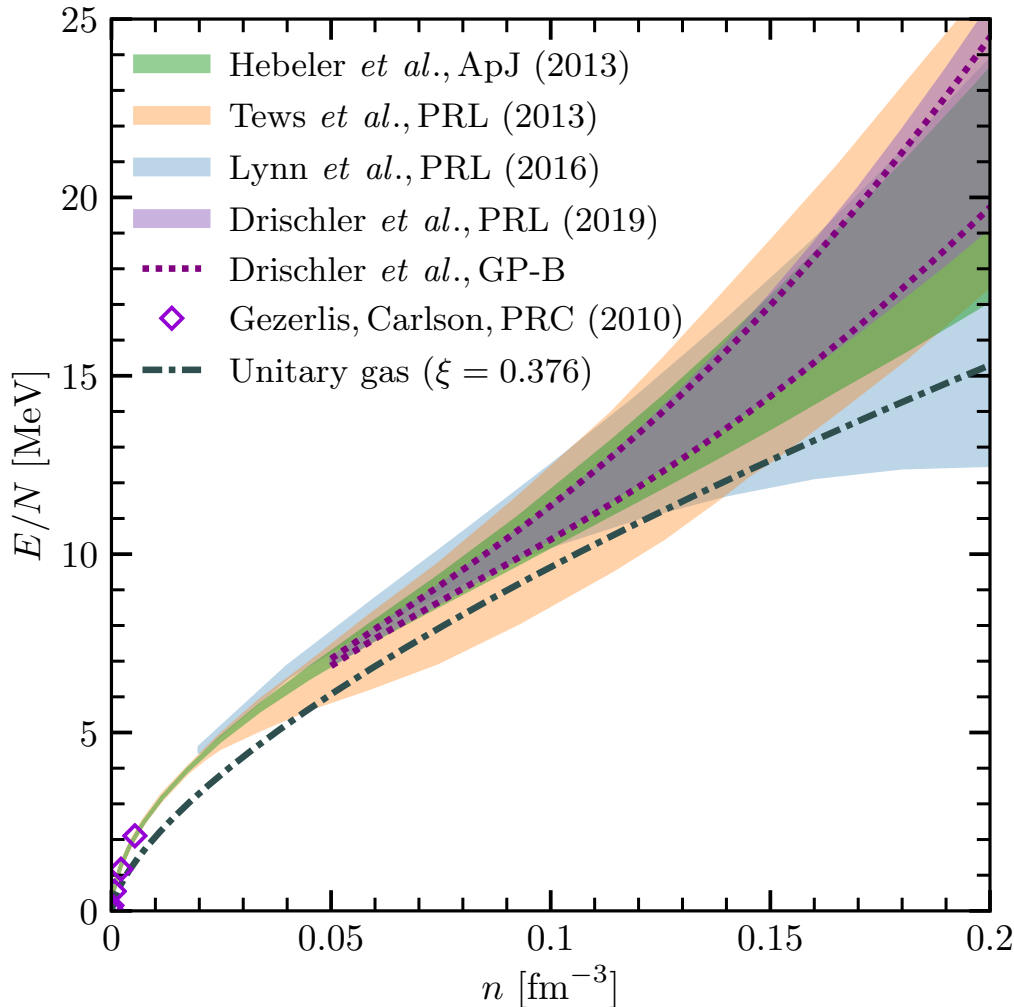
Tews, Krüger, Hebeler, AS, PRL (2013)

systematic improvement  
from N<sup>2</sup>LO to N<sup>3</sup>LO  
(EFT uncertainties  
+ cutoff, fit, reg uncertainties)



# Chiral EFT calculations of neutron matter

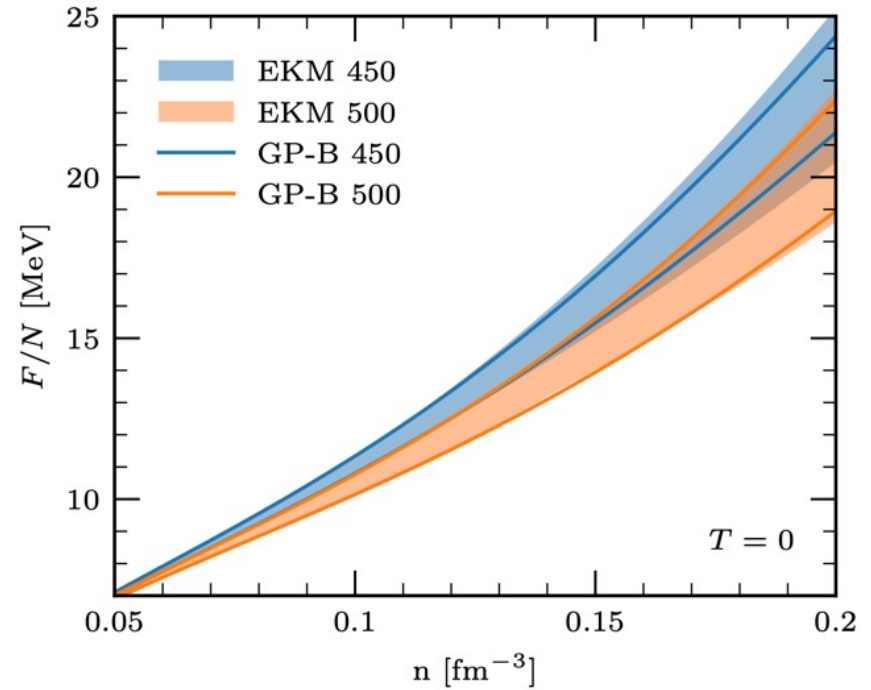
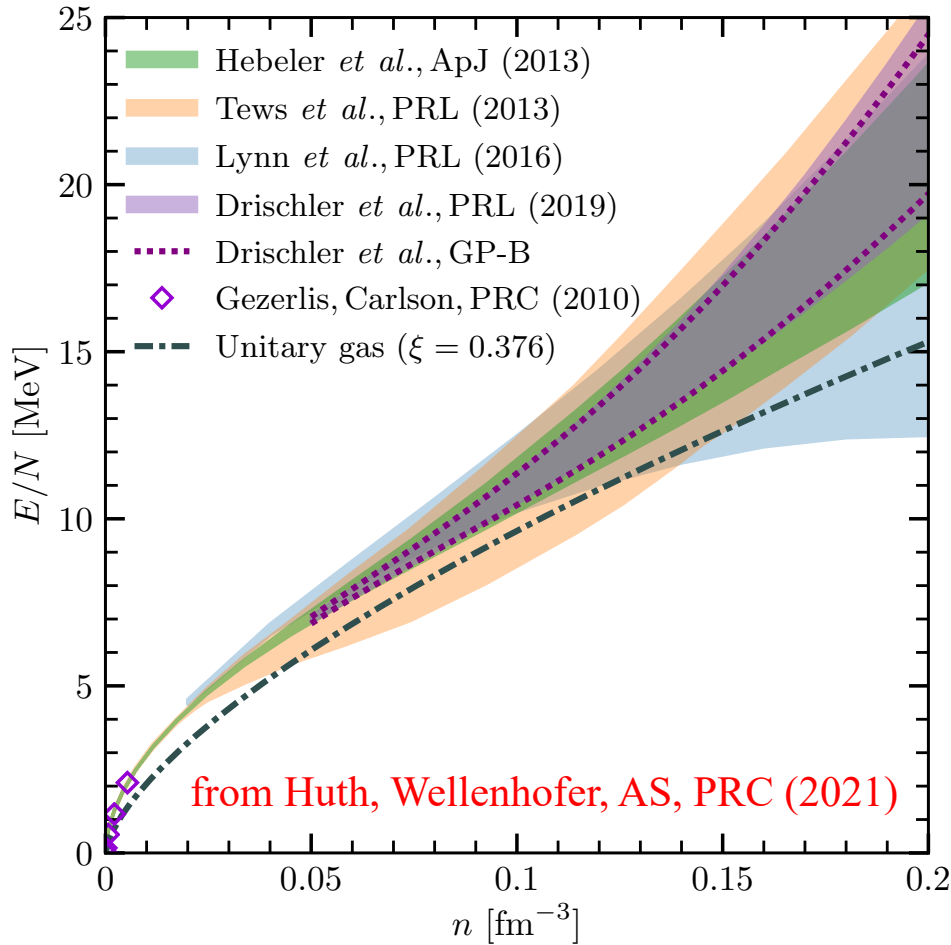
good agreement up to saturation density for neutron matter  
nonlocal/local int. and different calcs. (MBPT, QMC, SCGF, CC)



slope determines  
pressure of  
neutron matter

from Huth, Wellenhofer, AS, PRC (2021)

# Chiral EFT calculations of neutron matter - Uncertainties

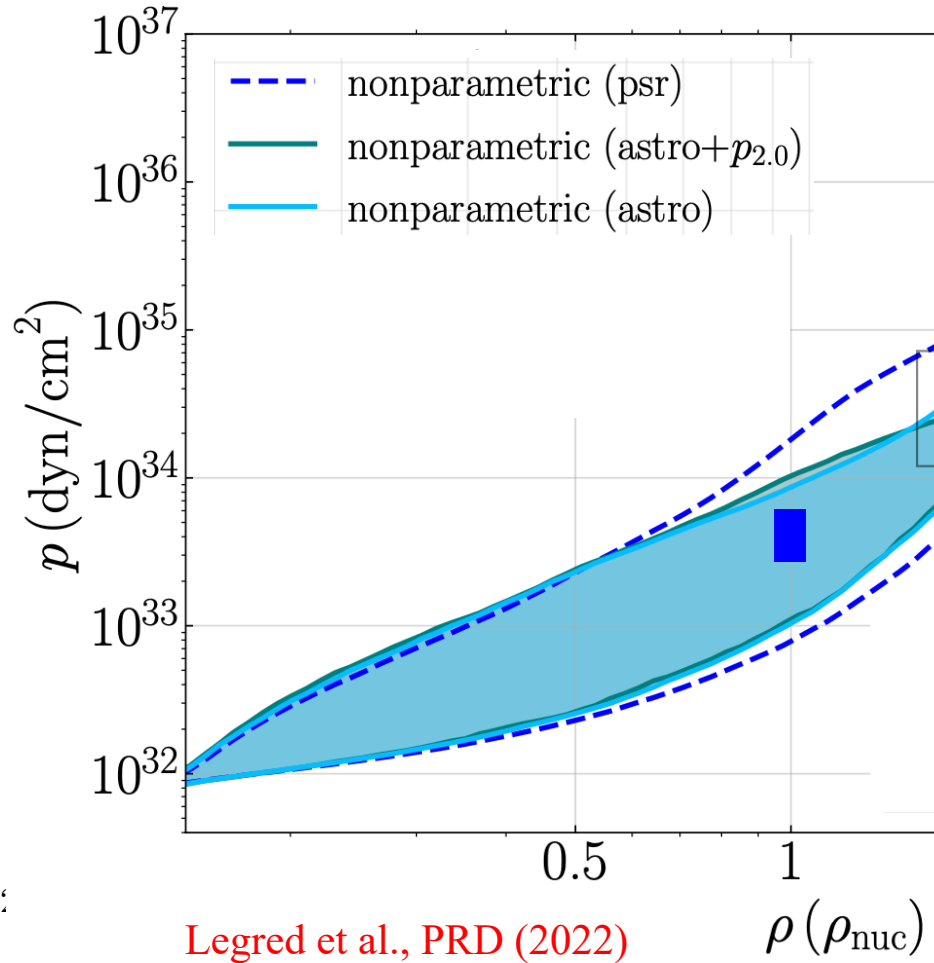
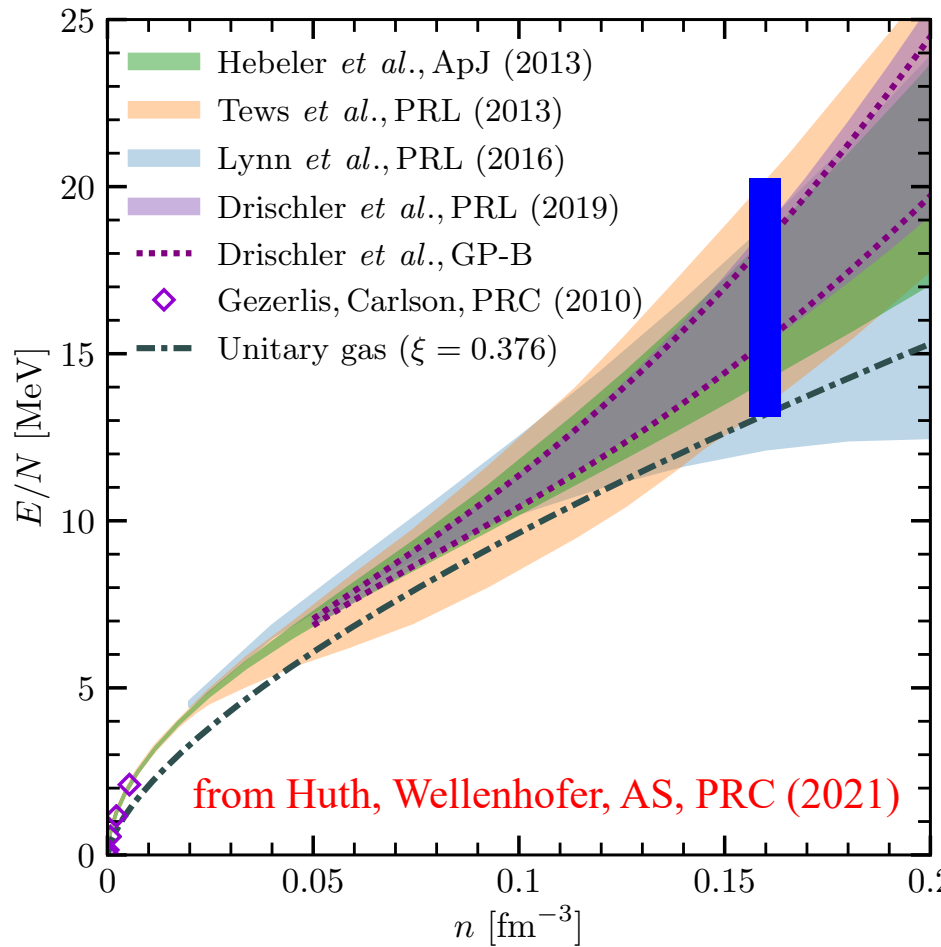


