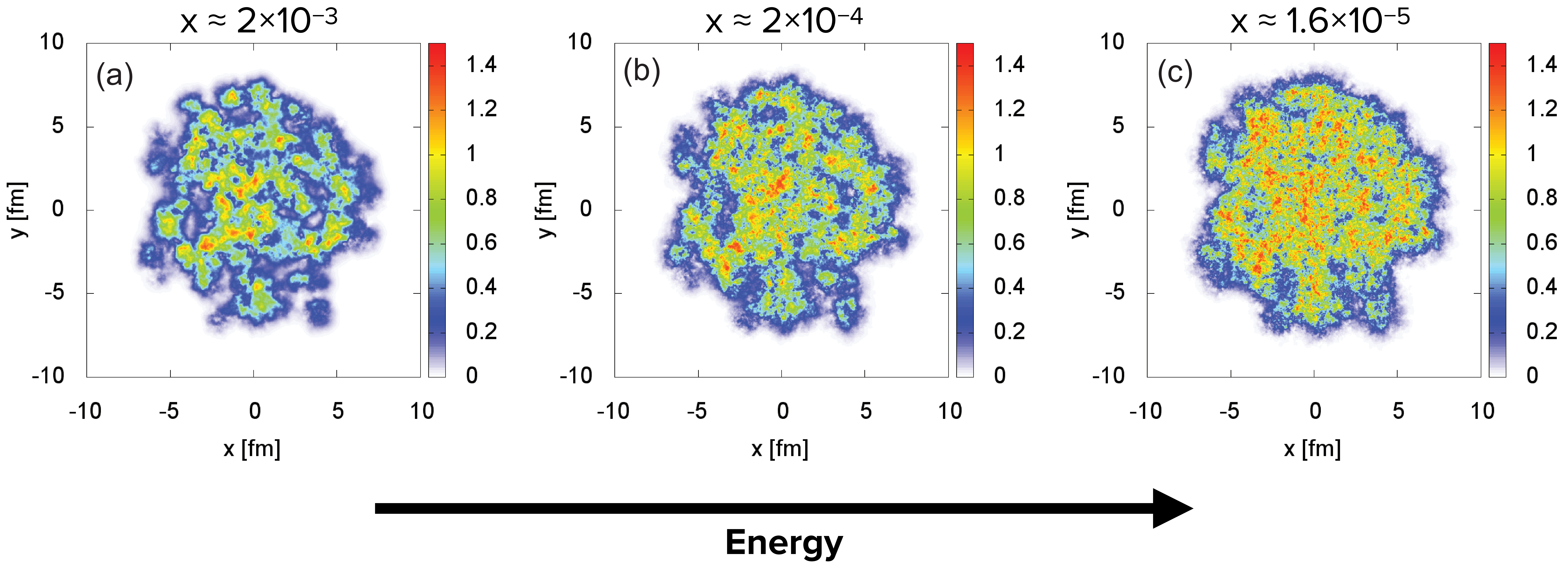
NUCLEAR DEFORMATION EFFECTS AT SMALL X

BJÖRN SCHENKE, BROOKHAVEN NATIONAL LABORATORY

Intersection of nuclear structure and high-energy nuclear collisions
Institute for Nuclear Theory
02/09/2023

Questions:

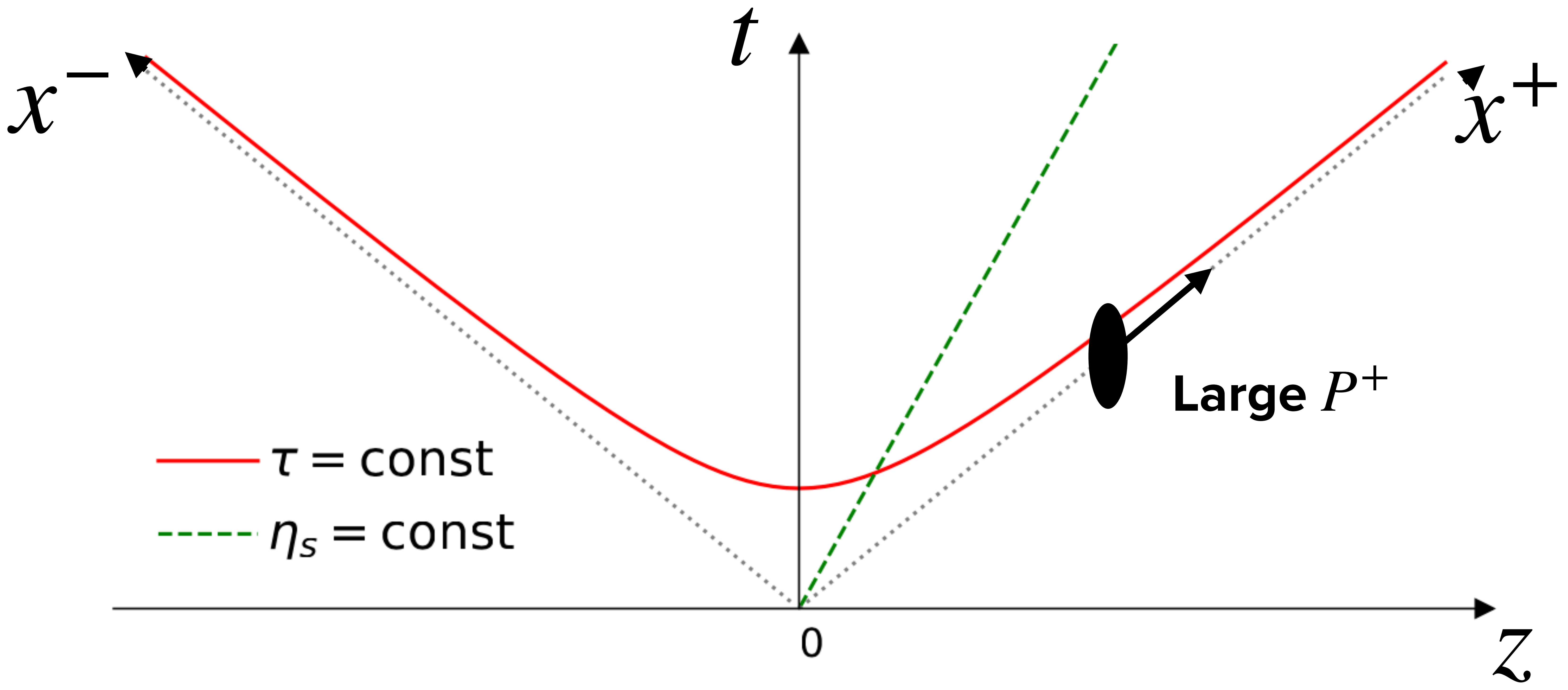
How does energy evolution affect the nuclear structure?



Are observables in high energy e+A scattering sensitive to nuclear deformation?

B. Schenke, S. Schlichting, Phys.Rev.C 94 (2016) 4, 044907

The scenario: Hadron moving at high momentum



Probe hadron (or nucleus) moving with large P^+ at scale $x_0 P^+$ with $x_0 \ll 1$

Separate partonic content based on longitudinal momentum $k^+ = xP^+$

Large $x > x_0$: Static and localized color sources ρ

Dynamic color fields

The moving color sources generate a current, independent of light cone time z^+:

$$J^{\mu,a}(z) = \delta^{\mu+}\rho^a(z^-, z_T)$$

a is the color index of the gluon

This current generates delocalized dynamical fields $A^{\mu,a}(z)$ described by the Yang-Mills equations

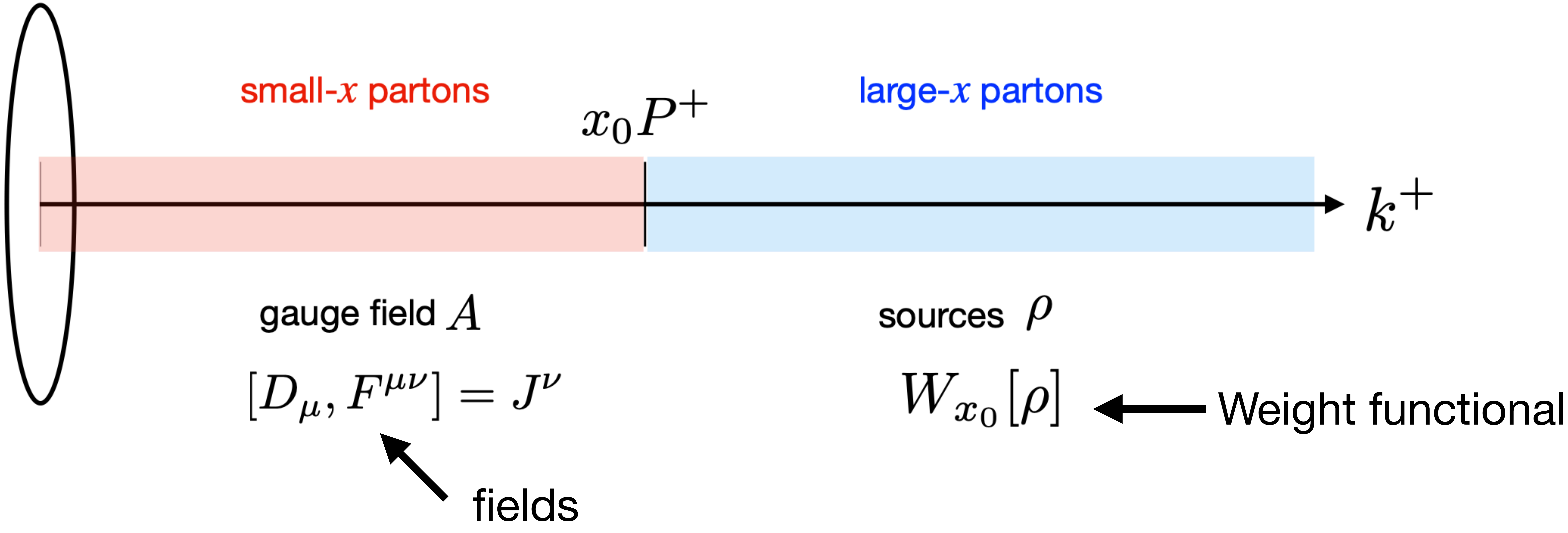
$$[D_\mu, F^{\mu\nu}] = J^\nu$$

with $D_\mu = \partial_\mu + igA_\mu$ **and** $F_{\mu\nu} = \frac{1}{ig}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$

These fields A are the small $x < x_0$ degrees of freedom

They can be treated classically, because their occupation number is large $\langle AA \rangle \sim 1/\alpha_s$

Color Glass Condensate (CGC): Sources and fields



When $x \lesssim x_0$ the path integral $\langle \mathcal{O} \rangle_\rho$ is dominated by classical solution and we are done

For smaller x we need to do quantum evolution

Wilson lines

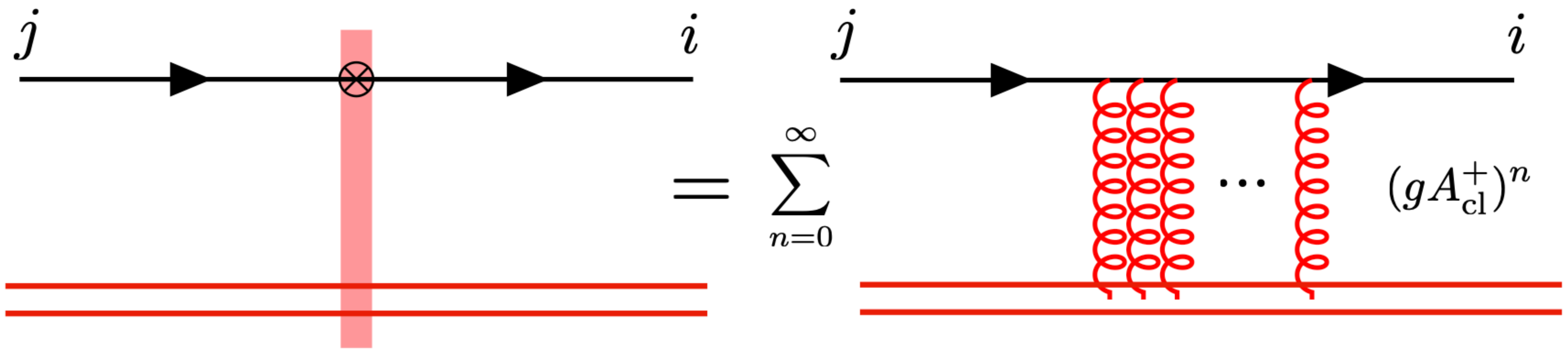
Interaction of high energy color-charged probe with large k^- momentum (and small $k^+ = \dfrac{k_T^2}{2k^-}$)

with the classical field of a nucleus can be described in the **eikonal approximation:**

The scattering rotates the color, but keeps k^-, transverse position $\vec{x}_T$, and any other quantum numbers the same.

The color rotation is encoded in a light-like Wilson line, which for a quark probe reads

$$V_{ij}(\vec{x}_T) = \mathscr{P}\left(ig \int_{-\infty}^{\infty} A^{+,c}(z^-, \vec{x}_T)\, t^c_{ij}\, dz^- \right)$$



MULTIPLE INTERACTIONS NEED TO BE RESUMMED, BECAUSE $A^+ \sim 1/g$

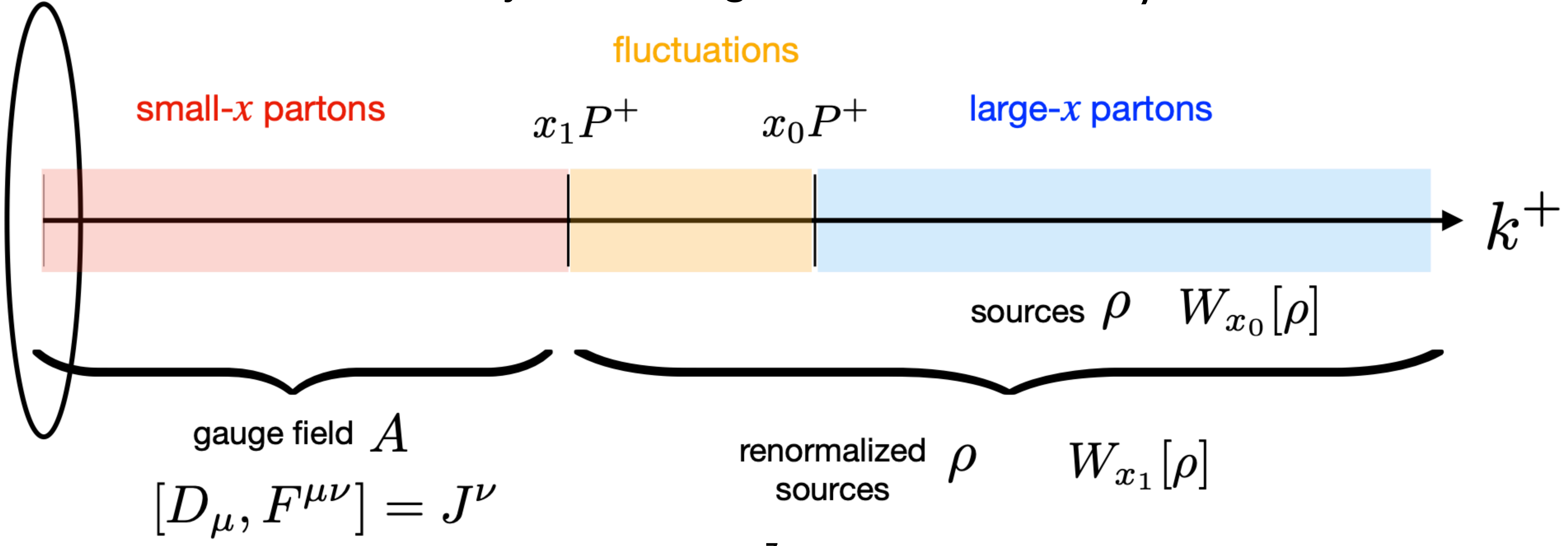
JIMWLK evolution

Jalilian-Marian, J.; Kovner, A.; McLerran, L.D.; Weigert, H., Phys. Rev. D 1997, 55, 5414–5428, [hep-ph/9606337]
Jalilian-Marian, J.; Kovner, A.; Weigert, H., Phys. Rev. D 1998, 59, 014015, [hep-ph/9709432]
Kovner, A.; Milhano, J.G.; Weigert, H., Phys. Rev. D 2000, 62, 114005, [hep-ph/0004014]
Iancu, E.; Leonidov, A.; McLerran, L.D., Nucl. Phys. A 2001, 692, 583–645,[hep-ph/0011241]
Iancu, E.; Leonidov, A.; McLerran, L.D., Phys. Lett. B 2001, 510, 133–144, [hep-ph/0102009]
Ferreiro, E.; Iancu, E.; Leonidov, A.; McLerran, L., Nucl. Phys. A 2002, 703, 489–538, [hep-ph/0109115]

LO Small-x evolution resums logarithmically enhanced terms $\sim \alpha_s \ln(x_0/x)$

$$\frac{dW_x[\rho]}{d\ln(1/x)} = -\mathscr{H}_{\mathrm{JIMWLK}}\, W_x[\rho]$$

Physically, one absorbs the quantum fluctuations in the interval $[x_0 - dx, x_0]$ into stochastic fluctuations of the color sources by redefining the color sources ρ



JIMWLK evolution

Jalilian-Marian, J.; Kovner, A.; McLerran, L.D.; Weigert, H., Phys. Rev. D 1997, 55, 5414–5428, [hep-ph/9606337]
Jalilian-Marian, J.; Kovner, A.; Weigert, H., Phys. Rev. D 1998, 59, 014015, [hep-ph/9709432]
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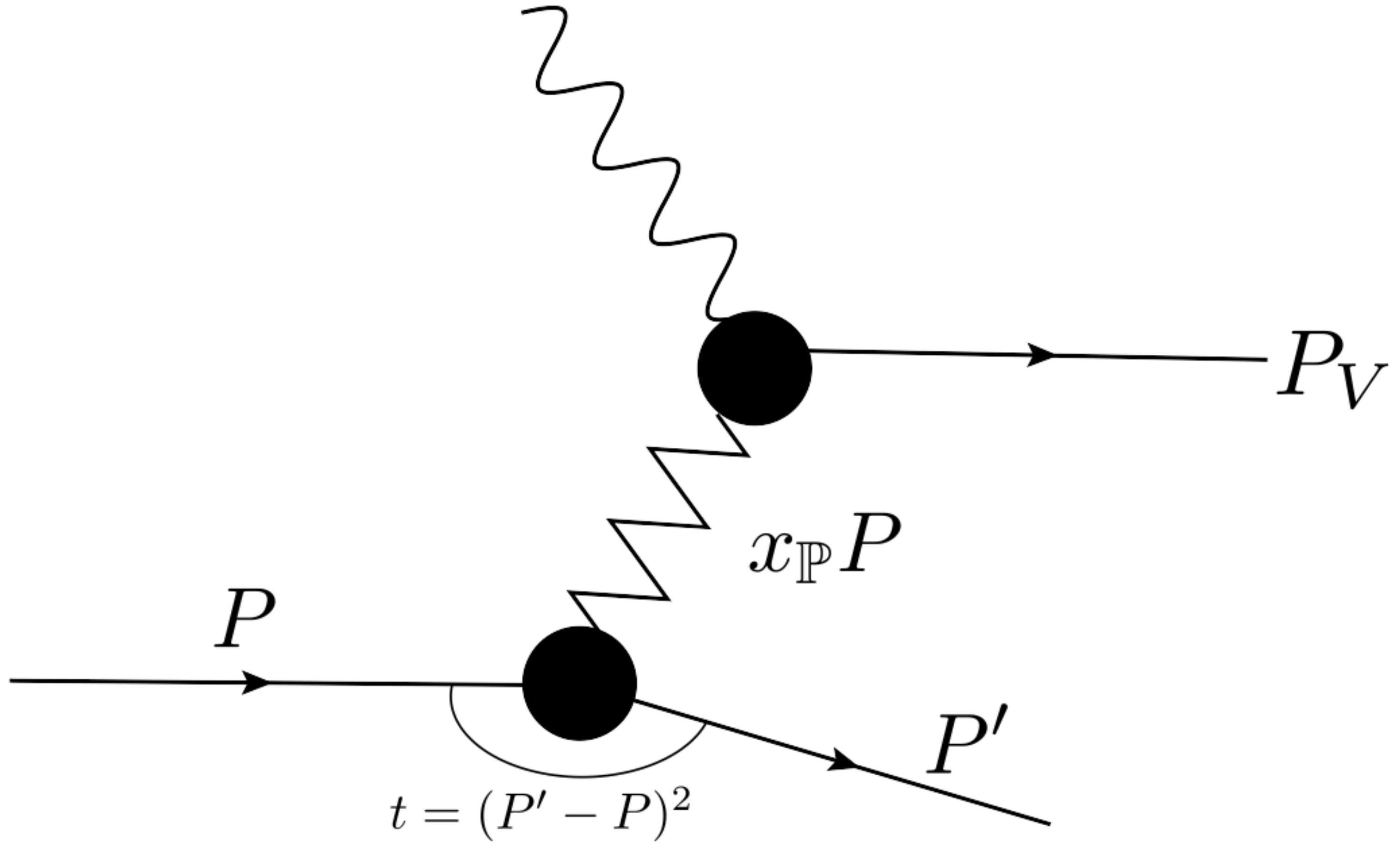
Evolution is done using the Langevin formulation of the JIMWLK equations on the level of Wilson lines

