Inferring breakdown scales in baryon chiral perturbation theory

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Nuclear Hamiltonians for Advancing Nuclear Physics and Beyond
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Hamiltonians based on chiral EFT

- Systematically improvable
- Connections between meson (ChPT), single-baryon (BChPT), few-nucleon (ChEFT) interactions
- Quantified uncertainties
- Expansion based on ratio of scales $Q = \Lambda_l/\Lambda_B$
  - Λ_l: light scale (pion mass, momentum)
  - Λ_B: breakdown scale (?)

What can we say about Λ_B?

Breakdown scale(s) in mesonic ChPT

$$\Lambda_B \approx \begin{cases} 4\pi F_\pi \approx 1\,\text{GeV} \\ m_\rho \approx 770\,\text{MeV} \end{cases}$$

Expansion parameter $M/\Lambda_B \approx 0.12 - 0.18$

Breakdown scale(s) in Baryon ChPT?

- New scales:
  - Nucleon mass in chiral limit: $\mathring{m} \approx 870\,\mathrm{MeV}$
  - Δ-nucleon mass splitting: $m_\Delta - m_N \approx 290\,\mathrm{MeV}$
- Loop factors of π
- Large numerical coefficients

See M. Hoferichter's INT-25-92W talk

Use chiral expansions of observables to determine Λ_B?

Chiral expansions of m_N

- Nucleon mass (one loop, heavy-baryon ChPT)

$$m_N = \mathring{m} - 4c_1 M^2 - \frac{3g_A^2 M^3}{32\pi F_\pi^2} + \left(k_1 \ln \frac{M}{m_N} + k_2\right) M^4$$

$$k_1 = -\frac{3}{32\pi^2 F_\pi^2 m_N} \left(g_A^2 + m_N \left(-8c_1 + c_2 + 4c_3\right)\right) \quad ,$$

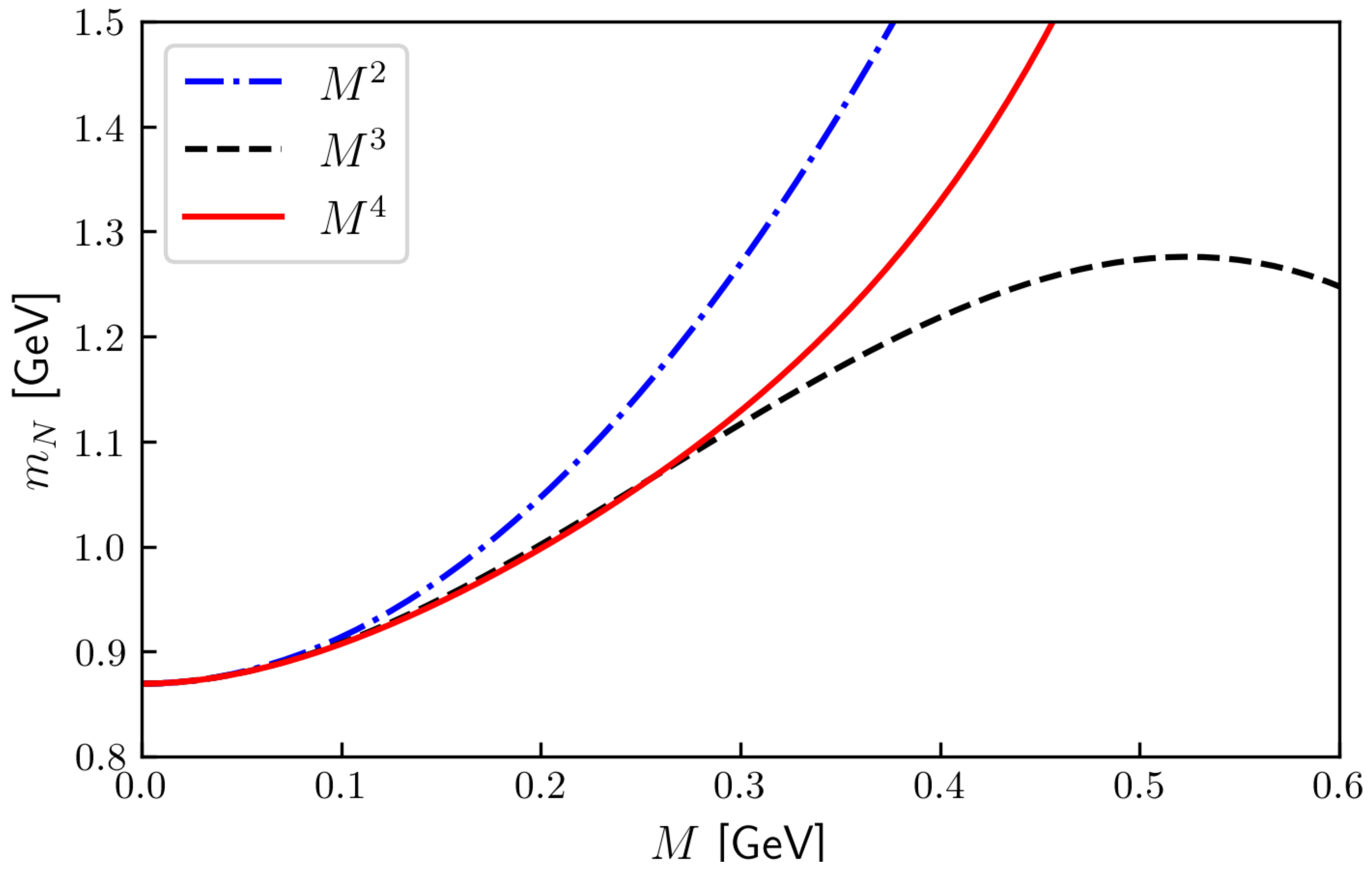
$$k_2 = \bar{e}_1 - \frac{3}{128\pi^2 F_\pi^2 m_N} \left(2g_A^2 - m_N c_2\right)$$