Keller, Wellenhofer, Hebeler, AS, PRC (2021)

**GP-B** (68%) gives similar bands as order-by-order EFT unc. (**EKM**)  
 from  $Q/\Lambda_b$  expansion  $\Delta X^{(j)} = Q \cdot \max(|X^{(j)} - X^{(j-1)}|, \Delta X^{(j-1)})$

interaction choices (cutoffs, reg, fit) add to GP-B/EKM uncertainties

# Chiral EFT calculations of neutron matter

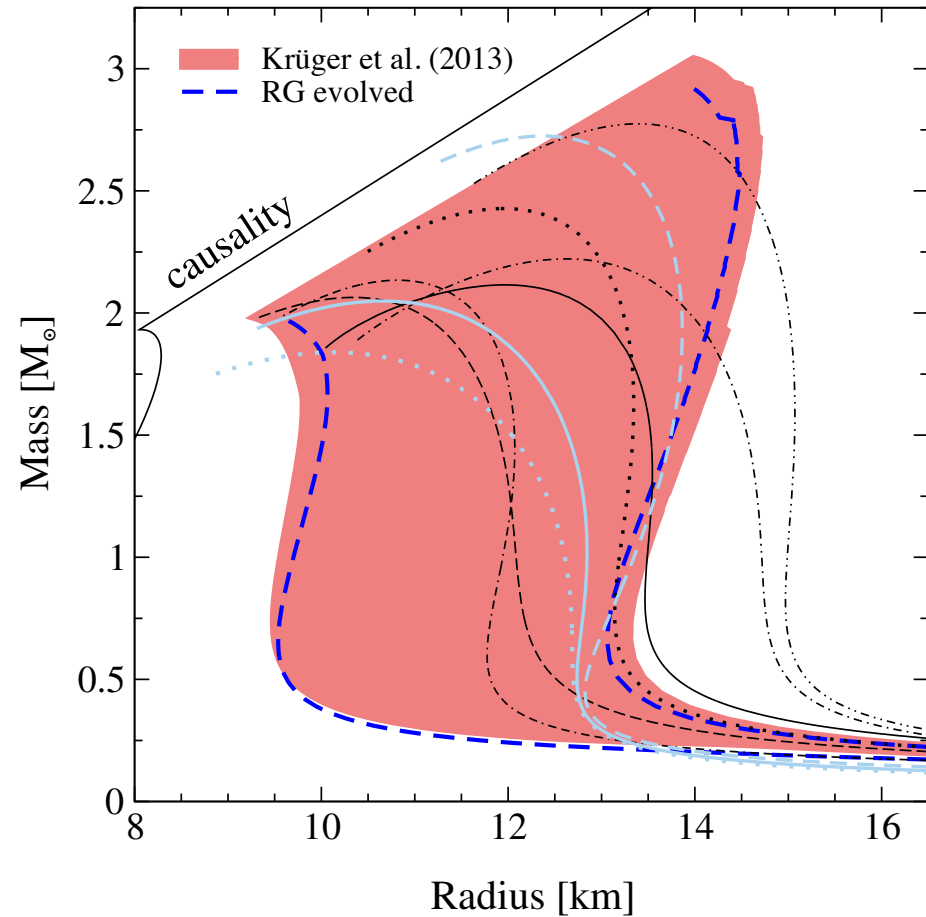
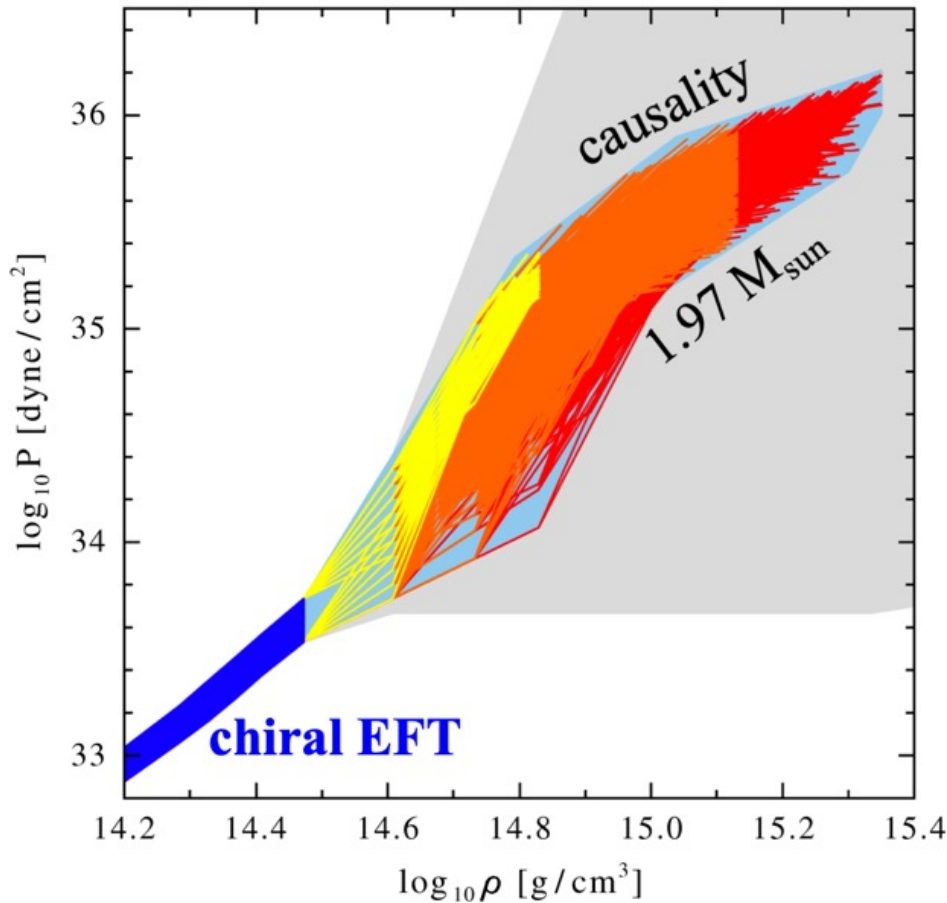


comparison to nonparametric EOS

suggests important nuclear physics constraints between nuclear crust at  $\sim 0.1 n_0$  and saturation density

# Impact on neutron stars (pre LIGO) Hebeler, Lattimer, Pethick, AS, ApJ (2013)

constrain high-density EOS by causality, require to support  $2 M_{\text{sun}}$  star



predicts neutron star radius: 9.7 - 13.9 km for  $M=1.4 M_{\text{sun}}$

1.8 - 4.4  $n_0$  modest central densities



# Neutron star radius from GW170817

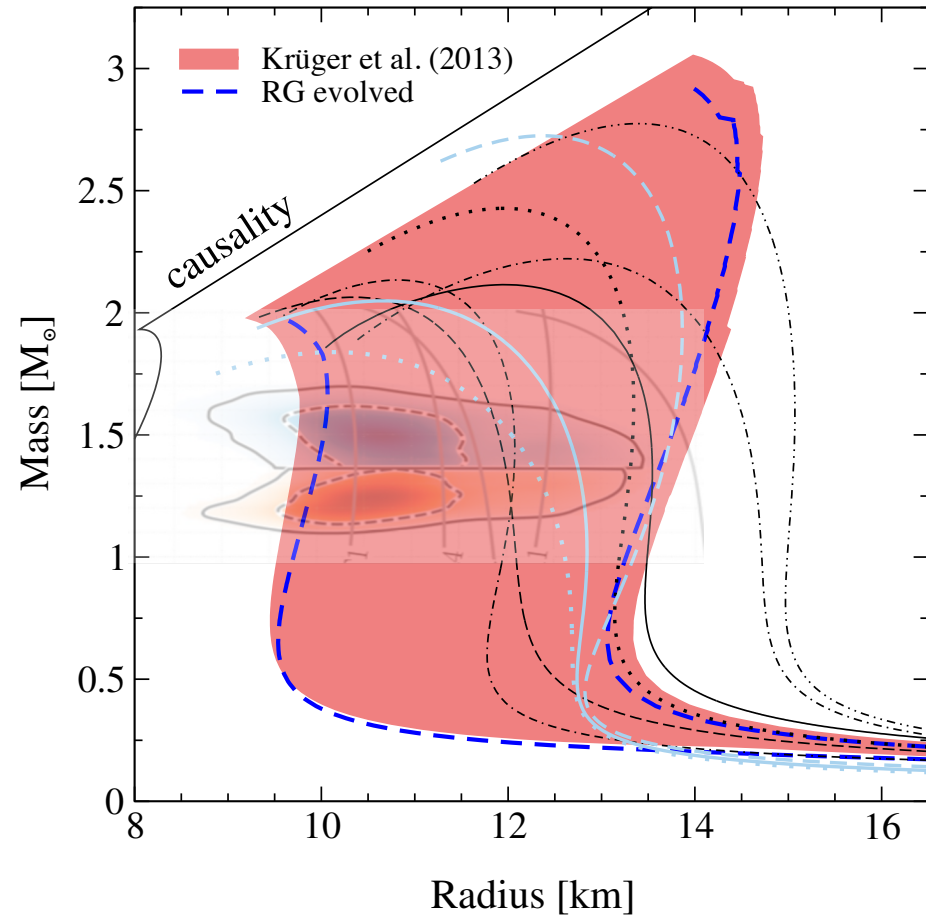
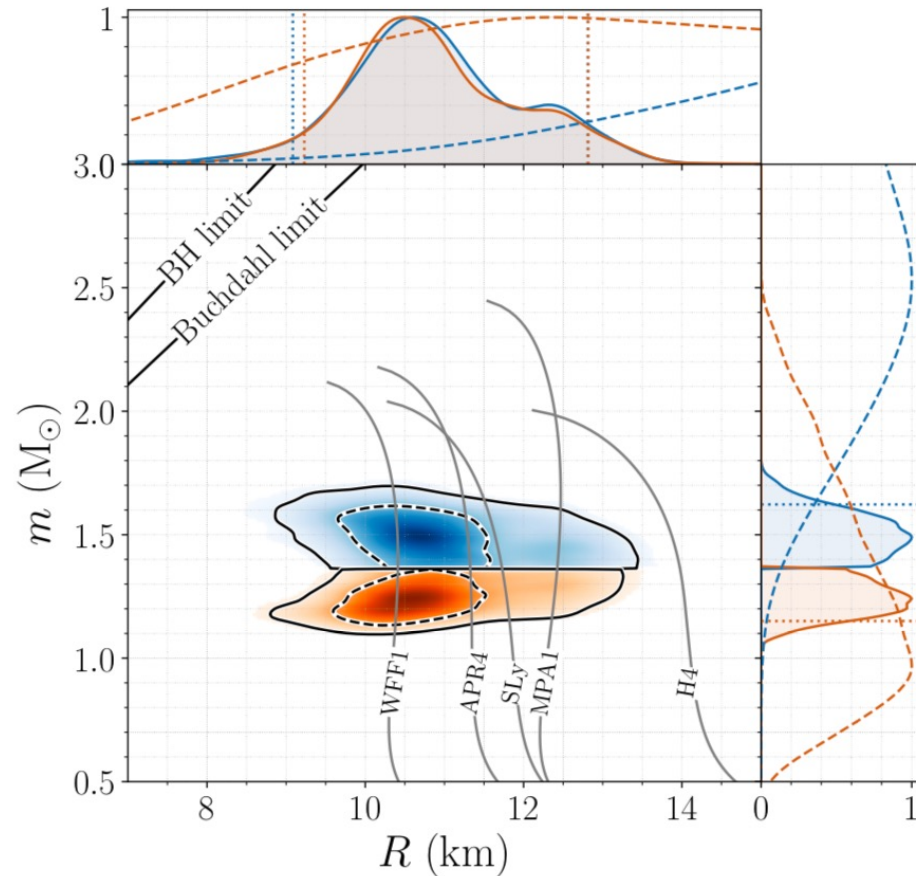
chiral EFT + general EOS extrapolation: 9.7 - 13.9 km for  $M=1.4 M_{\text{sun}}$

GW170817: Measurements of neutron star radii and equation of state

very consistent with

GW170817 from LIGO/Virgo

The LIGO Scientific Collaboration and The Virgo Collaboration  
(compiled 30 May 2018)

