Overview of Small-x Physics

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CFNS-INT Joint Program: Precision QCD with the Electron Ion Collider
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Outline

- CGC in a nutshell
- Theory developments

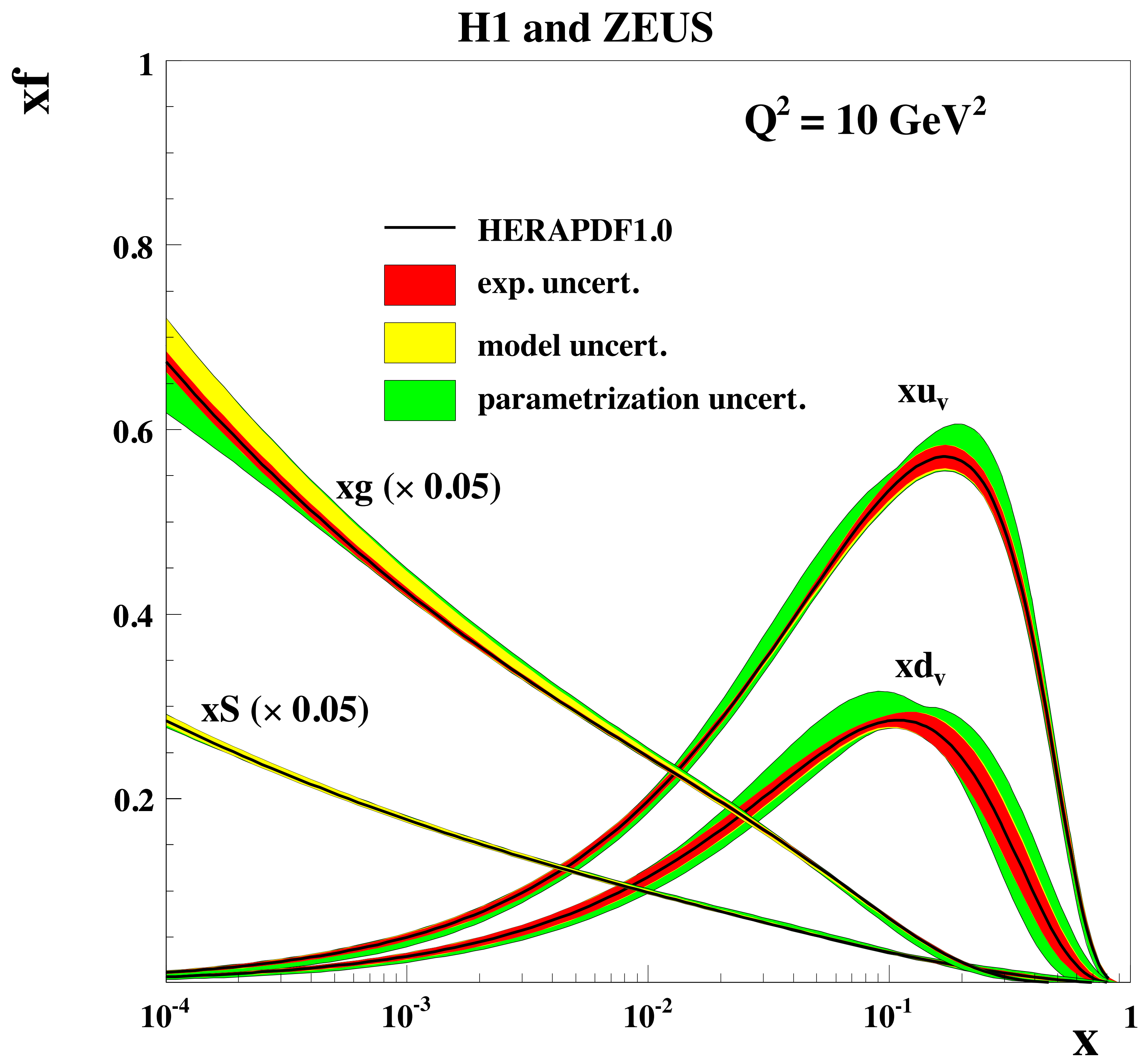
  Fixed order
  Small-x evolution
  CSS, DGLAP, BFKL/BK/JIMWLK resummation

- Observables at the EIC

  Inclusive: structure functions
  Semi-inclusive: two-particle correlations
  Exclusive: vector meson production

- Summary

Anatomy of high-energy QCD



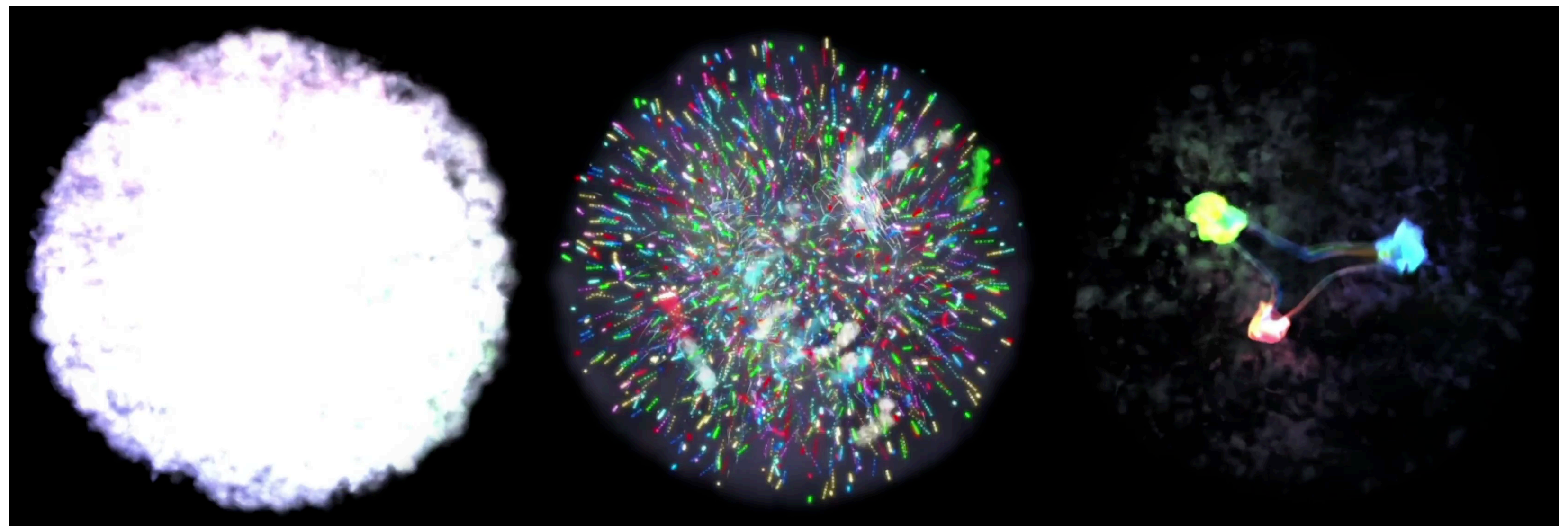
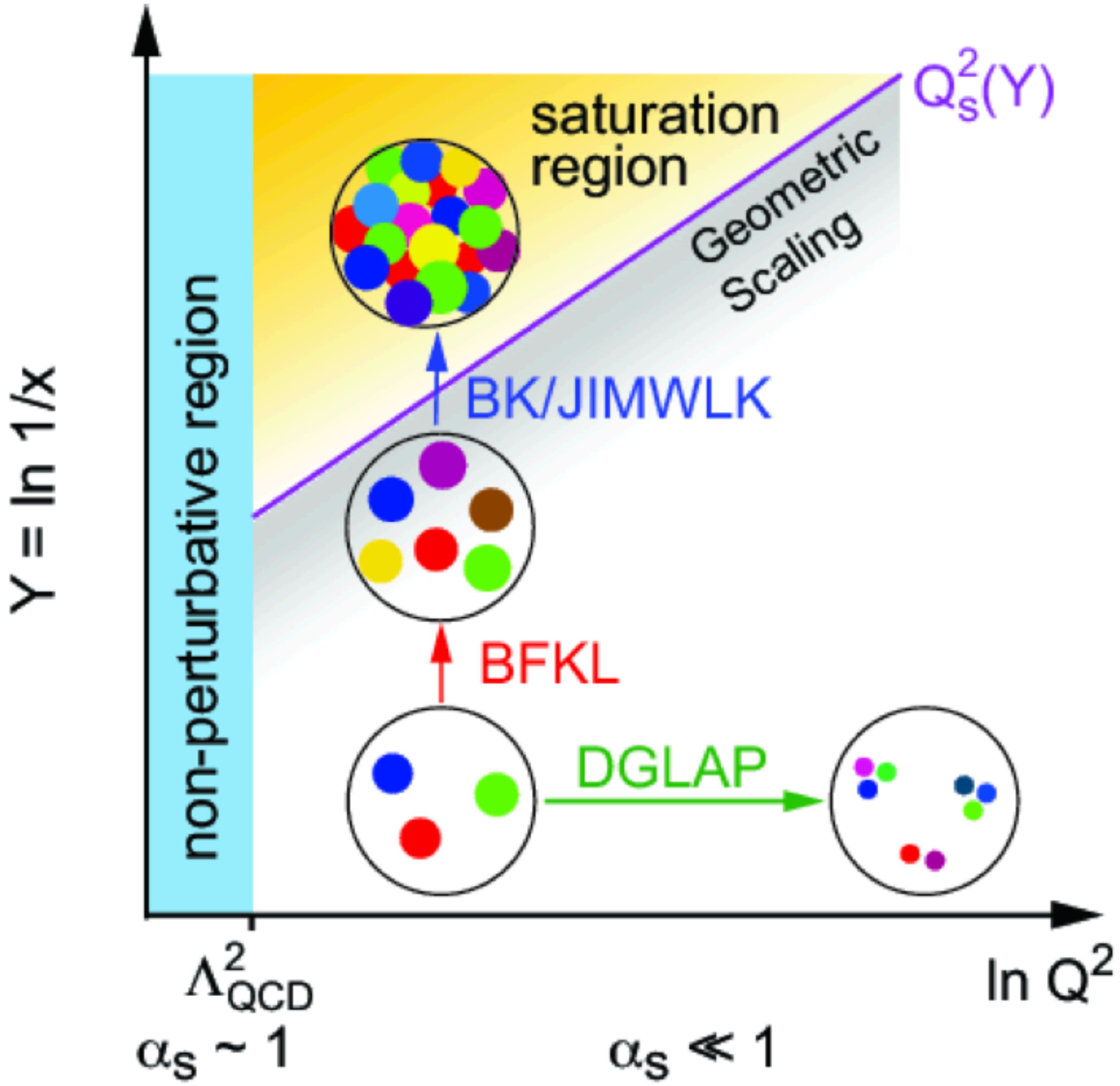


Figure from MIT/Jefferson Lab/Sputnik

Hadronic matter a vibrant QCD environment

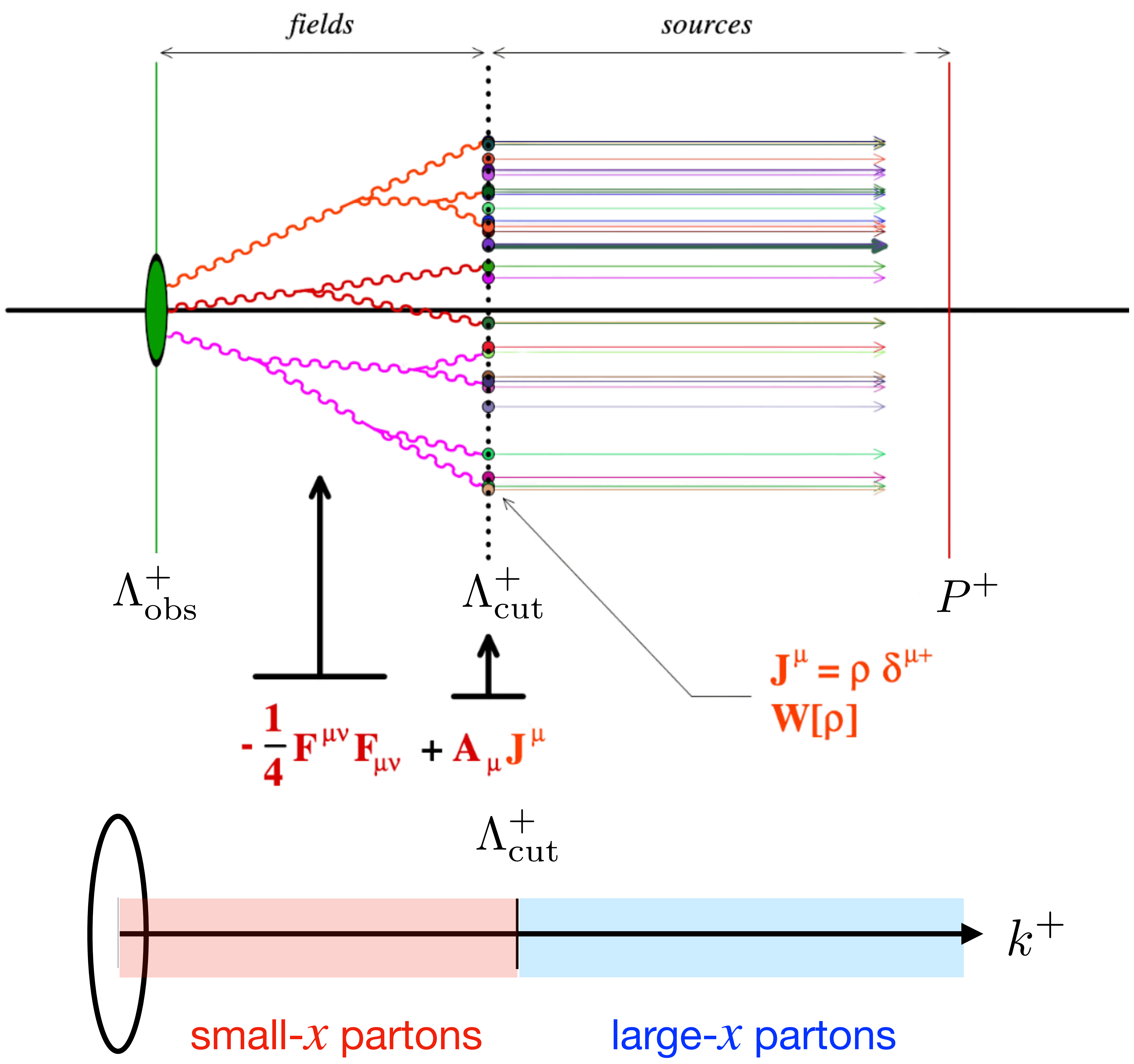


Partonic picture superseded by **strong color fields**

Emergence of x-dependent momentum scale $Q_s^2(x)$ allow for weakly coupled methods

Universality: unified description of QCD at high-energies

The Color Glass Condensate: fields and sources



Large-x partons

Integrated out, effectively treated as static and localized stochastic color charge density sources ρ, its correlations described by non-perturbative gauge invariant weight functional $W_{\Lambda^+_{\rm cut}}[\rho]$

Small-x partons

Dynamical gauge field A^μ generated by ρ. In the saddle point approximation related by classical Yang-Mills' equations

$$\langle\langle \mathcal{O} \rangle\rangle = \int [\mathcal{D}\rho]\, W_{\Lambda^+_{\rm cut}}[\rho]\, \frac{\int^{\Lambda^+_{\rm cut}} [\mathcal{D}A]\, \mathcal{O} e^{i\mathcal{S}[A,\rho]}}{\int^{\Lambda^+_{\rm cut}} [\mathcal{D}A]\, e^{i\mathcal{S}[A,\rho]}}$$

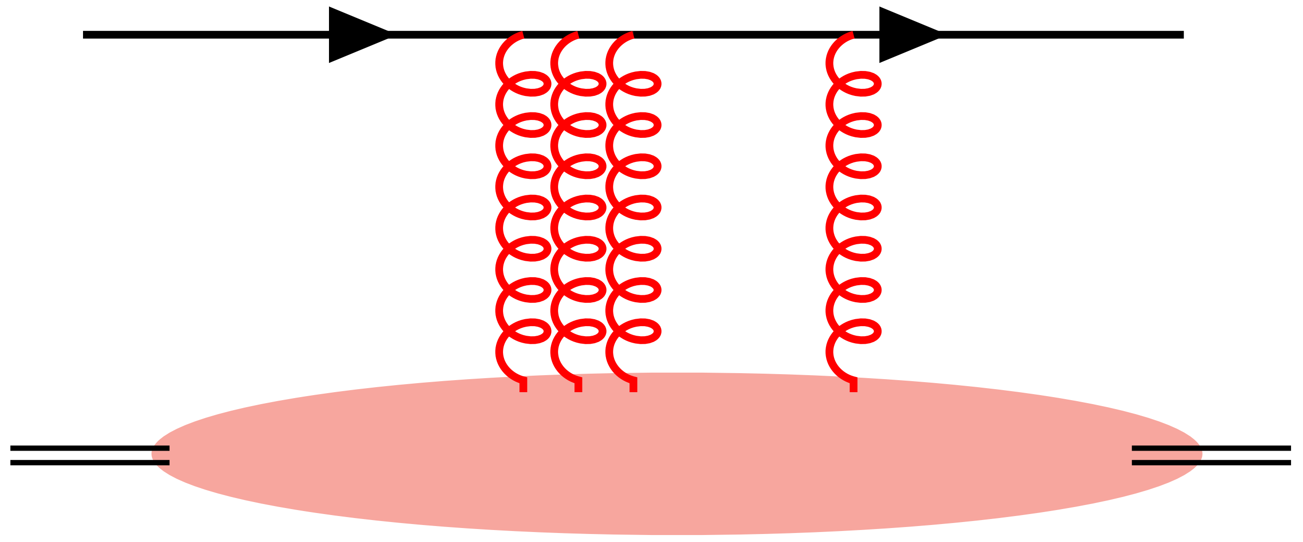
CGC average for ρ — Path integral in the presence of ρ

Color (Quantum Chromodynamics)
Glass (separation of time scales of d.o.f.s)
Condensate (highly occupied system)

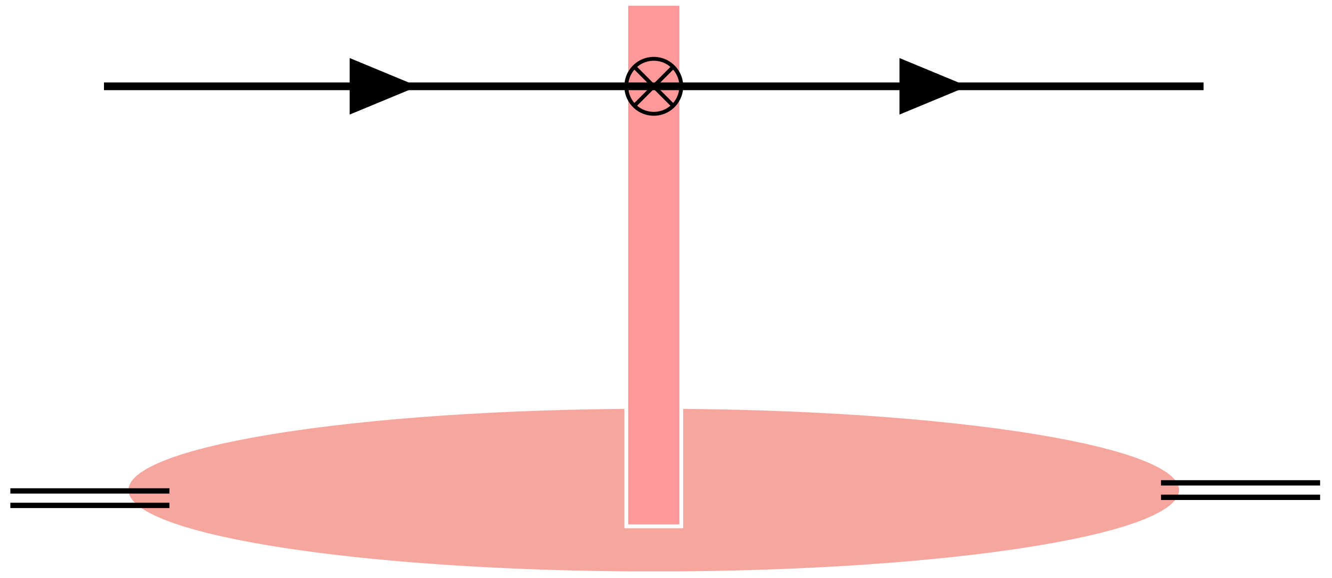
The Color Glass Condensate: multiple scattering

Shock-wave and Wilson lines

Multiple-eikonal scattering



Quark-shockwave vertex

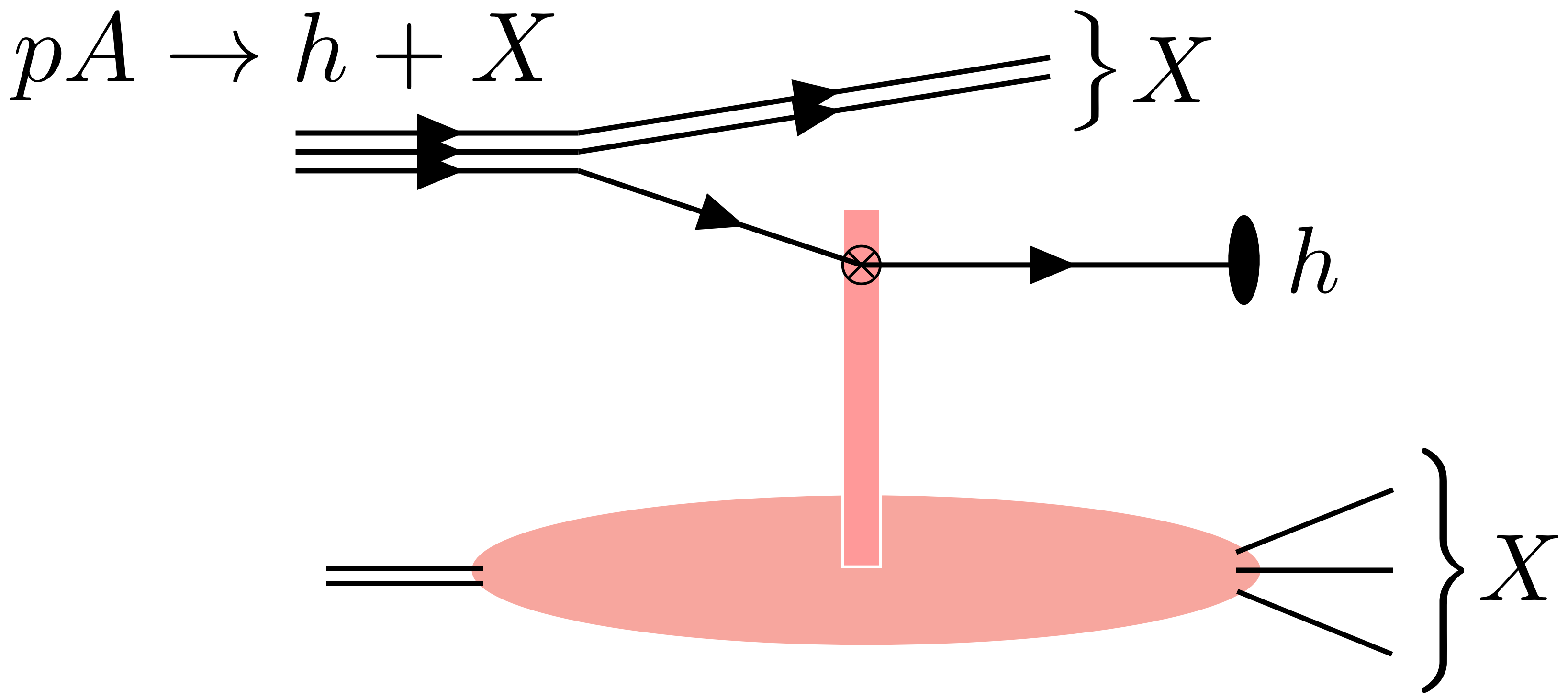


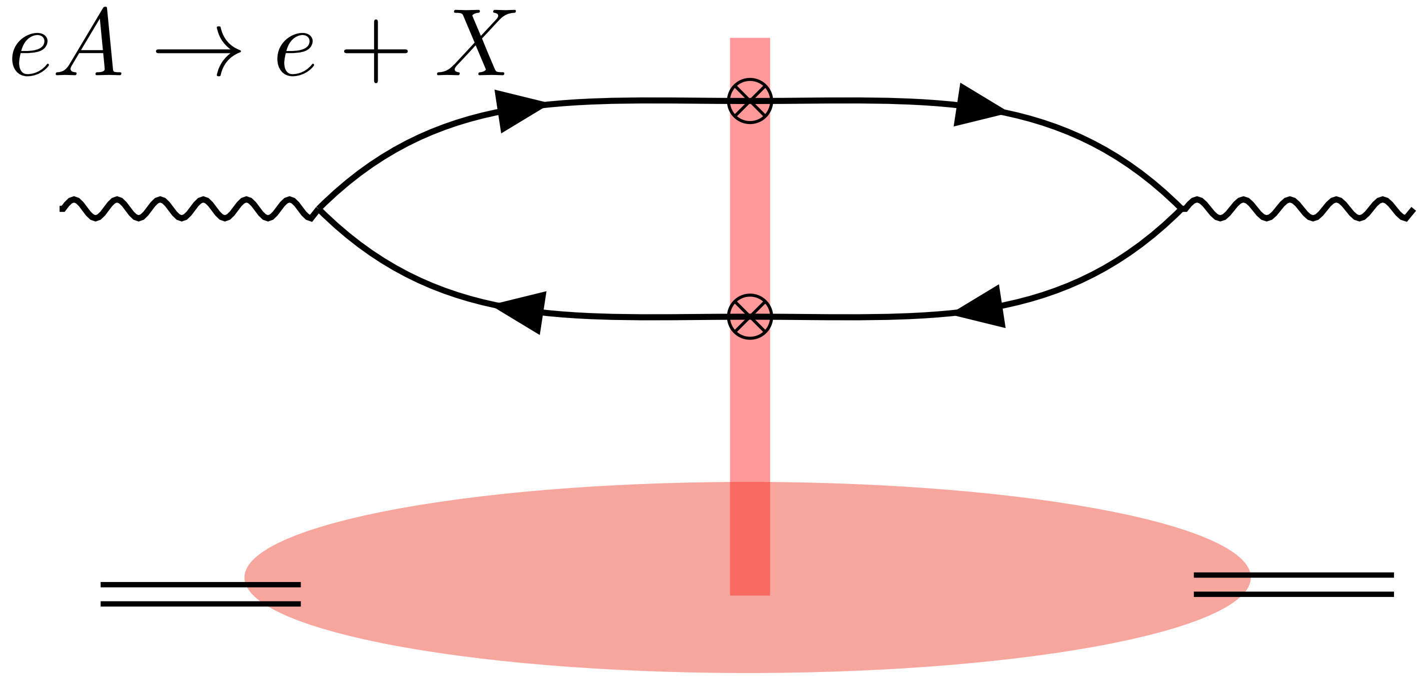
Effective vertex in terms of Light-like Wilson line

$$V_{ij}(\boldsymbol{x}) = P \exp\left\{ ig \int dx^- A_{cl}^{+,a}(\boldsymbol{x}, x^-) t^a \right\}$$

