Quarkonium transport in weakly and strongly coupled plasmas

Heavy Flavor Production in Heavy-Ion and Elementary Collisions
Institute for Nuclear Theory Workshop 22-3
University of Washington
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collaborators: Xiaojun Yao (UW) and Govert Nijs (MIT)
based on 2107.03945, 2205.04477, 2211.XXXXX

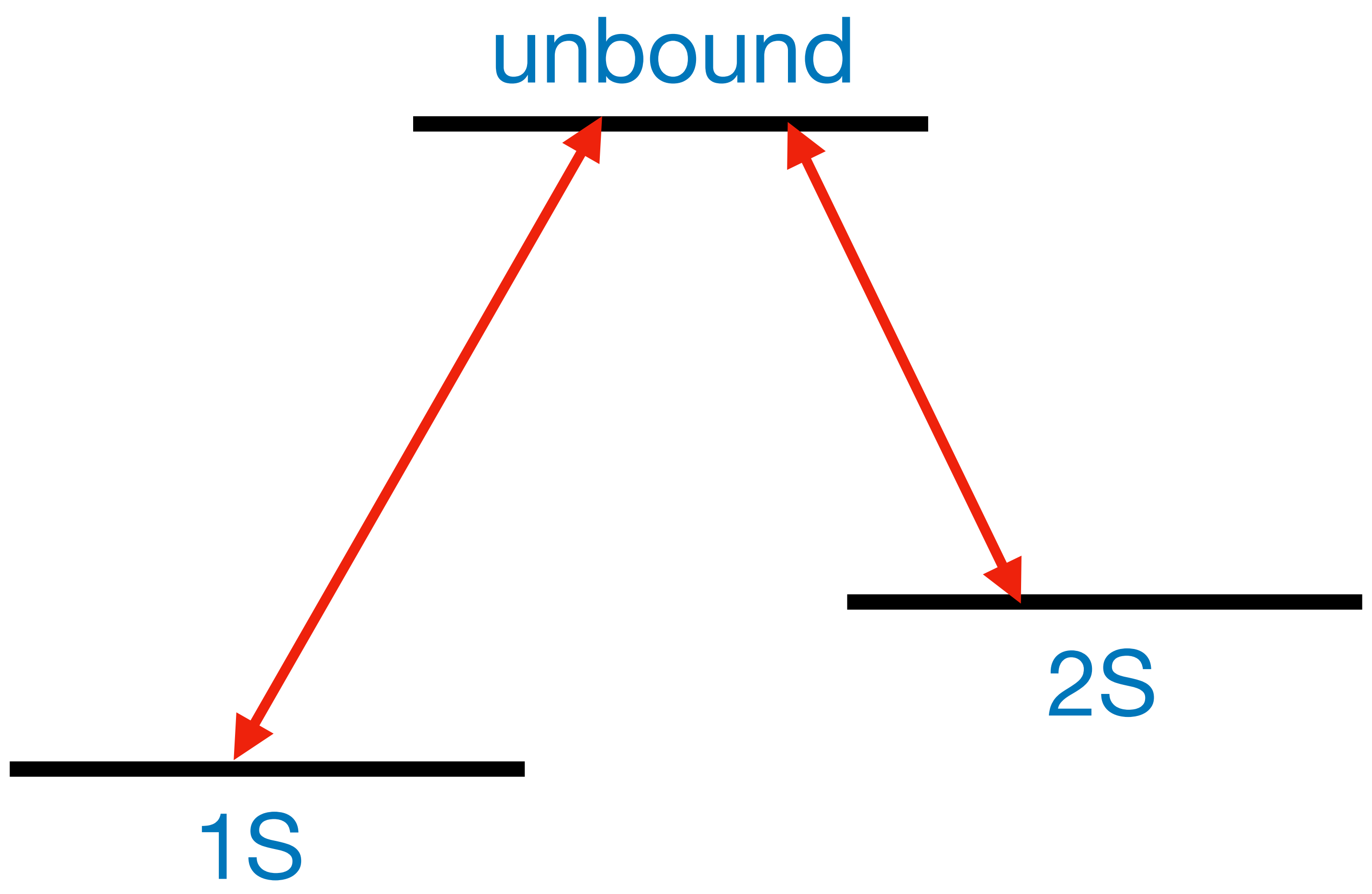
[*] N. Brambilla, A. Pineda, J. Soto, A. Vairo hep-ph/9707481, hep-ph/9907240, hep-ph/0410047

X. Yao, hep-ph/2102.01736

Time scales of quarkonia

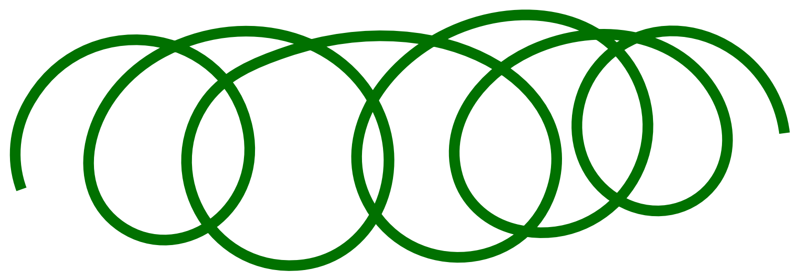
an open quantum system picture of pNRQCD [*]

Transitions between quarkonium energy levels (the system)



$$\frac{1}{\tau_S} \sim \Delta E_n \sim M v^2$$

Interaction with the environment



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QGP (the environment)

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\text{pNRQCD}} = \mathscr{L}_{\text{light quarks}} + \mathscr{L}_{\text{gluon}} + \int d^3r \text{Tr}_{\text{color}}\left[S^\dagger(i\partial_0 - H_s)S + O^\dagger(iD_0 - H_o)O + V_A\left(O^\dagger \mathbf{r}\cdot g\mathbf{E}S + \text{h.c.}\right) + \frac{V_B}{2}O^\dagger\left\{\mathbf{r}\cdot g\mathbf{E}, O\right\} + \cdots\right]$$

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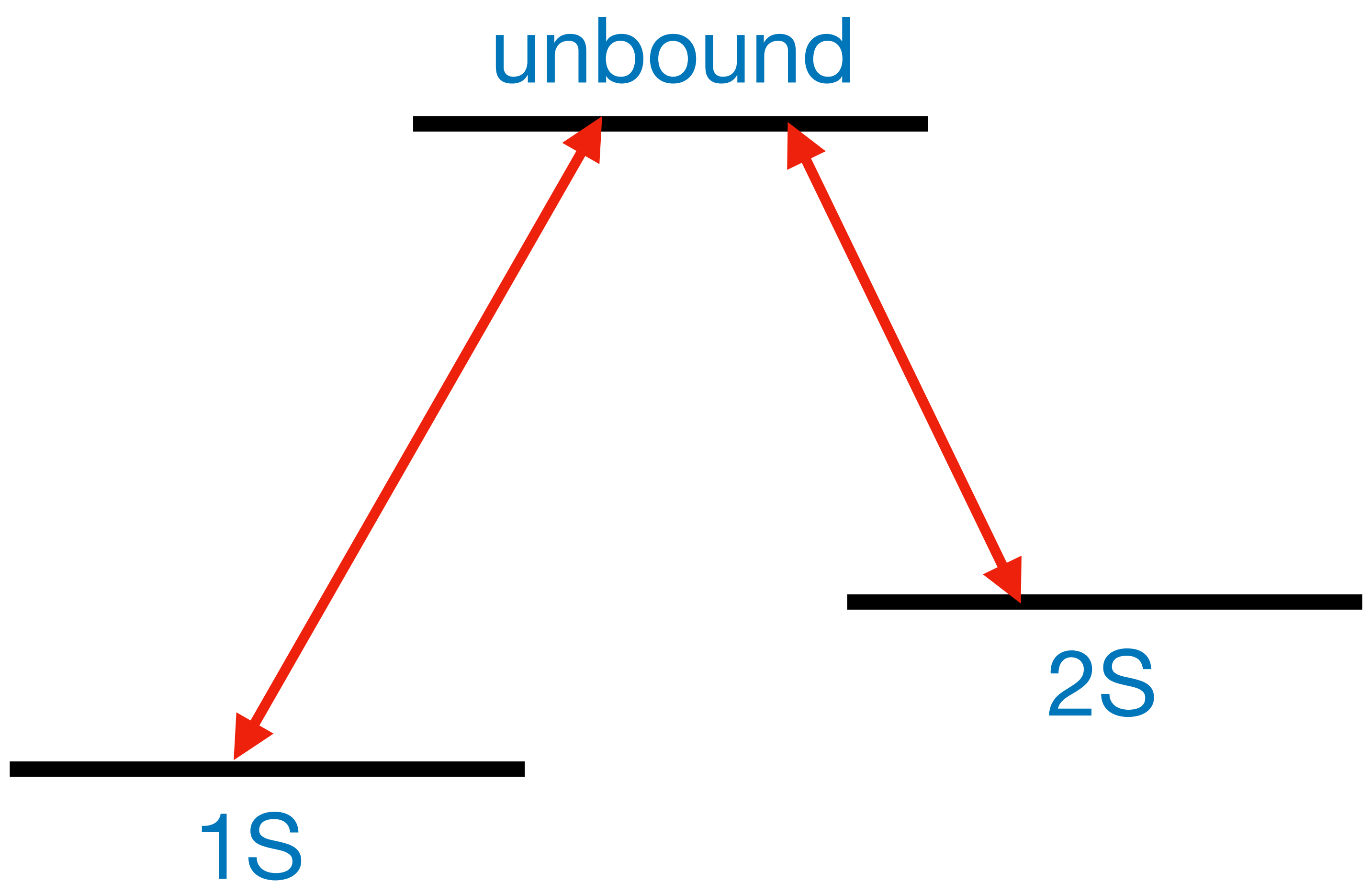

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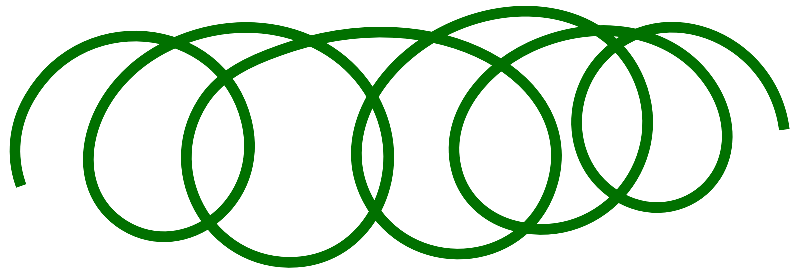

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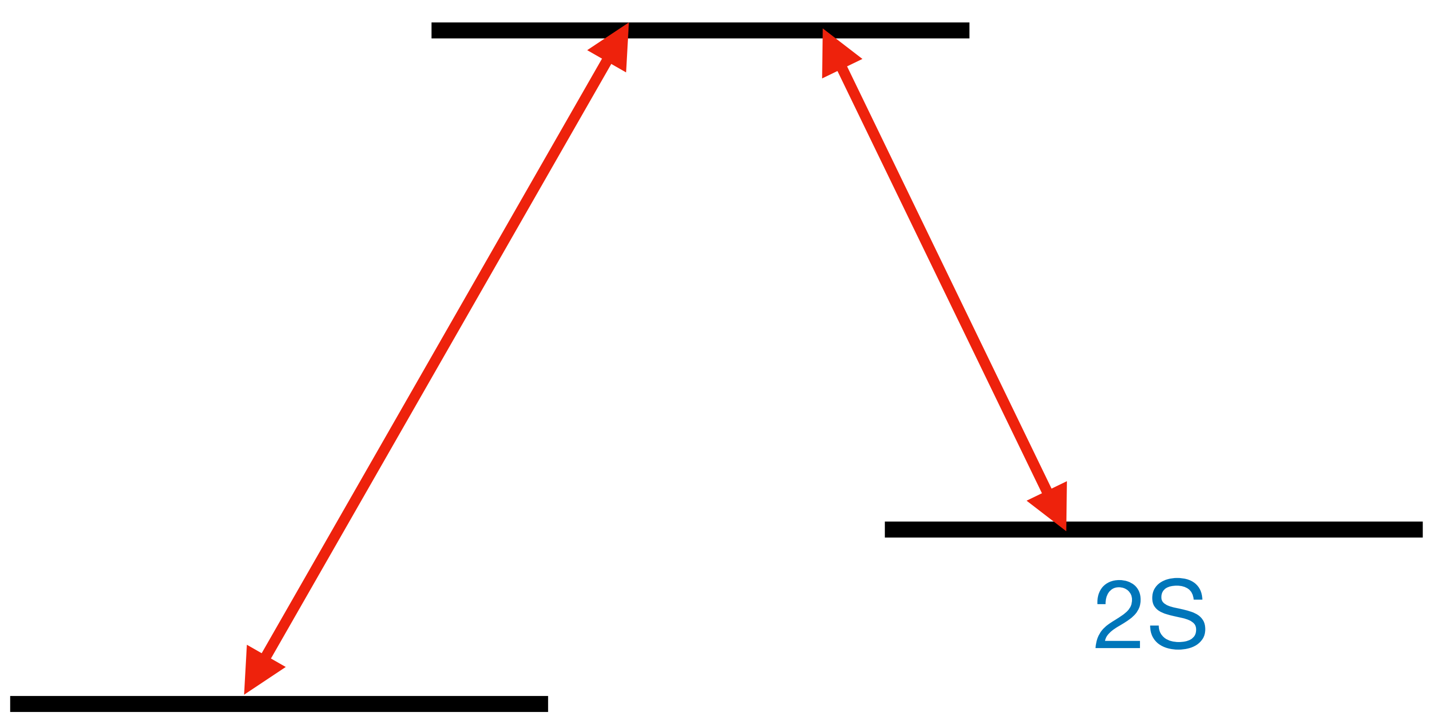

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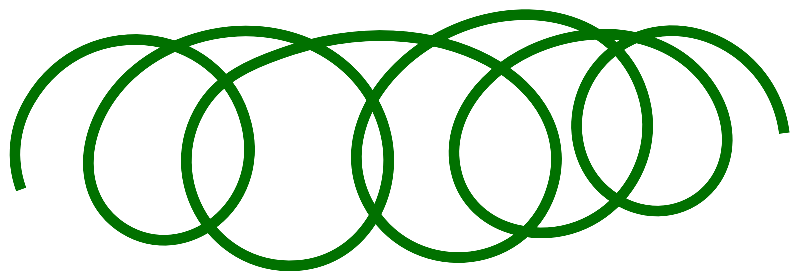

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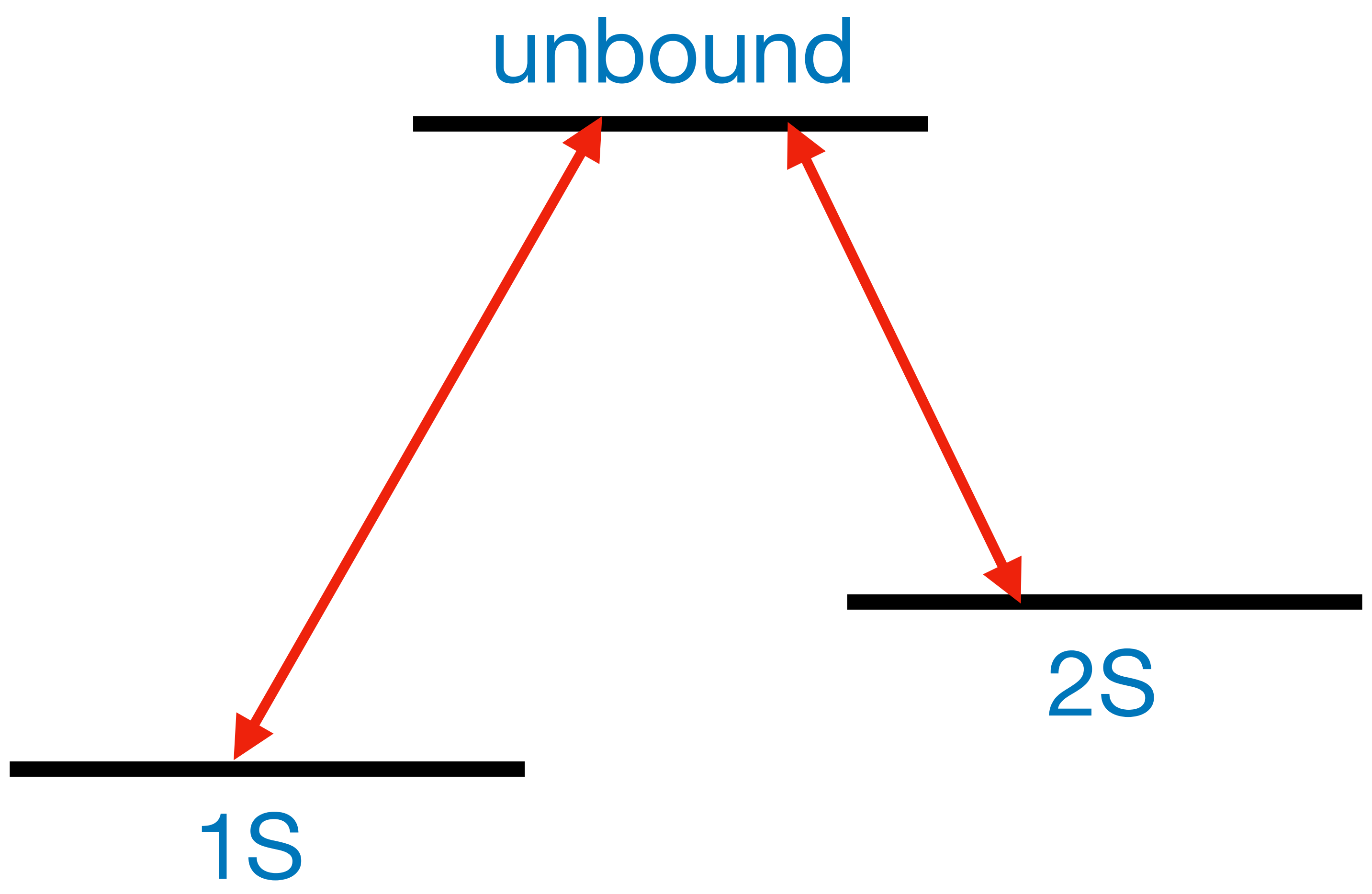

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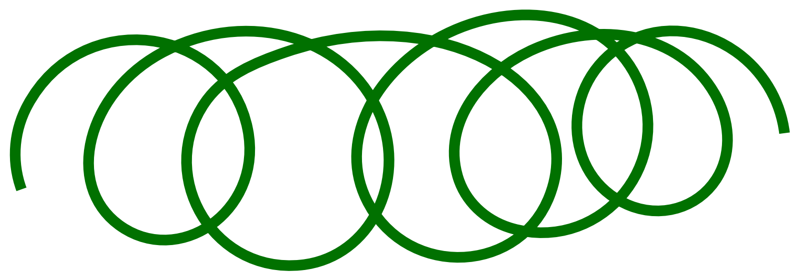

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Open quantum systems
“tracing/integrating out” the QGP

- Given an initial density matrix $\rho_{\text{tot}}(t=0)$, quarkonium coupled with the QGP evolves as

$$\rho_{\text{tot}}(t) = U(t)\rho_{\text{tot}}(t=0)U^{\dagger}(t).$$

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- Then, one derives an evolution equation for $\rho_S(t)$, assuming that at the initial time we have $\rho_{\text{tot}}(t=0) = \rho_S(t=0) \otimes e^{-H_{\text{QGP}}/T} / \mathcal{Z}_{\text{QGP}}$.

Lindblad equations for quarkonia at low T

quantum Brownian motion limit & quantum optical limit in pNRQCD

- After tracing out the QGP degrees of freedom, one gets a Lindblad-type equation:

$$\frac{\partial \rho}{\partial t} = - i[H_{\text{eff}}, \rho] + \sum_j \gamma_j \left(L_j \rho L_j^\dagger - \frac{1}{2} \left\{ L_j^\dagger L_j, \rho \right\} \right)$$

- This can be done in two different limits within pNRQCD:

Quantum Brownian Motion:

$$\tau_I \gg \tau_E$$
$$\tau_S \gg \tau_E$$

relevant for $Mv \gg T \gg Mv^2$

Quantum Optical:

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How does the QGP enter the dynamics?

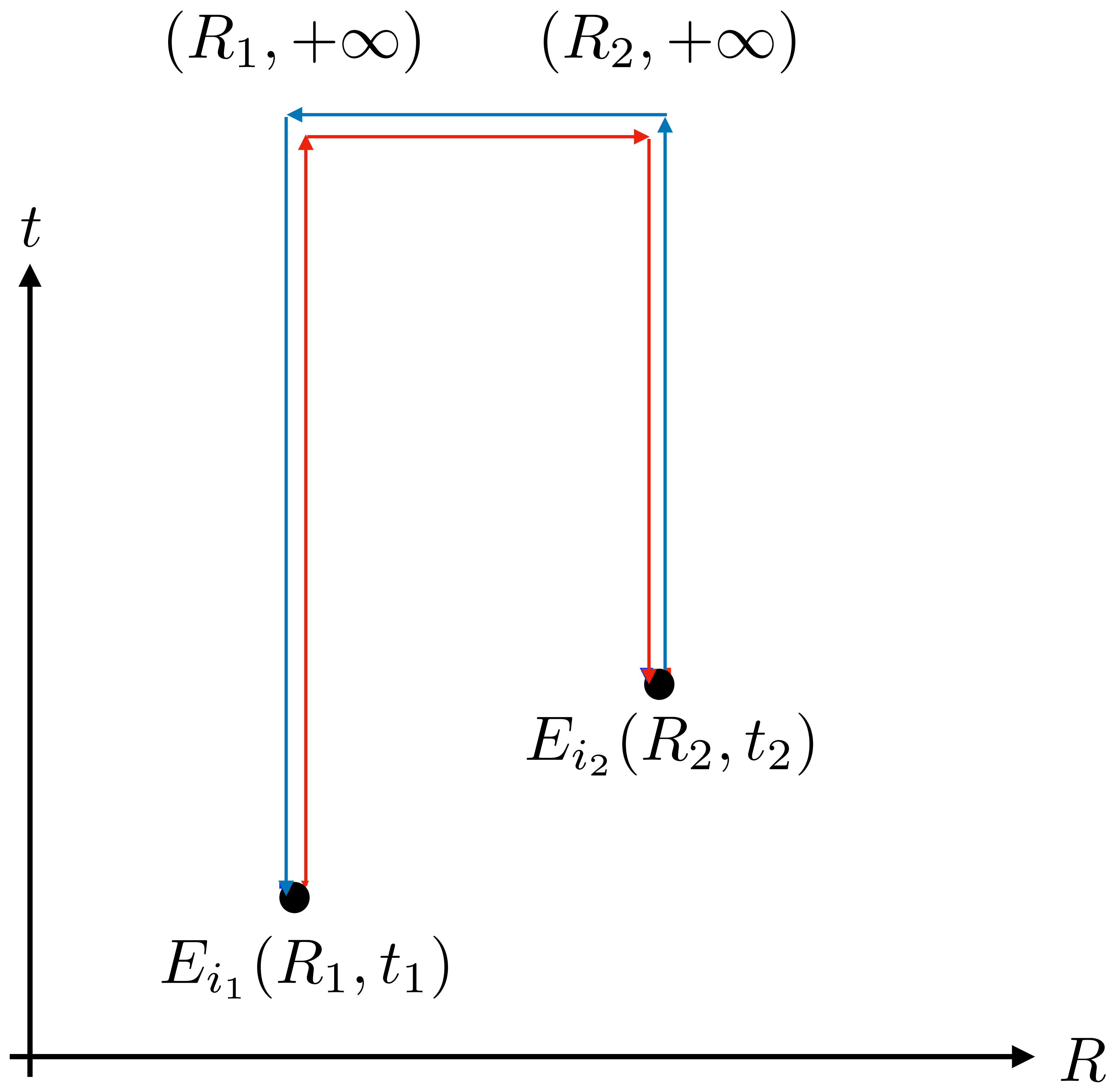
See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

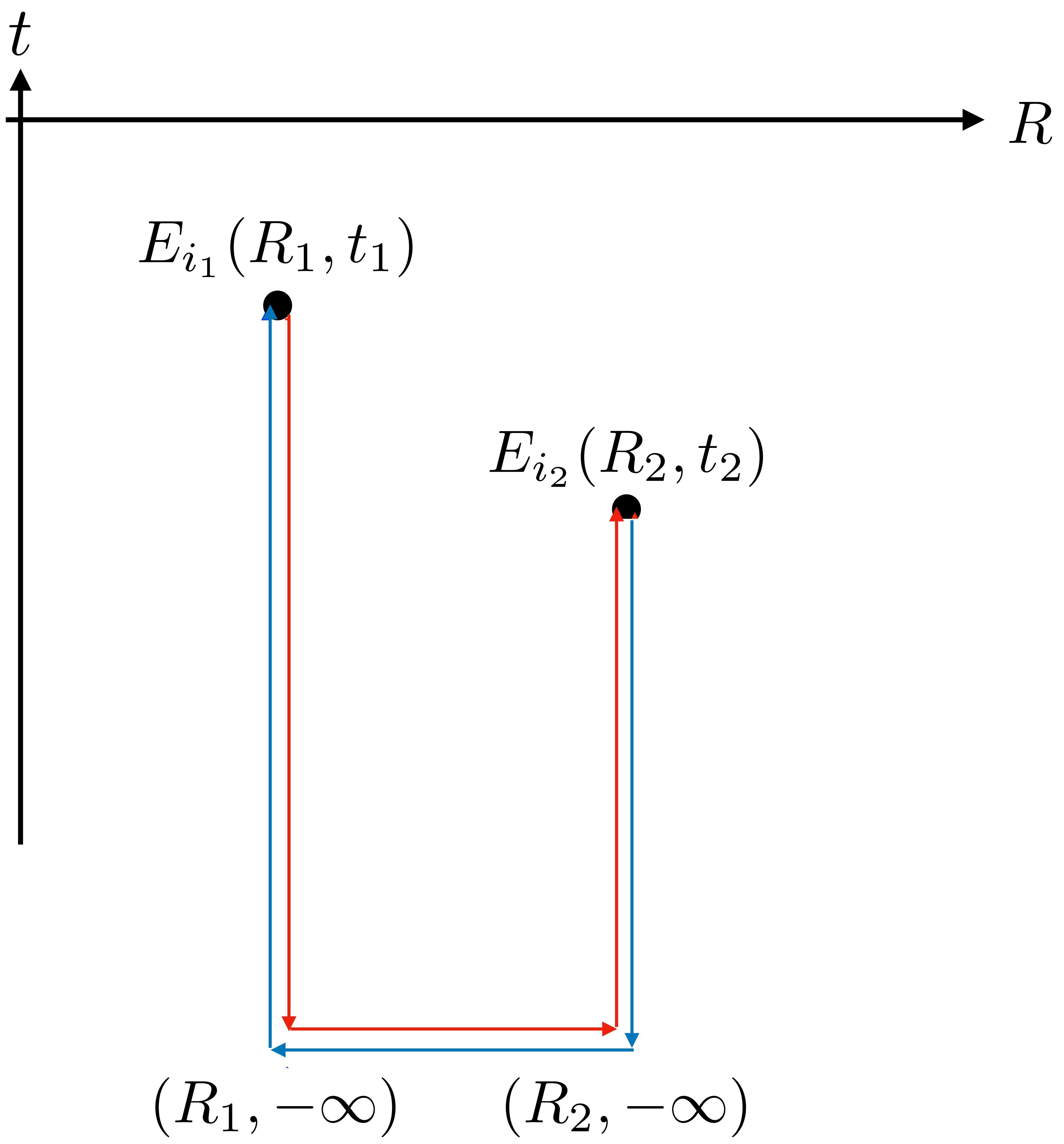
X. Yao and T. Mehen, hep-ph/2009.02408

QGP chromoelectric correlators

for quarkonia transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \Big\langle \big(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\big)^a \big(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\big)^a \Big\rangle_T$$