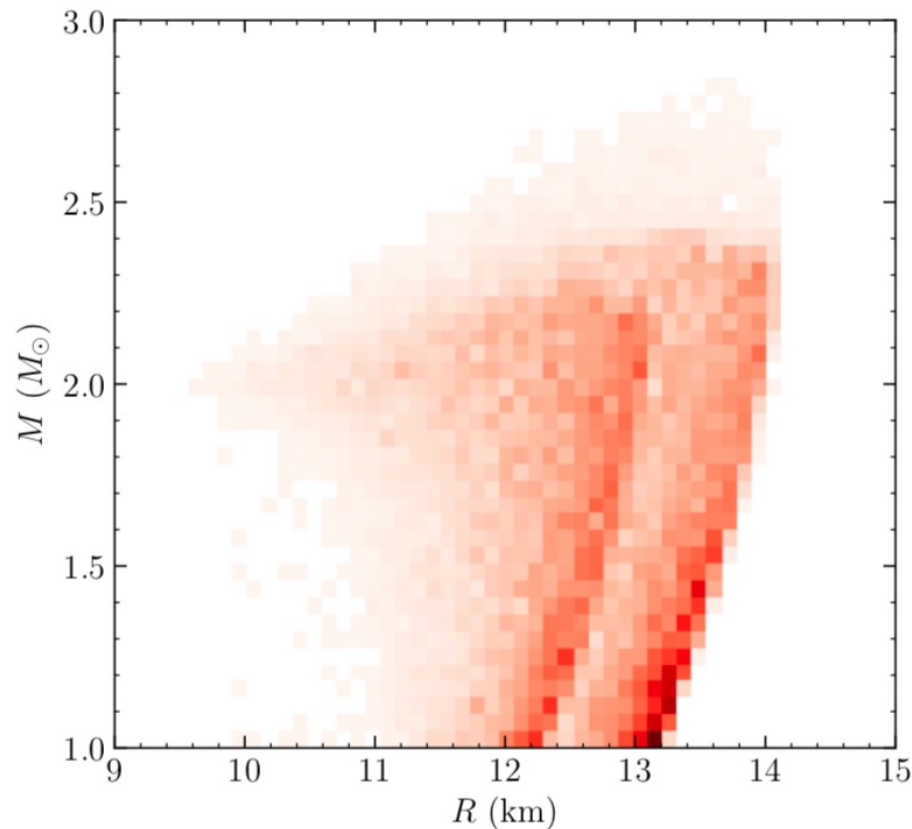
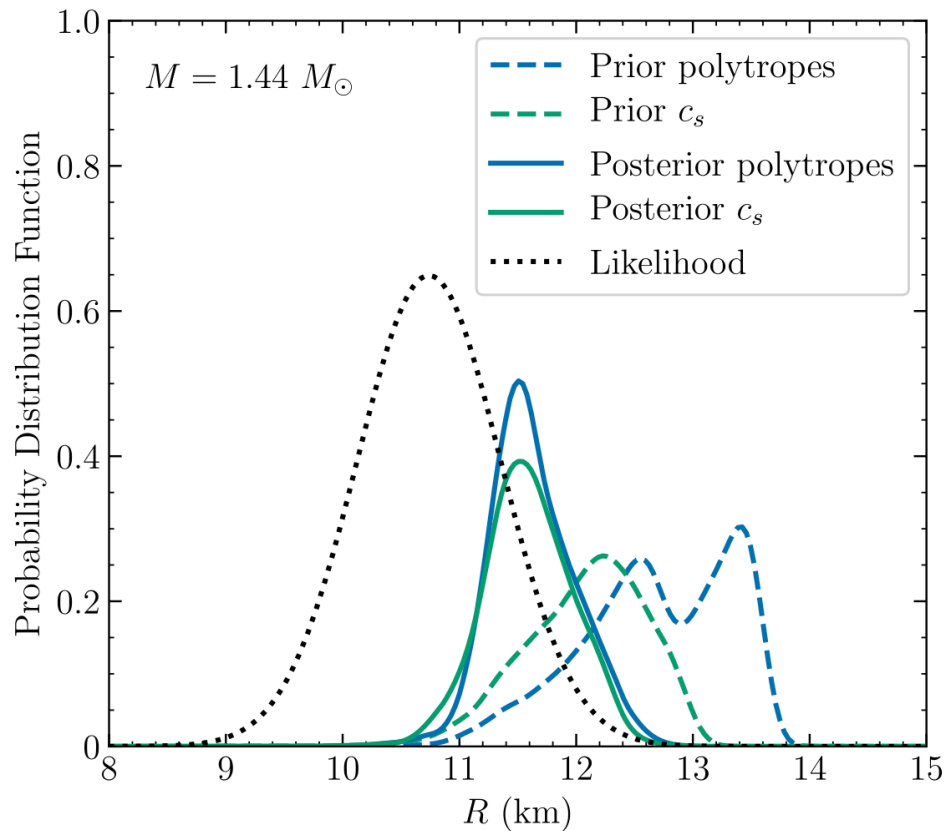


# Neutron star radius from GW170817

chiral EFT + general EOS extrapolation: 9.7 - 13.9 km for  $M=1.4 M_{\text{sun}}$

Bayesian inference vs. maximum extent, and prior sensitivities

Greif, Raaijmakers et al., MNRAS (2019)

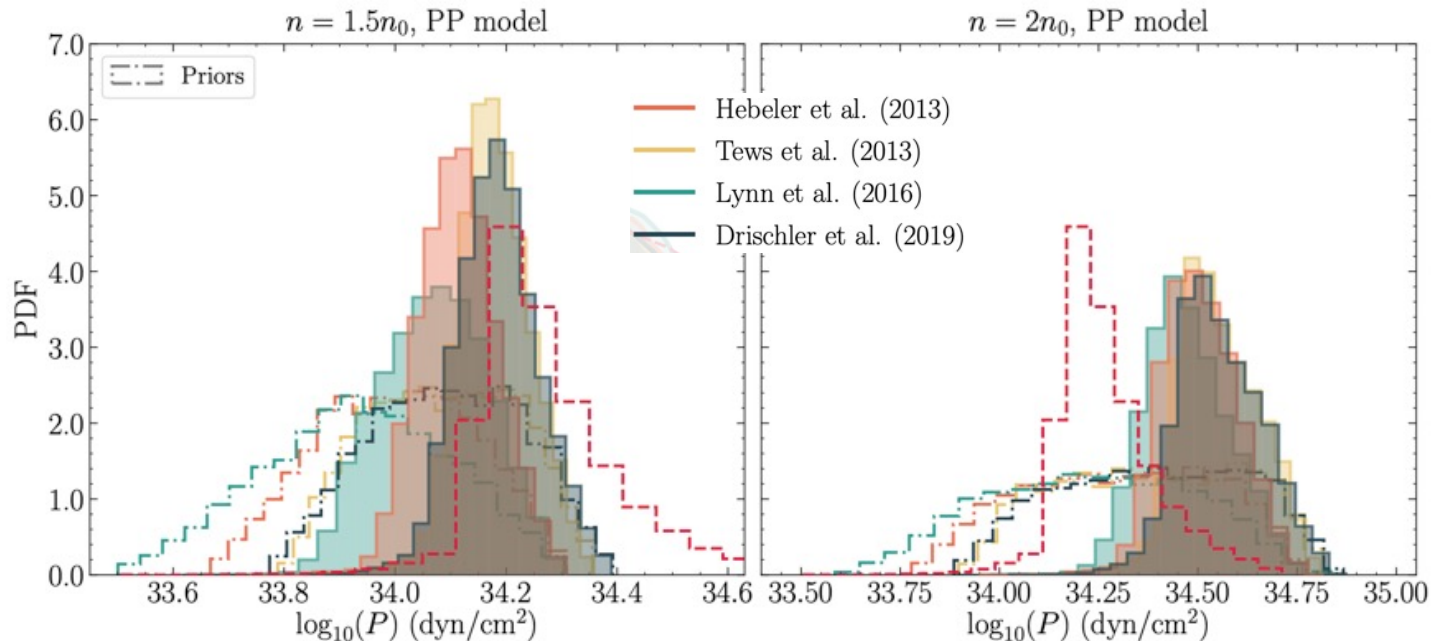
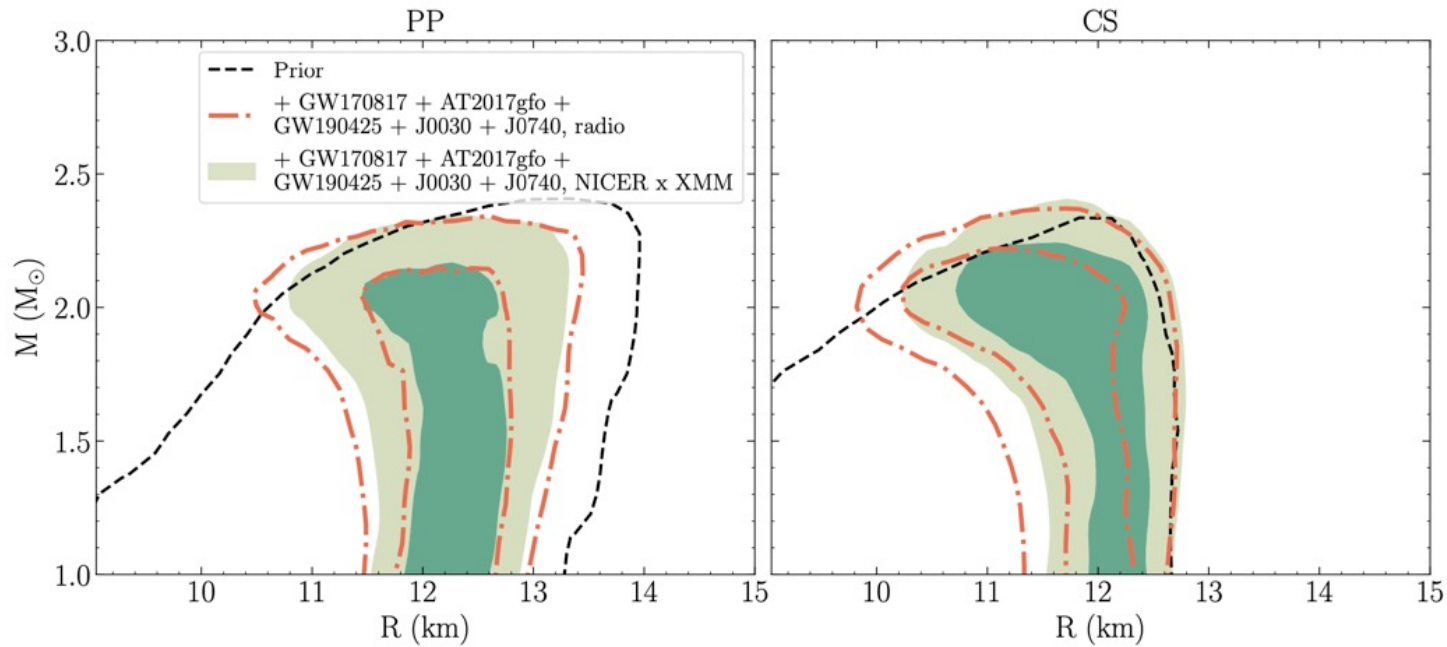
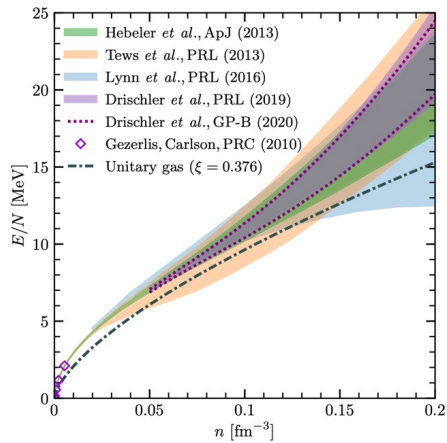


# Combined merger and NICER constraints

Raaijmakers et al.,  
ApJL (2020), (2021)  
for mass-radius

equation of state  
at  $1.5$  and  $2 n_0$

astro prefers  
higher pressures



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Chiral effective field theory for nuclear forces

Ab initio calculations of nuclear matter:  
cold, **finite temperature and arbitrary proton fraction**

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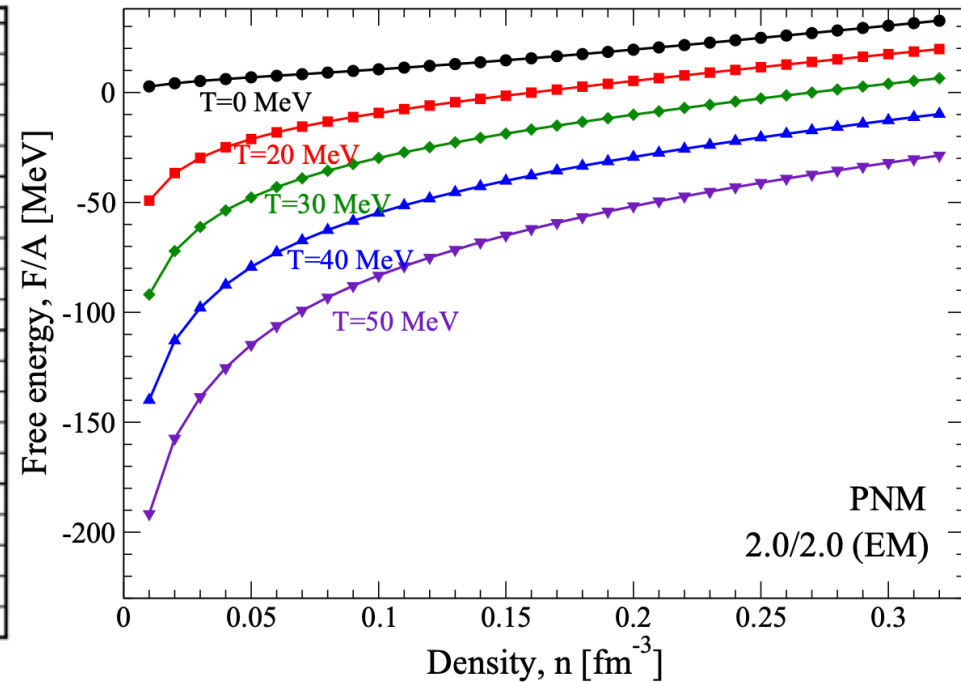
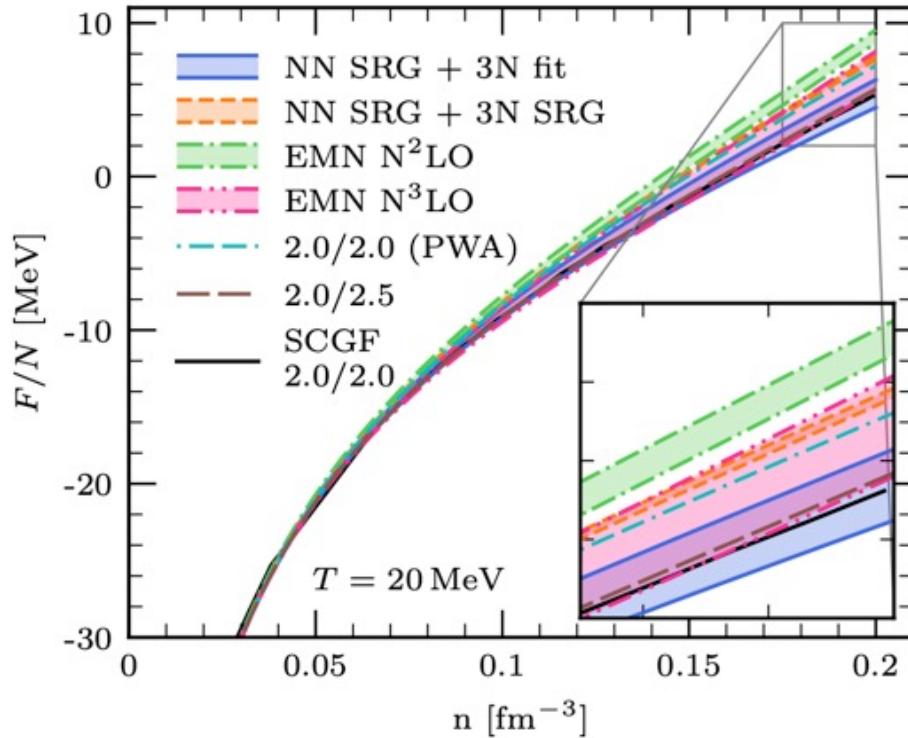
Nuclear experiments: dipole response, CREX, PREX