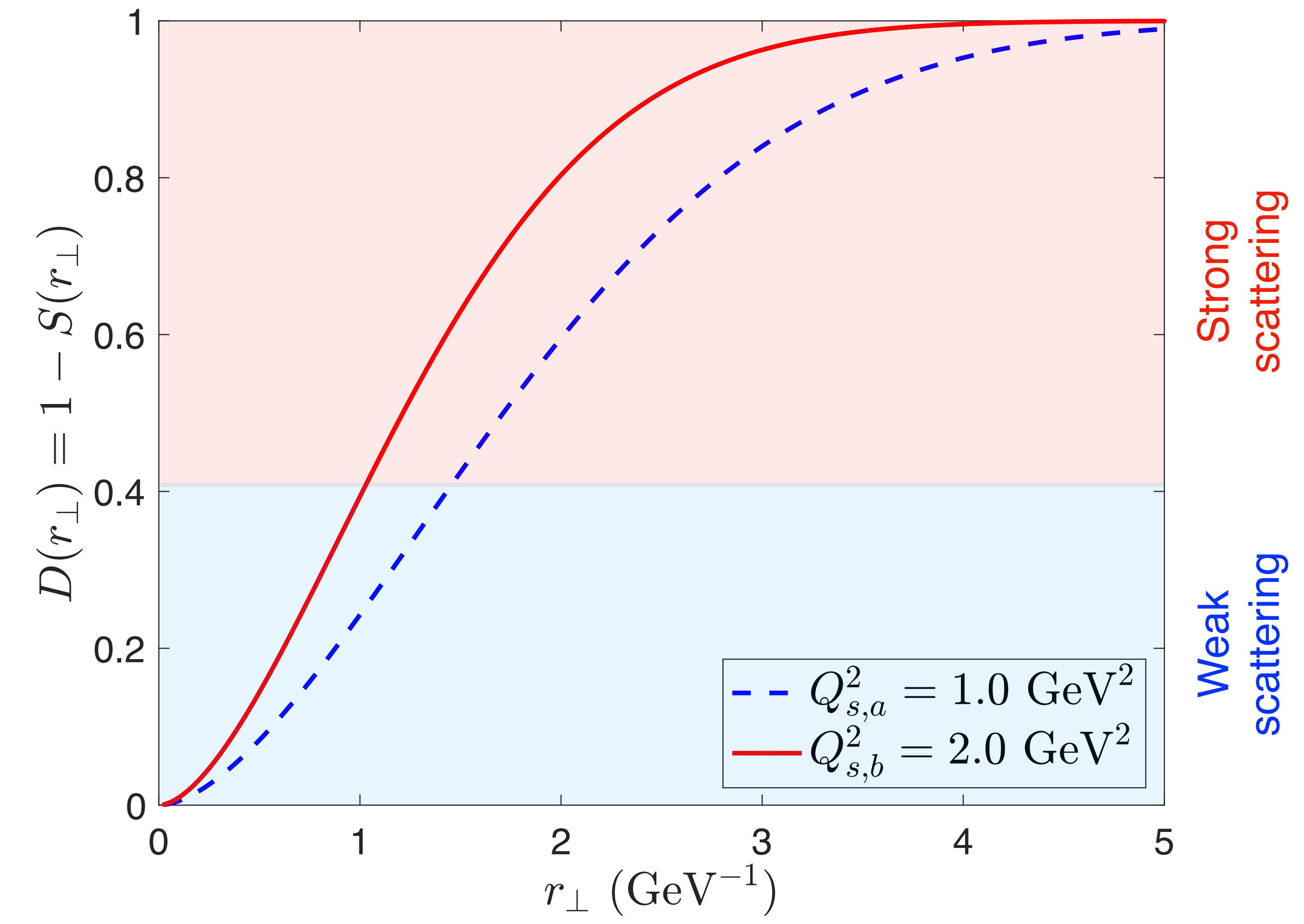
Observables built from Wilson lines and their derivatives, convoluted with perturbative factor (e.g. wavefunctions)

Universality: from proton-nucleus to electron-nucleus





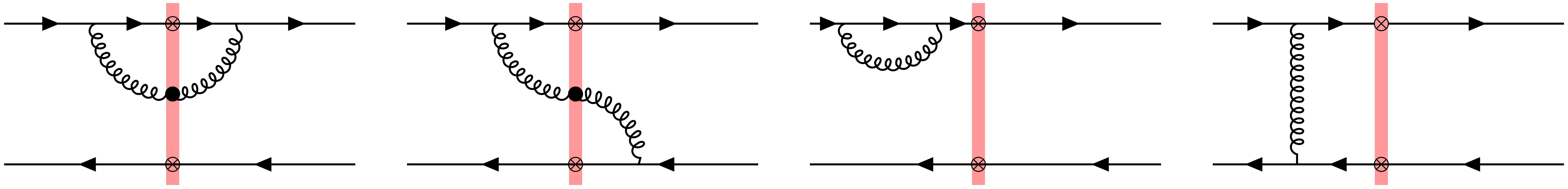
Both processes depend on the "dipole" $S(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = \langle \mathrm{Tr}[V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{y}_\perp)] \rangle$



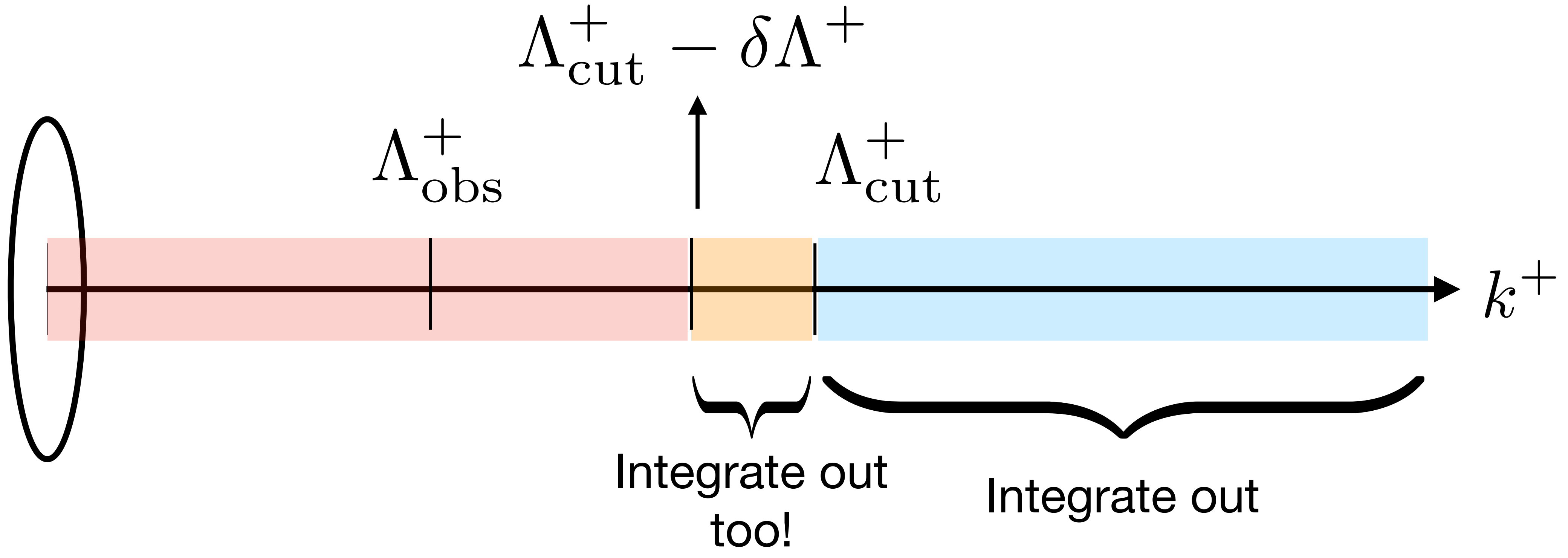
$$Q_s^2(x) \approx \Lambda_{QCD}^2 A^{1/3} (x_0/x)^\lambda$$

The Color Glass Condensate: quantum evolution

Quantum corrections beyond the semi-classical picture



Quantum corrections enhanced by large energy logarithms $\ln(\Lambda^+_{\rm obs}/\Lambda^+_{\rm cut})$



$$\langle\langle \mathcal{O} \rangle\rangle = \int [\mathcal{D}\rho]\, W_{\Lambda^+_{\rm cut}-\delta\Lambda^+}[\rho] \int^{\Lambda^+_{\rm cut}-\delta\Lambda^+} [\mathcal{D}A]\, \mathcal{O} e^{i\mathcal{S}[A,\rho]}$$

Absorb quantum gluons into classical sources

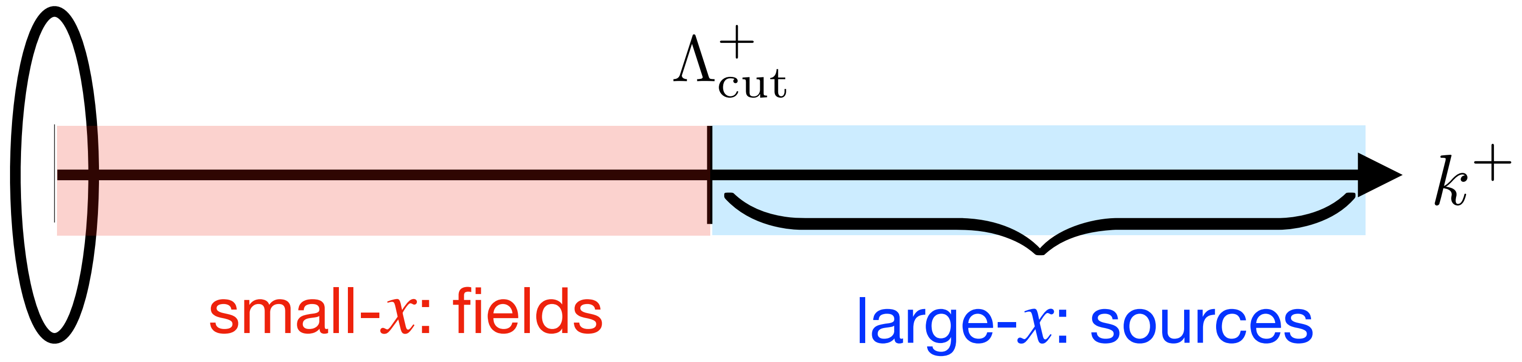
renormalization group evolution of $W[\rho]$

$$\frac{\delta W_{\Lambda^+}[\rho]}{\delta \ln(\Lambda^+)} = H_{\rm JIMWLK} W_{\Lambda^+}[\rho]$$

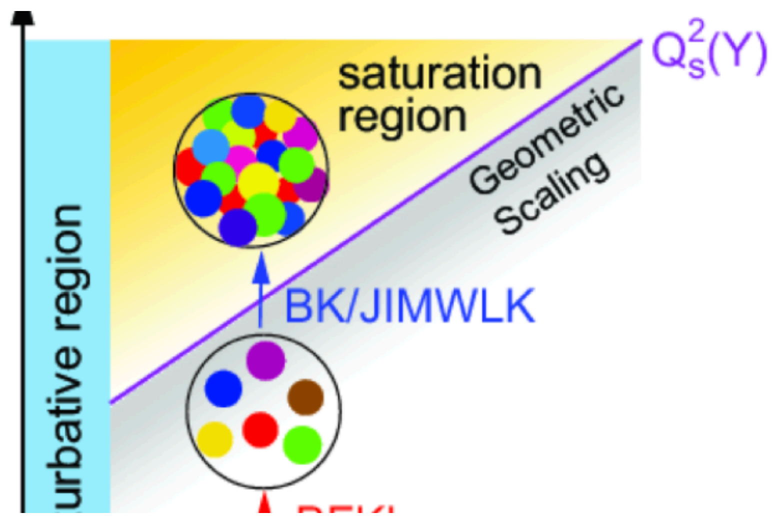
Non-linear RGE equations

The Color Glass Condensate in a nutshell

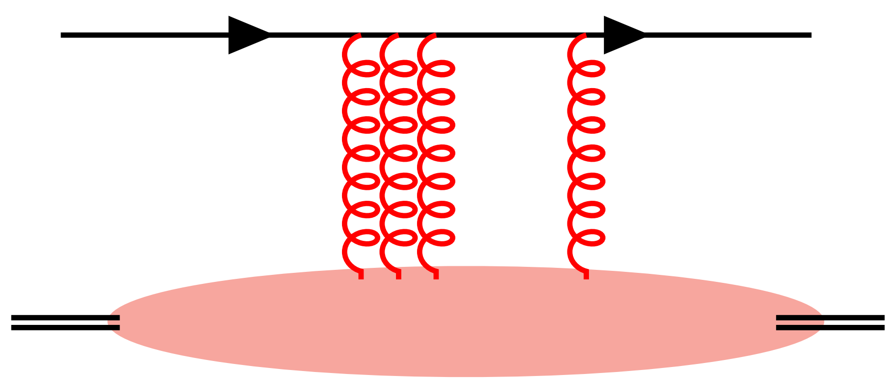
- Separation of degrees of freedom into sources and fields



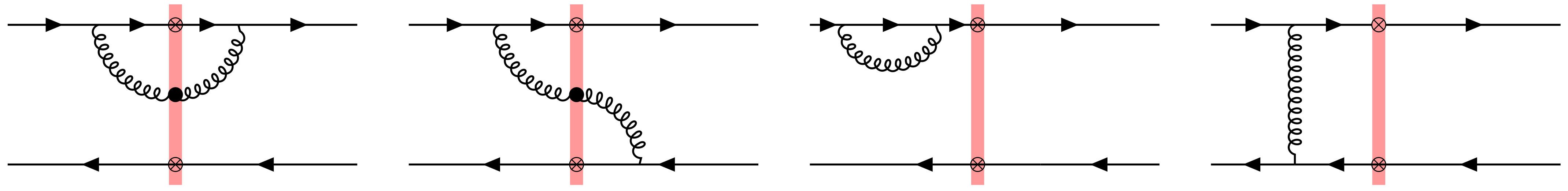
- CGC is an EFT of QCD providing a weak coupling approach for unitarization of cross-section



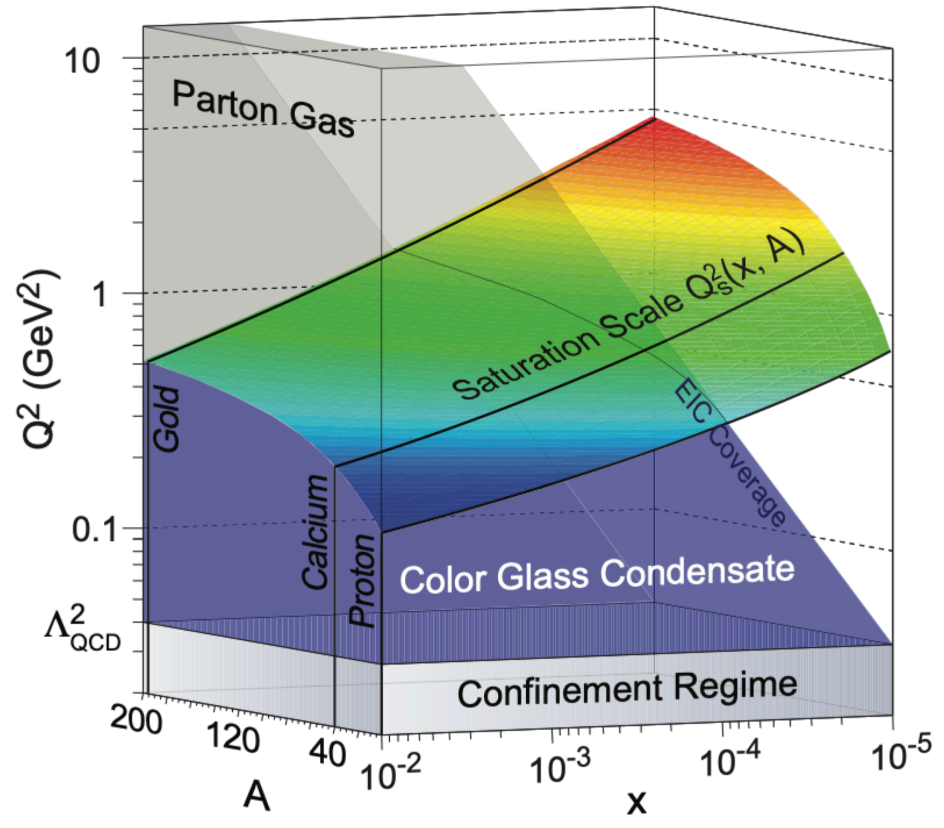
- Strong classical field -> multiple scattering via light-like Wilson lines -> broadening (Glauber)



- Small-x radiation -> quantum (non-linear) evolution of Wilson line correlators-> suppression (Gribov)



- Emergence of an x-dependent and A-dependent momentum scale:
$Q_s^2(x) \approx \Lambda_{QCD}^2 A^{1/3}(x_0/x)^\lambda$



- Saturation phenomena manifests in particle production of invariant mass $M^2 \lesssim Q_s^2(x)$

Power-counting in the CGC

Dilute-dilute: $Q_{sA}^2/k_{A\perp}^2 \ll 1$ and $Q_{sB}^2/k_{B\perp}^2 \ll 1$

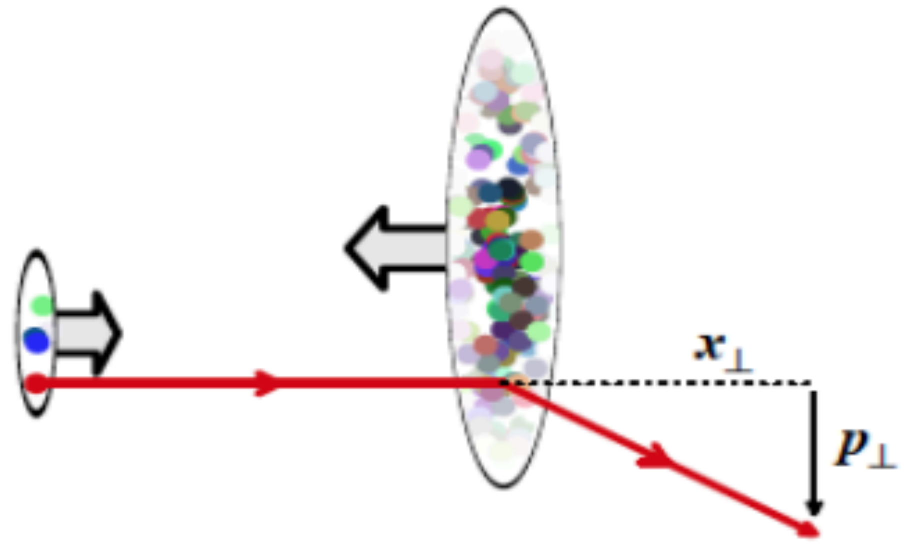
Match to pQCD computation of hard processes at small x

Hard production in hadron collisions

Dilute-dense: $Q_{sA}^2/k_{A\perp}^2 \ll 1$ and $Q_{sB}^2/k_{B\perp}^2 \sim 1$

Hybrid approach pQCD/CGC, advances at NLO and relation to TMD and GPDs

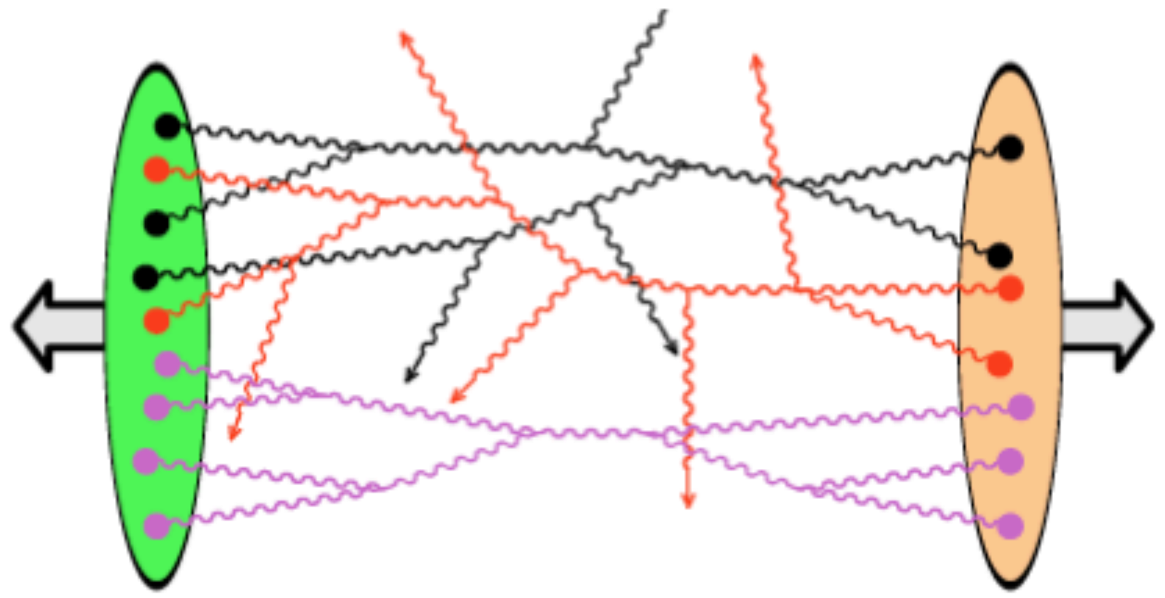
Semi-hard and forward particle production in proton-nucleus, electron-nucleus collisions



Dense-dense: $Q_{sA}^2/k_{A\perp}^2 \sim 1$ and $Q_{sB}^2/k_{B\perp}^2 \sim 1$

Solve classical YM equations numerically in 2+1 D (boost-invariant) /3+1 D

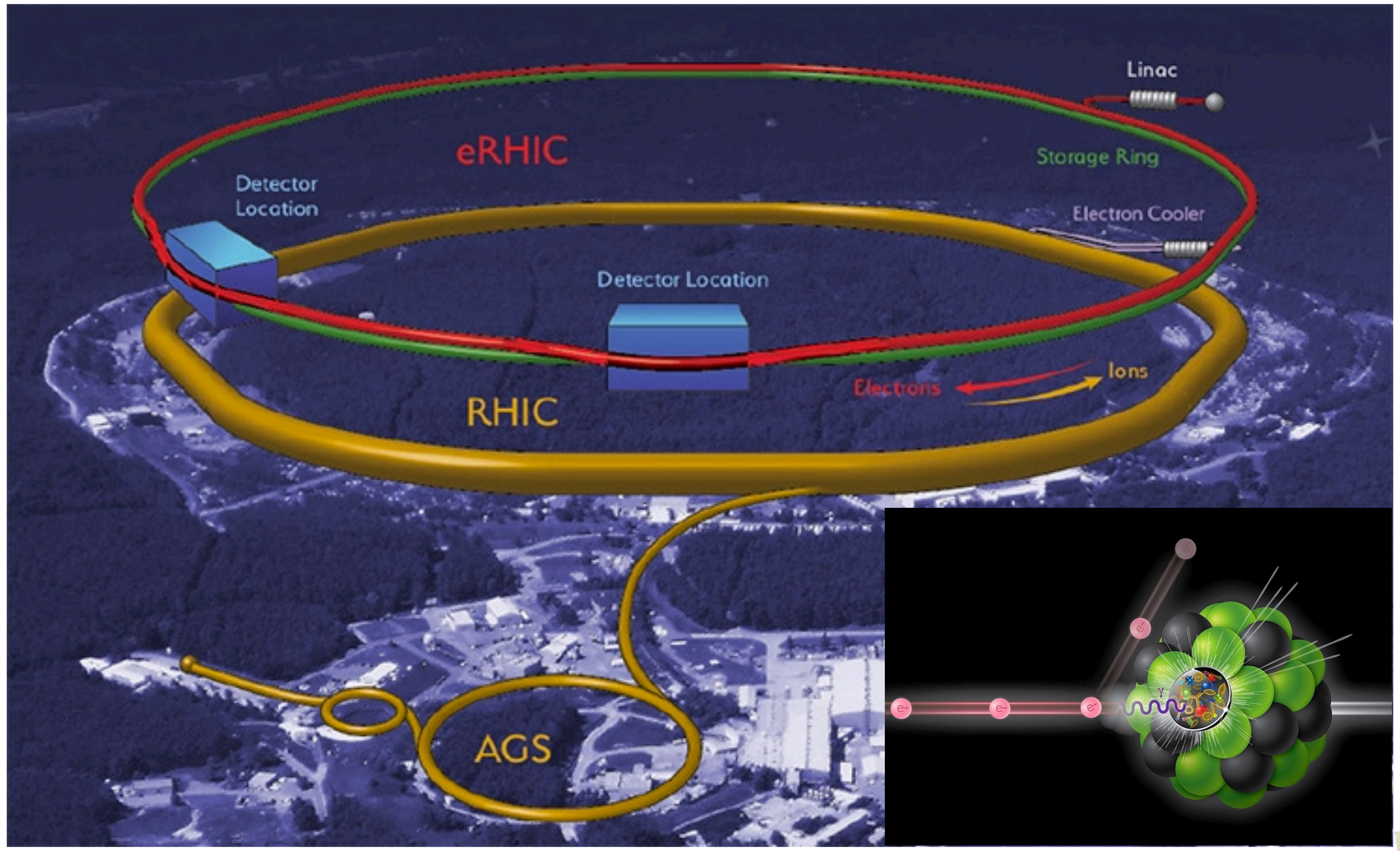
Semi-hard particle production in heavy-ion collisions



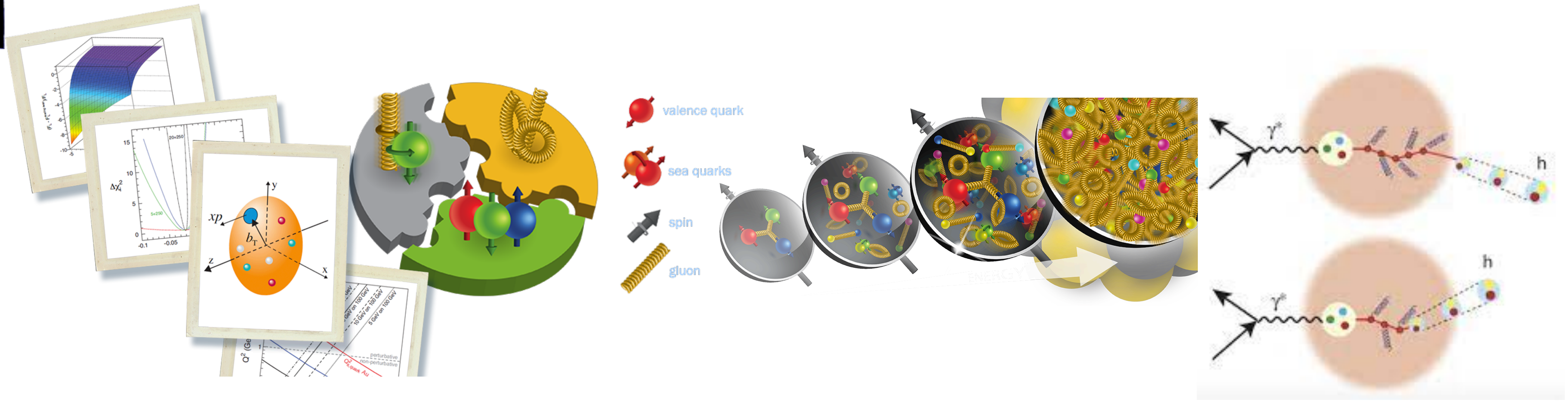
Regime is dictated by the colliding system, energy, centrality, rapidity, and transverse momentum of observed particles

Electron-Ion Collider Era

Capabilities and scientific case



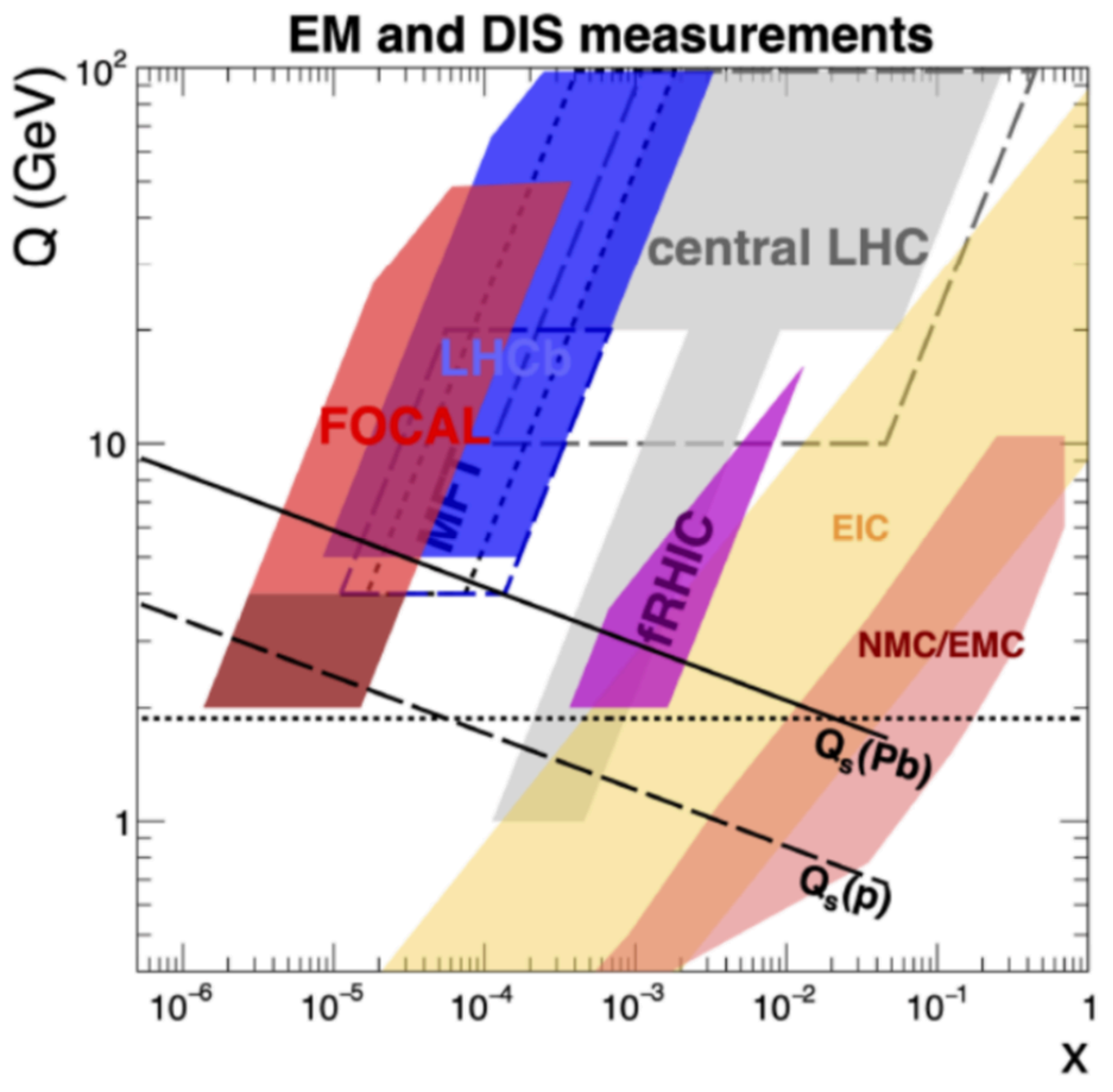
- High luminosity (high rate of collisions)
- Up to ~ 140 GeV center of mass energy
- Polarized beams of (light) ions and electrons
- Large ion species (from proton to gold)