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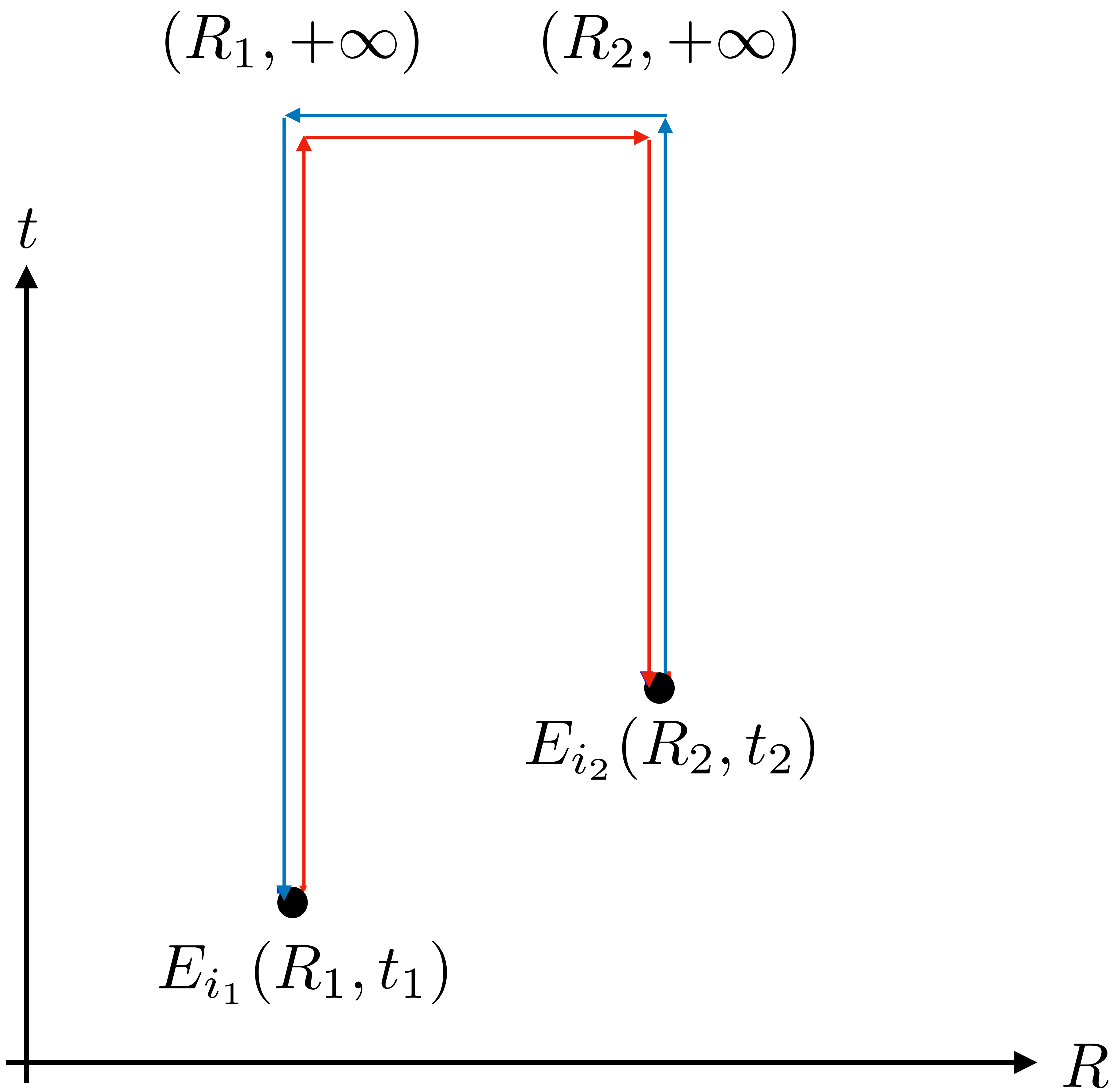

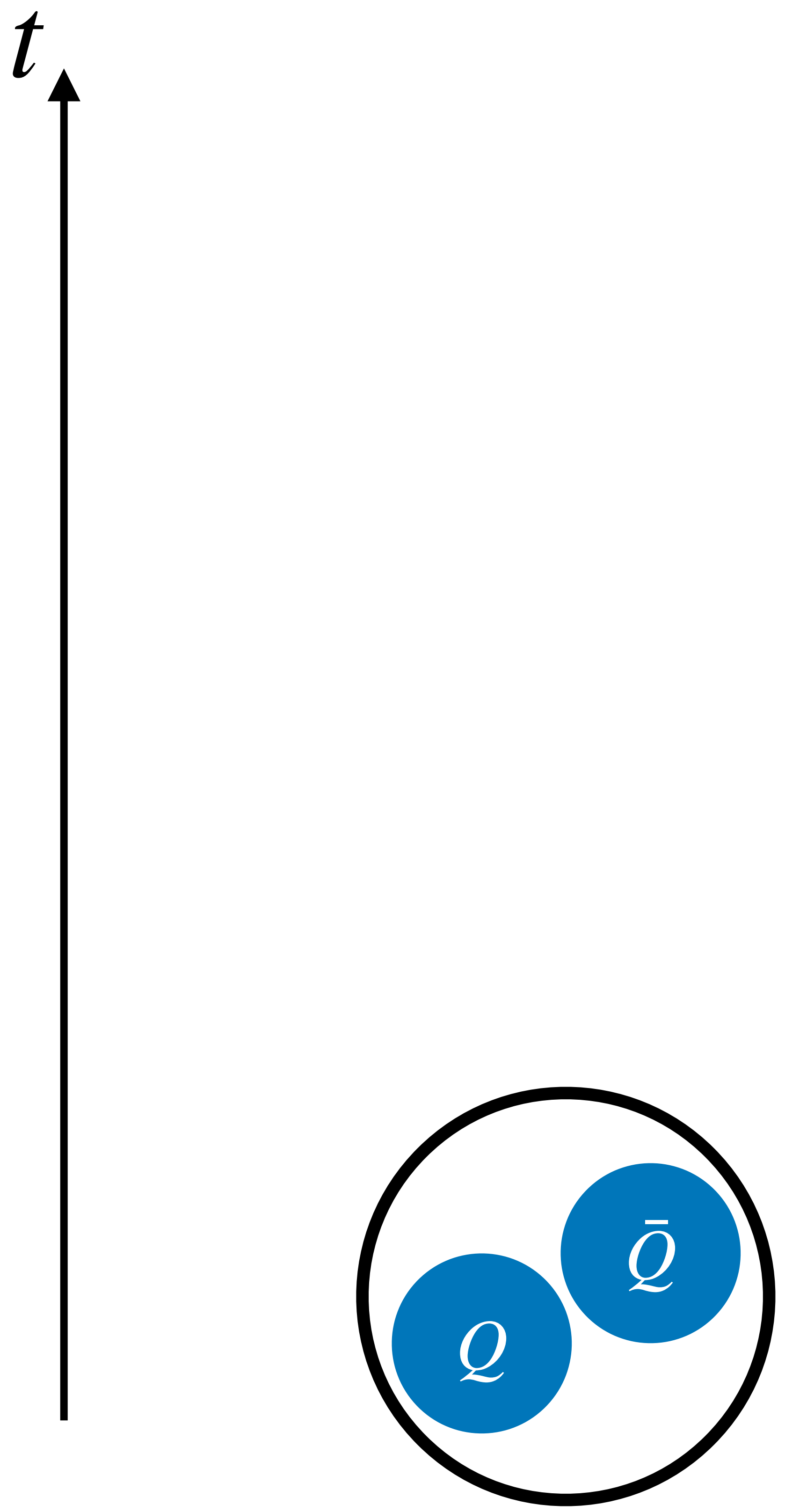

bound state:
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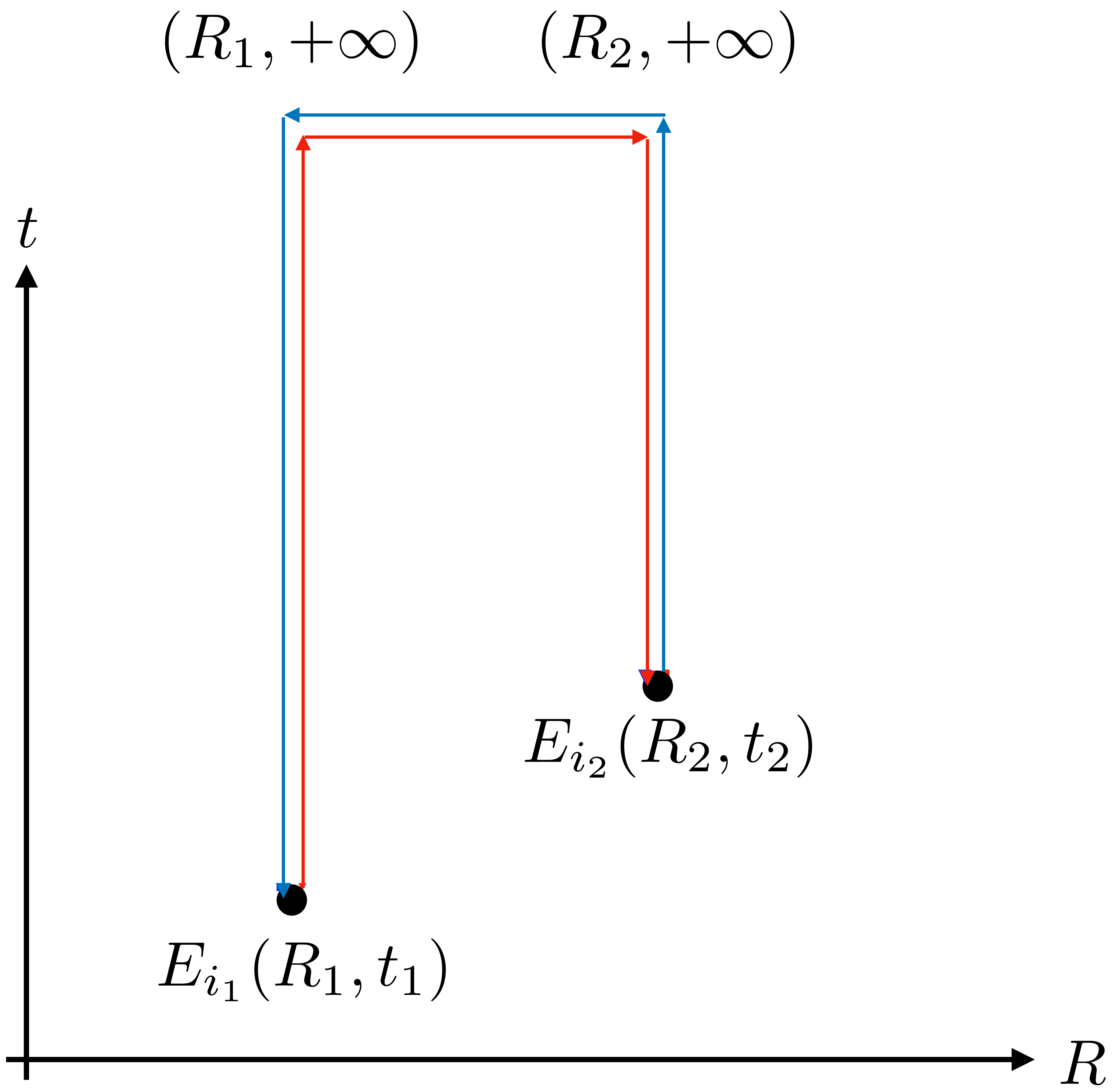

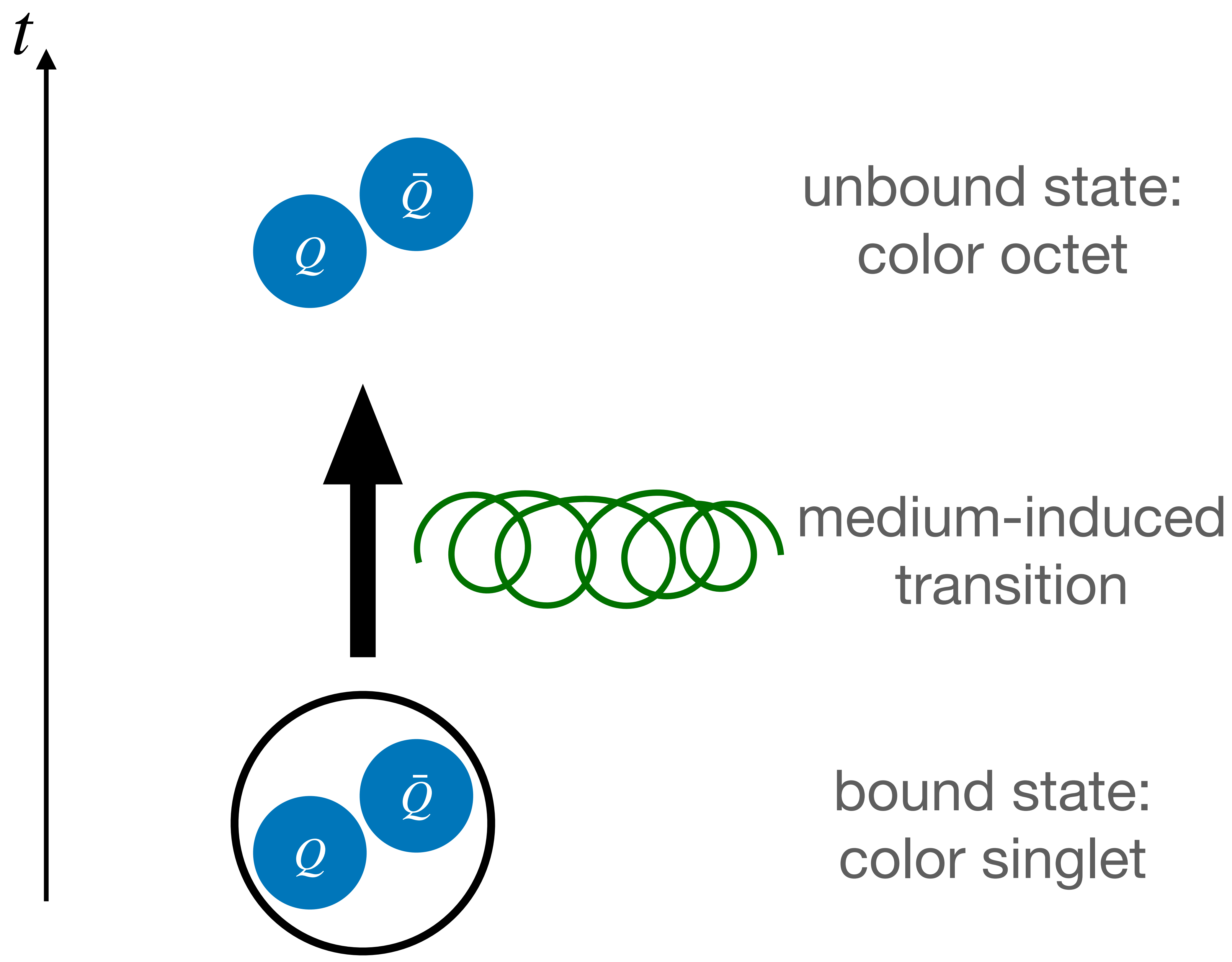

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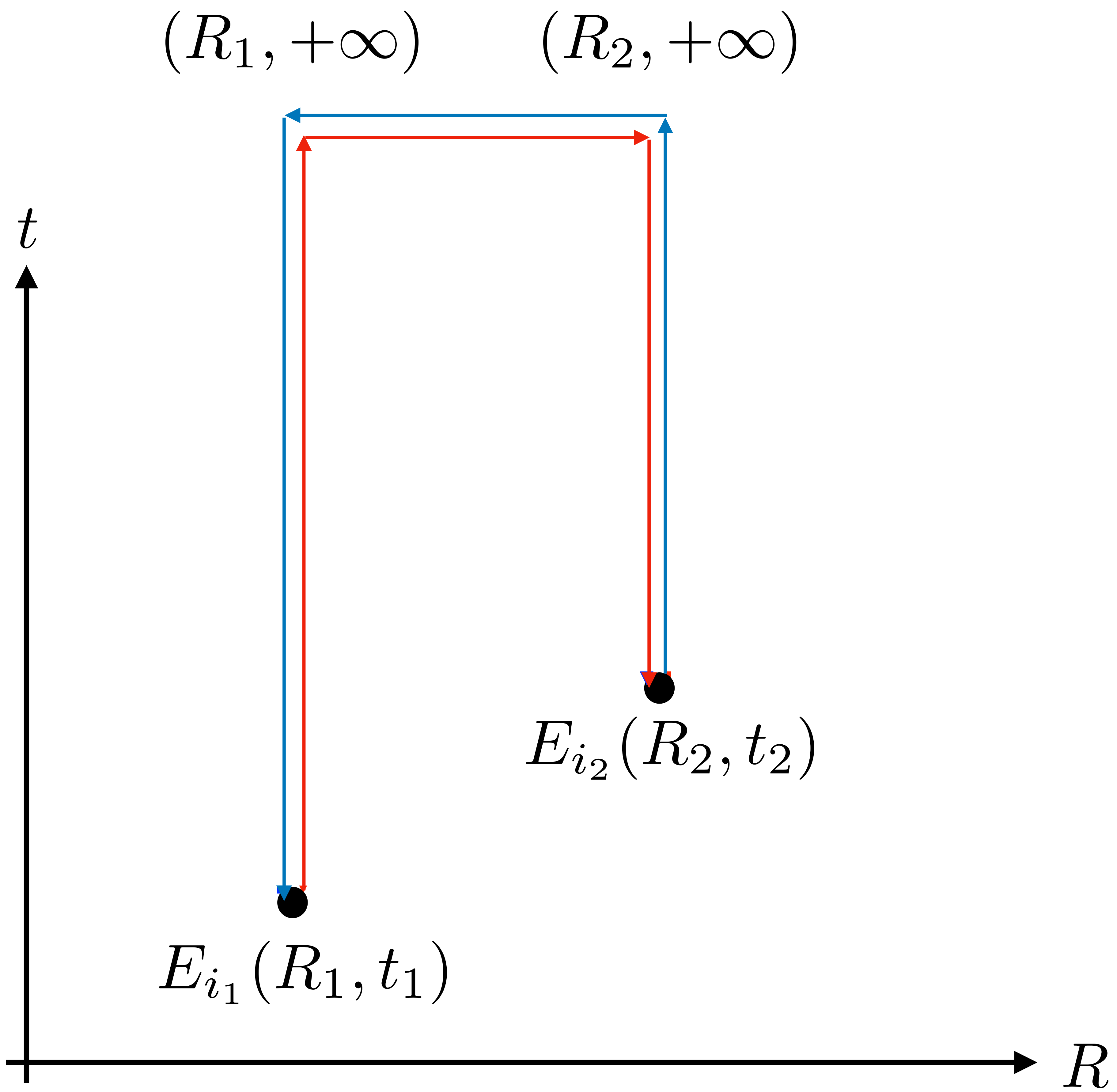