# Neutron matter at finite temperature

similar thermal effects for different NN+3N interactions

Keller, Wellenhofer, Hebeler, AS, PRC (2021)

SCGF (2.0/2.0): Carbone, AS, PRC (2019)

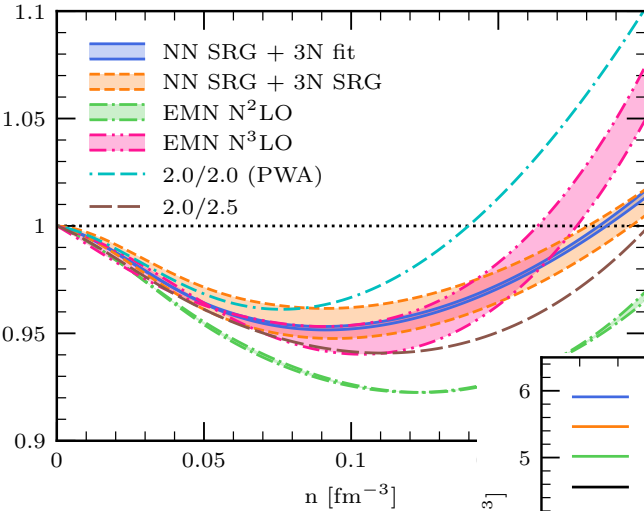
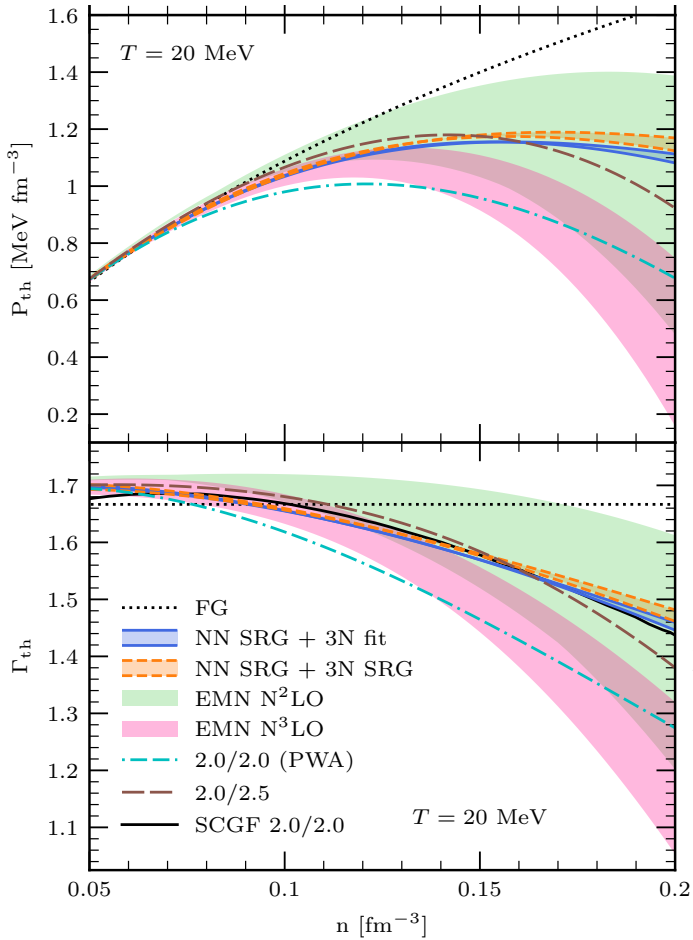


# Thermal effects governed by effective mass

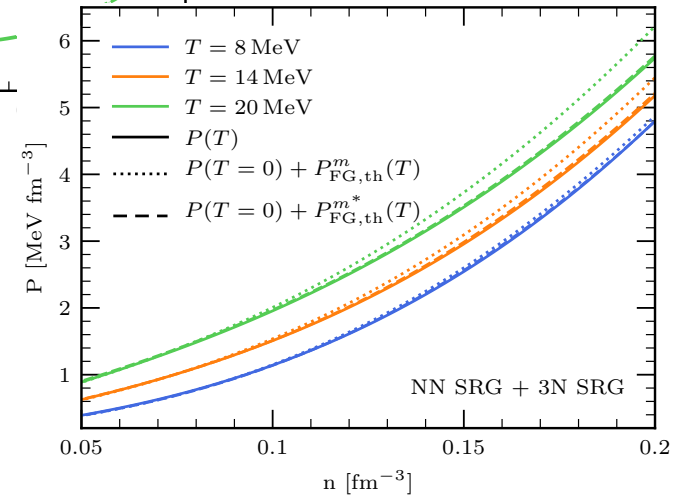
decreasing thermal pressure due to repulsive 3N contributions

increasing effective mass  $m^*$  beyond  $n_{\text{sat}}$

$$\Gamma_{\text{th}}^*(n) = \frac{5}{3} - \frac{n}{m_n^*} \frac{\partial m_n^*}{\partial n}$$



$$\Gamma_{\text{th}}(T, n) = 1 + \frac{P_{\text{th}}(T, n)}{\mathcal{E}_{\text{th}}(T, n)}$$



# Impact on core-collapse supernova simulations

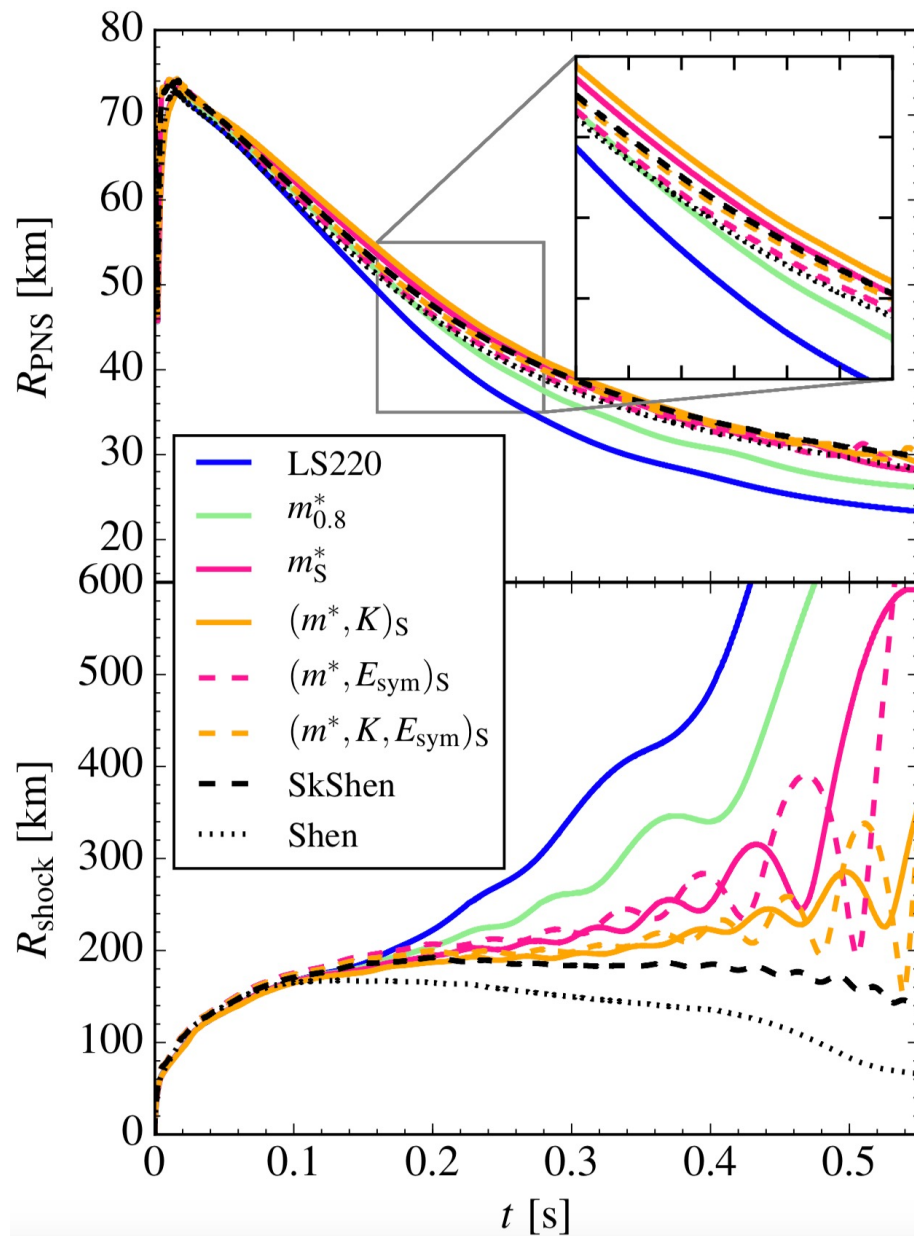
Yasin, Schäfer, Arcones, AS, PRL (2019)

constructed EOS that systematically vary nuclear matter properties between LS and Shen et al. EOS

	$m^*/m$	$K$	$E_{\text{sym}}$	$L$	$n_0$	$B$
LS220	1.0	220	29.6	73.7	0.155	16.0
Shen	0.634	281	36.9 <sup>a</sup>	110.8	0.145	16.3
Theo.	0.9(2)	215(40)	32(4)	51(19)	0.164(7)	15.86(57)

thermal contributions/ $m^*$  are key for proto-neutron star contraction

faster contraction aids supernova shock to more successful explosion



# EOS for arbitrary proton fraction and temperature

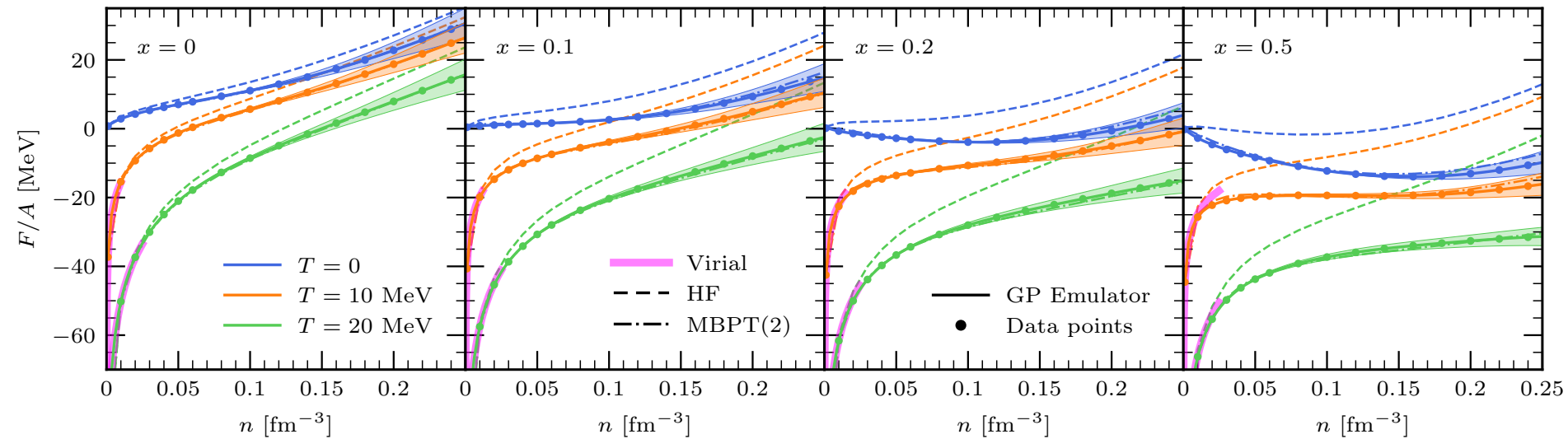
Keller, Hebeler, AS, arXiv:2204.14016

based on chiral EFT NN+3N interactions (EMN 450) to N<sup>3</sup>LO

order-by-order EFT uncertainties  $\Delta X^{(j)} = Q \cdot \max(|X^{(j)} - X^{(j-1)}|, \Delta X^{(j-1)})$   
(small) many-body uncertainties at MBPT(3)

