

# *Ab initio* calculations of beta decay recoil-order form factors for precision measurements

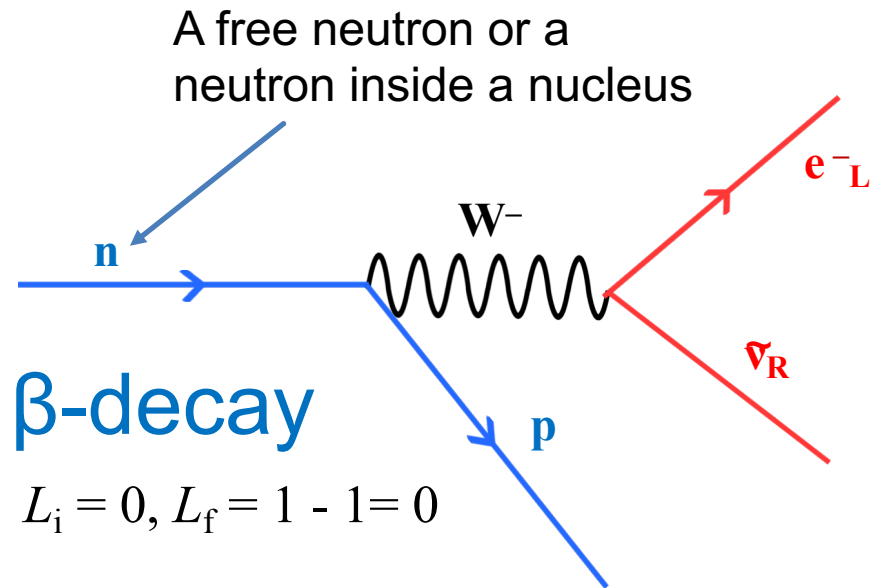
Grigor Sargsyan

INT 23-1b program, 9 May 2023,

Seattle, WA

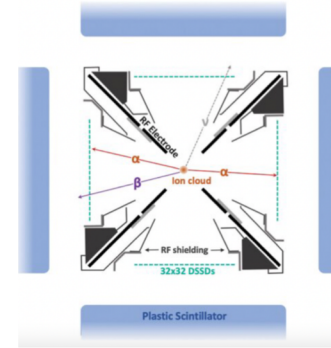


# Beta decay as a probe for BSM studies



Precision measurements need input from nuclear theory

- Beyond Standard Model (BSM) terms in weak interaction

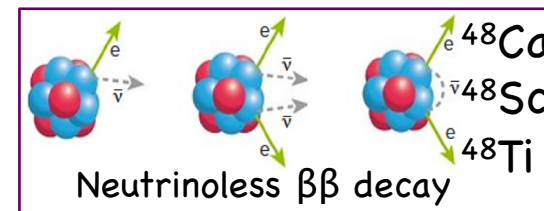


- Unitarity of the CKM quark mixing matrix

$$\begin{pmatrix} d_w \\ s_w \\ b_w \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

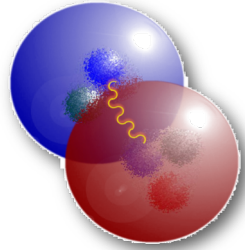
Weak states      **CKM mixing matrix**      Mass eigenstates

- Neutrinoless double beta decay



# From first principles to nuclear properties

## First Principles

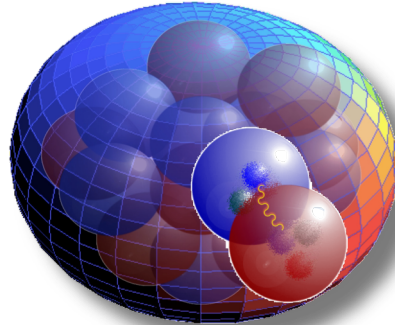


Realistic  
Interactions

E.g., from chiral effective field theory



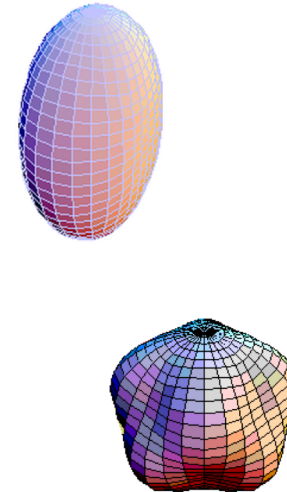
## Many-body Dynamics



E.g., Symmetry-  
Adapted No-Core  
Shell Model (SA-  
NCSM)



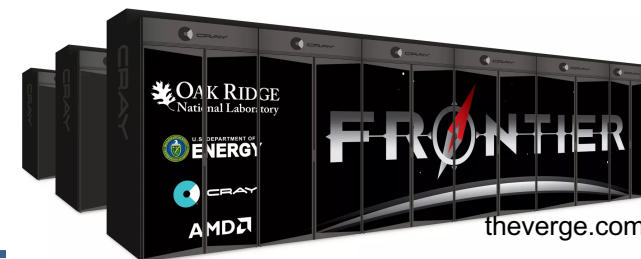
## Properties of Nuclei



E.g., deformation,  
excitation spectrum,  
clustering, etc.



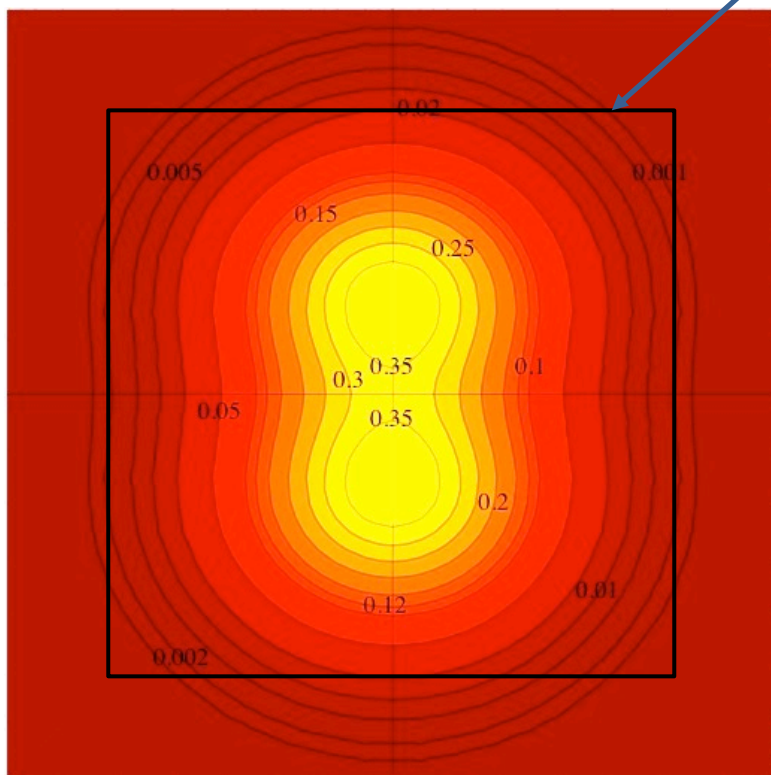
# Explosive growth of the model space



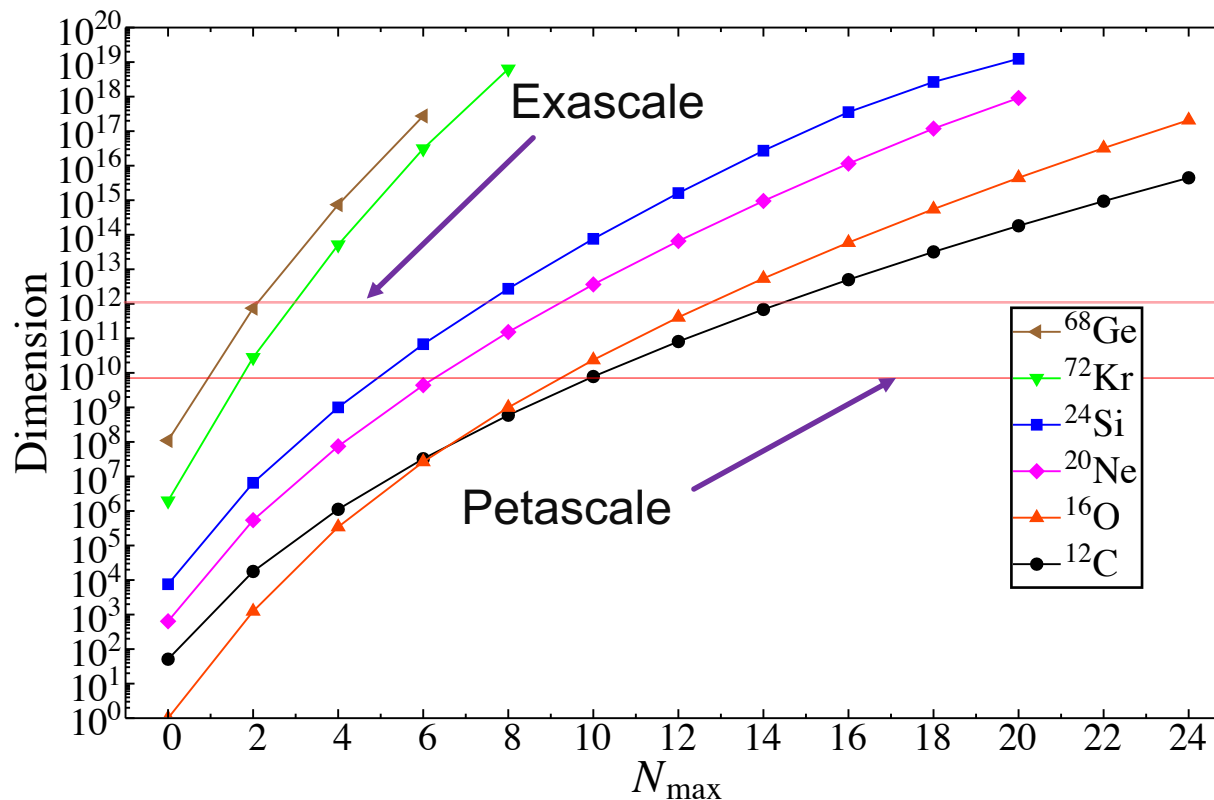
First exascale computer

## Conventional Shell Model

$N_{\max}$



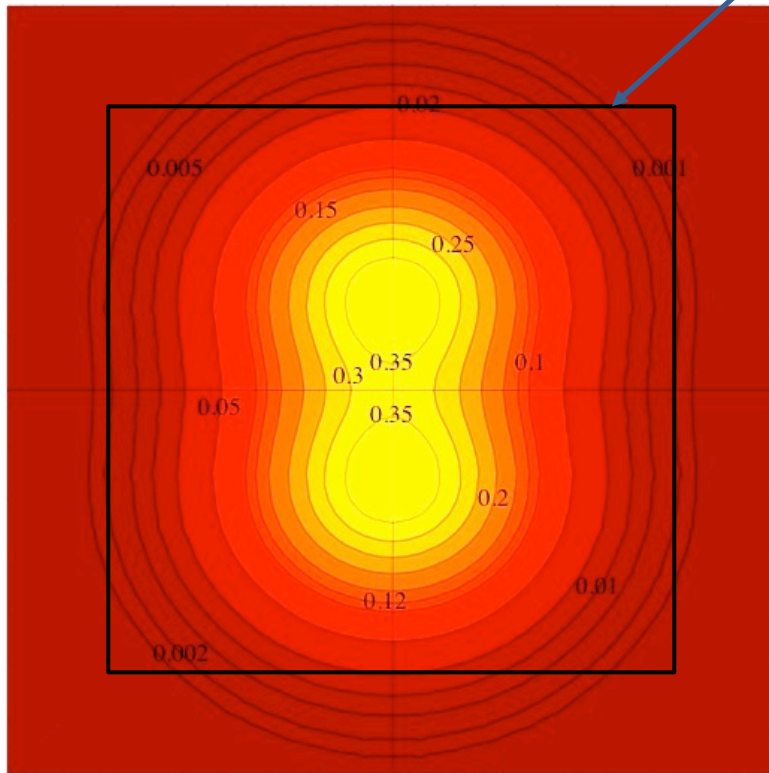
Nucleus in model space





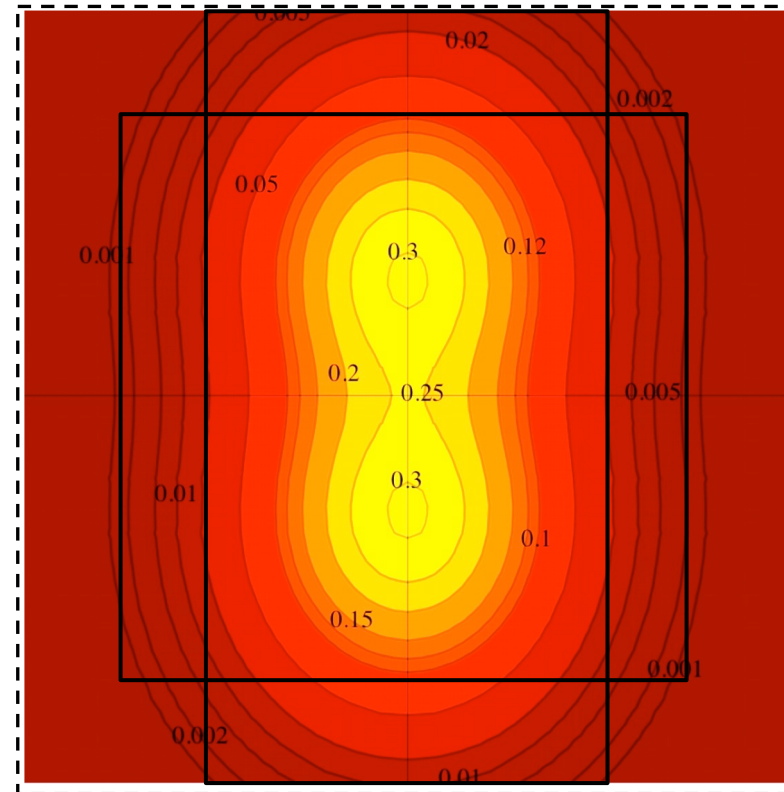
# Symmetry-adapted basis helps dramatically reduce the models space size