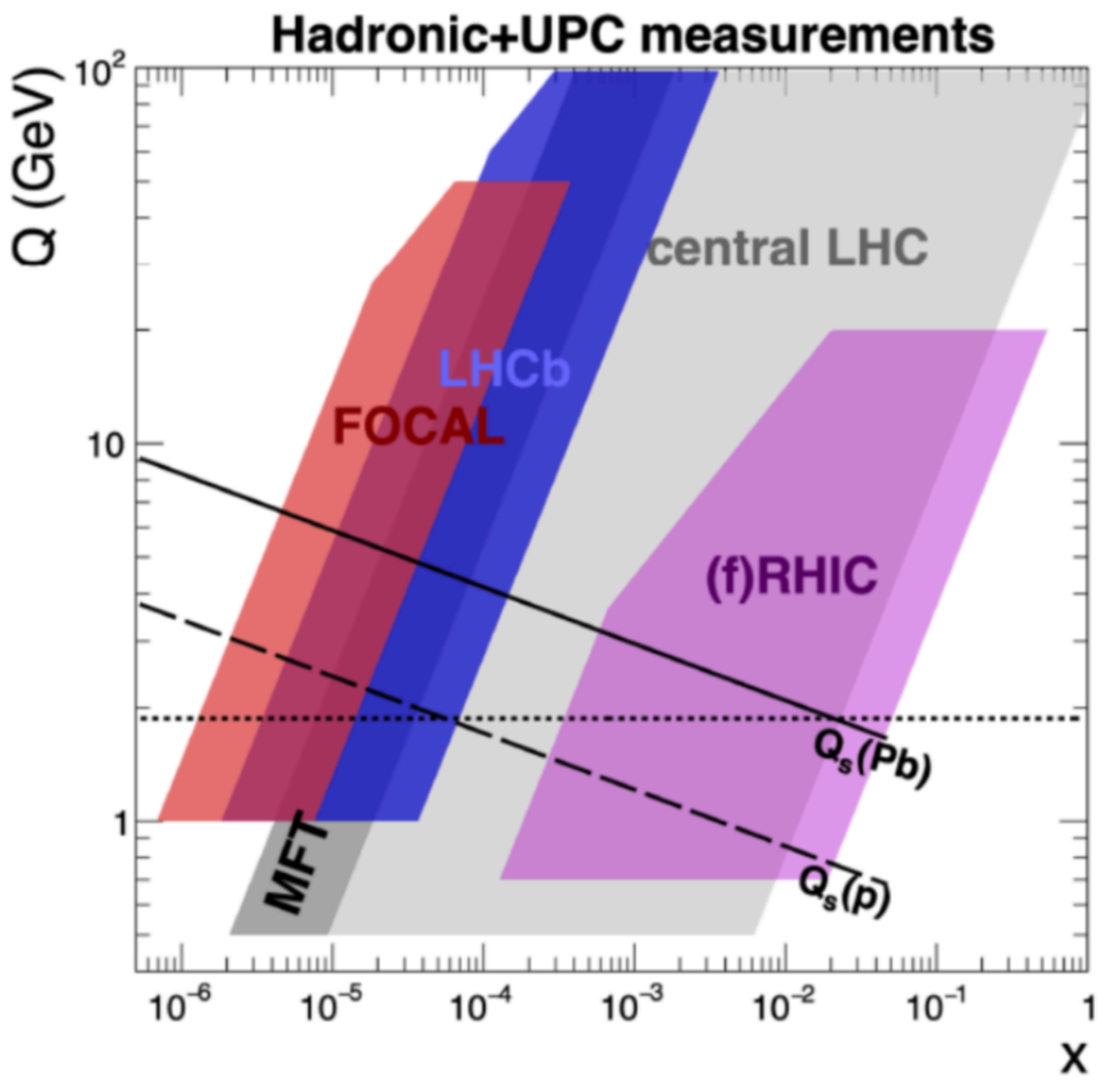


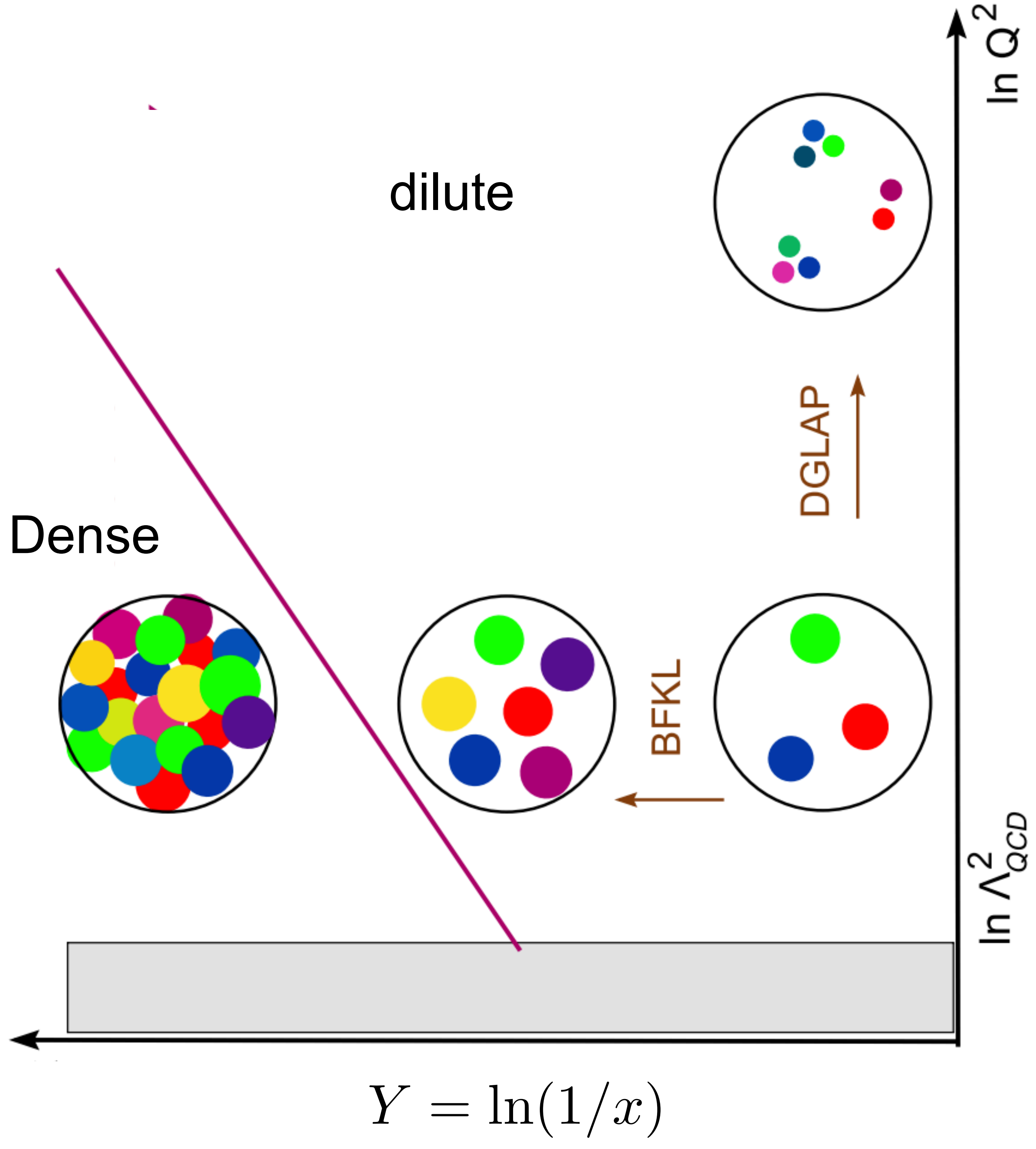
EIC goals: tomography, spin, gluon saturation, hadronization

Figures from https://www.bnl.gov/eic/science.php

Experimental prospects: LHC and EIC







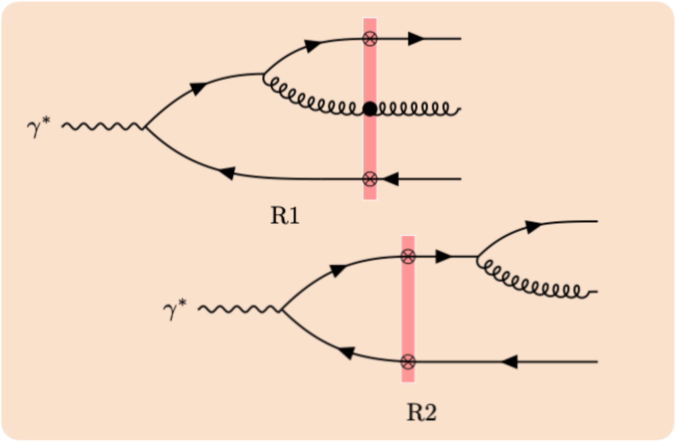
Complementarity between LHC and EIC

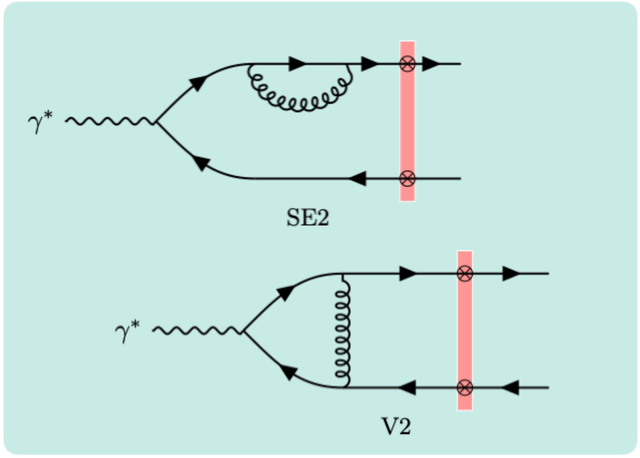
See Raju's talk on Wednesday
What progress do we need for the EIC?

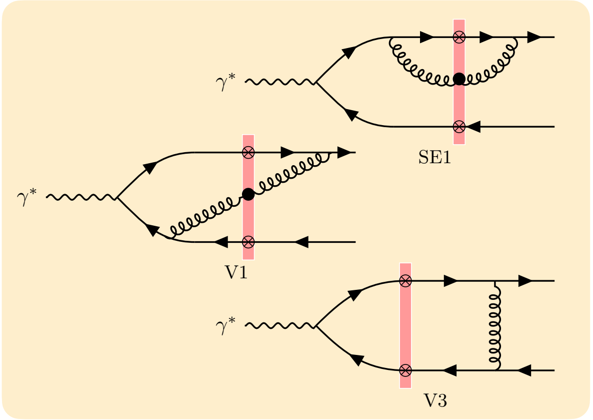
Theory developments

Fixed order calculations

FO calculations are much complicated (both analytical and numerically) than their collinear counterparts due to multiple scattering







Impact factors at one-loop

Structure functions

light quarks	Balitsky, Chirilli (2011) Beuf (2017) Hänninen, Lappi, Paatelainen (2017)
massive quarks	Beuf, Lappi, Paatelainen (2021,2022)

Diffractive processes in DIS

Structure function	Beuf, Lappi, Mäntysaari, Paatelainen, Penttala (2024)
dijets and light vector meson	Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2016)
vector meson	Mäntysaari, Penttala (2021, 2022)
single hadron	Fucilla, Grabovsky, Li, Szymanowski, Wallon (2023)

Semi-inclusive processes in DIS

dijet+photon	Roy, Venugopalan (2019)
dijets	Caucal, Salazar, Venugopalan (2021)
back-to-back limit	Caucal, Salazar, Schenke, Stebel, Venugopalan (2024)
dijets (photo-production)	Taels, Altinoluk, Beuf, Marquet (2022)
dihadron	Bergabo, Jalilian-Marian (2022)
back-to-back limit	Caucal, Salazar (2024)
jet SIDIS	Caucal, Ferrand, Salazar (2024) Caucal, Iancu, Mueller, Yuan (2024)
hadron SIDIS	Bergabo, Jalilian-Marian (2022) Altinoluk, Marquet, Shi (2025)

Semi-inclusive processes in pA

single hadron	Chirilli, Xiao, Yuan (2012) …. Mäntysaari, Tawabutr (2023)
single jet	Liu, Xie, Kang, Liu (2022)
Drell-Yan	Taels (2023)

Missing in the literature:

-Single and double inclusive open/closed heavy flavor in DIS

-Two particle correlations in pA (photons, hadrons, jets)

-Isolated photon production in pA

Automatization for higher-order loop computations?

Alternative regularization scheme?

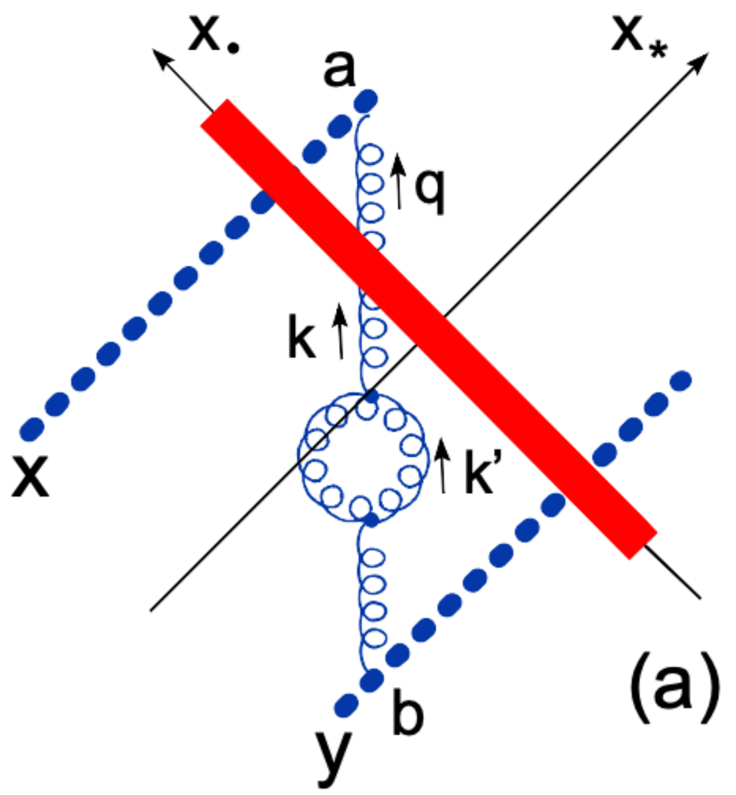
Small-x evolution

The evolution of the BK equation through the years

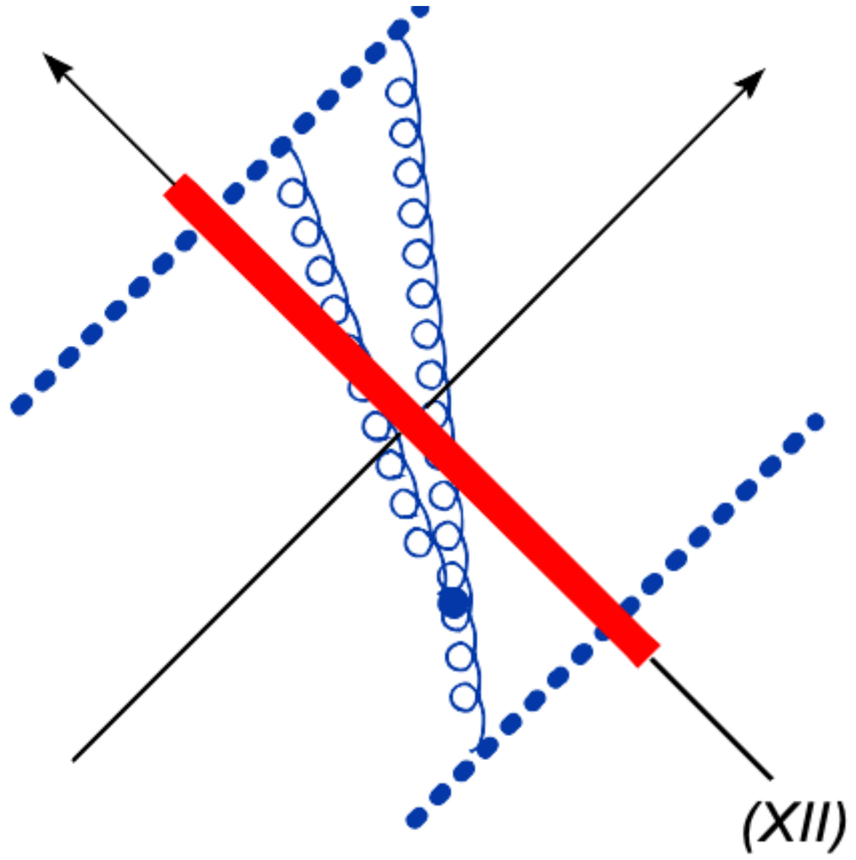
running coupling	Kovchegov, Weigert (2007) Balistky (2007)
NLL	Balistky, Chirilli (2008)
NLL is unstable	Lappi, Mäntysaari (2015)
NLL with resummation	Ducloue, Iancu, Mueller, Soyez, Triantafyllopoulos (2015)
NLL with resummation is stable	Lappi, Mäntysaari (2016)

and the JIMWLK equation

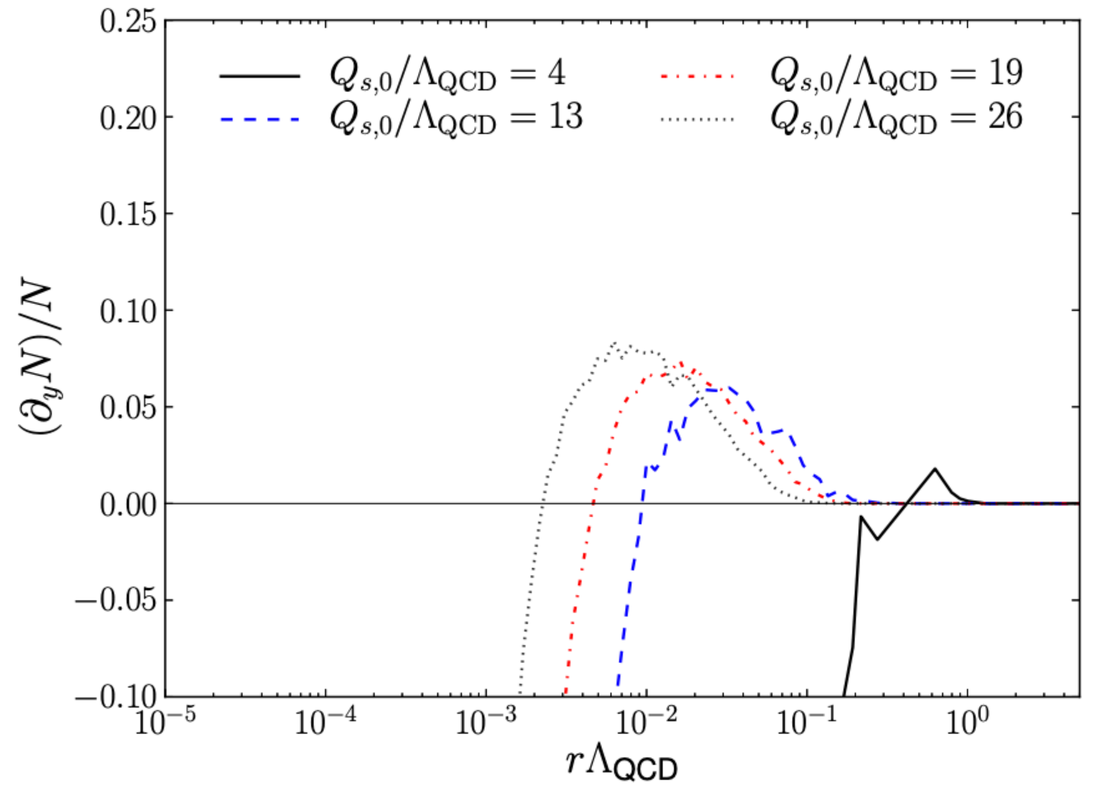
running coupling	Lappi, H. Mäntysaari (2013)
NLL	Balistky, Chirilli (2013) Kovner, Lublinky, Mulian (2014)
NLL with resummation	Hatta, Iancu (2016)
NLL with massive quarks	Dai, Lublinsky (2022)
running coupling revisited	Altinoluk, Beuf, Kovner, Lublinsky, Skokov (2023)
Langevin formulation at NLL	Korcyl, Motyka, Stebel (work in progress)



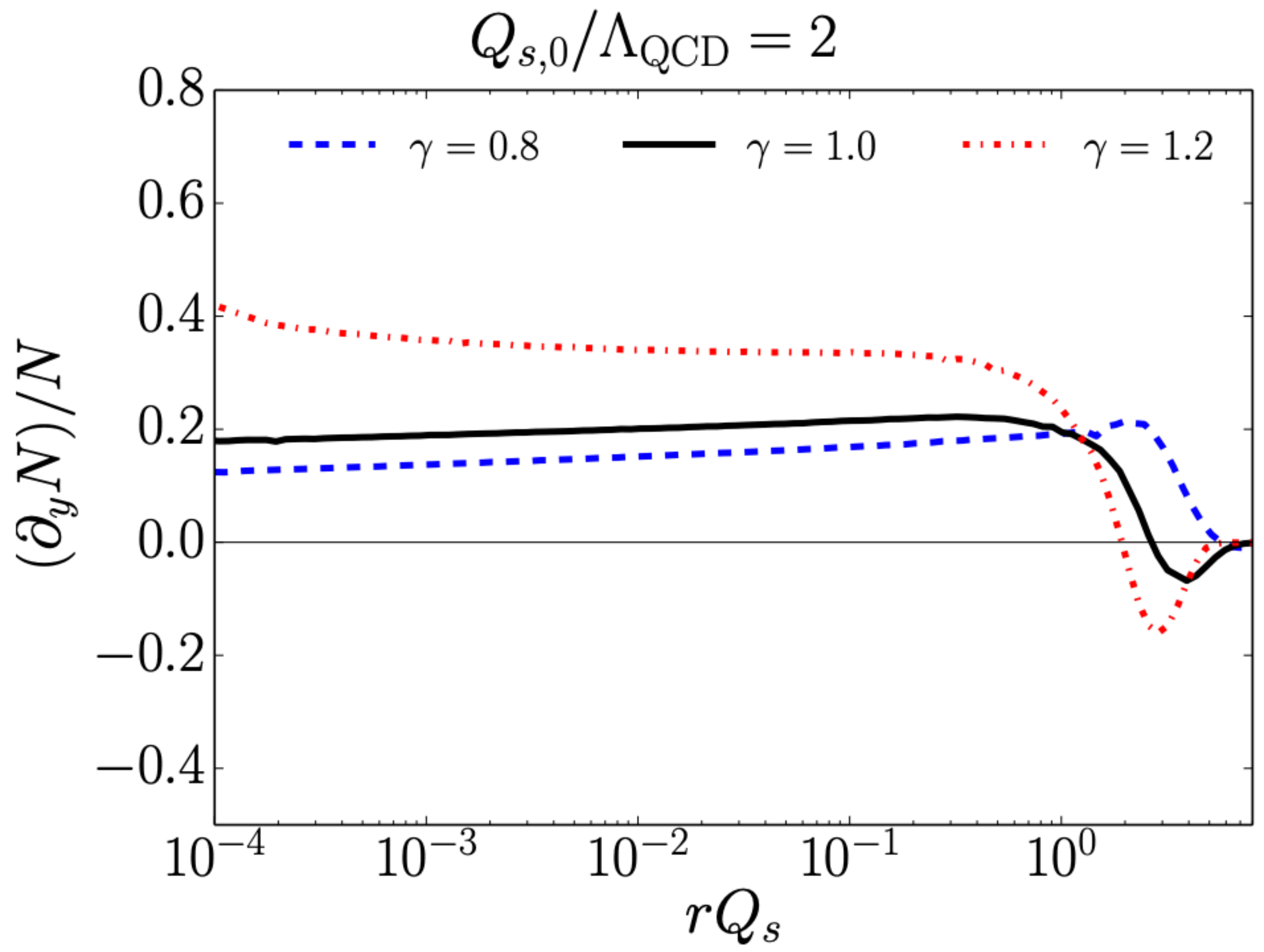
(a)



(XII)



(c) $\gamma = 1.0$



CSS, DGLAP, BFKL/BK/JIMWLK

Small-x TMD factorization first proposed by Dominguez, Marquet, Xiao, Yuan (2011)

In appropriate limit one can show that CGC result factorizes à la TMD, e.g. semi-inclusive forward dijet that are produced back-to-back in the transverse plane

The need for resummation of small-x and Sudakov (CSS) by Mueller, Xiao, Yuan (2013) and Xiao, Yuan, Zhou (2017)

See also Balitsky, Tarasov (2016) rapidity only factorization for TMD

Recent developments

Isolate Sudakov double logs (need kinematic constraint): dijet, dihadron, SIDIS jet — Taels, Altinoluk, Marquet, Beuf (2022); Caucal, Schenke, Salazar, Venugopalan (2022); Altinoluk, Jalilian-Marian, Marquet (2024)

Universality of sea-quark TMD factorization — Caucal, Iancu, Guerrero Morales, Salazar, Yuan (2025)

Factorization in terms of diffractive TMDs for SIDDIS and 2+1 jets — Iancu, Mueller, Triantafyllopolous (2021); Xiao, Yuan (2021)

Factorization in the target region in terms of jet fracture functions at small-x — Caucal, Salazar (2025)

Interplay between DGLAP, CSS and small-x evolution — Mukherjee, Skokov, Tarasov, Tiwari (2023); Duan, Kovner, Lublinsky (2024); Caucal, Iancu (2024)

More on factorization on Edmond's talk on Thursday

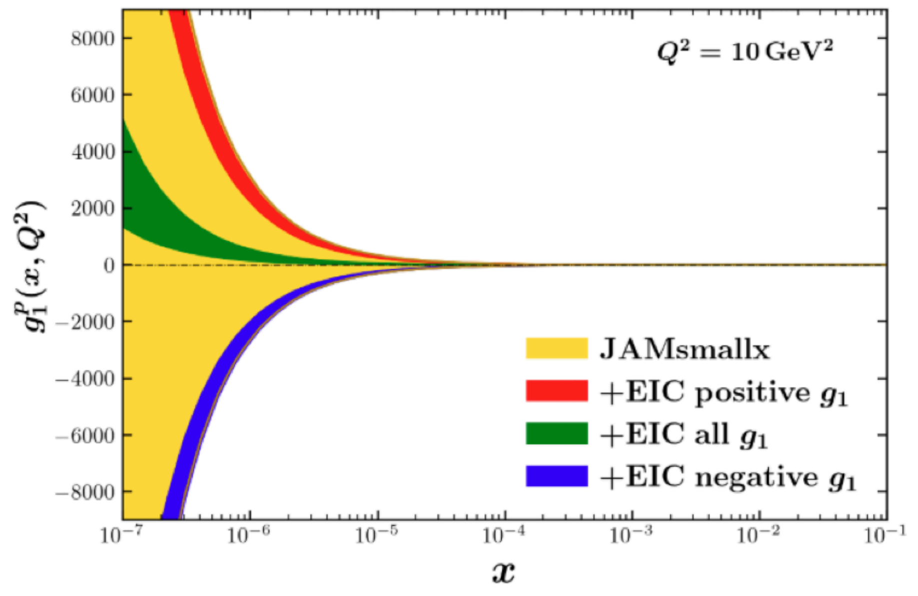
More on TMDs at small-x on Cyrille and Jamal's discussion on Thursday

Other theory developments

- Spin and small-x physics

  Kovchegov, Sievert, Pitonyak, …(2012-present)

  Adrian and Yuri’s discussion in the afternoon



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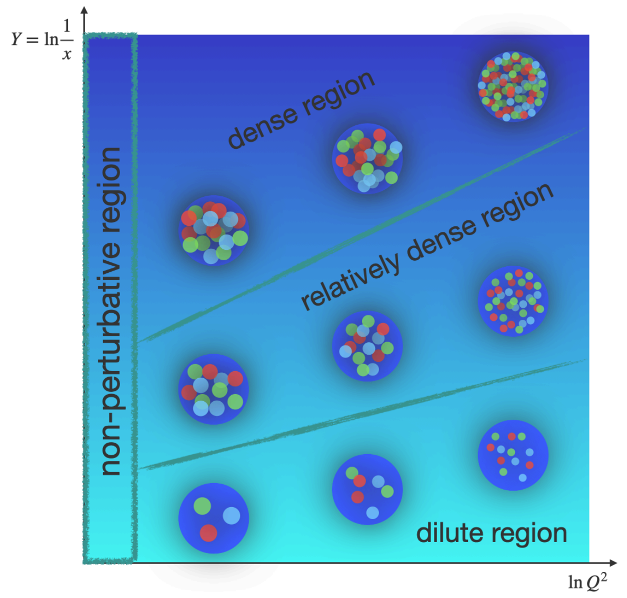
- Beyond eikonal: connecting small and moderate-x