with

- $\mathring{m}$: nucleon mass in chiral limit
- $c_i/\bar{e}_1$: low-energy constants from πN Lagrangian at order 2/4

Gasser, Sainio, Svarc (1988); Steininger, Meißner, Fettes (1998); Kambor, Mojžiš (1999)

Chiral expansions of m_N



LECs from Hoferichter et al. (2015, 2016)

Chiral expansions of g_A

- Axial-vector coupling (leading two loop, heavy-baryon ChPT)

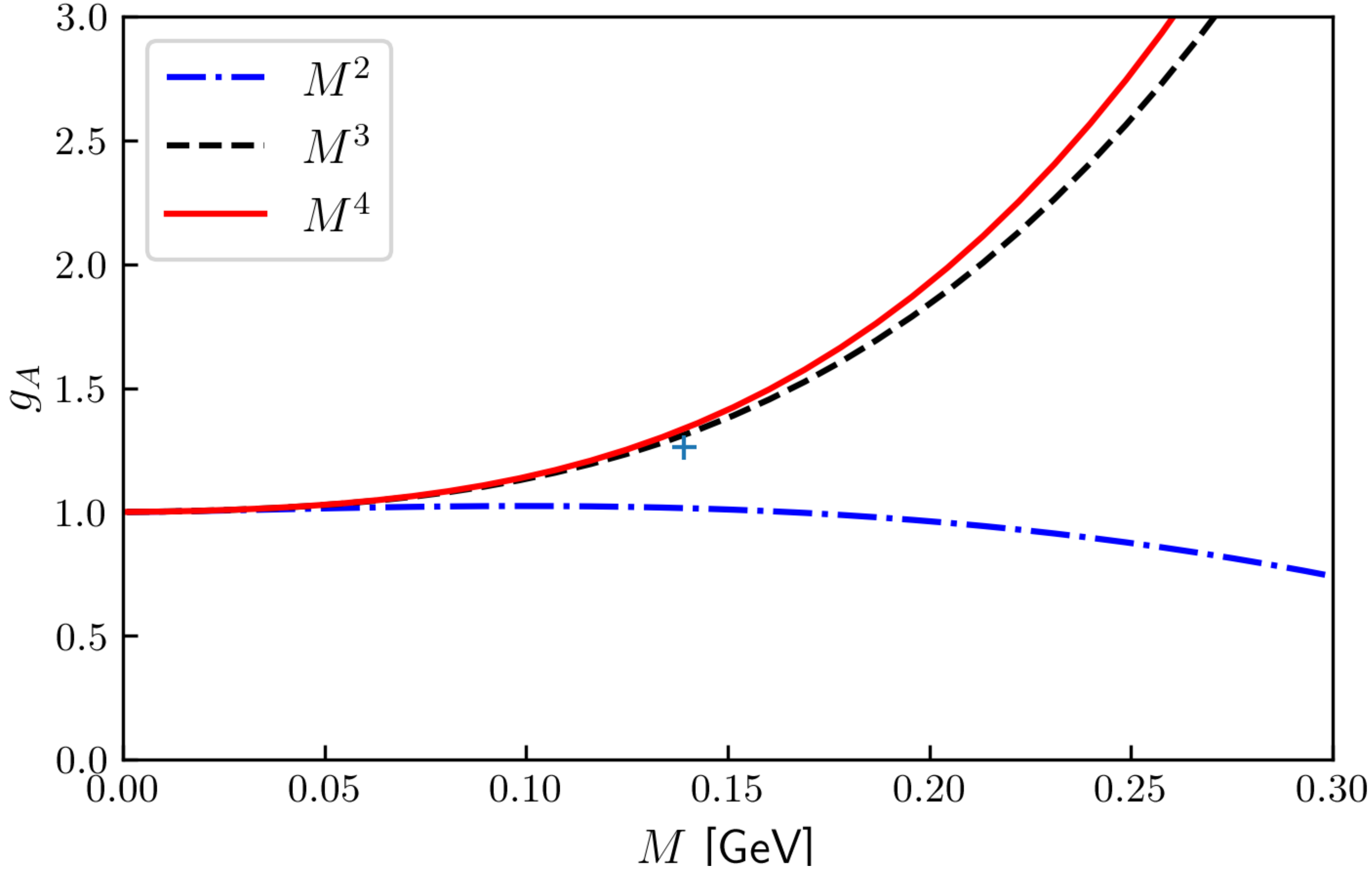
$$g_A = \mathring{g}\Bigg[1 + \left(\frac{\alpha_2}{(4\pi F)^2}\ln\left(\frac{M}{\mu}\right) + \beta_2\right)M^2 + \alpha_3 M^3 + \frac{1}{\mathring{g}}\left(\frac{\alpha_4}{(4\pi F)^4}\ln^2\left(\frac{M}{\mu}\right) + \frac{\gamma_4}{(4\pi F)^4}\ln\left(\frac{M}{\mu}\right) + \beta_4 + \mathring{g}C\right)M^4\Bigg]$$