Conventional Shell Model



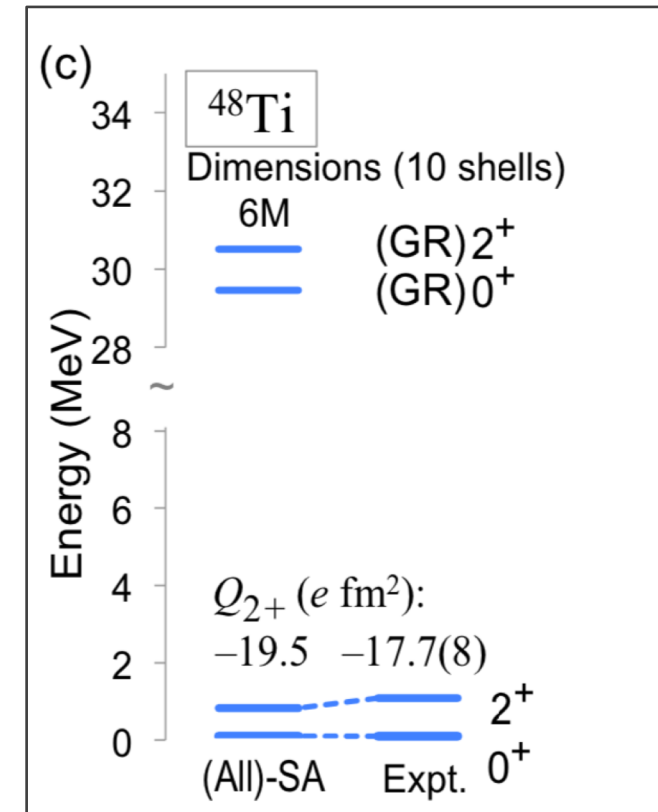
Nucleus in model space

*Ab initio* Symmetry-adapted No-core Shell Model (SA-NCSM)



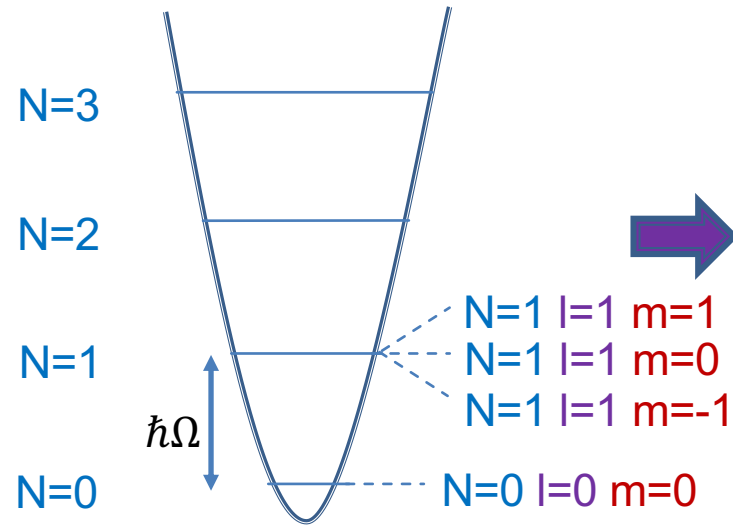
**SU(3) and symplectic symmetry**

Reaching medium-mass nuclei

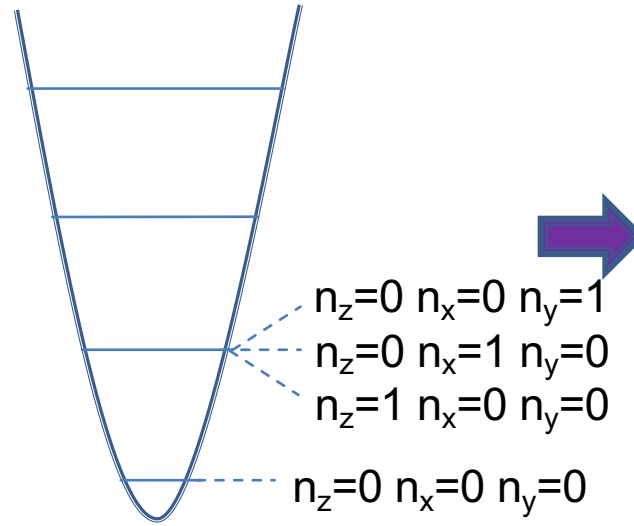


NNLO<sub>opt</sub>  $\hbar\Omega = 15 \text{ MeV}$

# Symmetry-adapted Basis: SU(3)-coupled

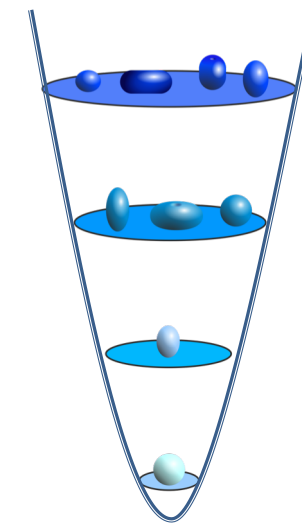


Spherical harmonic oscillator (HO): basis states given by  $\{N \mid m\}$



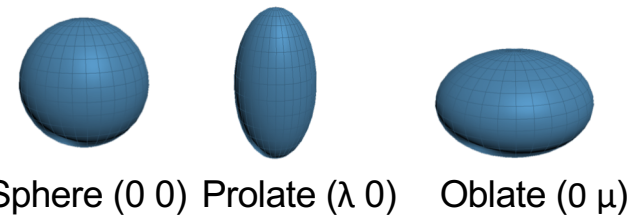
$N = n_z + n_x + n_y$   
Basis states given by  $\{n_x \ n_y \ n_z\}$

$$a_{Nlm}^+ \equiv a_{(N\ 0)lm}^+$$



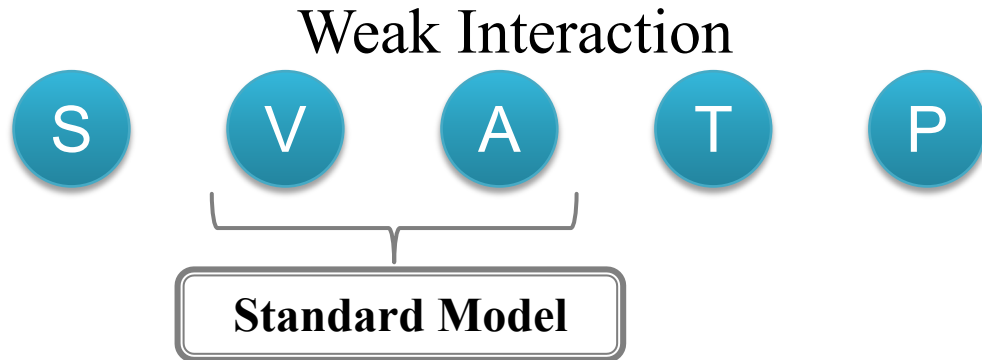
$\lambda = n_z - n_x, \ \mu = n_x - n_y$  (single particle)

Basis states given by  $(\lambda \ \mu)$  quantum numbers



Sphere (0 0) Prolate ( $\lambda$  0) Oblate (0  $\mu$ )

# Weak interaction in Standard Model



A series of  $\beta$ -decay experiments lead to the formulation of the V – A structure of the weak interaction:

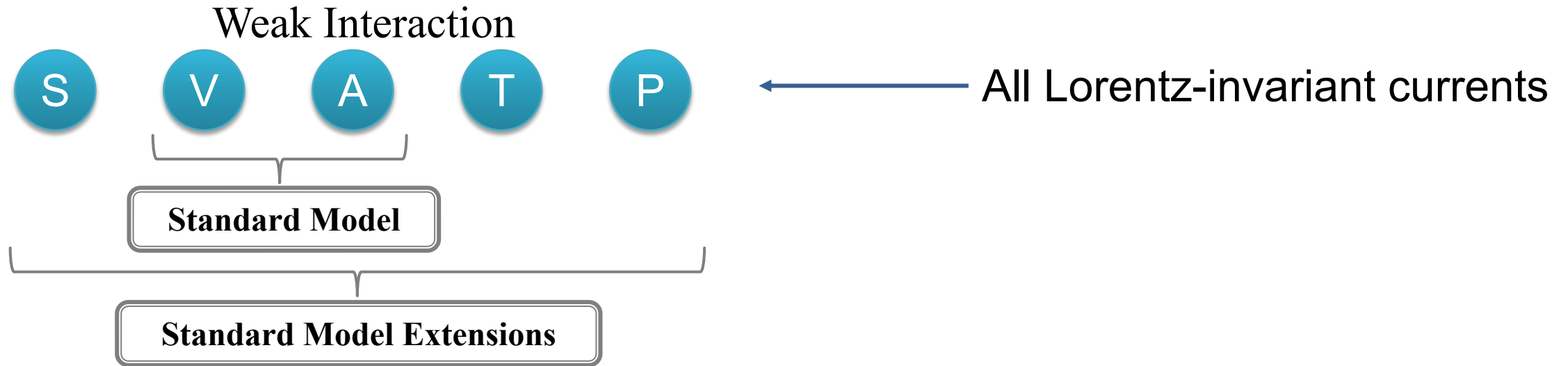
C. S. Wu, *et al.*, Phys. Rev. 105, 1413 (1957).

W. B. Herrmannsfeldt, *et al.*, Phys. Rev. 107, 641 (1957).

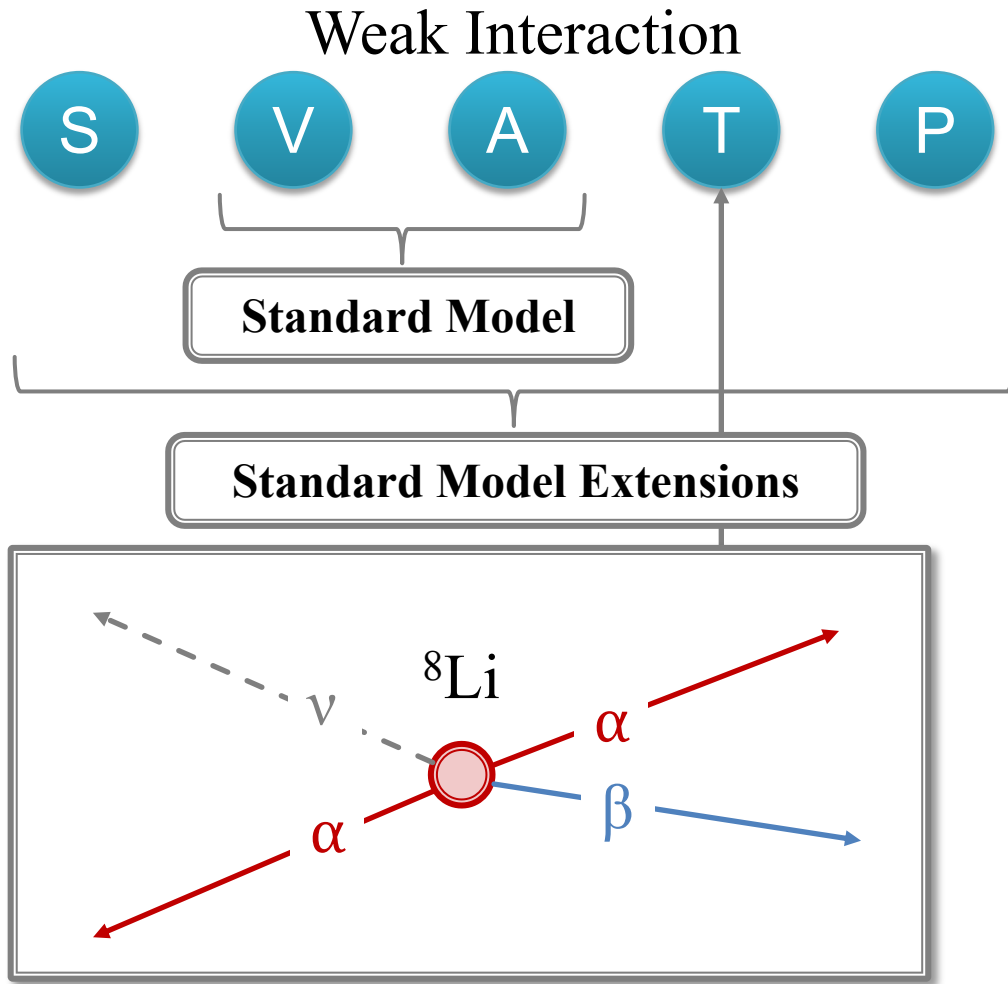
C. Johnson, *et al.*, Phys. Rev. 132, 1149 (1963).



