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Bayesian Refinement of RMF Models

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Introduction and Motivation

- Start with a energy density functional/Lagrangian

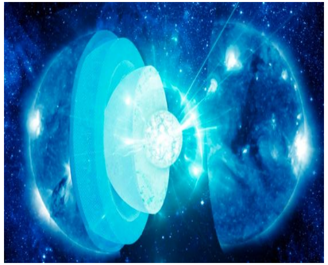
- Describe nuclear matter in low density finite systems all the way up to high density infinite matter

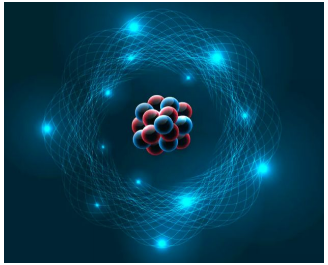
- Use experimental data from CREX/PREX and LIGO/NICER

- Better understand the ground state of nuclear matter and the corresponding interactions that take place

- Gain insight into the composition of neutron star matter

$$\mathcal{L}(x_1, x_2, \ldots x_n) = ?$$





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Previous Models — A starting point

- Consider nucleons interacting via meson exchanges

- For a large number of nucleons, use a mean field approximation to calculate ground state properties (BA, Rch, Rskin)

- For infinite nuclear matter meson fields are spatially independent

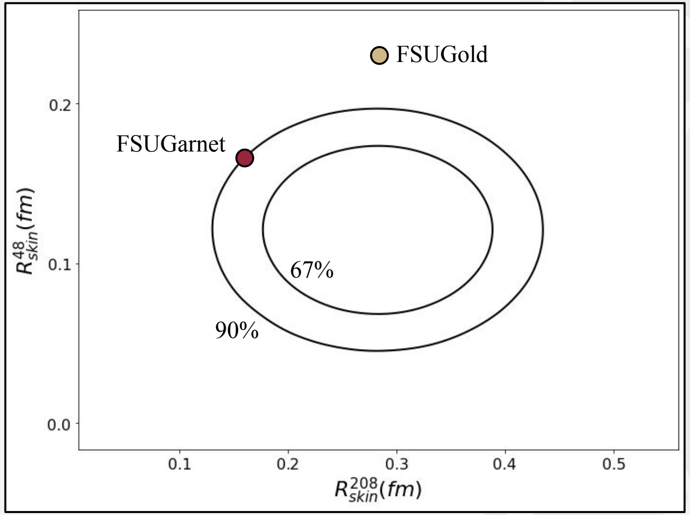
- Obtain nuclear matter EOS and calculate neutron star observables (M(R), Λ, I)