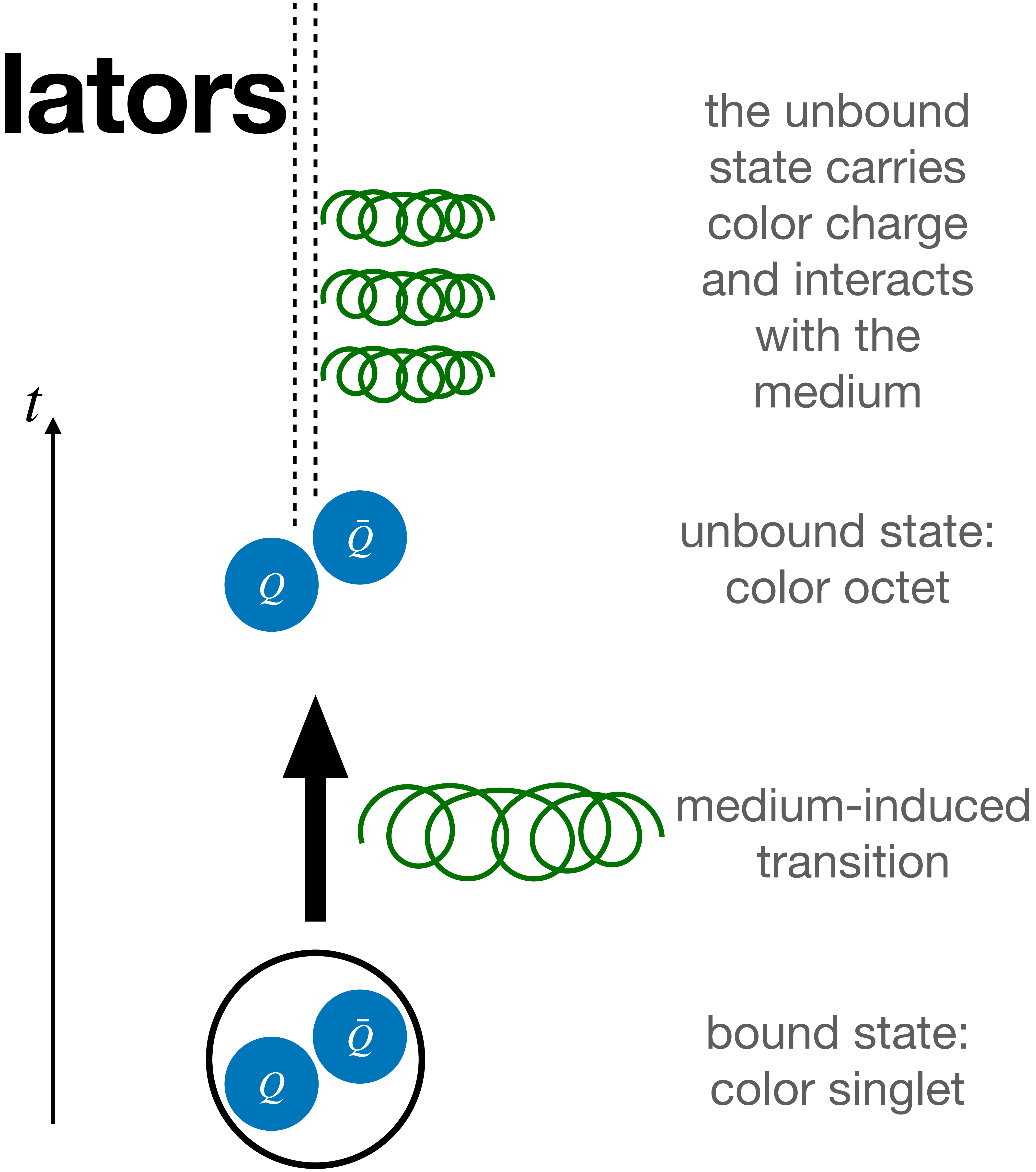

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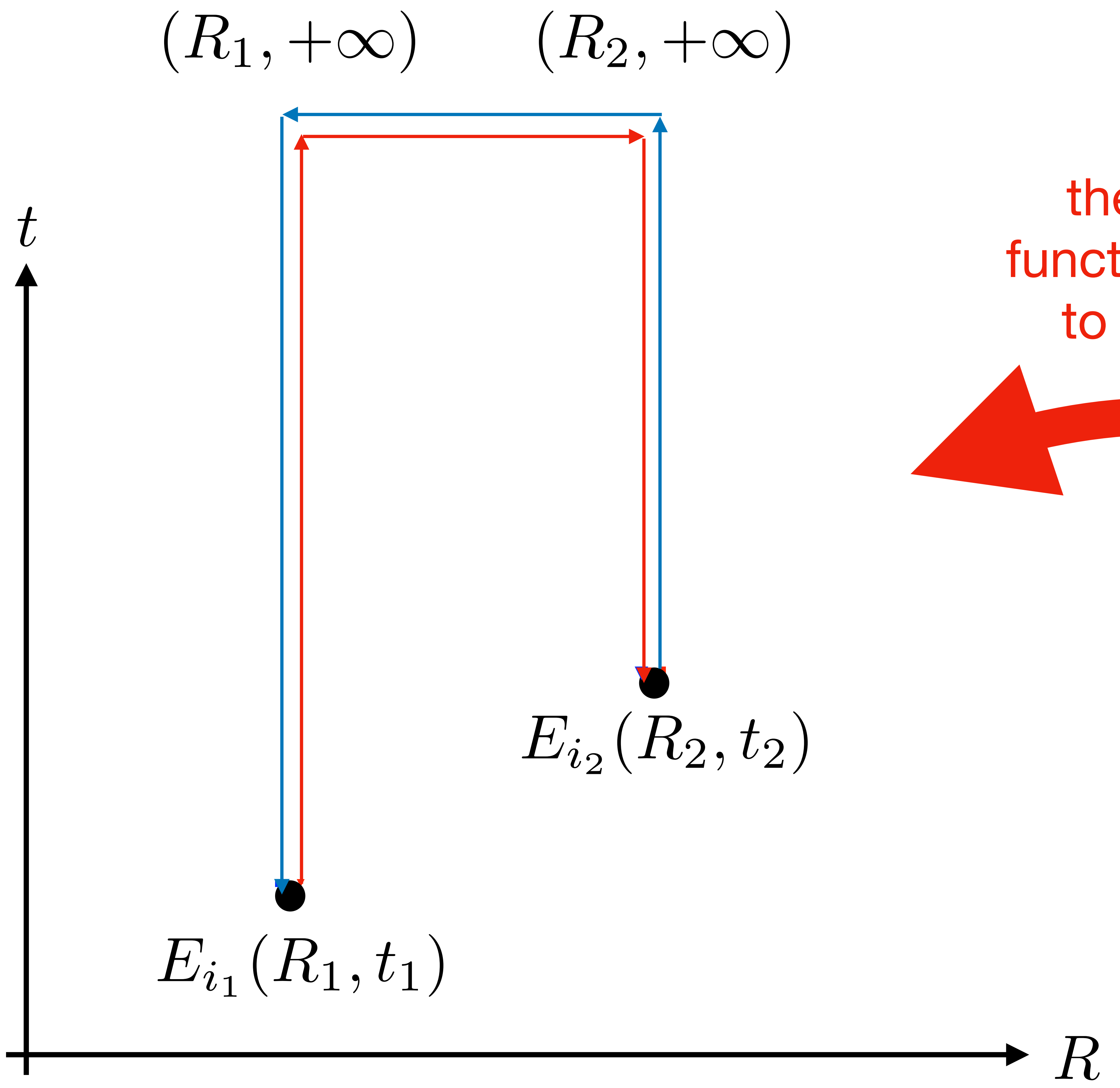
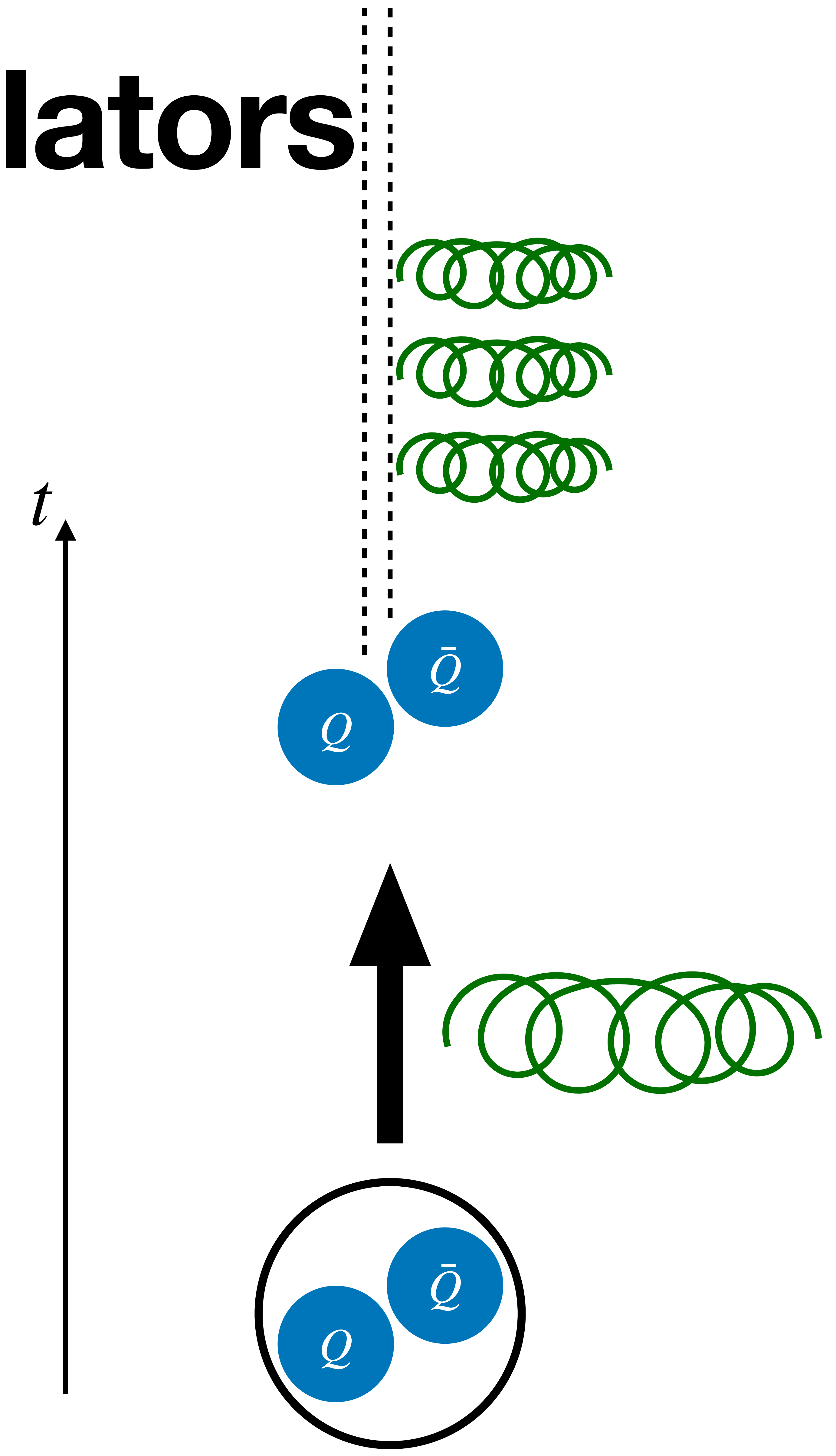


the correlation function associated to this process



the unbound state carries color charge and interacts with the medium

unbound state: color octet

medium-induced transition

bound state: color singlet

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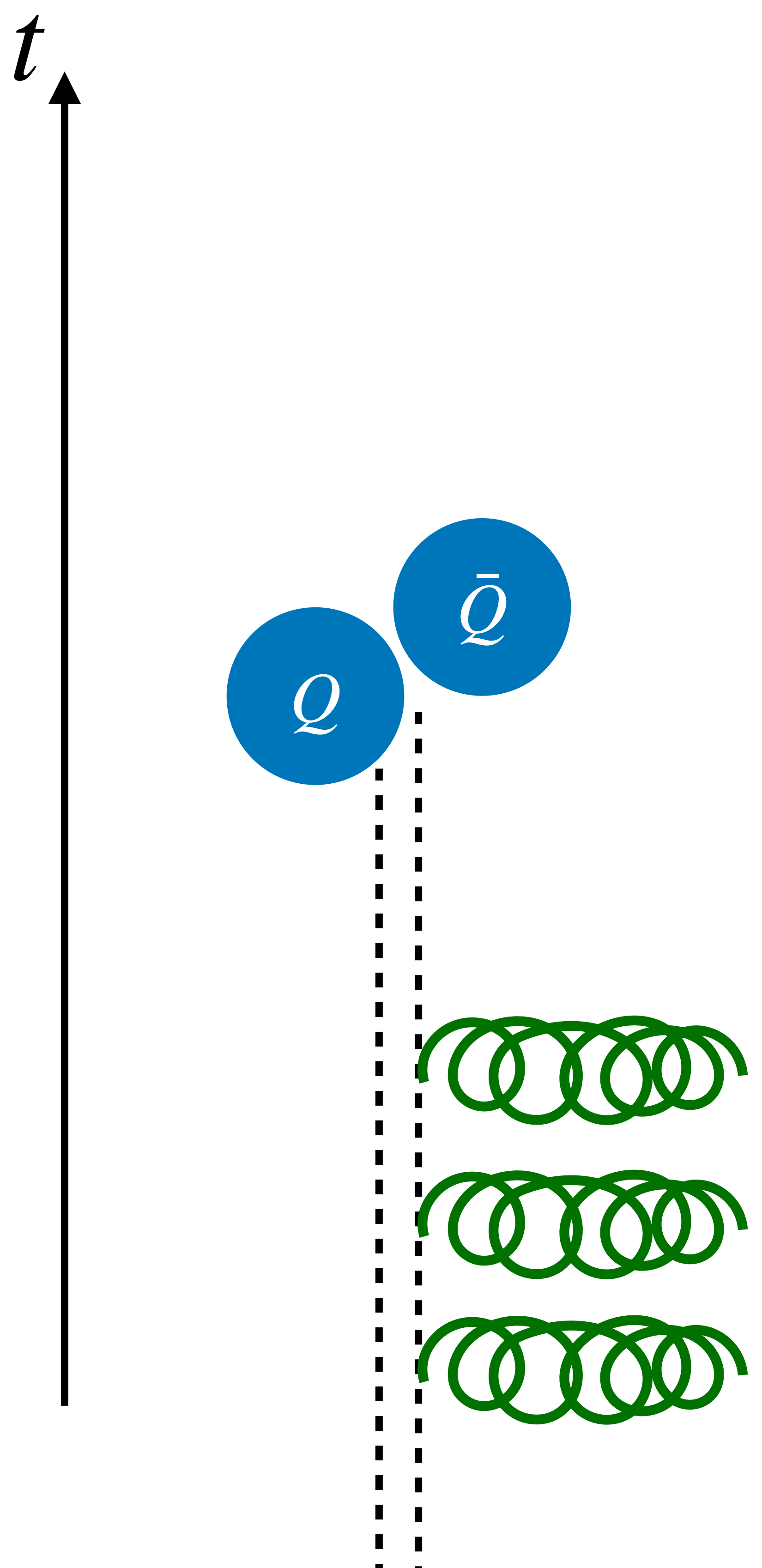

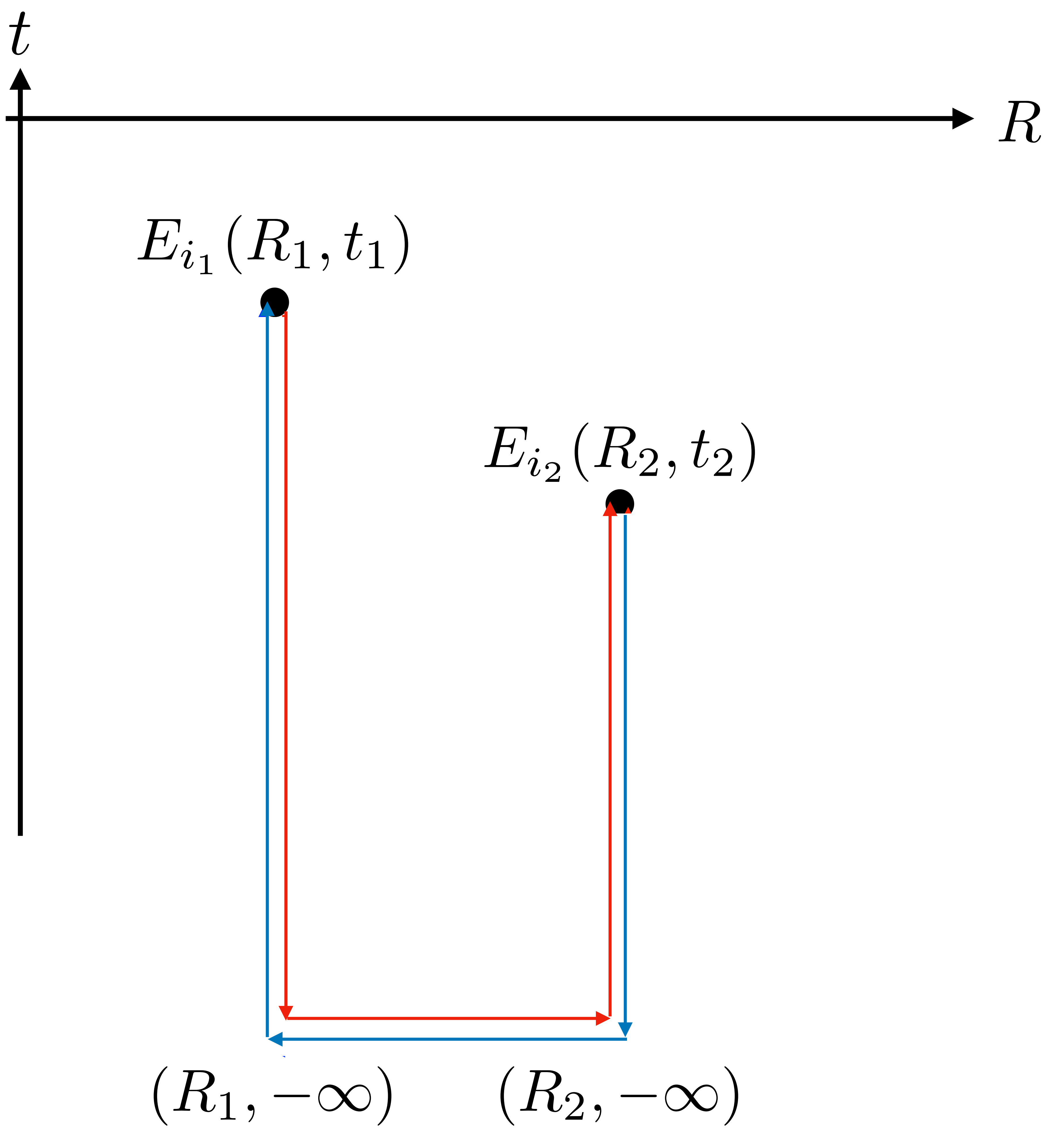

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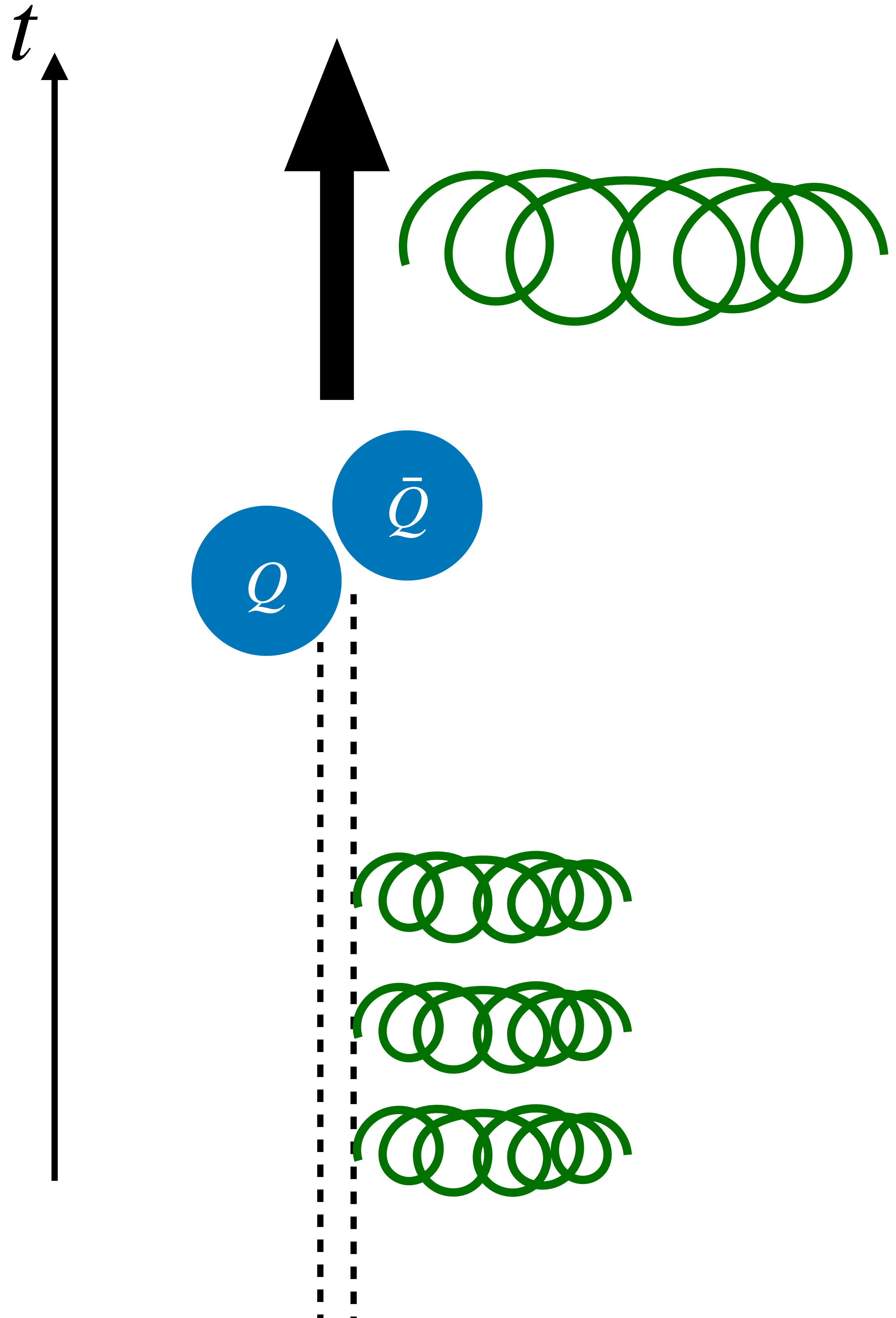

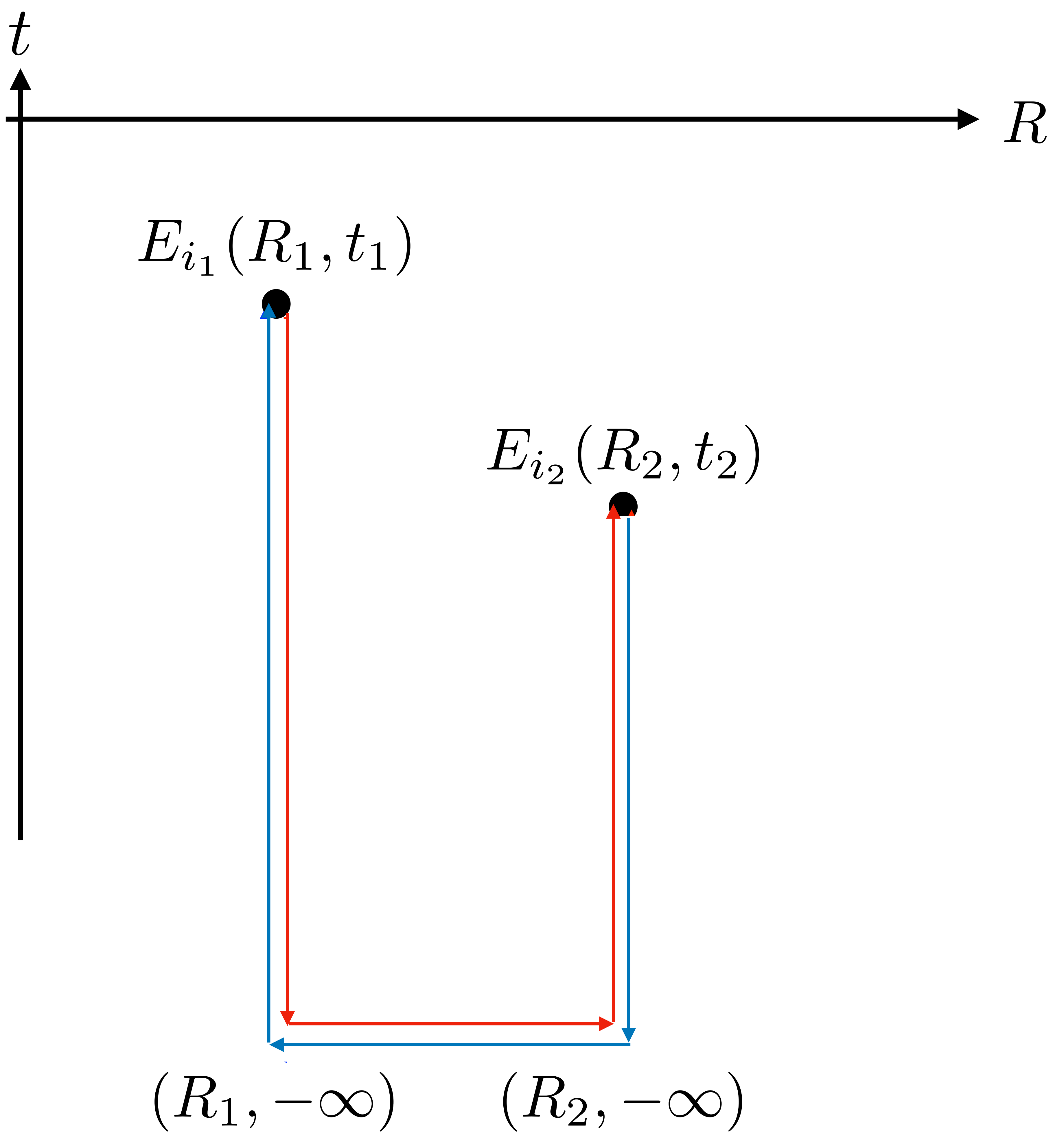

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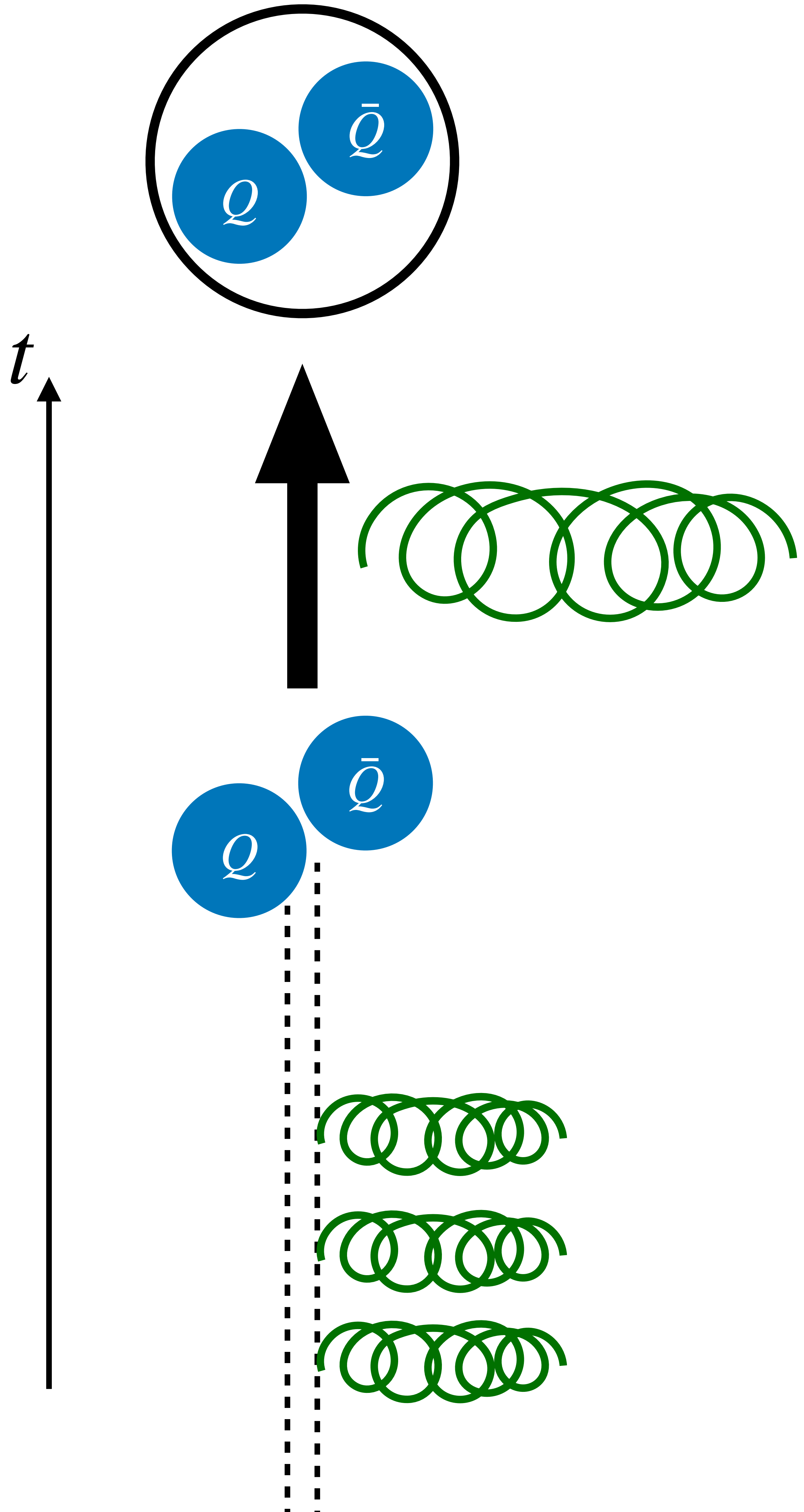