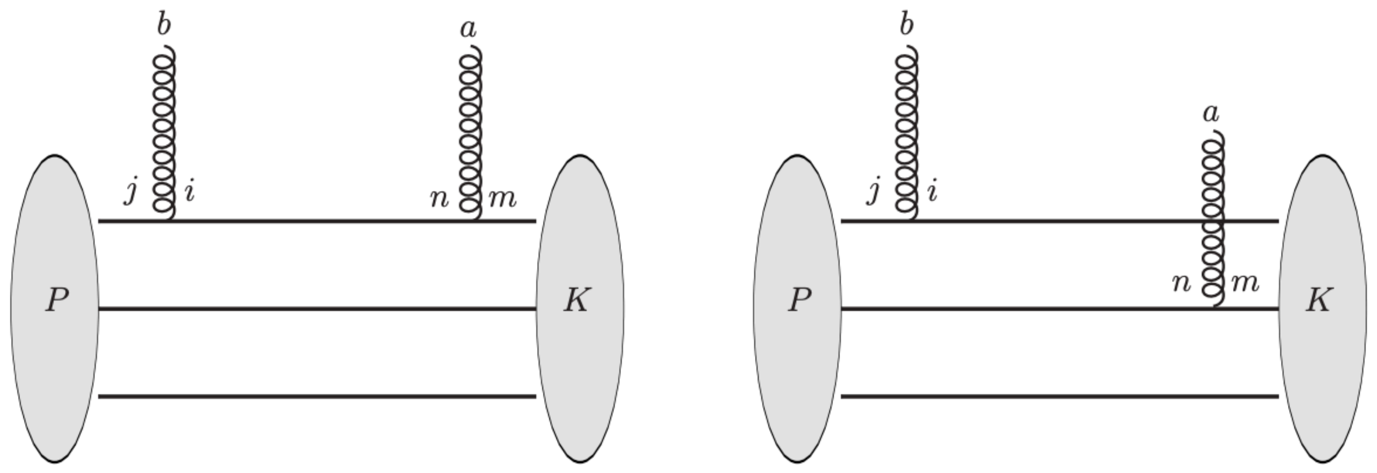
  Altinoluk, Armesto, Beuf, Chirilli, Marquet, Skokov …

  Vladi’s discussion on Friday



- Initial conditions for evolution (beyond MV model)

  Dumitri and Petreska (non-gaussianities)
  Dumitru et al (perturbative approach from valence quark)
  Penttala in progress (wider class of distribution, e.g. heavy-tailed)

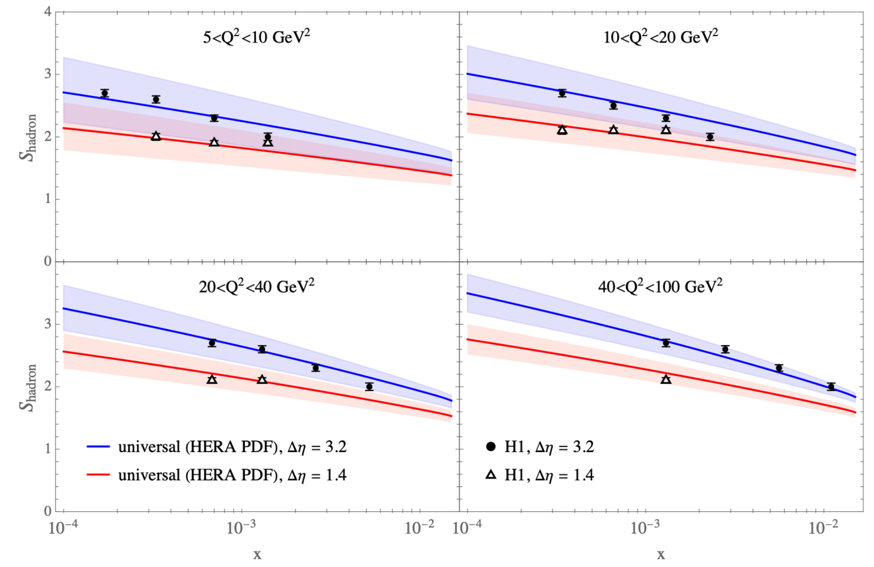


Other theory developments

- Entanglement/density matrix at small-x

Kharzeev, Levin, Kutak, Hentschinski, Tu (2012-present)

Armesto, Dominguez, Kovner, Lublinsky, Skokov (2019)



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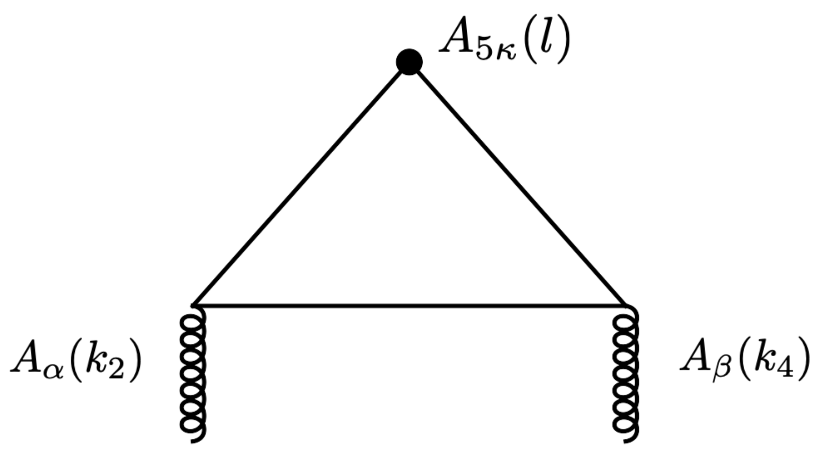
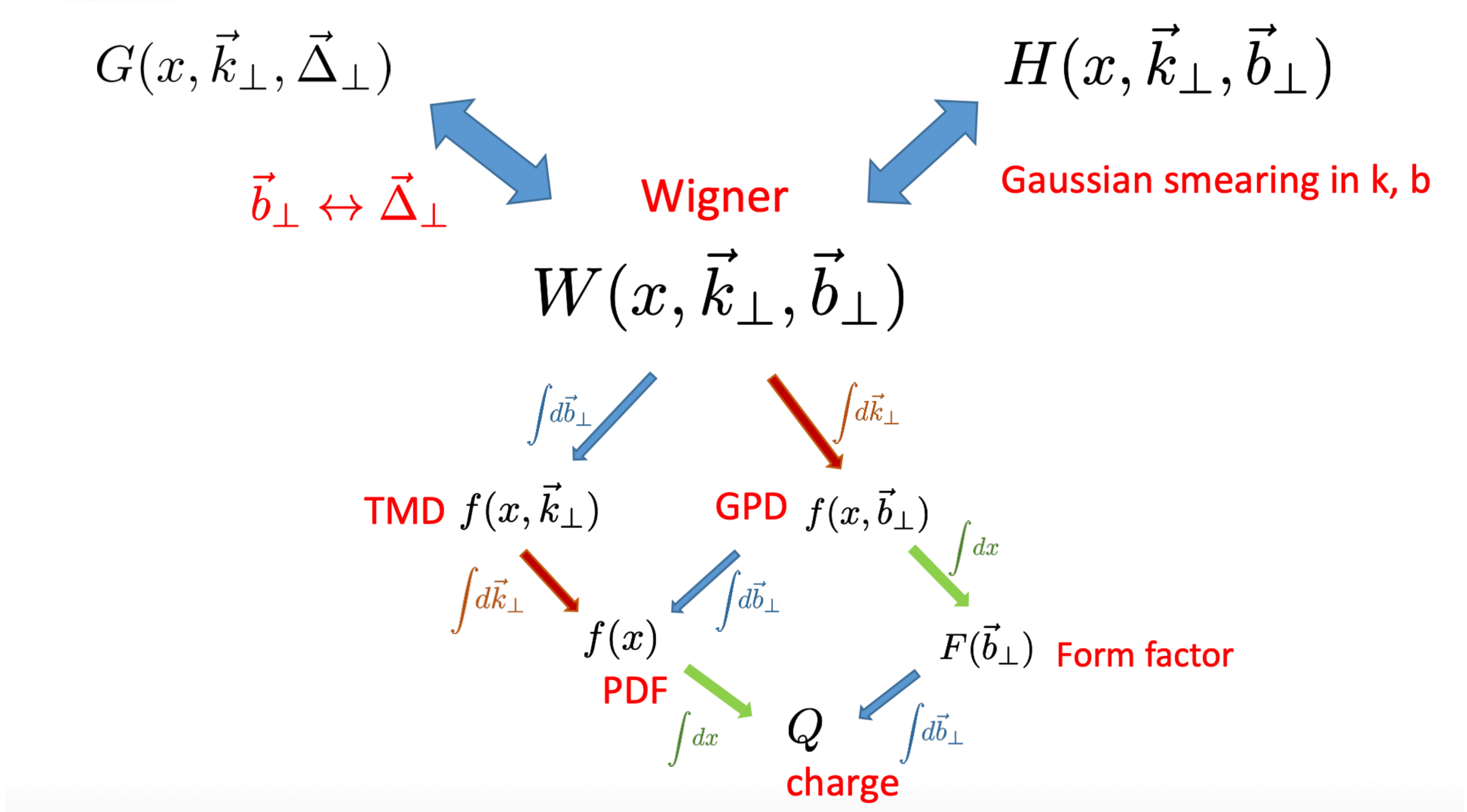
- Sphalerons at the EIC and interplay with chiral anomaly

Tarasov, Venugopalan (2020-present)



- Tomography at small-x: Wigner distribution, angular momentum…

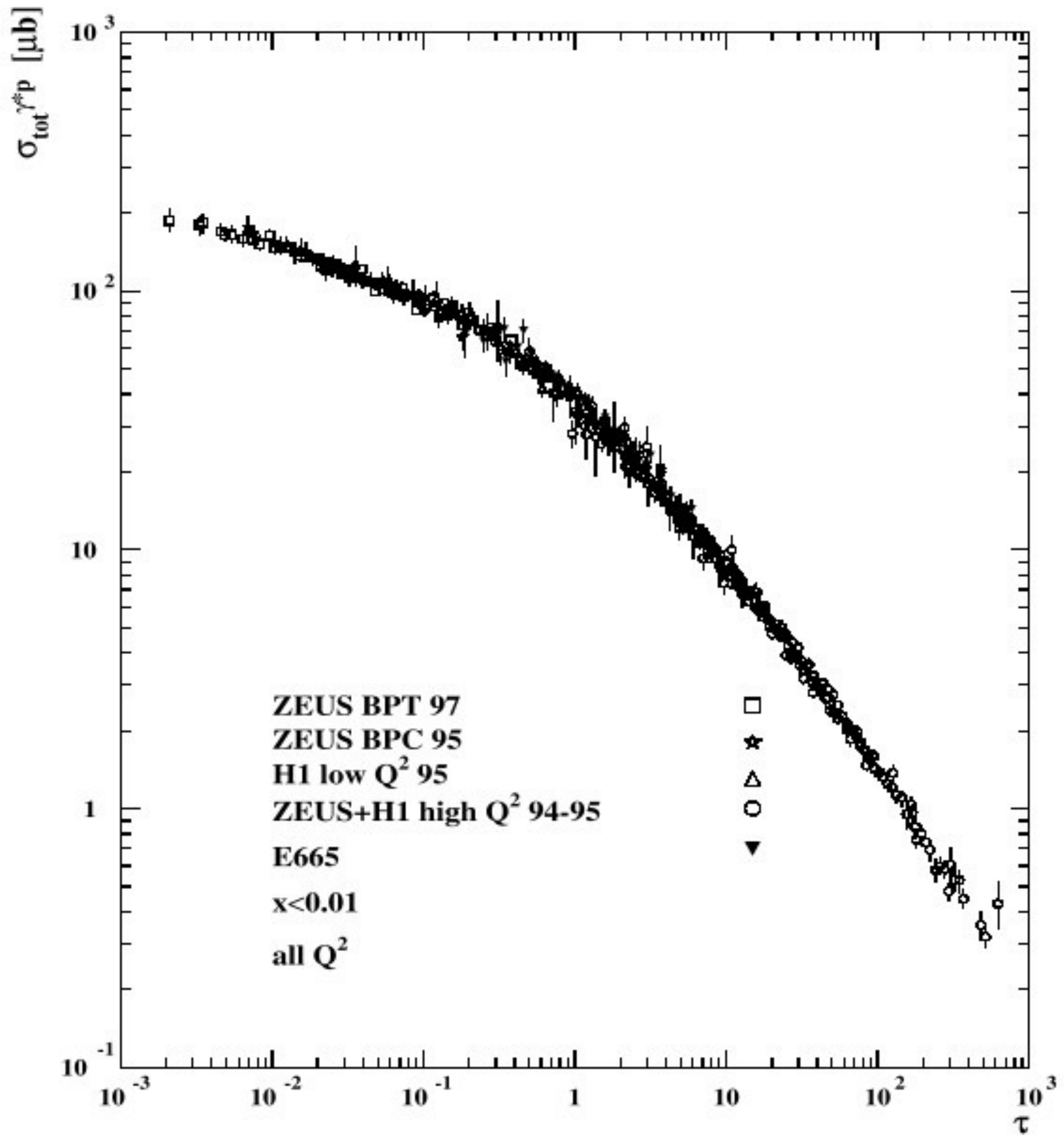
Bhattacharya, Boussarie, Hatta, Xiao, Yuan …



Observables at the EIC:

Inclusive

Structure functions: geometric scaling



$$\sigma_r(x, y, Q^2) = F_2(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L(x, Q^2)$$

- DIS cross-section generically depends on Q^2 and x
- HERA data shows signs of scaling: $\tau = Q^2/Q_s^2(x)$

$$Q_s^2(x) = Q_{s,0}^2(x_0/x)^\lambda$$

Stasto, Golec-Biernat, Kwiecinski (2000)

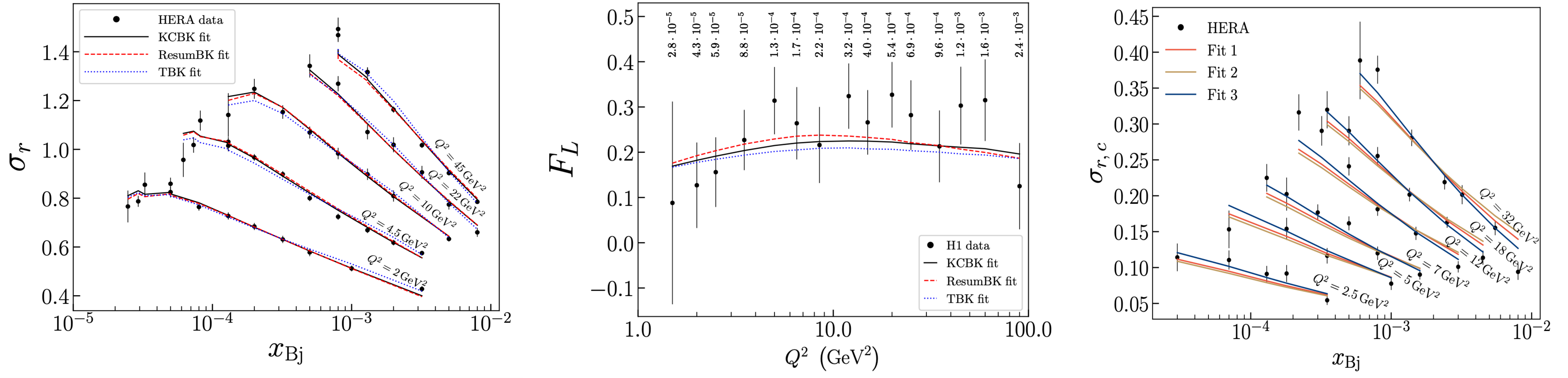
Iancu, Itakura, McLerran (2002)

- Can we observe geometric scaling for different nuclear species?
- Will we observe the nuclear size dependence of the saturation scale?

$$Q_s^2(x, A) = Q_{s,0}^2(x_0/x)^\lambda A^{1/3}$$

Structure functions: F_2 and F_L

- CGC at NLO provides a good simultaneous description of structure functions including charm



Beuf, Lappi, Hänninen, Mäntysaari (2020)

- However, F_2 has large non-perturbative contributions. It would be best to focus on F_L or $F_{2,c}$

For heavy flavor see Jani and Anna's discussion on Thursday

- Confront CGC to nuclear structrure functions at the EIC

Structure functions: linear vs non-linear evolution

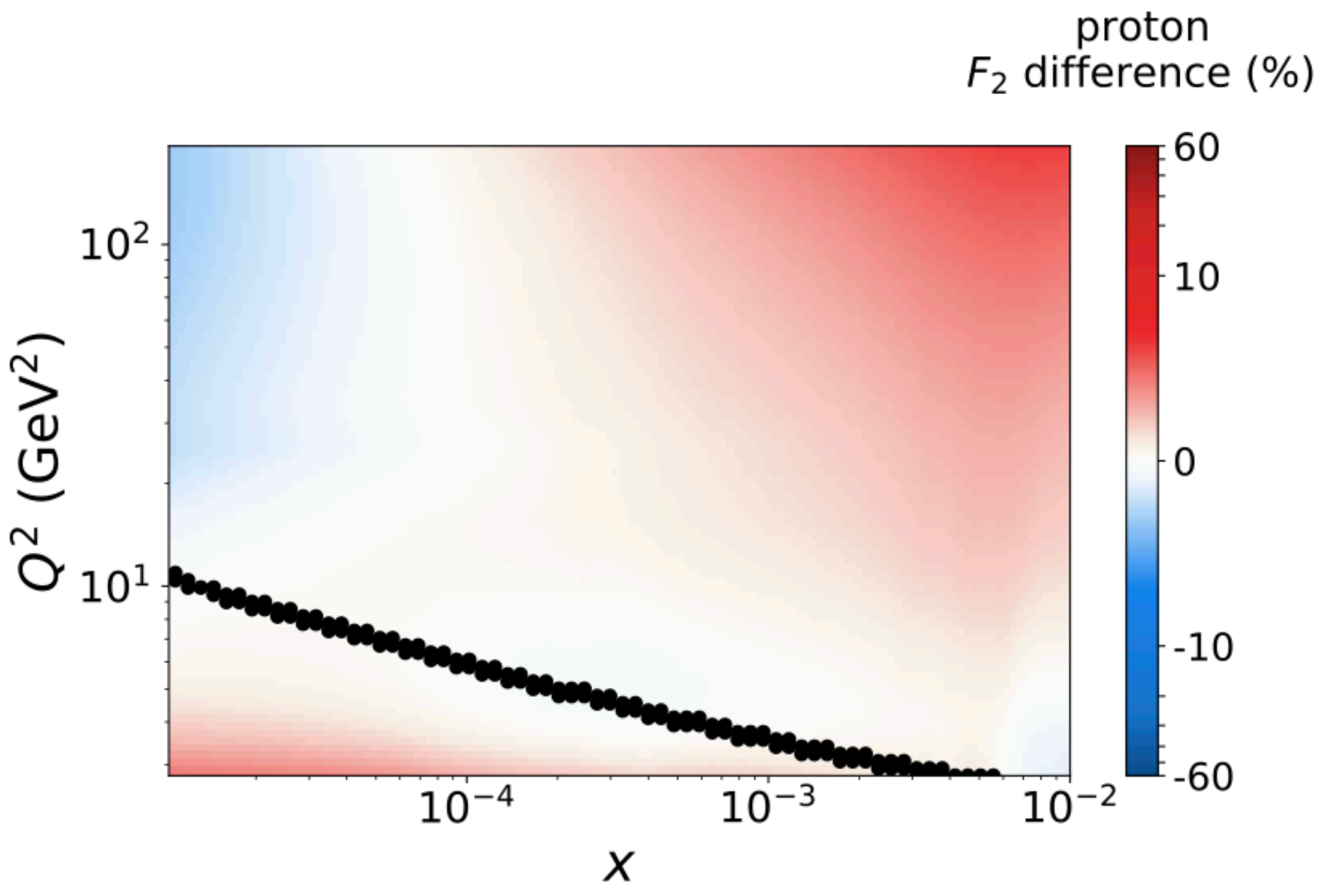
- Difference in predictions for $F_{2,L}$:
  linear (collinear/DGLAP)
  non-linear (dipole/Balitsky-Kovchegov)

$$(F^{\rm BK}_{2/L} - F^{\rm DGLAP,Rew}_{2/L})/F^{\rm BK}_{2/L}$$

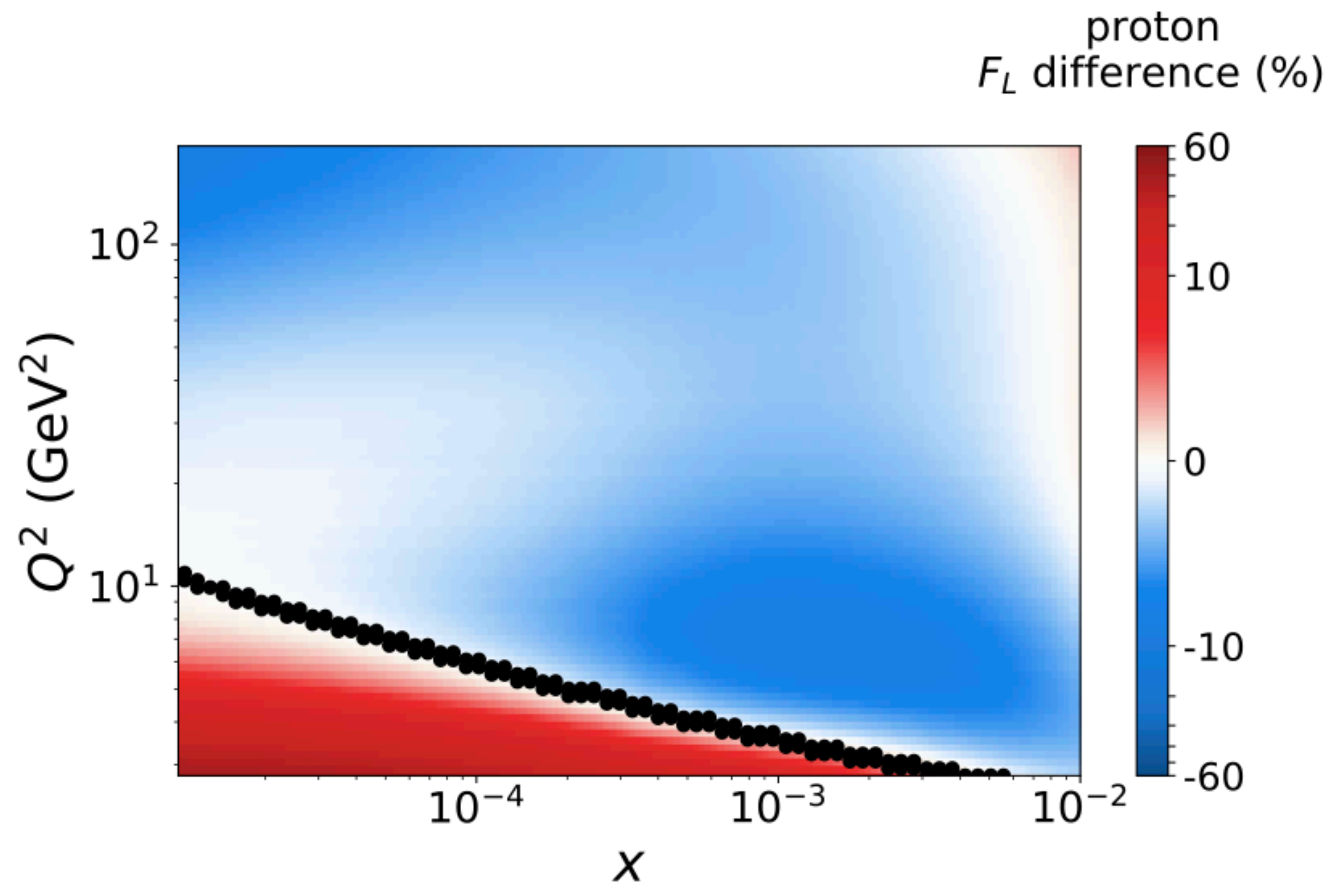
- Stronger effects for F_L than F_2

- Stronger effects for γAu than γp

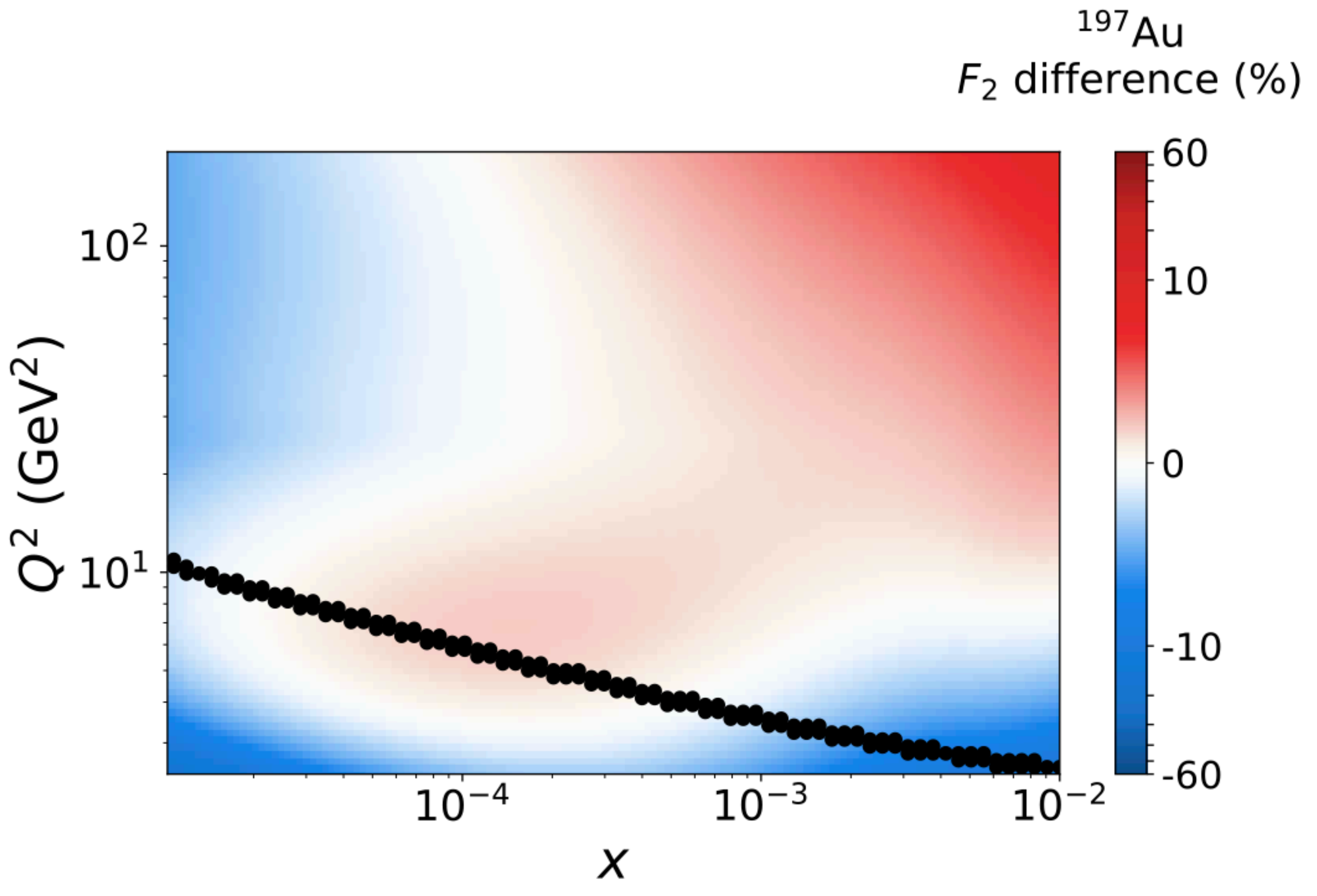
- It would be interesting to incorporate small-x evolution into DGLAP via BFKL, à la Ball, Bertone, Bonvini, Marazani, Rojo, Rottoli (2017), and compare with non-linear BK