# Weak interaction in Standard Model

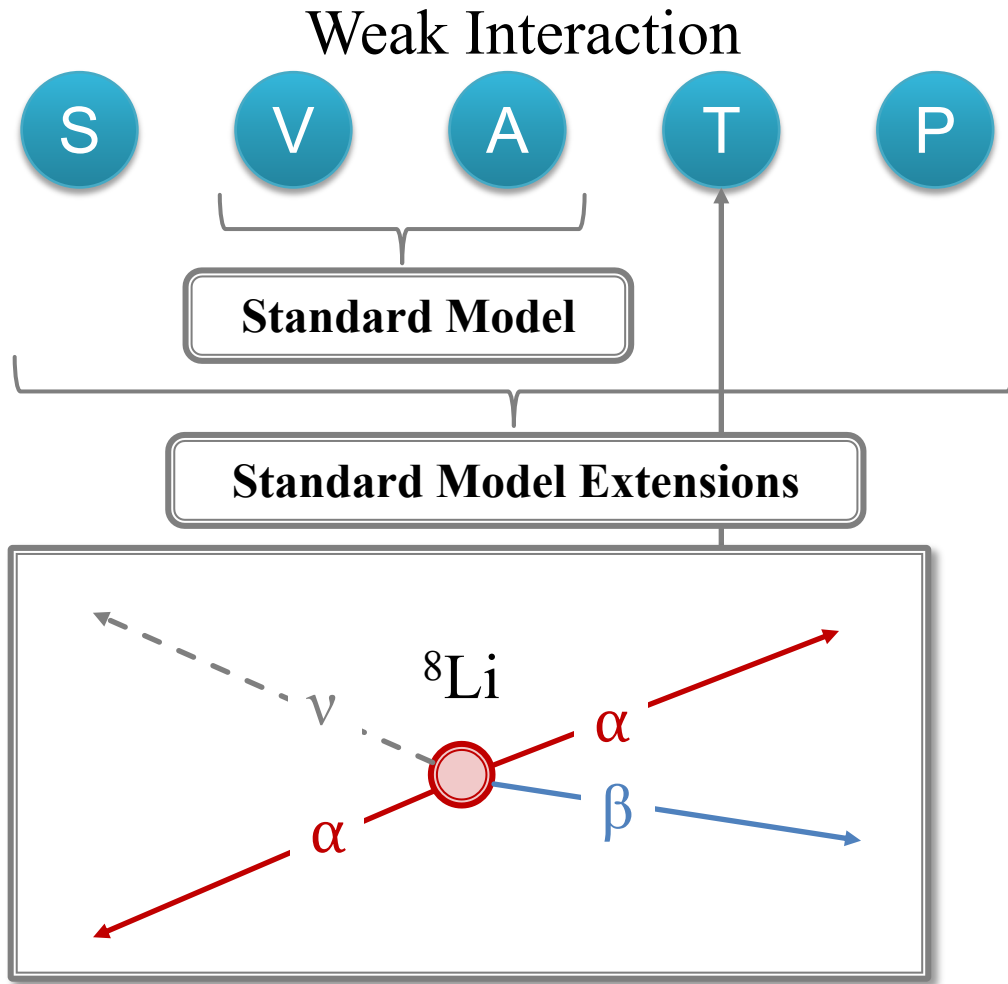


# Precision measurements of $^8\text{Li}$ beta decay to probe BSM physics



M. G. Sternberg, R. Segel, N. D. Scielzo, *et al.*, PRL **115**, 182501 (2015).  
MT Burkey, G Savard, AT Gallant, *et al.*, PRL **128** (20), 202502 (2022).

# Precision measurements of $^8\text{Li}$ beta decay to probe BSM physics



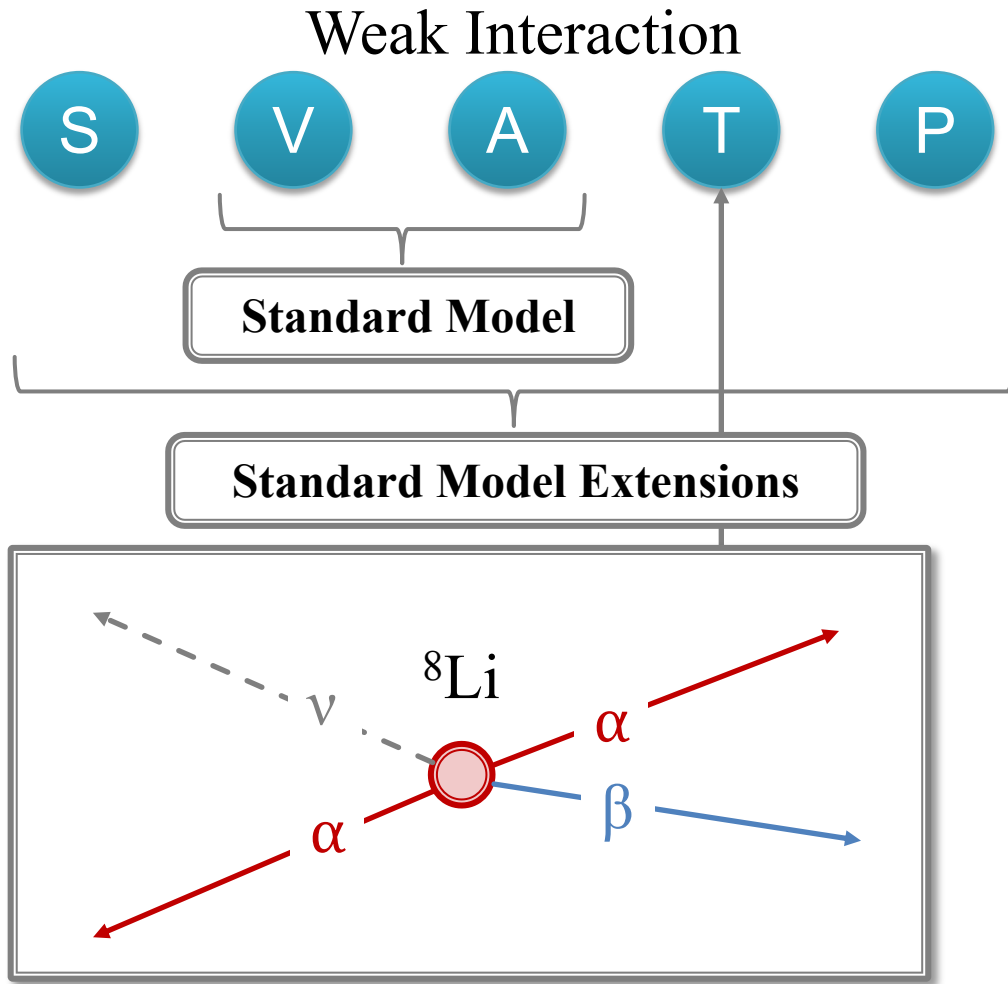
Systematic Uncertainty	$\Delta C_T/C_A ^2$
Calibration	$1.4 \times 10^{-4}$
$\alpha$ energy corrections	$1.17 \times 10^{-3}$
Cuts to the data	$1.25 \times 10^{-3}$
Radiative and recoil order terms	$3.36 \times 10^{-3}$
$\alpha$ Si detector lineshape	$6.3 \times 10^{-4}$
$\beta$ Scattering	$5.0 \times 10^{-4}$
<b>Total</b>	<b><math>3.62 \times 10^{-3}</math></b>

From Mary Burkey's PhD Thesis (U. Chicago/ANL/LLNL, 2019)

M. G. Sternberg, R. Segel, N. D. Scielzo, *et al.*, PRL **115**, 182501 (2015).  
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$$\langle A \rangle = \underbrace{a \times 1}_{\text{Leading order (Gamow-Teller)}} + \underbrace{b \times \frac{q}{M} + c \times \frac{q^2}{M^2} + \dots}_{\text{Recoil-order}}$$

For  $^8\text{Li}$  beta decay  
 $q/M \sim 0.002$

M. G. Sternberg, R. Segel, N. D. Scielzo, *et al.*, PRL **115**, 182501 (2015).  
 MT Burkey, G Savard, AT Gallant, *et al.*, PRL **128** (20), 202502 (2022).

# Experiment needs reliable $\beta$ -decay recoil-order terms

Matrix elements in impulse approximation

${}^8\text{Be}^*$   $\xrightarrow{\beta\text{-decay}}$   ${}^8\text{Li}$

$$j_K \propto \langle \Psi_f || \sum_i^A \tau_i^\pm [\sigma_i \times \hat{Q}_2(\hat{r}_i)]^K || \Psi_0 \rangle$$

Nuclear recoil form-factor

$$c_0 \propto \langle \Psi_f || \sum_i^A \tau_i^\pm \sigma_i || \Psi_0 \rangle$$

Gamow-Teller matrix element

B. R. Holstein, Rev. Mod. Phys. 46, 789 (1974)

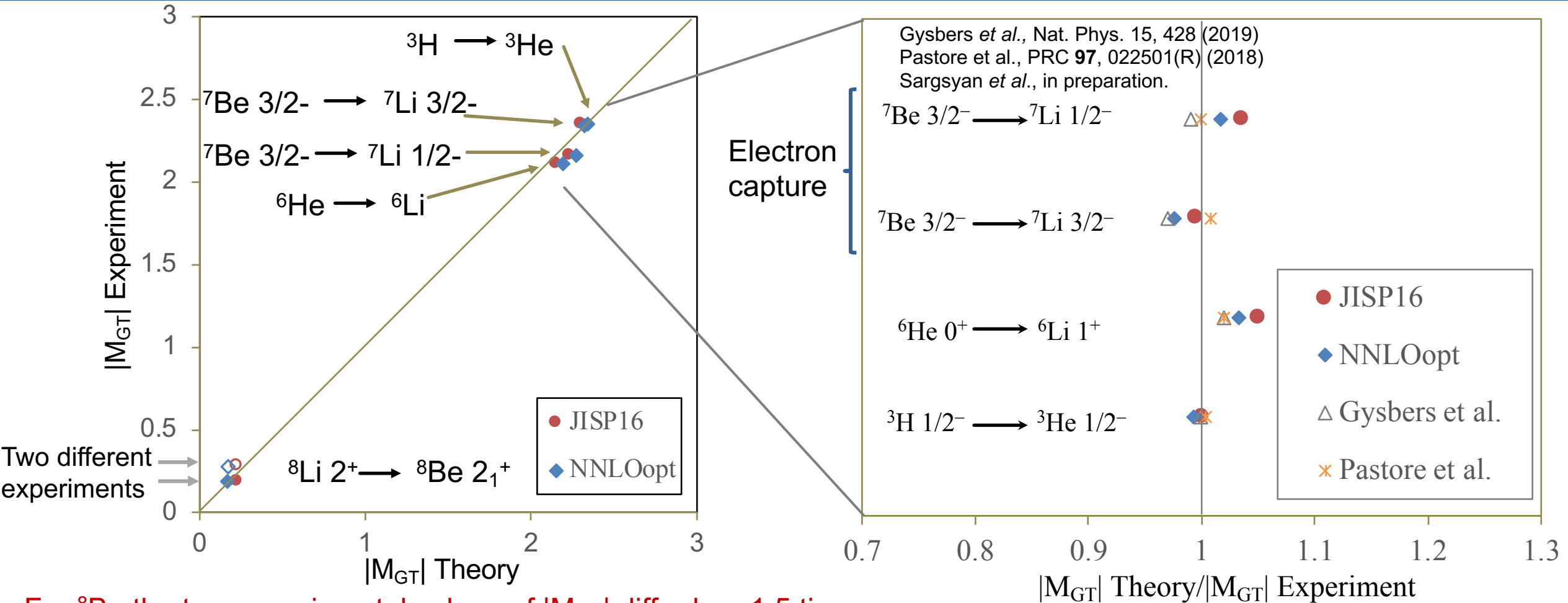
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From Mary Burkey's PhD Thesis (U. Chicago/ANL/LLNL, 2019)