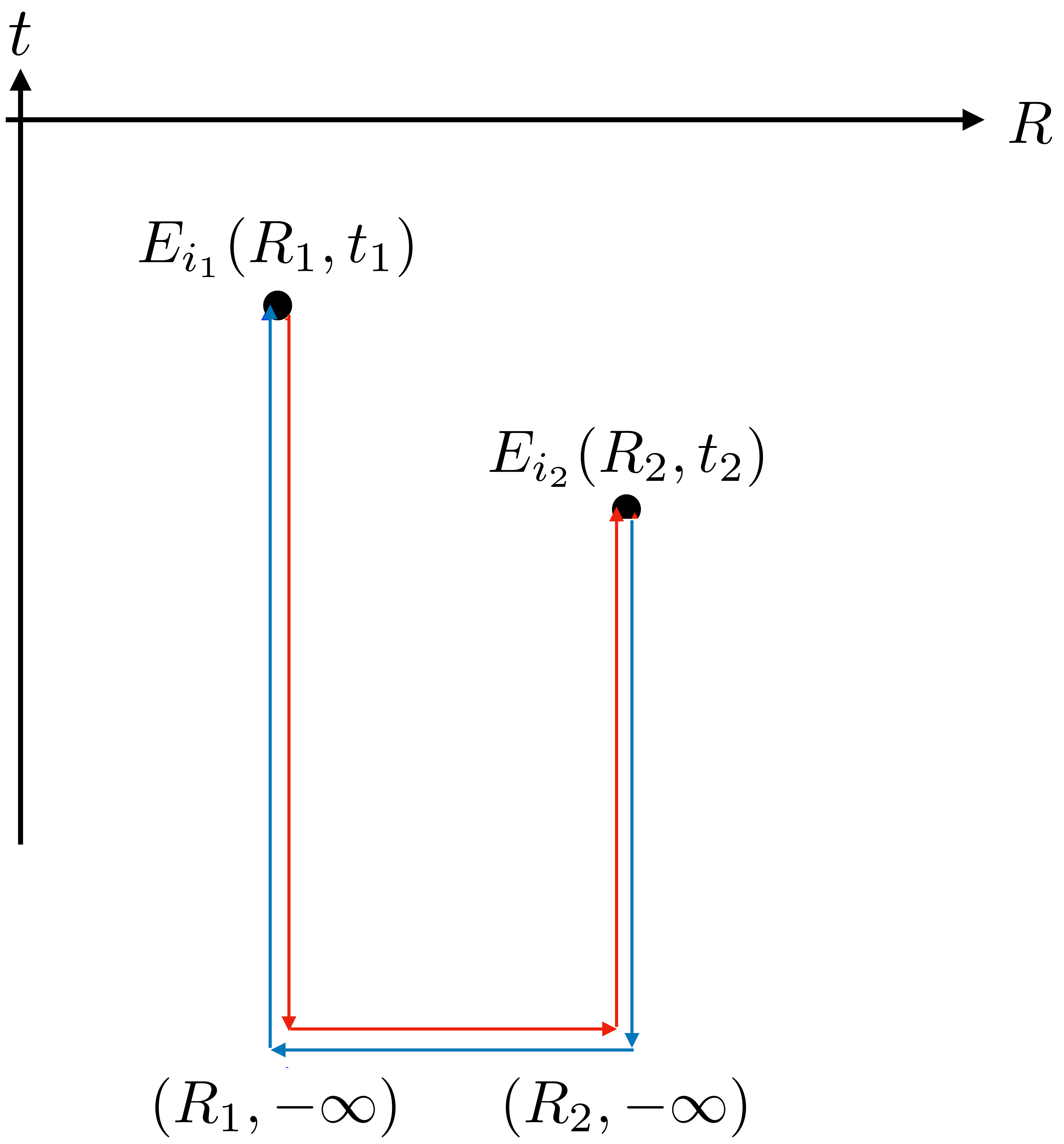

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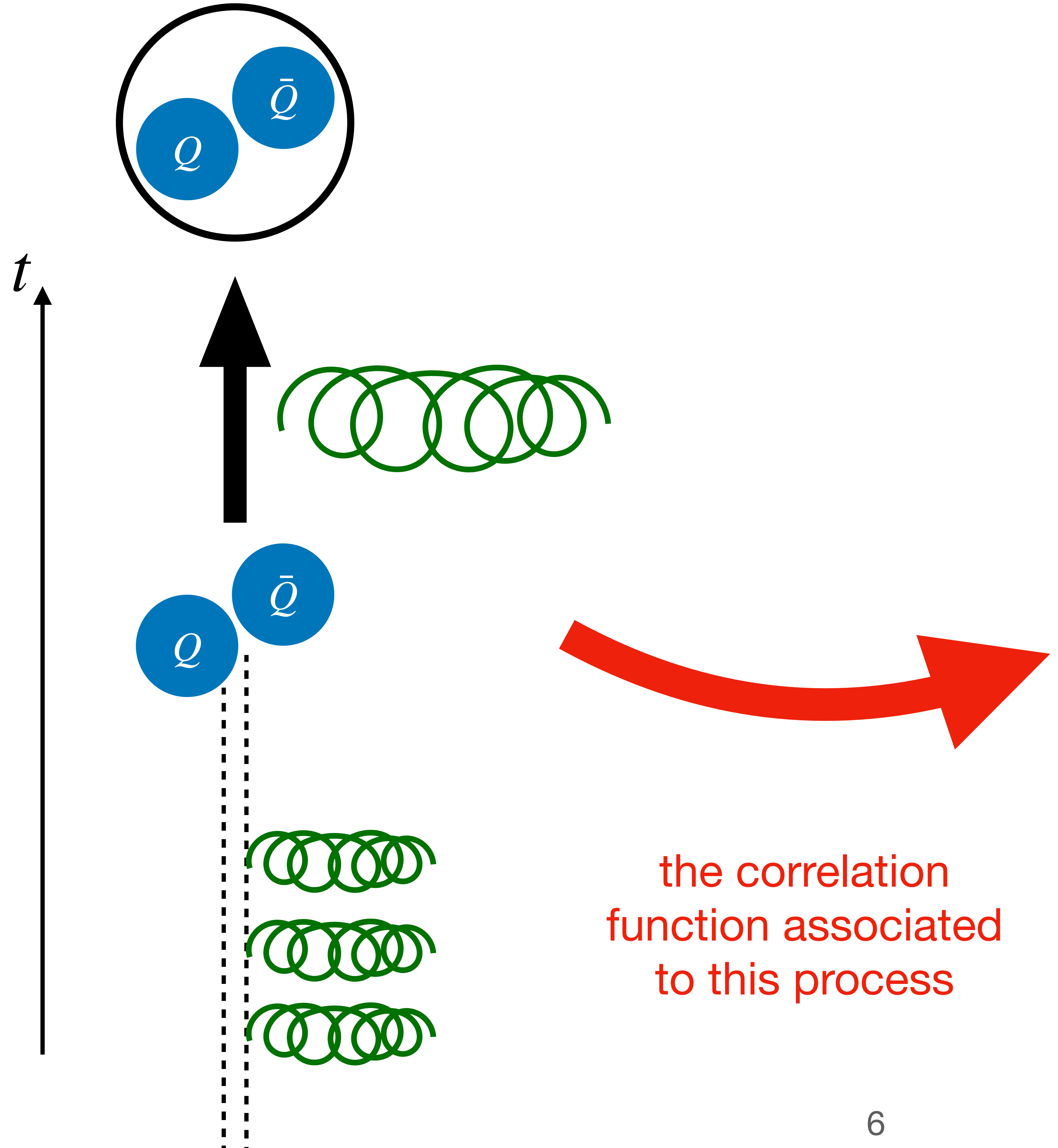

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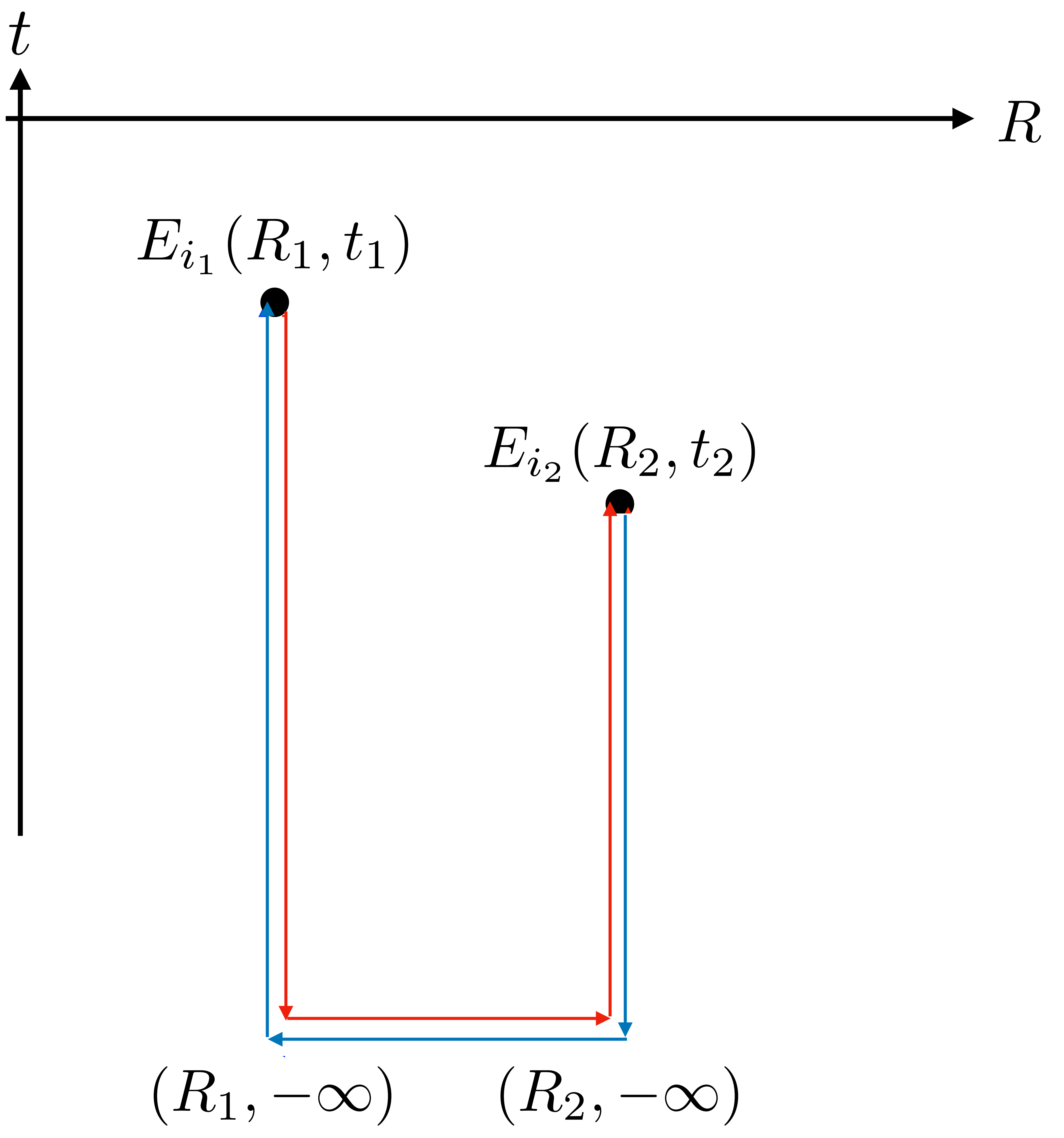

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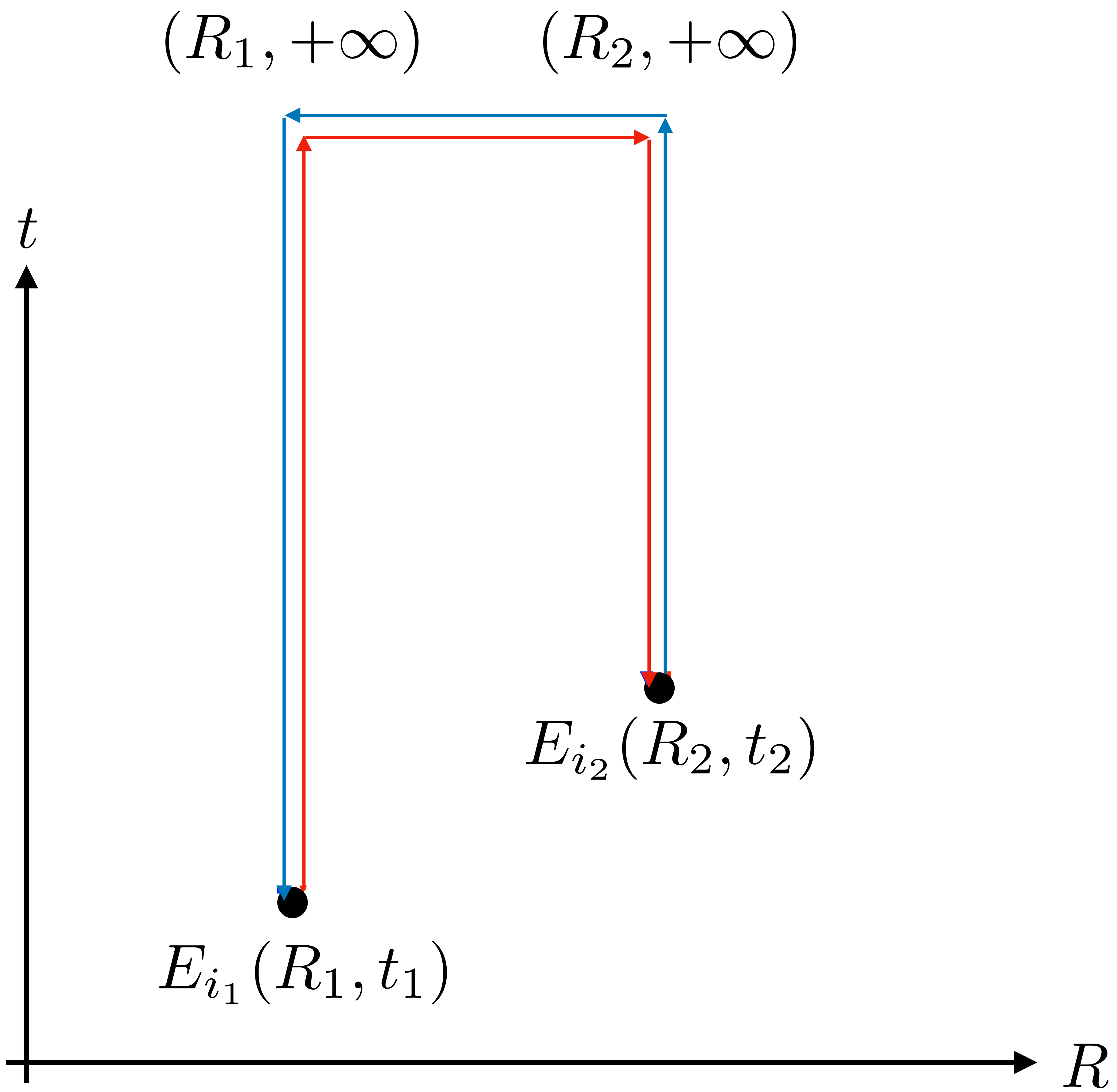

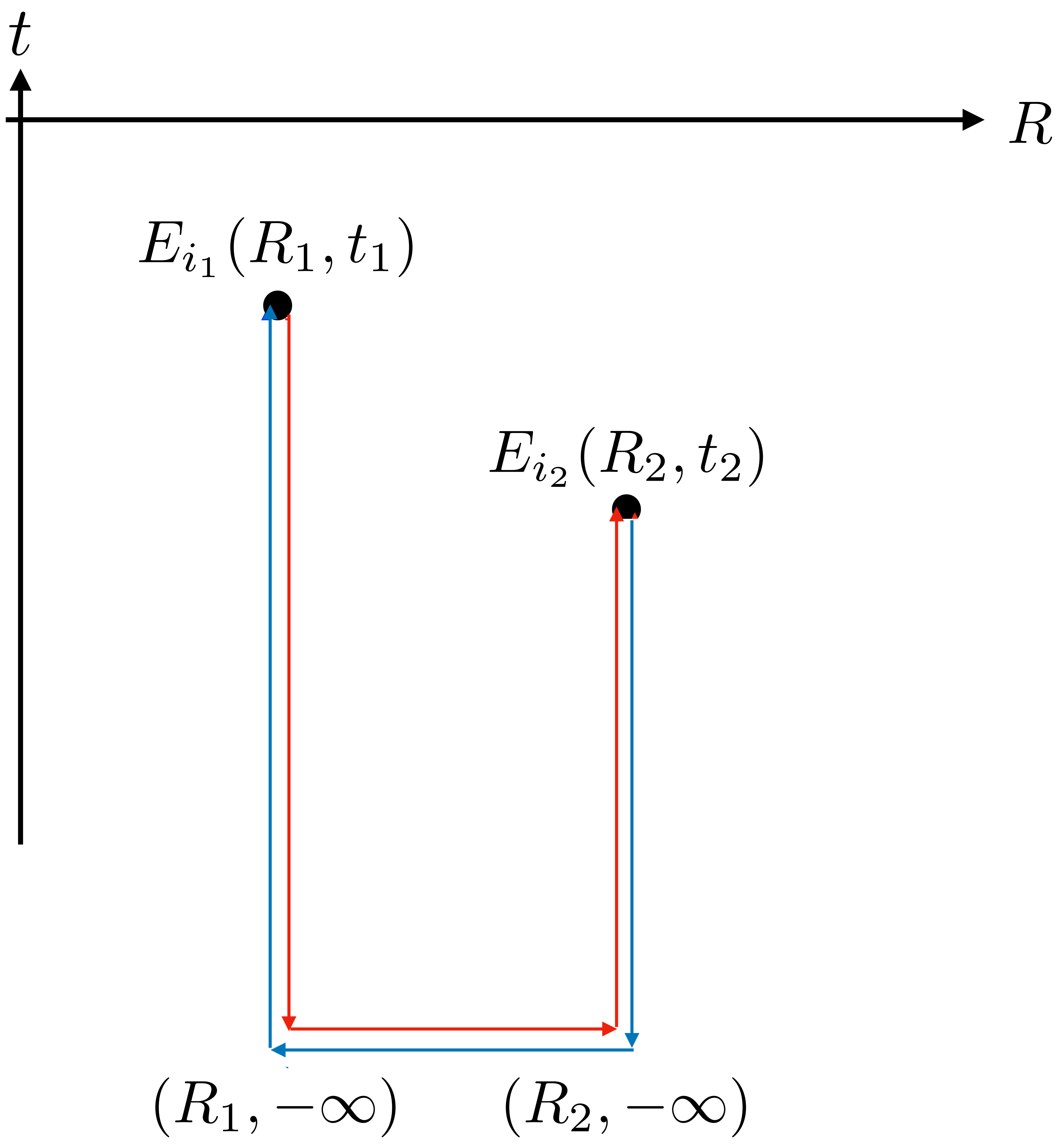

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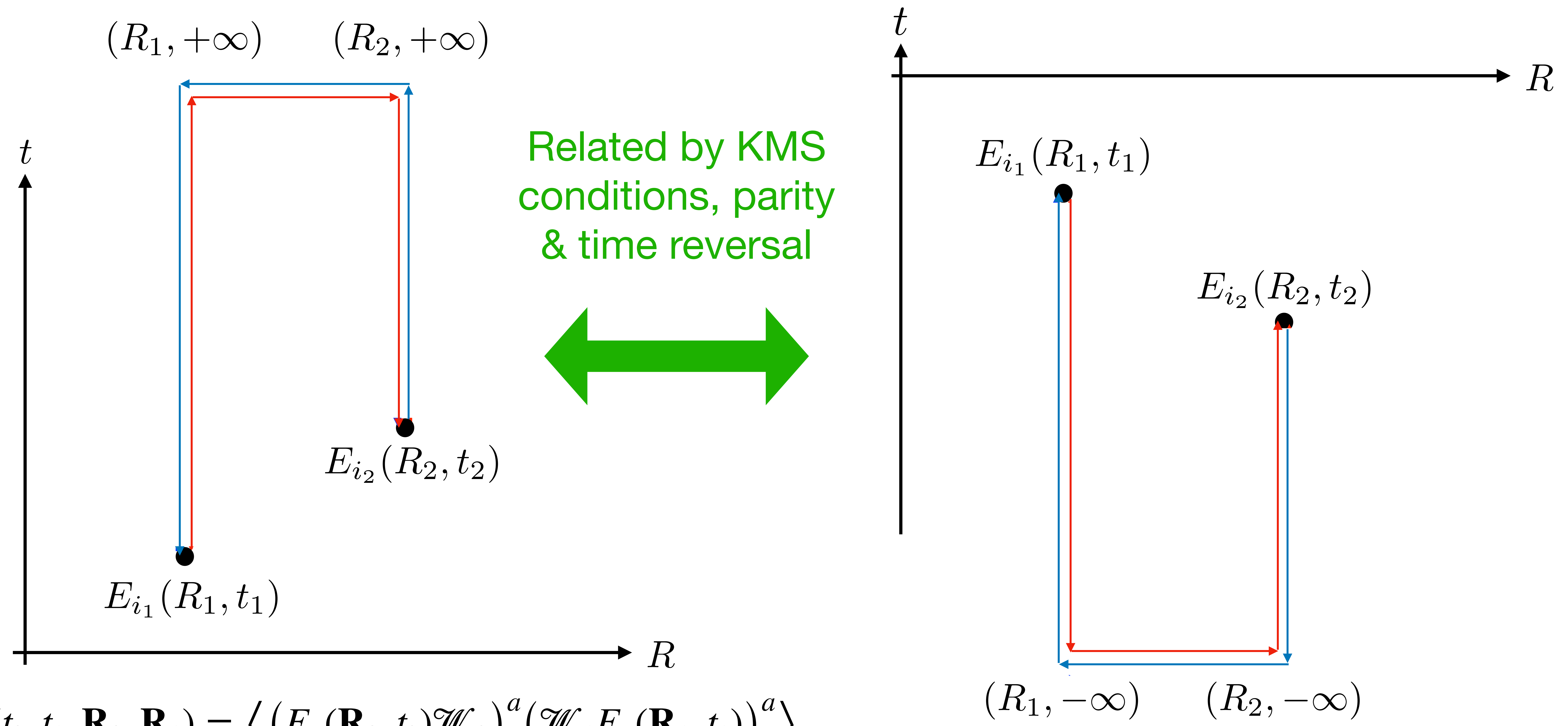

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Why are these correlators interesting?

These determine the dissociation and formation rates of quarkonia in the quantum optical limit:

$$\Gamma^{\mathrm{diss}} \propto \int \frac{\mathrm{d}^3\mathbf{p}_{\mathrm{rel}}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\,|\langle\psi_{\mathscr{B}}|\,\mathbf{r}\,|\Psi_{\mathbf{p}_{\mathrm{rel}}}\rangle|^2\,[g_E^{++}]^{>}_{ii}\left(q^0 = E_{\mathscr{B}} - \frac{\mathbf{p}_{\mathrm{rel}}^2}{M}, \mathbf{q}\right),$$

$$\Gamma^{\mathrm{form}} \propto \int \frac{\mathrm{d}^3\mathbf{p}_{\mathrm{cm}}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{p}_{\mathrm{rel}}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\,|\langle\psi_{\mathscr{B}}|\,\mathbf{r}\,|\Psi_{\mathbf{p}_{\mathrm{rel}}}\rangle|^2\,[g_E^{--}]^{>}_{ii}\left(q^0 = \frac{\mathbf{p}_{\mathrm{rel}}^2}{M} - E_{\mathscr{B}}, \mathbf{q}\right)$$
$$\times f_{\mathscr{S}}(\mathbf{x}, \mathbf{p}_{\mathrm{cm}}, \mathbf{r} = 0, \mathbf{p}_{\mathrm{rel}}, t)\,.$$

They are also directly related to the correlators that define the transport coefficients in the quantum brownian motion limit (see Michael Strickland's talk on 10/03):

$$\gamma \equiv \frac{g^2}{6N_c} \mathrm{Im} \int_{-\infty}^{\infty} ds \, \langle \mathcal{T} \, E^{a,i}(s, \mathbf{0}) E^{a,i}(0, \mathbf{0}) \rangle \, ,$$

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$$\gamma \equiv \frac{g^2}{6N_c} \mathrm{Im} \int_{-\infty}^{\infty} ds \, \langle \mathcal{T} \, E^{a,i}(s,\mathbf{0}) \mathscr{W}^{ab}[(s,\mathbf{0}),(0,\mathbf{0})] E^{b,i}(0,\mathbf{0}) \rangle \, ,$$

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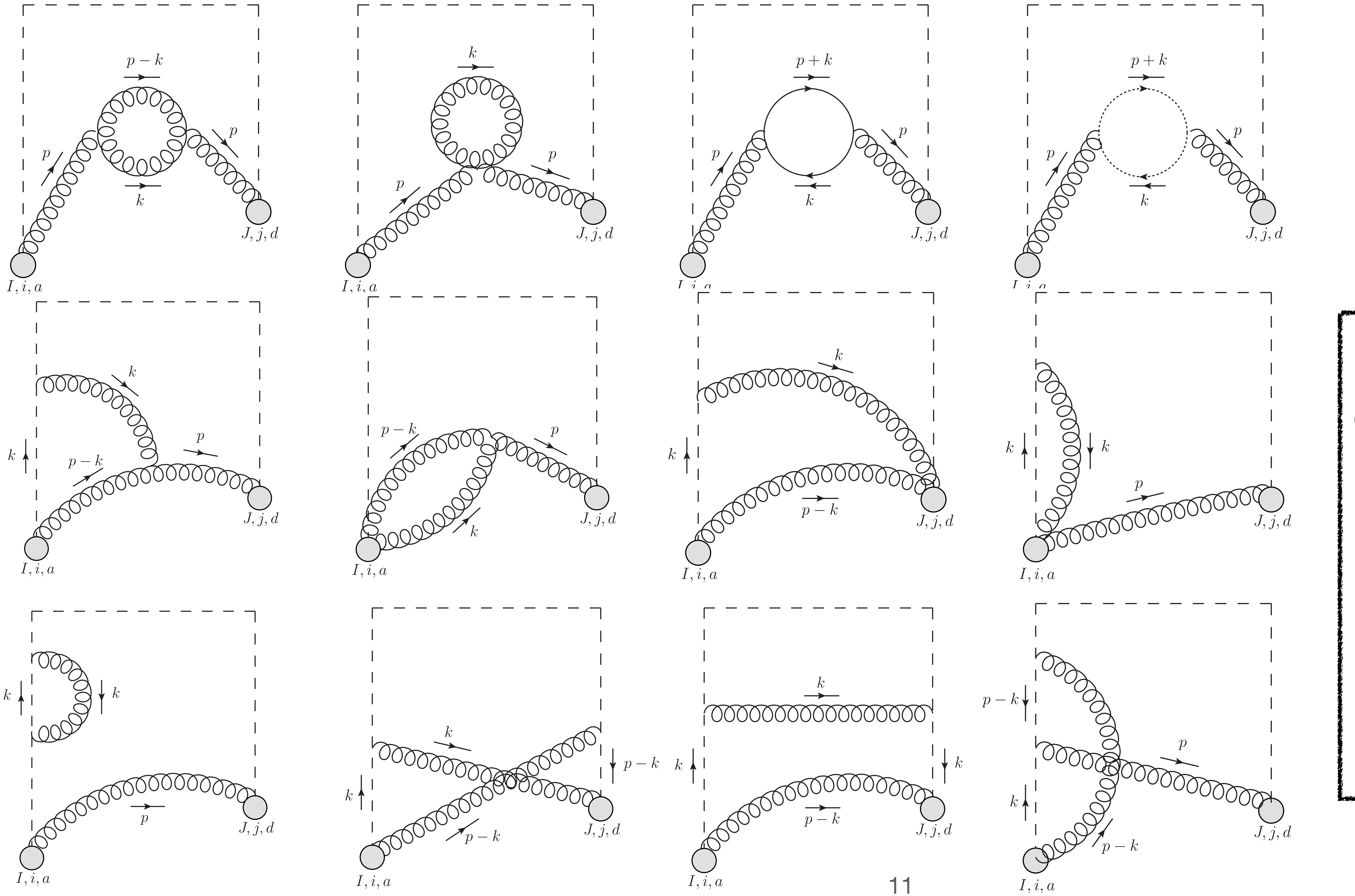
Adjoint representation Wilson lines!