K. Rummukainen and H. Weigert Nucl. Phys. A739 (2004) 183; T. Lappi, H. Mäntysaari, Eur. Phys. J. C73 (2013) 2307

Long distance tales are tamed by imposing a regulator in the JIMWLK kernel, *m*

S. Schlichting, B. Schenke, Phys.Lett. B739 (2014) 313-319

Diffractive vector meson production



- Coherent diffraction:
  Target stays intact

$$\frac{d\sigma^{\gamma^* p \to Vp}}{dt} = \frac{1}{16\pi} \left| \left\langle A^{\gamma^* p \to Vp}\left(x_P, Q^2, \vec{\Delta}\right) \right\rangle \right|^2$$

sensitive to the average size of the target

- Incoherent diffraction:
  Target breaks up

$$\frac{d\sigma^{\gamma^* p \to Vp^*}}{dt} = \frac{1}{16\pi} \left(\left\langle \left| A^{\gamma^* p \to Vp}\left(x_P, Q^2, \vec{\Delta}\right) \right|^2 \right\rangle - \left| \left\langle A^{\gamma^* p \to Vp}\left(x_P, Q^2, \vec{\Delta}\right) \right\rangle \right|^2 \right)$$

sensitive to fluctuations (including geometric ones)

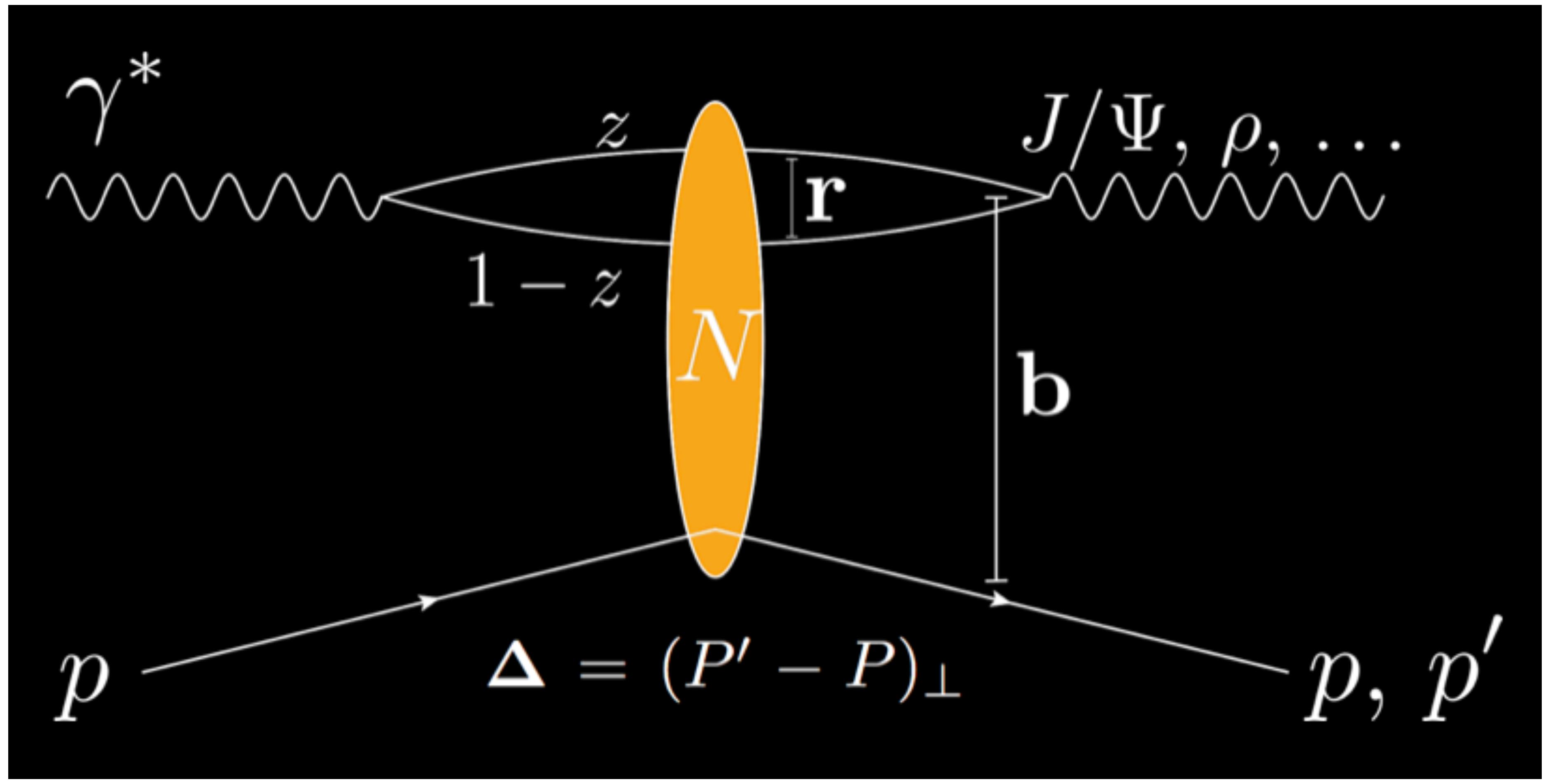
M. L. Good and W. D. Walker, Phys. Rev. 120 (1960) 1857
H. I. Miettinen and J. Pumplin, Phys. Rev. D18 (1978) 1696
Y. V. Kovchegov and L. D. McLerran, Phys. Rev. D60 (1999) 054025
A. Kovner and U. A. Wiedemann, Phys. Rev. D64 (2001) 114002

Dipole picture: Scattering amplitude

H. Mäntysaari, B. Schenke, Phys. Rev. Lett. 117 (2016) 052301; Phys.Rev. D94 (2016) 034042

High energy factorization:

- $\gamma^* \to q\bar{q} : \psi^\gamma(r, Q^2, z)$
- $q\bar{q}$ dipole scatters with amplitude N
- $q\bar{q} \to V : \psi^V(r, Q^2, z)$



$$A \sim \int d^2b\, dz\, d^2r\, \psi^* \psi^V(\vec{r}, z, Q^2) e^{-i\vec{b}\cdot\vec{\Delta}} N(\vec{r}, x, \vec{b})$$

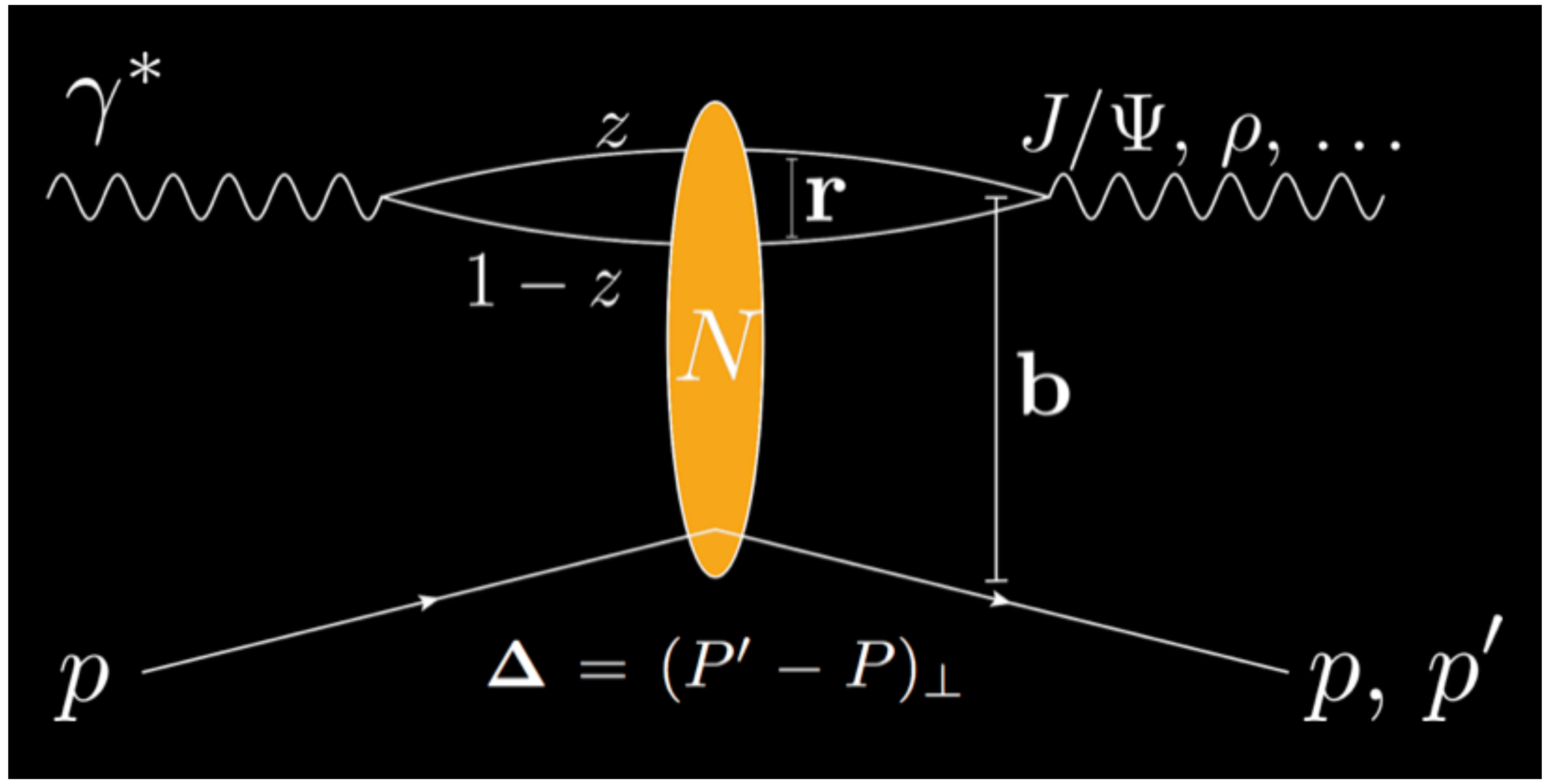
- Impact parameter ***b*** is the Fourier conjugate of transverse momentum transfer ***Δ*** → Access to spatial structure $(t = -\Delta^2)$
- Total F_2: forward scattering amplitude (***Δ***=0) for V=γ (same N)

Color glass condensate formalism

H. Mäntysaari, B. Schenke, Phys.Rev.D 98 (2018) 3, 034013

Compute the Wilson lines using color charges whose correlator depends on $\vec{b}_\perp$

$$\langle \rho^a(\mathbf{b}_\perp)\rho^b(\mathbf{x}_\perp)\rangle = g^2\mu^2(x, \mathbf{b}_\perp)\delta^{ab}\delta^{(2)}(\mathbf{b}_\perp - \mathbf{x}_\perp)$$



$$N(\vec{r}, x, \vec{b}) = N(\vec{x} - \vec{y}, x, (\vec{x} + \vec{y})/2) = \mathrm{Tr}(\mathrm{V}(\vec{\mathrm{x}})\mathrm{V}^\dagger(\vec{\mathrm{y}}))/\mathrm{N_c}$$

Model impact parameter dependence (proton, nucleon)

H. Mäntysaari, B. Schenke, Phys. Rev. Lett. 117 (2016) 052301; Phys.Rev. D94 (2016) 034042

1) Assume Gaussian proton shape:

$$T(\vec{b}) = T_{\rm p}(\vec{b}) = \frac{1}{2\pi B_{\rm p}} e^{-b^2/(2B_{\rm p})}$$

2) Assume Gaussian distributed and Gaussian shaped hot spots:

$$P(b_i) = \frac{1}{2\pi B_{\rm qc}} e^{-b_i^2/(2B_{\rm qc})}$$ (angles uniformly distributed)

$$T_{\rm p}(\vec{b}) = \frac{1}{N_{\rm q}} \sum_{i=1}^{N_{\rm q}} T_{\rm q}(\vec{b}-\vec{b}_i)$$ with $N_{\rm q}$ hot spots; $$T_{\rm q}(\vec{b}) = \frac{1}{2\pi B_{\rm q}} e^{-b^2/(2B_{\rm q})}$$

Diffractive J/ψ production in e+p at HERA

Nucleon parameters B_{q}, B_{qc}, can be constrained by e+p scattering data from HERA

Exclusive diffractive J/Ψ production in e+p:

Incoherent x-sec sensitive to fluctuations

H. Mäntysaari, B. Schenke, Phys. Rev. Lett. 117 (2016) 052301

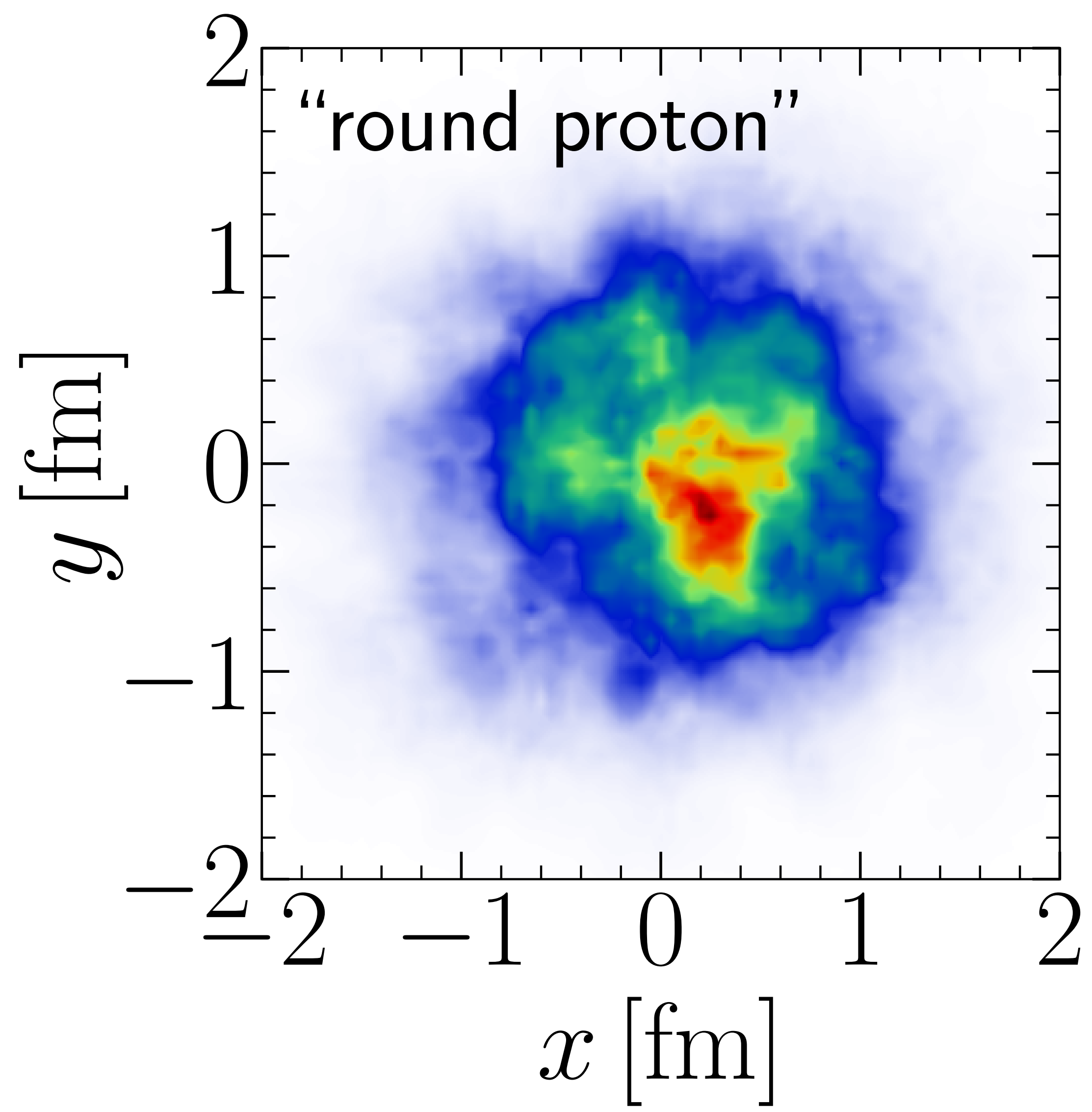
Phys.Rev. D94 (2016) 034042

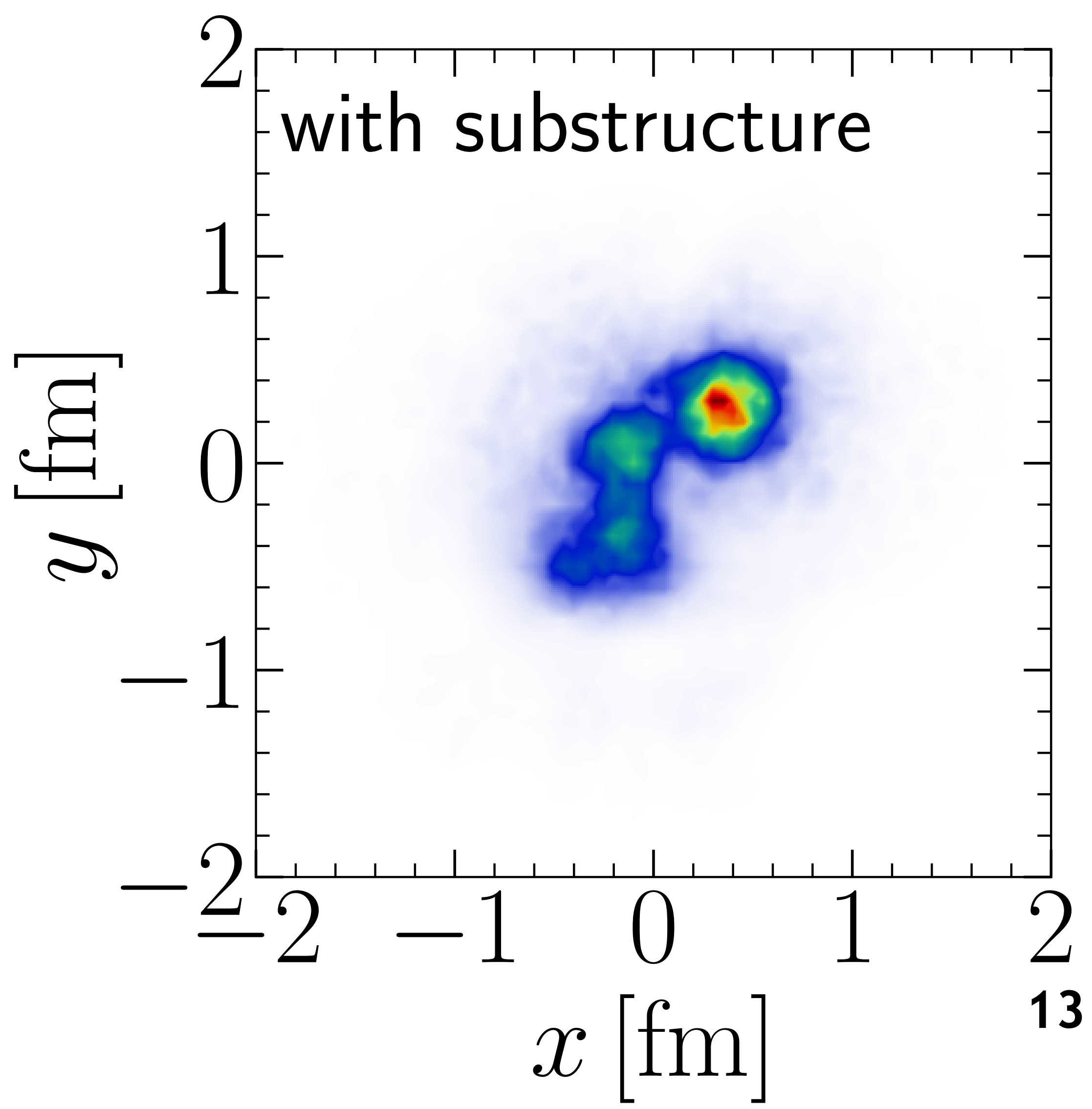
also see:

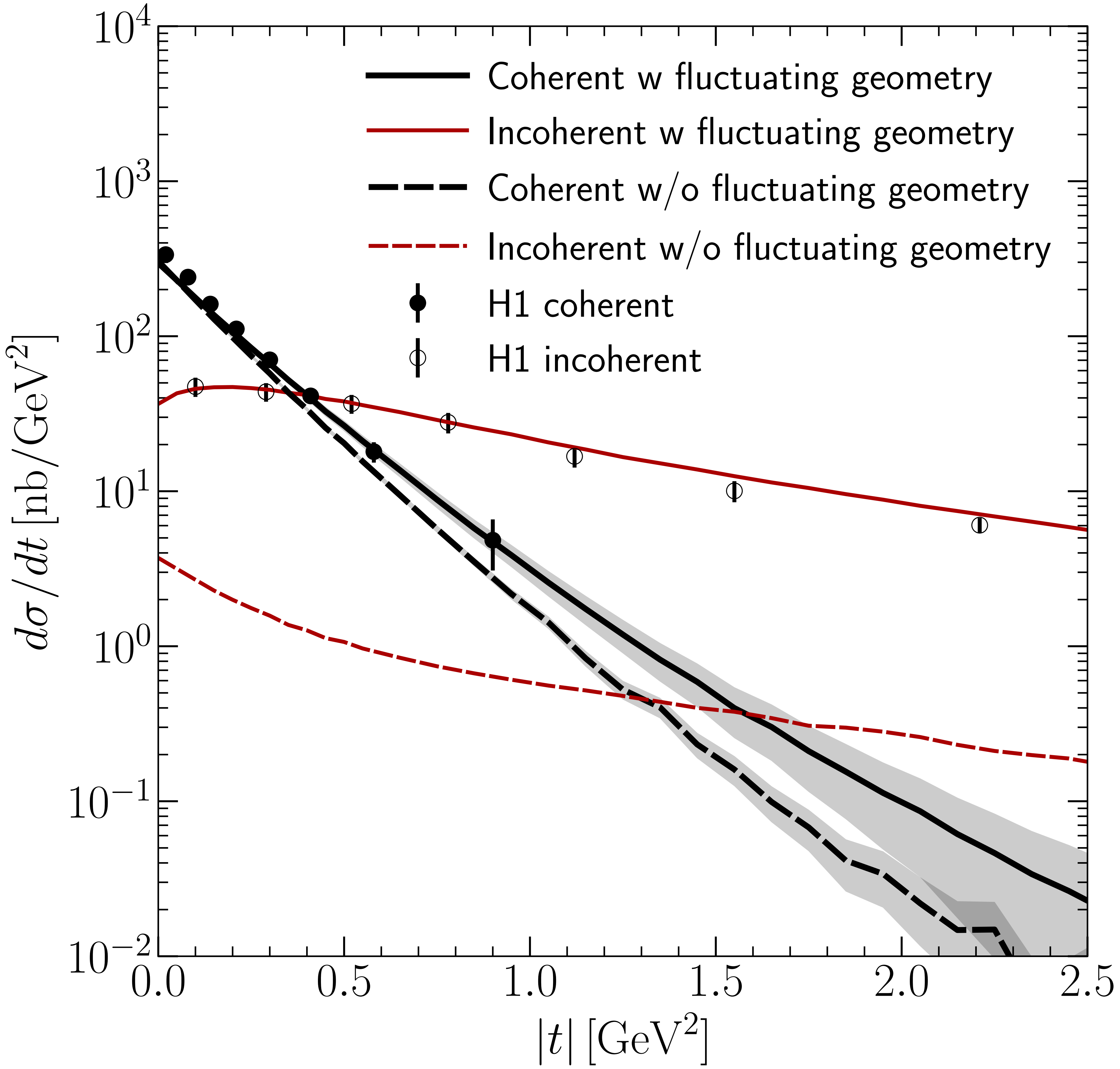
S. Schlichting, B. Schenke, Phys.Lett. B739 (2014) 313-319

H. Mäntysaari, Rep. Prog. Phys. 83 082201 (2020)

B. Schenke, Rep. Prog. Phys. 84 082301 (2021)

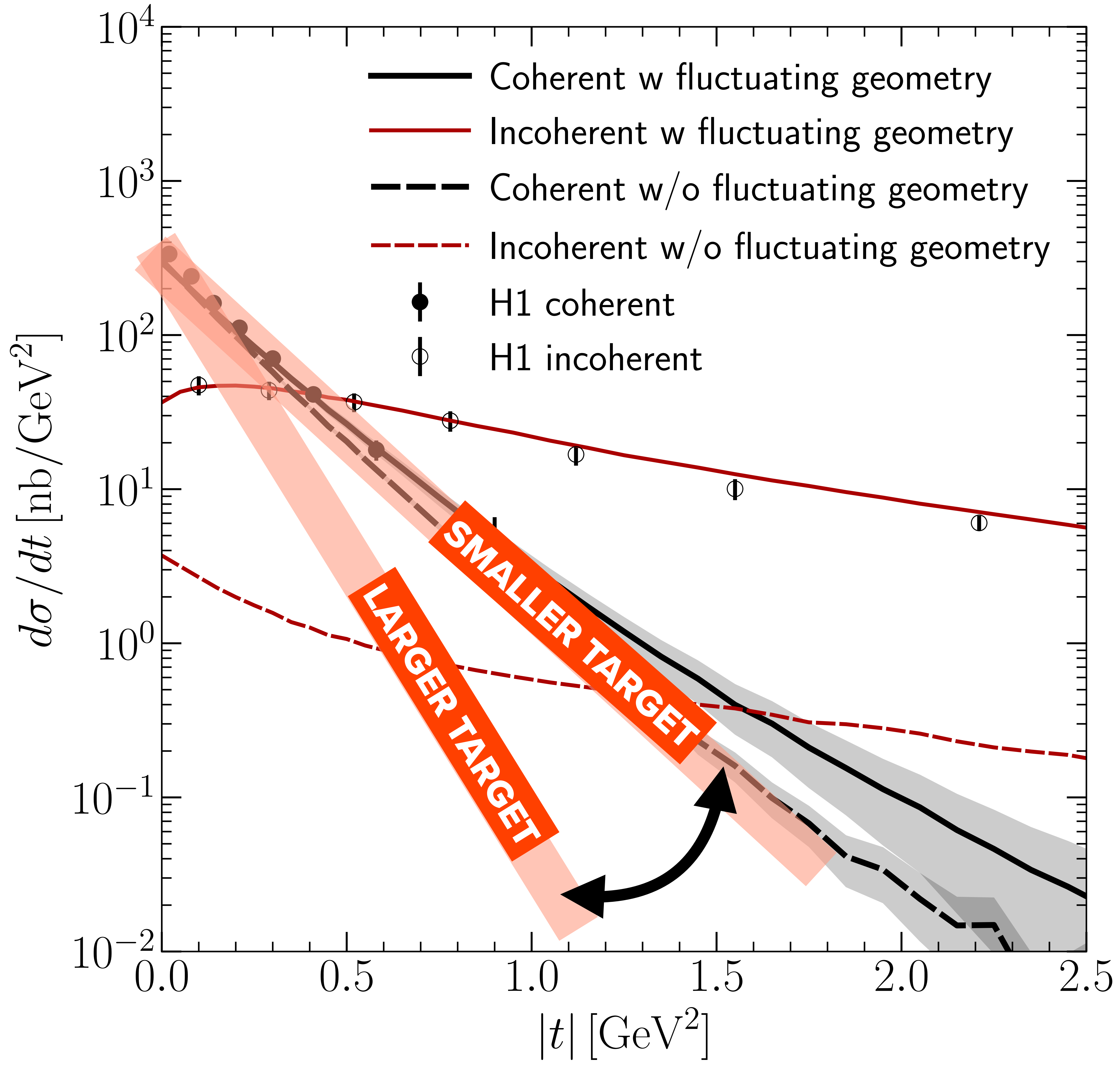






H1 Collaboration, Eur. Phys. J. C73 (2013) no. 6 2466

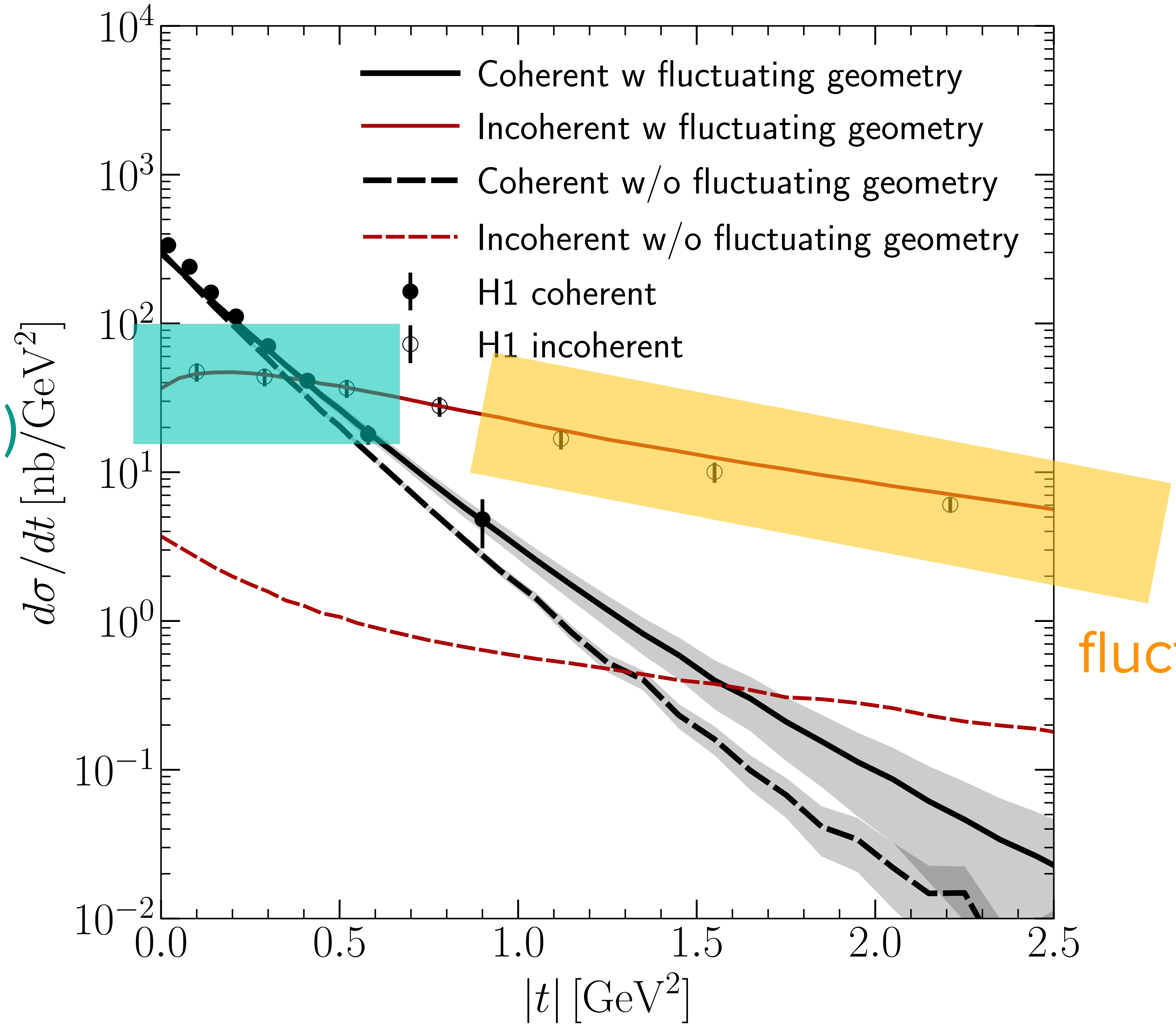
Information in the diffractive cross sections



H1 Collaboration, Eur. Phys. J. C73 (2013) no. 6 2466

Information in the diffractive cross sections

larger scale fluctuations (>0.2 fm)



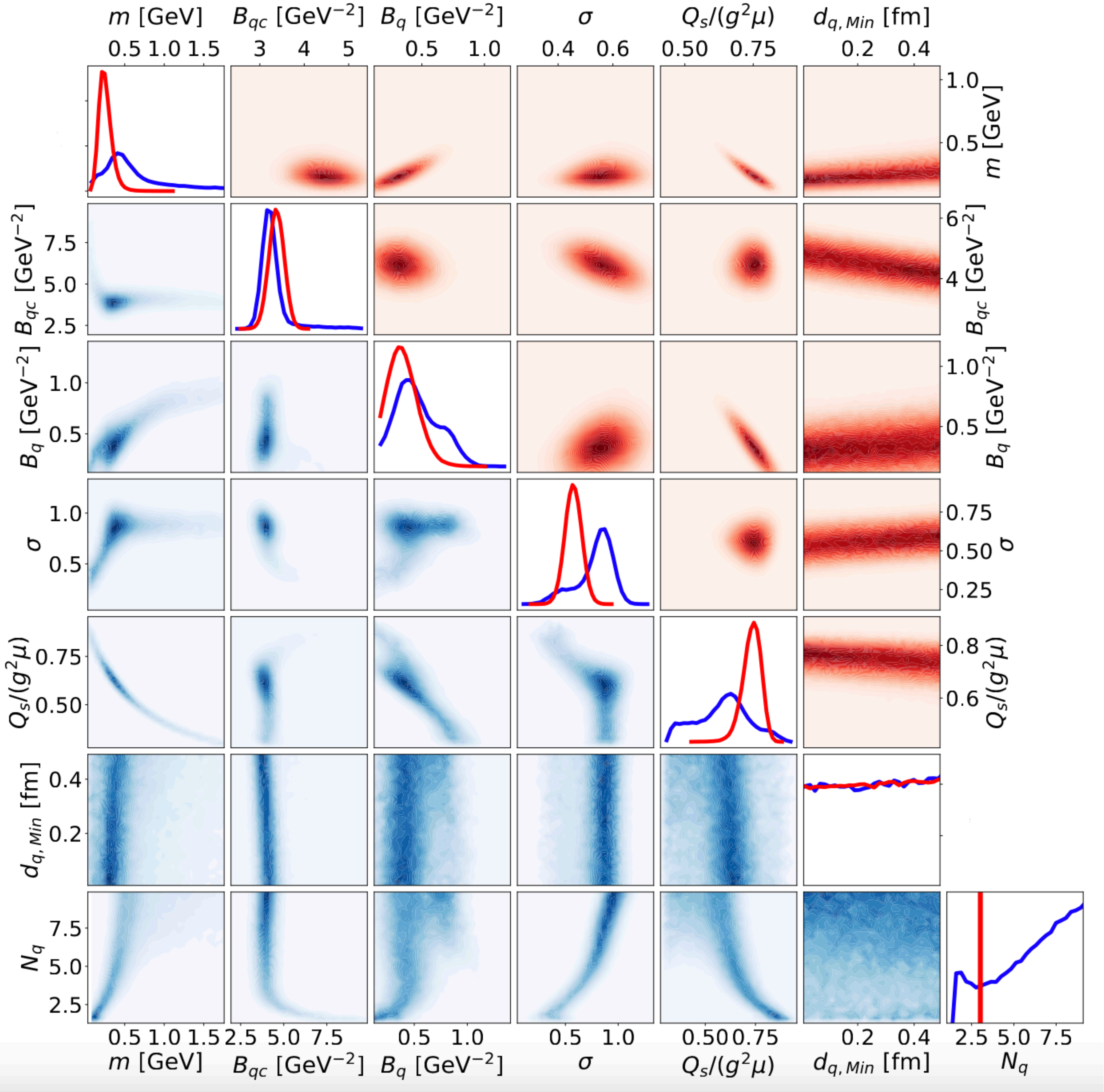
short scale fluctuations (<0.2 fm)

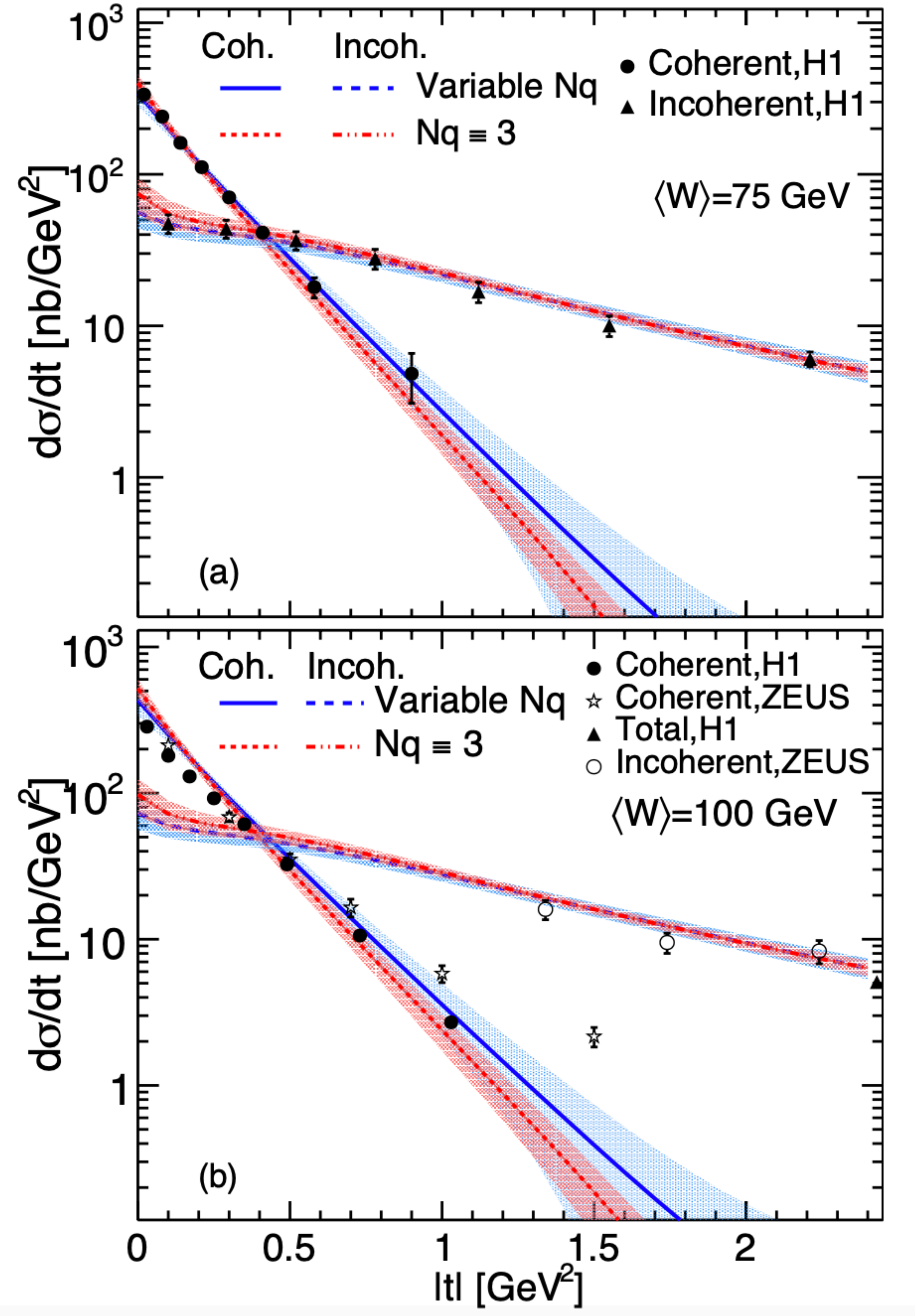
H1 Collaboration, Eur. Phys. J. C73 (2013) no. 6 2466