excellent reproduction of free energy data by Gaussian process

agrees with model-indep. virial EOS Horowitz, AS, NPA (2006) at low densities

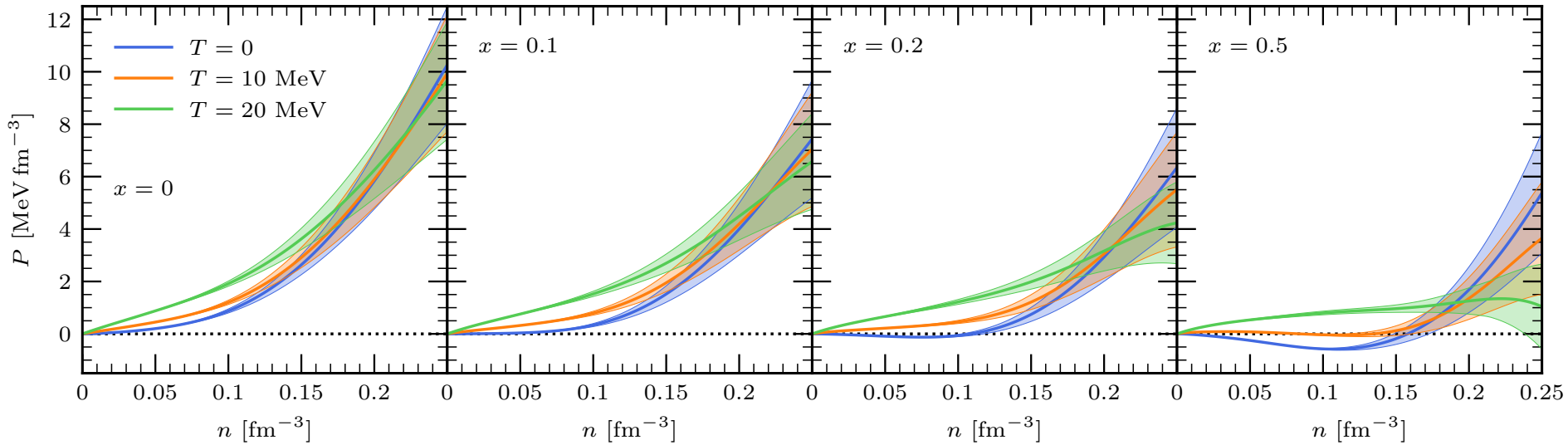




# EOS for arbitrary proton fraction and temperature

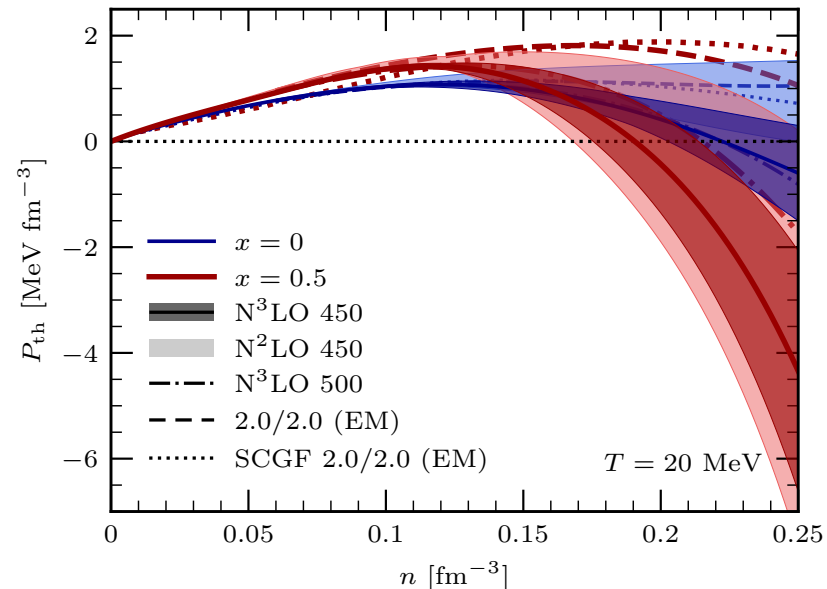
Keller, Hebeler, AS, arXiv:2204.14016

GP emulator to calculate pressure (thermodyn. consistent derivatives)



pressure isothermals cross at higher densities  $\rightarrow$  negative thermal expansion

thermal part of pressure decreases with increasing density, observed for different chiral orders, cutoffs and interactions



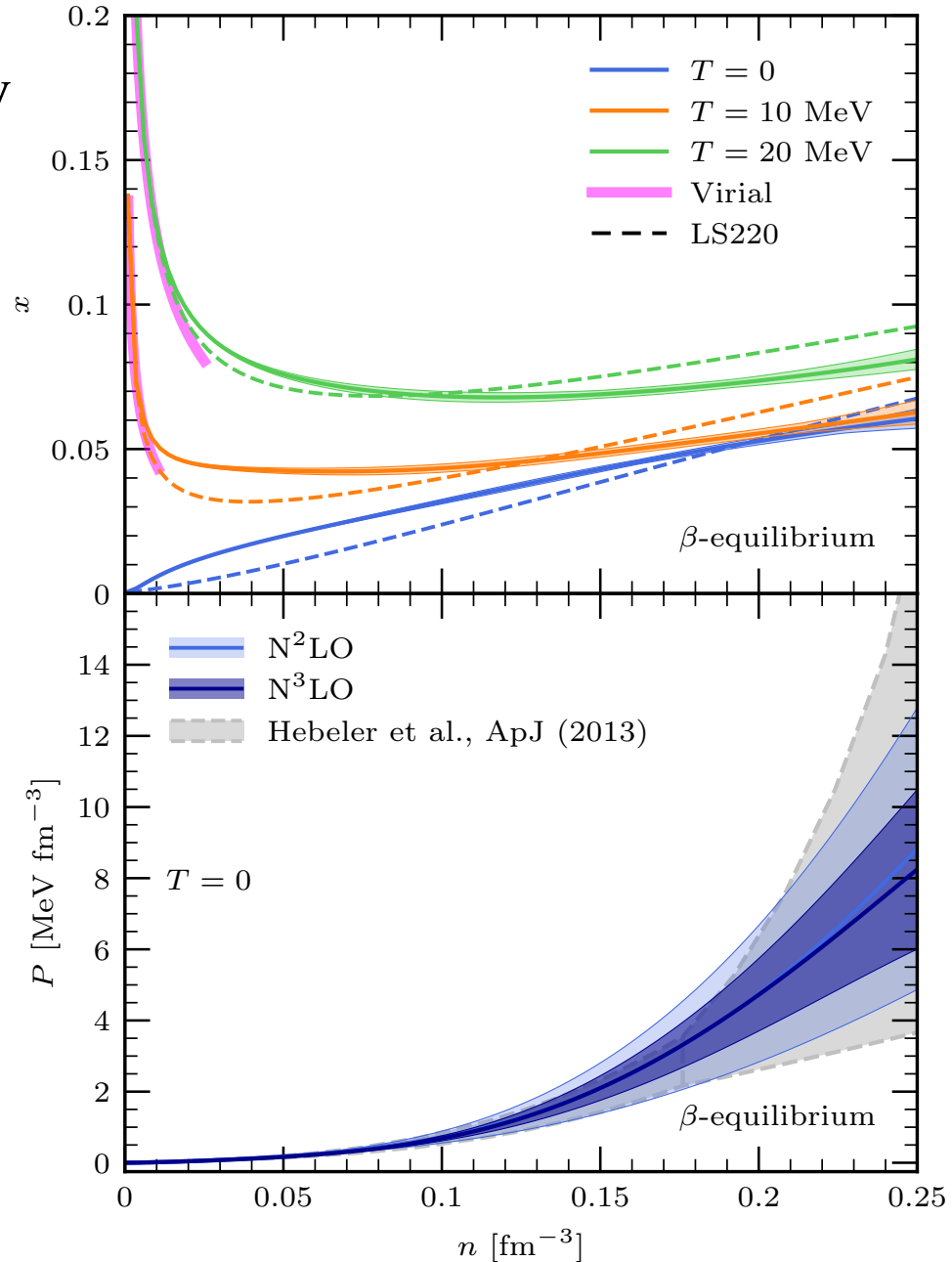
# EOS for neutron star matter in beta equilibrium

Keller, Hebeler, AS, arXiv:2204.14016

use GP emulator to access arbitrary  
proton fraction,  
solve for beta equilibrium

EOS of neutron star matter  
at  $N^2\text{LO}$  and  $N^3\text{LO}$ ,  
no indication of EFT breakdown

$N^3\text{LO}$  band prefers higher  
pressures, improvement over  
older calculations



# Applications to speed of sound and symmetry free energy

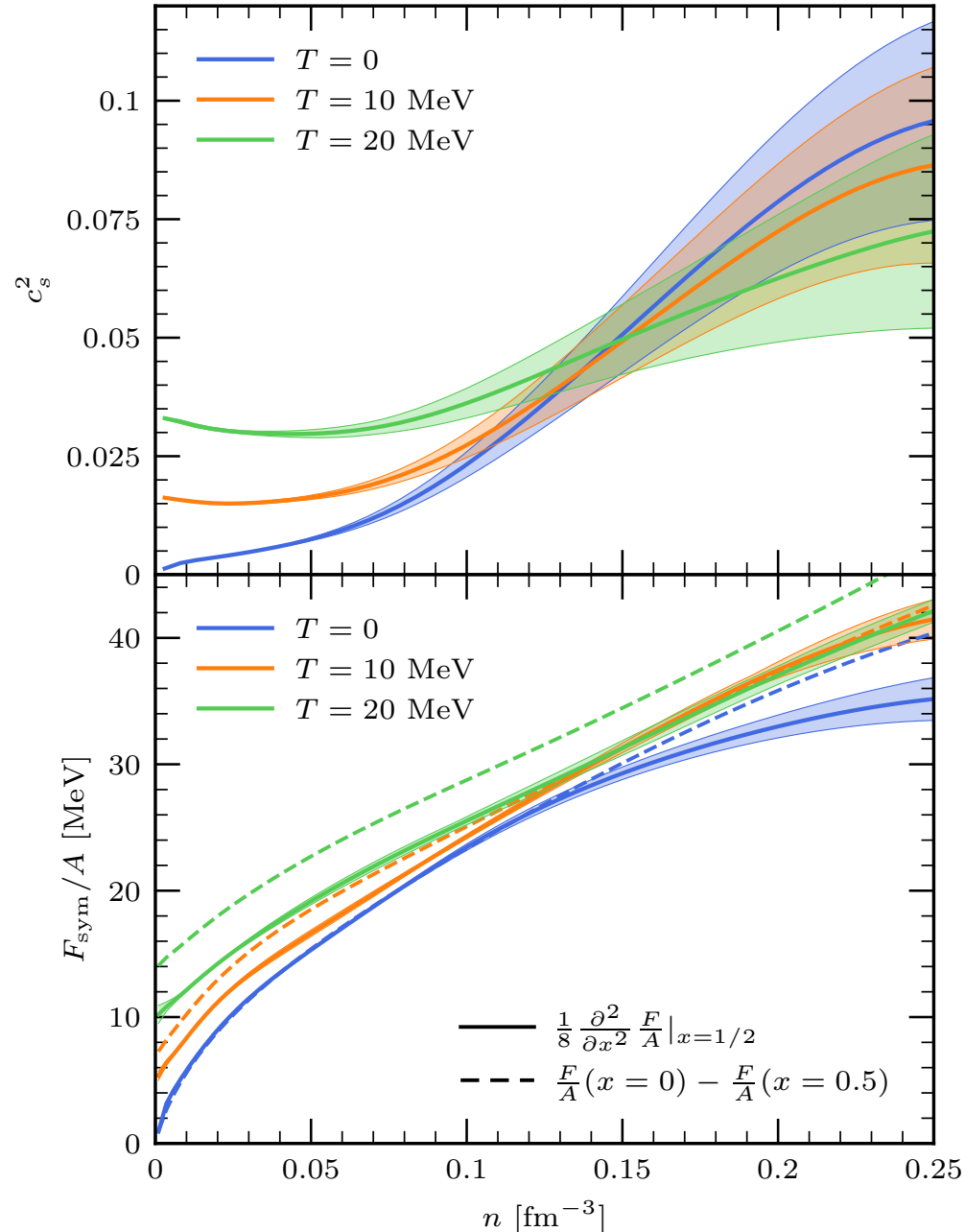
Keller, Hebeler, AS, arXiv:2204.14016

speed of sound for neutron matter  
at constant entropy

high-density behavior with  
increasing T similar to pressure

symmetry energy tightly  
constrained at fixed density

difference in definition mainly  
due to kinetic energy ( $m^*$ )



# Outline

Chiral effective field theory for nuclear forces

Ab initio calculations of nuclear matter:  
cold, finite temperature and arbitrary proton fraction

**Constraints at intermediate densities:  
astro, heavy-ion collisions, functional RG**

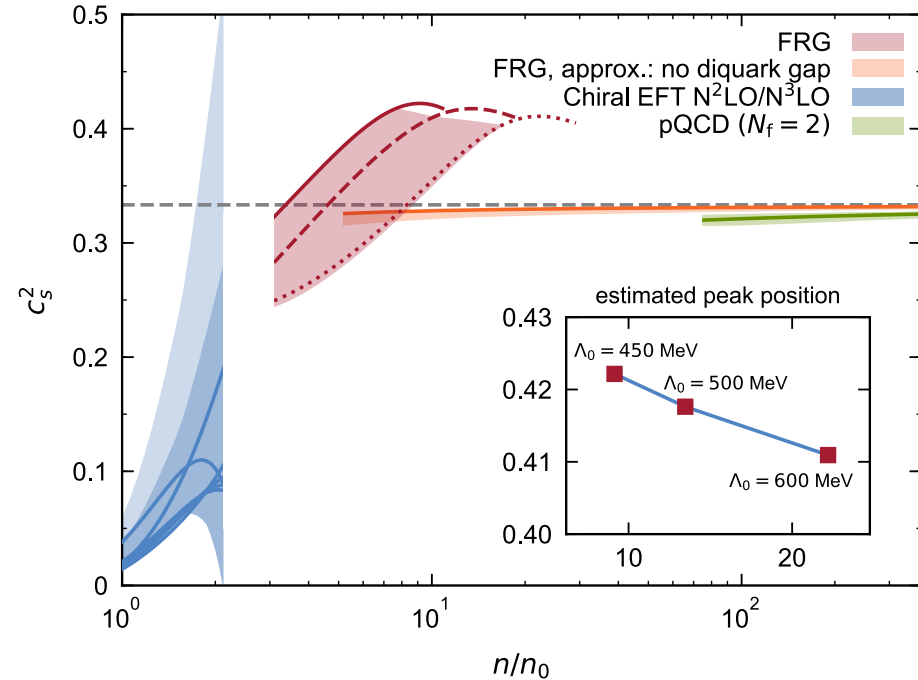
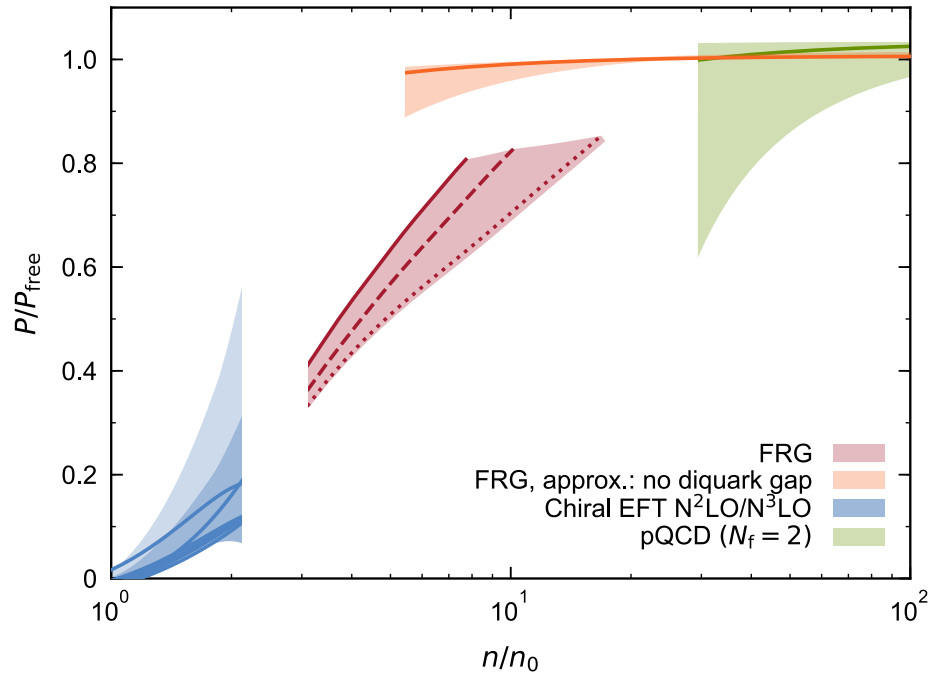
Nuclear experiments: dipole response, CREX, PREX