(as appropriate for color octet states)

So, let's calculate

T. Binder, K. Mukaida, B. Scheihing-Hitschfeld and X. Yao, hep-ph/2107.03945

Weakly coupled calculation in QCD



The real-time calculation proceeds by evaluating these diagrams (+ some permutations of them) on the Schwinger-Keldysh contour

The spectral function at NLO

It is simplest to write the integrated spectral function:

$$\varrho_E^{++}(p_0) = \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\delta^{ad}\delta_{ij}\big[\rho_E^{++}\big]^{da}_{ji}(p_0,\mathbf{p})\,.$$

We found

$$g^2\varrho_E^{++}(p_0) = \frac{g^2(N_c^2-1)p_0^3}{(2\pi)^3}\left\{4\pi^2 + g^2\left[\left(\frac{11}{12}N_c - \frac{1}{3}N_f\right)\ln\left(\frac{\mu^2}{4p_0^2}\right) + \left(\frac{149}{36}+\frac{\pi^2}{3}\right)N_c - \frac{10}{9}N_f + F\left(\frac{p_0}{T}\right)\right]\right\}$$

T. Binder, K. Mukaida, B. Scheihing-Hitschfeld and X. Yao, hep-ph/2107.03945

The spectral function at NLO

and a comparison with its heavy quark counterpart

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and the heavy quark counterpart is, with the same T-dependent function $F(p_0/T)$,

Y. Burnier, M. Laine, J. Langelage and L. Mether, hep-ph/1006.0867

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But they look so similar…

Heavy quark and quarkonia correlators

a small, yet consequential difference

The heavy quark diffusion coefficient can be defined from the real-time correlator J. Casalderrey-Solana and D. Teaney, hep-ph/0605199; see also A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

$$\left\langle \mathrm{Tr}_{\mathrm{color}} \left[U(-\infty, t) E_i(t) U(t,0) E_i(0) U(0, -\infty) \right] \right\rangle_T ,$$

whereas for quarkonia the relevant quantity is

$$T_F \left\langle E_i^a(t) \mathscr{W}^{ab}(t,0) E_i^b(0) \right\rangle_T .$$

Heavy quark and quarkonia correlators

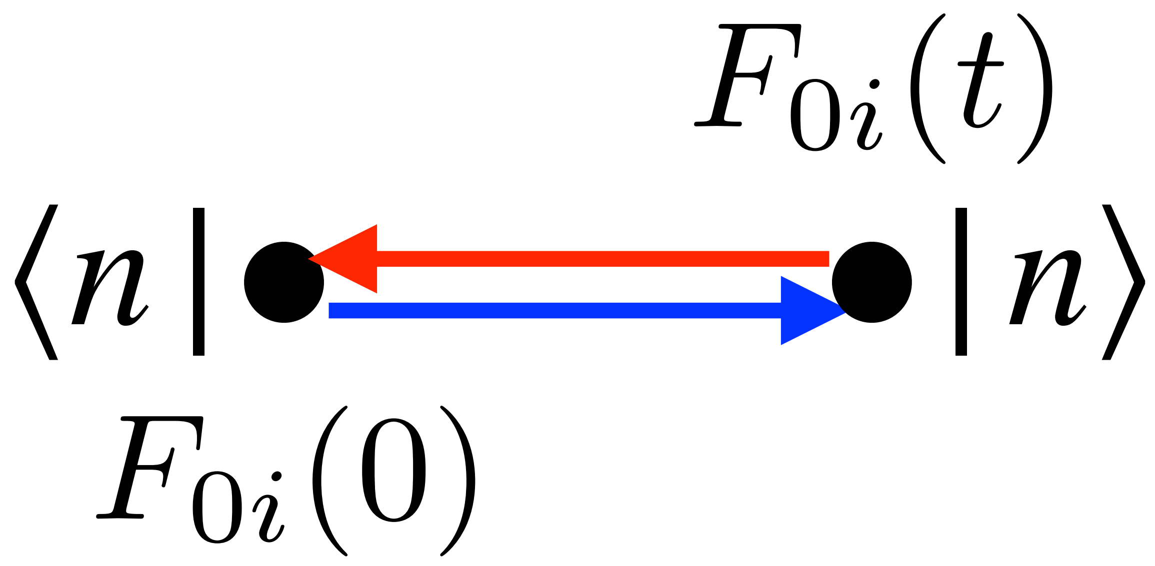
a small, yet consequential difference

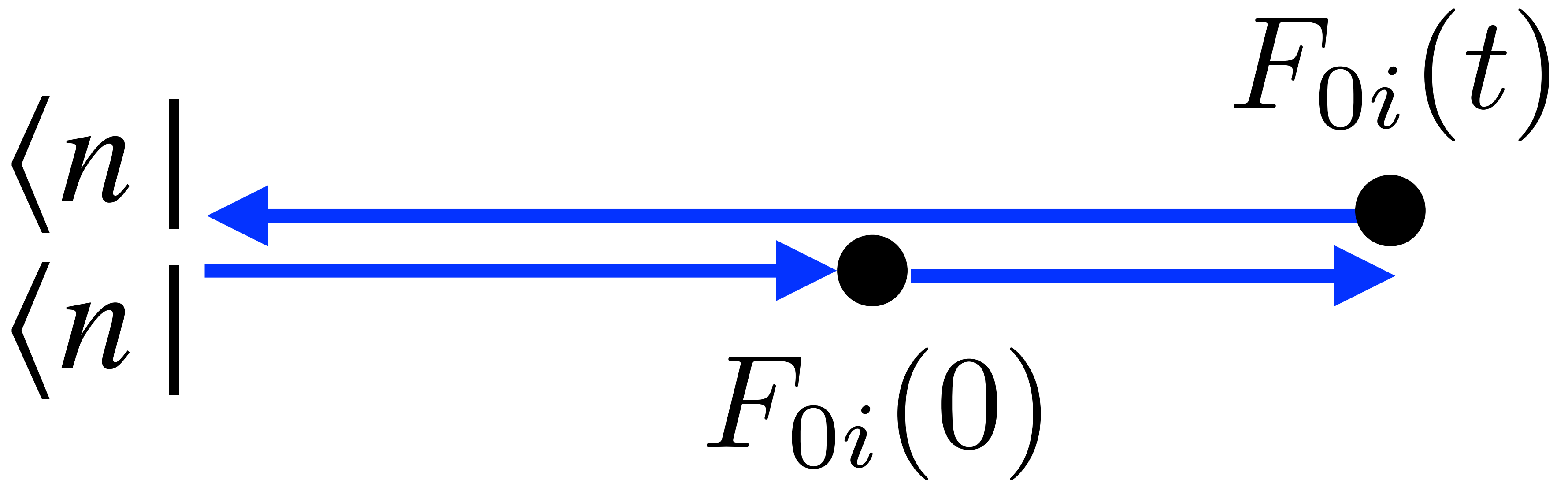
A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

What we just found, and had been noticed even earlier by Eller, Ghiglieri and Moore, is simply stating that:

They compared M. Eidemuller and M. Jamin, hep-ph/9709419 with
Y. Burnier, M. Laine, J. Langelage and L. Mether, hep-ph/1006.0867

$$T_F \left\langle E_i^a(t) \mathscr{W}^{ab}(t,0) E_i^b(0) \right\rangle_T \neq \left\langle \mathrm{Tr}_{\text{color}} \left[U(-\infty, t) E_i(t) U(t,0) E_i(0) U(0, -\infty) \right] \right\rangle_T$$





An axial gauge puzzle

an apparent (but not actual) inconsistency

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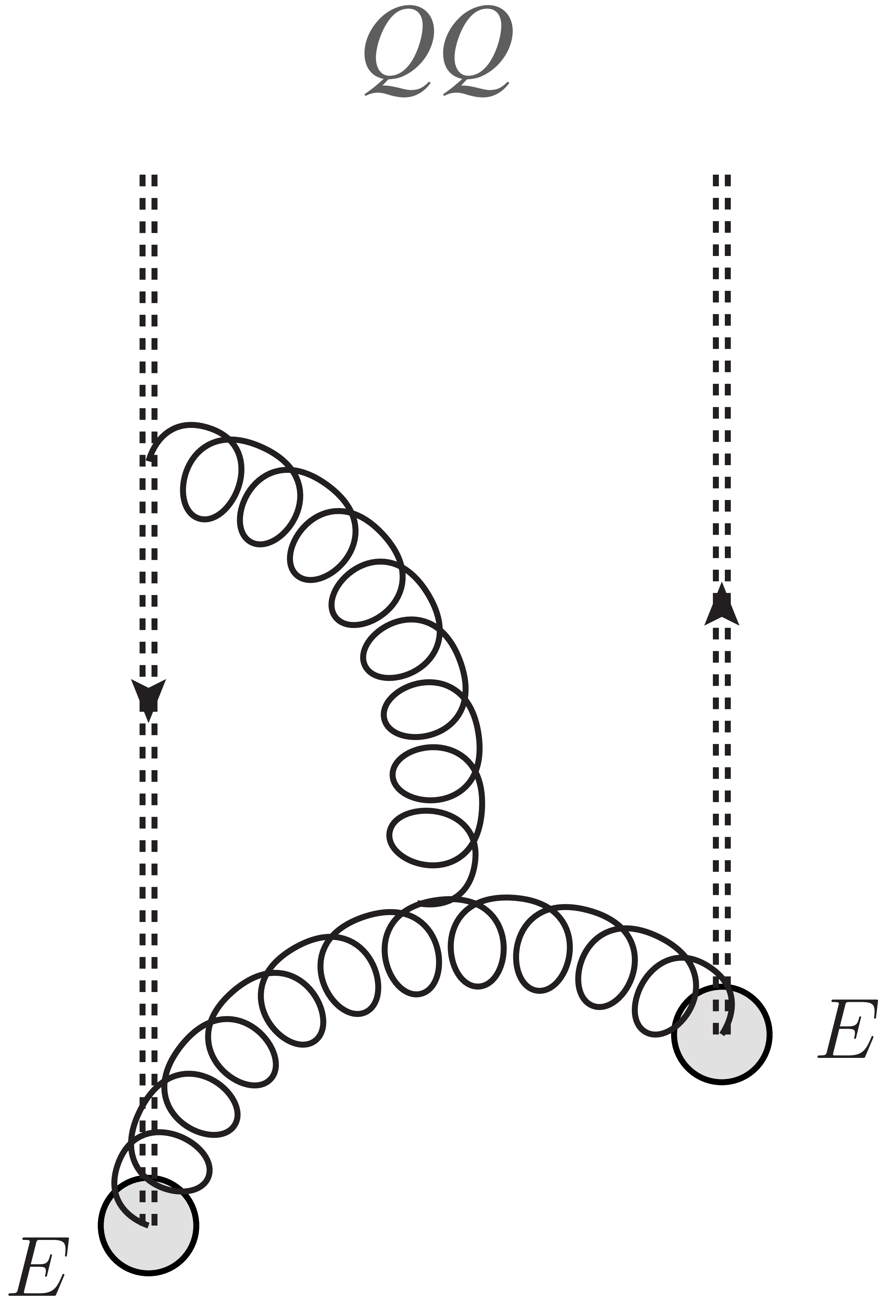
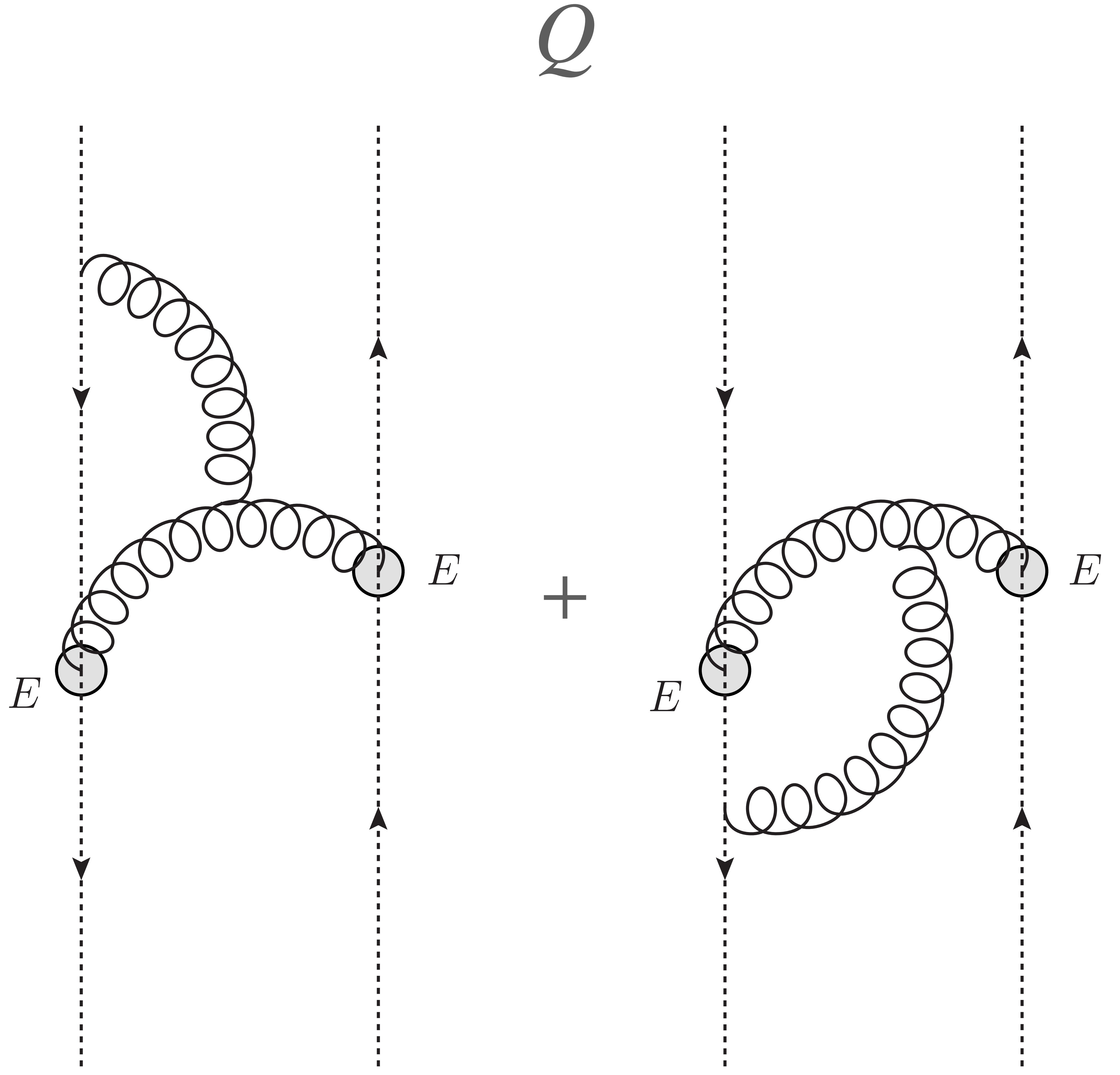
The difference in terms of diagrams

operator ordering is crucial!



Perturbatively, one can isolate the difference between the correlators to these diagrams.



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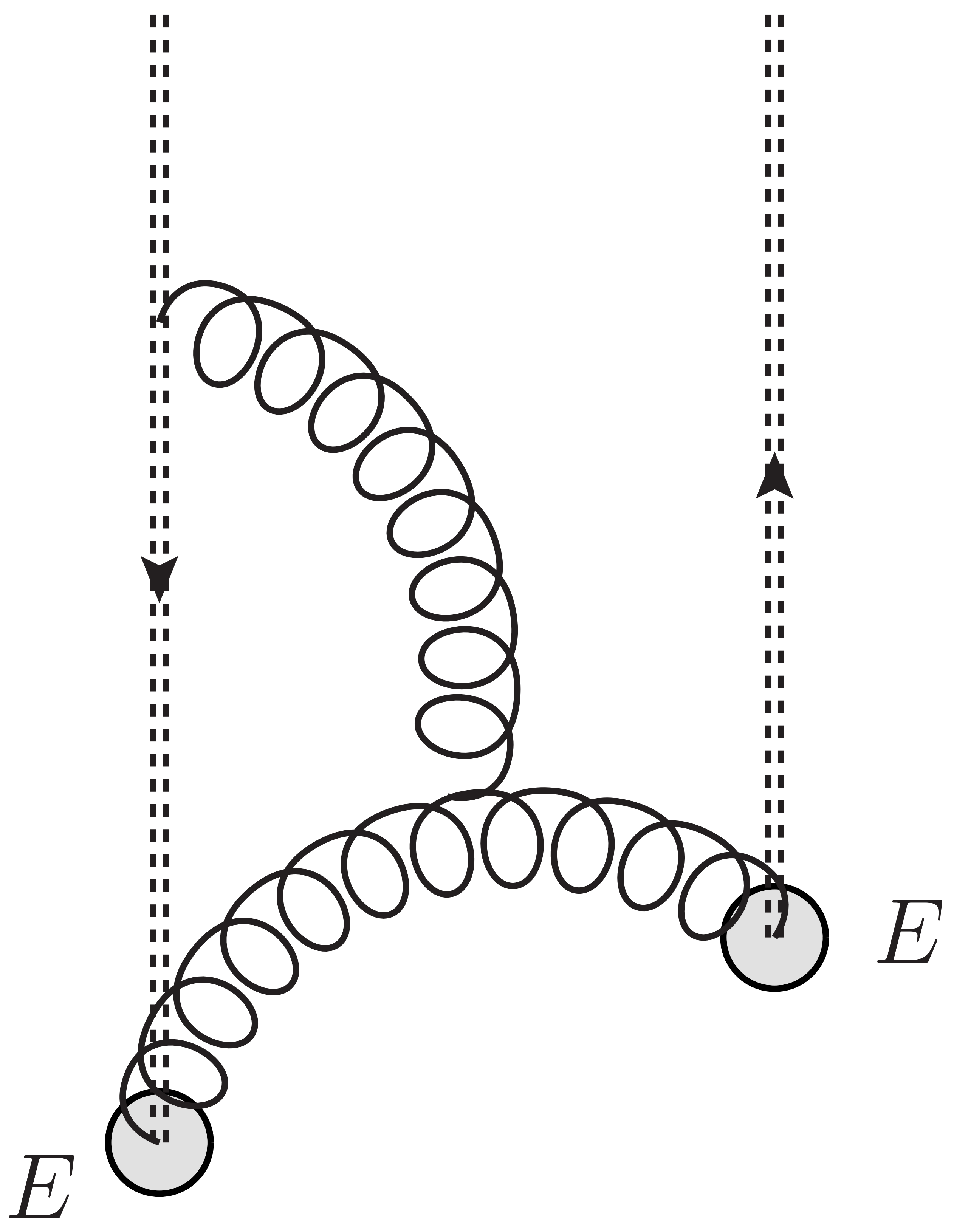
$Q\bar{Q}$



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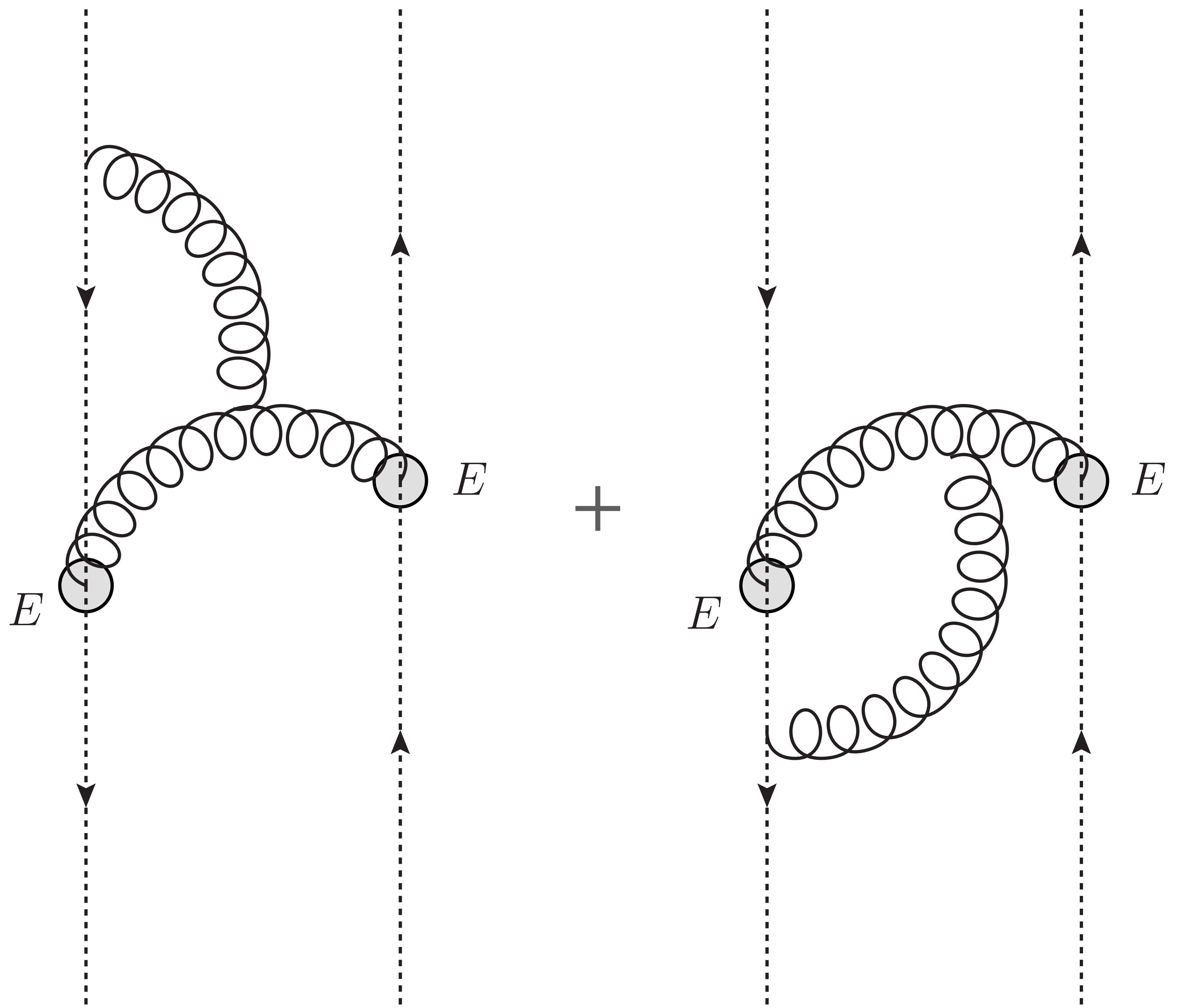
The difference is due to different operator orderings (different possible gluon insertions).

Q



Gauge independence of the difference

explicit gauge interpolation

- We performed an explicit calculation of the difference between the correlators in vacuum at NLO, with a gauge condition $G_M^a[A] = \frac{1}{\lambda} A_0^a(x) + \partial^\mu A_\mu^a(x)$.

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$\Longrightarrow$ The $Q\bar{Q}$ and Q correlators are different, gauge invariant quantities.

So, we understand the weakly coupled limit in QCD.

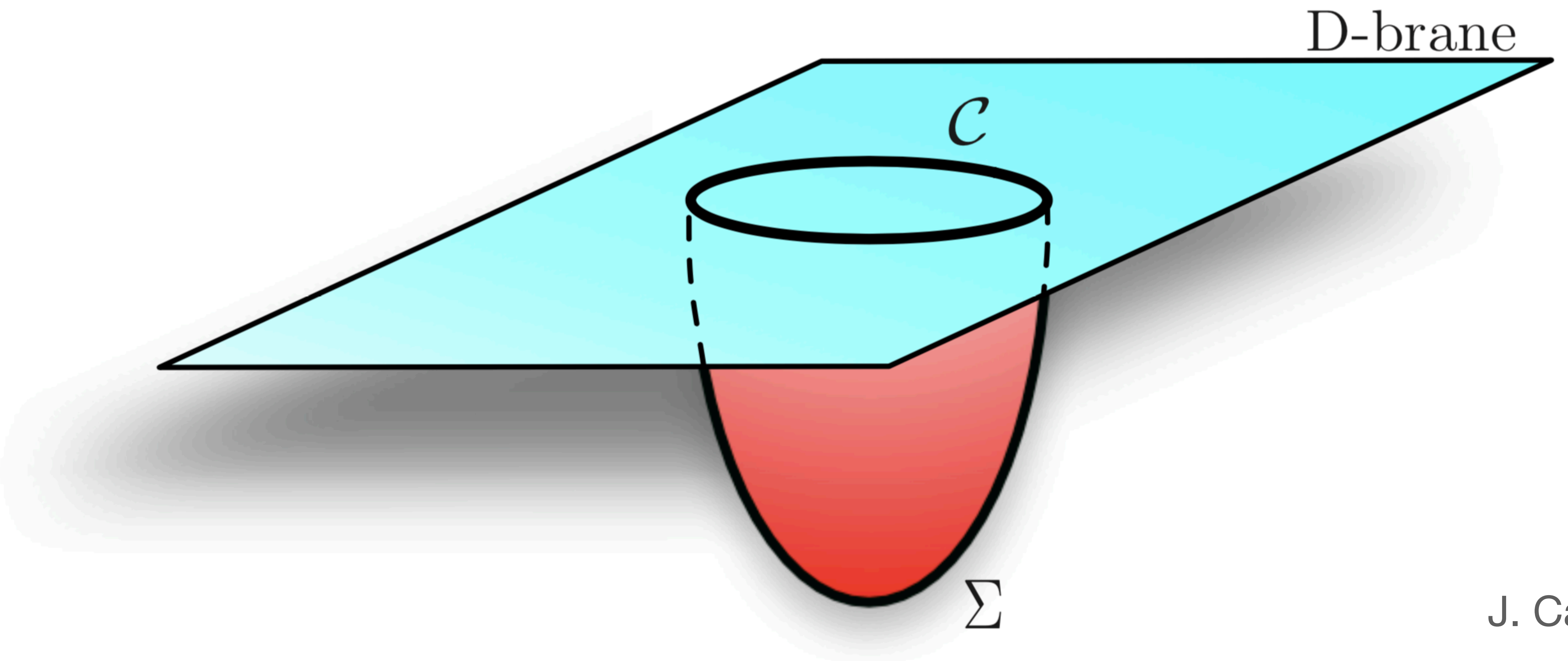
What about at strong coupling?