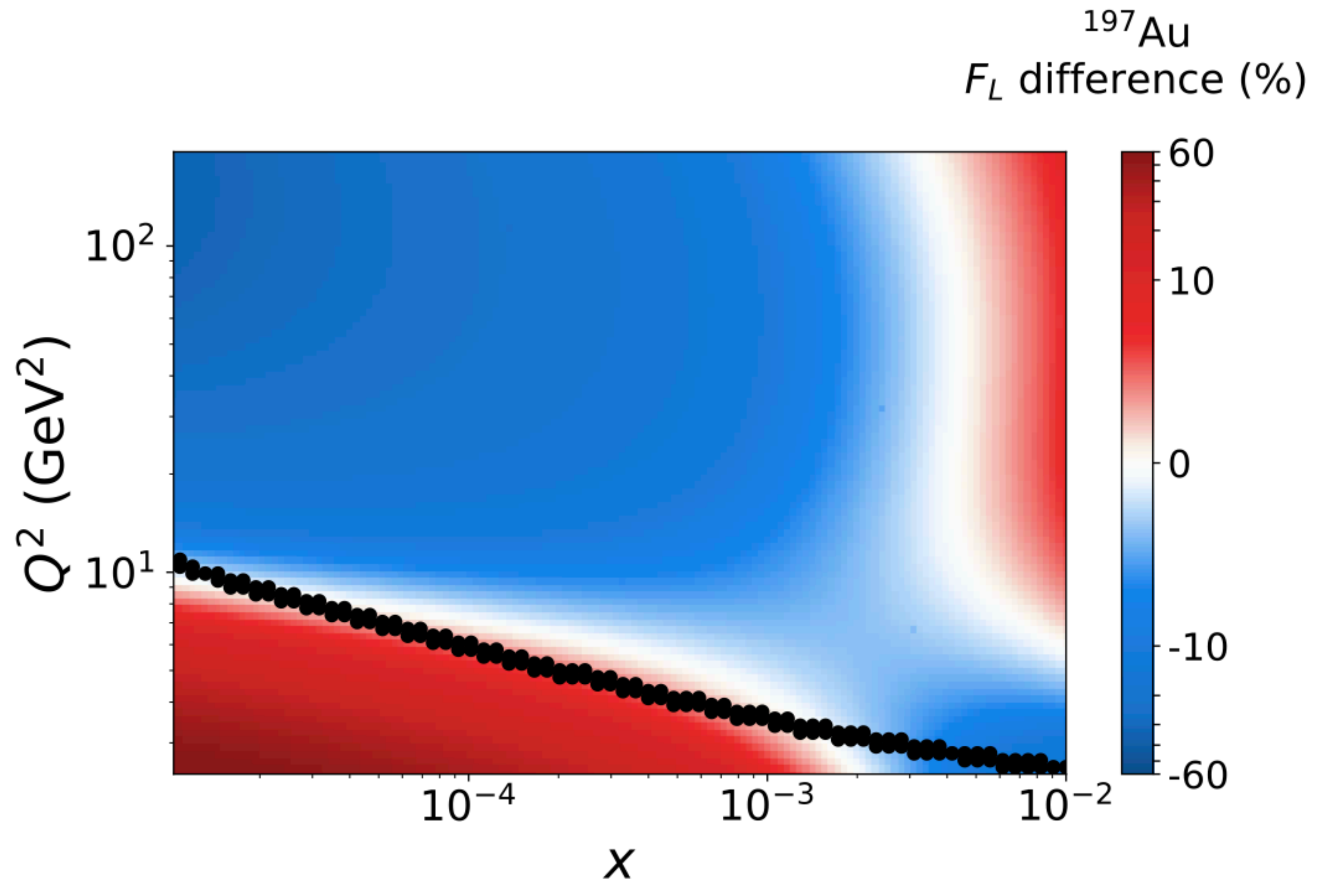
(a) F_2



(b) $F_{\rm L}$



(a) F_2

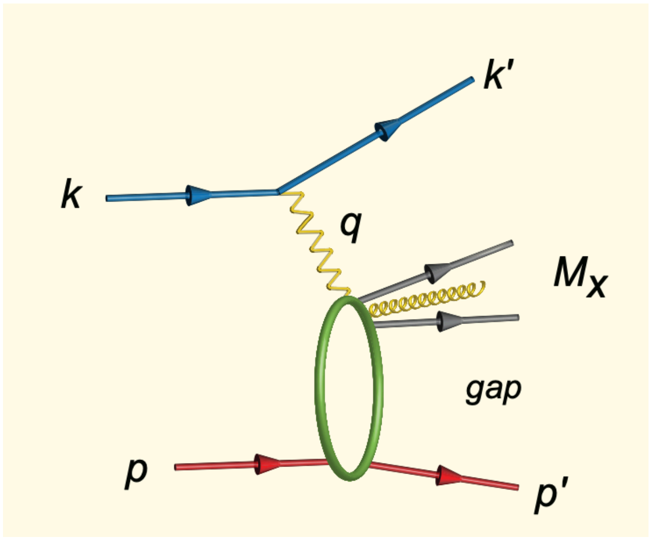


(b) $F_{\rm L}$

Armesto, Lappi, Mäntysaari, Paukkunen, Tevio (2022)

See also Marquet, Moldes, Zurita (2017)

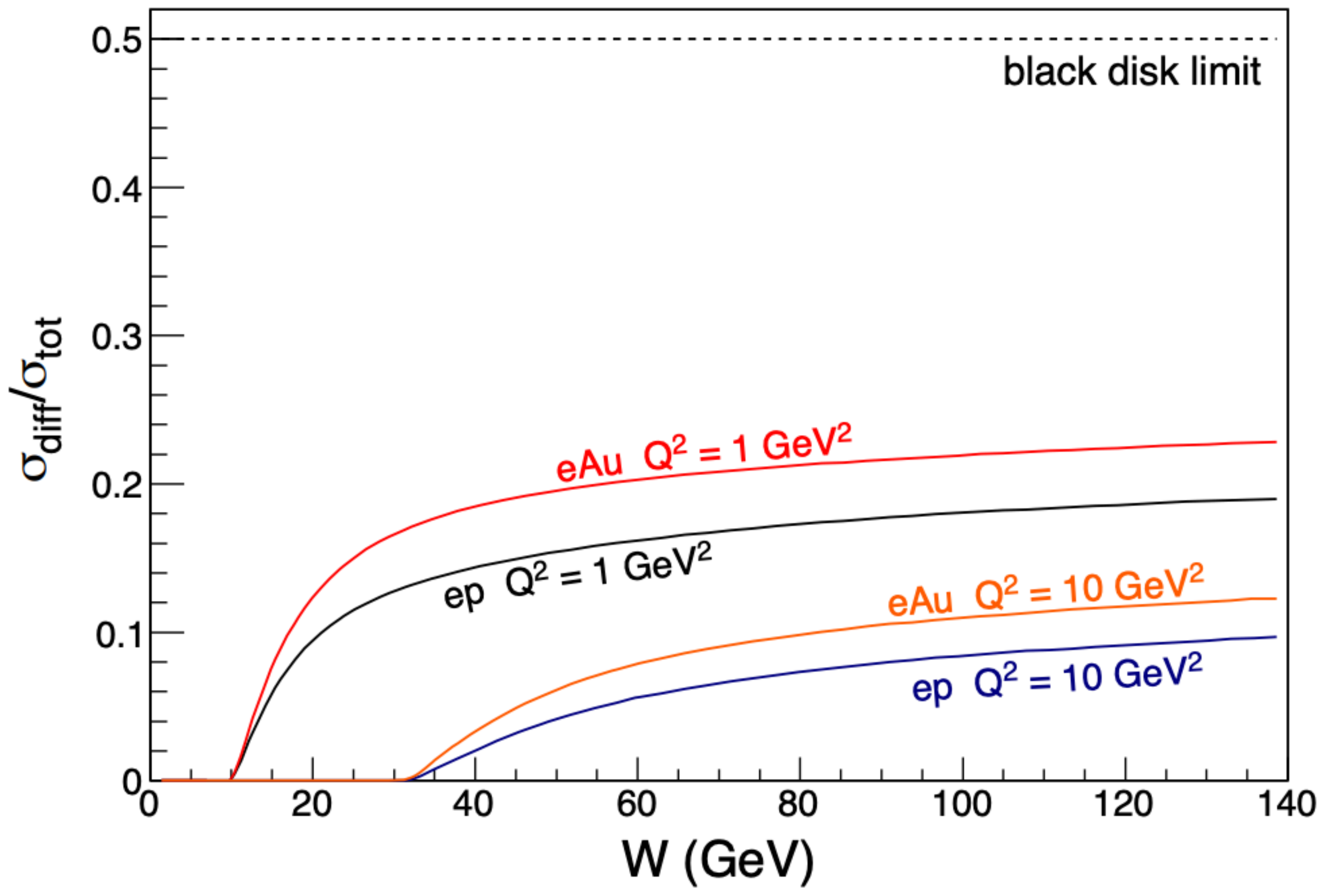
Diffractive structure functions



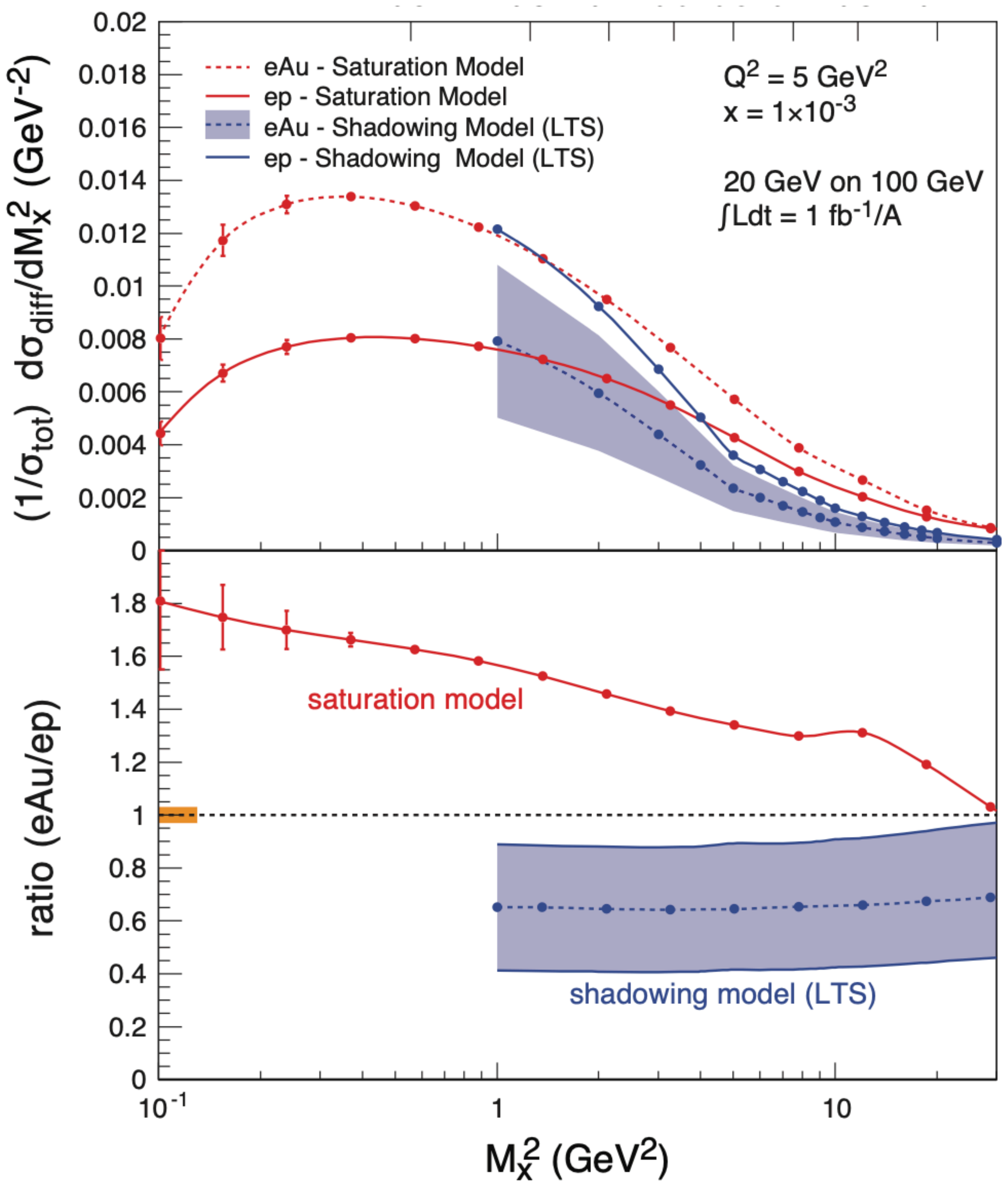
Diffractive events are characterized by rapidity gap

Neutral color exchange requires at least two-gluons (enhanced sensitivity to gluon sat)

Ratio of diffractive and total cross-section in ep and eAu collisions



Diffractive events enhanced at lower Q^2 and have weak dependence on energy



Clear difference between saturation models and leading twist shadowing (LTS)

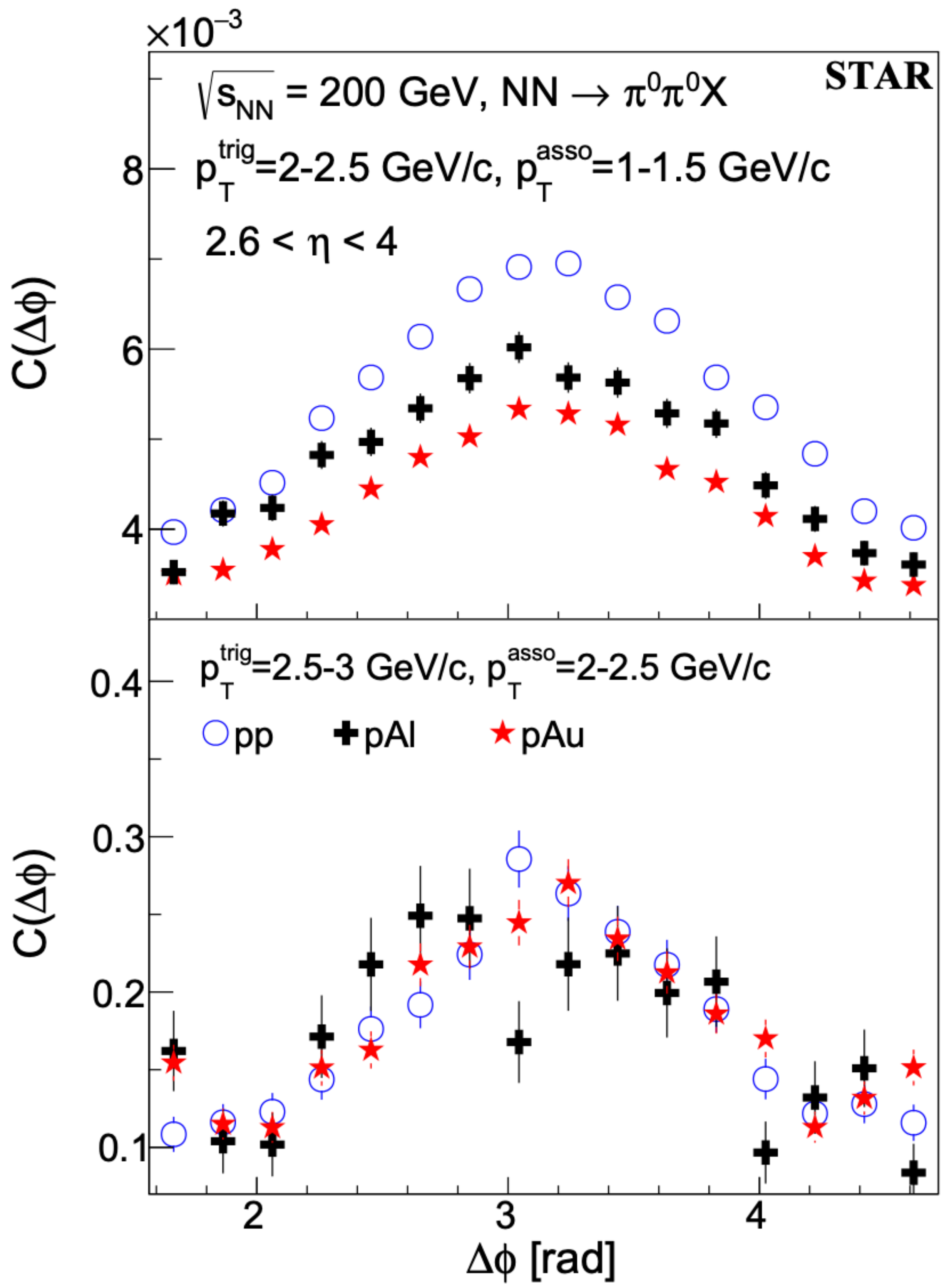
EIC White paper (2012)

Observables at the EIC:

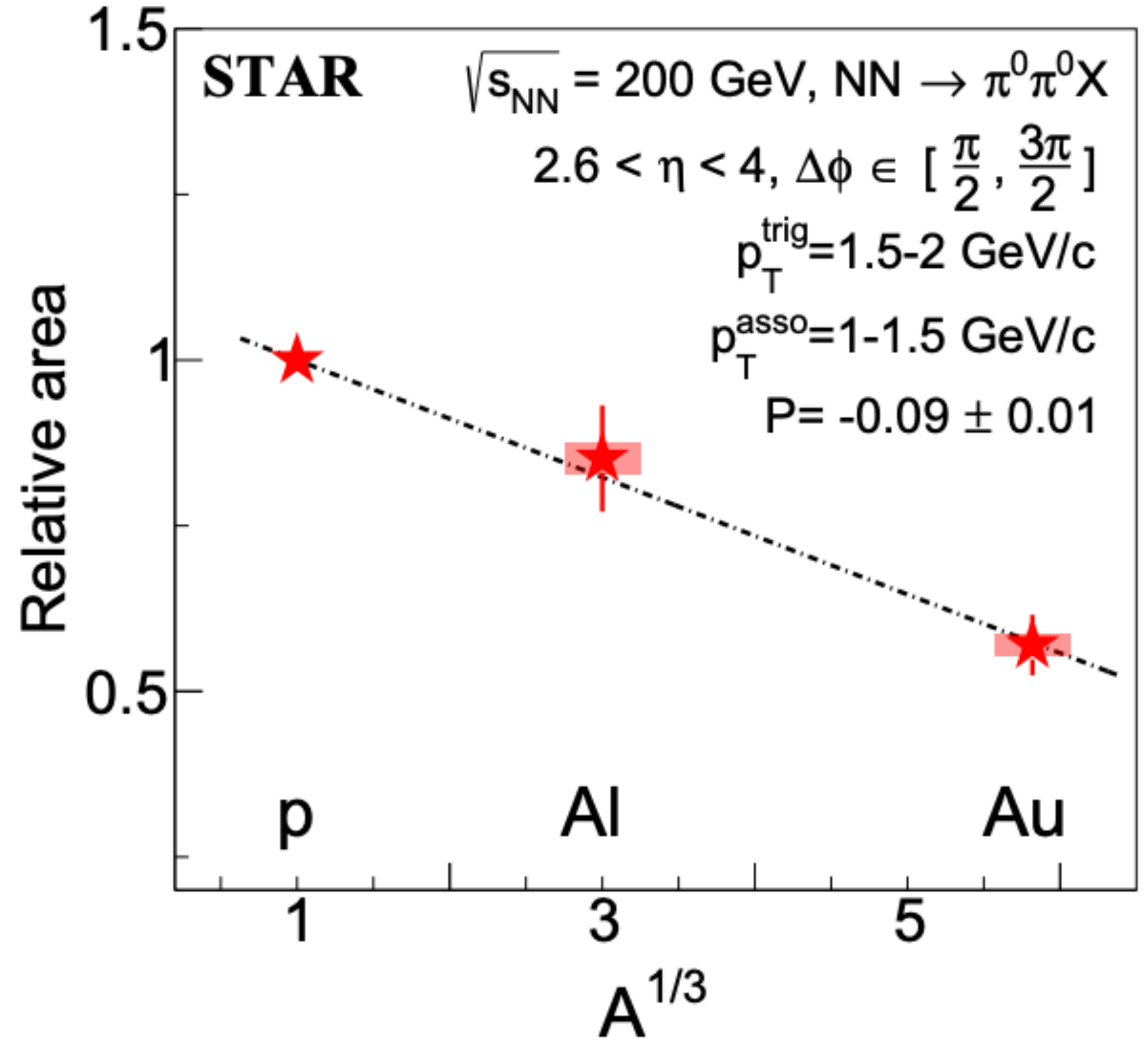
Semi-inclusive

Two particle correlations at RHIC

Evidence for Nonlinear Gluon Effects in QCD and Their Mass Number Dependence at STAR

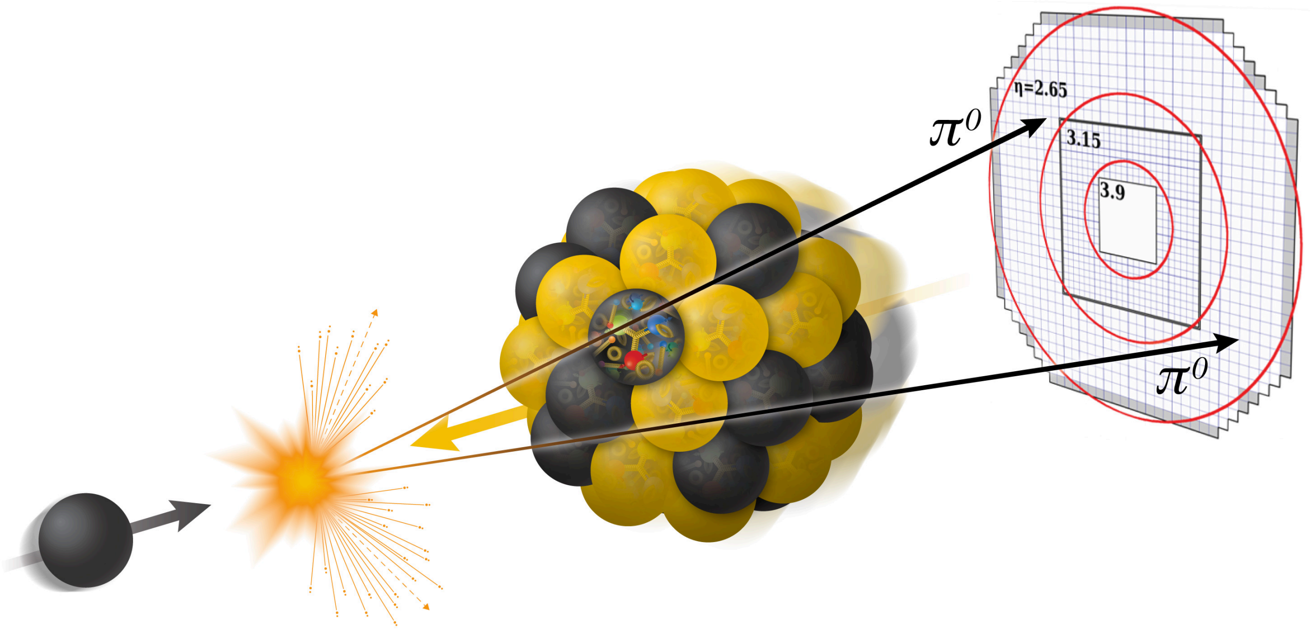


STAR Collaboration
Phys. Rev. Lett. 129, 092501 (2022)



Suppression characteristic of saturation

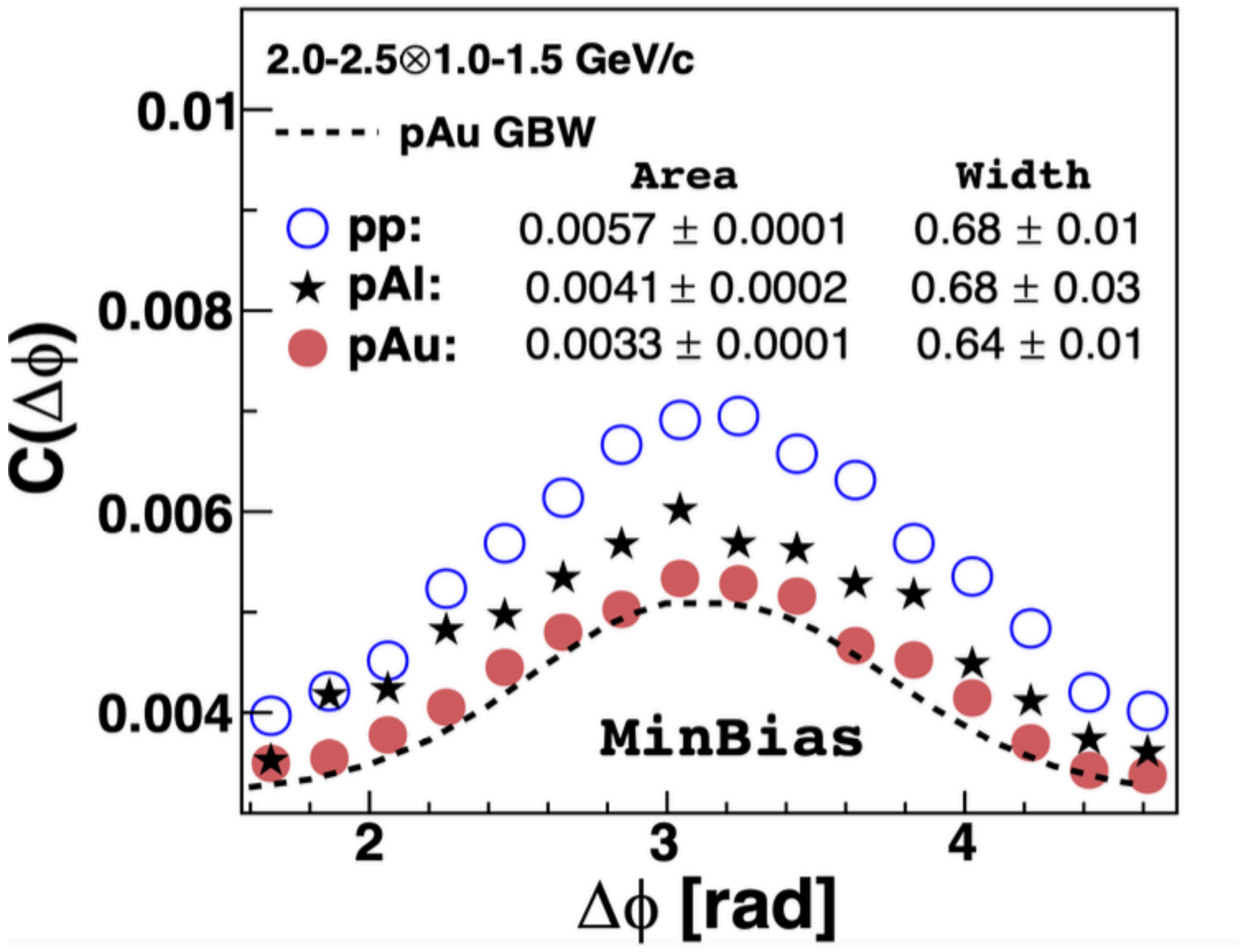
$$Q_s^2 \propto A^{1/3}$$



Xiaoxuan Chu and Elke Aschenauer

Two particle correlations at RHIC

nPDF or saturation?