Extracting parameters using Bayesian inference

SEE WENBIN ZHAO'S TALK AT THIS INT PROGRAM

H. Mäntysaari, B. Schenke, C. Shen, W. Zhao, Phys.Lett.B 833 (2022) 137348

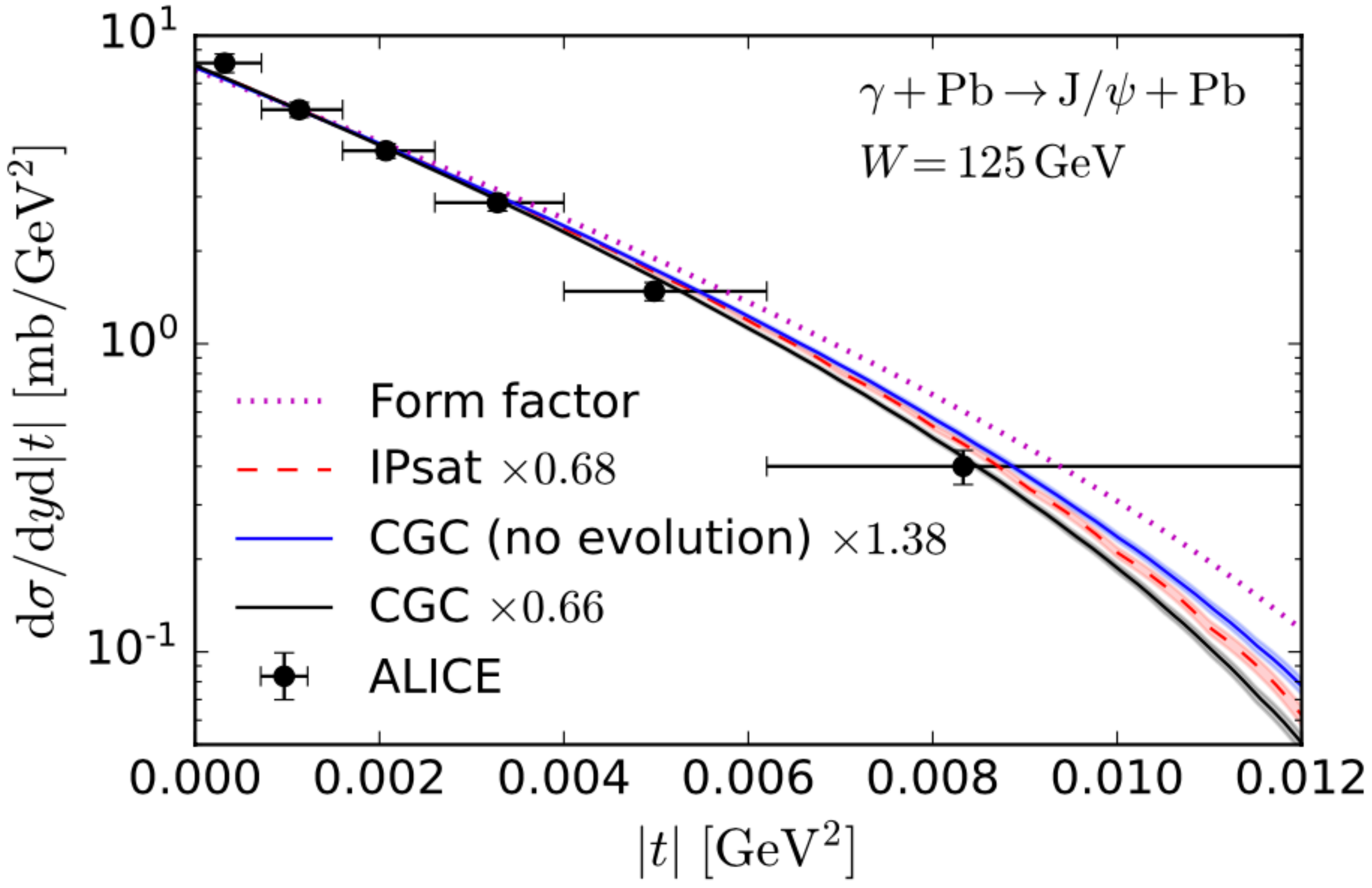


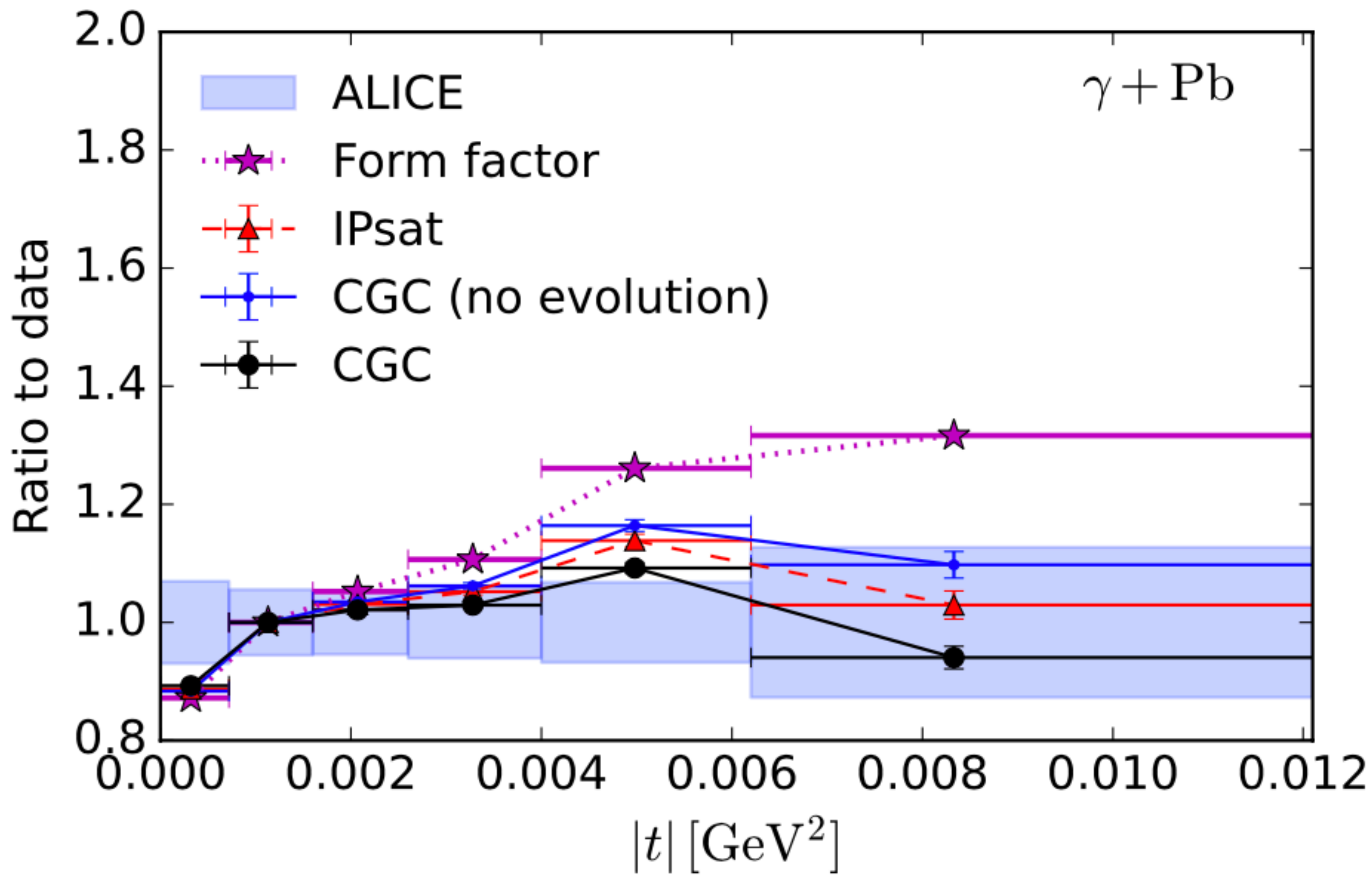


UPCs: γ+Pb measurement - Role of saturation effects

H. Mäntysaari, F. Salazar, B. Schenke, Phys.Rev.D 106 (2022) 7, 074019

Here, ALICE removed interference and photon k_T effects to get the γ+Pb cross section





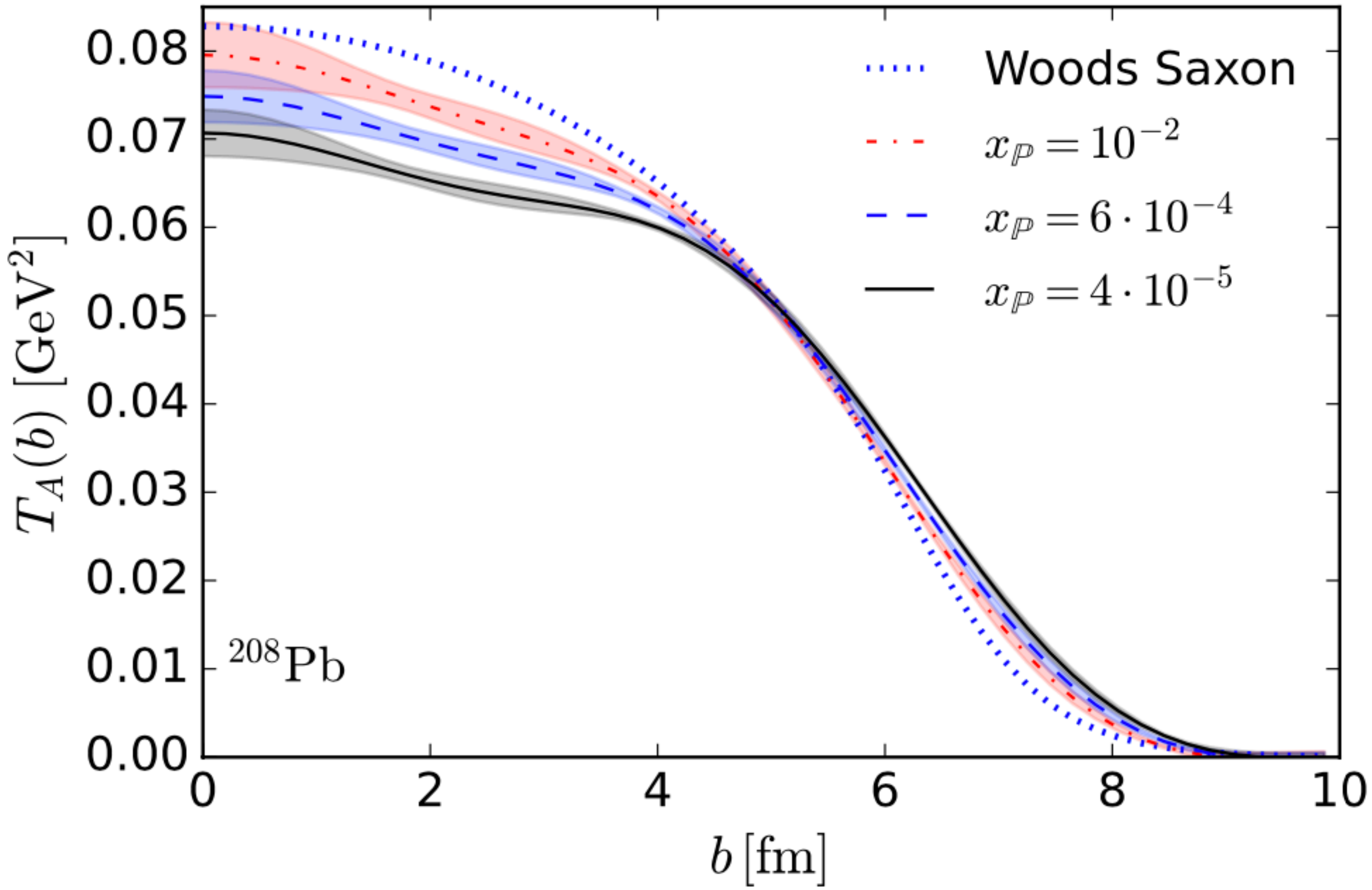
ALICE Collaboration, Phys.Lett.B 817 (2021) 136280

Saturation effects improve agreement with experimental data significantly

Saturation effects on nuclear geometry

H. Mäntysaari, F. Salazar, B. Schenke, Phys.Rev.D 106 (2022) 7, 074019

Fourier transform to coordinate space



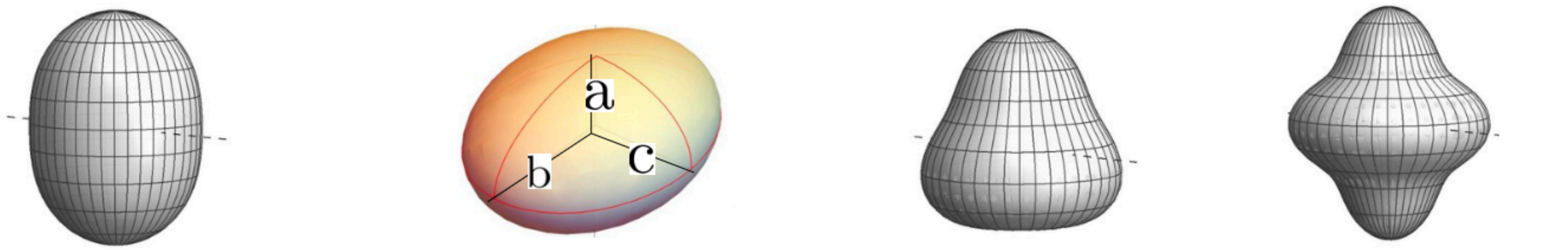
JIMWLK evolution leads to growth of the nucleus towards small x and depletion near the center (normalized so $\int d^2b T_A(b) = 208$)

Effects of deformation on diffractive cross sections

H. Mäntysaari, B. Schenke, C. Shen, W. Zhao, in progress

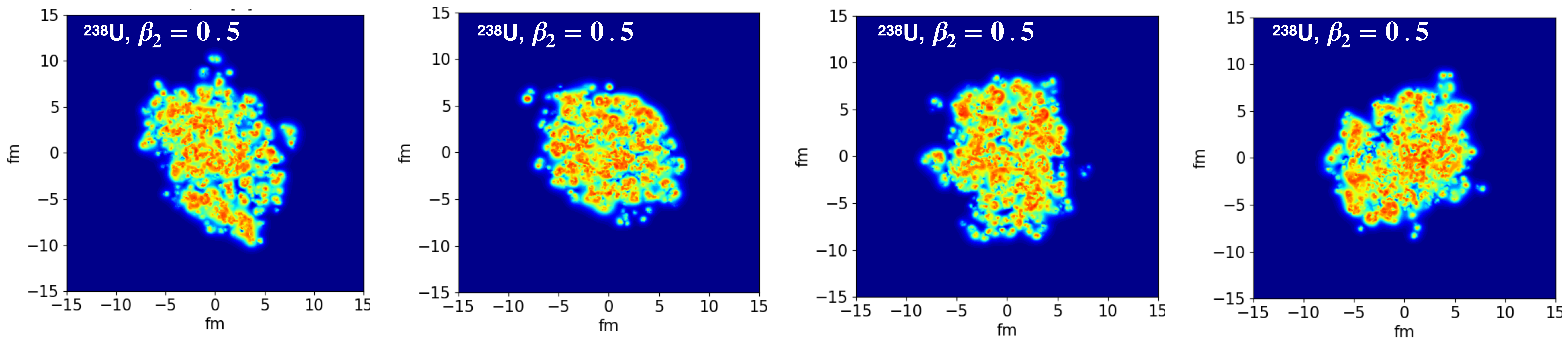
Implement deformation in the Woods-Saxon distribution:

$$\rho(r,\Theta,\Phi) \propto \frac{1}{1+\exp\left(\left[r-R(\Theta,\Phi)\right]/a\right)}\;, \quad R(\Theta,\Phi) = R_0\left[1+\beta_2\Big(\cos\gamma Y_{20}(\Theta)+\sin\gamma Y_{22}(\Theta,\Phi)\Big)+\beta_3 Y_{30}(\Theta)+\beta_4 Y_{40}(\Theta)\right]$$



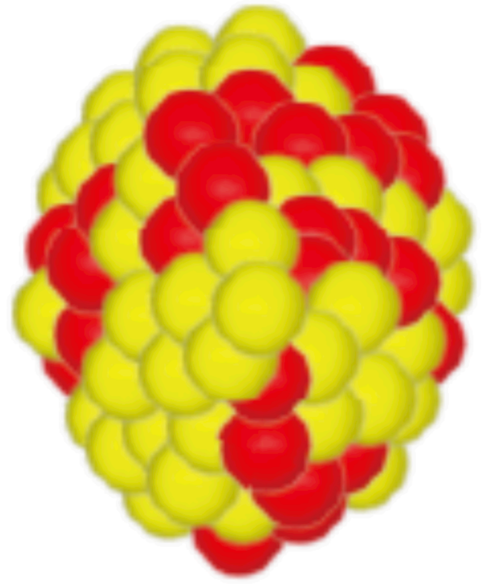
from G. Giacalone

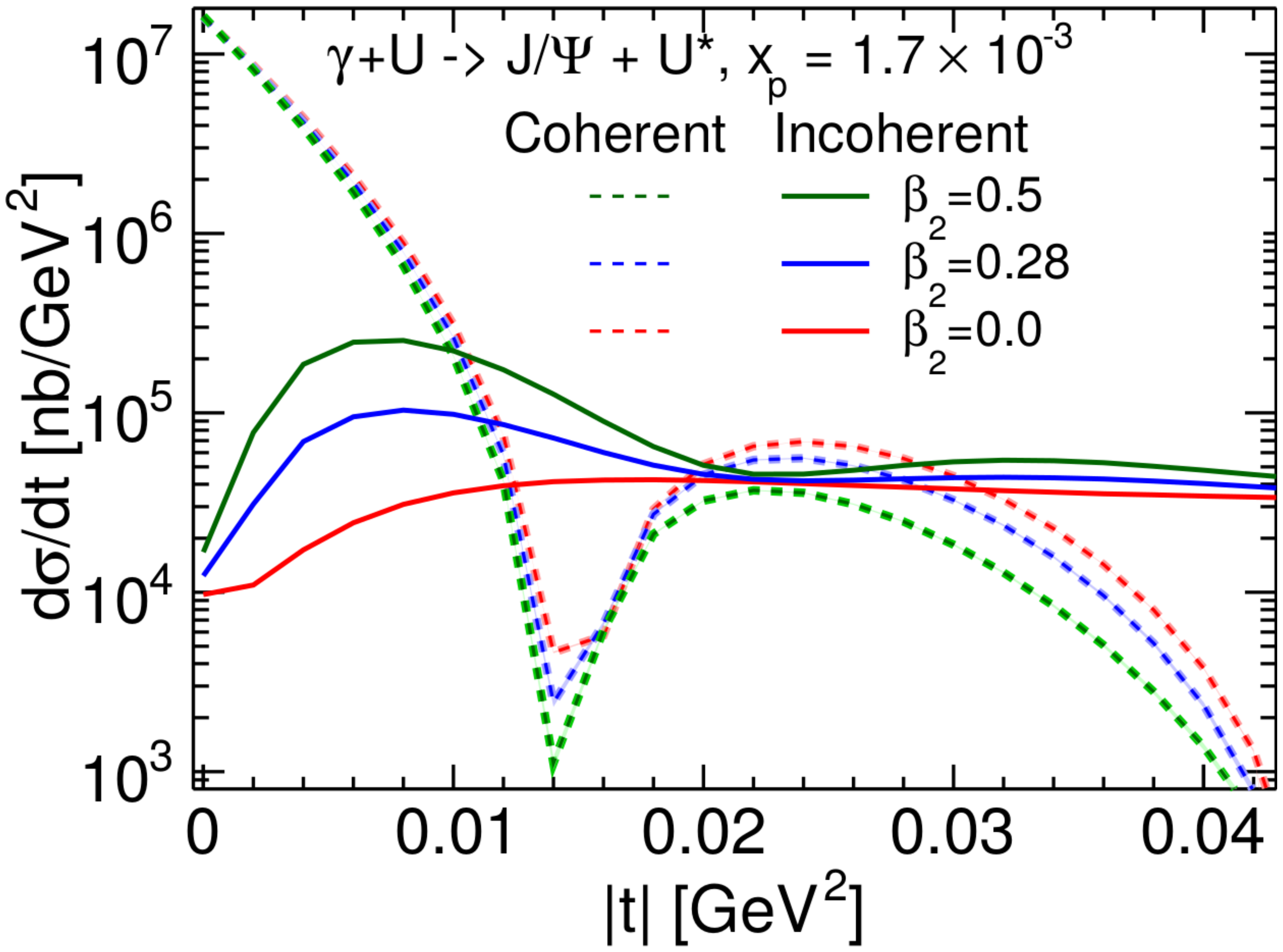
Deformed nuclei exhibit larger fluctuation in the transverse projection:



Effects of deformation on diffractive cross sections: Uranium

H. Mäntysaari, B. Schenke, C. Shen, W. Zhao, in progress



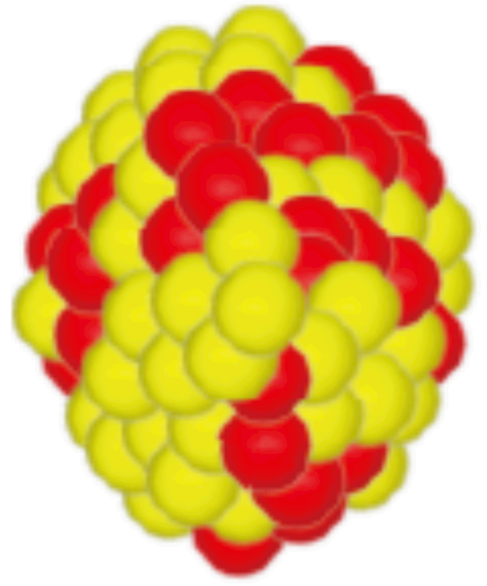


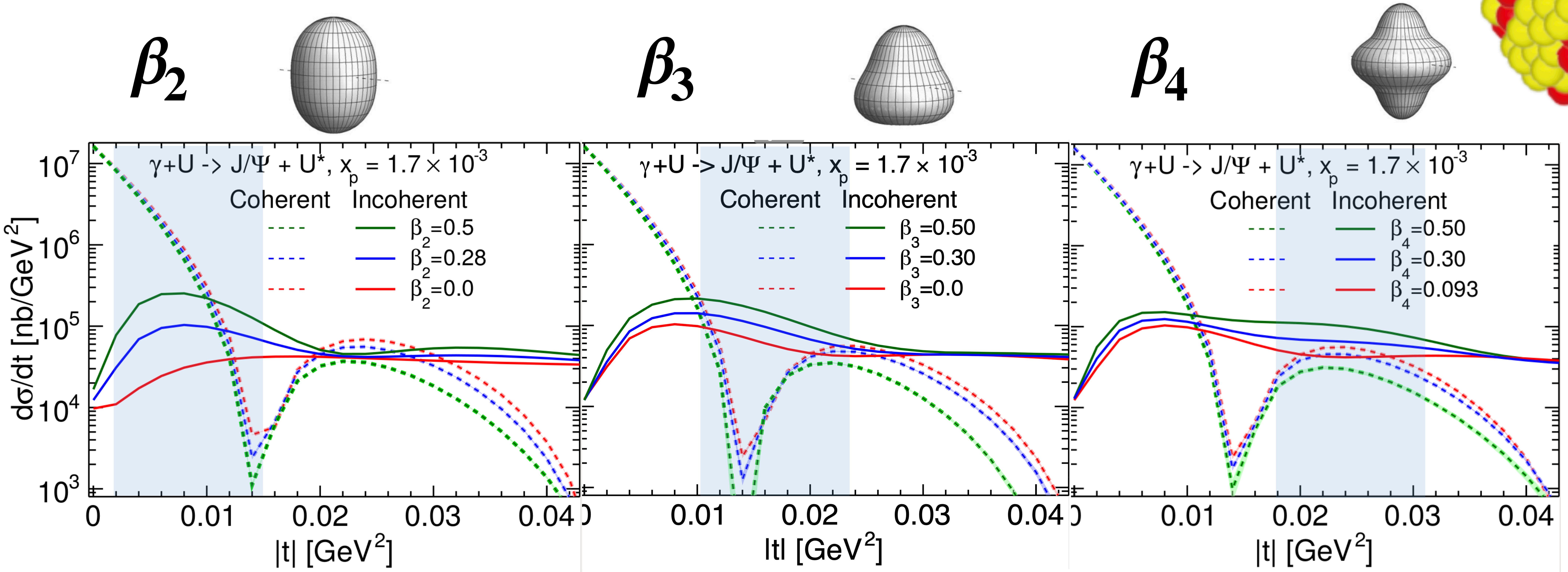
Deformation of the nucleus affects incoherent cross section at small $|t|$ (large length scales)

This observable provides direct information on the small x structure

Effects of deformation on diffractive cross sections: Uranium

H. Mäntysaari, B. Schenke, C. Shen, W. Zhao, in progress

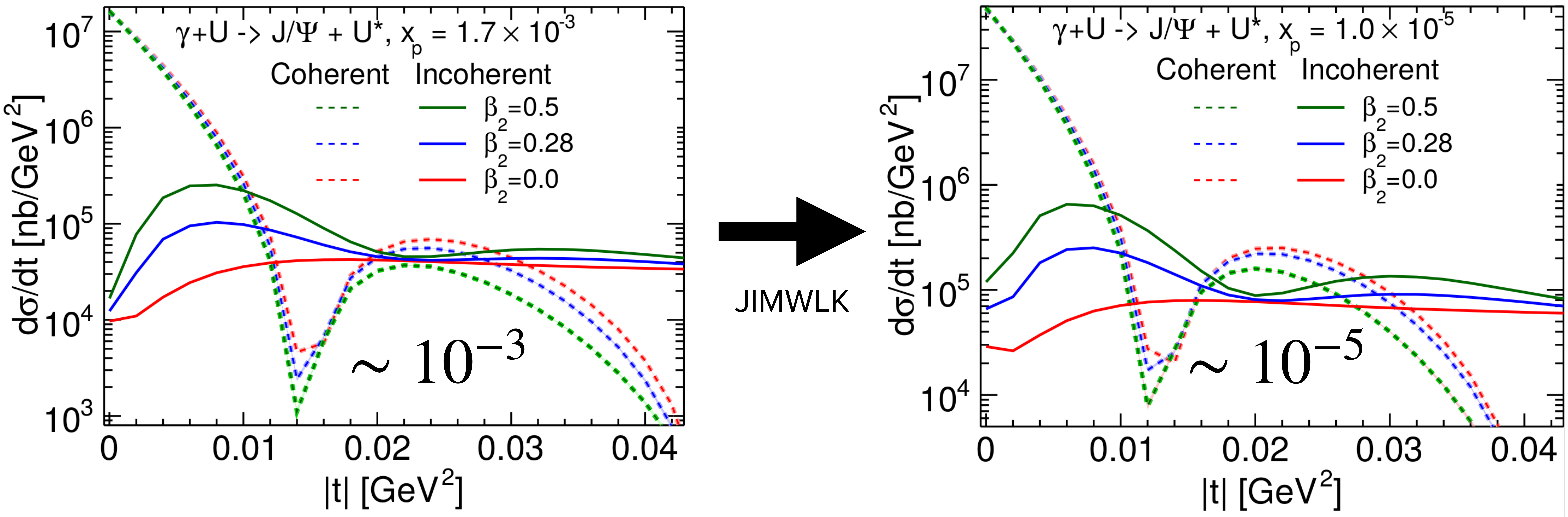




- β_2 , β_3 and β_4 modify fluctuations at different length scales: Change incoherent cross section in different |t| regions
- Different values of deformation do not affect the location of the first minimum of the coherent cross sections (average size remains the same)

Towards smaller *x*: Do deformation effects survive?

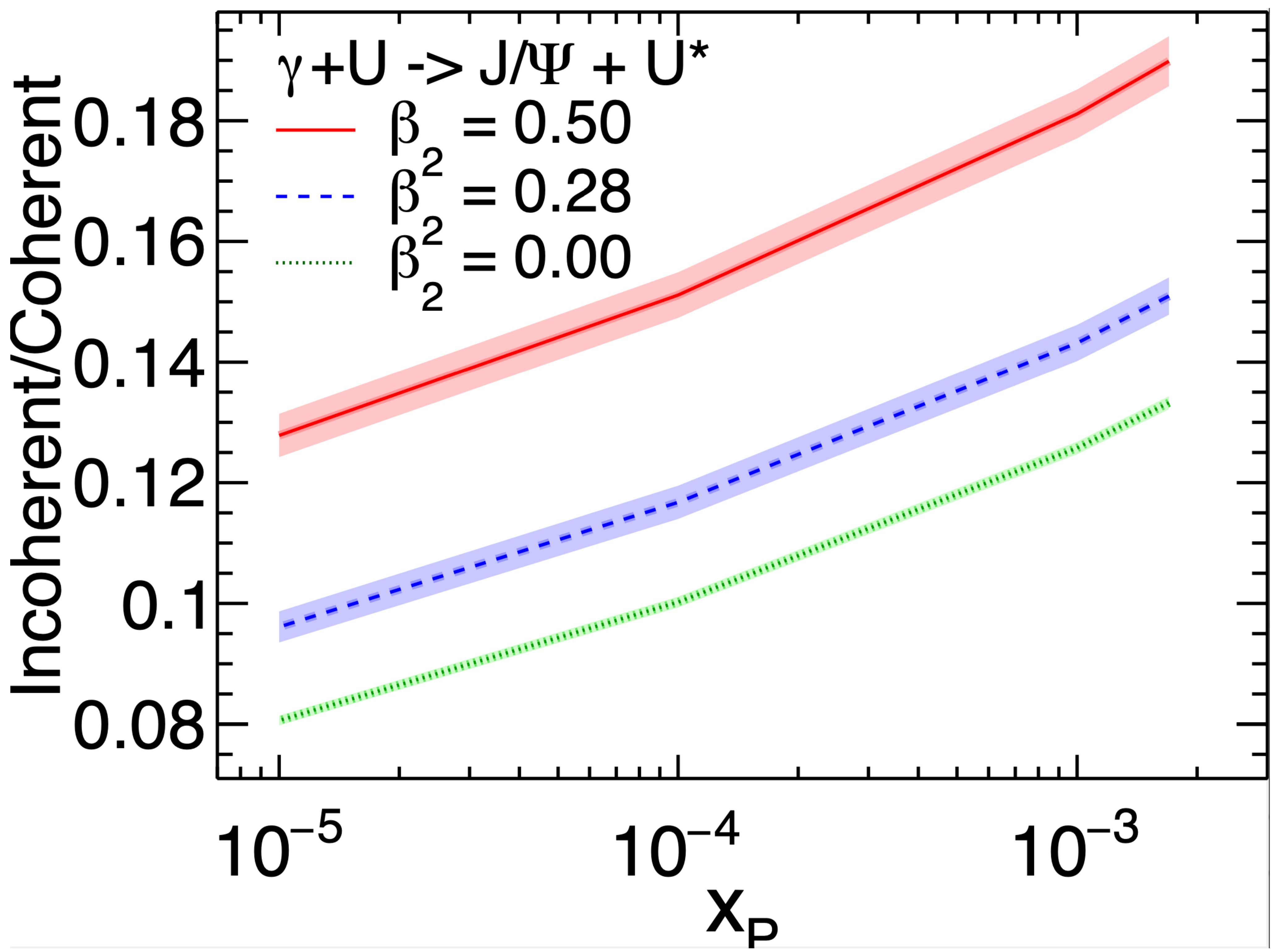
H. Mäntysaari, B. Schenke, C. Shen, W. Zhao, in progress

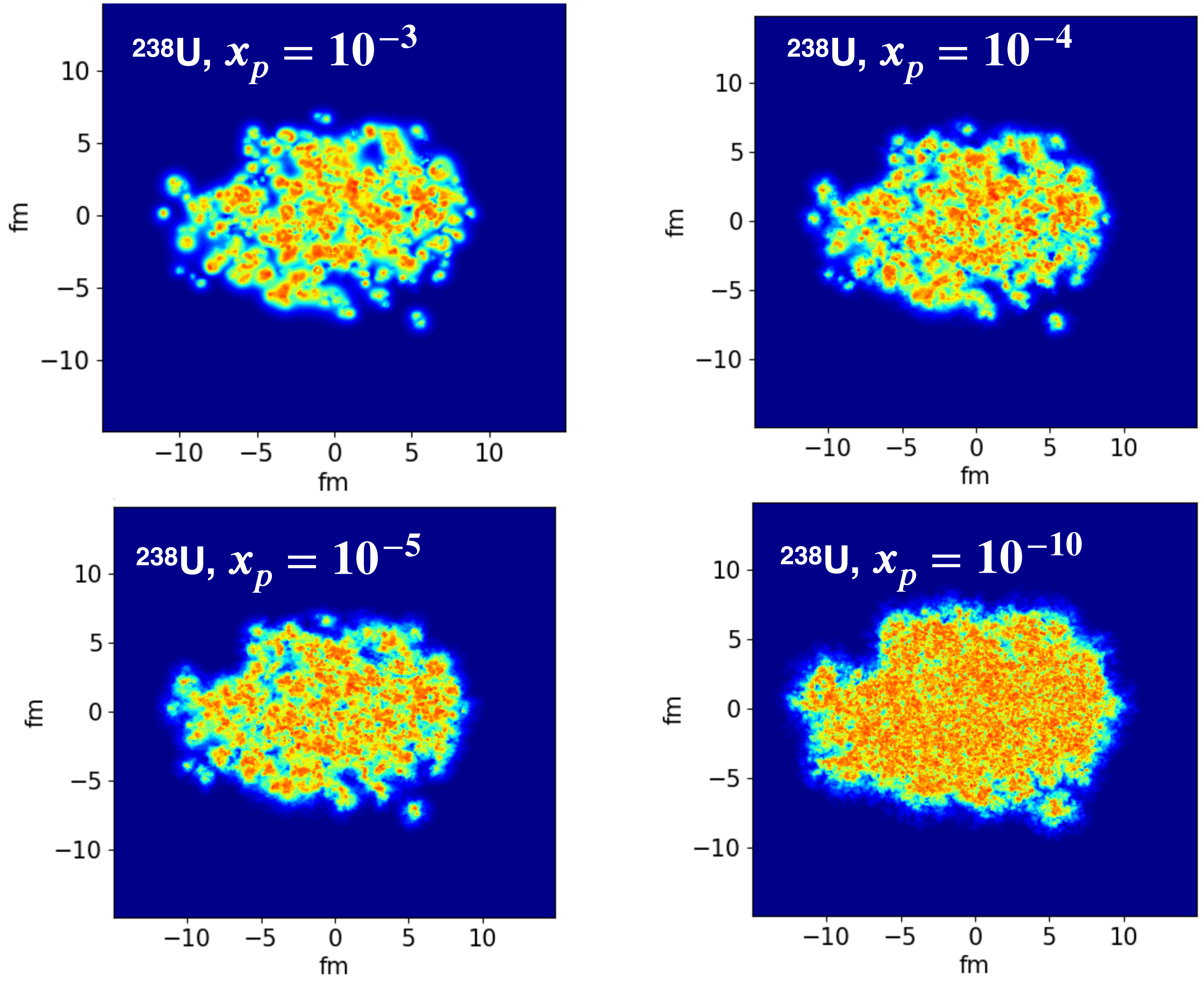


Some changes in the cross section, but deformation effects survive

Towards smaller *x*: Incoherent / coherent ratio

H. Mäntysaari, B. Schenke, C. Shen, W. Zhao, in progress



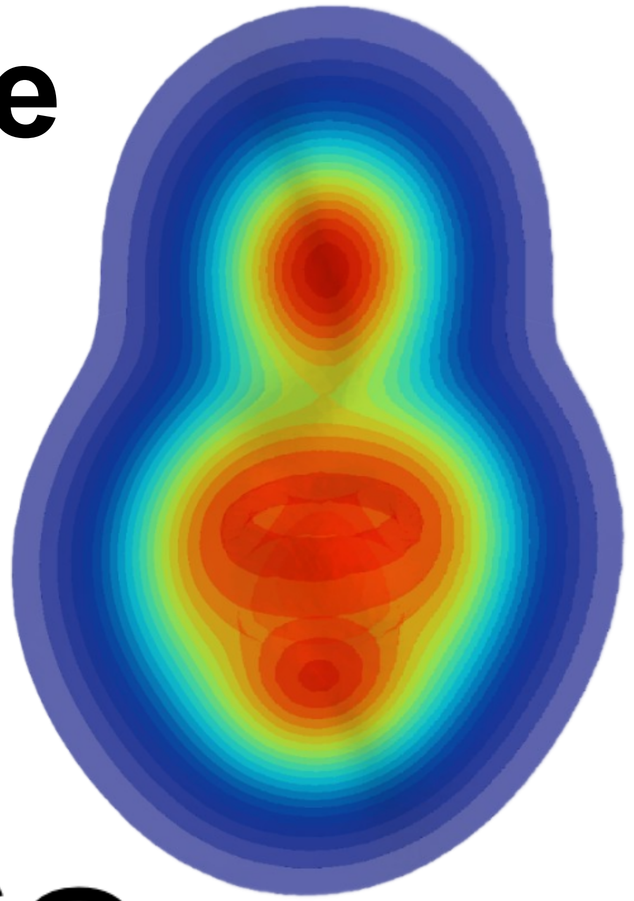


- Both cross sections grow for decreasing *x*
- Because fluctuations are reduced, incoherent/coherent ratio decreases
- Effects of deformation not noticeably reduced

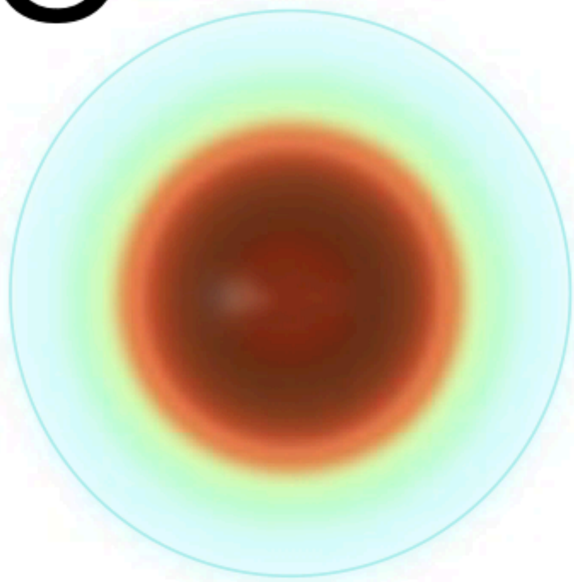
Comparing Neon and oxygen

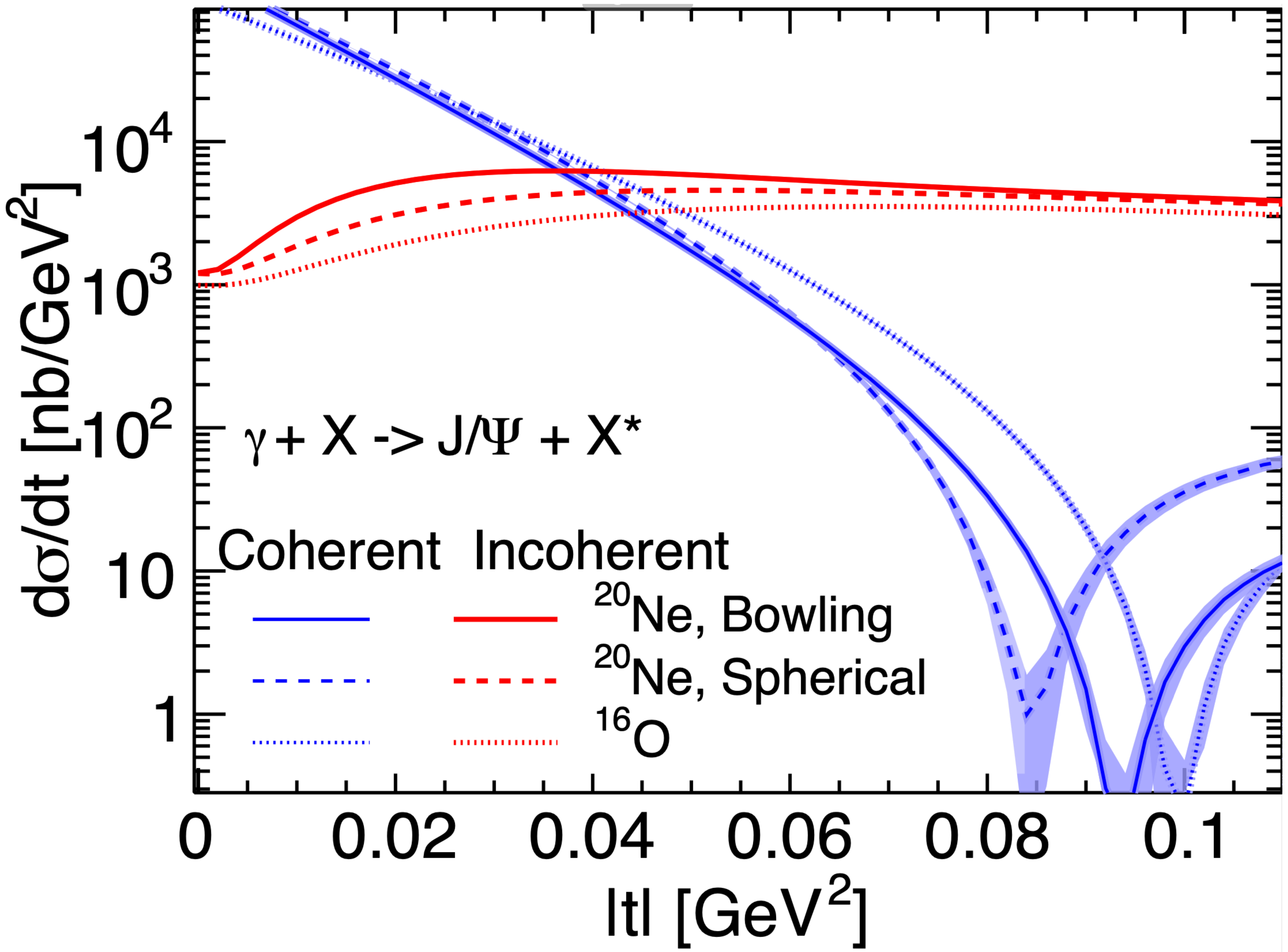
H. Mäntysaari, B. Schenke, C. Shen, W. Zhao, in progress