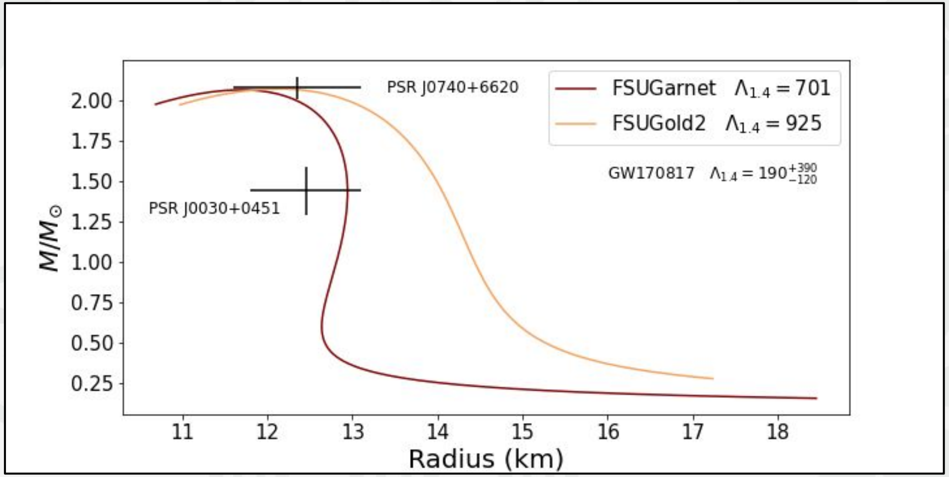
FSUGarnet and FSUGold Models

$$\mathcal{L}_{\rm nuc} = \bar{\psi}_\alpha\left[\gamma_\beta(i\partial^\beta - g_\omega\omega^\beta) - (m - g_\sigma\sigma) - \frac{1}{2}g_\rho\gamma_\beta(\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\beta)\right]\psi_\alpha + \frac{1}{2}(\partial_\beta\sigma\partial^\beta\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\beta\nu}\omega^{\beta\nu} + \frac{1}{2}m_\omega^2\omega^\beta\omega_\beta - \frac{1}{4}\boldsymbol{\rho}_{\beta\nu}\cdot\boldsymbol{\rho}^{\beta\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\beta\cdot\boldsymbol{\rho}^\beta - \frac{1}{3!}\kappa(g_\sigma\sigma)^3 - \frac{1}{4!}\lambda(g_\sigma\sigma)^4 + \frac{1}{4!}\zeta(g_\omega^2\omega^\beta\omega_\beta)^2 + \Lambda_V(g_\rho^2\boldsymbol{\rho}_\beta\cdot\boldsymbol{\rho}^\beta)(g_\omega^2\omega^\nu\omega_\nu)$$

σ-meson, ω-meson, ρ-meson

Previous Model Predictions — Motivation for Refinement





PSR J0030+0451	$M = 1.44 \pm 0.15\ M_{\odot}$ $R = 12.45 \pm 0.65$ km
PSR J0740+6620	$M = 2.08 \pm 0.07\ M_{\odot}$ $R = 12.35 \pm 0.75$ km

Before Refinement — An important mapping

- Adjusting coupling constants can make convergence difficult

- Much easier to map couplings to some bulk properties

- Bulk properties are obtained by extrapolating to infinite nuclear matter

- Bulk properties can be used to calculate couplings and vice versa

$$(g_\sigma, g_\omega, g_\rho, \kappa, \lambda, \zeta, \Lambda_V) \leftrightharpoons (BA, \rho_0, m^*, K, J, L, \zeta)$$

$$E(\varrho,\alpha) \approx E_{\mathrm{SNM}}(\varrho) + \frac{1}{\varrho}\frac{\partial \epsilon}{\partial \alpha}\bigg|_{\alpha=0} \alpha + \frac{1}{2\varrho}\frac{\partial^2 \epsilon}{\partial \alpha^2}\bigg|_{\alpha=0} \alpha^2 + \ldots$$

$$E_{\mathrm{SNM}}(x) \approx \frac{\epsilon}{\varrho}\bigg|_{\varrho_0} - m + \frac{9\varrho_0}{2}\left(\frac{d^2\epsilon}{d\varrho^2}\right)_{\varrho_0} x^2 + \ldots$$
$$\approx B/A + \frac{1}{2}Kx^2 + \ldots$$

$$\alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$S(\varrho) \approx S(\varrho_0) + 3\varrho_0 \frac{dS}{d\varrho}\bigg|_{\varrho_0} x + \frac{9\varrho_0^2}{2}\frac{d^2S}{d\varrho^2}\bigg|_{\varrho_0} x^2 + \ldots$$
$$S(\varrho) \approx J + Lx + K_{\mathrm{sym}}x^2 + \ldots$$

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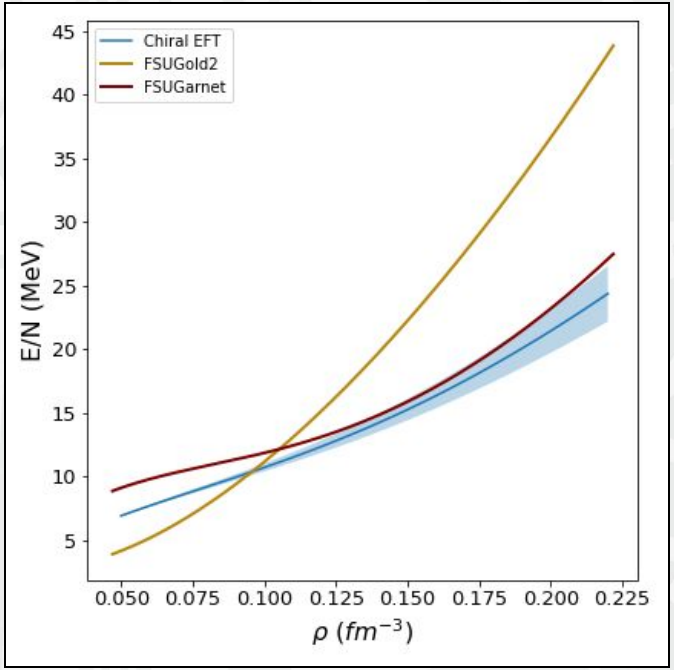
Bayesian Refinement

Use Bayesian Analysis to update the prior EOS's

$$P(\mathcal{M}|D) = \frac{P(D|\mathcal{M})P(\mathcal{M})}{P(D)} \Longrightarrow P(\mathcal{M}|D) \propto \mathcal{L}(\mathcal{M}, D)P(\mathcal{M})$$

The new posterior distribution can be sampled using Markov Chain Monte Carlo (MCMC).