[**] J. Maldacena, hep-th/9711200

Wilson loops in AdS/CFT
setup

- The holographic duality provides a way to formulate the calculation of analogous correlators in strongly coupled theories. [**]
    - Wilson loops can be evaluated by solving classical equations of motion:

$$\langle W[\mathscr{C} = \partial\Sigma]\rangle_T = e^{iS_{\mathrm{NG}}[\Sigma]}$$



J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, hep-ph/1101.0618

Strongly coupled calculation in $\mathcal{N} = 4$ SYM

setup

- Field strength insertions along a Wilson loop can be generated by taking variations of the path $\mathscr{C}$:

$$\frac{\delta}{\delta f^\mu(s_2)}\frac{\delta}{\delta f^\nu(s_1)}W[\mathscr{C}_f]\bigg|_{f=0} = (ig)^2\mathrm{Tr}_{\mathrm{color}}\Big[U_{[1,s_2]}F_{\mu\rho}(\gamma(s_2))\dot{\gamma}^\rho(s_2)U_{[s_2,s_1]}F_{\nu\sigma}(\gamma(s_1))\dot{\gamma}^\sigma(s_1)U_{[s_1,0]}\Big]$$

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- Same in spirit as the lattice calculation of the heavy quark diffusion coefficient:

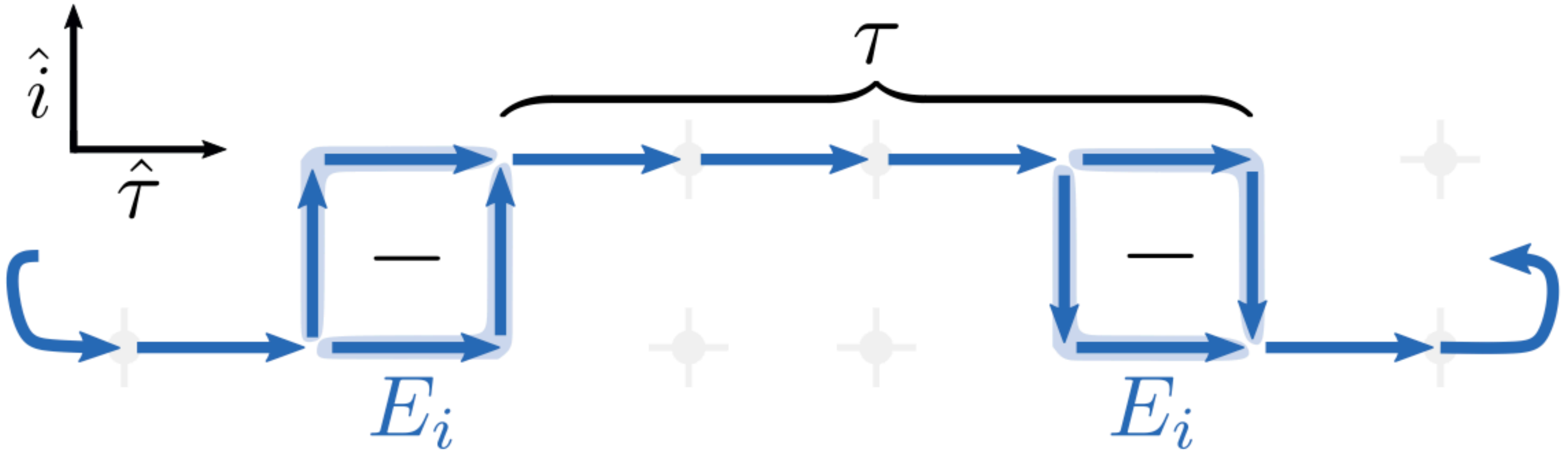
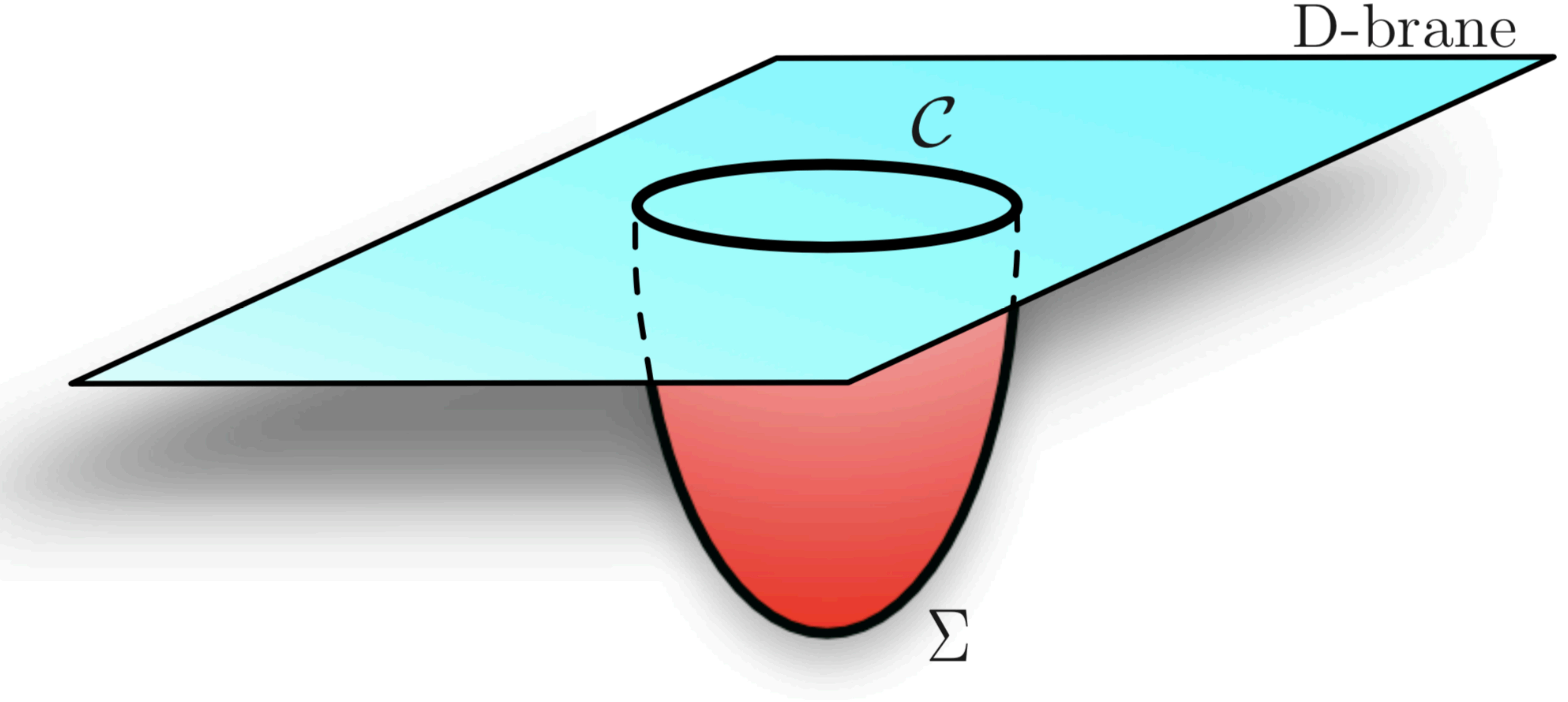
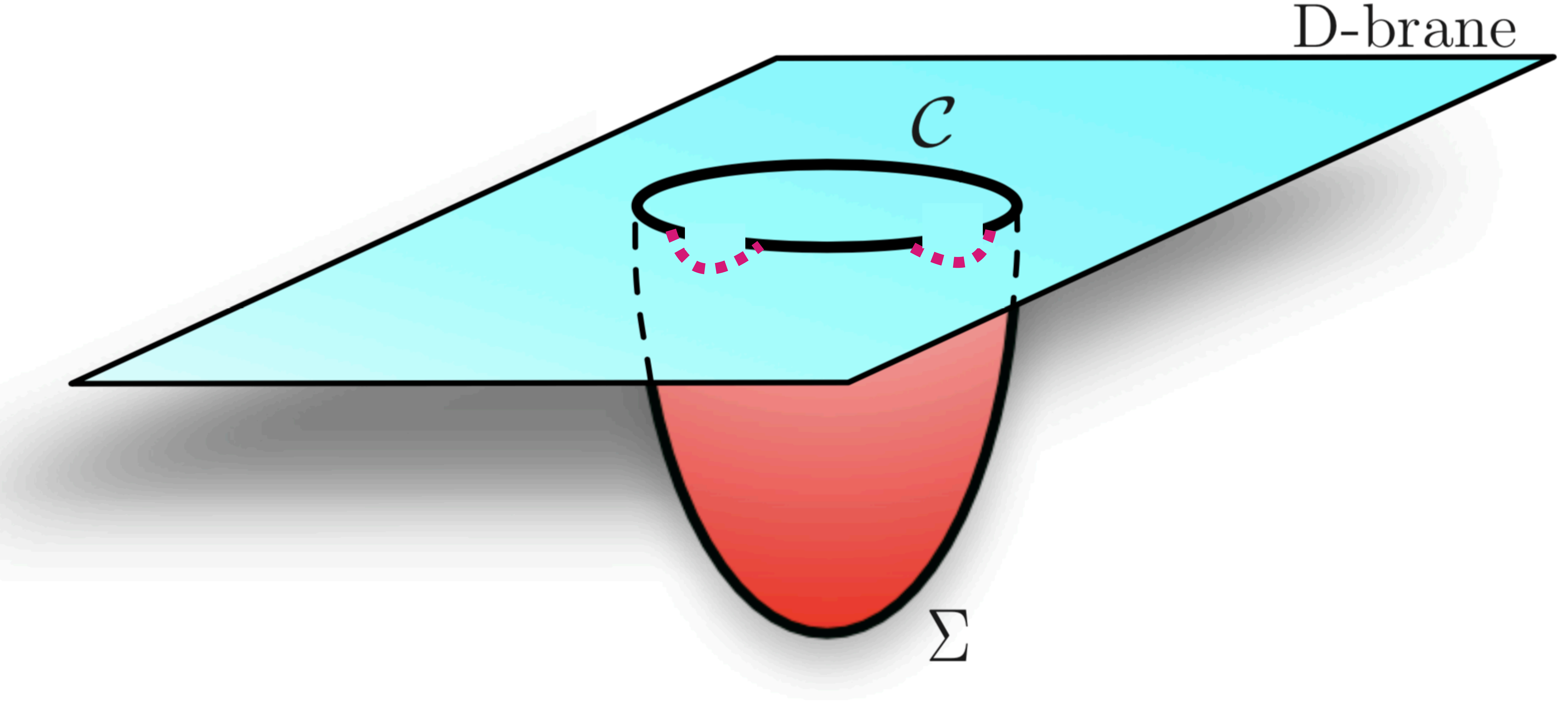


Figure from Luis Altenkort's talk on 10/19


D-brane
$\mathcal{C}$
Σ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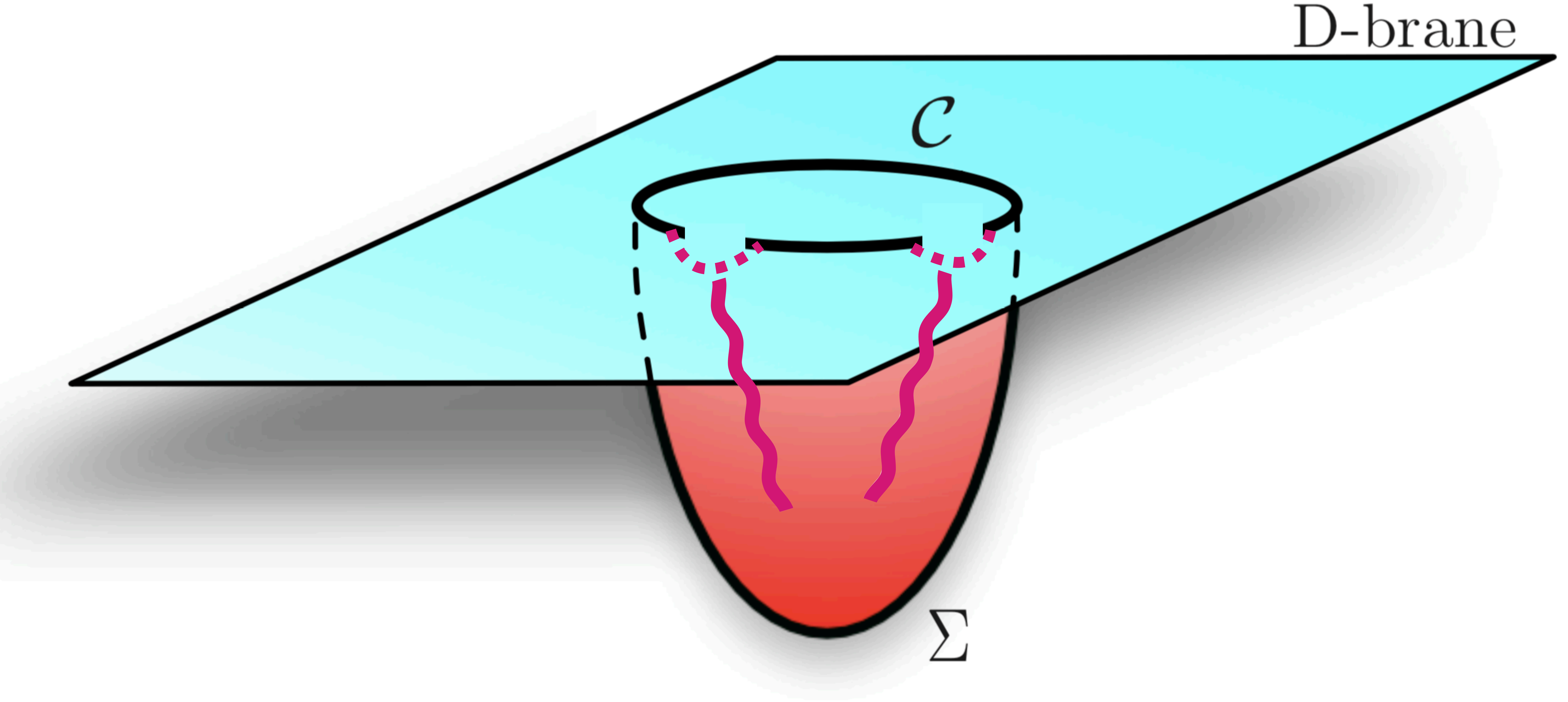
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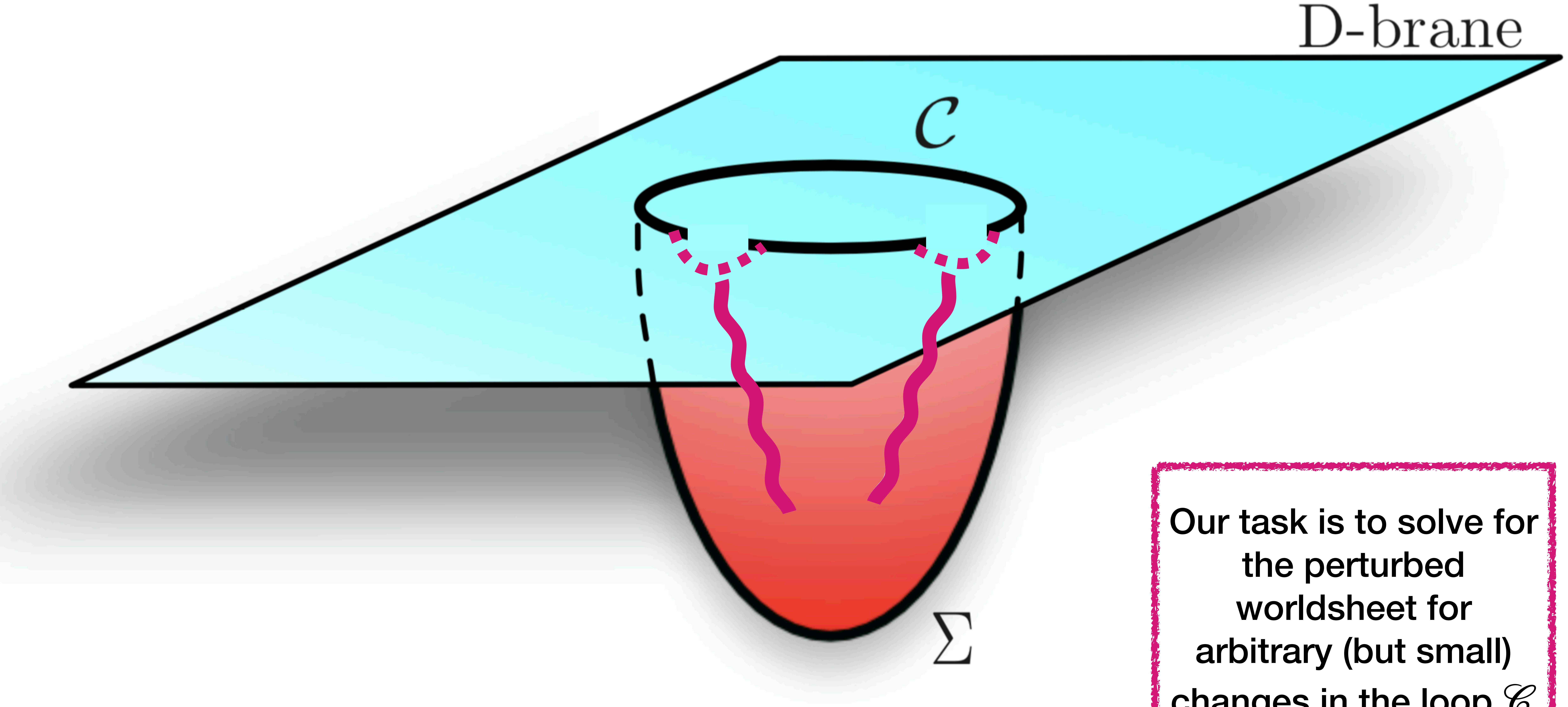


Our task is to solve for the perturbed worldsheet for arbitrary (but small) changes in the loop $\mathscr{C}$

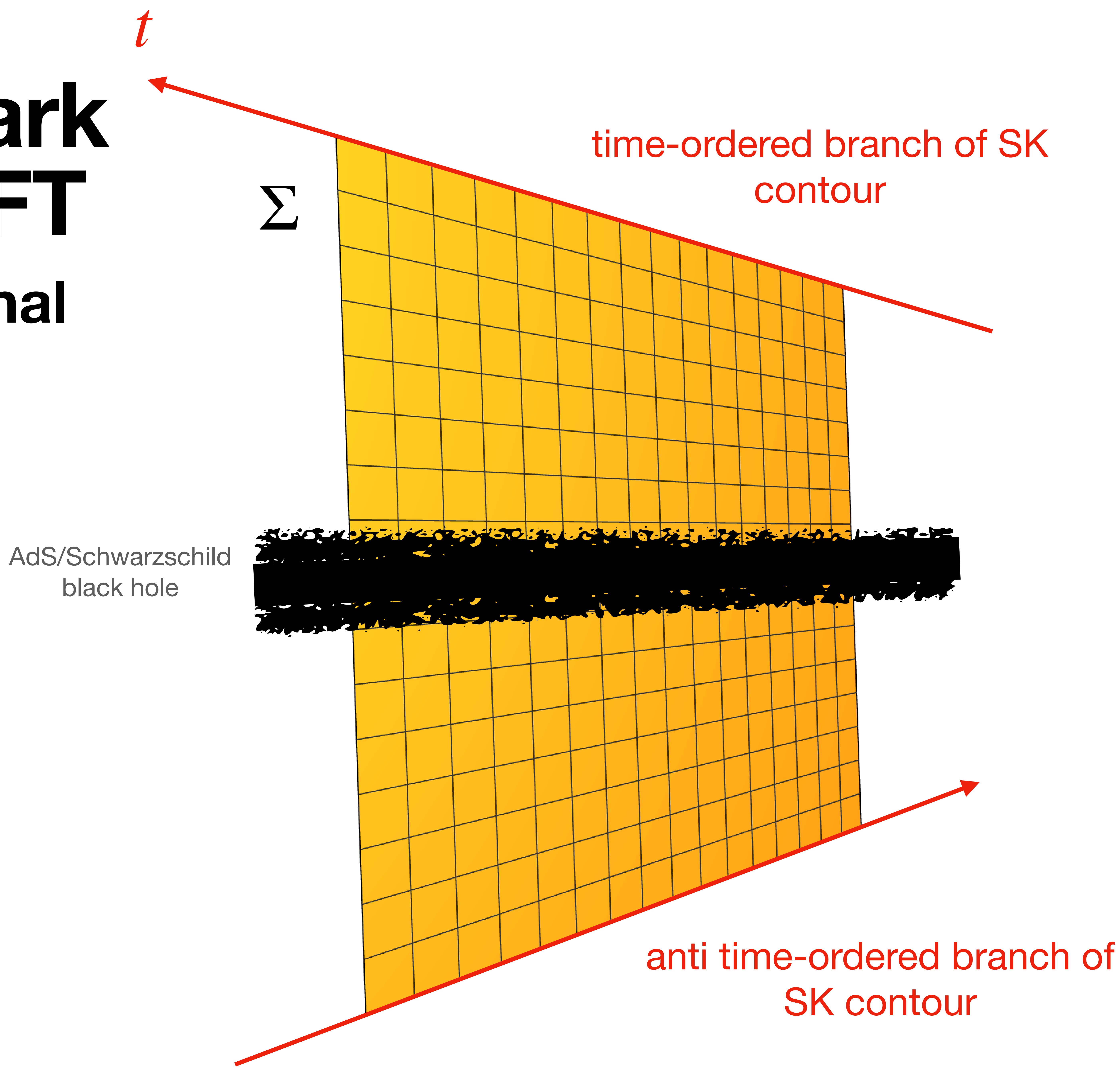
[***] J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

Review: Heavy Quark Diffusion in AdS/CFT

using the same computational technique

Steps of the calculation:

1. Find the appropriate background solution



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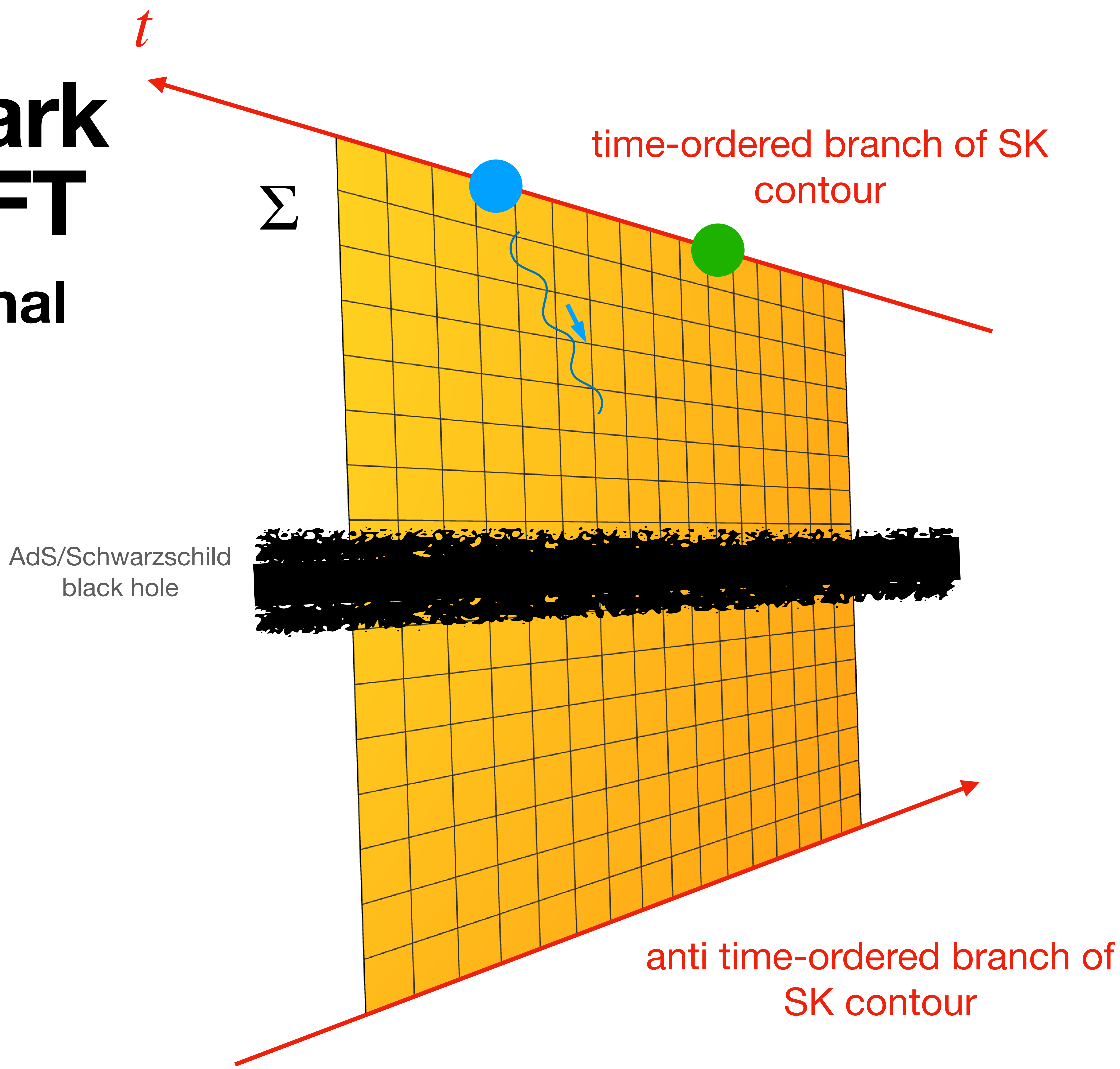

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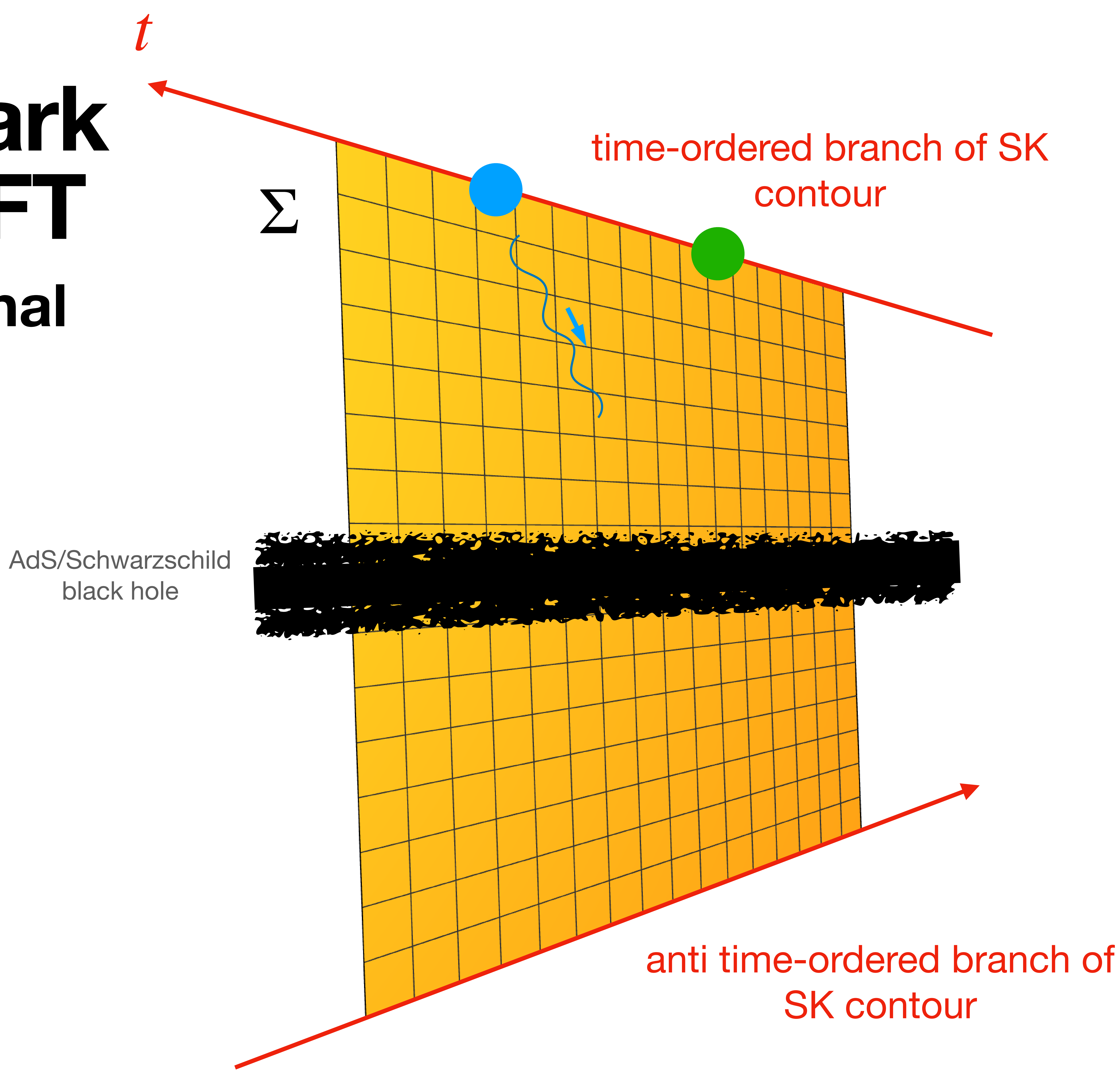