Need more accurate and precise  $j_{2,3}/A^2 c_0$  and other recoil-order terms

Use ab initio methods to calculate them

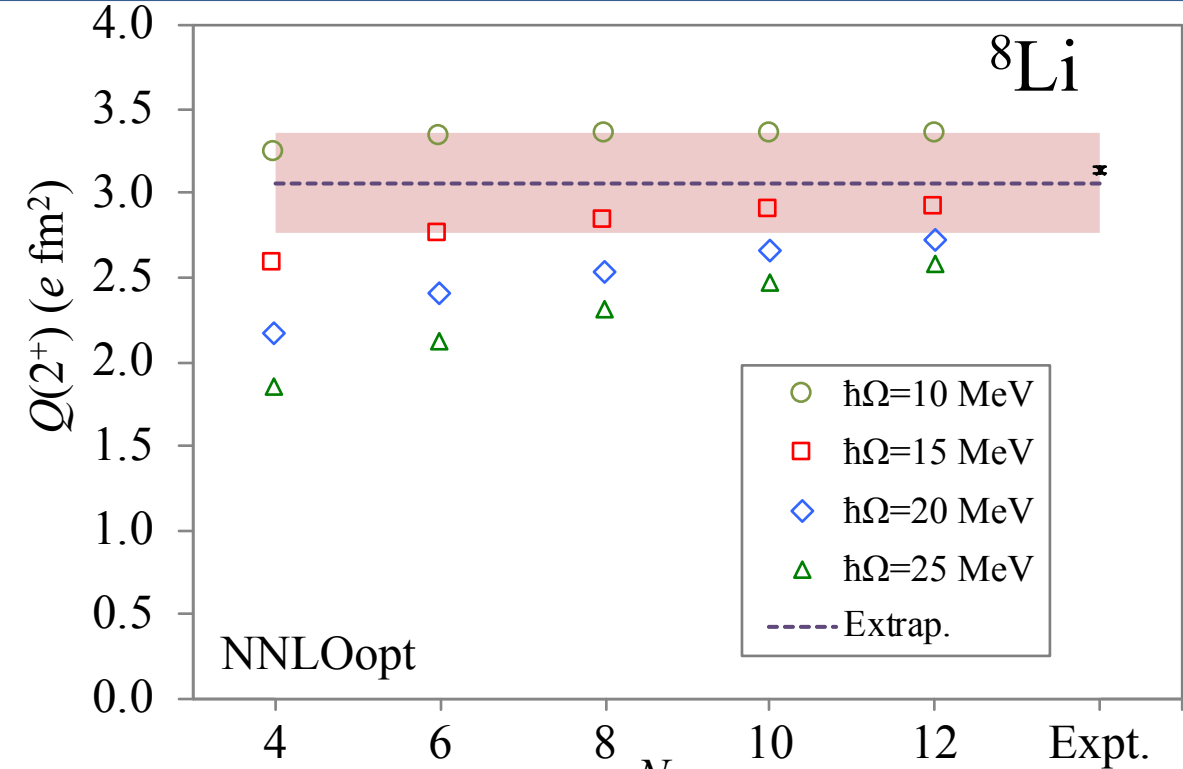
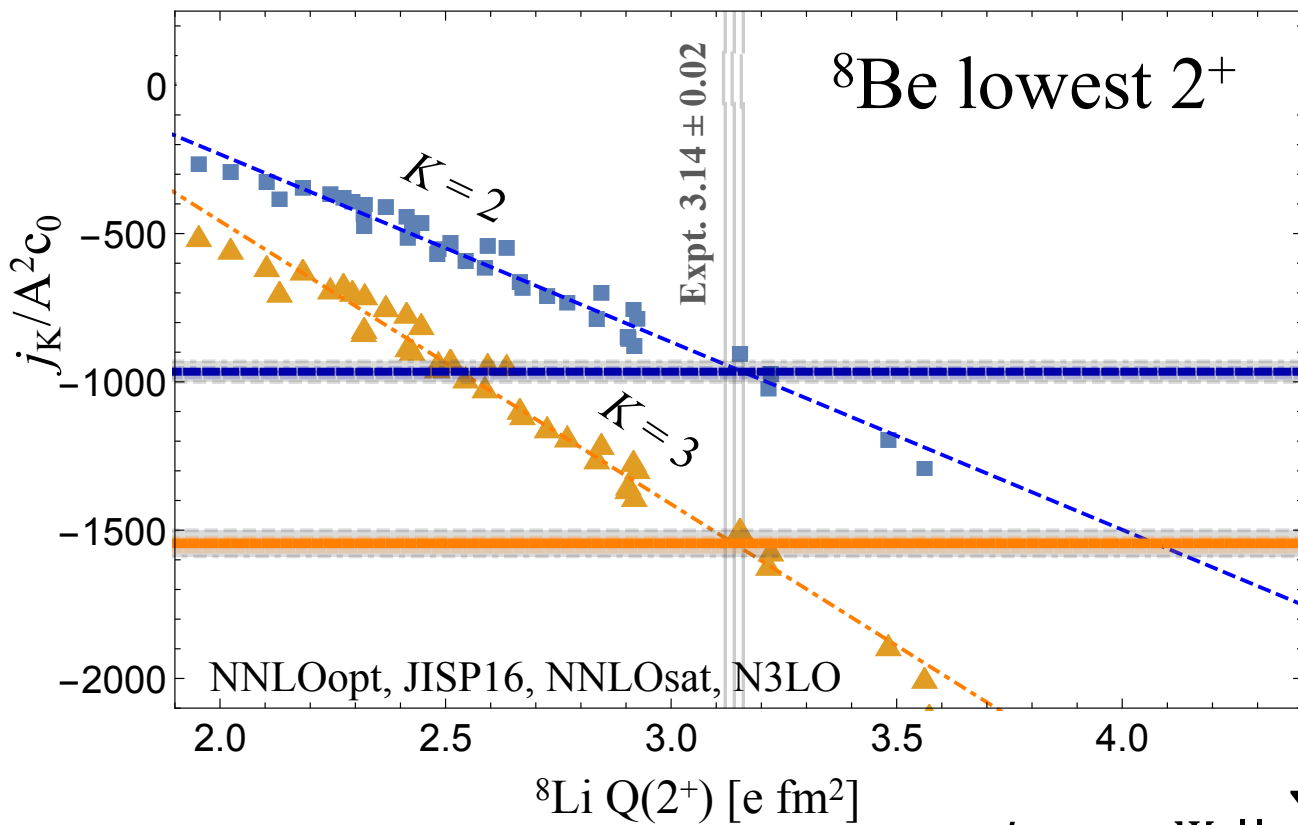
# Beta decays with SA-NCSM



For  ${}^8\text{Be}$  the two experimental values of  $|M_{GT}|$  differ by  $\sim 1.5$  times  
 Non-renormalized interactions, unquenched  $g_A$



# Correlation between $j_K$ and $Q$ helps constrain recoil order terms

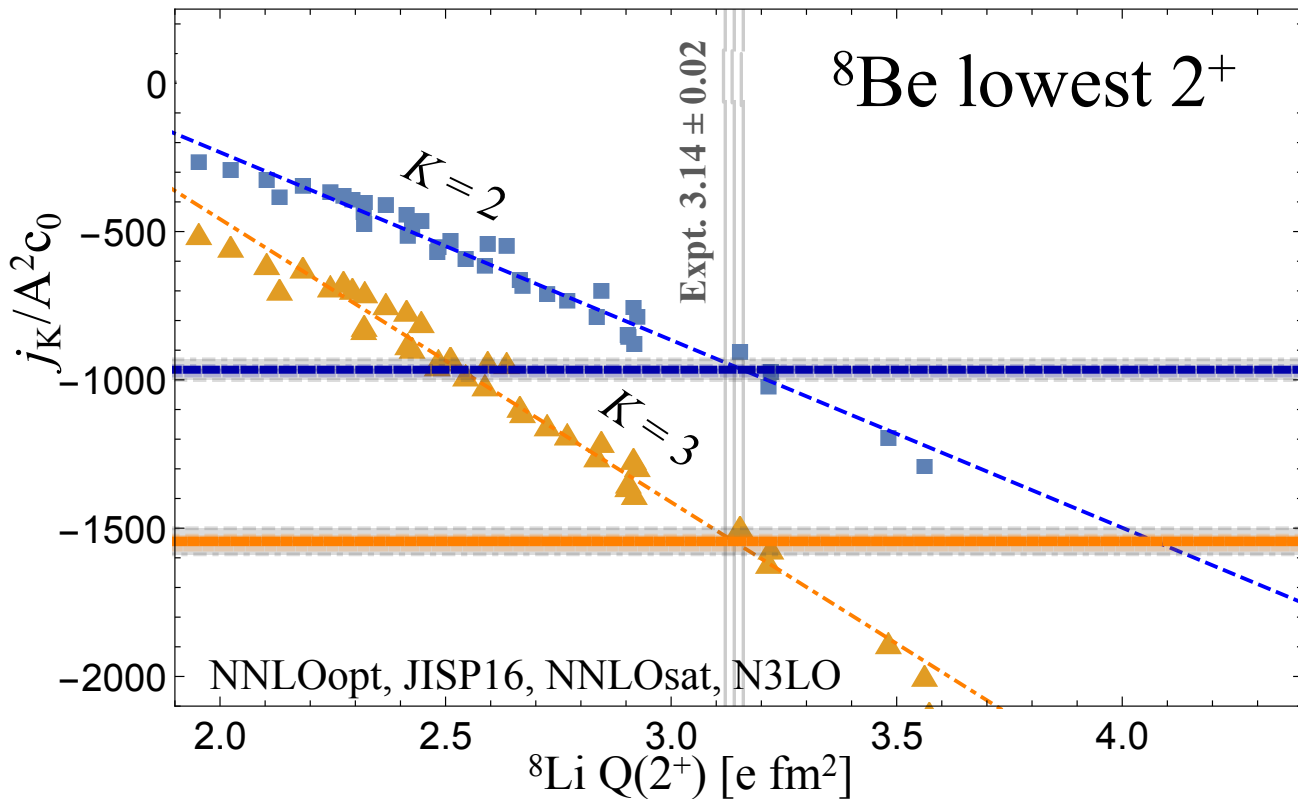


$$j_K \propto \langle \Psi_f | \sum_i^A \tau_i^\pm [\sigma_i \times \hat{Q}_2(\hat{r}_i)]^K | \Psi_0 \rangle \longleftarrow N_{\max} \text{ Recoil-order term}$$

$$c_0 \propto \langle \Psi_f | \sum_i^A \tau_i^\pm \sigma_i | \Psi_0 \rangle \longleftarrow \text{Gamow-Teller matrix element}$$

Sargsyan, Launey, Burkey, *et al.*, PRL128 (20), 202503 (2022)

# Most precise beta-decay measurement of its type in 50 years!



Mary Burkey's PhD Thesis (U. Chicago/ANL/LLNL, 2019)

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Improved by nearly 50%!

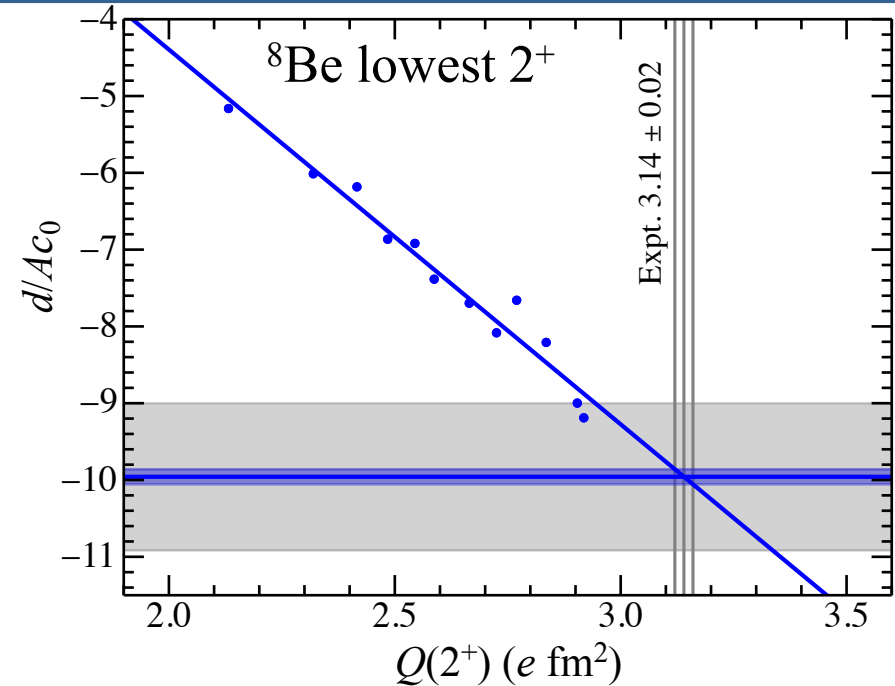
TABLE I. Summary of dominant systematic uncertainties, listed at  $1\sigma$ .