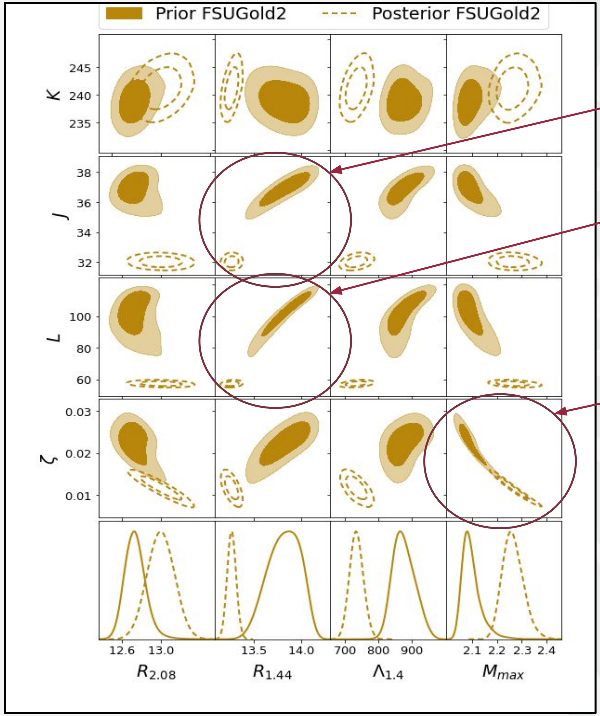
Additional input into likelihood using predictions made by Chiral EFT on the EOS for Pure Neutron Matter



Drischler C, Furnstahl R, Melendez J, Phillips D. Phys. Rev. Lett. 125:202702 (2020)

$$P(\theta|\mathcal{D}) = \prod_{i=1}^{N_{MR}} \mathcal{L}_{MR}(\theta, D_i) \times \prod_{j=1}^{N_{GW}} \mathcal{L}_{GW}(\theta, D_j) \times \prod_{k=1}^{N_\chi} \mathcal{L}_{\chi_{EFT}}(\theta, D_k) \times P(\theta)$$

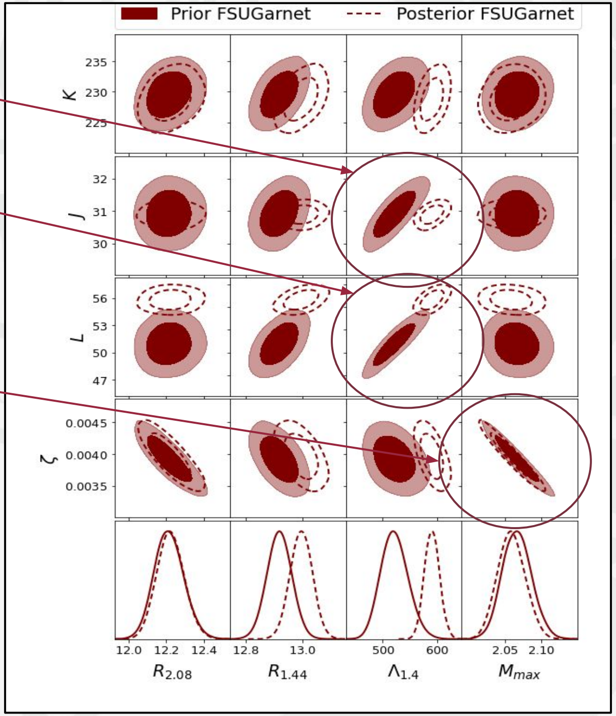
Emergent Correlations from Bayesian Analysis