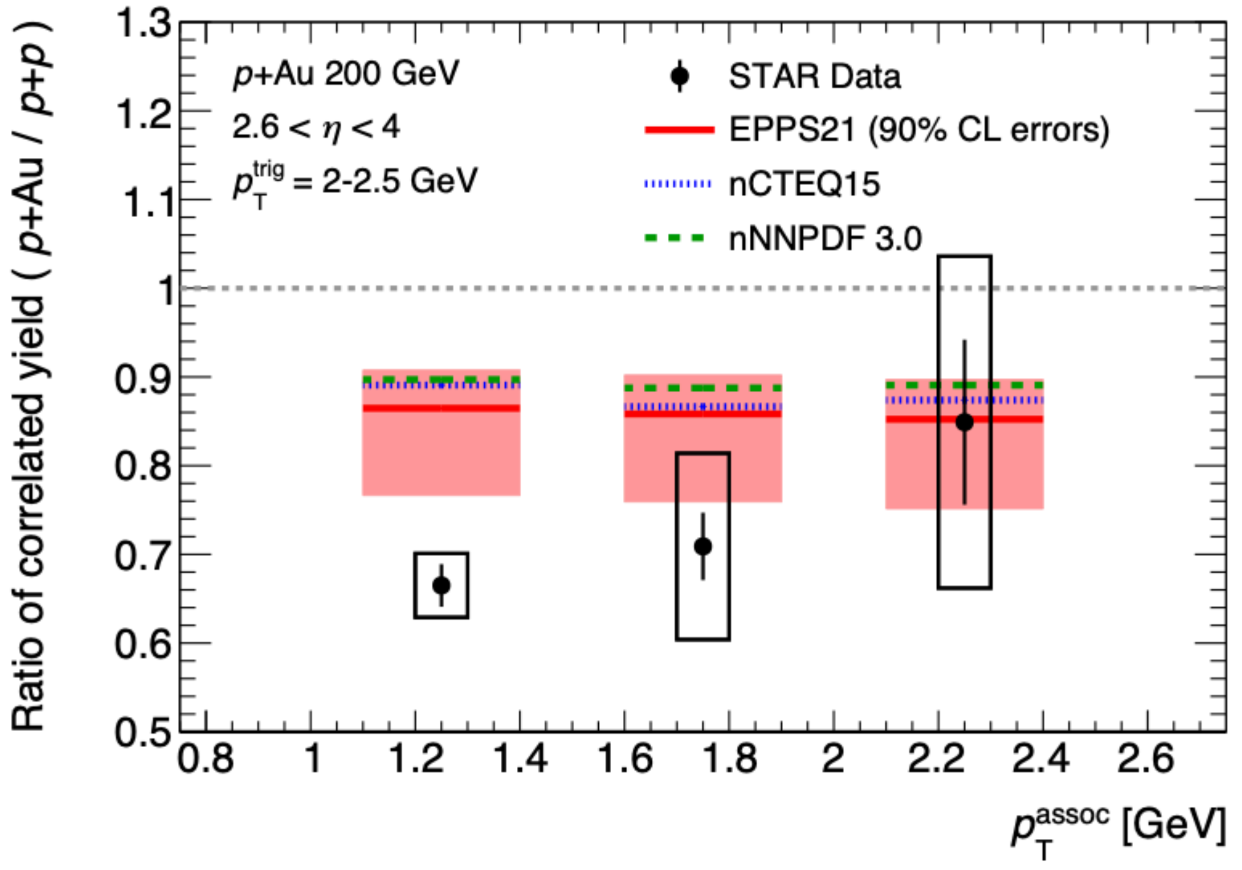
STAR (2021)

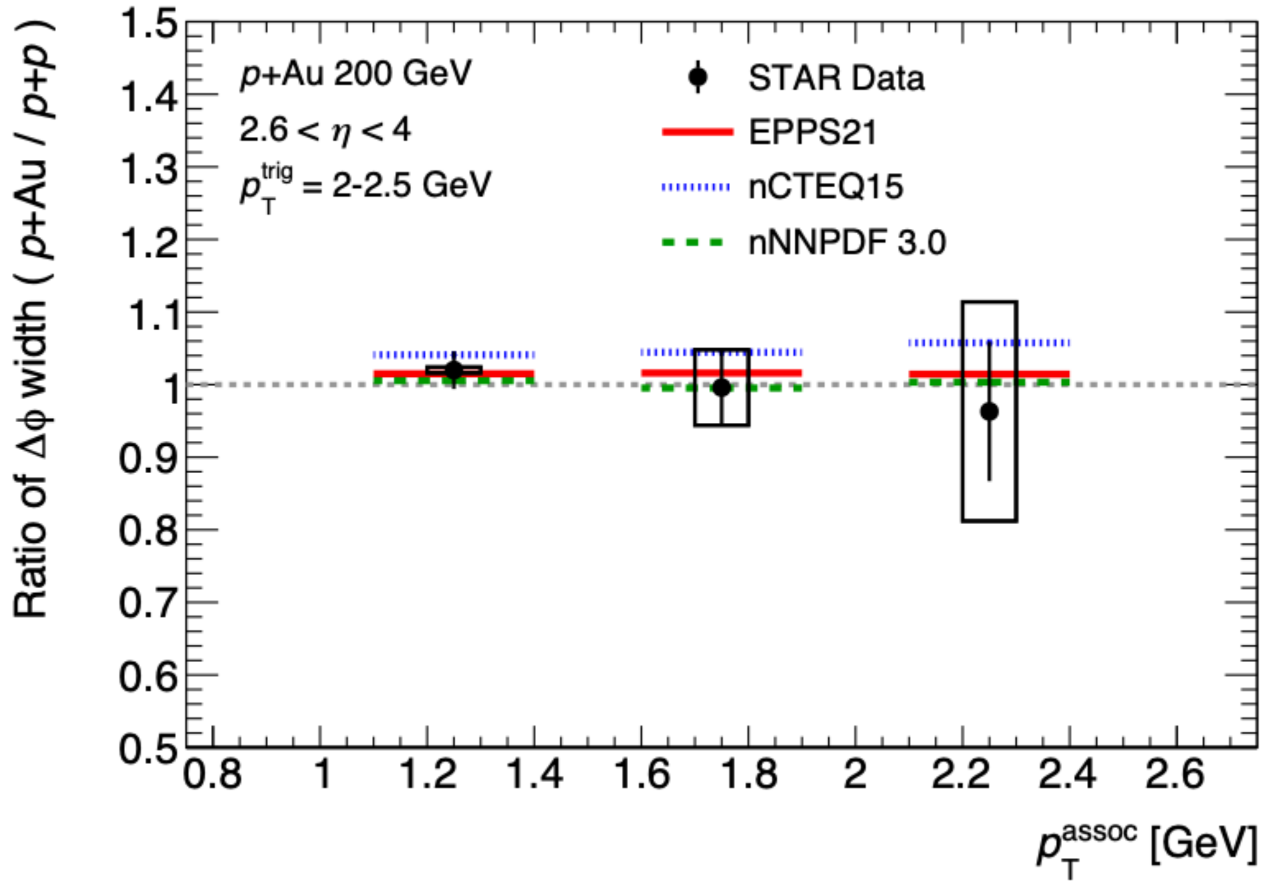
- CGC approach (work in progress Zhao et al)
  -Small-x evolution -> $p_\perp$**-dependent suppression**
  -Soft gluon radiation -> **similar width of correlation in pp and pA** (i.e. not much broadening) hints of this in full NLO calculation in DIS Caucal, Salazar, Schenke, Stebel, Venugopalan (2024)

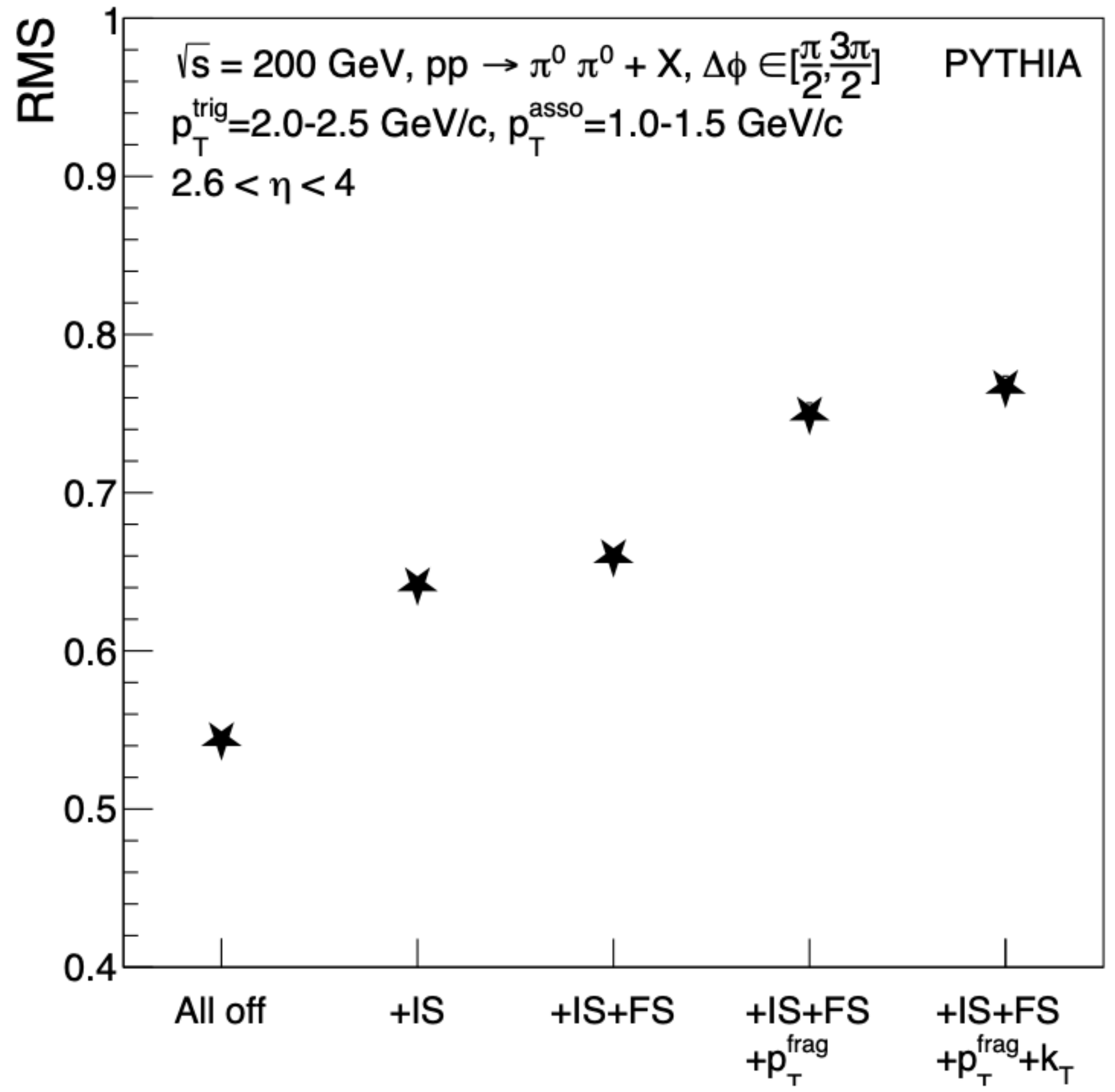
[More on two-particle correlations at RHIC and EIC at Elke and Xiaoxuan's discussion on Wednesday]

[Also Wenbin's discussion?]

- nPDFs approach: Perepelitsa (2025)
  di-hadron RHIC data shows **nuclear size dependent suppression** but **no significant broadening**







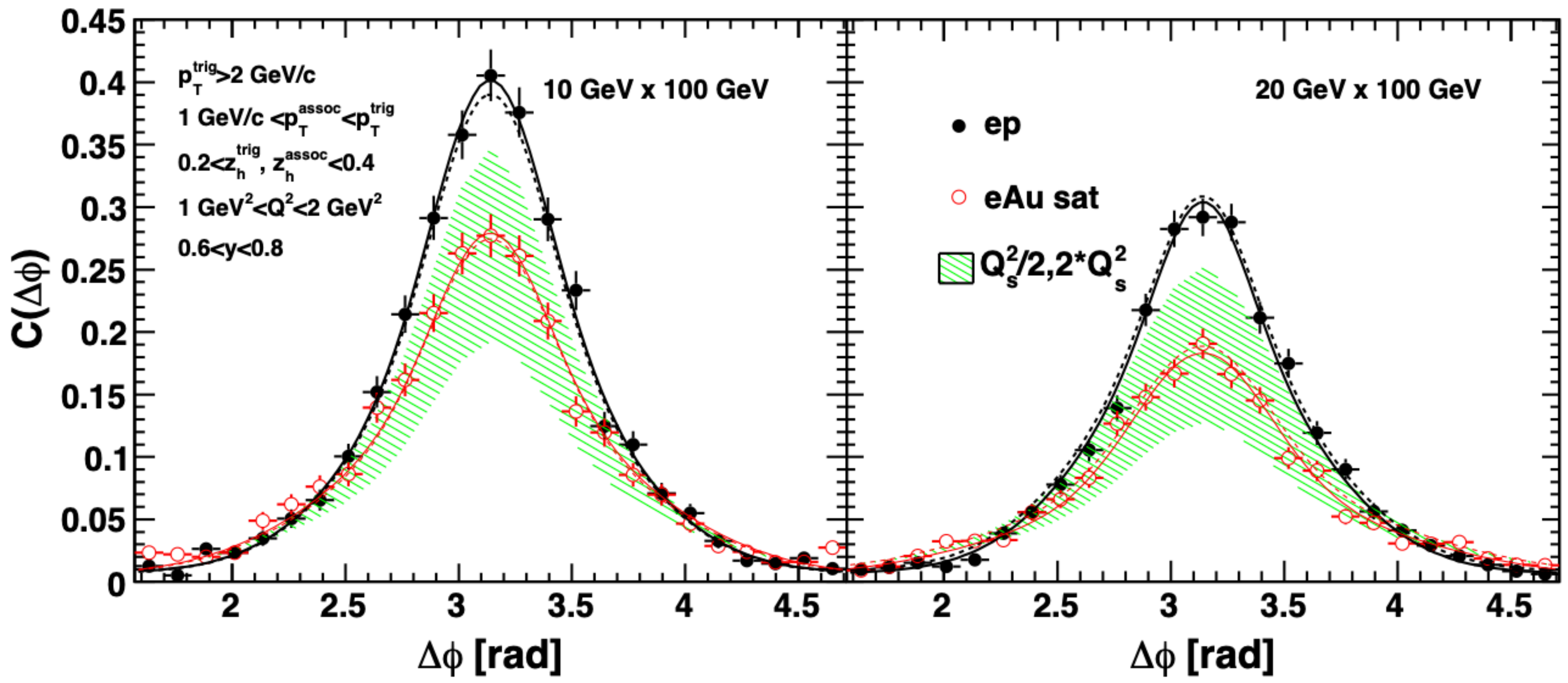
- Cassar, Wang, Chu, Aschenauer (2025)

Parton shower + hadron fragmentation control width of correlation

Absence of broadening is not necessarily challenge to the saturation paradigm

Two particle correlations at EIC

Dihadron suppression
back-to-back peak at EIC



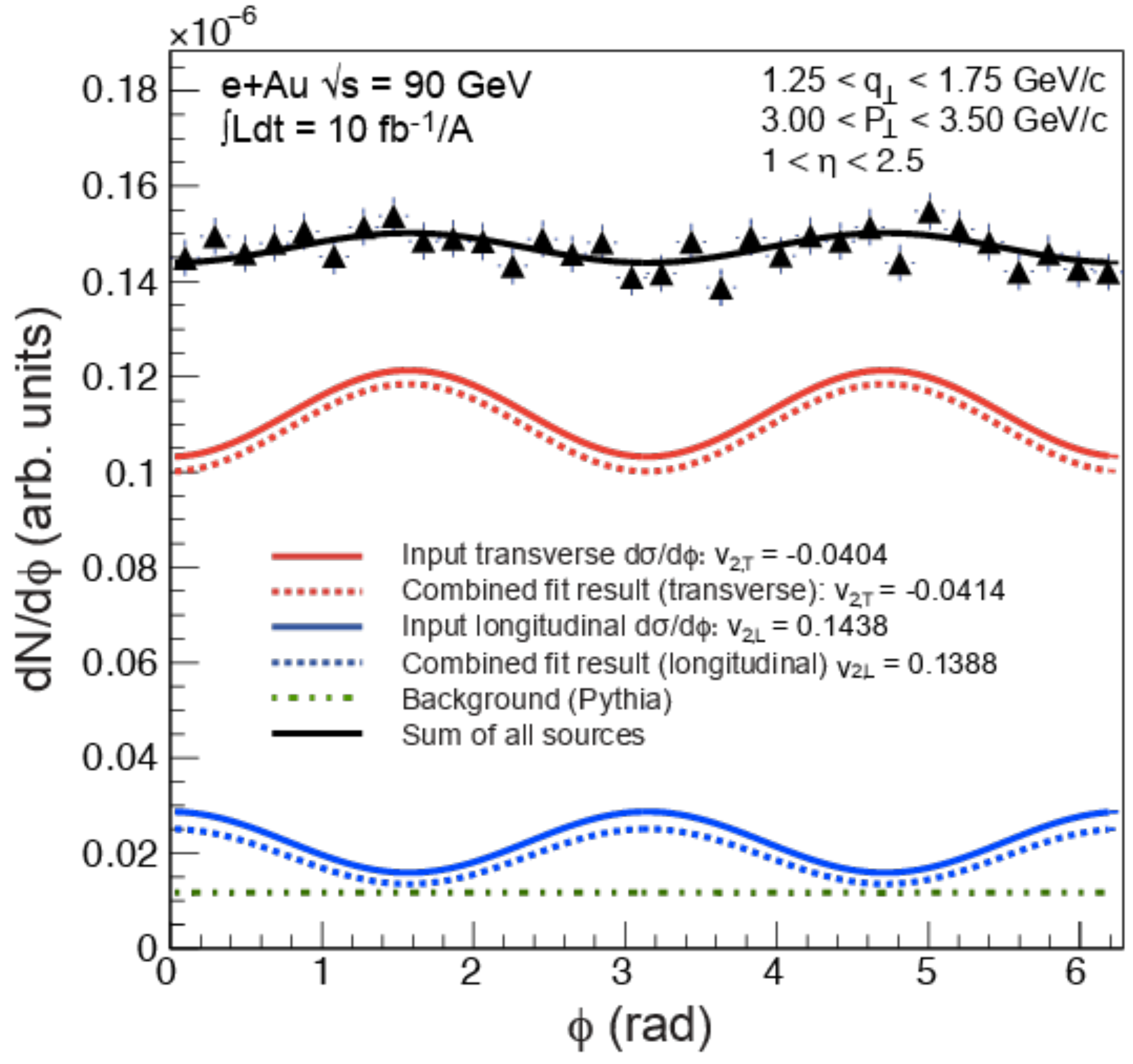
Zheng, Aschenauer, Lee, Xiao (2014)

Suppression of particle production for momentum imbalance $\lesssim Q_s$

Further suppression in eAu than ep due to larger sat scale

NLO calculation
Caucal, Salazar, Schenke, Stebel, Venugopalan (2024)
Caucal, Salazar (2025)

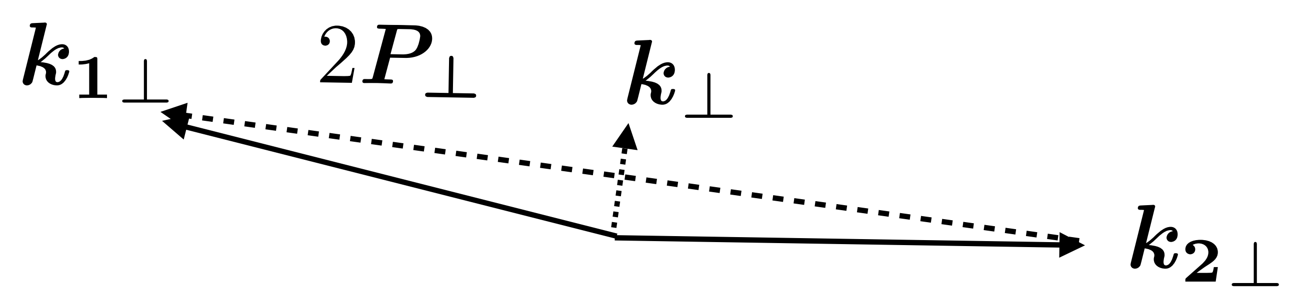
Dijet momentum imbalance
azimuthal correlations



Dumitru, Skokov, Ullrich (2018)

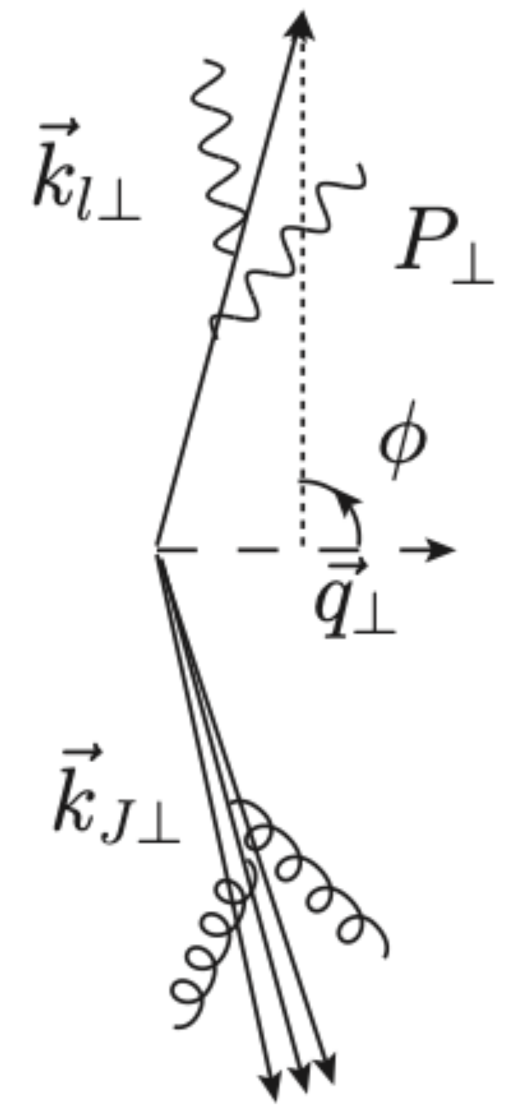
Sensitivity to linearly polarized gluons

ϕ angle between $P_\perp$ and $k_\perp$

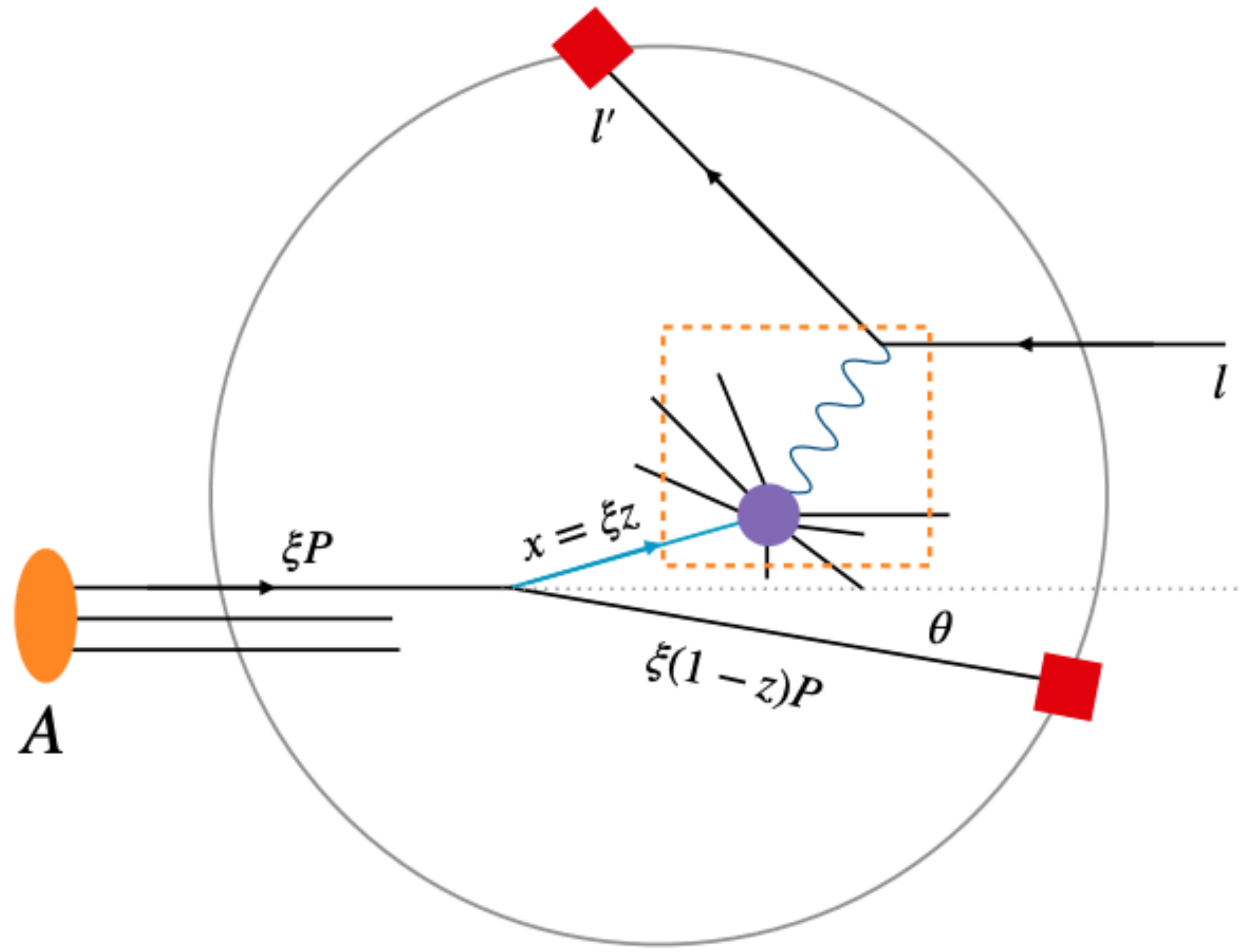


Can also be measured with direct J/ψ production in DIS
Cheung, Kang, Salazar, Vogt (2024)

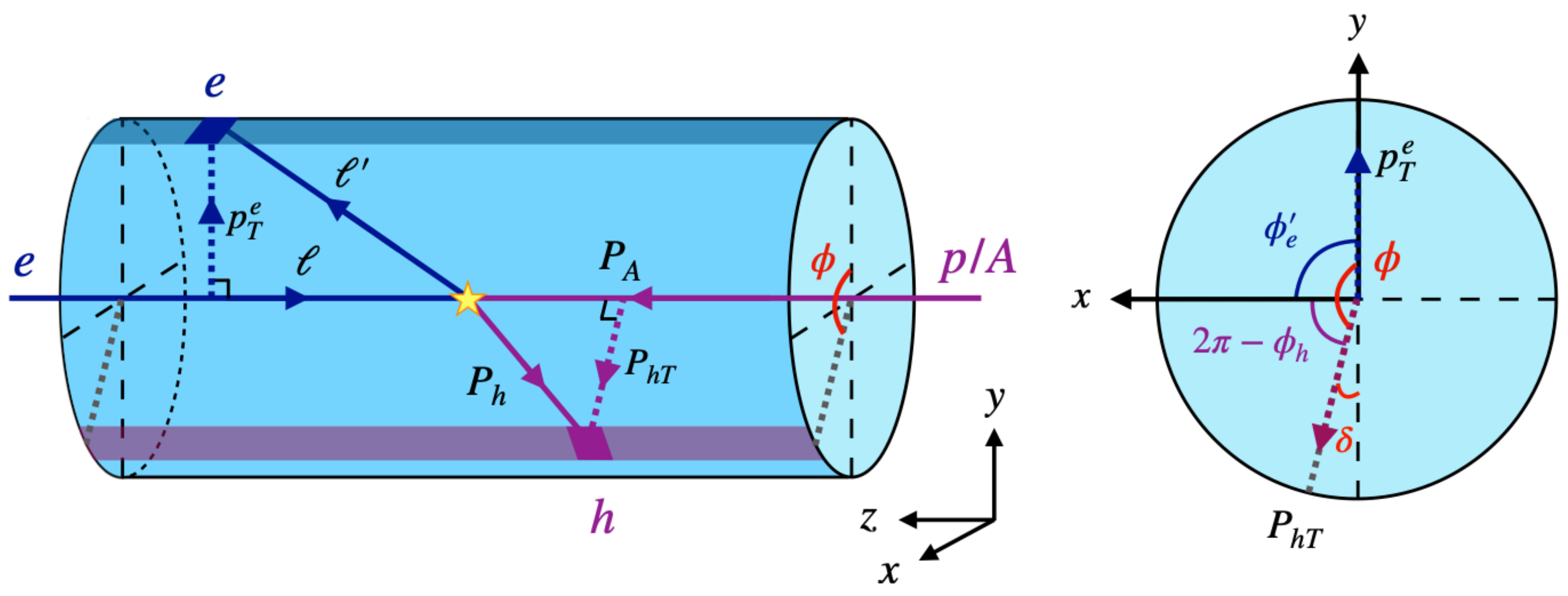
Other two-particle correlations: lepton-jet and nucleon-energy energy correlators



Lepton-jet correlations
Tong, Xiao, Zhang (2022)



Nucleon energy correlator
Liu, Pan, Yuan, Zhu (2023)



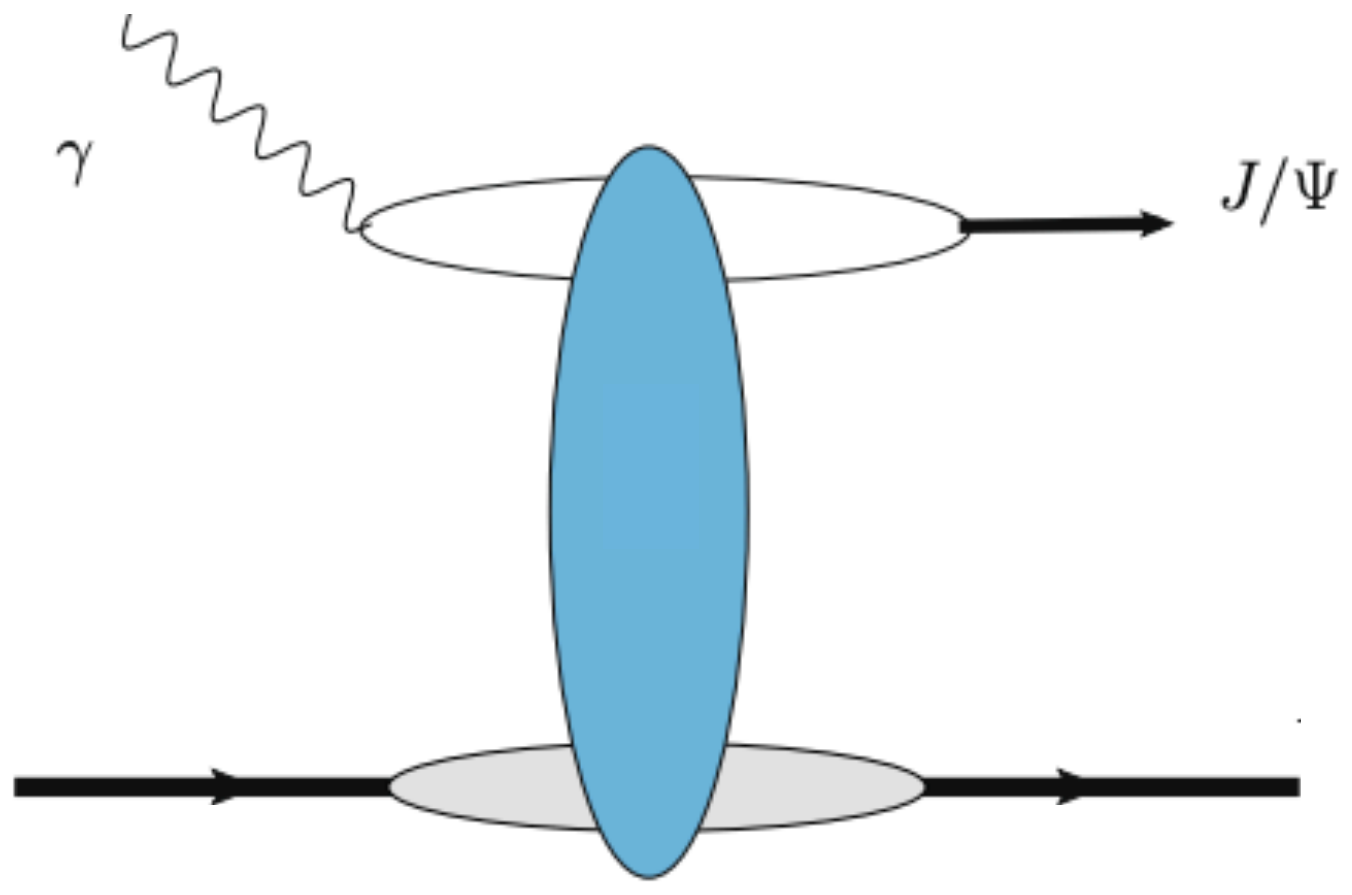
Transverse energy-energy correlators
Kang, Penttala, Zhao, Zhou (2024)