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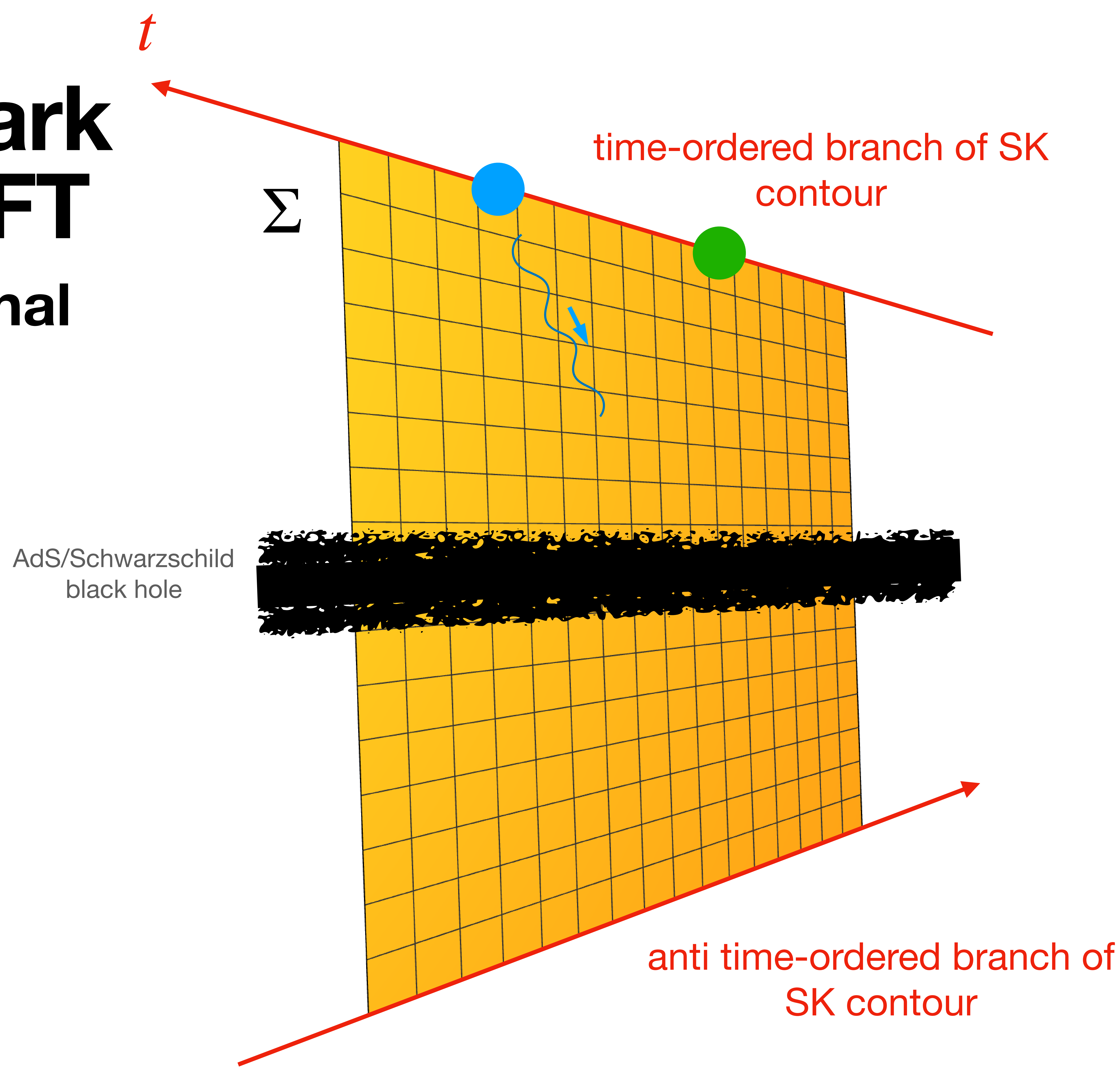
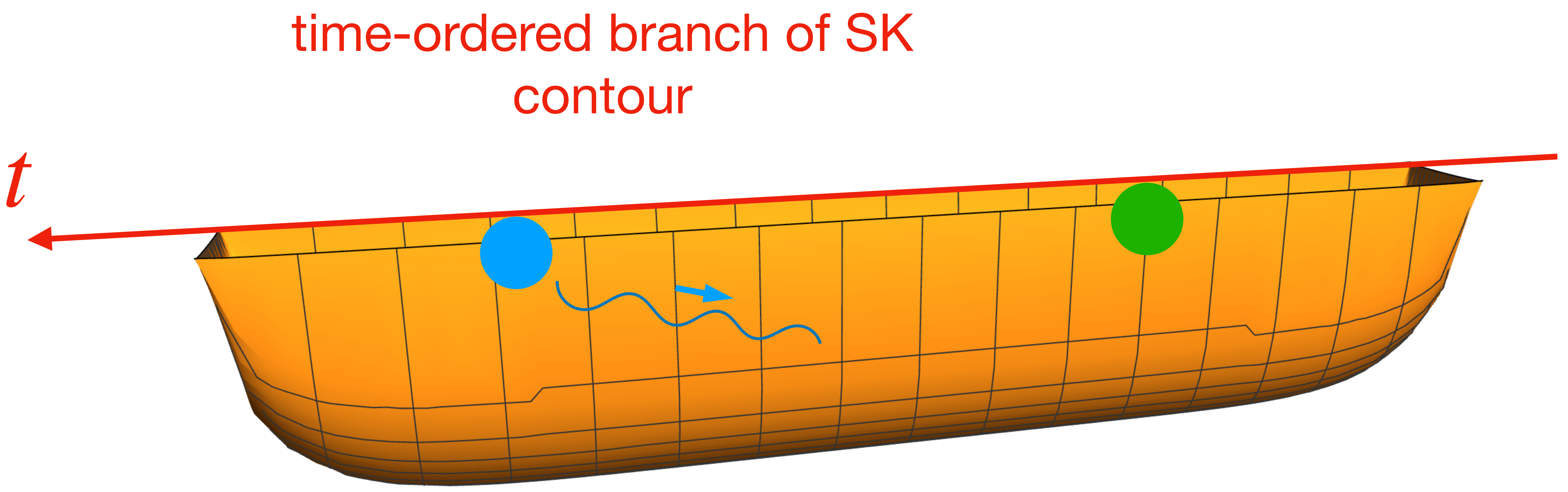
From here: $\kappa = \pi\sqrt{g^2 N_c}\, T^3$



Quarkonia correlator in AdS/CFT

Quarkonium transport in AdS/CFT

Conceptually, same steps as before.



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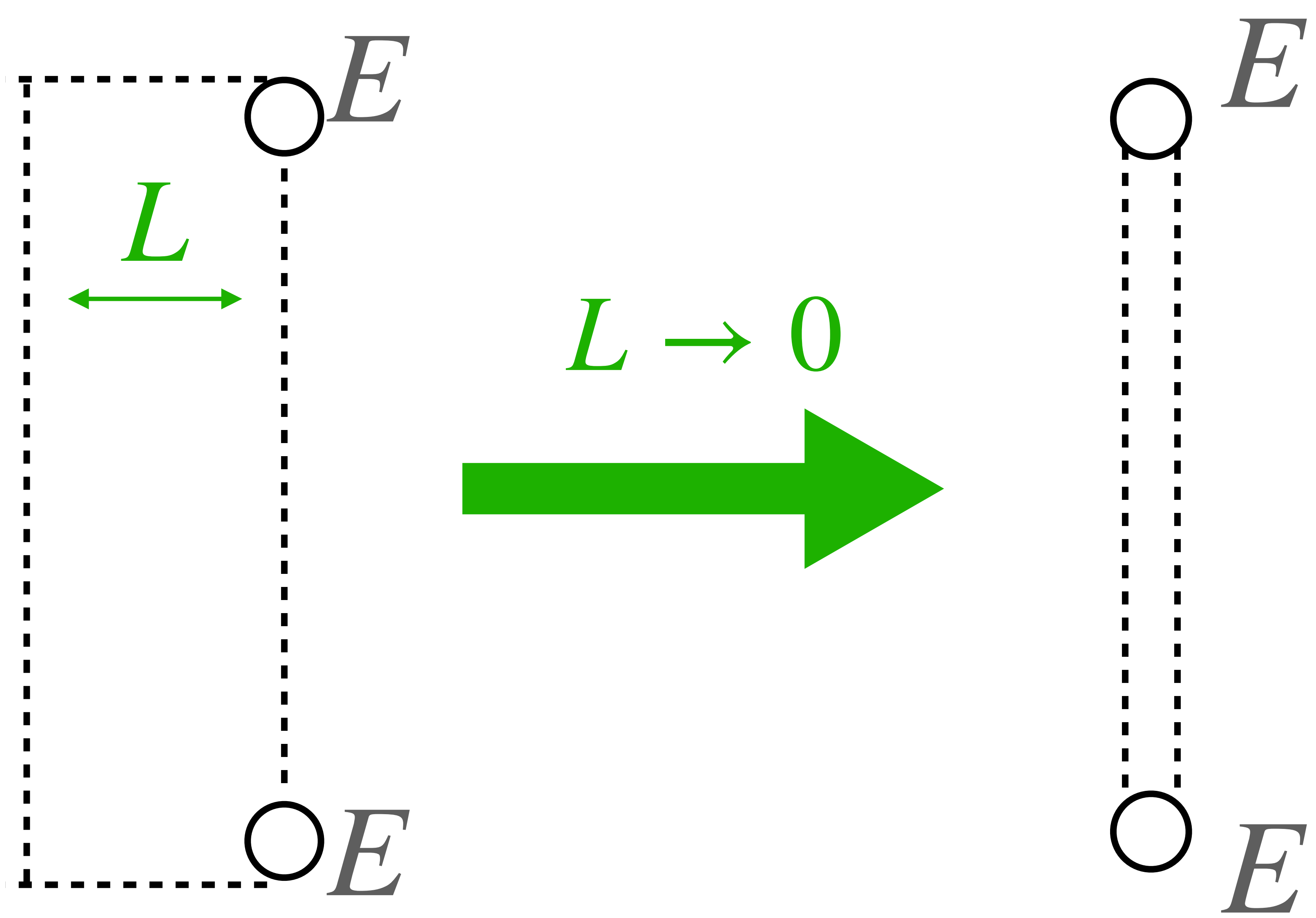
AdS/ Schwarzschild black hole



G. Nijs, B. Scheihing-Hitschfeld and X. Yao, hep-ph/2211.XXXXX

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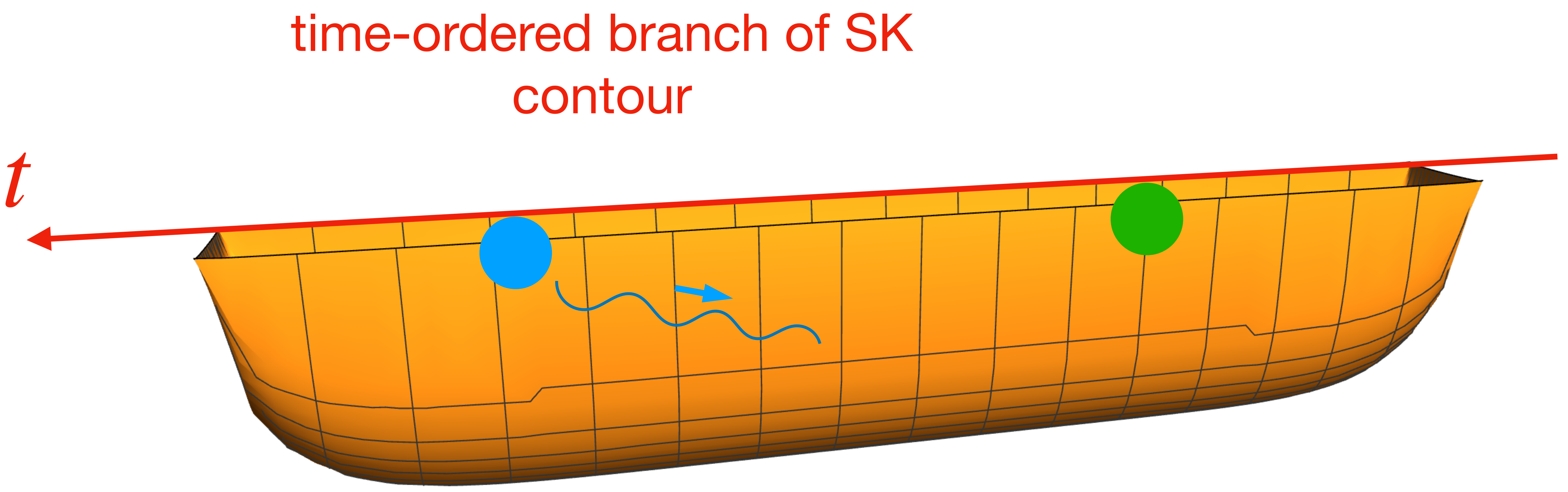

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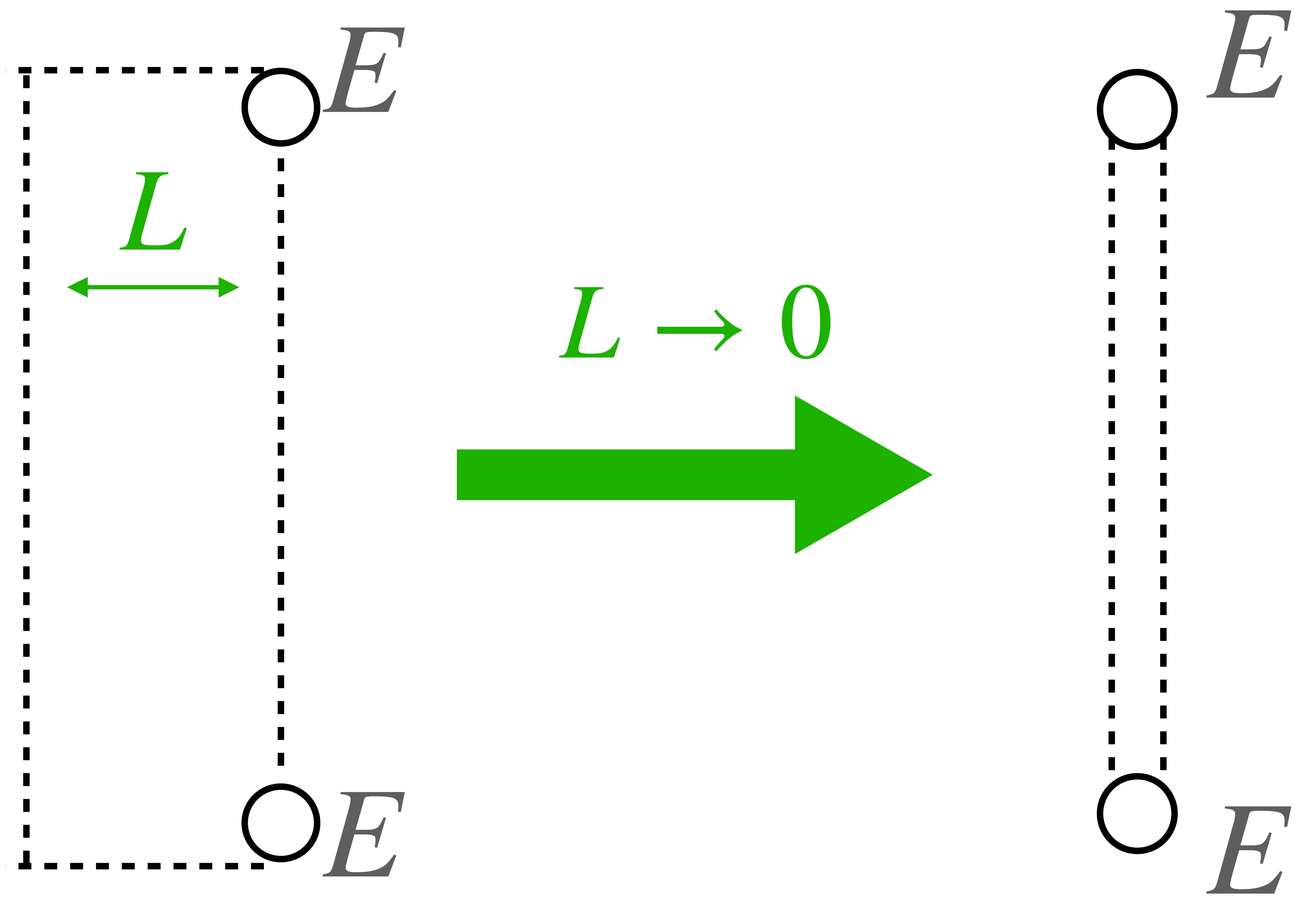

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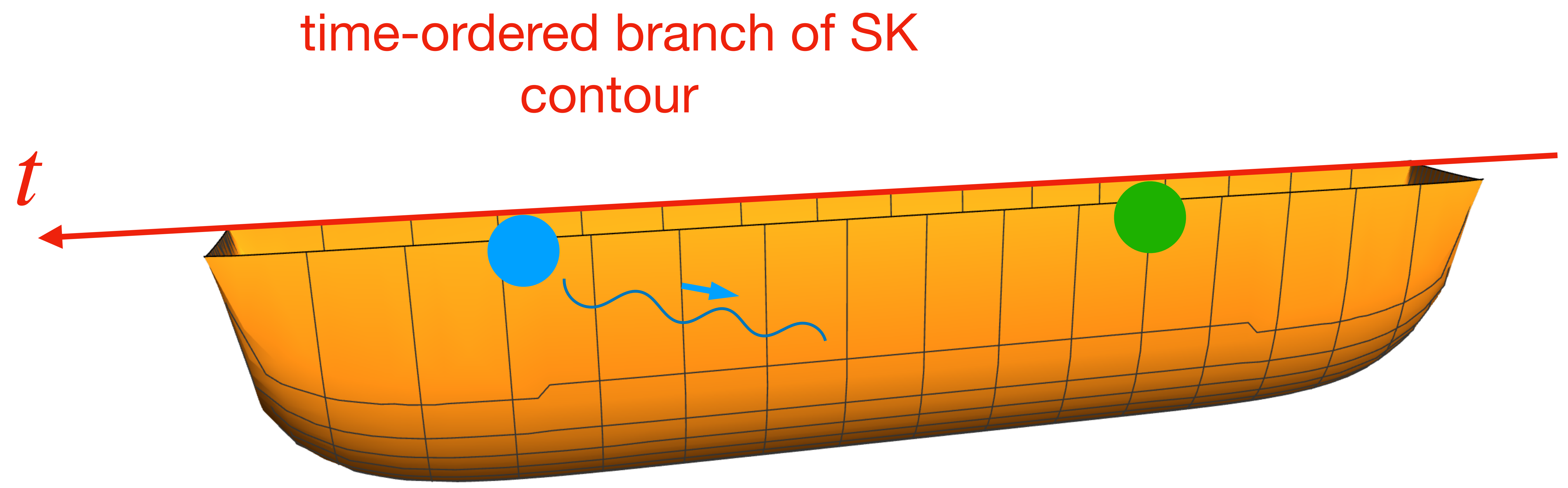

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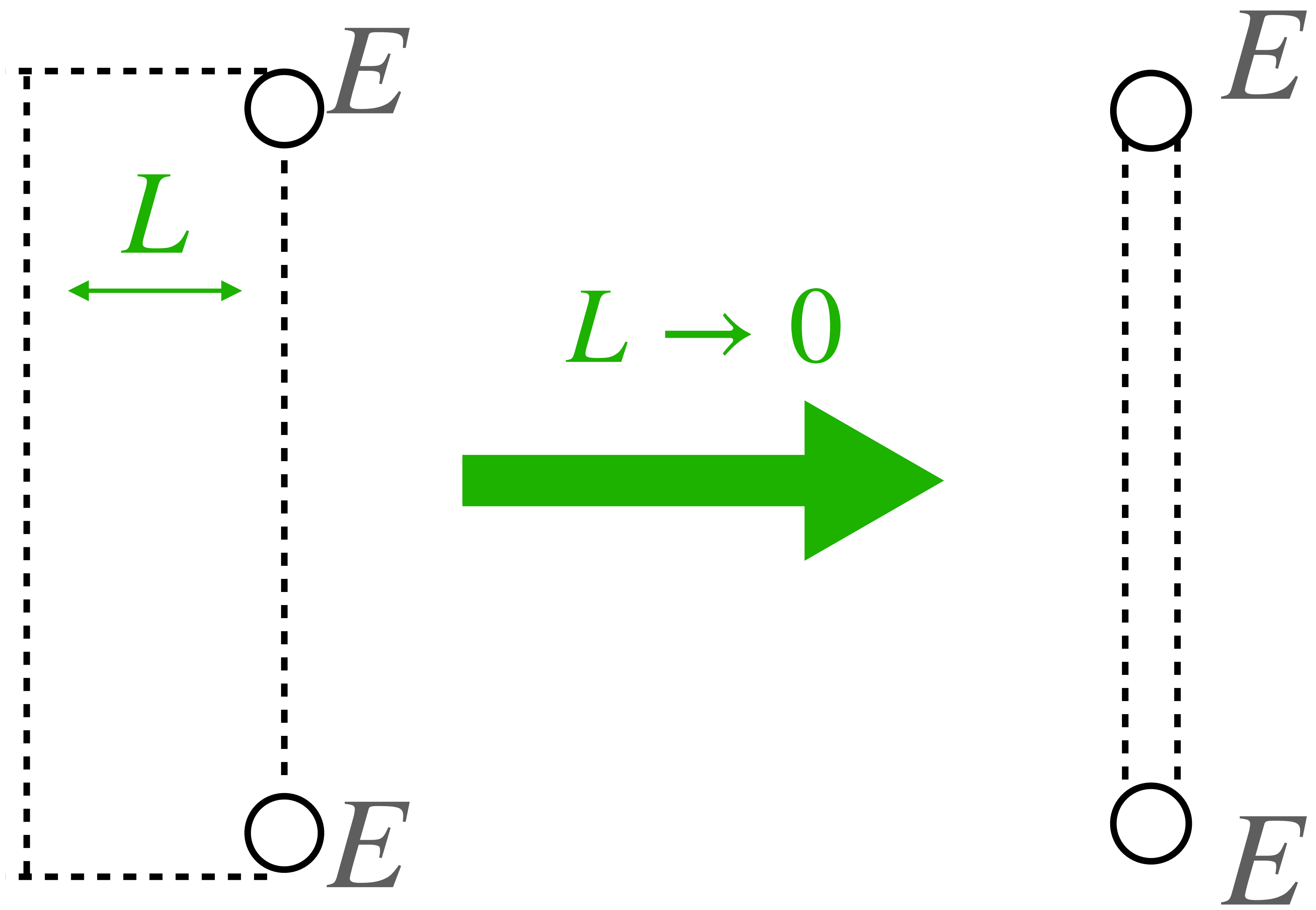

G. Nijs, B. Scheihing-Hitschfeld and X. Yao, hep-ph/2211.XXXXX

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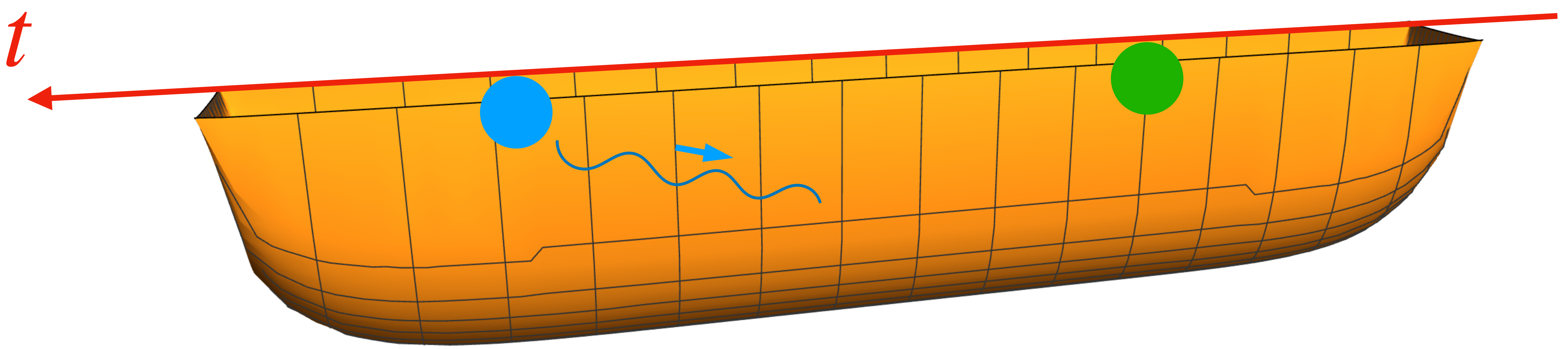