$J \propto R_{1.44}$ & $J \propto \Lambda_{1.4}$

$L \propto R_{1.44}$ & $L \propto \Lambda_{1.4}$

$\zeta \propto -M_{max}$



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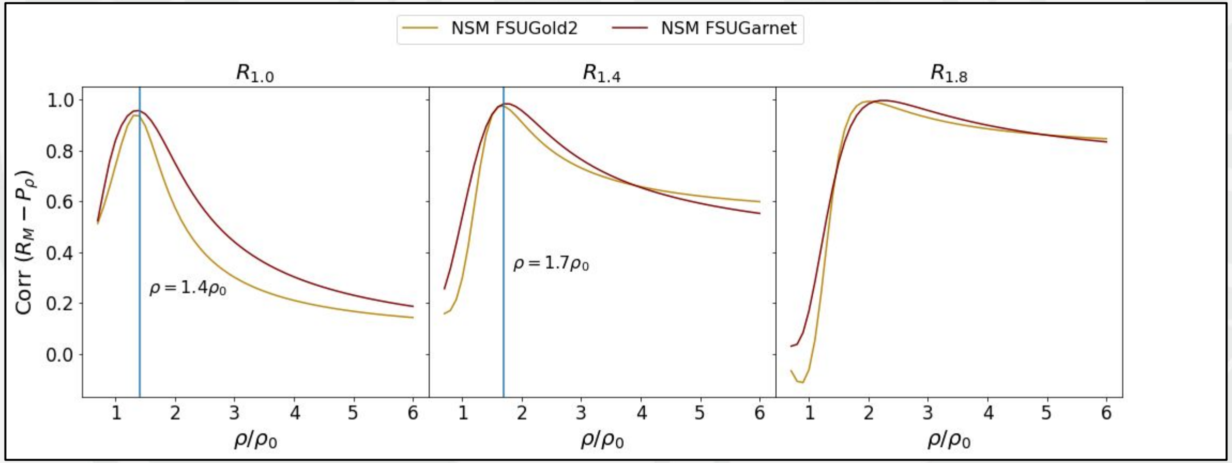
More Insightful Correlations

- Precise measurements of R1.0 and R1.4 could provide tighter constraints on the EOS at densities between 1 and 2 times saturation

- Radius measurements of large mass neutron stars seem unimportant for EOS at high densities



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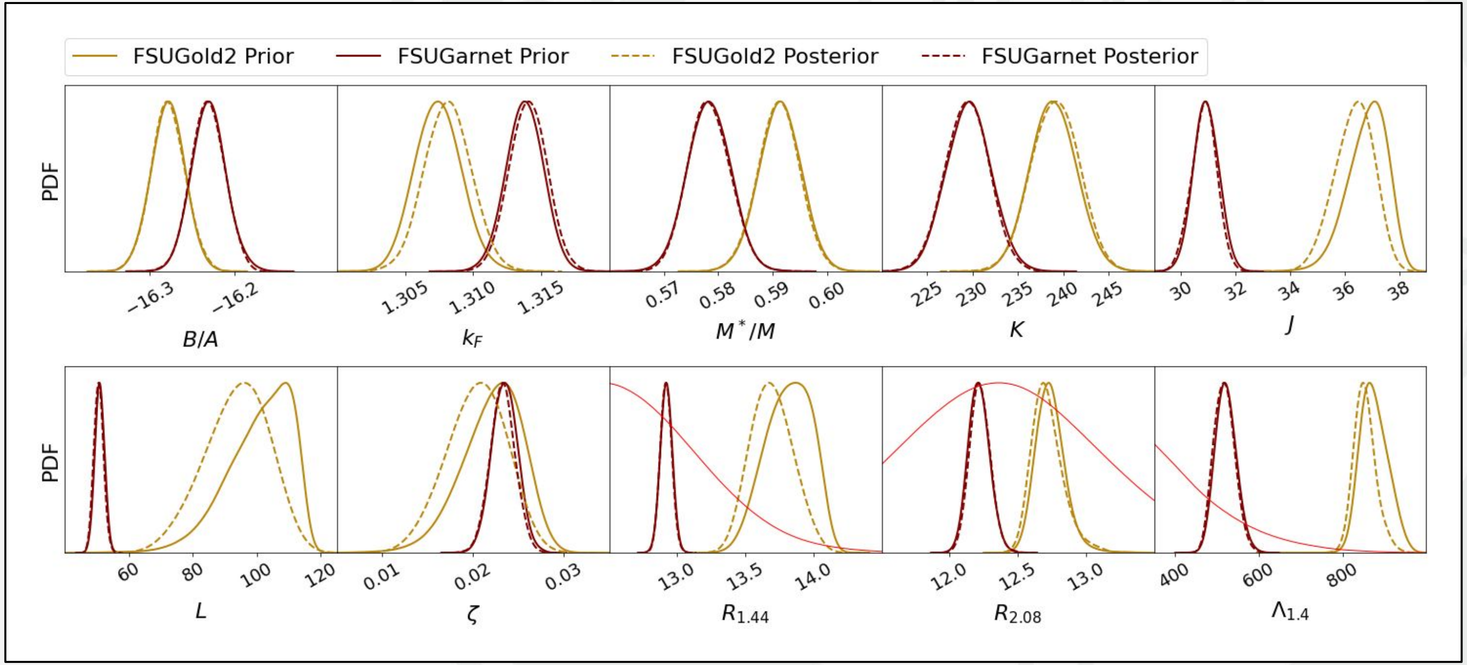
Astrophysics Impact