	Systematic Uncertainty	$\Delta C_T/C_A ^2$
Theory	Intruder State (added linearly)	0.0005
	Recoil-Order Terms & Radiative Corrections	0.0015
Experiment	$\alpha$ -Energy Calibration	0.0007
	Detector Lineshape	0.0009
	Data Cuts	0.0009
	$\beta$ Scattering	0.0010
	<b>Total</b>	<b>0.0028</b>

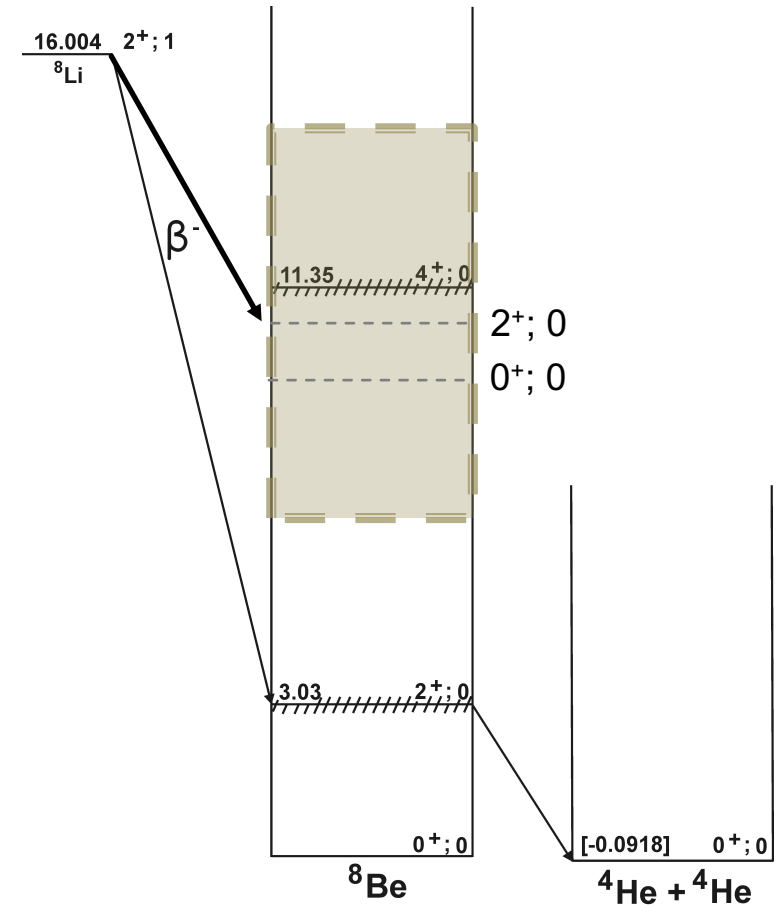
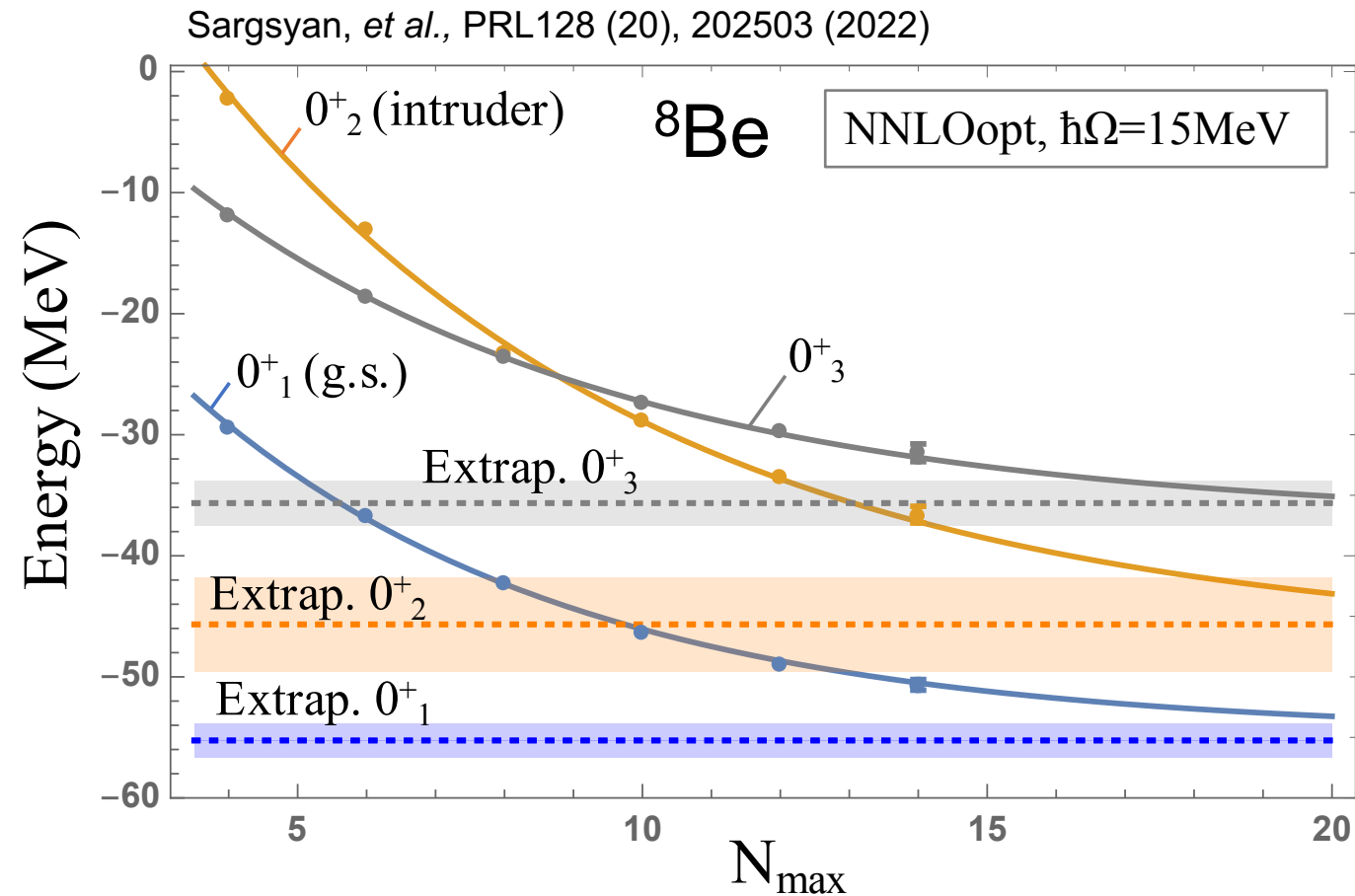
Most precise beta-decay measurement of its type in 50 years!  
 MT Burkey, G Savard, AT Gallant, *et al.*, PRL 128 (20), 202502 (2022).

# Weak magnetism and induced tensor recoil-order terms

- Weak magnetism ( $b$ ) and induced tensor ( $d$ ) recoil terms: next significant after  $j_2$  and  $j_3$
- Important for the tests of conserved vector current (CVC) hypothesis and existence of second class currents
- With SA-NCSM we can calculate these beta decay recoil-order terms for up to intermediate mass nuclei



# Possible intruder states in ${}^8\text{Be}$ can explain the discrepancy in ${}^8\text{Li}$ beta decay



Adapted from <https://nucldata.tunl.duke.edu>

# 0<sup>+</sup> and 2<sup>+</sup> intruder states in <sup>8</sup>Be

PHYSICAL REVIEW C, VOLUME 64, 051301(R)

## Intruder states in <sup>8</sup>Be

E. Caurier,<sup>1</sup> P. Navrátil,<sup>2</sup> W. E. Ormand,<sup>2</sup> and J. P. Vary<sup>3</sup>

<sup>1</sup>Institut de Recherches Subatomiques, IN2P3-CNRS-Université Louis Pasteur, Bâtiment 27/11, F-67037 Strasbourg Cedex 2, France

<sup>2</sup>Lawrence Livermore National Laboratory, L-414, P. O. Box 808, Livermore, California 94551

<sup>3</sup>Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011

(Received 11 July 2001; published 4 October 2001)

Low-lying intruder  $T=0$  states in <sup>8</sup>Be have been posited and challenged. To address this issue, we performed *ab initio* shell model calculations in model spaces consisting of up to  $10\hbar\Omega$  excitations above the unperturbed ground state with the basis state dimensions reaching  $1.87 \times 10^8$ . To gain predictive power we derive and use effective interactions from realistic nucleon-nucleon ( $NN$ ) potentials in a way that guarantees convergence to the exact solution with increasing model space. Our  $0\hbar\Omega$  dominated states show good stability when the model space size increases. At the same time, we observe a rapid drop in excitation energy of the  $2\hbar\Omega$  dominated  $T=0$  states. In the  $10\hbar\Omega$  space the intruder  $0^+$  state falls below 18 MeV of excitation and, also, below the lowest  $0^+1$  state. Our extrapolations suggest that this state may stabilize around 12 MeV. We hypothesize that these states might be the broad resonance intruder states needed in  $R$ -matrix analysis of  $\alpha$ - $\alpha$  elastic scattering. In addition, we present our predictions for the  $A=8$  binding energies with the CD-Bonn  $NN$  potential.

## Measurement of the full excitation spectrum of the <sup>7</sup>Li( $p, \gamma$ ) $\alpha\alpha$ reaction at 441 keV

Michael Munch\*, Oliver Sølund Kirsebom, Jacobus Andreas Swartz, Karsten Riisager, Hans Otto Uldall Fynbo

Department of Physics and Astronomy, Aarhus University, Denmark



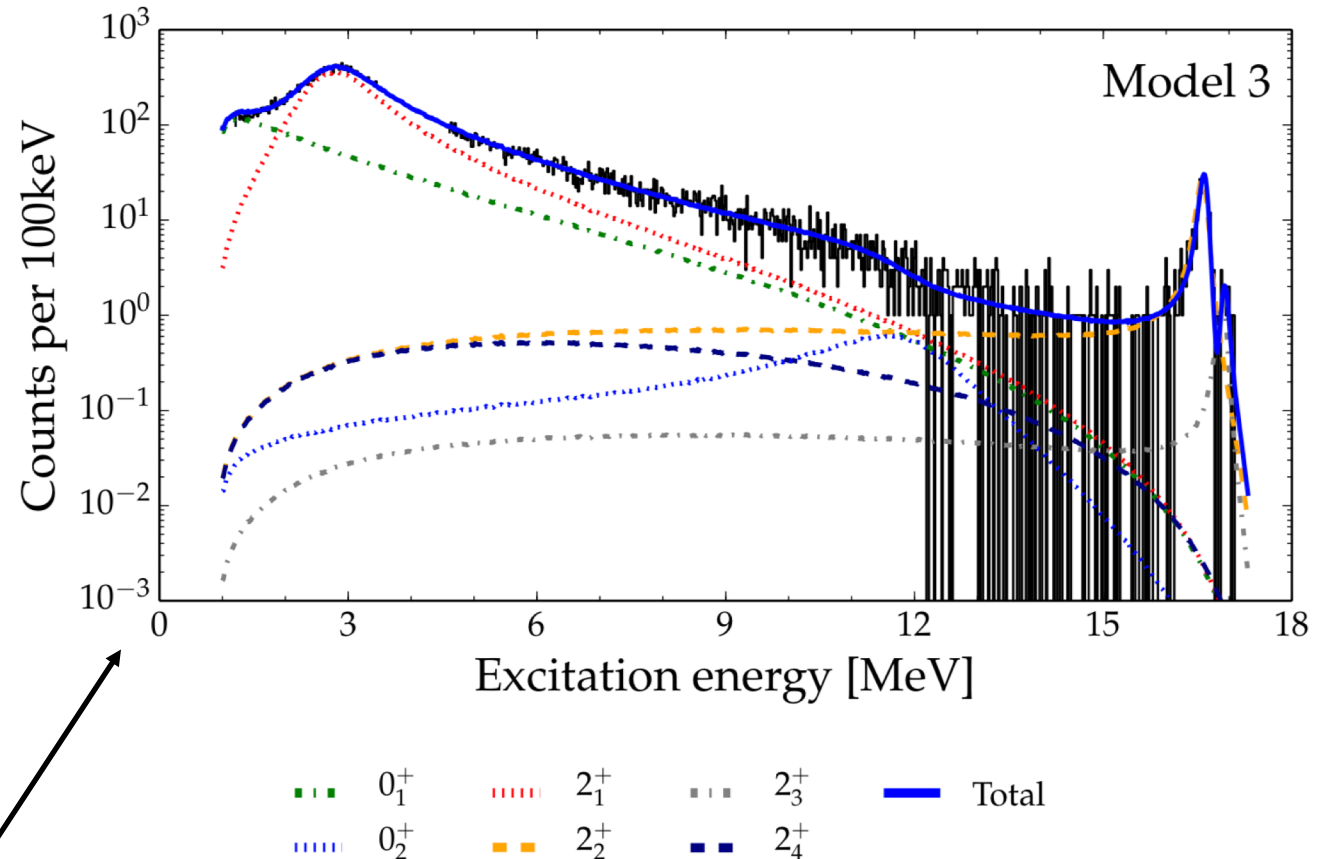
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<sup>8</sup>Be  
Radiative decay width  
Light nuclei