# Functional RG: From QCD to intermediate densities

based on QCD at high densities

symmetric matter ( $m_u=m_d$ , no s quark, no electroweak interactions)

Leonhardt, Pospiech, Schallmo, Braun et al., PRL (2020)



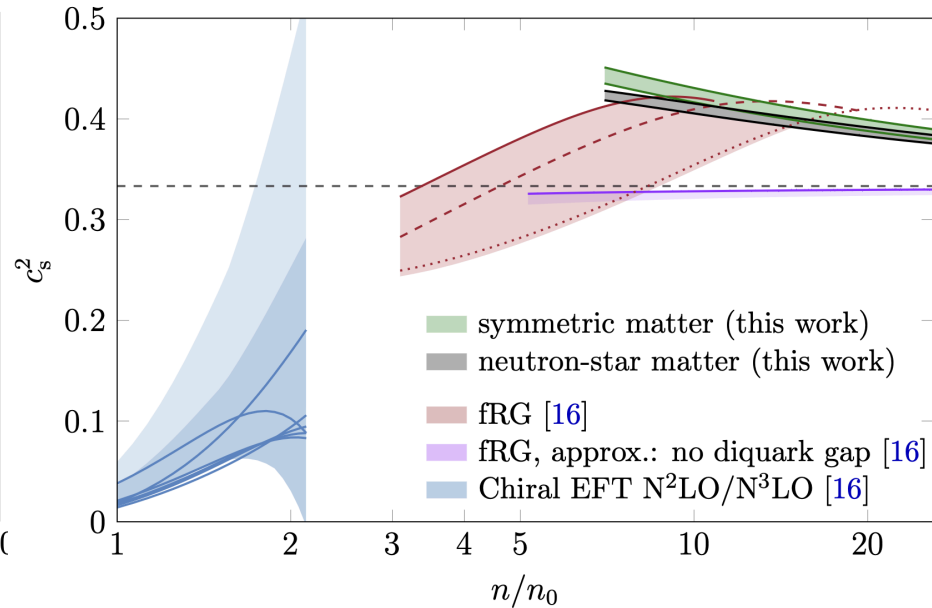
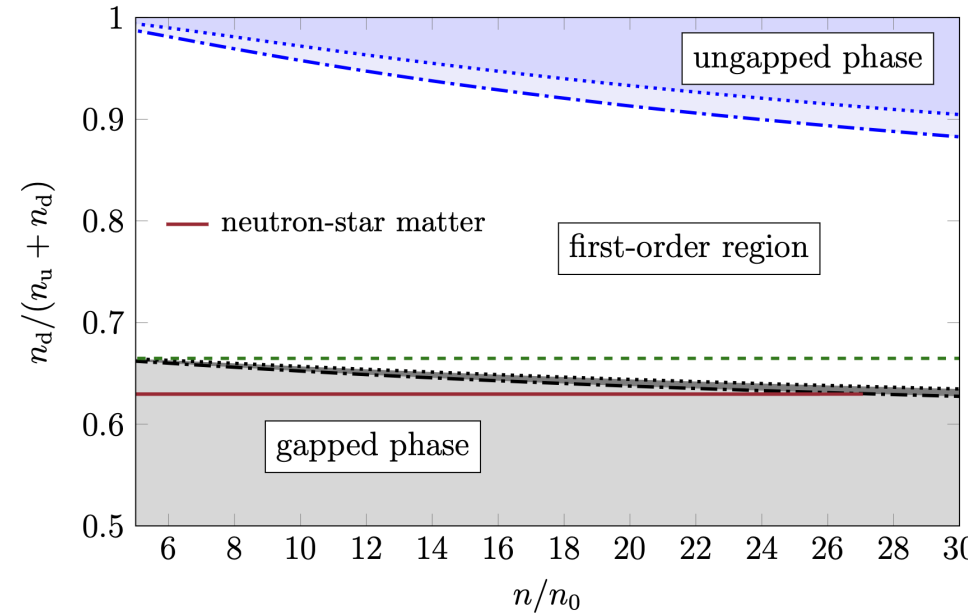
promising consistency between  
chiral EFT and FRG and pQCD

diquark correlations crucial  
for intermediate densities  
and high speed of sound

# Functional RG: From QCD to intermediate densities

neutron star matter ( $m_u=m_d$ , no s quark)

Braun, Schallmo, arXiv:2204.00358



suggests ungapped quark matter phase is at very high densities

diquark gap gives large speed of sound up to high densities

may push validity of pQCD to higher densities

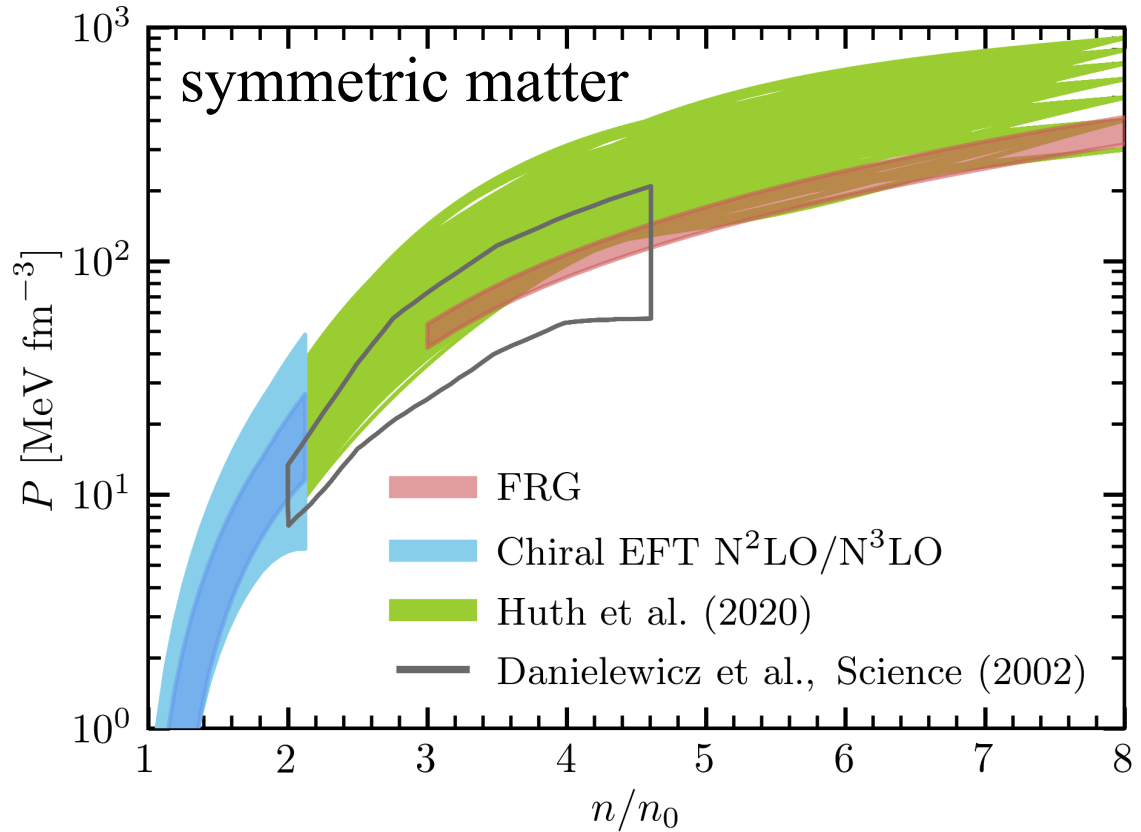
# Symmetric matter: From chiral EFT to functional RG

comparison to **new EOS functionals** with  $2 M_{\text{sun}} + \text{LIGO/Virgo, NICER}$

Huth, Wellenhofer, AS, PRC (2021)

comparison to  
heavy-ion constraint

Danielewicz et al., Science (2002)



# Symmetric matter: From chiral EFT to functional RG

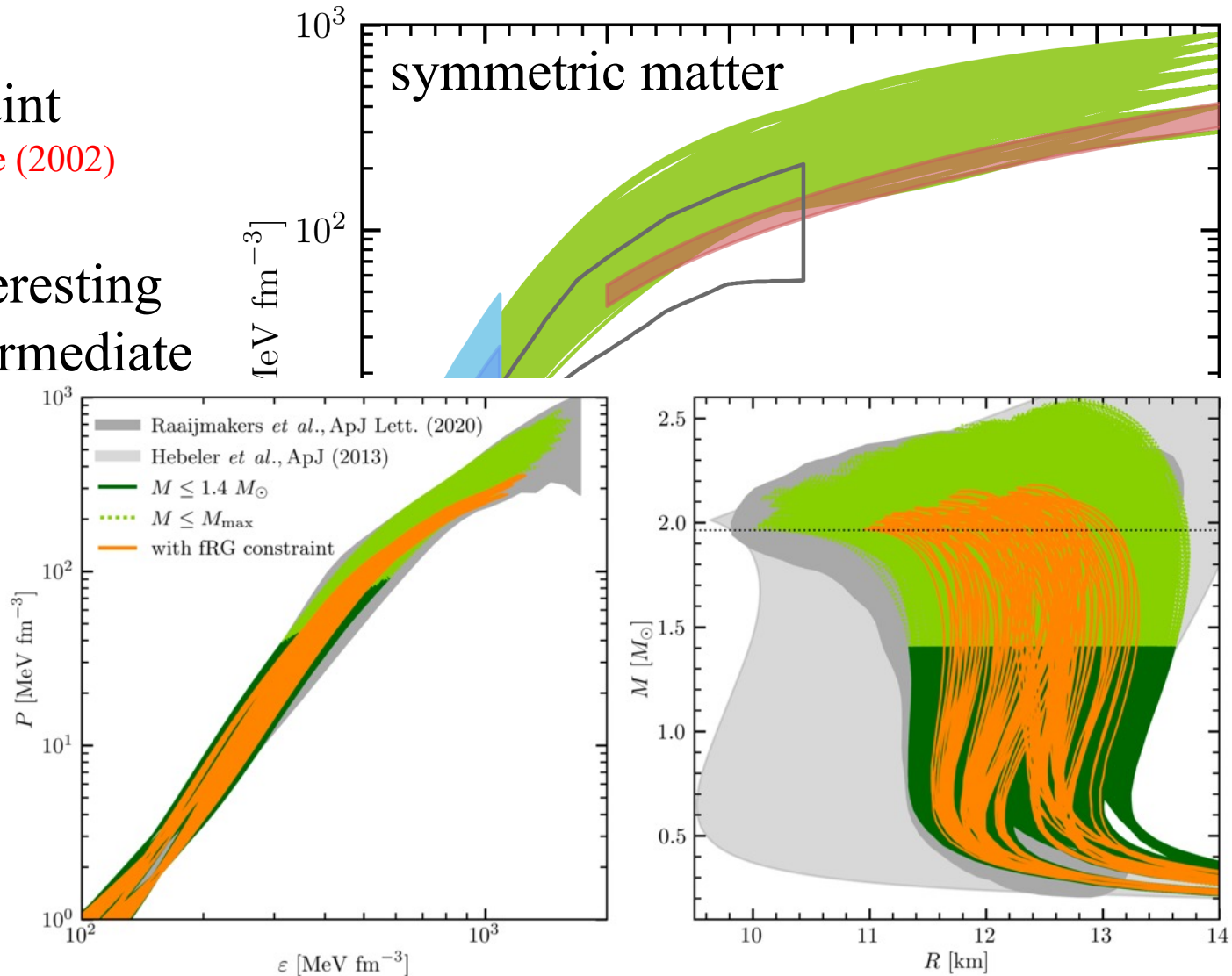
comparison to **new EOS functionals** with  $2 M_{\text{sun}} + \text{LIGO/Virgo, NICER}$

Huth, Wellenhofer, AS, PRC (2021)