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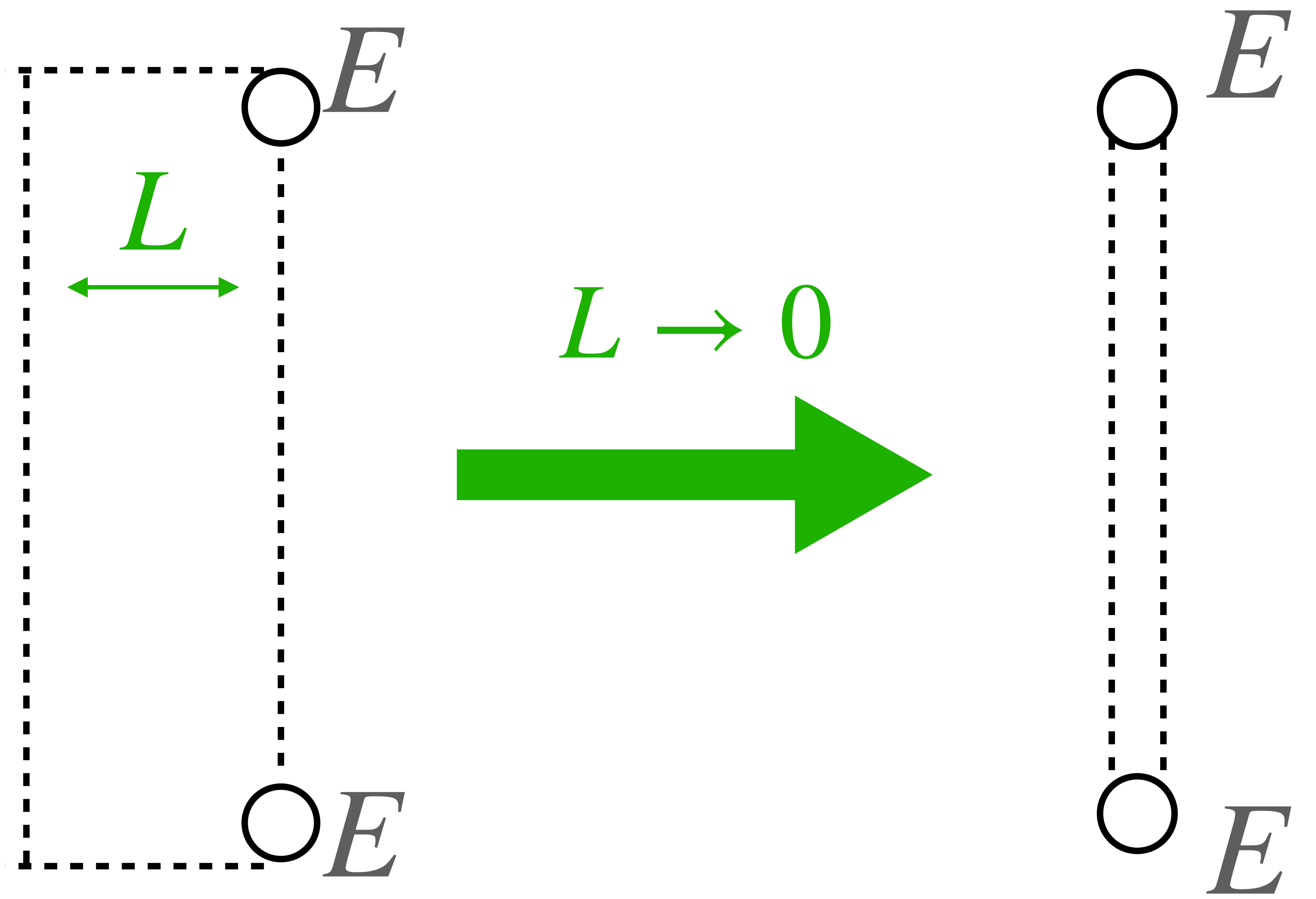

T. Binder, K. Mukaida, B. Scheihing-Hitschfeld and X. Yao, hep-ph/2107.03945

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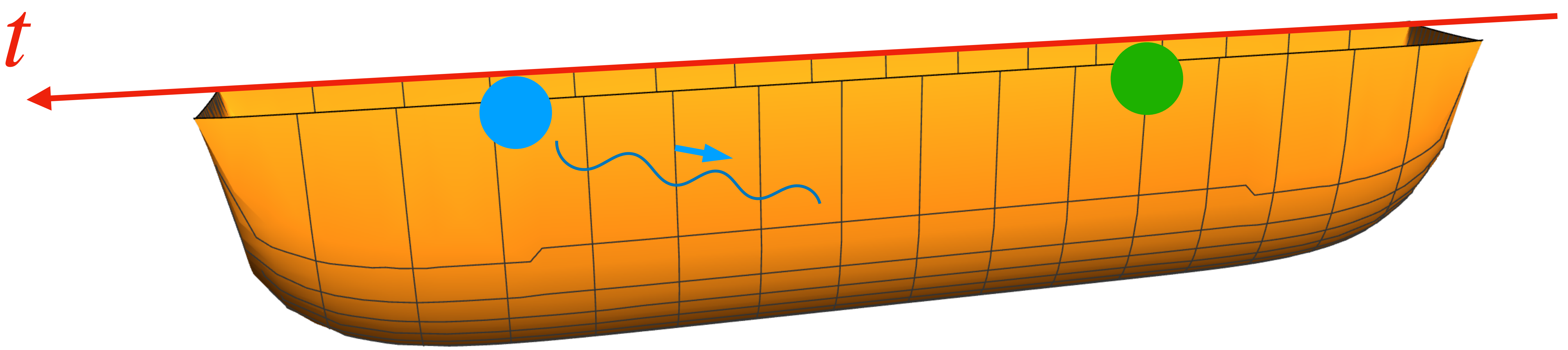

Ongoing calculation!

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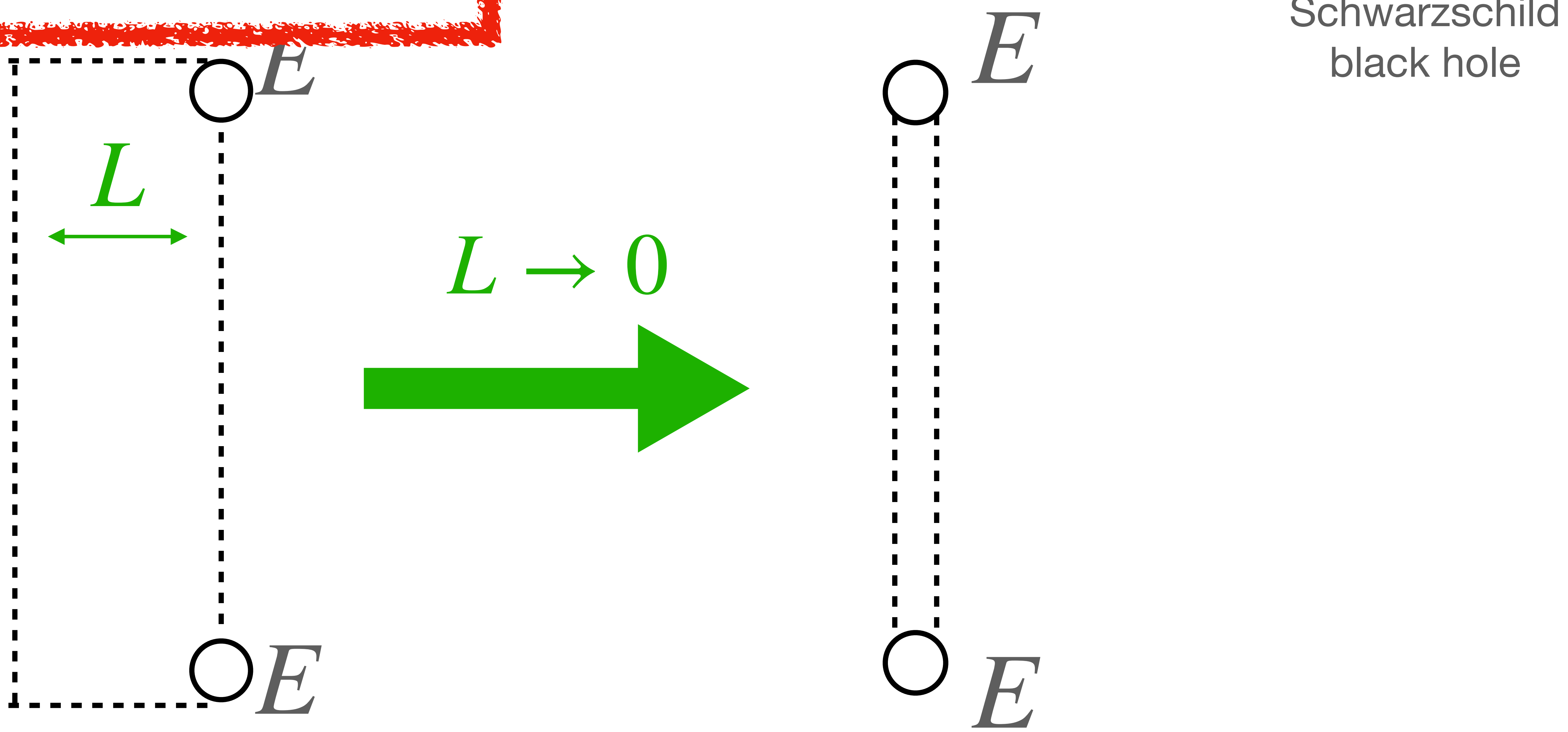

T. Binder, K. Mukaida, B. Scheihing-Hitschfeld and X. Yao, hep-ph/2107.03945

How the calculation proceeds

what equations do we need to solve?

- The classical, unperturbed equations of motion from the Nambu-Goto action to determine Σ:

$$S_{\text{NG}} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{-\det\left(g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\right)}\ .$$

- The classical, linearized equation of motion with perturbations in order to be able to calculate derivatives of $\langle W[\mathscr{C}_f]\rangle_T = e^{iS_{\text{NG}}[\Sigma_f]}$:

$$S_{\text{NG}}[\Sigma_f] = S_{\text{NG}}[\Sigma] + \int dt_1 dt_2 \left.\frac{\delta^2 S_{\text{NG}}[\Sigma_f]}{\delta f(t_1)\delta f(t_2)}\right|_{f=0} f(t_1)f(t_2) + O(f^3)\ .$$

- In practice, the equations are only numerically stable in Euclidean signature, so we have to solve them and analytically continue back.

Summary and conclusions

- We have discussed how to calculate the chromoelectric correlators of the QGP that govern quarkonium transport

  A. at weak coupling in QCD

  B. at strong coupling in $\mathcal{N} = 4$ SYM

- Relevant for both quantum Brownian motion and quantum optical limits

- Next steps:

  - $\langle B_i^a \mathcal{W}^{ab} B_i^b \rangle_T$ correlator at strong coupling in $\mathcal{N} = 4$ SYM

  - Use them as input for quarkonia transport codes

- Stay tuned!

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Thank you!

Extra slides

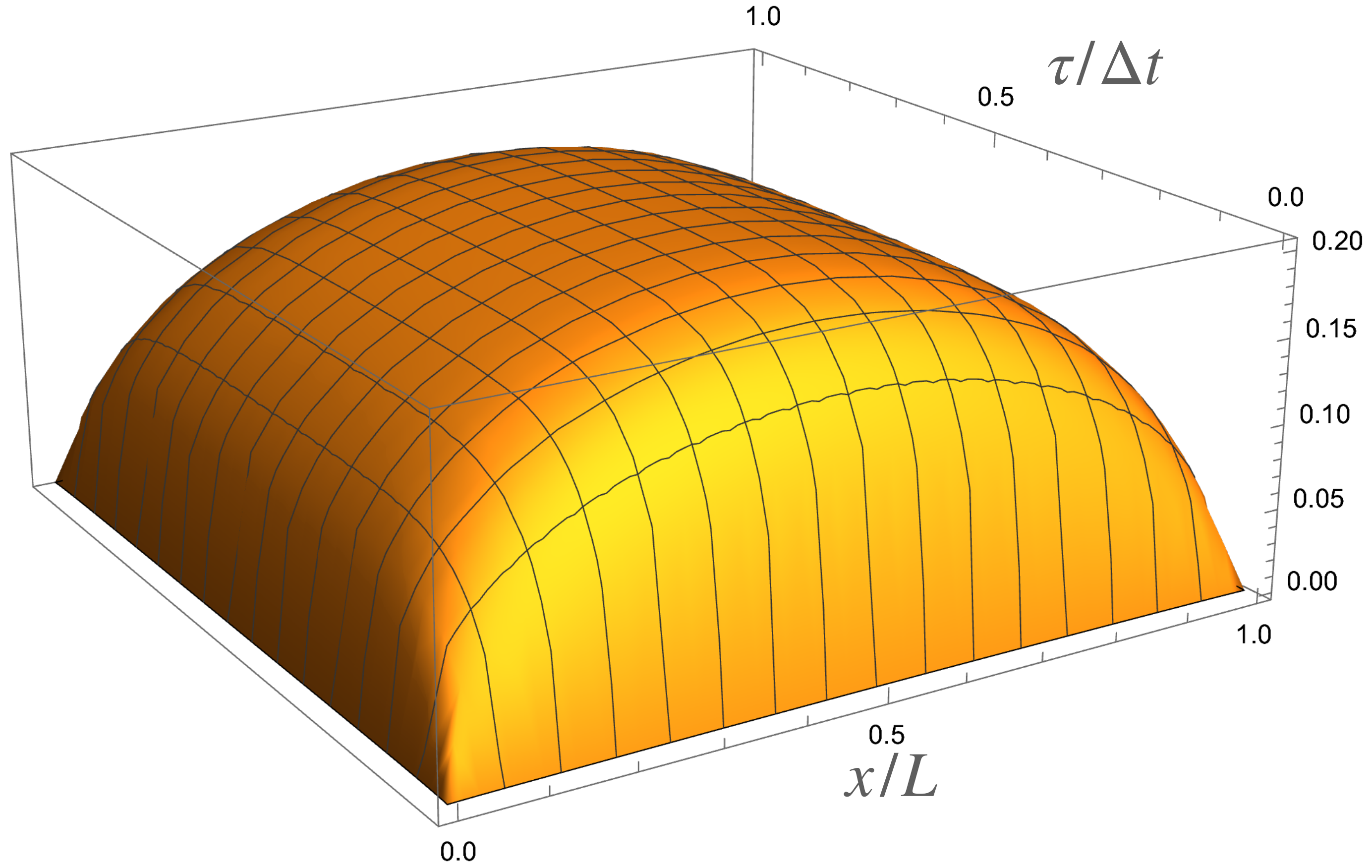
G. Nijs, B. Scheihing-Hitschfeld and X. Yao, hep-ph/2211.XXXXX

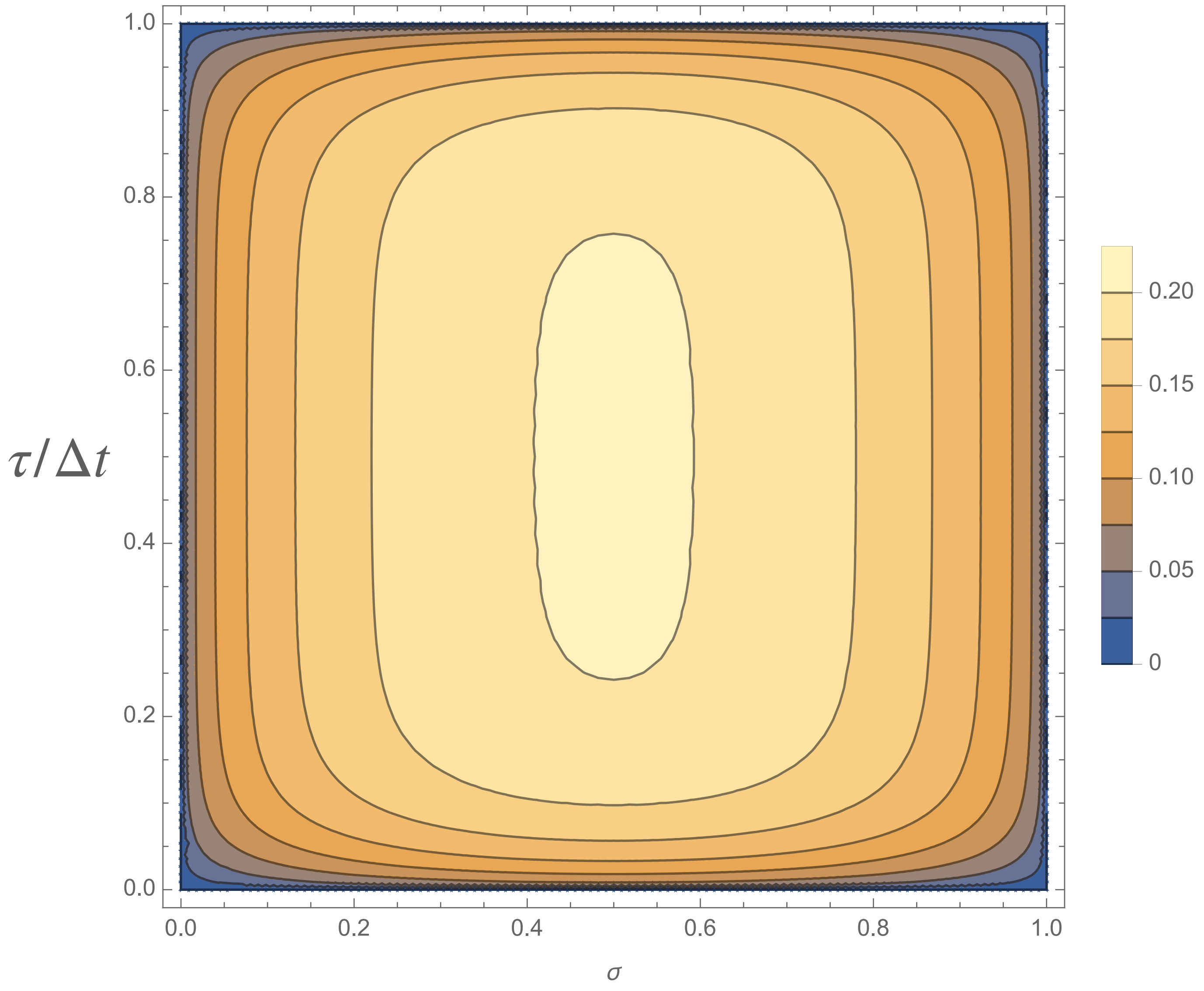
Extracting the EE correlator for quarkonia

the pipeline

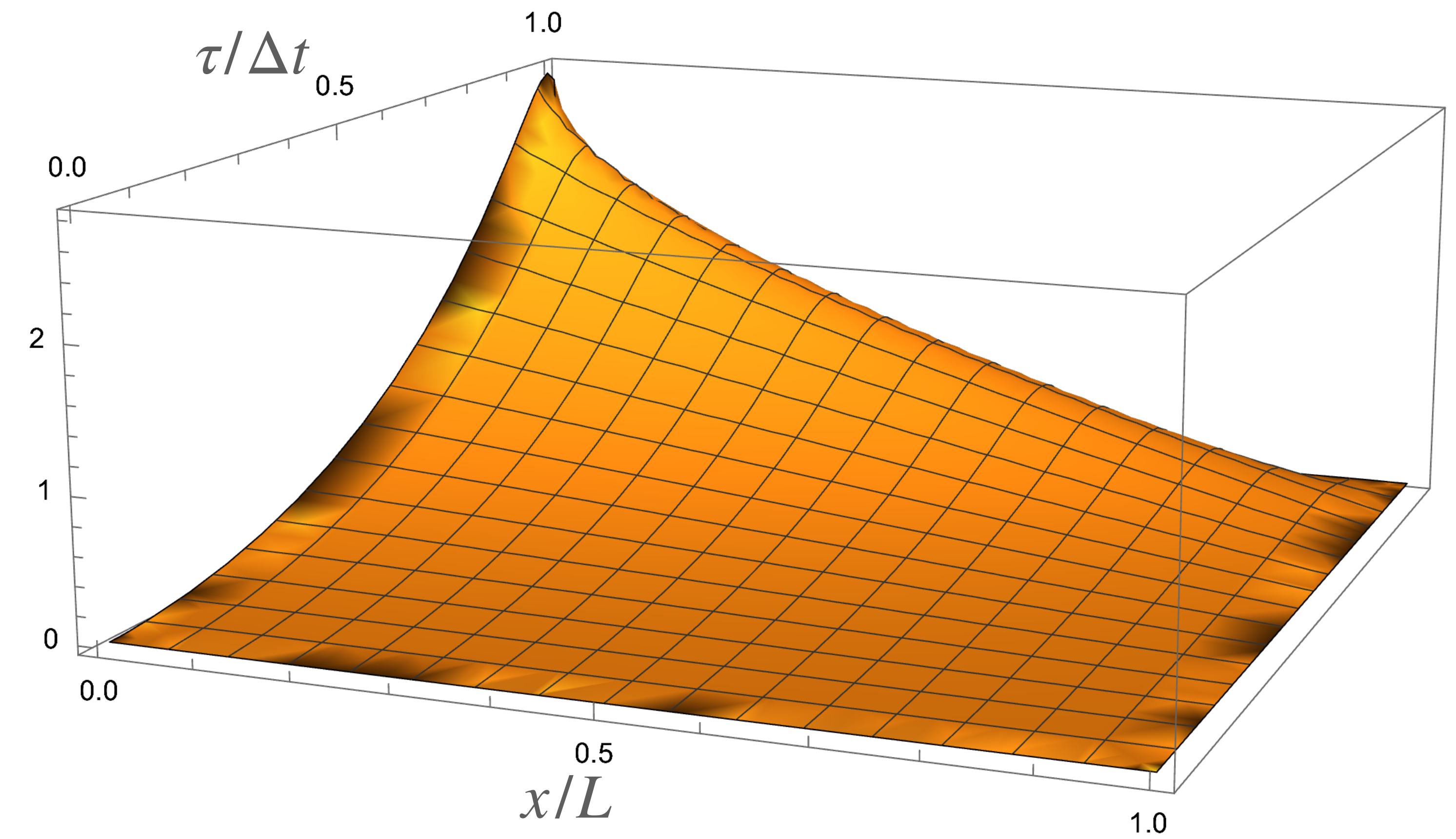
J.P. Boyd, "Chebyshev and Fourier Spectral Methods," Dover books on Mathematics (2001)

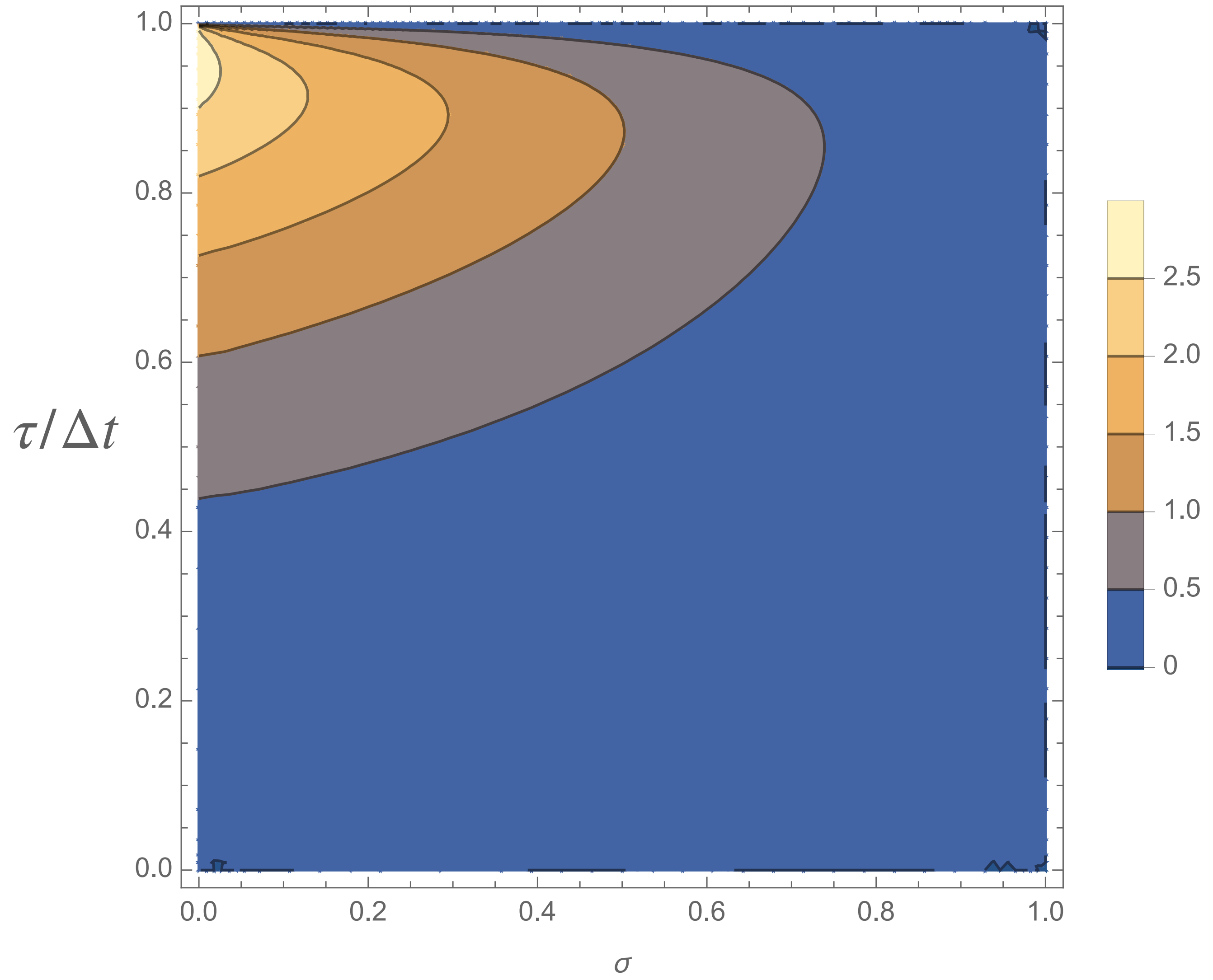
1) Solve for the background worldsheet solution:





2) Solve for the fluctuations with a source as a boundary condition:





3) Extrapolate in the limit $L \to 0$:

The spectral function of quarkonia

symmetries and KMS relations

The KMS conjugates of the previous correlators are such that

$$[g_E^{++}]_{ji}^{>}(q) = e^{q^0/T}[g_E^{++}]_{ji}^{<}(q)\,, \quad [g_E^{--}]_{ji}^{>}(q) = e^{q^0/T}[g_E^{--}]_{ji}^{<}(q)\,,$$

and one can show that they are related by

$$[g_E^{++}]_{ji}^{>}(q) = [g_E^{--}]_{ji}^{<}(-q)\,, \quad [g_E^{--}]_{ji}^{>}(q) = [g_E^{++}]_{ji}^{<}(-q)\,.$$

The spectral functions $[\rho_E^{++/--}]_{ji}(q) = [g_E^{++/--}]_{ji}^{>}(q) - [g_E^{++/--}]_{ji}^{<}(q)$ are not necessarily odd under $q \leftrightarrow -q$. However, they do satisfy:

$$[\rho_E^{++}]_{ji}(q) = -\,[\rho_E^{--}]_{ji}(-q)\,.$$