### ABSTRACT

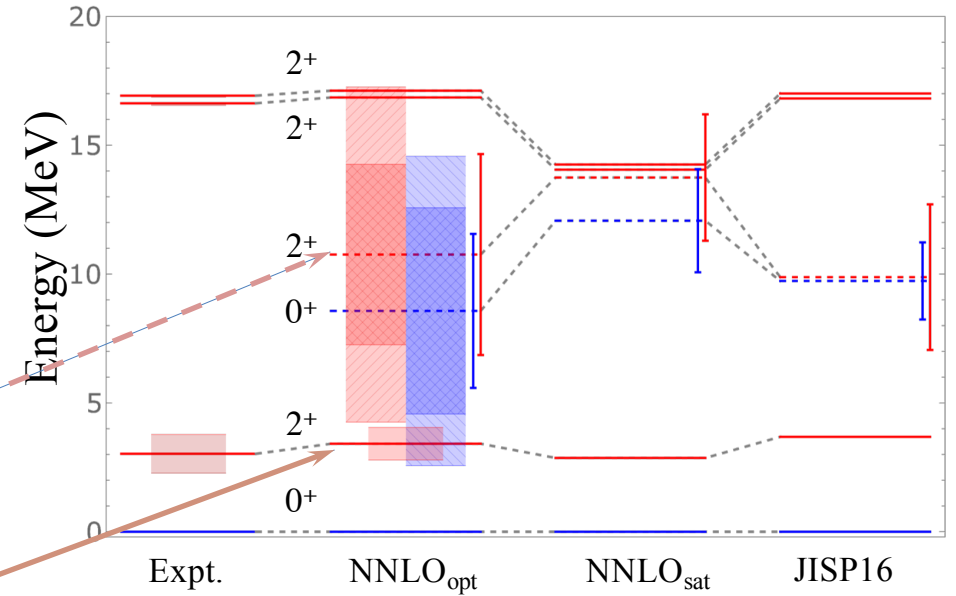
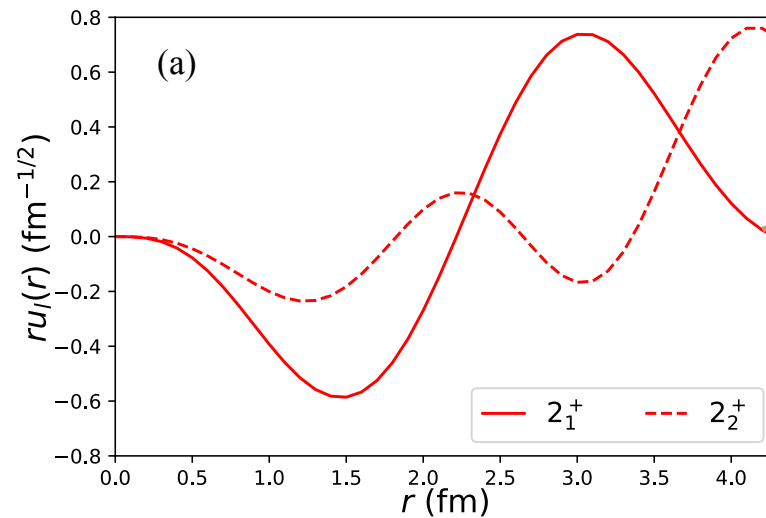
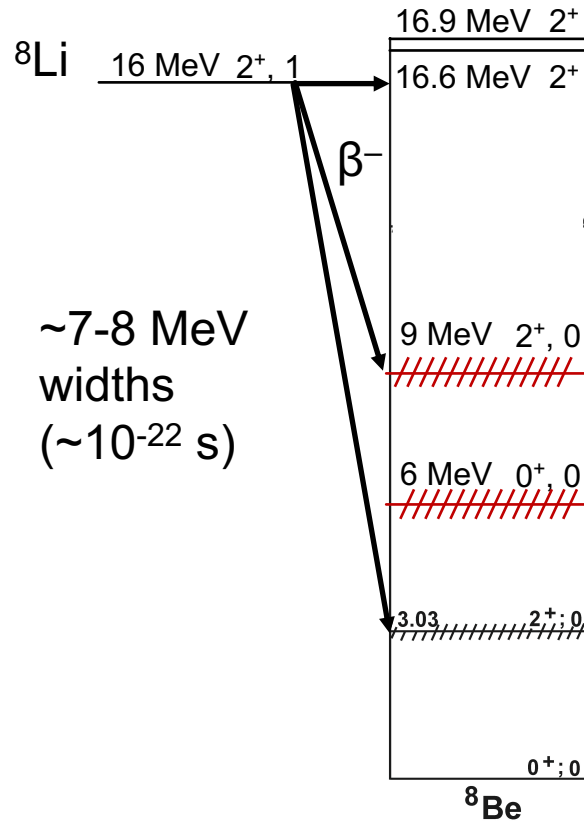
A current challenge for *ab initio* calculations is systems that contain large continuum contributions such as <sup>8</sup>Be. We report on new measurements of radiative decay widths in this nucleus that test recent Green's function Monte Carlo calculations. Traditionally,  $\gamma$  ray detectors have been utilized to measure the high energy photons from the <sup>7</sup>Li( $p, \gamma$ ) $\alpha\alpha$  reaction. However, due to the complicated response function of these detectors it has not yet been possible to extract the full  $\gamma$  ray spectrum from this reaction. Here we present an alternative measurement using large area Silicon detectors to detect the two  $\alpha$  particles, which provides a practically background free spectrum and retains good energy resolution. The resulting spectrum is analyzed using a many-level multi channel R-matrix parametrization. Improved values for the radiative widths are extracted from the R-matrix fit. We find evidence for significant non-resonant continuum contributions and **tentative evidence for a broad  $0^+$  resonance at 12 MeV**.  
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Munch et al., Phys. Lett. B 782 (2018) 779–784

# $^8\text{Be}$ low-lying $0^+$ and $2^+$ states not confirmed in experiments

➤ First proposed by Barker in 1968-69

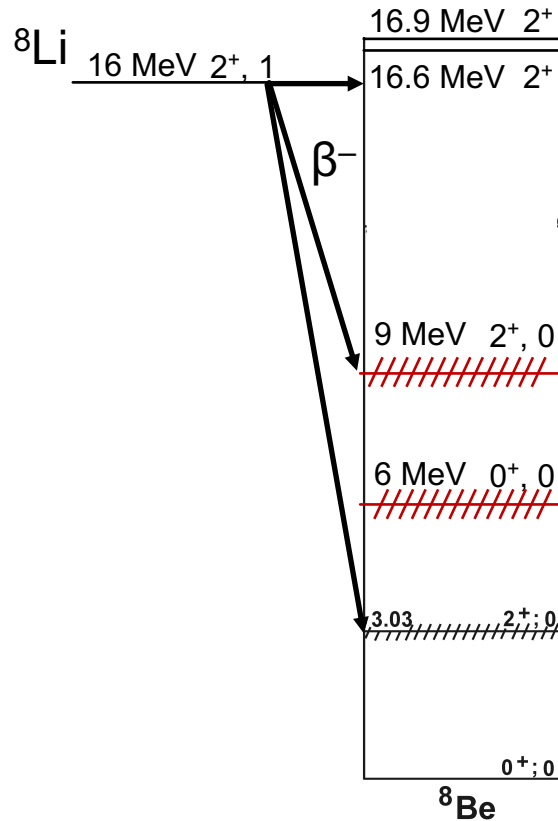


Sargsyan, Launey, Burkey, *et al.*,  
PRL128 (20), 202503 (2022)

F. C. Barker. Australian Journal of Physics, vol. 21, 239–257, 1968.  
F. C. Barker. Australian Journal of Physics, vol. 22, 293–316, 1969.



# Recoil terms for all ${}^8\text{Li}$ $\beta$ -decay accessible states



04-2014

${}^8\text{Be}$ states	$j_2/A^2 c_0$	$j_3/A^2 c_0$	$d/Ac_0$	$b/Ac_0$
$2_1^+$	$-966 \pm 36$	$-1546 \pm 44$	$10.0 \pm 1.0$	$6.0 \pm 0.4$
$2_2^+$ (new)	$-10 \pm 10$	$-80 \pm 30$	$-0.5 \pm 0.5$	$3.7 \pm 0.4$
$2_3^+$ (doublet 1)	$12 \pm 5$	$-60 \pm 15$	$0.3 \pm 0.2$	$3.8 \pm 0.2$
$2_4^+$ (doublet 2)	$11 \pm 3$	$-65 \pm 11$	$0.2 \pm 0.2$	$3.8 \pm 0.2$

- $j_2/A^2 c_0$  and  $j_3/A^2 c_0$  values for the lowest  $2^+$  are much larger than for other states
- $b/Ac_0$  and  $d/Ac_0$  values are also important for tests of conserved vector current hypothesis

# Previous state-of-the-art values for $j_2$ and $j_3$

PHYSICAL REVIEW C **83**, 065501 (2011)

## Test of the conserved vector current hypothesis by a $\beta$ -ray angular distribution measurement in the mass-8 system

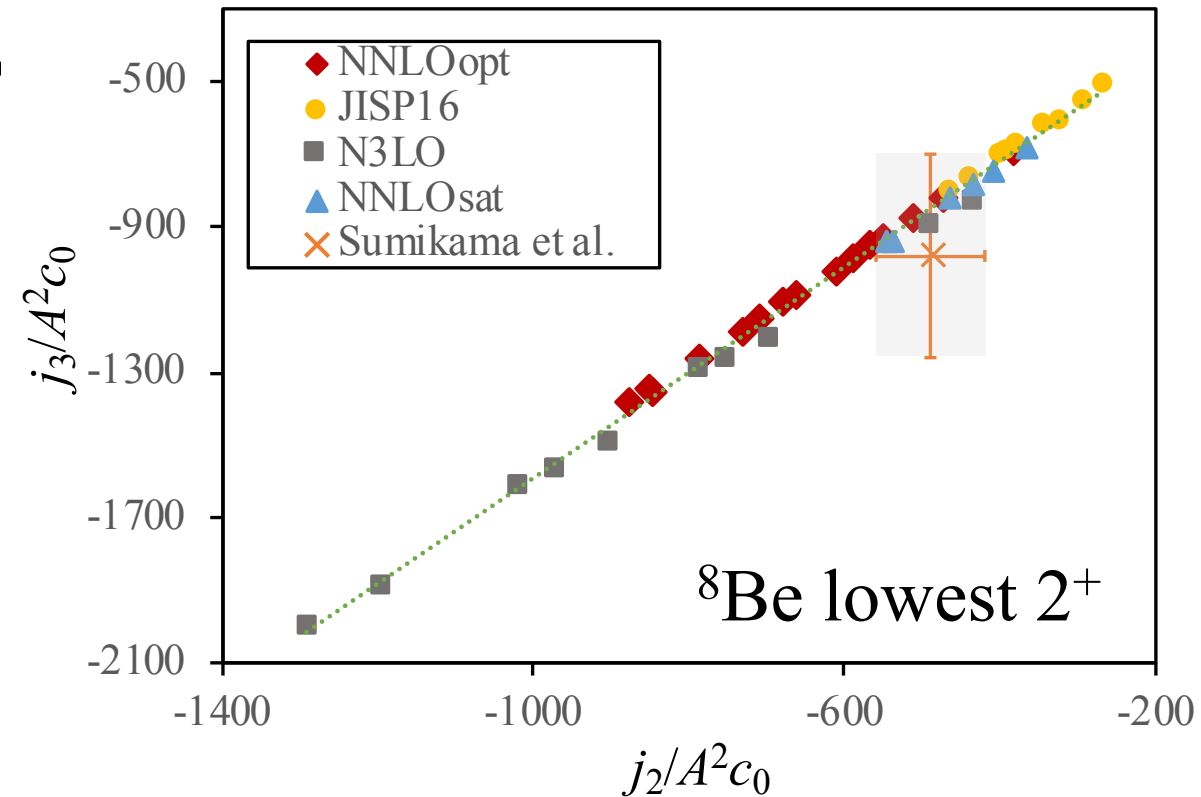
T. Sumikama,<sup>1,2</sup> K. Matsuta,<sup>1</sup> T. Nagatomo,<sup>3</sup> M. Ogura,<sup>1</sup> T. Iwakoshi,<sup>1</sup> Y. Nakashima,<sup>1</sup> H. Fujiwara,<sup>1</sup> M. Fu M. Mihara,<sup>1</sup> K. Minamisono,<sup>4</sup> T. Yamaguchi,<sup>5</sup> and T. Minamisono<sup>6</sup>