with

- $\alpha_i, \beta_i, \gamma_i$: linear combinations of LECs from $\mathcal{L}_{\pi N}^{(1,2,3)}$
- C: linear combination of LECs from $\mathcal{L}_{\pi N}^{(5)}$ (set $C = 0$ in following)

Bernard, Gegelia, Ghosh, Meißner (2025)

Chiral expansions of g_A



LECs from Siemens et al. (2015, 2016)

Inferring Λ_B

- Posterior pdf for Λ_B given data $\mathbf{y}$ and assumptions I:

$$p(\Lambda_B|\mathbf{y}, I)$$

- Data $\mathbf{y}$: chiral expansion of observable $Y \in \{m_N, g_A\}$ at increasing orders

$$Y^{(n)} = Y_{\text{ref}} \sum_{i=0}^{n} c_i Q^i$$

  with $Q = M/\Lambda_B$, $Y_{\text{ref}} = Y_{\text{LO}}$ and $c_0 = 1$

- Assumptions I: naturalness of expansion coefficients c_i

Inferring Λ_B

- Bayes theorem

$$p(\Lambda_B|\mathbf{y}, I) \propto p(\mathbf{y}|\Lambda_B, I) \cdot p(\Lambda_B|I)$$

- $p(\mathbf{y}|\Lambda_B, I)$: Likelihood of data → change of variables to $\{c_i\}$