^{20}Ne



^{16}O

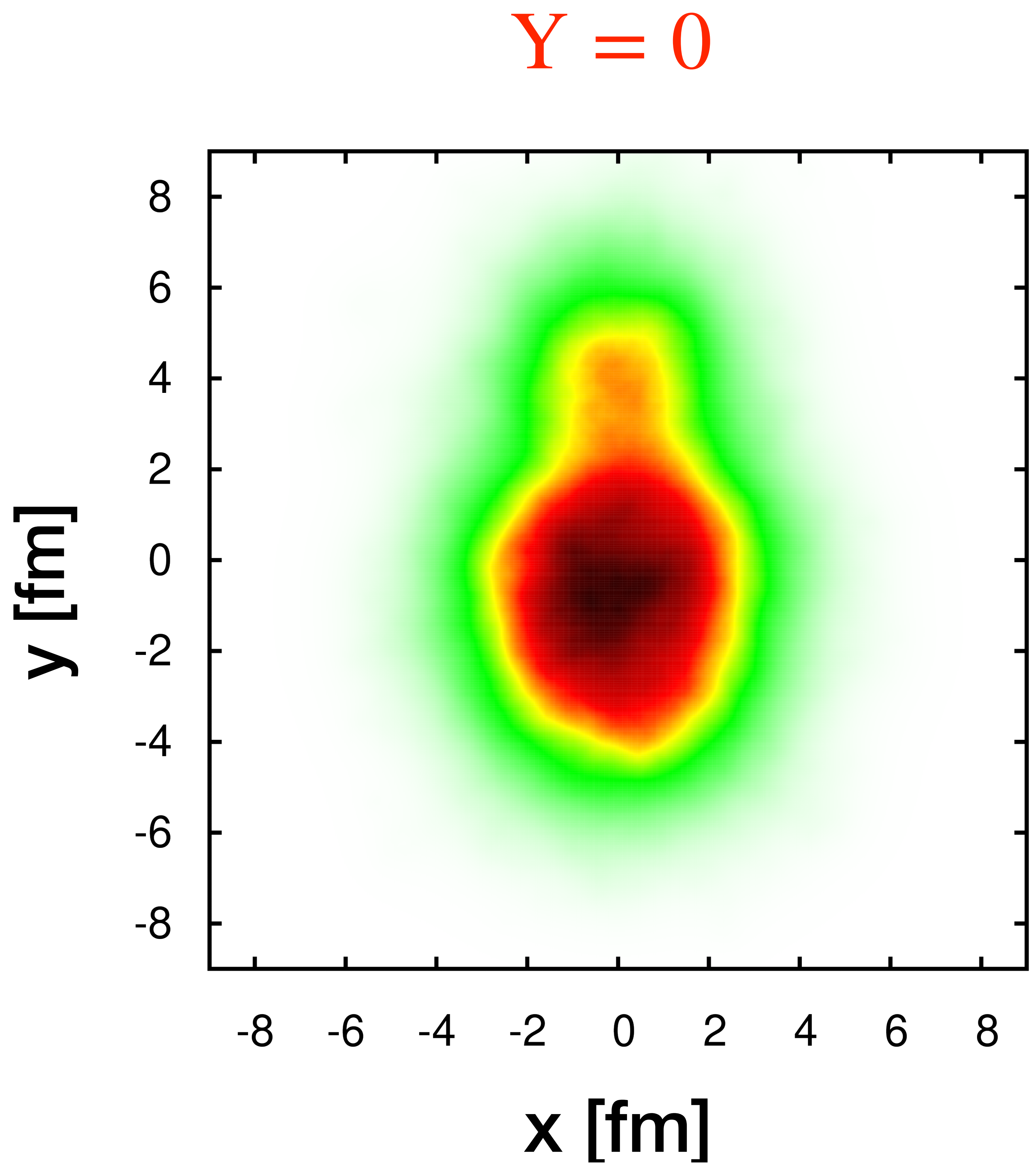


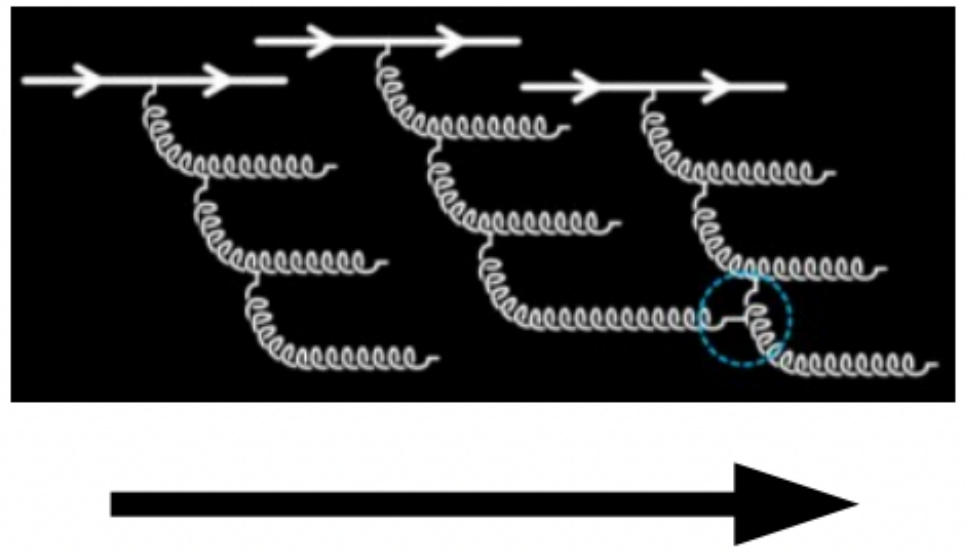


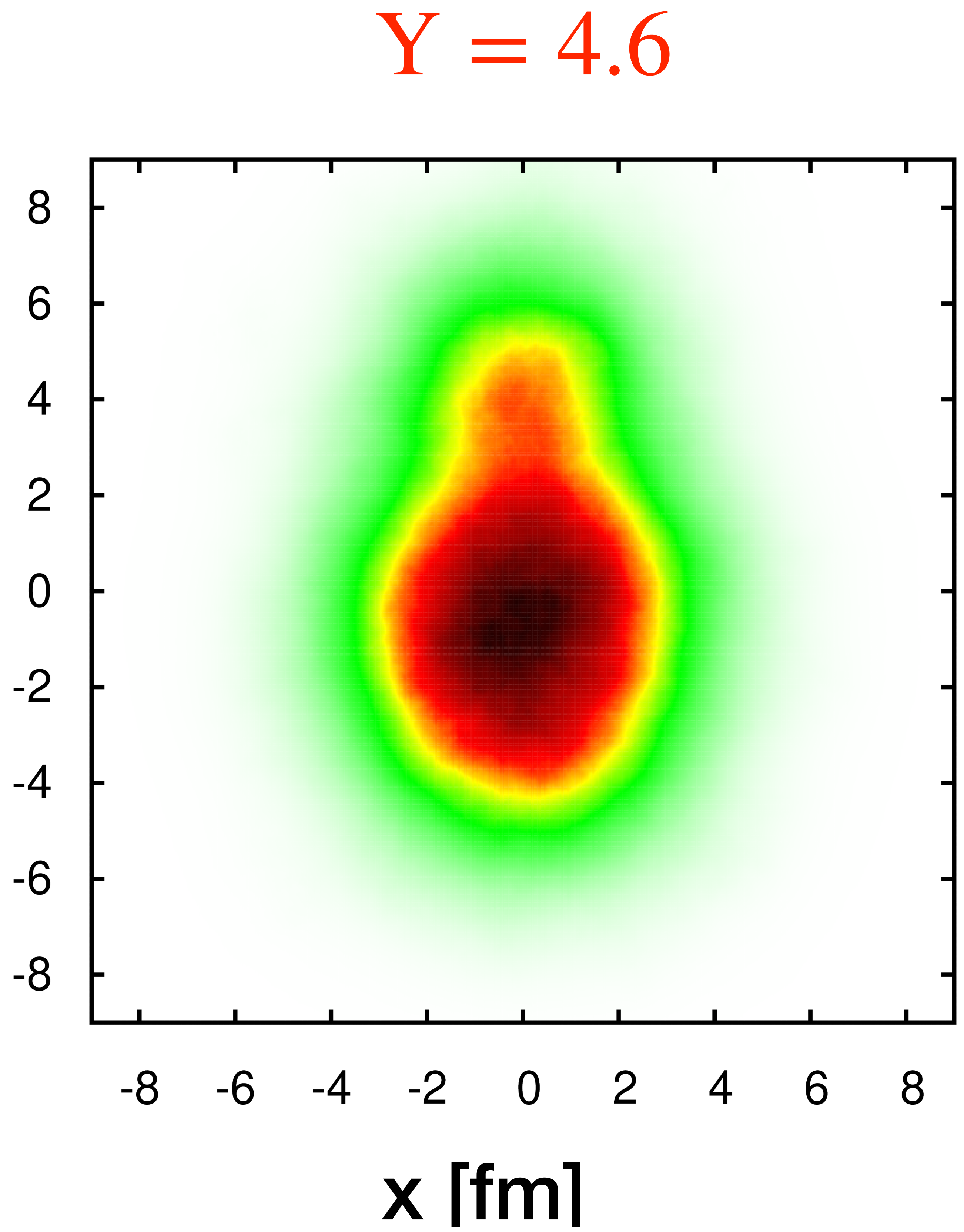
- Incoherent cross section at small |t| captures the deformation of ^{20}Ne
- Significant difference between ^{20}Ne and ^{16}O diffractive cross sections

Neon - JIMWLK evolution

G. Giacalone, B. Schenke, S. Schlichting, P. Singh, in progress





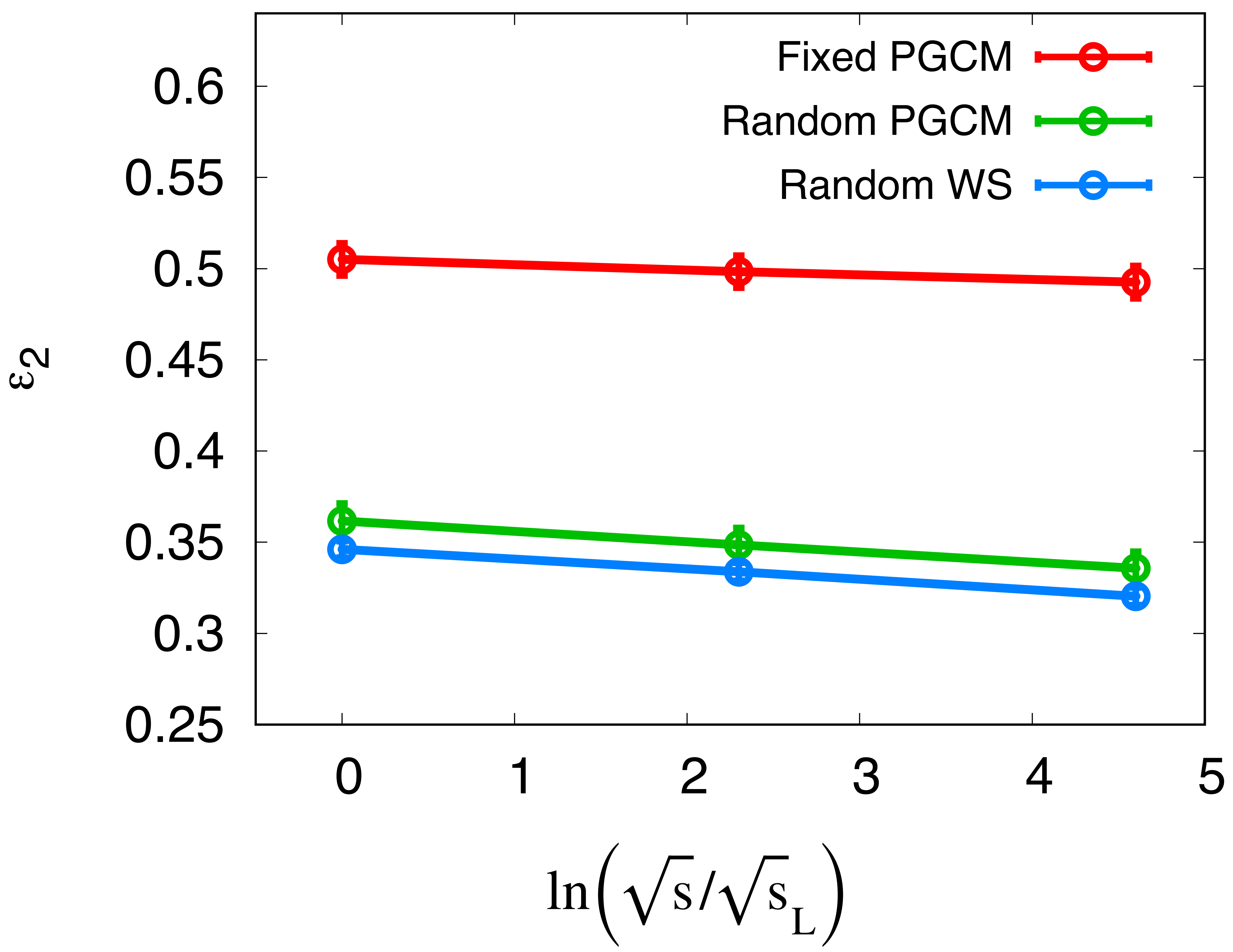


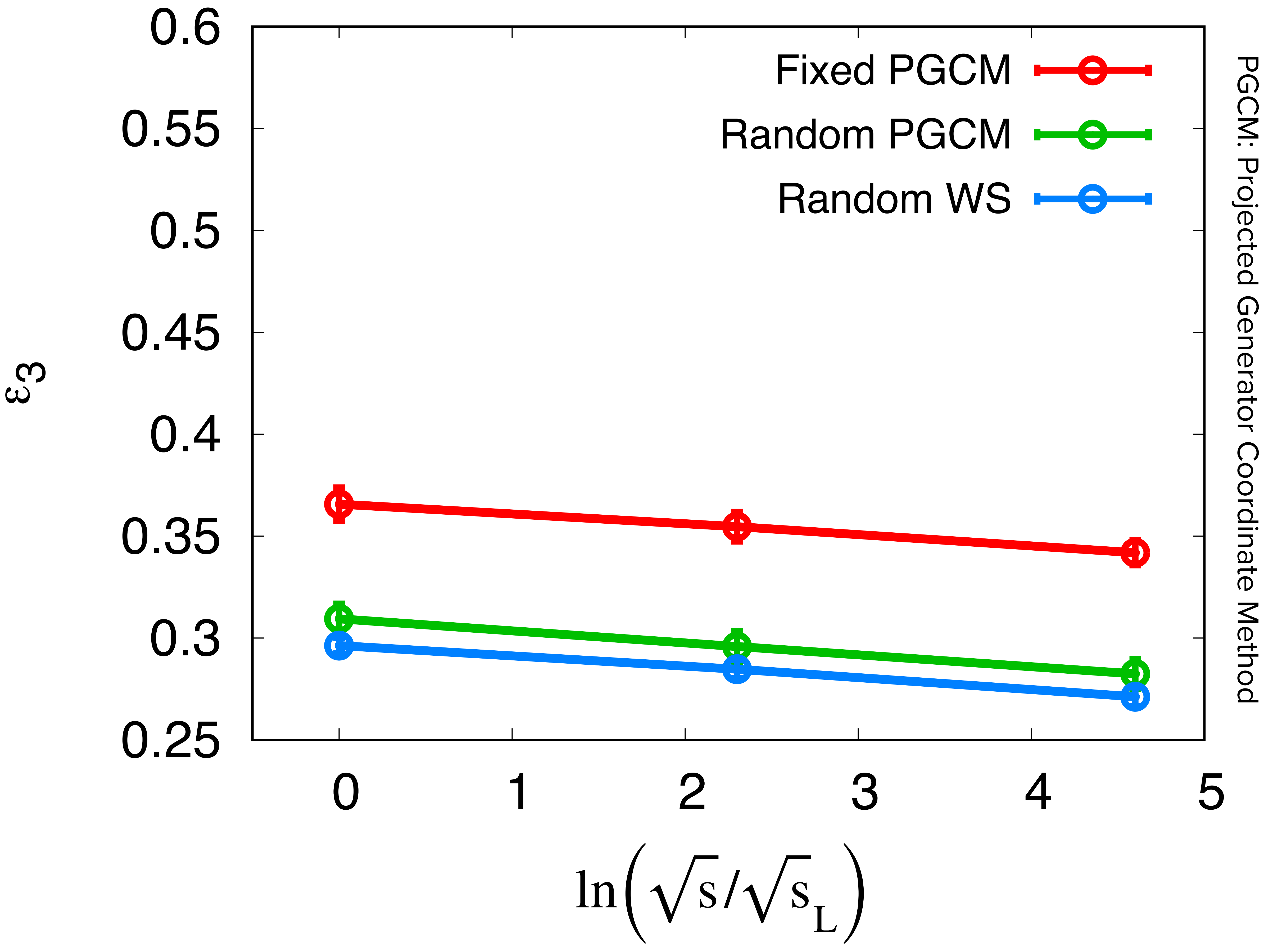
- Small-x evolution does not melt the bowling pin shape

Neon+Neon collisions - JIMWLK evolution

G. Giacalone, B. Schenke, S. Schlichting, P. Singh, in progress

- After the collision at different energies (x), measure the spatial eccentricities