$$j_2/A^2c_0 = -490 \pm 70$$

$$j_3/A^2c_0 = -980 \pm 280$$

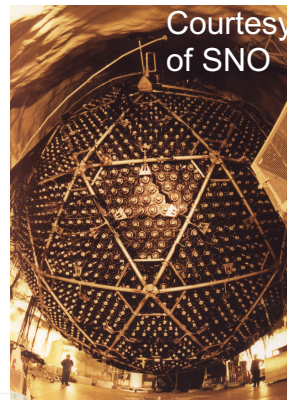
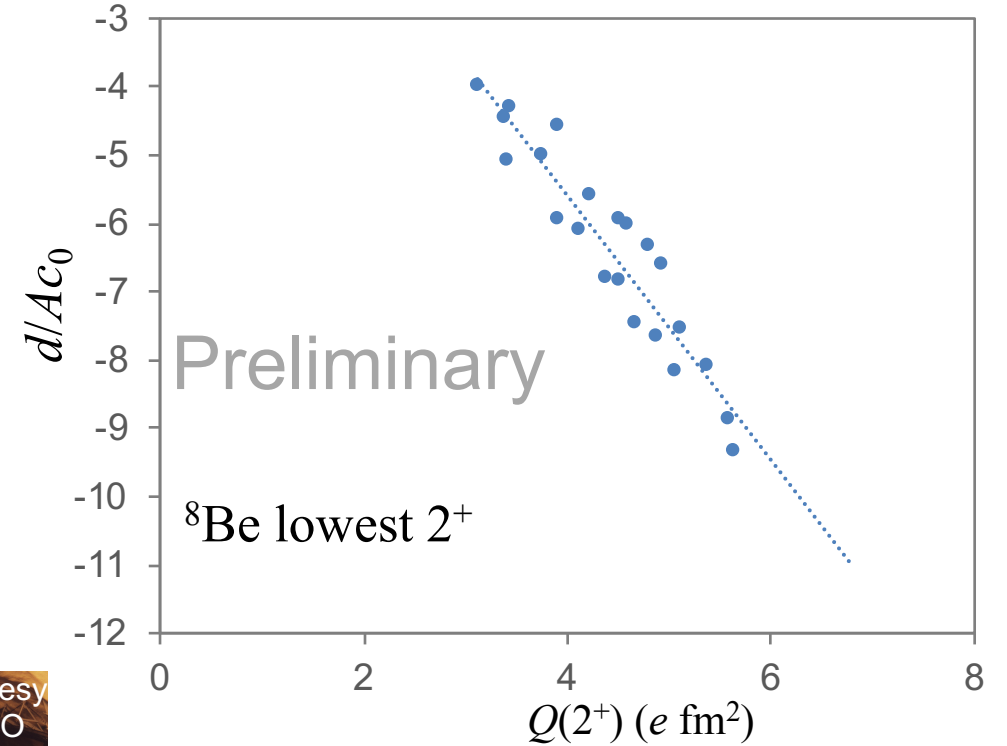
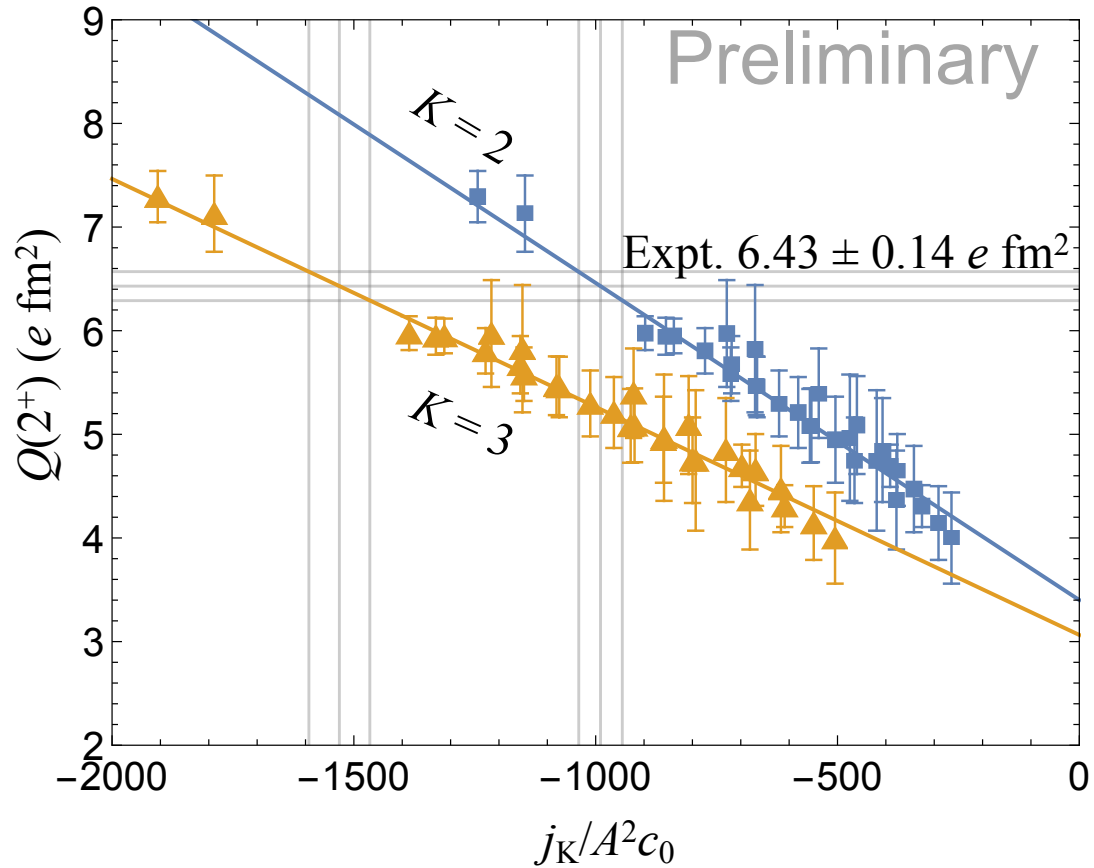
- Extremely difficult measurements  $\Rightarrow$  Large uncertainties
- $j_2$  and  $j_3$  values were considered constant over the entire beta decay energy range

Strong correlation between  $j_2$  and  $j_3$  recoil-order terms



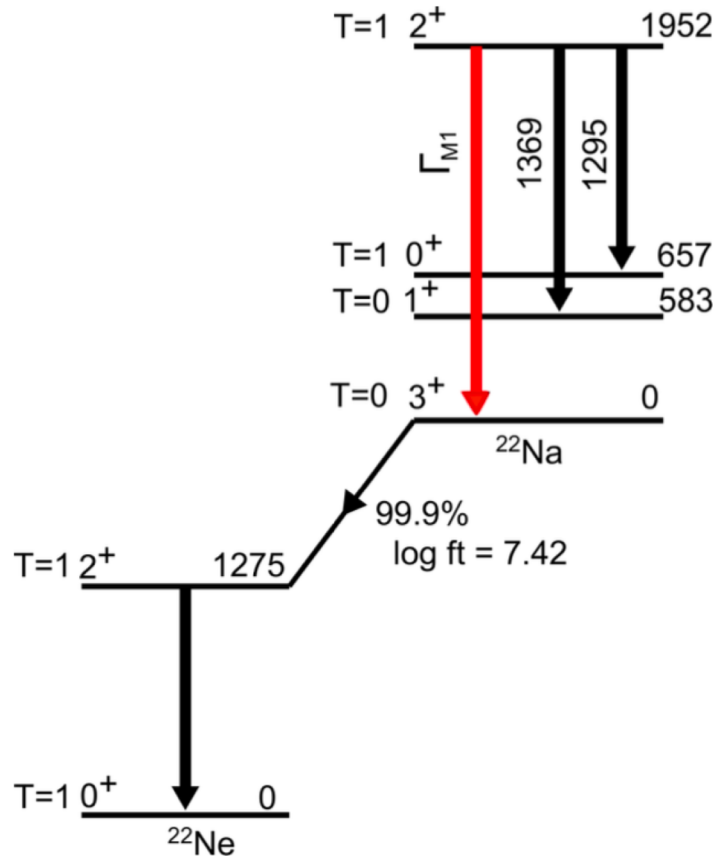
Sargsyan, *et al.*, PRL128 (20), 202503 (2022)

# Recoil terms for ${}^8\text{B}$ to inform precision beta decay experiments



Also important for  
precision measurements of  
solar neutrinos!  
See talk by B. Longfellow

# Weak magnetism and induced tensor terms in $^{22}\text{Na}$



S. Triambak, *et al.*, Phys. Rev. C 95, 035501 (2017)

Using CVC they determined

$$|b/Ac| = 8.9 \pm 1.2$$

$$|d/Ac| = 21 \pm 6$$

Which disagree with shell model calculations

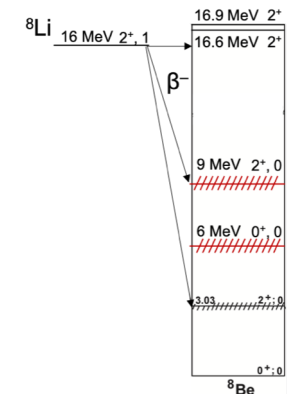
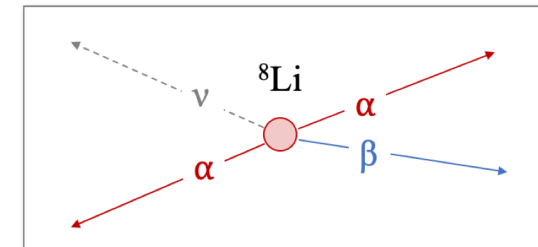
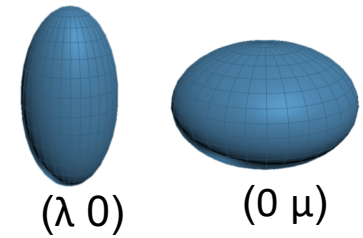
Form factor	Calculated value
Weak magnetism $b/Ac_1$	-19
Second-order axial vector $c_2/c_1 R^2$	-0.37
First-class induced tensor $d/Ac_1$	-3.2

R. B. Firestone, W. C. McHarris, and B. R. Holstein, Phys. Rev. C **18**, 2719 (1978)

- If the sign of  $b/Ac$  is different from shell model prediction, then  $|d/Ac| = 3 \pm 6$
- Our preliminary calculations favor this scenario

# Summary

- The SA-NCSM employs emergent symmetries in nuclei to decrease the dimensionality of the model space, thus allowing us to reach heavier nuclei and large model spaces
- Our calculations of  ${}^8\text{Li}$  beta decay recoil-order terms helped experiment to constrain BSM tensor currents in the weak interaction
- The calculated  $b/Ac_0$  and  $d/Ac_0$  values are important for tests of conserved vector current hypothesis
- Low-lying intruder states in  ${}^8\text{Be}$  can have important implications for  $A=8$  beta decays and related precision measurements





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Czech Academy of Sciences

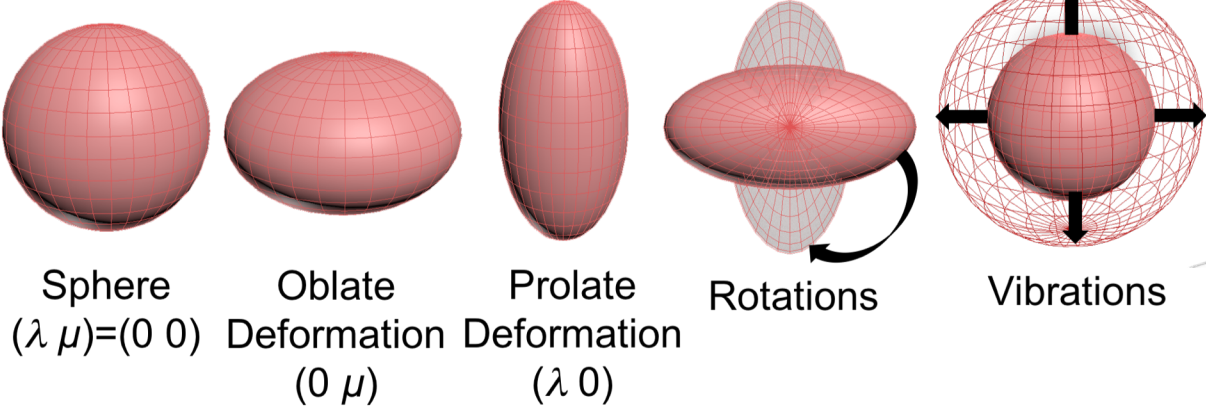
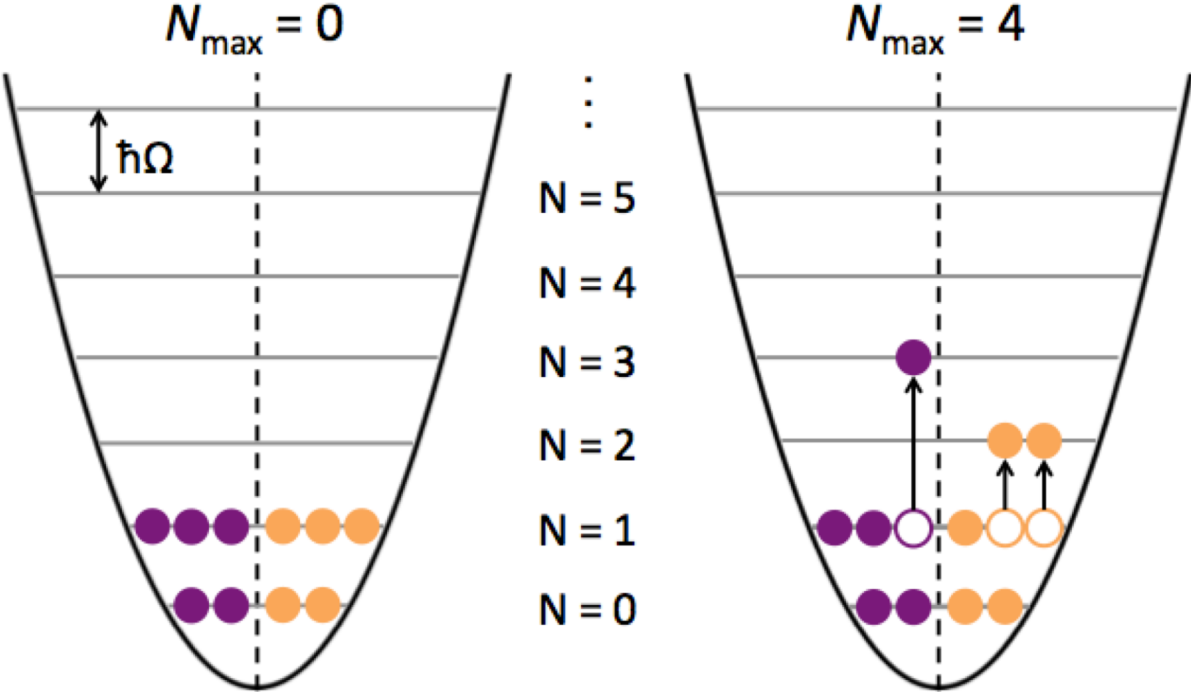