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Results Summary

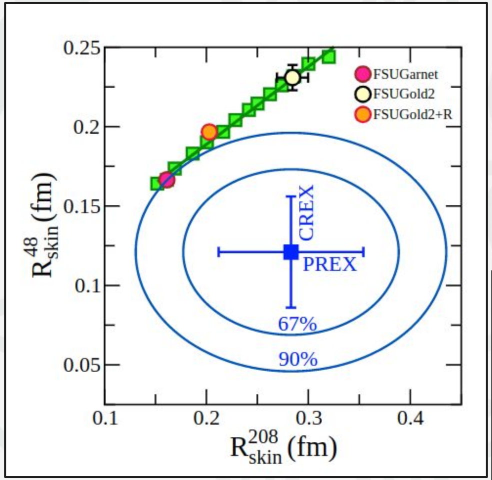
- Better agreement with NICER/LIGO

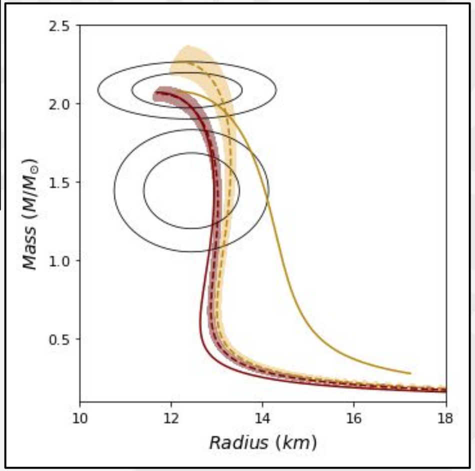
- Low density region of the EOS needs improvement

- XEFT/LIGO predict a soft EOS at low densities

- PREX favors stiff EOS and CREX favors soft EOS

- Radius measurements do not provide enough precision to tightly constrain the EOS





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Possible Solutions

- More precise measurements on the neutron skin thickness could push models in or out of agreement

- Re-examine the current lagrangian to add additional interactions or phase transitions

- Use Bayesian analysis to calibrate similar or entirely different models

- Adding an scalar isoscalar delta meson a0(980) seems promising in resolving CREX/PREX puzzle at low densities, but needs improvement in higher density regions (https://arxiv.org/abs/2305.19376)

- Consider effects of tensor couplings (ex. Omega, rho meson)

$$\mathcal{L}_{\text{new}} = \mathcal{L}_{\text{old}} + g_\delta \bar{\psi}(\boldsymbol{\tau} \cdot \boldsymbol{\delta})\psi + \ldots$$

$$\mathcal{L}_{\text{int}} = \frac{f_\omega}{2}\bar{\psi}\frac{\sigma^{\mu\nu}}{4m}\psi\partial_\nu\omega_\mu \quad ?$$

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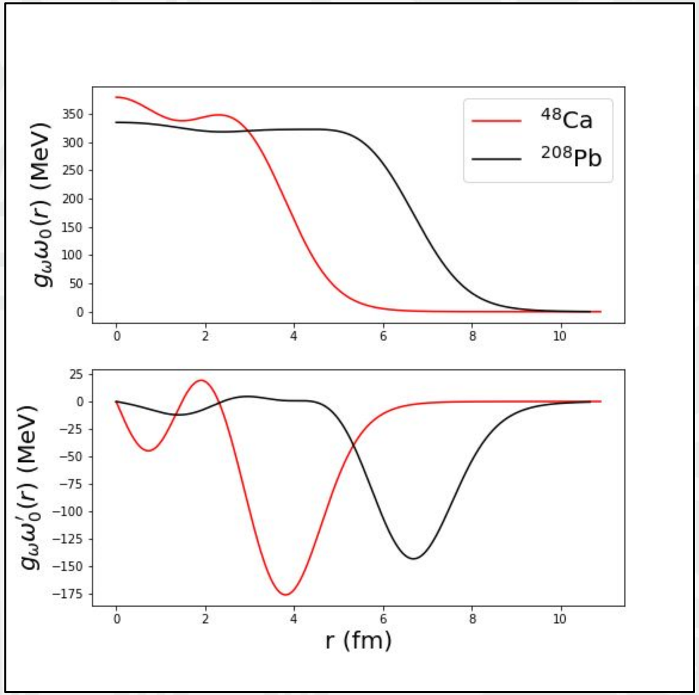
Tensor Coupling

- Usually never considered in RMF lagrangians

- Derivative coupling to the meson field vanishes in infinite nuclear matter/only effects finite nuclei

- Should theoretically have a stronger impact on ^{48}Ca than on ^{208}Pb

- Lowers skin thickness at the cost of increase in binding energy



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