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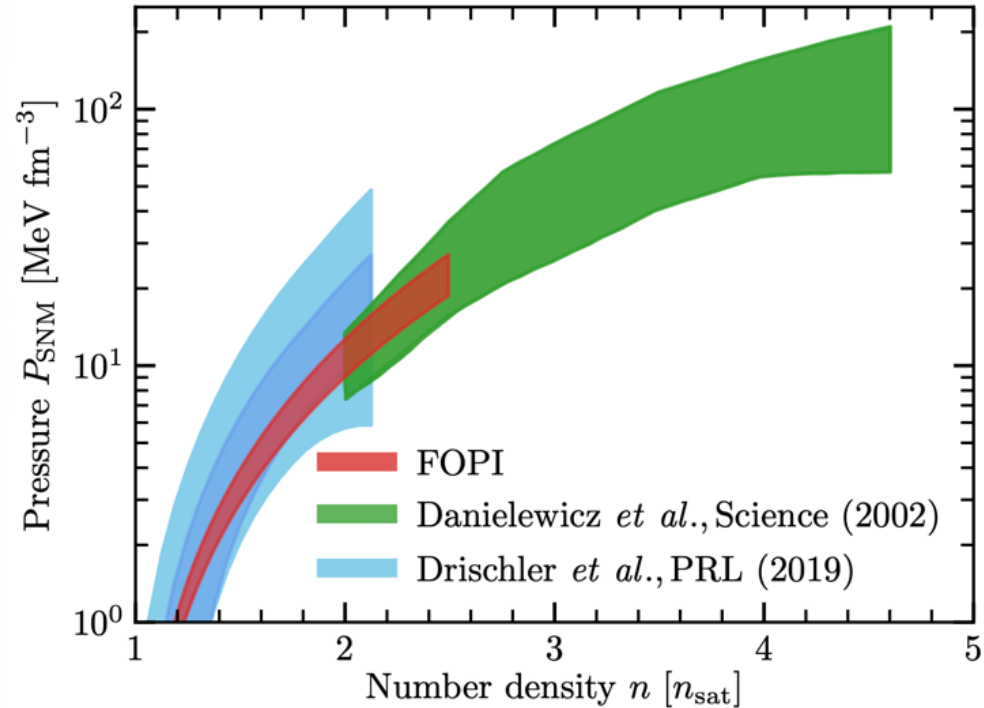
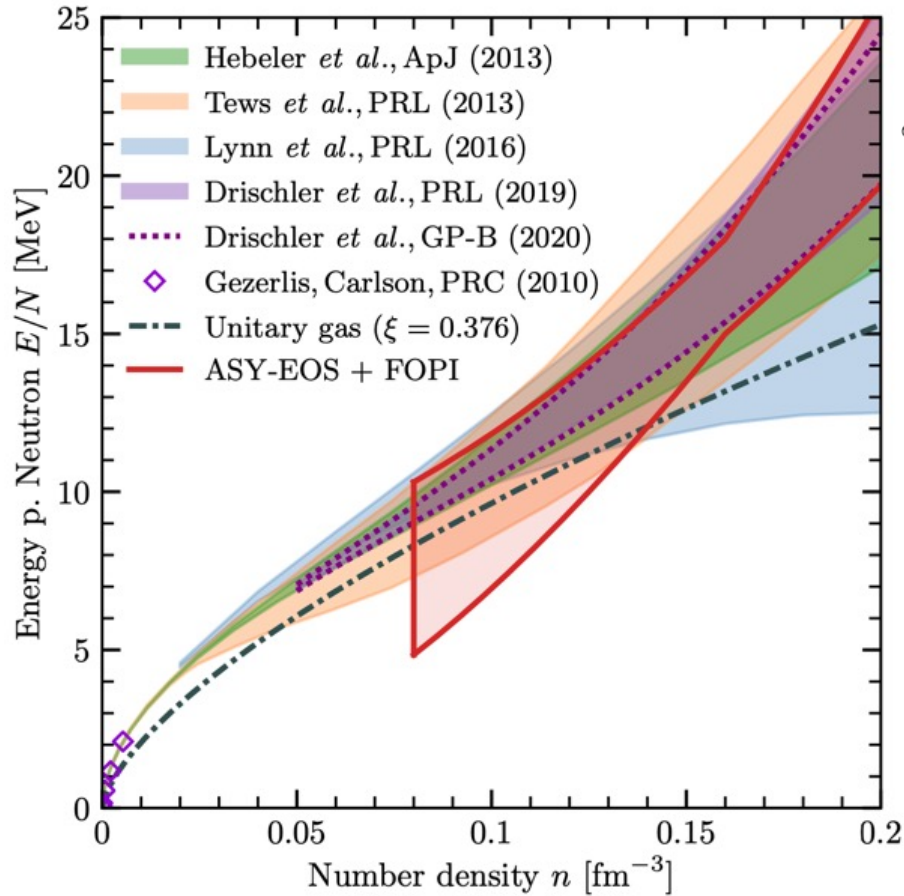
**fRG** provides interesting  
constraints at intermediate  
densities





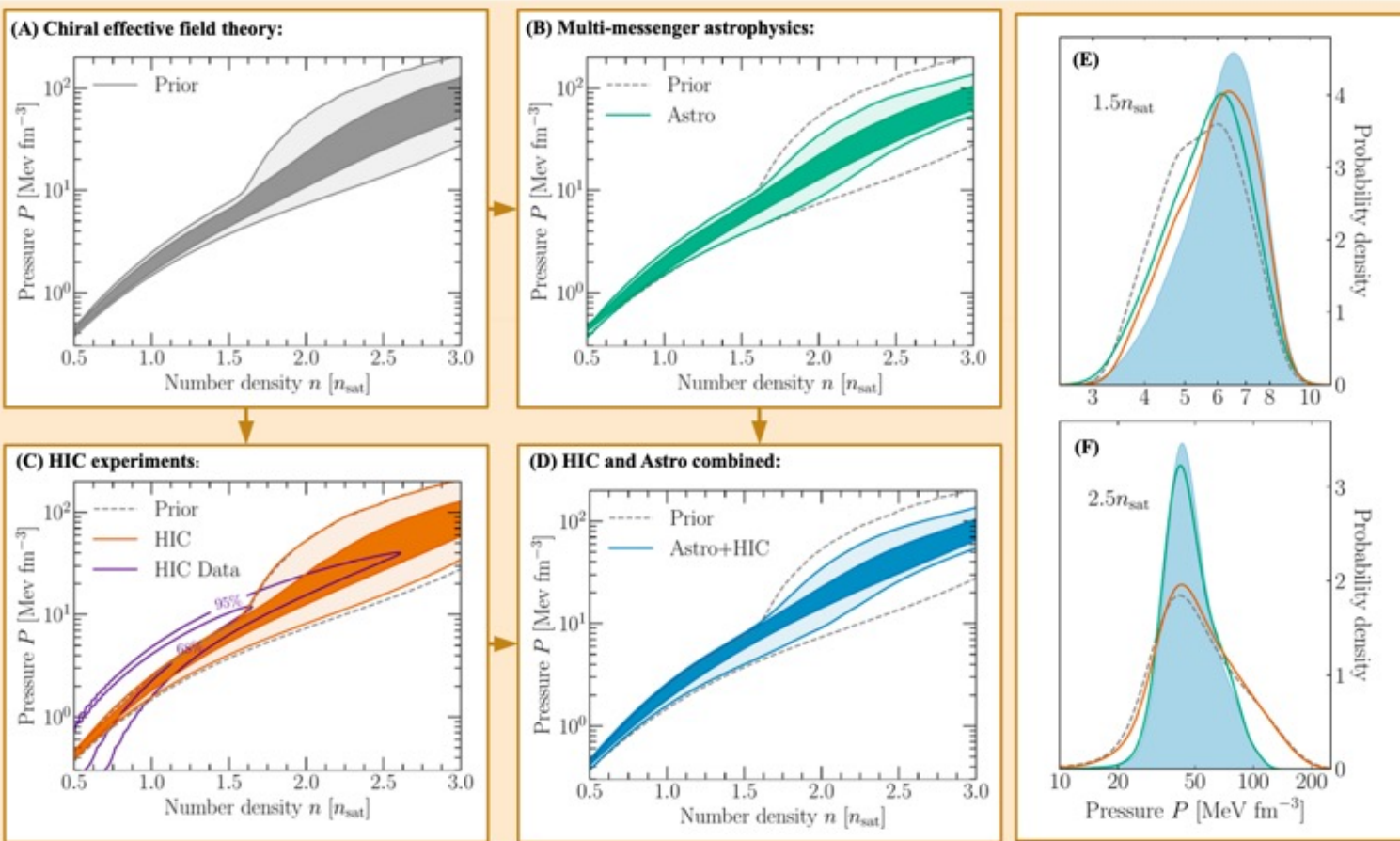
# Constraints from heavy-ion collisions Huth, Pang et al., Nature (2022)

include in addition to chiral EFT: constraints from ASY-EOS and FOPI for neutron and symmetric matter with different functionals



# Constraints from heavy-ion collisions Huth, Pang et al., Nature (2022)

inclusion of HIC constraints prefers higher pressures, similar to NICER, overall remarkable consistency with chiral EFT and astro constraints



improves HIC constraints for interm. densities very interesting

	Prior	Astro only	HIC only	Astro + HIC
$P_{1.5n_{\text{sat}}}$	$5.59^{+2.04}_{-1.97}$	$5.84^{+1.95}_{-2.26}$	$6.06^{+1.85}_{-2.04}$	$6.25^{+1.90}_{-2.26}$
$R_{1.4}$	$11.96^{+1.18}_{-1.15}$	$11.93^{+0.80}_{-0.75}$	$12.06^{+1.13}_{-1.18}$	$12.01^{+0.78}_{-0.77}$

# Outline

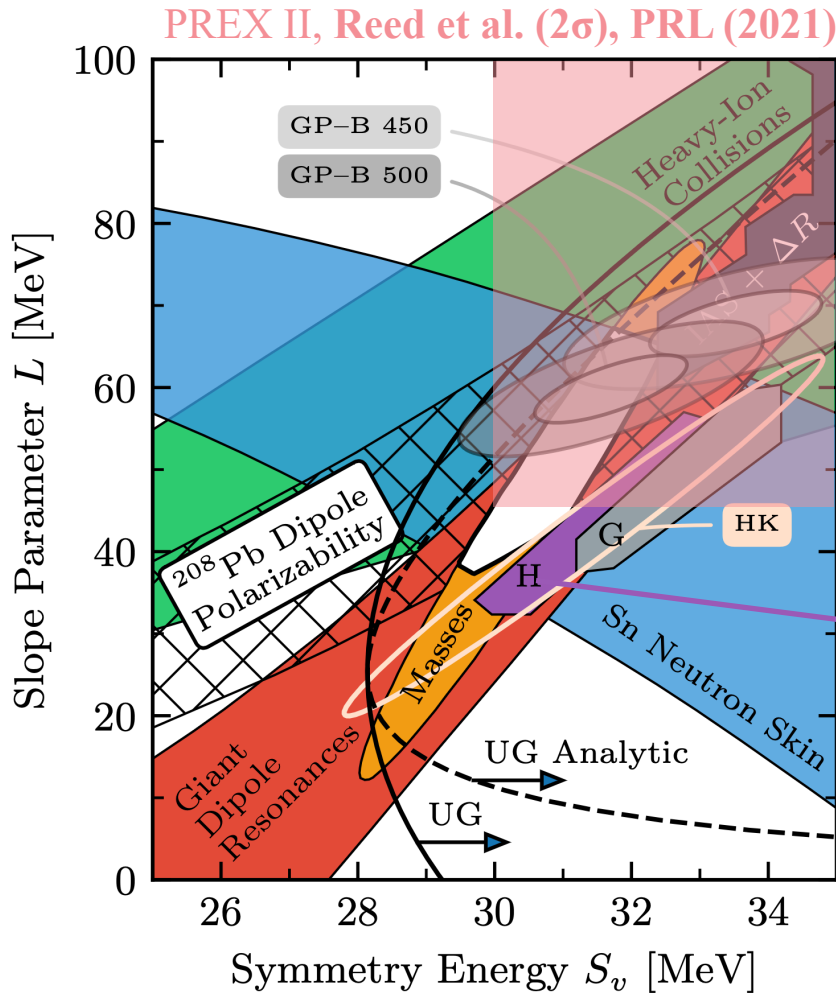
Chiral effective field theory for nuclear forces

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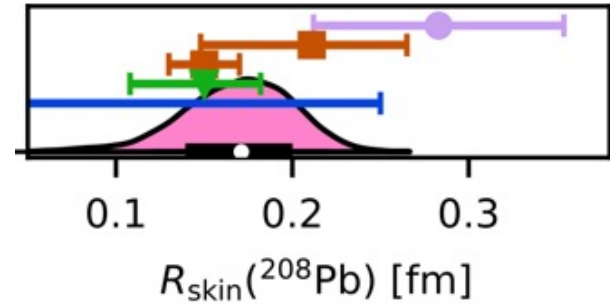
Constraints at intermediate densities:  
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**Nuclear experiments: dipole response, CREX, PREX**

# Symmetry energy vs. L parameter based on Lattimer, Lim, ApJ (2013)



Ab initio calc of  $^{208}\text{Pb}$  neutron skin  
 Hu, Jiang, Miyagi et al., arXiv:2112.01125



**Region H** corresponds to  
 $^{208}\text{Pb}$  neutron skin: 0.14-0.20 fm  
 Hebeler, Lattimer, Pethick, AS, PRL (2010)

MREX precision goal  $\pm 0.03$  fm  
 very important (ideally better)

from Drischler, Holt, Wellenhofer, AS, ARNPS (2021)

Note: not all regions are at same saturation density

# Impact of PREX and $^{208}\text{Pb}$ dipole polarizability

Essick, Landry, AS, Tews, PRL, PRC (2021)

