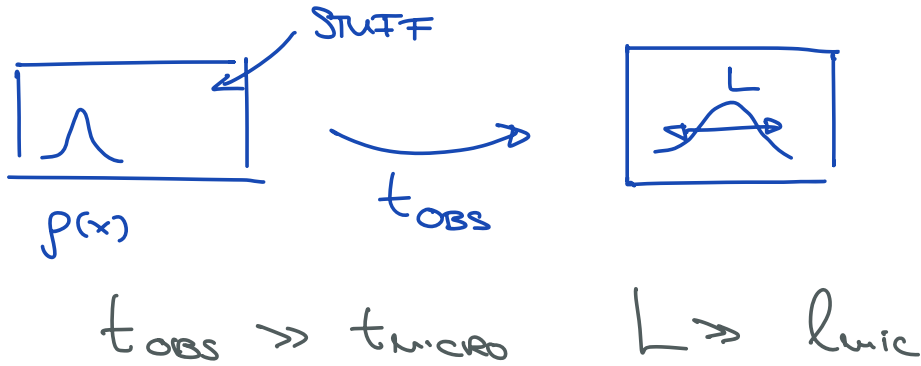


FLUCTUATING FLUID DYNAMICS

FLUID DYNAMICS



OBSERVATION:

- NON-CONSERVED DENSITY $\tau_{DEC} \sim t_{micro}$
- CONSERVED DENSITY $\tau_{DEC} \sim c_s L \gg t_{mic}$

$$\left. \begin{aligned} \partial_0 \rho + \nabla \cdot \mathbf{j} &= 0 \\ \partial_0 \pi_i + \nabla_j \pi_{ij} &= 0 \\ \partial_0 \varepsilon + \nabla \cdot \mathbf{j} \varepsilon &= 0 \end{aligned} \right\} \partial_\mu T^{\mu\nu} = 0$$

CONST. RELATIONS: GRAD. EXP.

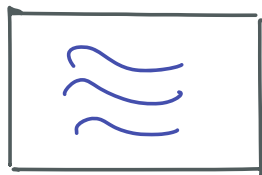
$$\pi_{ij} = \pi_{ij}^0 + \pi_{ij}^1 + \dots$$

$$\Pi_{ij}^0 = \delta_{ij} P + \rho \sigma_i \sigma_j$$

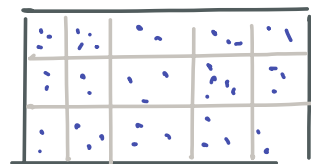
$$\Pi_{ij}^1 = \gamma \left(\partial_i \sigma_j + \partial_j \sigma_i - \frac{2}{3} \delta_{ij} \partial \cdot \sigma \right) + \dots$$

WHY FLUCTUATIONS?

COARSE
GRAIN



$\hat{=}$



$$\langle (\rho_j)^2 \rangle \sim \frac{k_B T \chi}{(\Delta V)}$$

IMPORTANT IF (ΔV) SMALL OR
 χ LARGE \leadsto PHASE TRANSITIONS

FORMAL REASON: RESPECT FD-RELATIONS
(\leadsto REQUIRED FOR CORRECT $t \rightarrow \infty$
BEHAVIOR)

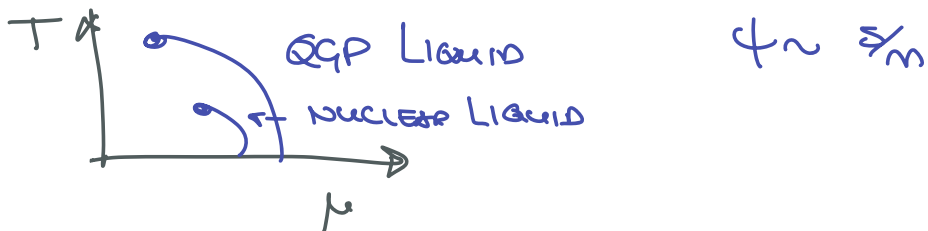
How? LANDAU - LIFSHITZ

$$\partial_0 \pi_i + \nabla_j \pi_{ij} = 0$$

$$\pi_{ij} = \pi_{ij}^0 + \pi_{ij}^1 + \pi_{ij}^2 + \dots$$

$$\langle \pi_{ij} \rangle = 0 \quad \langle \pi_{ij} \pi_{kl} \rangle = \int T \Delta_{ijkl} \delta^3(x-x') \delta(t-t')$$

EXAMPLE: (NON) CRITICAL DIFFUSION



$$\partial_0 \phi + \nabla \cdot j = 0 \quad j = -D \nabla \phi$$

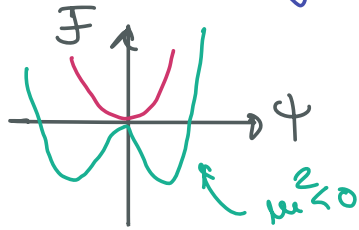
↑
Fick's Law

NON-LIN. STOCH. DIFFUSION

$$\partial_0 \phi = \kappa \nabla^2 \frac{\delta F}{\delta \phi} + \xi$$

↑ CONDUCTIVITY ← FREE ENERGY ↑ NOISE

$$F = \int d^3x \left\{ \frac{1}{2} (\nabla\phi)^2 + \frac{1}{2} \mu^2 \phi^2 + \lambda \phi^4 \right\}$$



$$\hookrightarrow D = \omega \mu^2$$

$$\chi = \frac{1}{\mu^2}$$

$$\xi \approx \frac{1}{\mu}$$

$$\langle \phi \rangle = 0$$

$$\langle \phi \phi \rangle = \omega_T \nabla^2 \delta^2(x-x') \delta(t-t')$$

I) ANALYT. APPROACH

COR. FCT (OR RESP. FCT)

$$\langle \phi \phi \rangle = \int \mathcal{D}\phi e^{-\phi \mathcal{L}_0 \phi} \quad \text{NOISE: } \mathcal{L}_0 = \omega_T \nabla^2$$

$$\cdot \int \mathcal{D}\phi \phi \delta(\omega_T \phi - \omega_T \nabla^2 \frac{\delta F}{\delta \phi} - \phi)$$