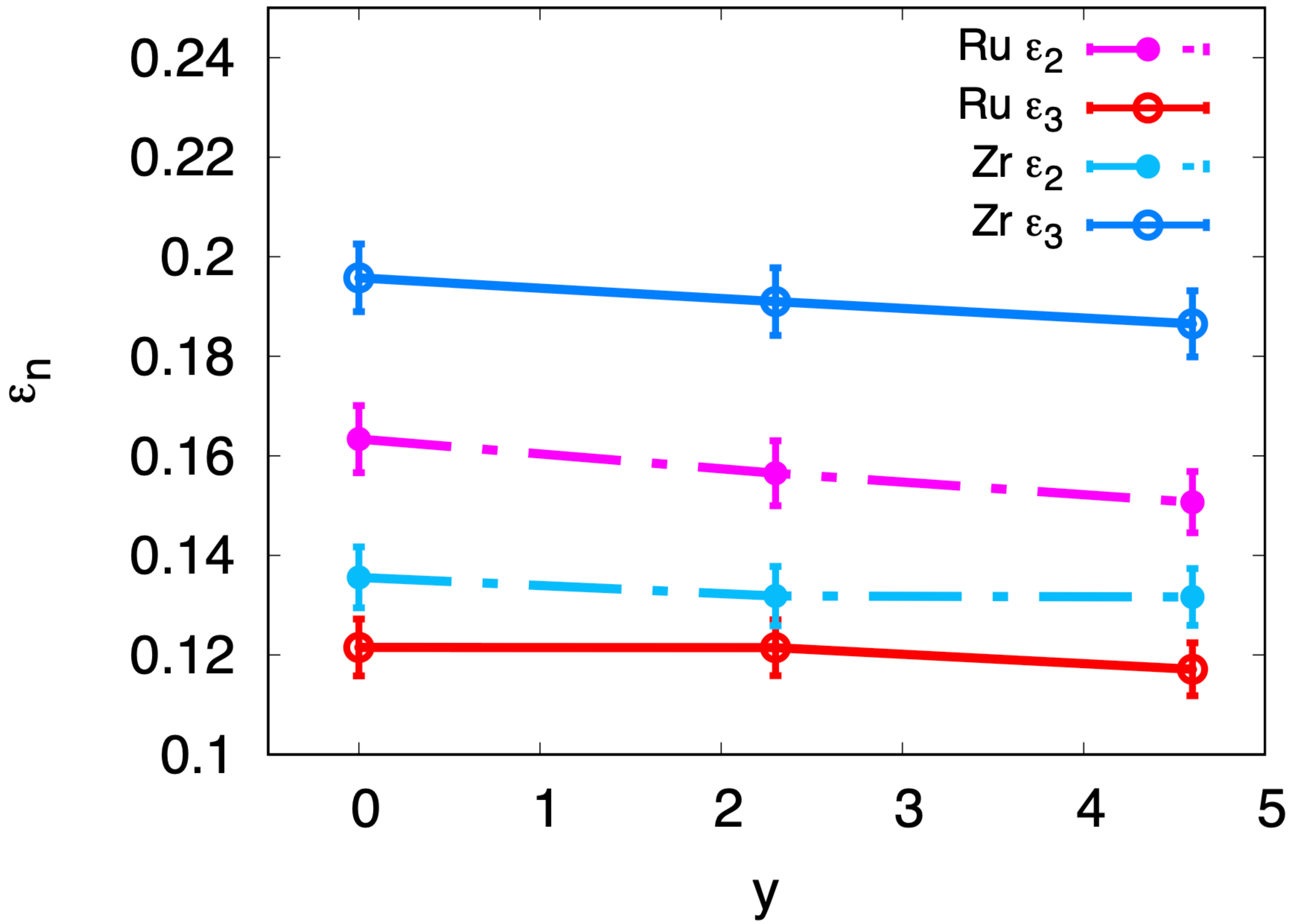


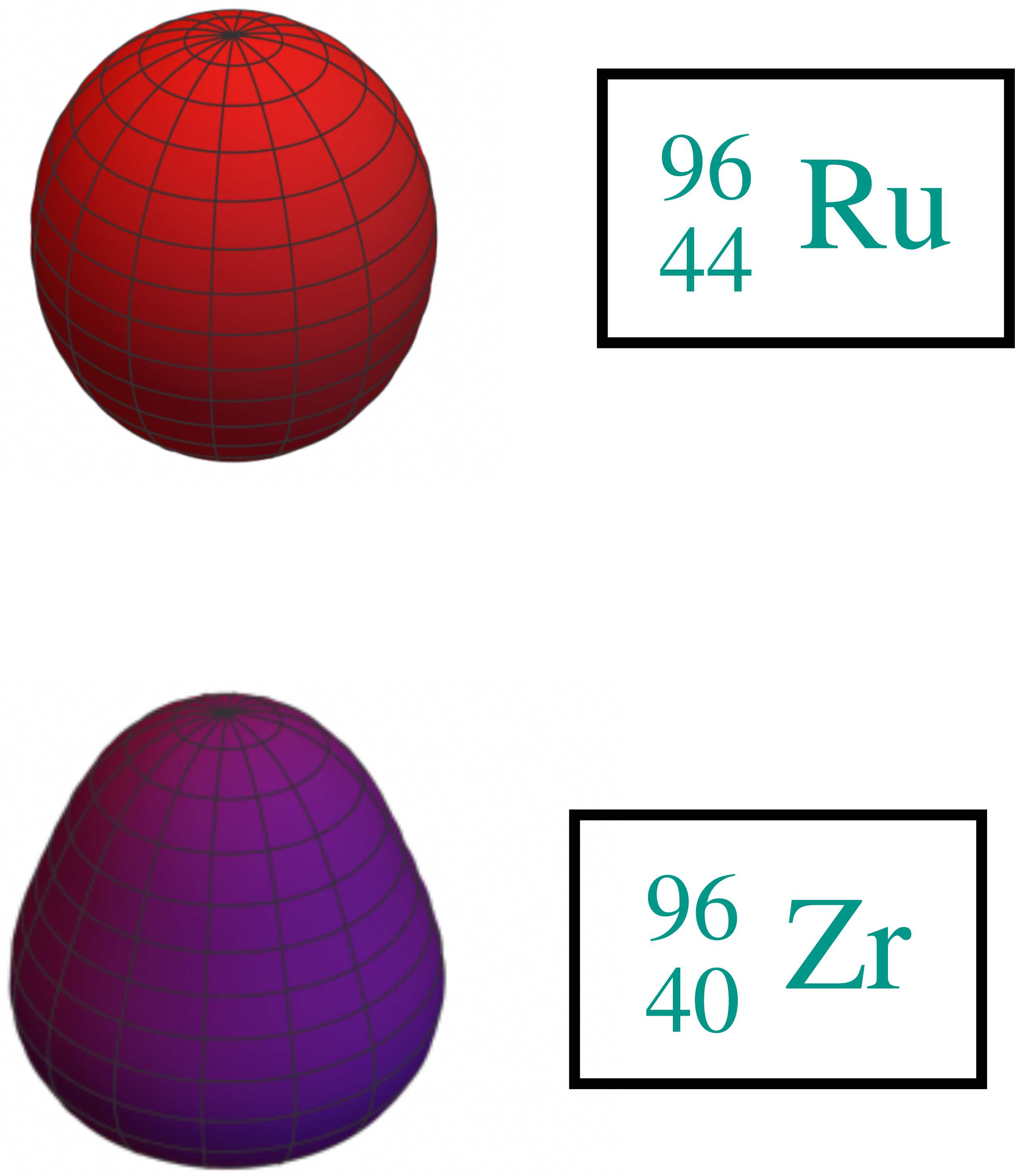


- Expected reduction - smoother distributions, but no large change

Isobar shapes - JIMWLK evolution

G. Giacalone, B. Schenke, S. Schlichting, P. Singh, in progress





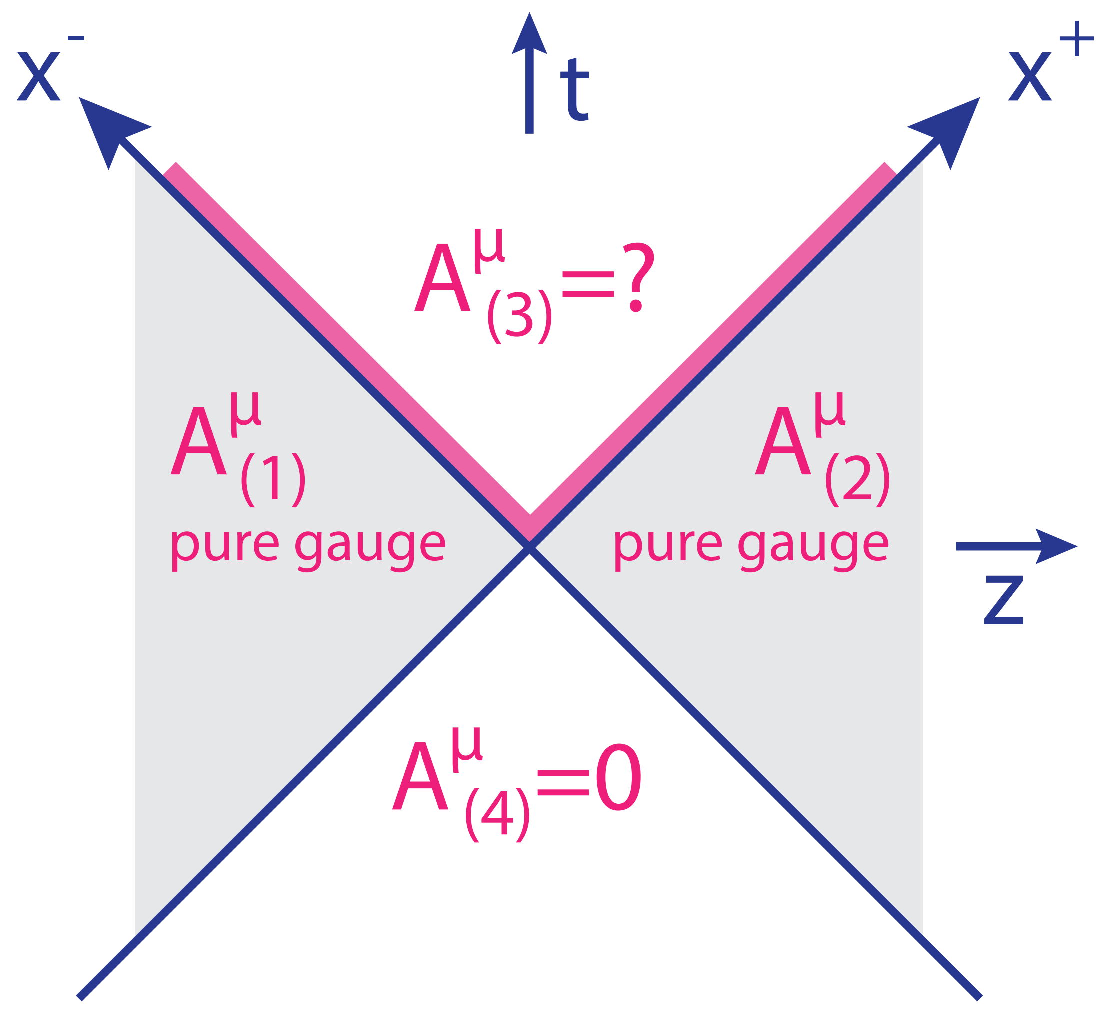
SUMMARY

- The x evolution leads to a growing gluonic radius as well as saturation effects in the center of the nucleus
- The x evolution does not significantly modify the nuclear deformation for large nuclei
- Deformation of a nuclear target affects the incoherent diffractive cross section for VM production - needs to be taken into account
- Differences between ^{20}Ne and ^{16}O also appear in the incoherent cross section
- Caveats: Growth depends on IR regulator *m*. Evolution speed depends on α_s and *m.* Evolution between $x \sim 0.1$ to $x \sim 0.01$ not included

    - Please see talks by Pragya Singh and Wenbin Zhao for more details

BACKUP

Heavy ion collision



Compute gluon fields after the collision using light cone gauge:
$A^+ = 0$ for a right moving nucleus, $A^- = 0$ for a left moving nucleus

gauge transformation: $A_\mu(x) \to V(x)\left(A_\mu(x) - \frac{i}{g}\partial_\mu\right)V^\dagger(x)$

using our Wilson lines $V^\dagger(x^-, \mathbf{x}_\perp) = \mathscr{P}\exp\left(-ig\int_{-\infty}^{x^-} dz^- A^+(z^-, \mathbf{x}_\perp)\right)$ (for the right moving nucleus)

Then, the gauge fields read (choosing $A^\mu = 0$ for the quadrant for $x^- < 0$ and $x^+ < 0$)

$$A^i(x) = \theta(x^+)\theta(x^-)\alpha^i(\tau, \mathbf{x}_\perp) + \theta(x^-)\theta(-x^+)\alpha^i_P(\mathbf{x}_\perp) + \theta(x^+)\theta(-x^-)\alpha^i_T(\mathbf{x}_\perp)$$

$$A^\eta(x) = \theta(x^+)\theta(x^-)\alpha^\eta(\tau, \mathbf{x}_\perp)$$

with $\alpha^i_P(\mathbf{x}_\perp) = \frac{1}{ig}V_P(\mathbf{x}_\perp)\partial^i V^\dagger_P(\mathbf{x}_\perp)$ and $\alpha^i_T(\mathbf{x}_\perp) = \frac{1}{ig}V_T(\mathbf{x}_\perp)\partial^i V^\dagger_T(\mathbf{x}_\perp)$

$A^\tau = 0$, because we chose Fock-Schwinger gauge $x^+A^- + x^-A^+ = 0$

Heavy ion collision

Plugging this ansatz

$$A^i(x) = \theta(x^+)\theta(x^-)\alpha^i(\tau, \mathbf{x}_\perp) + \theta(x^-)\theta(-x^+)\alpha_P^i(\mathbf{x}_\perp) + \theta(x^+)\theta(-x^-)\alpha_T^i(\mathbf{x}_\perp)$$

$$A^\eta(x) = \theta(x^+)\theta(x^-)\alpha^\eta(\tau, \mathbf{x}_\perp)$$

into YM equations leads to singular terms on the boundary from derivatives of θ-functions

Requiring that the singularities vanish leads to the solutions

$$\alpha^i = \alpha_P^i + \alpha_T^i \qquad \alpha^\eta = -\frac{ig}{2}\left[\alpha_{Pj}, \alpha_T^j\right] \qquad \begin{aligned} \partial_\tau \alpha^i &= 0 \\ \partial_\tau \alpha^\eta &= 0 \end{aligned}$$

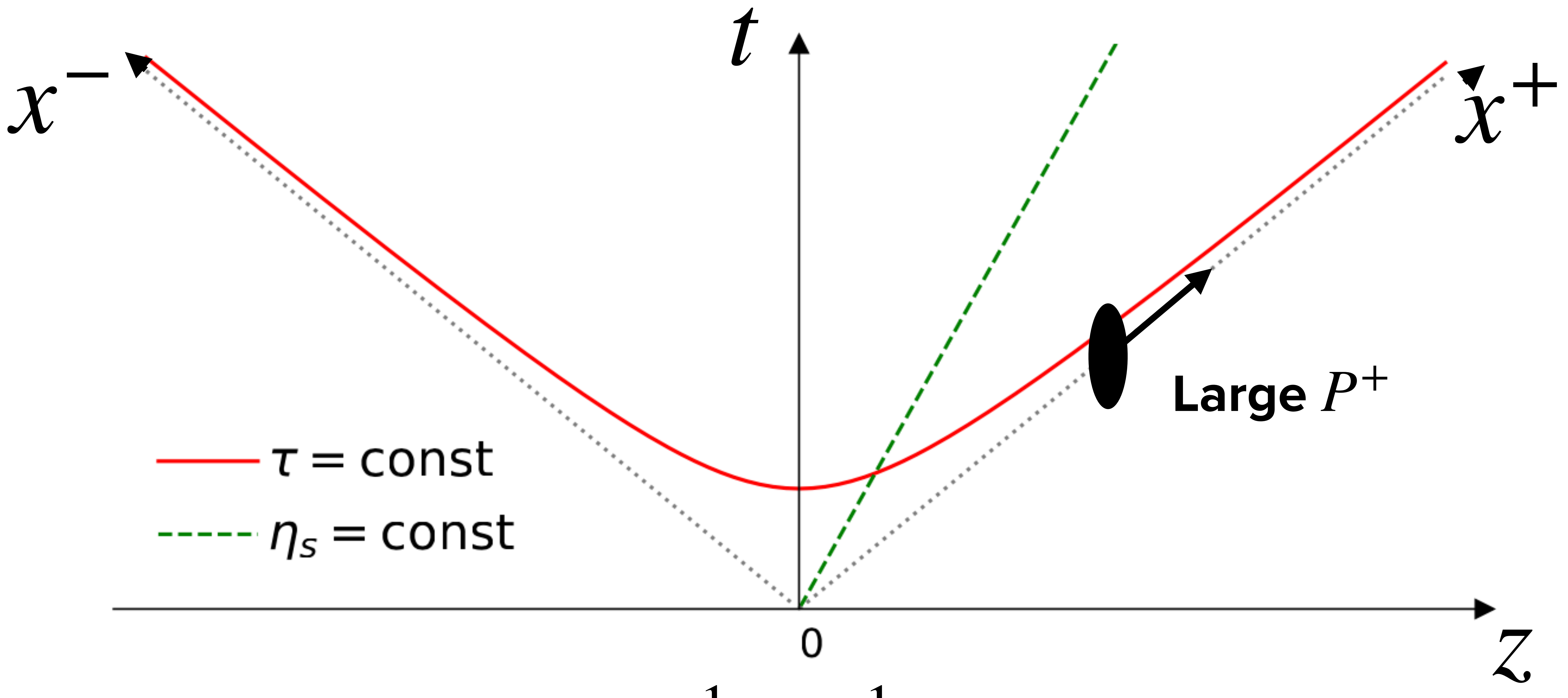
These are the gauge fields in the forward light cone.

We can compute $T^{\mu\nu}$ from it, providing an initial condition for hydrodynamics.

Geometry, fluctuations, …

- All the information on geometry and nucleon and sub-nucleon fluctuations is contained in the distribution of color charges $\rho^a_{P/T}(x^{\mp}, \mathbf{x}_\perp)$
- Typically, use the MV model, which gives $\langle \rho^a(\mathbf{b}_\perp)\rho^b(\mathbf{x}_\perp)\rangle = g^2\mu^2(x, \mathbf{b}_\perp)\delta^{ab}\delta^{(2)}(\mathbf{b}_\perp - \mathbf{x}_\perp)$
- The color charge distribution $g^2\mu(x, \mathbf{b}_\perp)$ depends on the longitudinal momentum fraction x and the transverse position $\mathbf{b}_\perp$. The latter needs to be modeled, the former can be modeled or obtained from e.g. JIMWLK evolution
- We factorize $\mu(x, \mathbf{b}_\perp) \sim T(\mathbf{b}_\perp)\mu(x)$ and constrain the impact parameter $\mathbf{b}_\perp$ dependence using input from a process sensitive to geometry, such as diffractive VM production
- The cross section for that process can be expressed with the Wilson lines of the target The same quantities we have used to initialize the heavy ion collision

Color sources

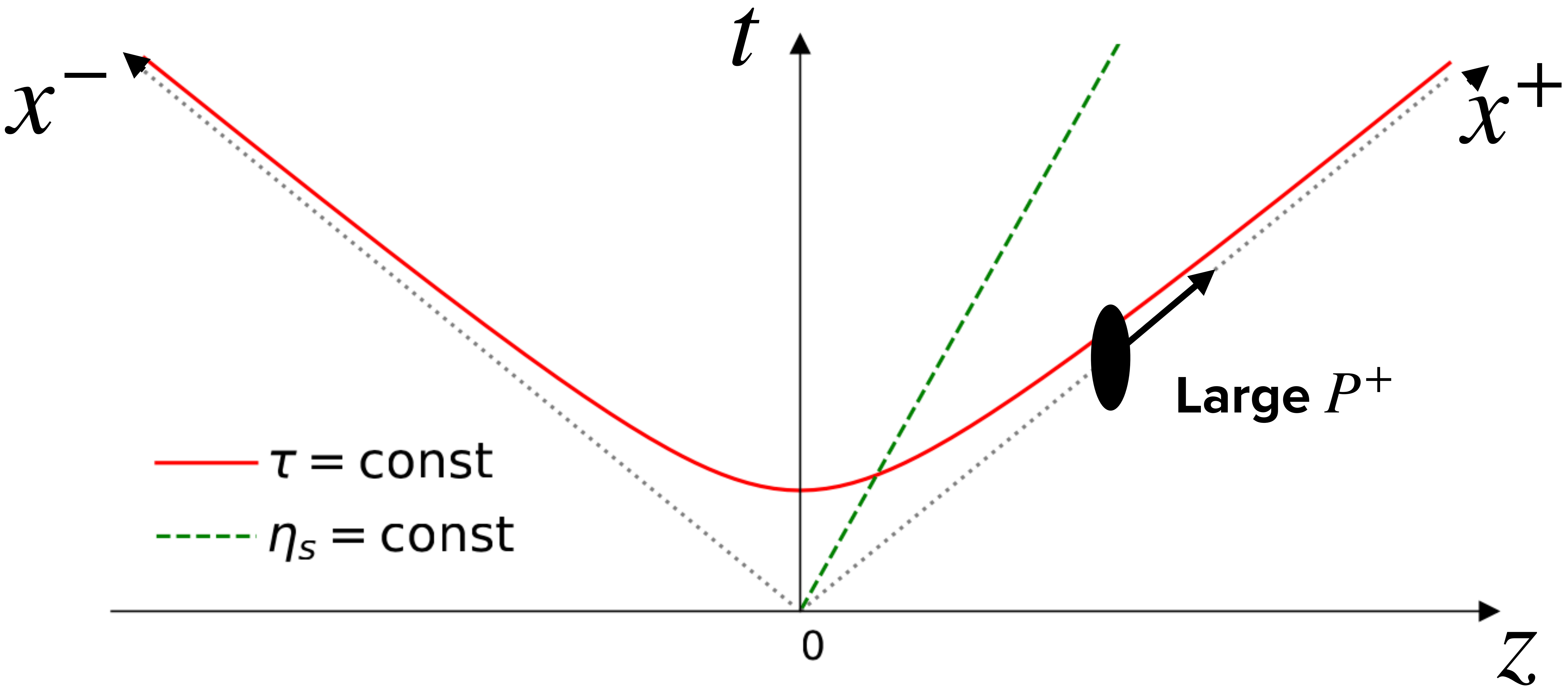


How localized are these sources? $\Delta z^- \sim \dfrac{1}{k^+} = \dfrac{1}{xP^+}$

What is the resolution scale of the probe? $\dfrac{1}{x_0 P^+} > \dfrac{1}{xP^+}$ **for** $x > x_0$