Nonparametric Prior

Nonparametric Astro Posterior

$\chi\text{EFT}+\text{Astro}$  Posterior

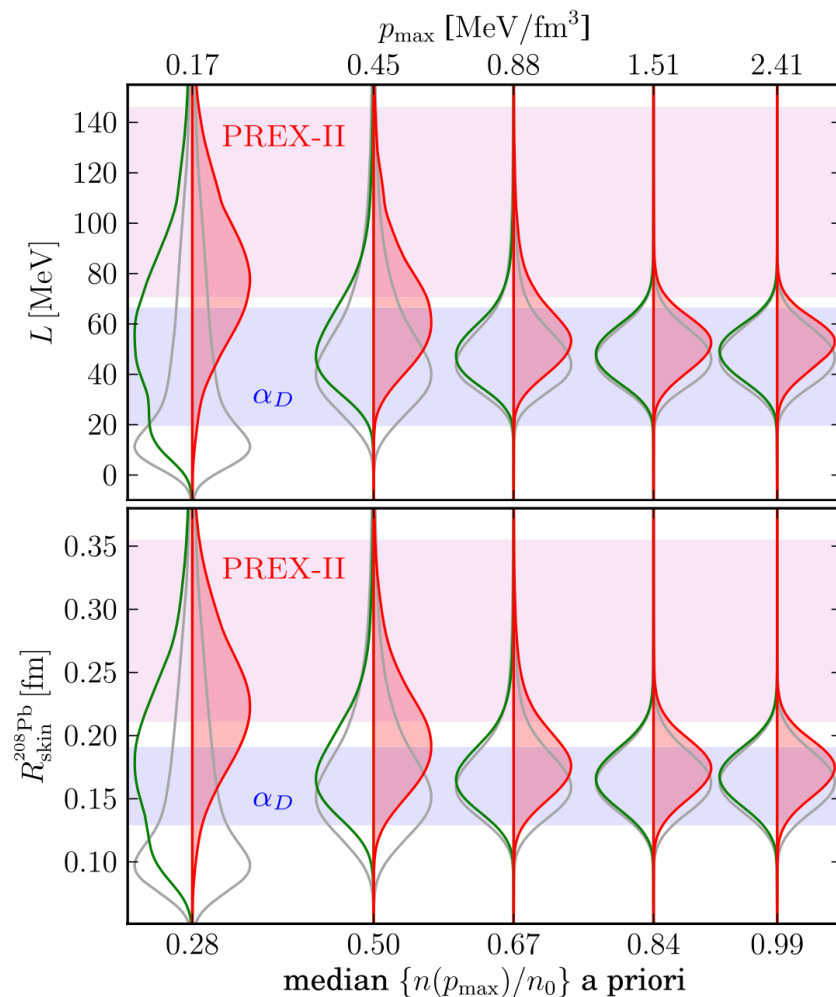
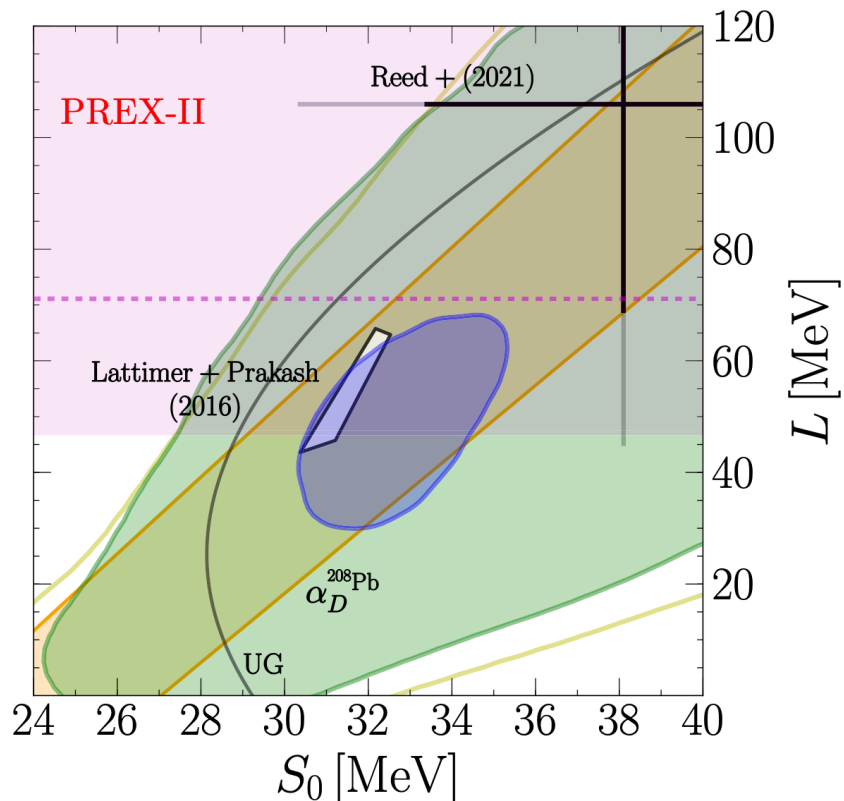


FIG. 2. Prior (gray, unshaded), Astro posterior (green, left-unshaded), and Astro + PREX-II posterior (red, right-shaded)

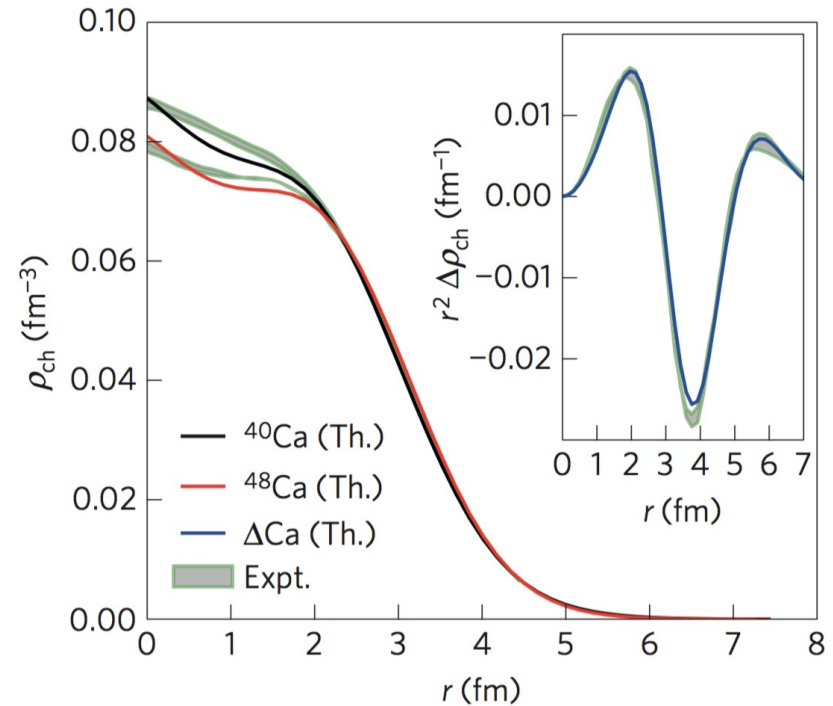
$^{208}\text{Pb}$  dipole polarizability Tamii et al., PRL (2021)

very consistent with  $\chi\text{EFT}+\text{Astro}$  posterior

# Neutron skin and dipole polarizability of $^{48}\text{Ca}$

Hagen et al., Nature Phys. (2015)

ab initio calculations lead to charge distributions consistent with exp



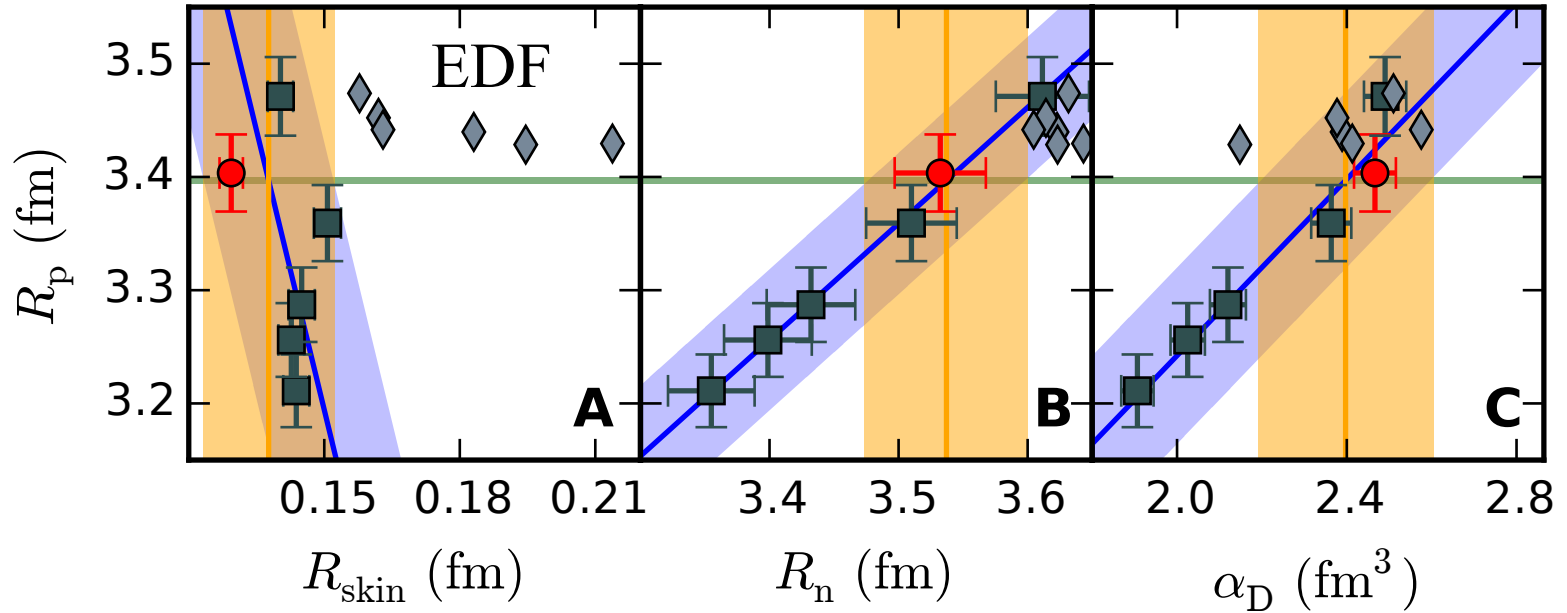
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predict **small neutron skin**

**dipole polarizability  $\alpha_D$**



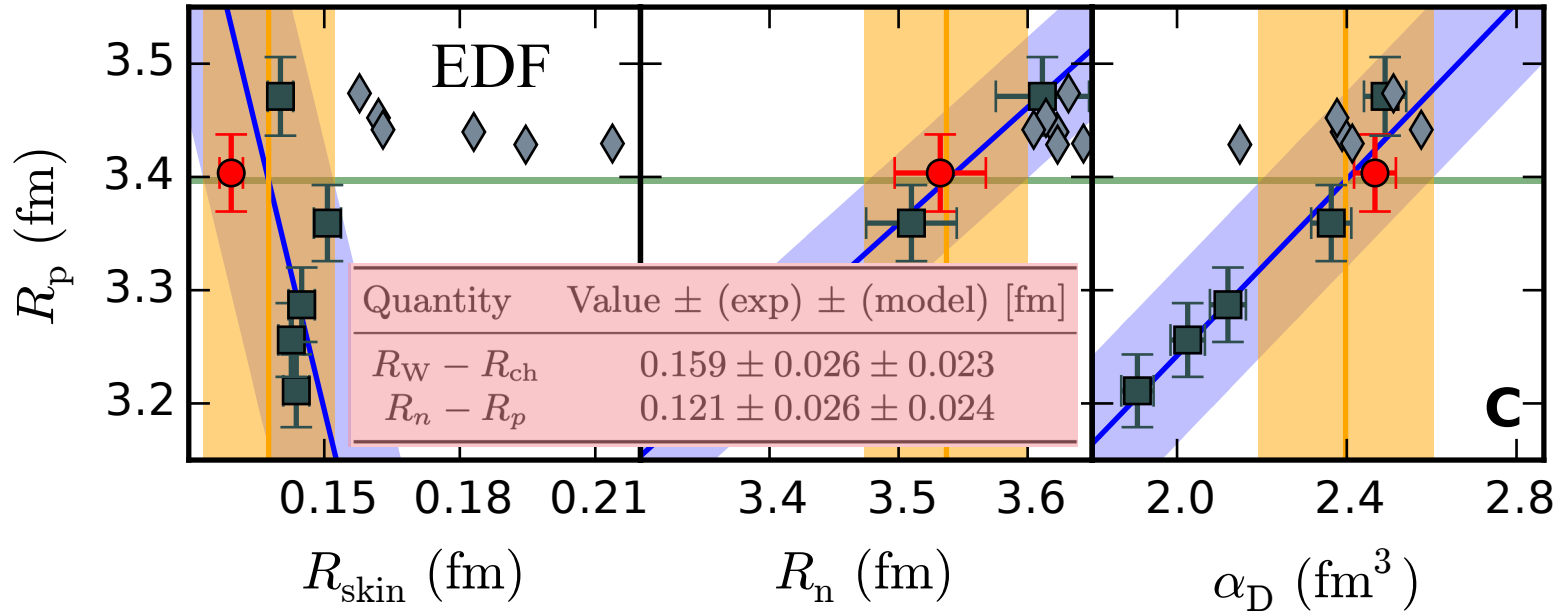
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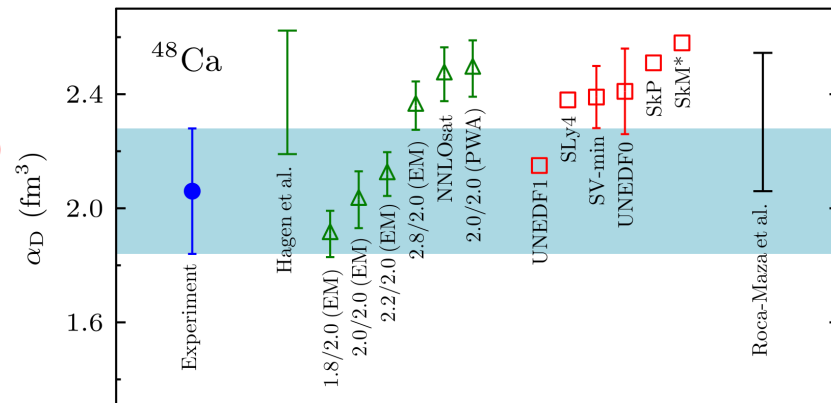
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agrees with dipole polarizability from  
Darmstadt-Osaka exp Birkhan et al., PRL (2017)

+ with CREX result Adhikari et al., PRL (2022)





# Summary

Thanks to: J. Braun, T. Dietrich, C. Drischler, R. Essick, S. Greif, K. Hebeler, S. Huth, P. Landry, J. Lattimer, A. Le Fèvre, J. Keller, P. Pang, C. Pethick, G. Raaijmakers, I. Tews, W. Trautmann, A. Watts, C. Wellenhofer

ab initio calculations based on chiral EFT interactions for many nuclei and matter: cold, finite T, arbitrary proton fraction

reliable results up to  $1-1.5 n_0$  with controlled uncertainties: EFT truncation + systematic uncertainties

astro, heavy-ion collisions, functional RG provides insights to intermediate densities

chiral EFT agrees with many experiments for (n-rich) nuclei +  $^{48}\text{Ca}$  dipole polarizability, CREX – precise PREX/MREX needed