↑
EQU. OF MOTION

AUXILIARY FIELD

$$\delta(\omega \omega) = \int \mathcal{D}(i\tilde{\phi}) e^{\tilde{\phi}(\omega \omega)}$$

$$= \int \mathcal{D}\phi \int \mathcal{D}\psi e^{-\int \mathcal{L}_0} e^{-\int \tilde{\mathcal{L}}(\phi, \psi, \dots)}$$

$$= \int \mathcal{D}\phi \mathcal{D}\psi e^{-\mathcal{S}}$$

$$\mathcal{S} = \int \tilde{\mathcal{L}}(\phi, \psi) - \kappa \int \tilde{\mathcal{L}} \nabla^2 \tilde{\mathcal{L}} - \kappa \int \tilde{\mathcal{L}} \nabla^2 \psi^2$$

GREEN Fcn

$$\begin{pmatrix} \langle \tilde{\mathcal{L}} \tilde{\mathcal{L}} \rangle & \langle \tilde{\mathcal{L}} \psi \rangle \\ \langle \psi \tilde{\mathcal{L}} \rangle & \langle \psi \psi \rangle \end{pmatrix} = \begin{pmatrix} 0 & G_R \\ G_A & G_S \end{pmatrix}$$

→ ANALYT. JR. OF US GREEN Fcn

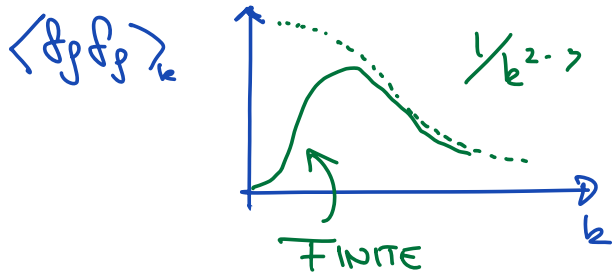
RESULTS: (NON-CRIT)

$$\rightarrow e^{-t D k^2}$$

$$\rightarrow \sim \frac{1}{t^{3/2}} \text{ "LONG TIME TAIL"}$$

CRTT. FLUID

$$\tau \sim L^2 \quad z \approx 3$$



QUENCH RATE \rightarrow k^2 -SCALING

II) NUMERICAL SIMULATION

CONSIDER MODEL A: $\partial_t \psi = -\kappa \frac{\delta F}{\delta \psi} + \xi$

$$\psi(t+\Delta t) = \psi(t) + (\Delta t) \left[\kappa \nabla^2 \frac{\delta F}{\delta \psi} \right]$$

$$+ \left(\frac{\kappa T}{a^3 \Delta t} \right)^{1/2} \xi$$

$\xi \sim \langle \xi^2 \rangle = 2$

\rightarrow NOISE DIVERGES; BUT: IN EQUILIBRIUM
 $P[\psi] \sim \exp(-F/k_B T)$

IDEA: METROPOLIS ALGORITHM

$$\psi(t+\Delta t) = \psi(t) + \sqrt{2kT\Delta t} \cdot \xi$$

$$P_{\text{Acc}} = \text{MIN} \left(1, e^{-\Delta F/k_B T} \right)$$

$$P_{\text{Res}} = 1 - P_{\text{Acc}}$$

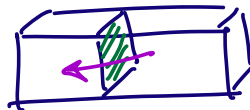
DIFFUSIVE

$$\Rightarrow \langle [\psi(t+\Delta t) - \psi(t)] \rangle = -(\Delta t) k \frac{\partial F}{\partial \psi}$$

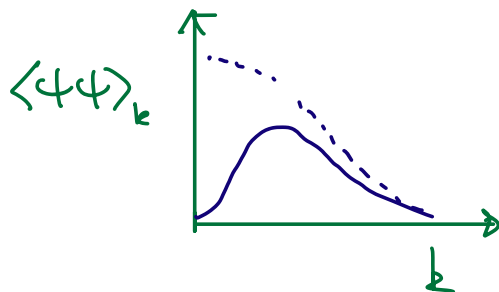
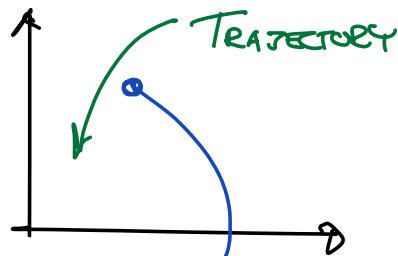
$$\langle [\psi(t+\Delta t) - \psi(t)]^2 \rangle = (\Delta t) 2kT$$

↑ NOISE

FOR MODEL B: APPLY METROPOLIS TO
FLUXES



APPLICATION:



OUTLOOK: Full
(REL?) FLUX
DYNAMICS