→Color sources look fully localized to the probe in z^-

Color sources



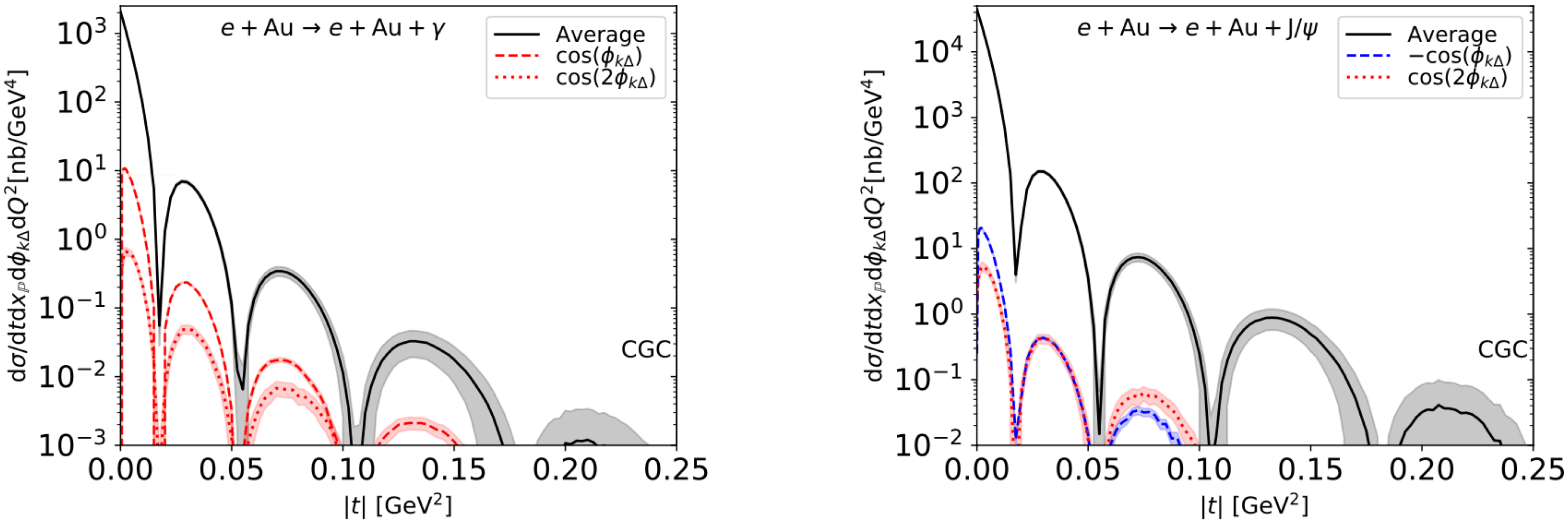
How fast do they evolve? $\Delta z^+ \sim \dfrac{1}{k^-} = \dfrac{2k^+}{k_T^2} = \dfrac{2xP^+}{k_T^2}$ (because $a_\mu b^\mu = a^+b^- + a^-b^+ - \vec{a}_T \cdot \vec{b}_T$)

What is the time scale of the probe? $\tau \approx \dfrac{2x_0P^+}{k_T^2} < \dfrac{2xP^+}{k_T^2}$

→Color sources look static to the probe in light cone time z^+

Predictions for e-Au at the future EIC

DVCS and exclusive J/ψ : Spectra and azimuthal modulations

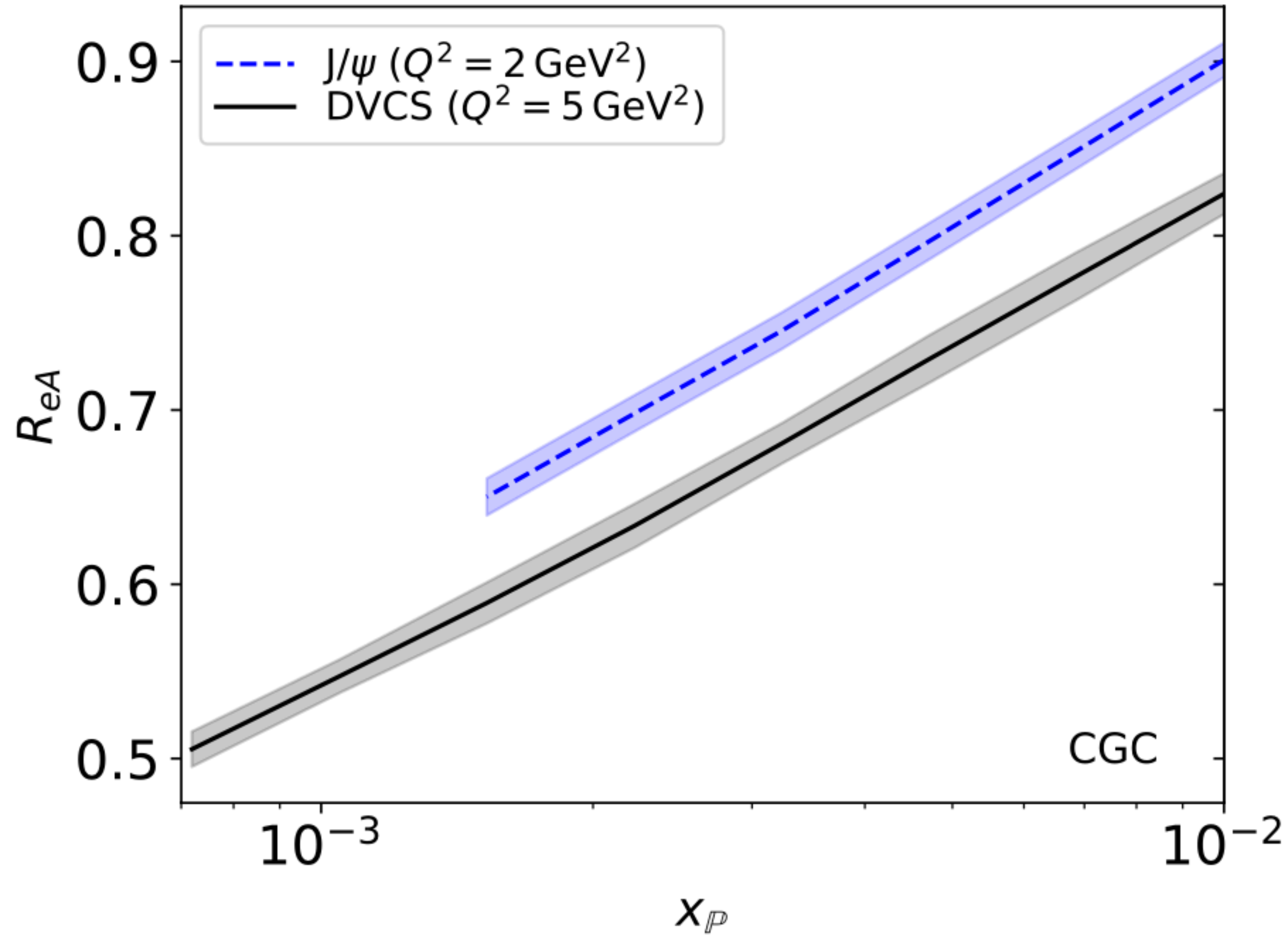


Characteristic dips in spectra due to Woods-Saxon nuclear profile

Azimuthal modulations v_n a few percent for DVCS, and less than 1% for J/ψ

Predictions for e-Au at the future EIC

Nuclear suppressions factor for DVCS and exclusive J/ψ



$$R_{eA} = \left. \frac{\mathrm{d}\sigma^{e+A\to e+A+V}/\mathrm{d}t\mathrm{d}Q^2\mathrm{d}x_{\mathbb{P}}}{A^2\mathrm{d}\sigma^{e+p\to e+p+V}/\mathrm{d}t\mathrm{d}Q^2\mathrm{d}x_{\mathbb{P}}} \right|_{t=0}$$

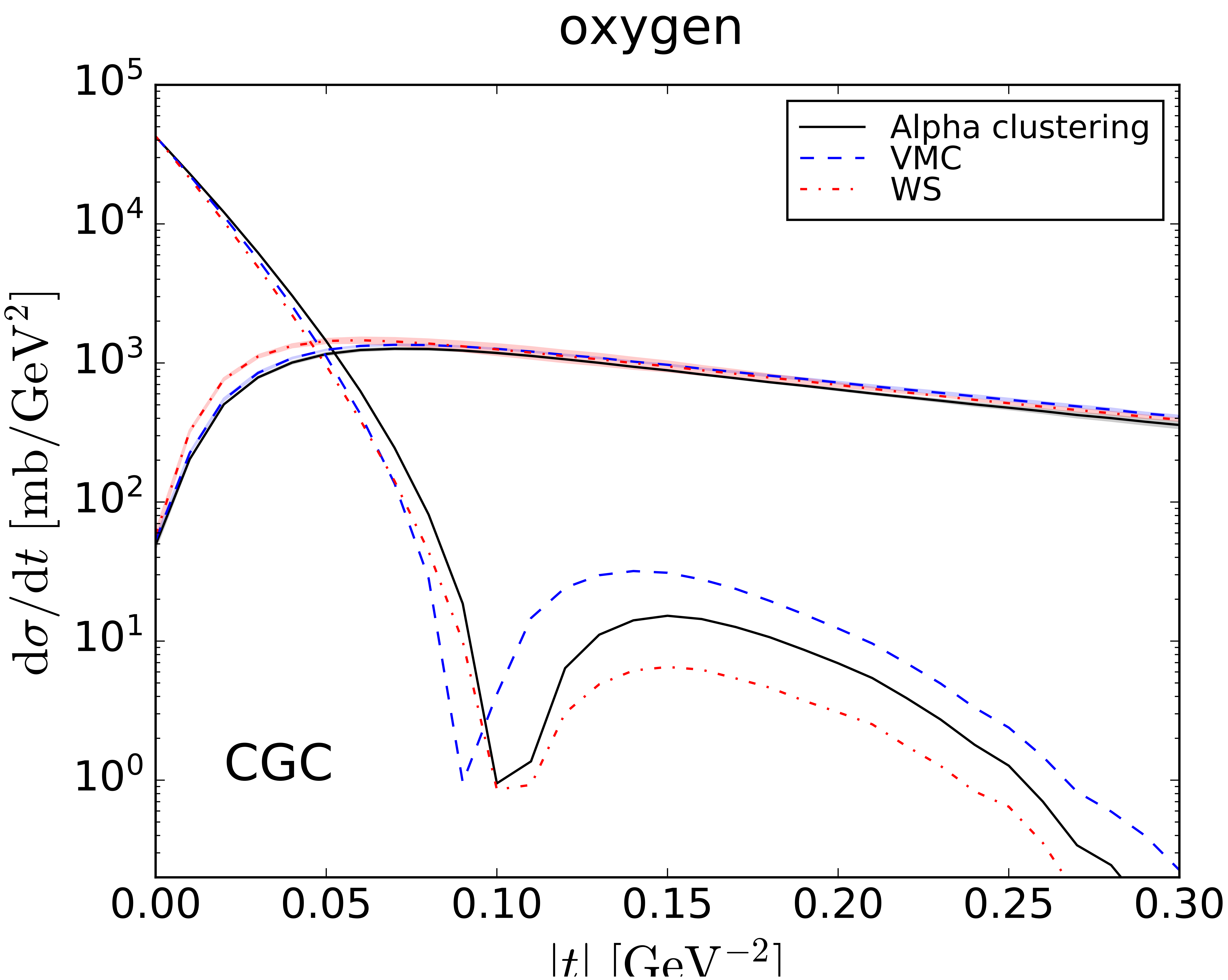
Expect $R_{eA} = 1$ in the dilute limit.

Mäntysaari, Venugopalan. 1712.02508

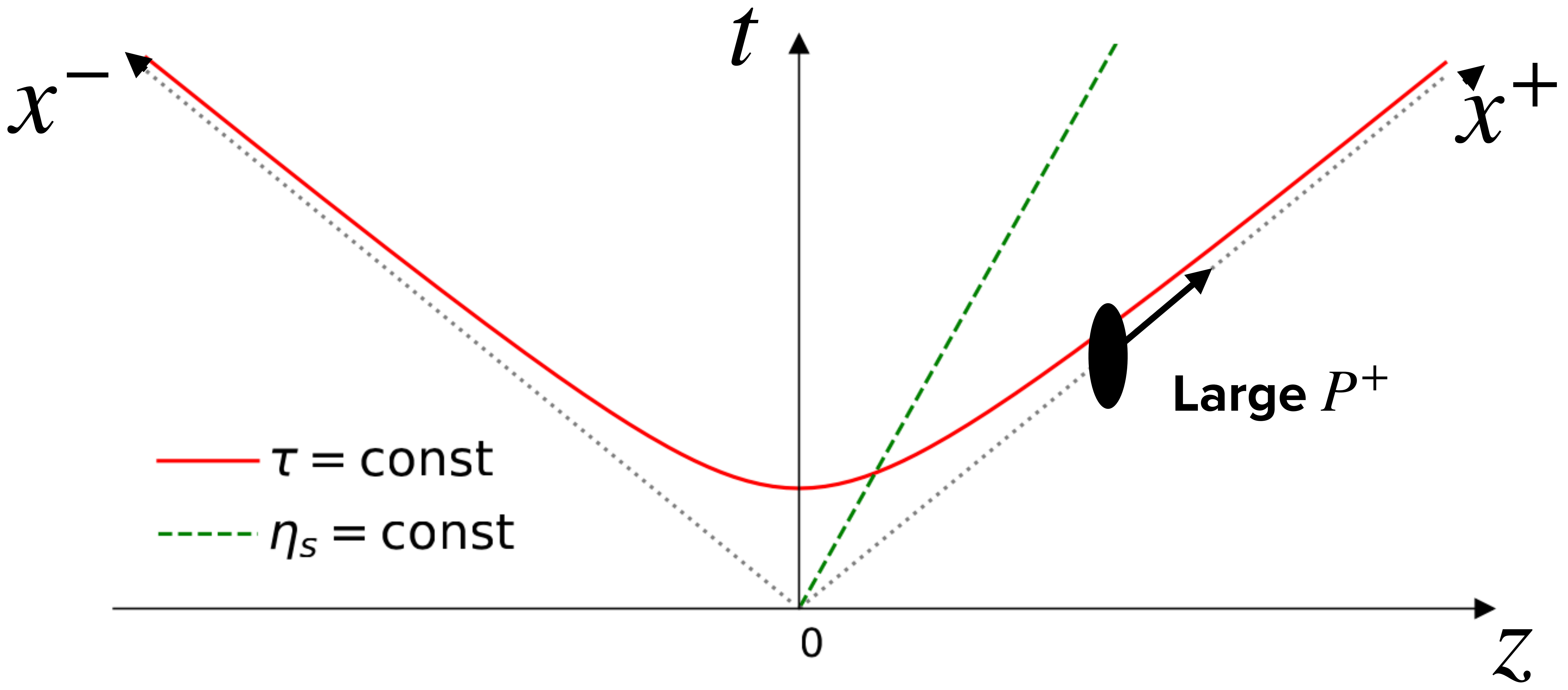
Significant suppression that evolves with energy/ $x_{\mathbb{P}}$

Larger suppression for DVCS due to larger dipole contributions.

e+O: Oxygen wave function dependence



Light cone



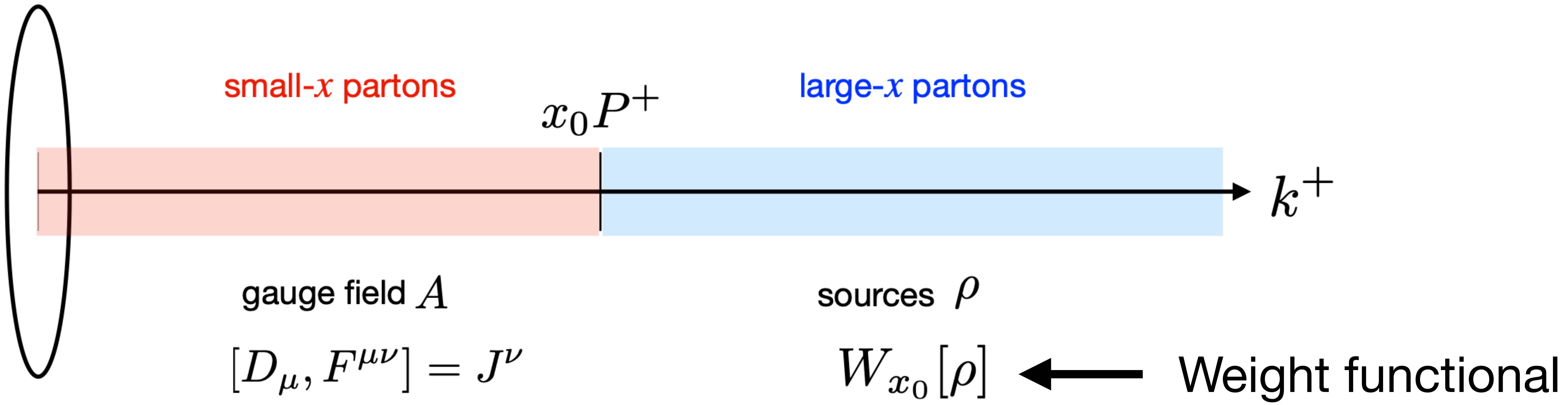
Light cone coordinates $v^{\pm} = (v^0 \pm v^3)/\sqrt{2}$

In the future light cone define $x^+ = \dfrac{\tau}{\sqrt{2}} e^{+\eta}$, and $x^- = \dfrac{\tau}{\sqrt{2}} e^{-\eta}$

or inverted $\tau = \sqrt{2x^+x^-}$, and $\eta = \dfrac{1}{2} \ln\left(\dfrac{x^+}{x^-}\right)$

Weight functional

https://arxiv.org/pdf/hep-ph/0406169.pdf



What is the weight functional?

Need to model. E.g. the McLerran-Venugopalan model:

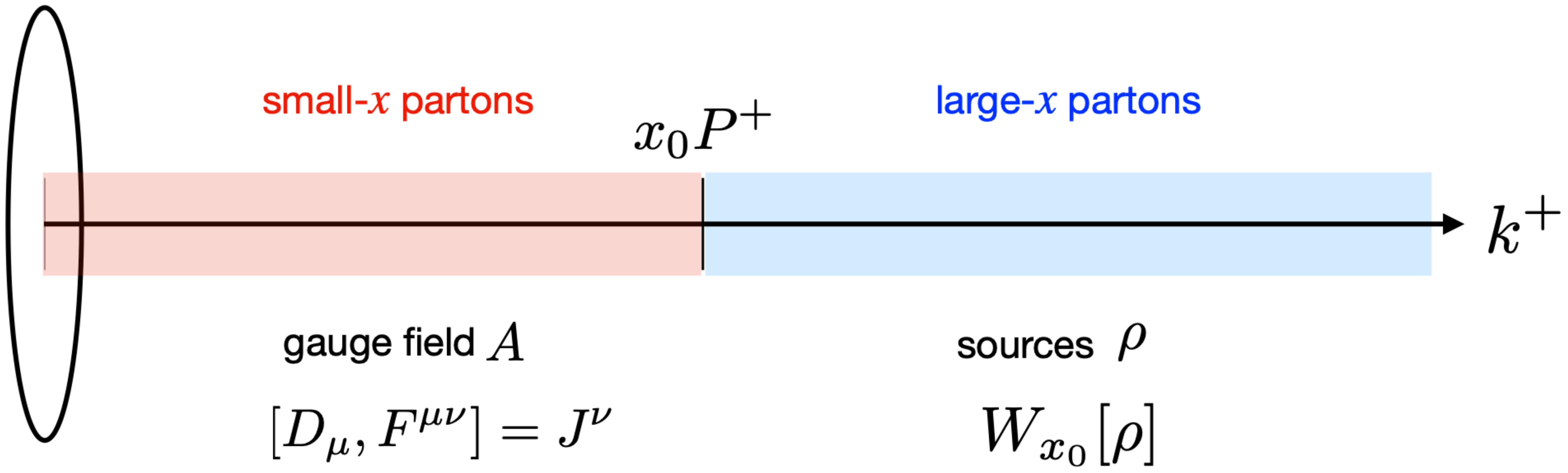
Assume a large nucleus, invoke central limit theorem. All correlations of ρ^a are Gaussian

$$W_{x_0}[\rho] = \mathcal{N} \exp\left(-\frac{1}{2} \int dx^- d^2 x_T \frac{\rho^a(x^-, x_T)\rho^a(x^-, x_T)}{\lambda_{x_0}(x^-)} \right)$$

where $\lambda_{x_0}(x^-)$ is related to the transverse color charge density distribution of the nucleus

Weight functional

https://arxiv.org/pdf/hep-ph/0406169.pdf



…where $\lambda_{x_0}(x^-)$ is related to the transverse color charge density distribution of the nucleus

$$\mu^2 = \int dx^- \lambda_{x_0}(x^-) = \frac{(g^2 C_F)(AN_c)}{\pi R_A^2} \frac{1}{N_c^2 - 1} = \frac{g^2 A}{2\pi R_A^2} \sim A^{1/3}$$

That color charge density is related to Q_s, the saturation scale.

normalized per color degree of freedom