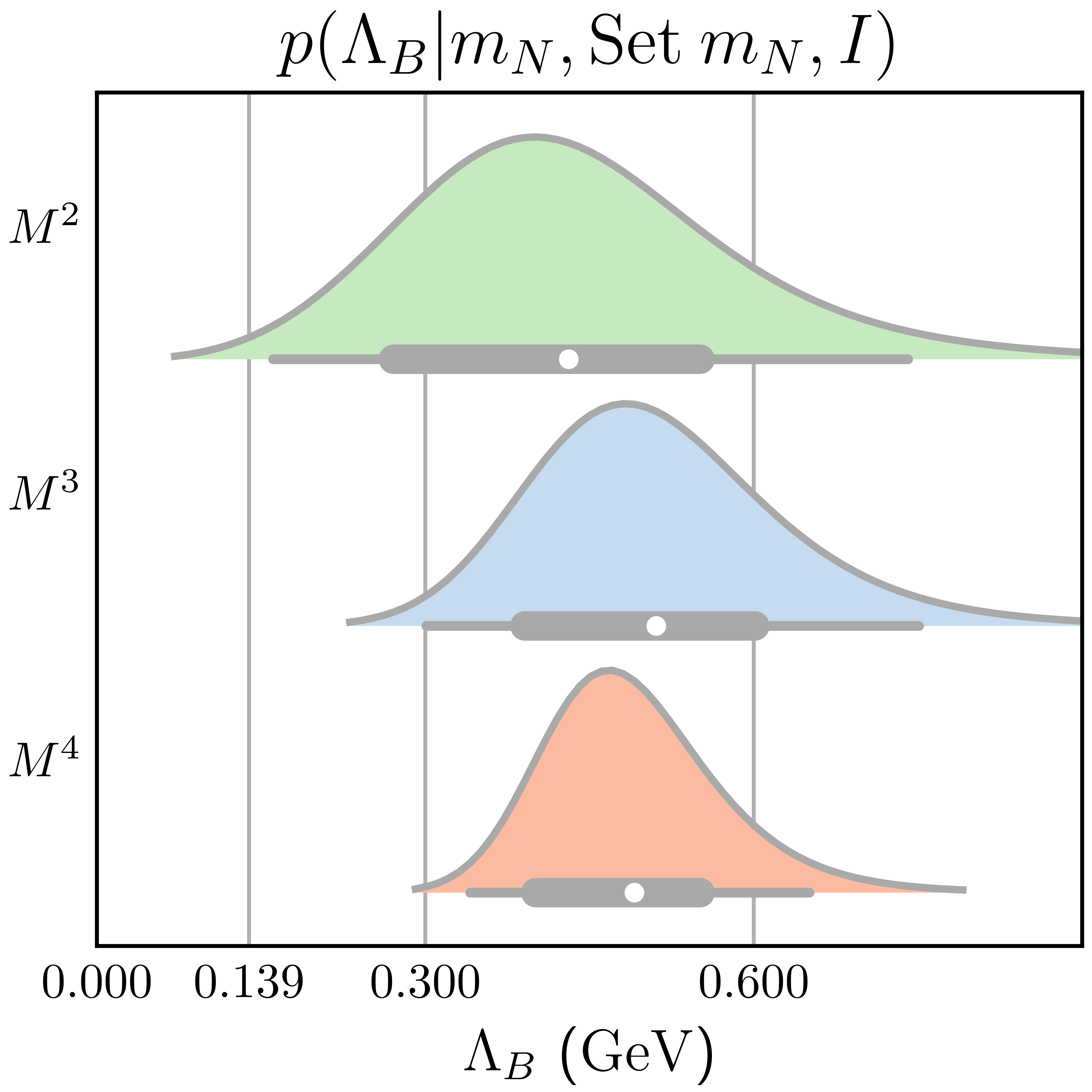
- Priors on expansion coefficients: $c_i \overset{\text{iid}}{\sim} \mathcal{N}(0,\bar{c}^2)$ with variance $\bar{c}^2 \sim \chi^{-2}(\nu_0 = 2,\tau_0^2 = 1)$
  → ≈ 67% of probability for $\bar{c}^2$ in [1/3,3]

- Prior on Λ_B: log-uniform in $[M_\pi^{(\text{phys})}/40, 40M_\pi^{(\text{phys})}]$