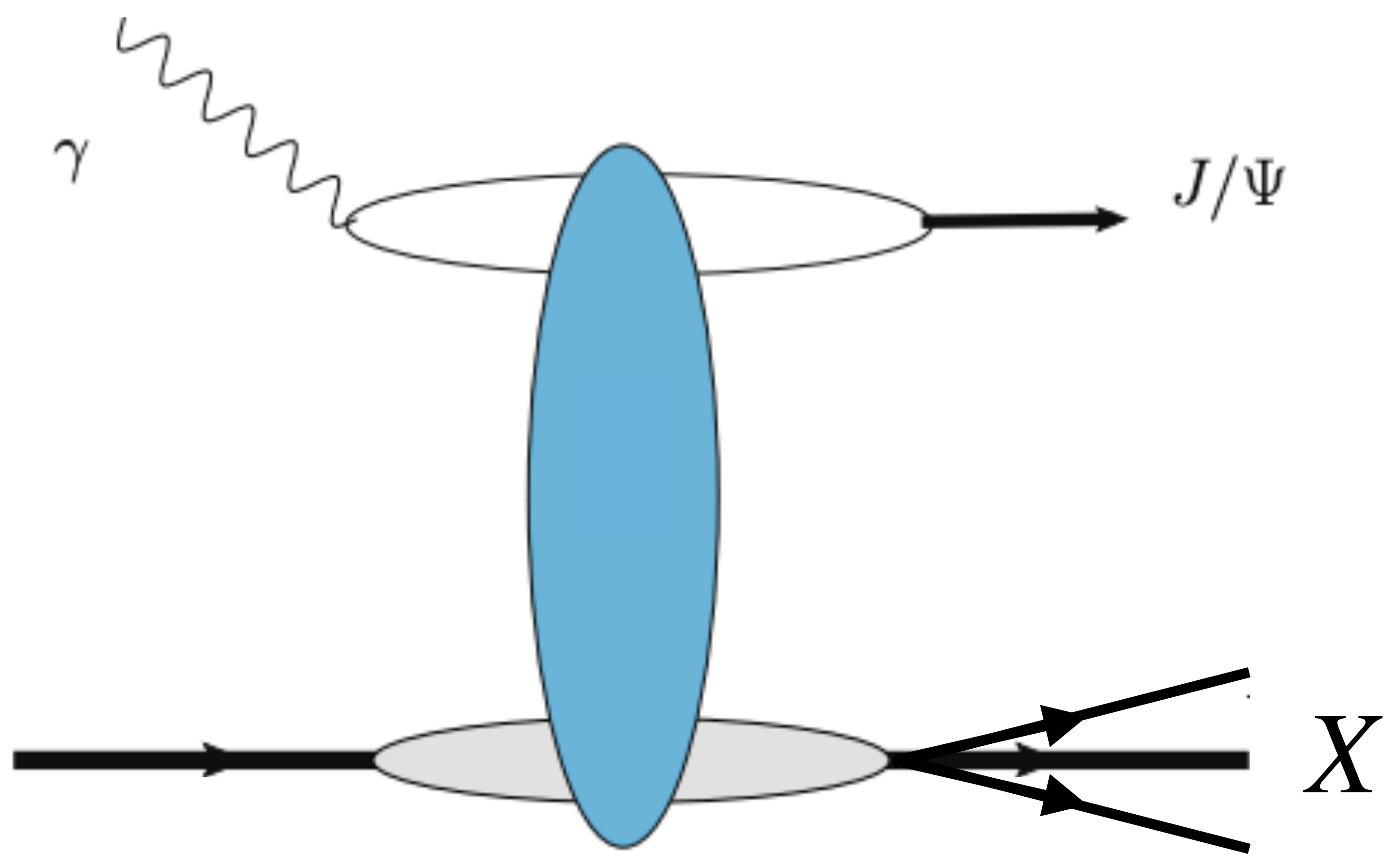
For jet observables see Jani and Anna's discussion on Thursday

Observables at the EIC: Exclusive

Exclusive vector meson production

Coherent and incoherent reactions





$$\mathrm{d}\sigma_{\mathrm{coh}} \propto \langle \mathcal{A}^{\dagger}(\mathbf{\Delta}_{\perp})\rangle\langle \mathcal{A}(\mathbf{\Delta}_{\perp})\rangle$$

- Nuclear target remains intact
- t-dependence gives information on spatial distribution gluons in transverse plane (to the beam)
- Connection to GPDs

$$\mathrm{d}\sigma_{\mathrm{incoh}} \propto \langle \mathcal{A}^{\dagger}(\mathbf{\Delta}_{\perp})\mathcal{A}(\mathbf{\Delta}_{\perp})\rangle - \langle \mathcal{A}^{\dagger}(\mathbf{\Delta}_{\perp})\rangle\langle \mathcal{A}(\mathbf{\Delta}_{\perp})\rangle$$

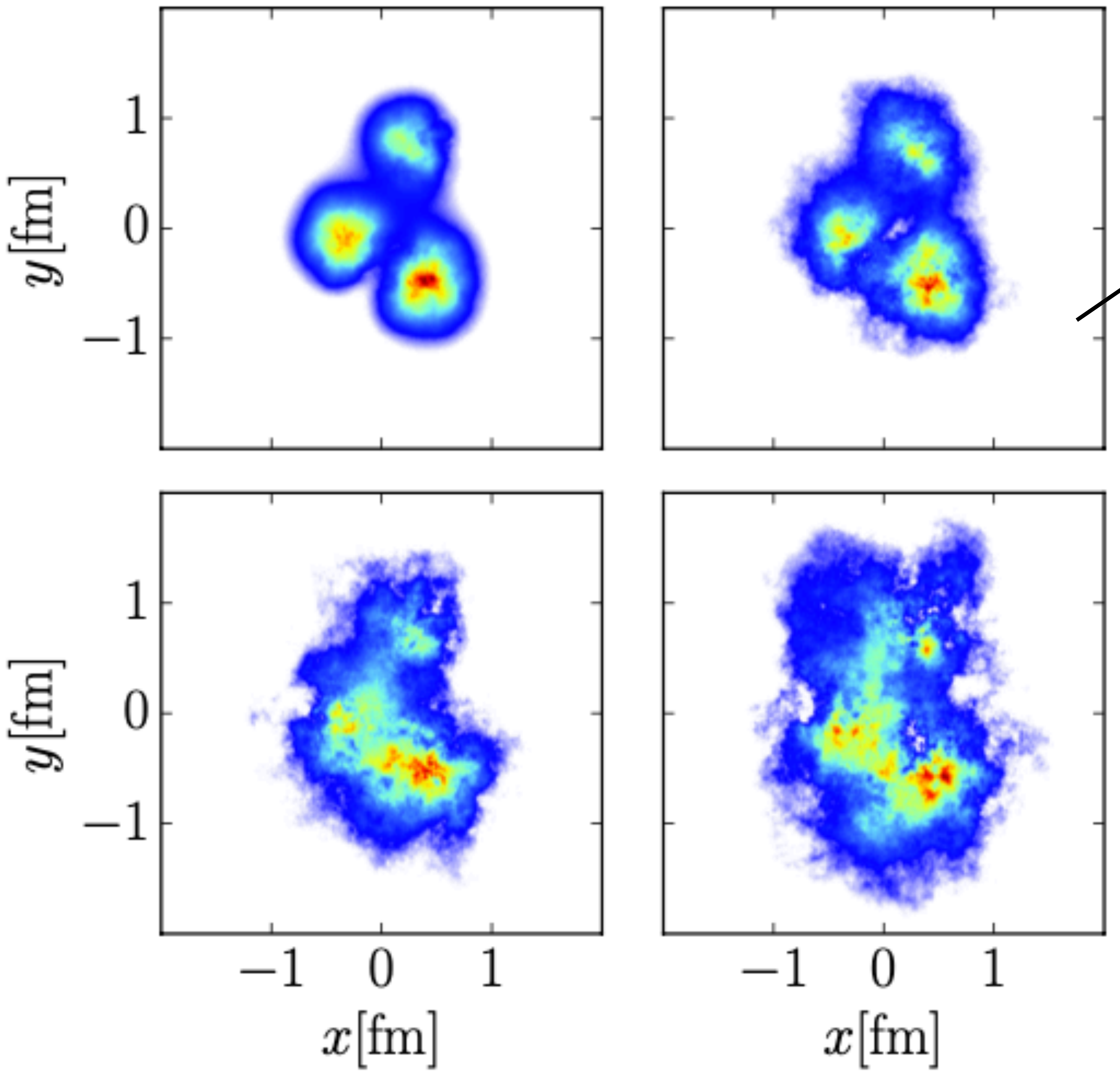
- Target breaks up, but one has a rapidity gap
- Sensitive to fluctuations: color charge, sub-nucleon, nucleon

Exclusive vector meson production

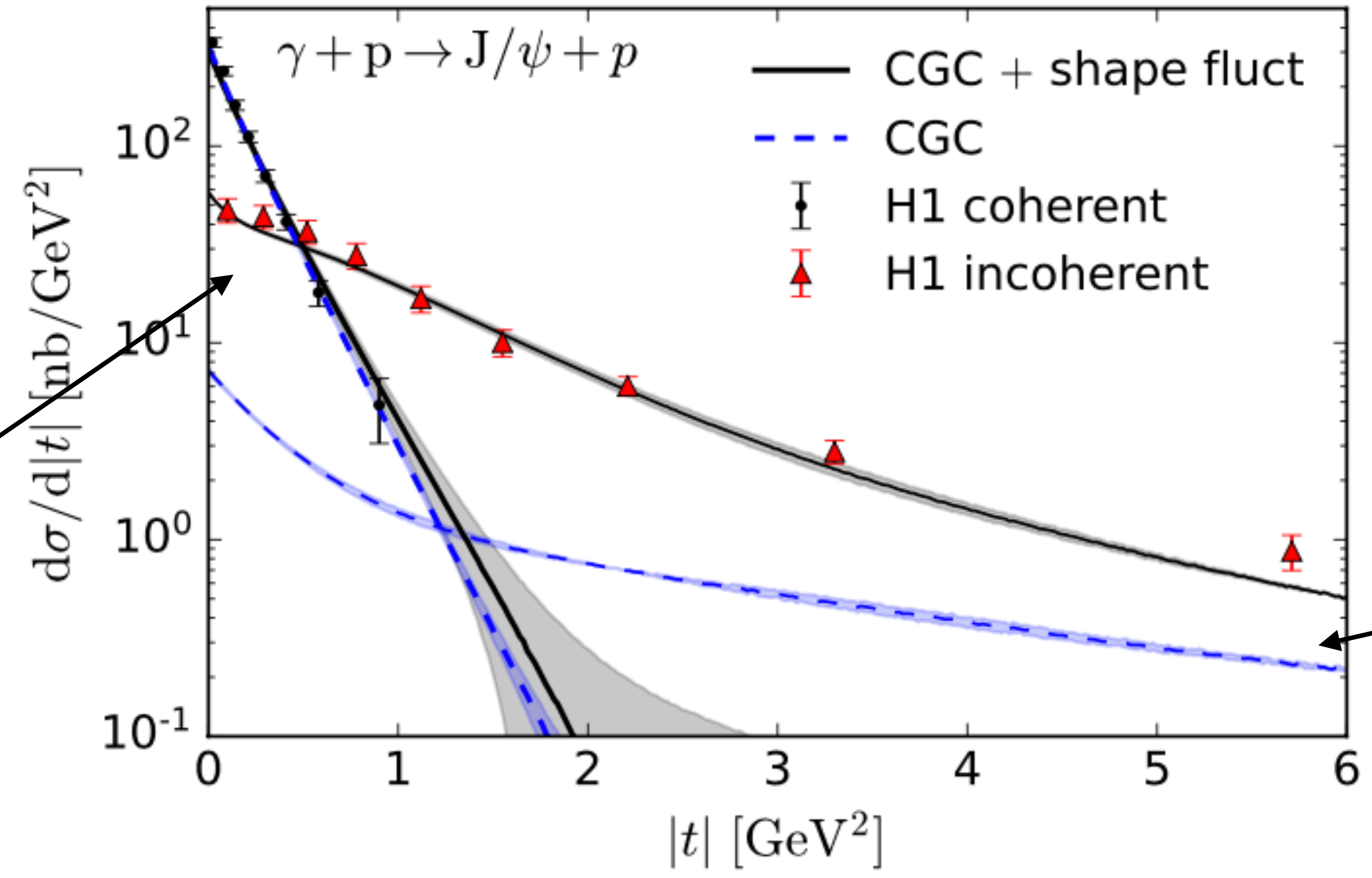
Event-by-event sub-nuclear fluctuations

Introduce sub-nucleon structure

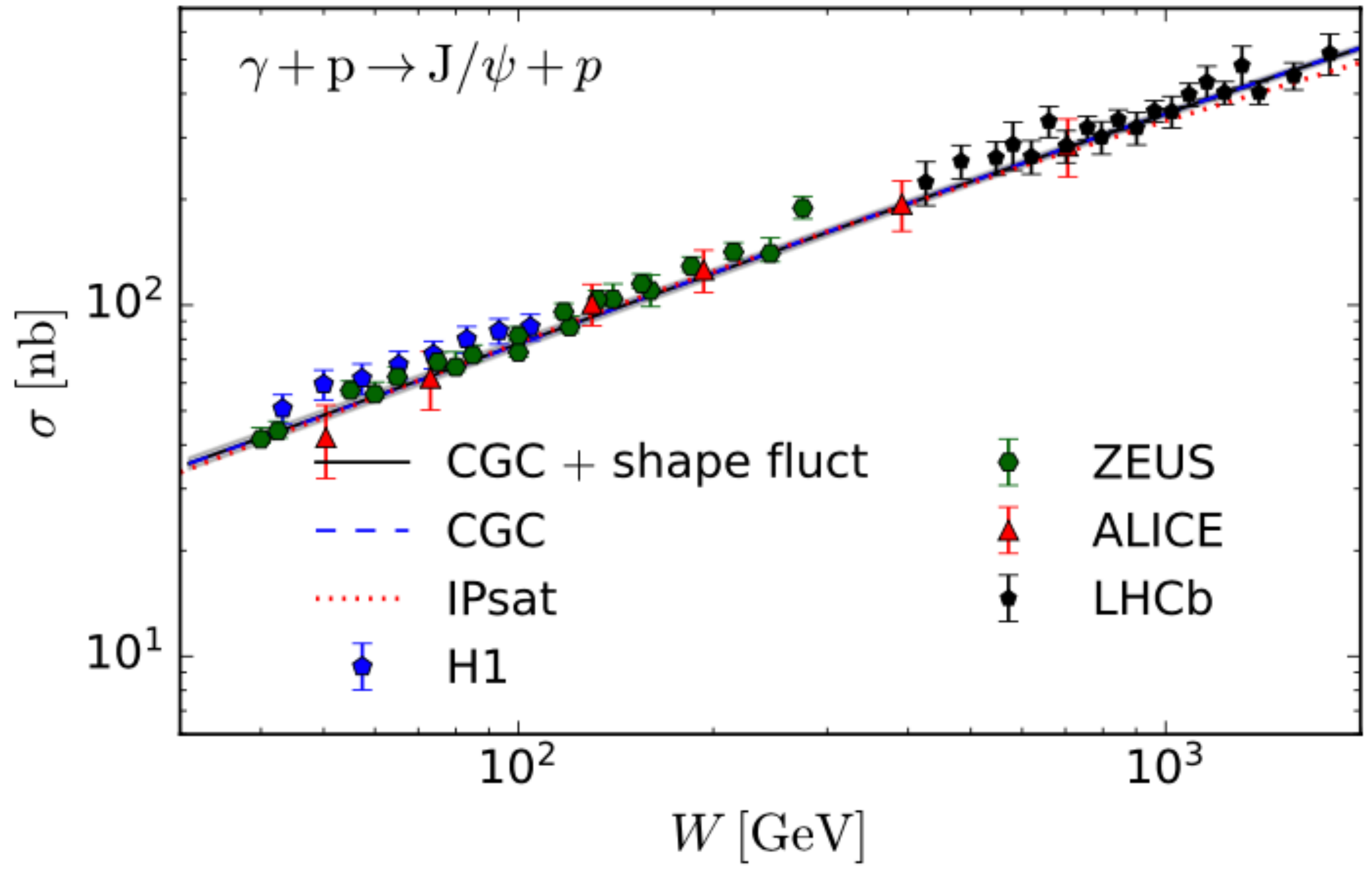
Mäntysaari, Schenke (2016)



Mäntysaari, Schenke (2018)
Mäntysaari, Salazar, Schenke (2022)

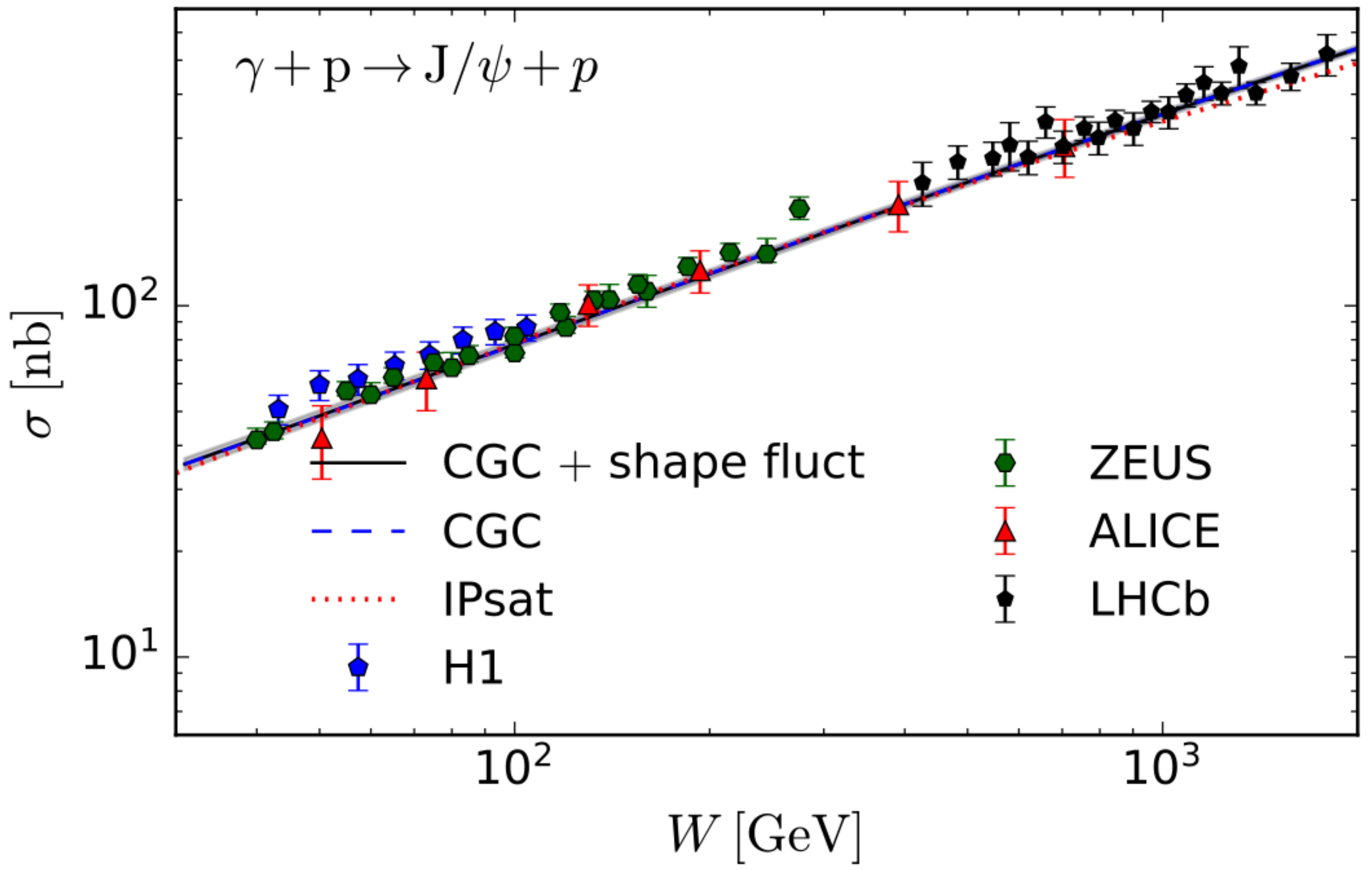


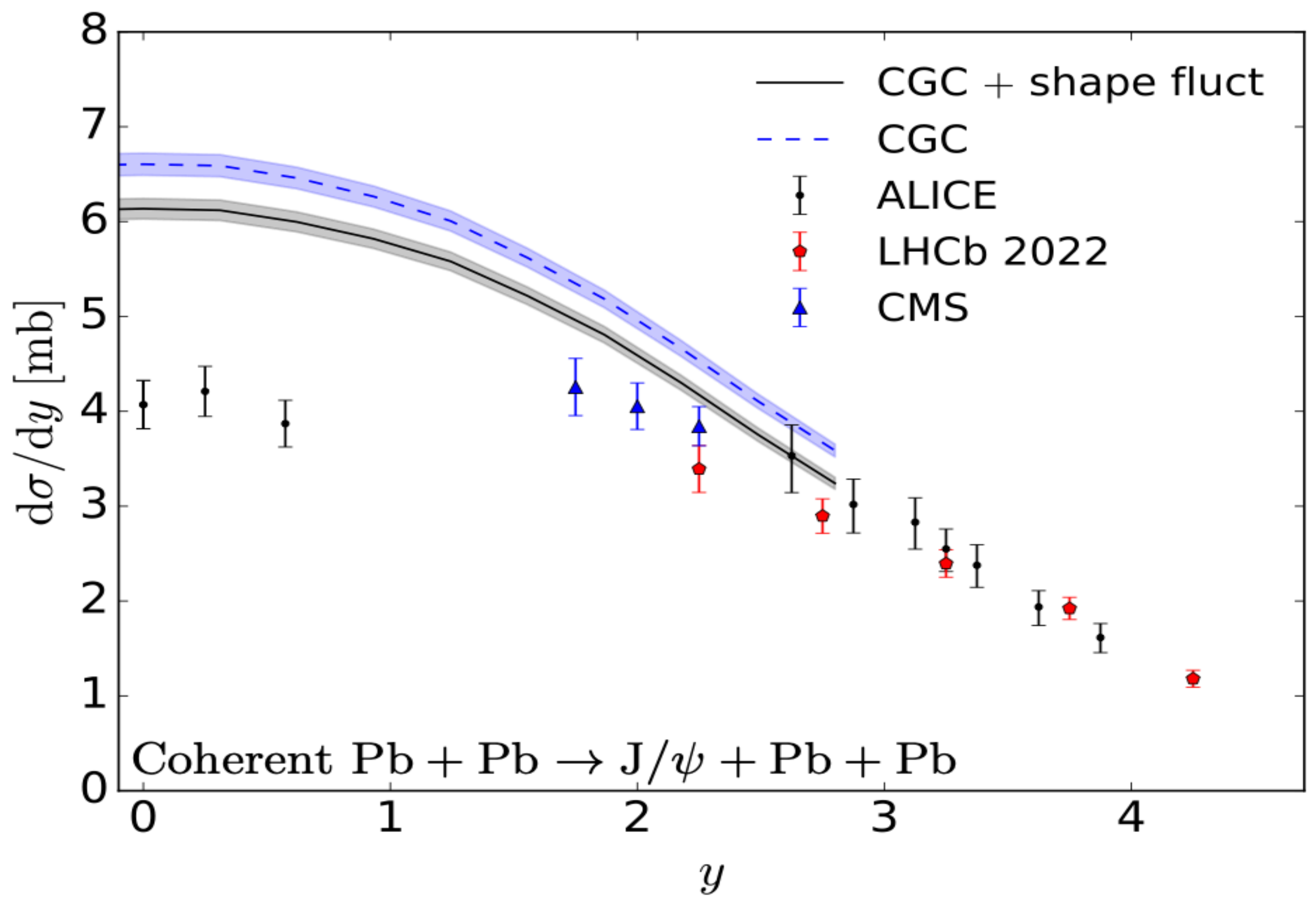
No subnucleon fluctuations

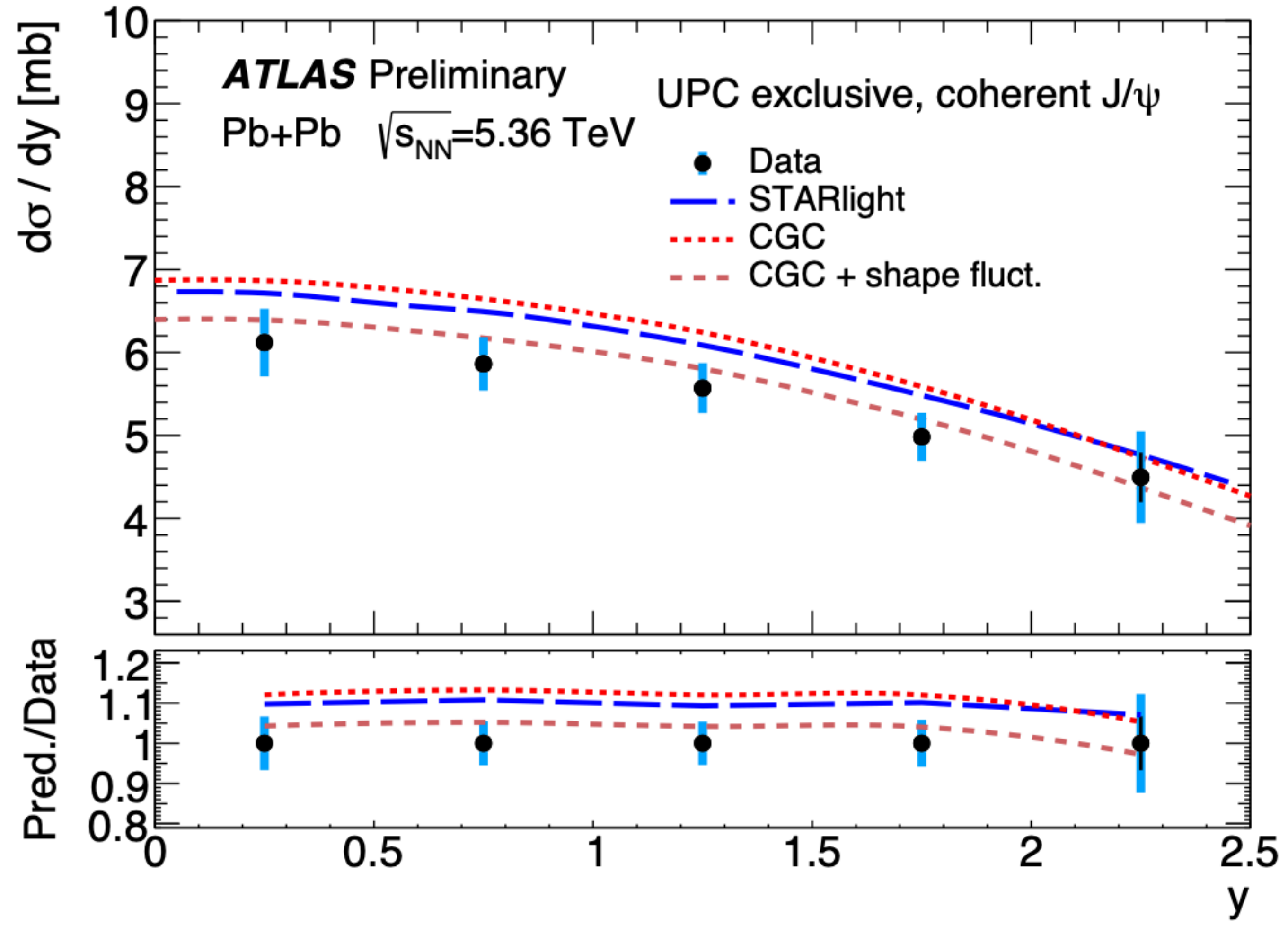


Exclusive vector meson production in UPCs

Coherent production γp and γA



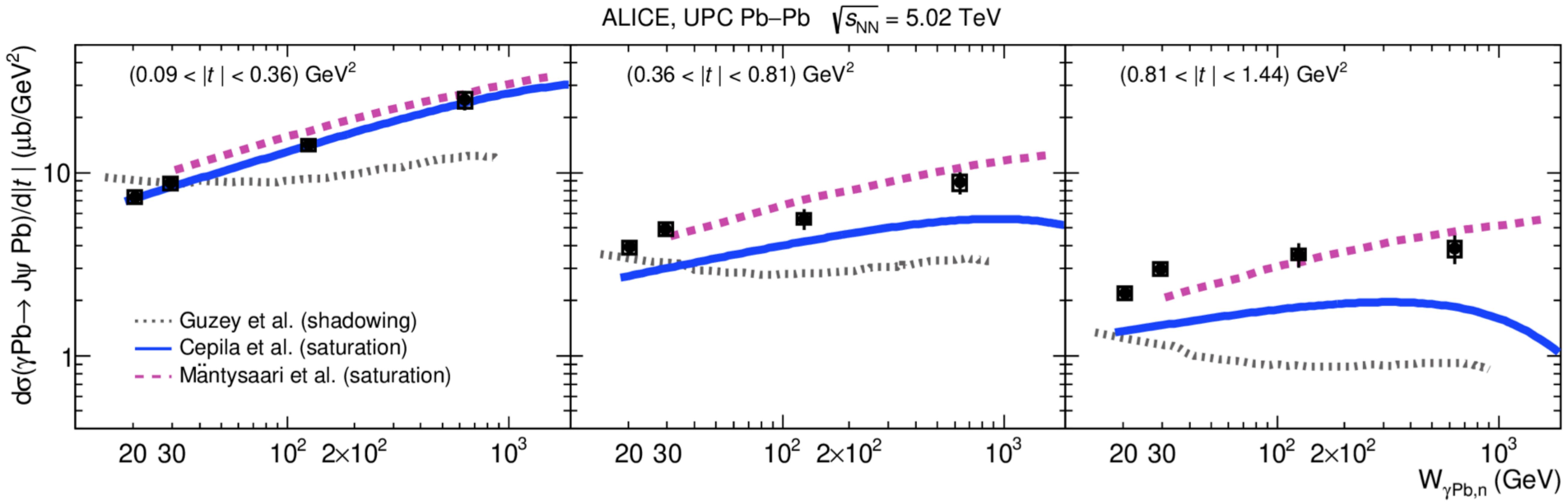




Mäntysaari, Salazar, Schenke (2022, 2024)