# Thank you!

Experimental crew at the ATLAS facility at Argonne National Lab



# Backup slide zone



# Recoil-order terms in $\beta$ -decay

Beta decay rate:

$$d\Gamma \propto |T|^2$$

$T$  matrix in SM (V-A):

$$T \propto l^\mu \langle \beta | V_\mu - A_\mu | \alpha \rangle$$

$$\begin{aligned}
 l^\mu \langle \beta, J' M' | A_\mu | \alpha, J M \rangle &= C_{J' M' 1 k}^{J M} \epsilon_{ijk} \epsilon_{ij\lambda\eta} \frac{1}{4M} [c(q^2) l^\lambda P^\eta - d(q^2) l^\lambda q^\eta \\
 &+ \frac{1}{(2M)^2} h(q^2) q^\lambda P^\eta \mathbf{q} \cdot \mathbf{l}] \\
 &+ C_{J' M' 2 k}^{J M} C_{1 n 2 n'}^{2 k} l_n (4\pi/5)^{1/2} Y_{2 n'}(\hat{q}) \frac{q^2}{(2M)^2} j_2(q^2) \\
 &+ C_{J' M' 3 k}^{J M} C_{1 n 2 n'}^{3 k} l_n (4\pi/5)^{1/2} Y_{2 n'}(\hat{q}) \frac{q^2}{(2M)^2} j_3(q^2) + \dots
 \end{aligned}$$

Systematic Uncertainty	$\Delta C_T/C_A ^2$
Calibration	$1.4 \times 10^{-4}$
$\alpha$ energy corrections	$1.17 \times 10^{-3}$
Cuts to the data	$1.25 \times 10^{-3}$
Radiative and recoil order terms	$3.36 \times 10^{-3}$
$\alpha$ Si detector lineshape	$6.3 \times 10^{-4}$
$\beta$ Scattering	$5.0 \times 10^{-4}$
<b>Total</b>	<b><math>3.62 \times 10^{-3}</math></b>

From Mary Burkey's PhD Thesis (U. Chicago/ANL/LLNL, 2019)

# Recoil-order terms in $\beta$ -decay

Beta decay rate:

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$$T \propto l^\mu \langle \beta | V_\mu - A_\mu | \alpha \rangle$$

Axial current matrix element

Leading order (Gamow-Teller)

Recoil-order  $q/M$

$$l^\mu \langle \beta, J' M' | A_\mu | \alpha, J M \rangle$$

$$= C_{J'M'1k}^{JM} \epsilon_{ijk} \epsilon_{ij\lambda\eta} \frac{1}{4M} [c(q^2) l^\lambda P^\eta - d(q^2) l^\lambda q^\eta] + \frac{1}{(2M)^2} h(q^2) q^\lambda P^\eta \mathbf{q} \cdot \mathbf{1} + C_{J'M'2k}^{JM} C_{1n2n'}^{2k} l_n (4\pi/5)^{1/2} Y_{2n'}(\hat{q}) \frac{q^2}{(2M)^2} j_2(q^2) + C_{J'M'3k}^{JM} C_{1n2n'}^{3k} l_n (4\pi/5)^{1/2} Y_{2n'}(\hat{q}) \frac{q^2}{(2M)^2} j_3(q^2) + \dots$$

Lepton current matrix element

Recoil-order  $(q/M)^2$

For  ${}^8\text{Li}$  and  ${}^8\text{B}$  beta decay  $q/M \sim 0.002$

Systematic Uncertainty	$\Delta C_T/C_A ^2$
Calibration	$1.4 \times 10^{-4}$
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# Recoil-order terms $b/Ac_0$ and $d/Ac_0$

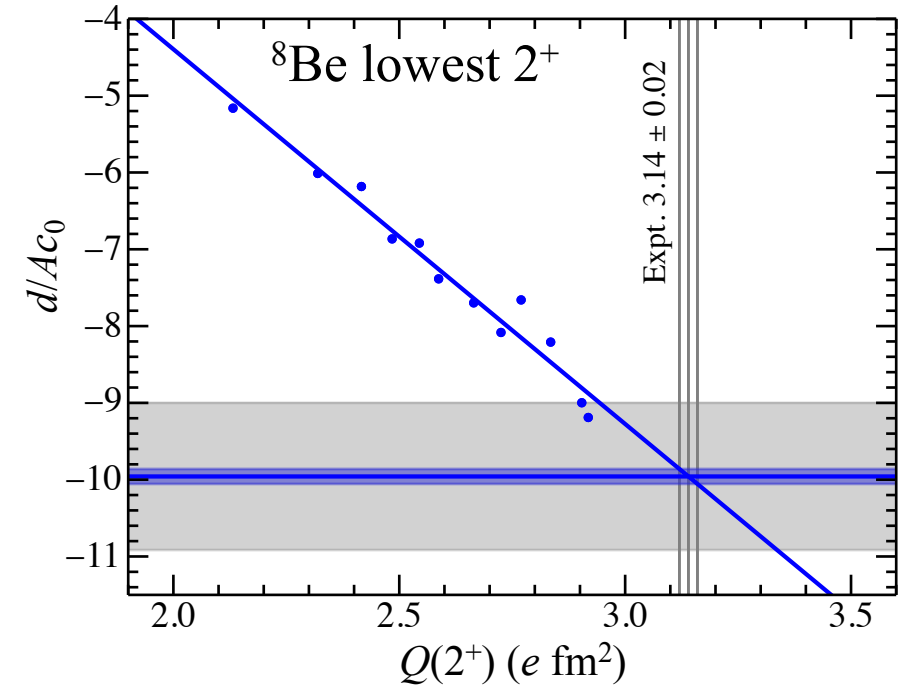
- Weak magnetism ( $b$ ) and induced tensor ( $d$ ) recoil terms become the next significant sources of uncertainty with more precise  $j_2$  and  $j_3$ .

$$b \propto g_M \langle \Psi_f || \sum_i^A \tau_i^\pm \sigma_i || \Psi_0 \rangle + g_V \langle \Psi_f || \sum_i^A \tau_i^\pm L_i || \Psi_0 \rangle$$

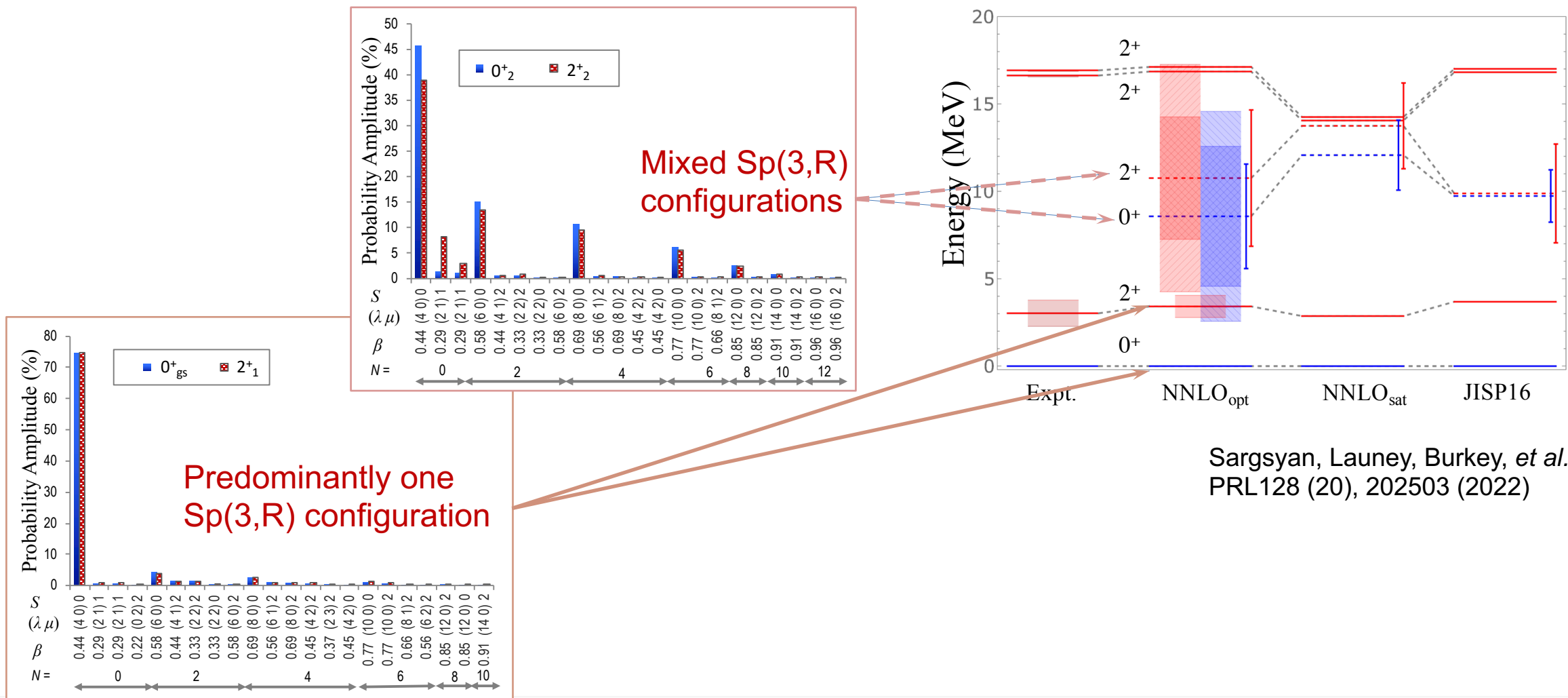
$$d \propto \langle \Psi_f || \sum_i^A \tau_i^\pm \sqrt{2} [L_i \times \sigma_i]^1 || \Psi_0 \rangle$$

$g_M(0) = 4.7$  Weak magnetism coupling

$g_V(0) = 1.0$  Vector coupling



# $^8\text{Be}$ low-lying $0^+$ and $2^+$ states not confirmed in experiments



Sargsyan, Launey, Burkey, *et al.*,  
PRL128 (20), 202503 (2022)

$$\begin{aligned}
j_K(q^2) &= -(-)^{J'-J} \frac{2}{3} \frac{g_A(q^2)}{\sqrt{2J+1}} \frac{M^2 c^4}{(\hbar c)^2} \langle J' || \sqrt{\frac{16\pi}{5}} \sum_i^A \tau_i^\pm r_i^2 [Y_2(\hat{r}_i)]^K \times \sigma_i || J \rangle \\
&= -(-)^{J'-J} \frac{2}{3} \frac{g_A(q^2)}{\sqrt{2J+1}} \frac{(Am_N)^2 c^4}{(\hbar c)^2} b^2 \langle J' || \sum_i^A \tau_i^\pm [\hat{Q}_2(\hat{r}_i) \times \sigma_i]^K || J \rangle \\
&= -(-)^{J'-J} \frac{2}{3} \frac{g_A(q^2)}{\sqrt{2J+1}} \frac{A^2 m_N c^2}{\hbar \Omega} \langle J' || \sum_i^A \tau_i^\pm [\hat{Q}_2(\hat{r}_i) \times \sigma_i]^K || J \rangle,
\end{aligned}$$