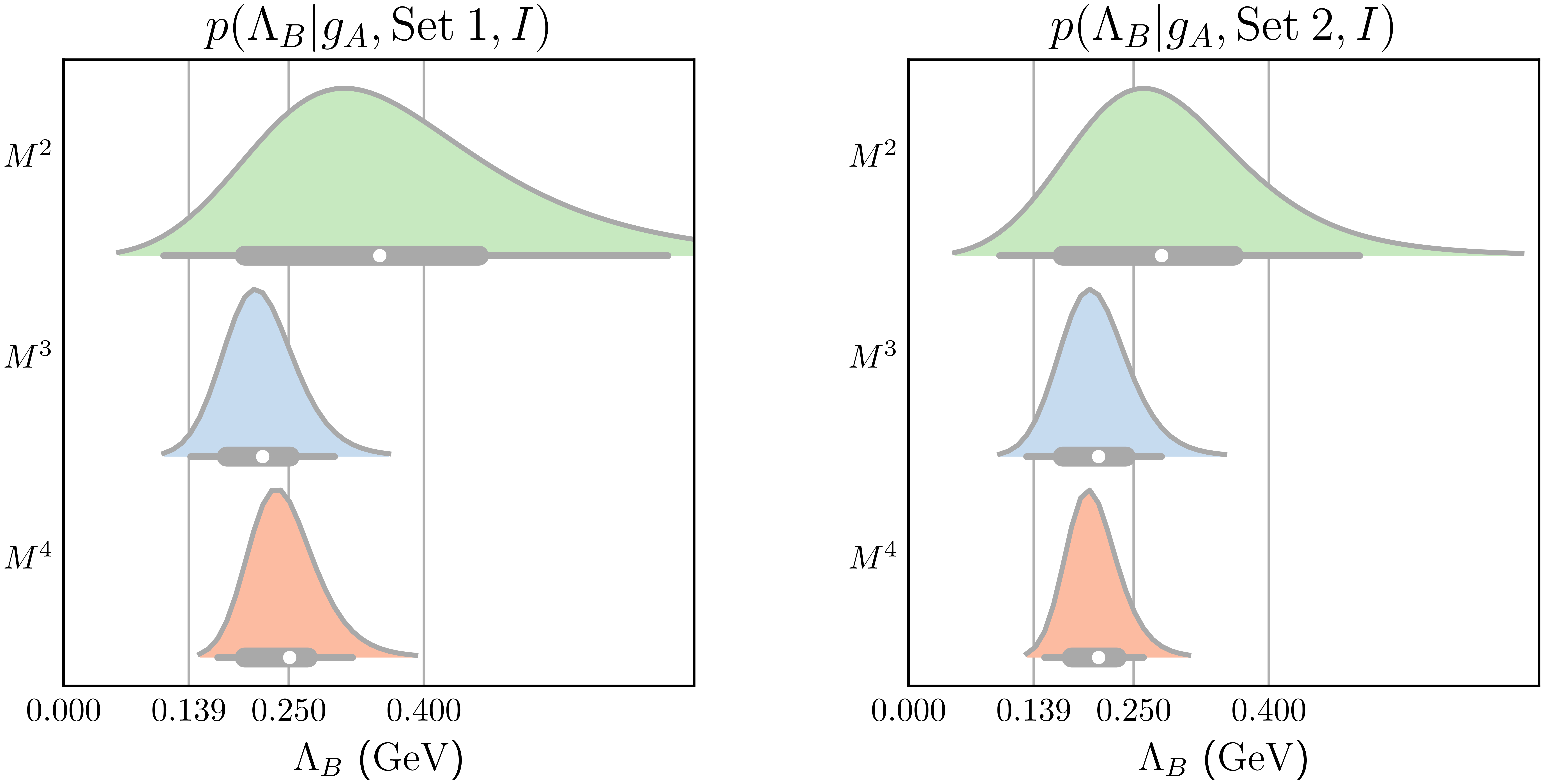
- Pointwise method: $M = [9.85\,\text{MeV}, 301.84\,\text{MeV}]$

Melendez et al. (2019)

Λ_B from m_N



Λ_B from g_A



Results for Λ_B

- Median values and 68% degree of belief intervals (highest posterior density)

Order	$\Lambda_B(g_A;\text{Set 1})$	$\Lambda_B(g_A;\text{Set 2})$	$\Lambda_B(m_N;\text{Set } m_N)$
M^2	351 (201,461)	281 (171,361)	431 (271,551)
M^3	221 (181,251)	211 (171,241)	511 (391,601)
M^4	251 (201,271)	211 (181,231)	491 (401,551)

- Stable under: variations in priors, values of M

Conclusions

- Bayesian inference of breakdown scales in baryon chiral perturbation theory
  - Nucleon mass m_N: $\Lambda_{B,m_N} \approx 490\,\mathrm{MeV}$
  - Axial-vector coupling g_A: $\Lambda_{B,g_A} \approx 230\,\mathrm{MeV}$
- Consistent with earlier estimates
- For g_A: inclusion of Δ required? (Not enough - large numerical factors?)
- Not considered: uncertainties in LECs

Djukanovic, Gegelia, Scherer (2006); McGovern, Birse (2006); MRS et al. (2006); Bernard, Meißner (2006)

Conclusions

- Λ_B for individual observables
- Not "the breakdown scale of BChPT"
- Naturalness of $c_i \neq$ naturalness of LECs
- Impact on two- and few-body interactions?
  - ChEFT in NN scattering, A=3,4, neutron matter: $\Lambda_B \approx 600\,\mathrm{MeV}$
  - Axial currents?
- Momentum dependence?