ATLAS (preliminary)

Double-differential incoherent γA



← ALICE

CGC-based Bayesian analysis for J/ψ data
γp and γA work in progress

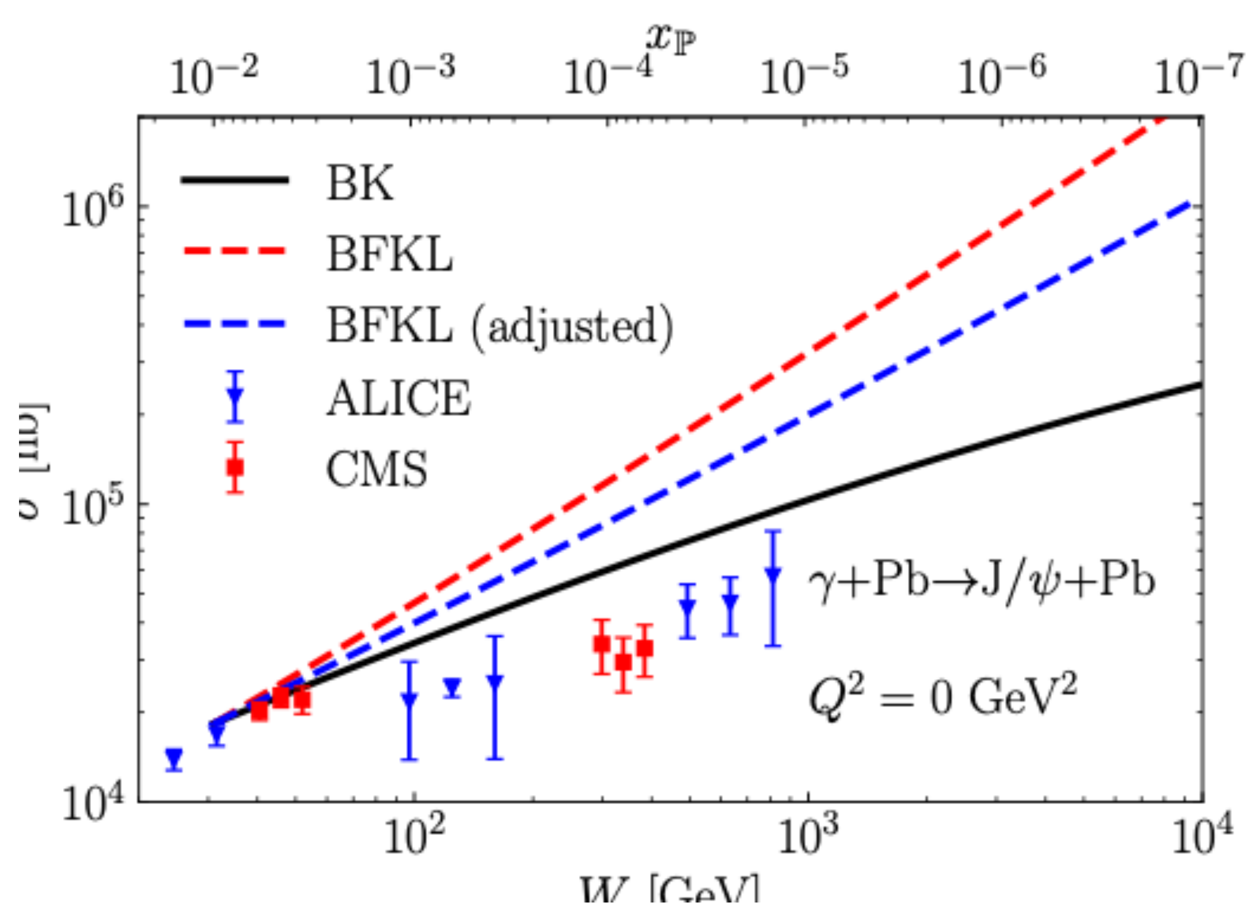
Exclusive vector meson production in UPCs

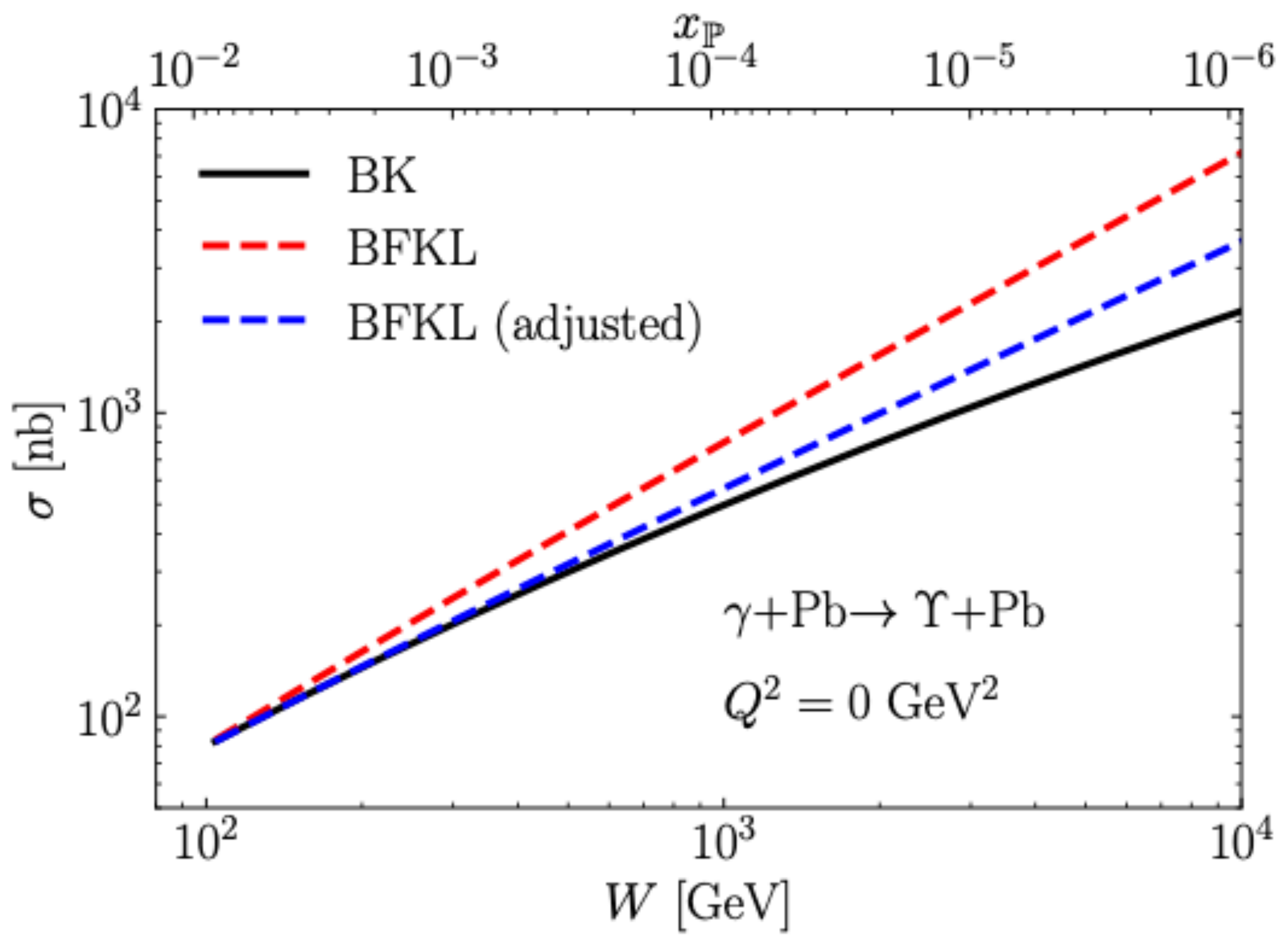
- Stronger saturation effects(more nuclear suppression) :

  -for larger nuclei and larger energy (smaller-x)

  $$Q_s^2(x) \approx \Lambda_{QCD}^2 A^{1/3}(x_0/x)^\lambda$$

  -for less massive vector meson $M_V^2 \lesssim Q_s^2(x)$





Penttala, Royon (2024)

Map Q² varying VM mass

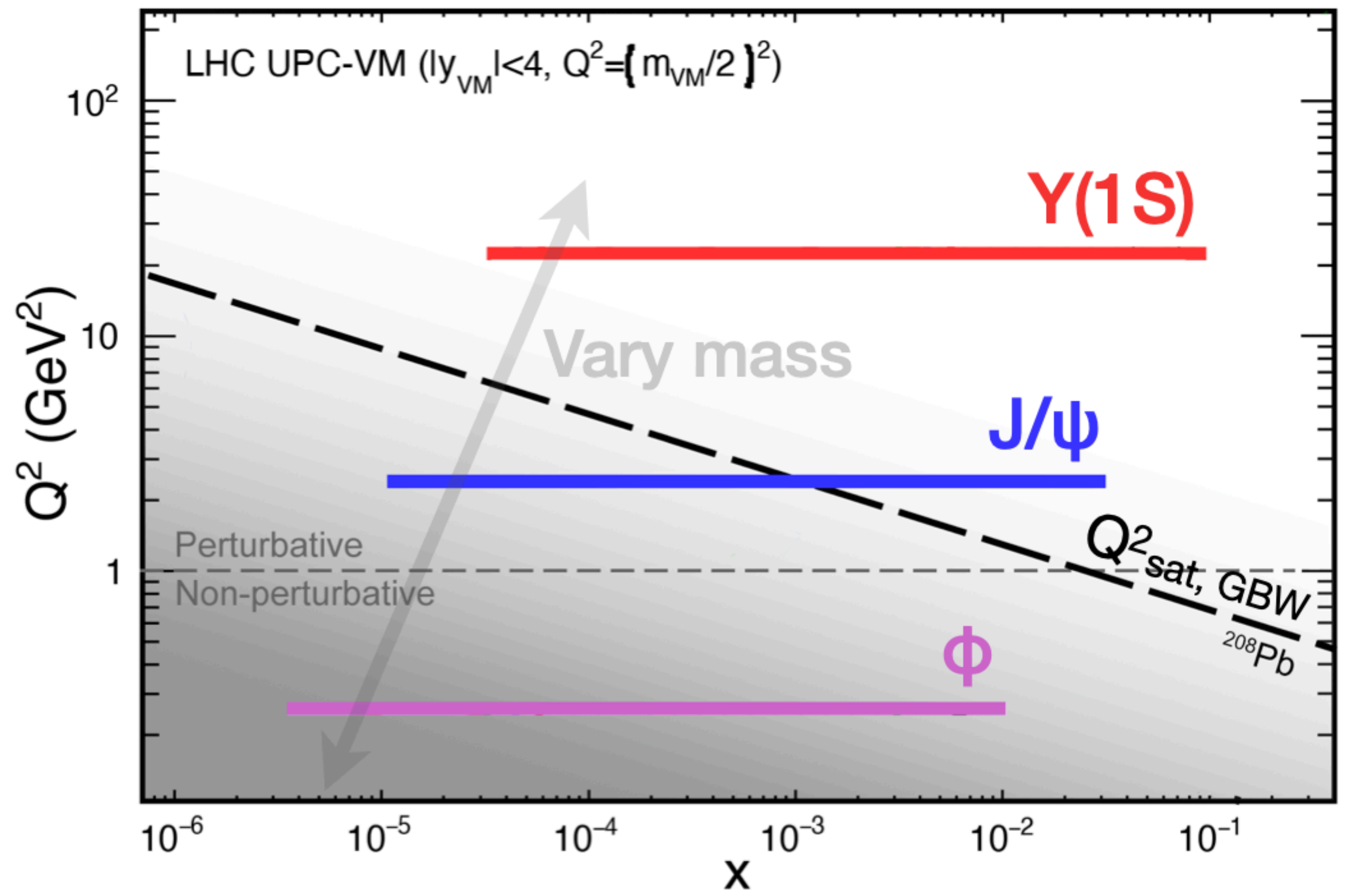


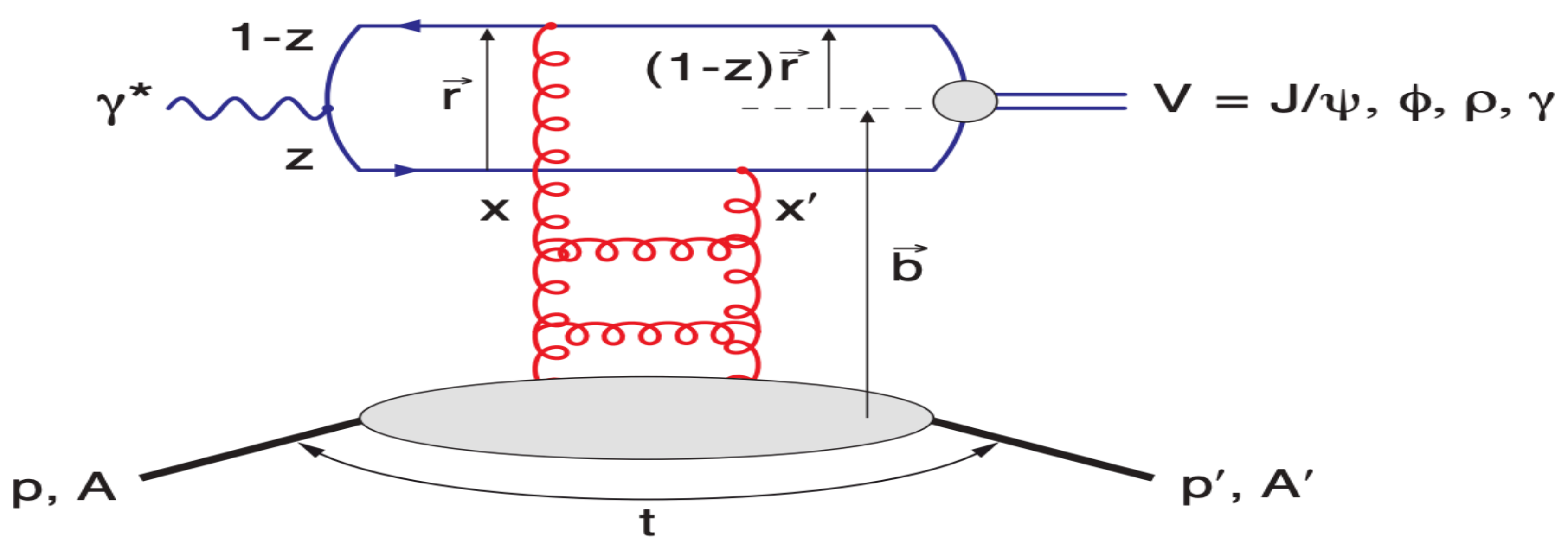
Figure from CMS (preliminary talk at QM)

- Also results for ϕ from CMS (and upcoming from STAR). CGC predictions for ϕ in UPC not very reliable due to non-perturbative effects.
- Preliminary CMS data shows more suppression for Υ than expected from CGC

- At EIC we can perform a scan on the virtuality of the photon Q^2, low Q^2 saturation regime, high Q^2 dilute regime

For heavy flavor see Jani and Anna's discussion on Thursday

Exclusive vector meson production in UPCs



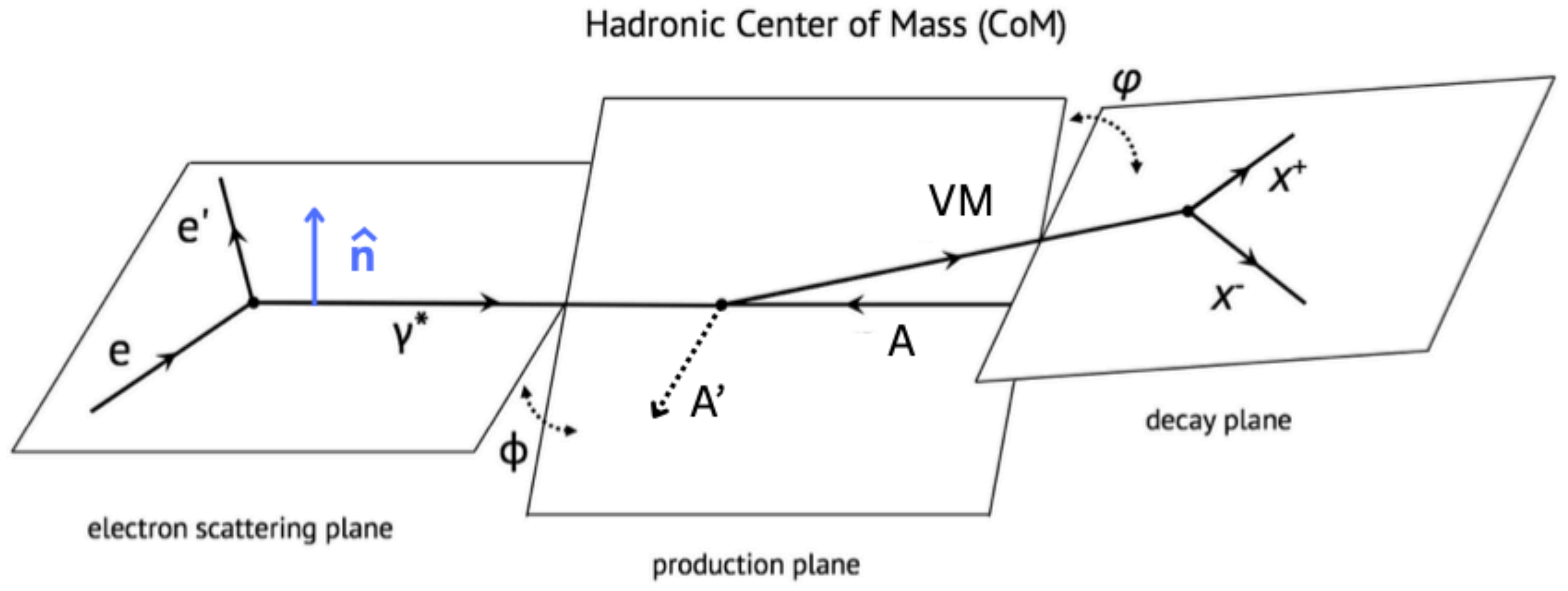
Sensitive to spatial distribution (tomography)

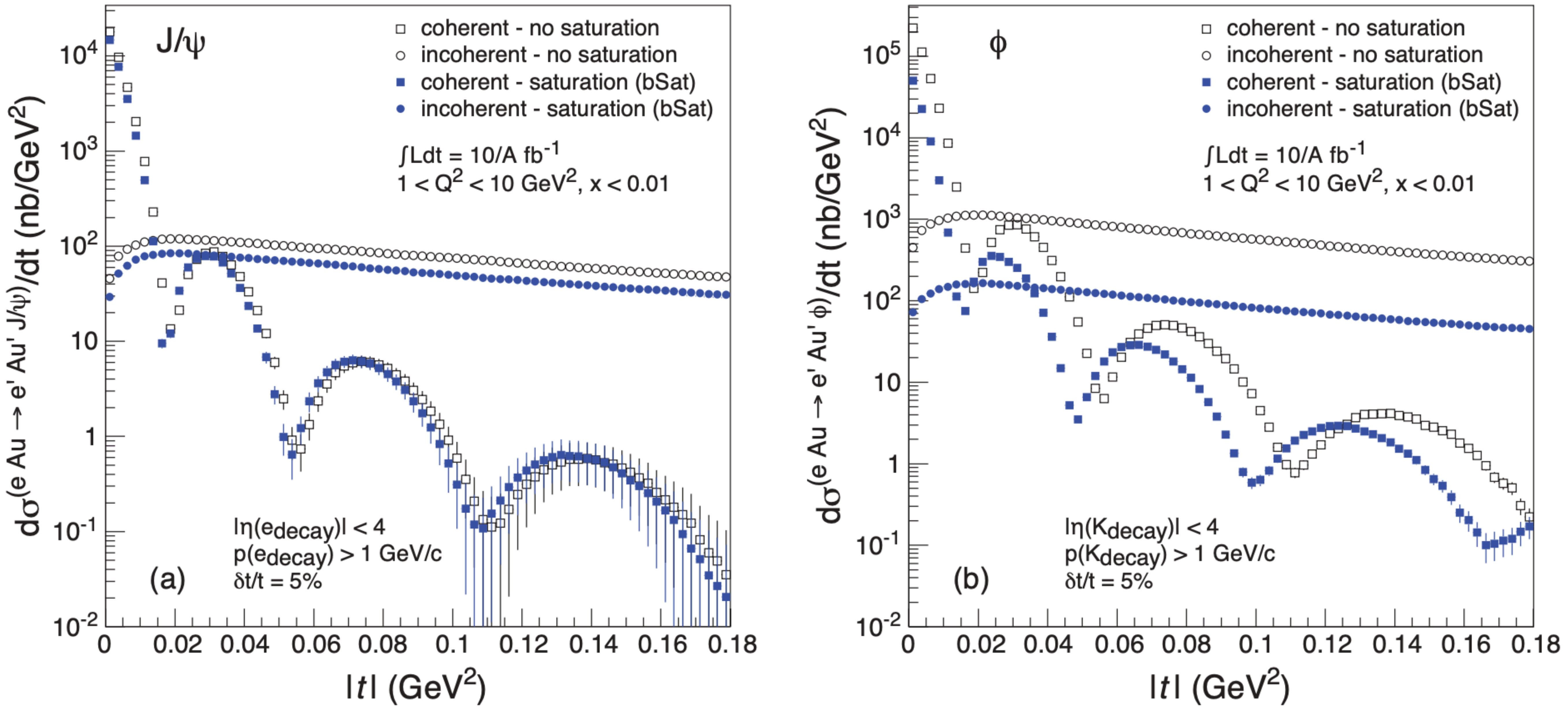
$$t = -\Delta_\perp^2$$

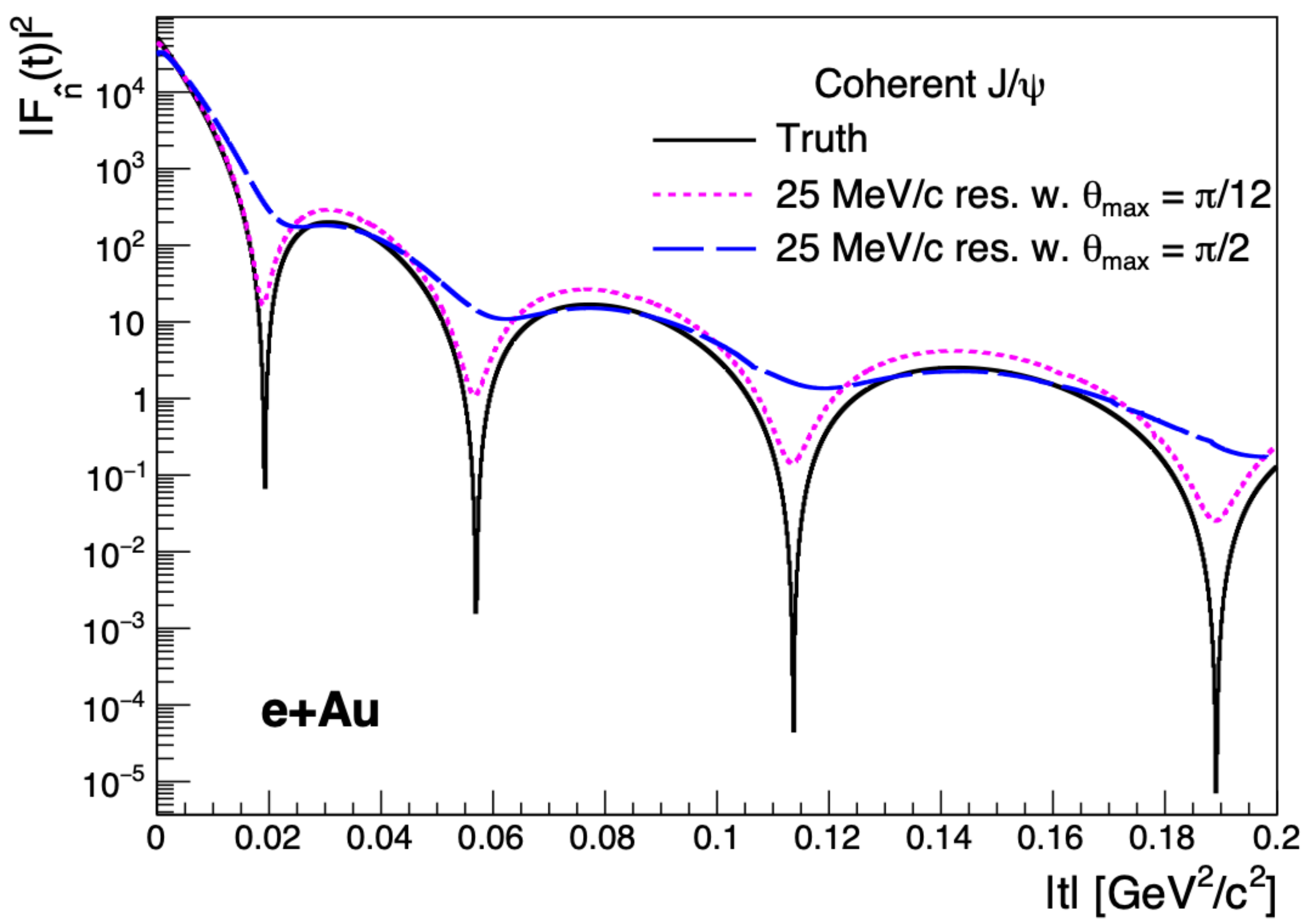
$$\Delta_\perp \leftrightarrow b_\perp$$

- Disentangle coherent from incoherent with polarized electron

Kesler, Ikbal Sheikh, Ma, Tu, Ullrich, Xu (2025)







- Sartre event generator (bSat & bNonSat = linearized bSat)
- Saturation has an imprint on the spectrum. Large difference for φ less so for J/ψ

T. Toll, T. Ullrich (2012)

Summary

- Search for gluon saturation is one of major goals of the EIC, and future upgrades to the LHC (e.g. ALICE FoCal)

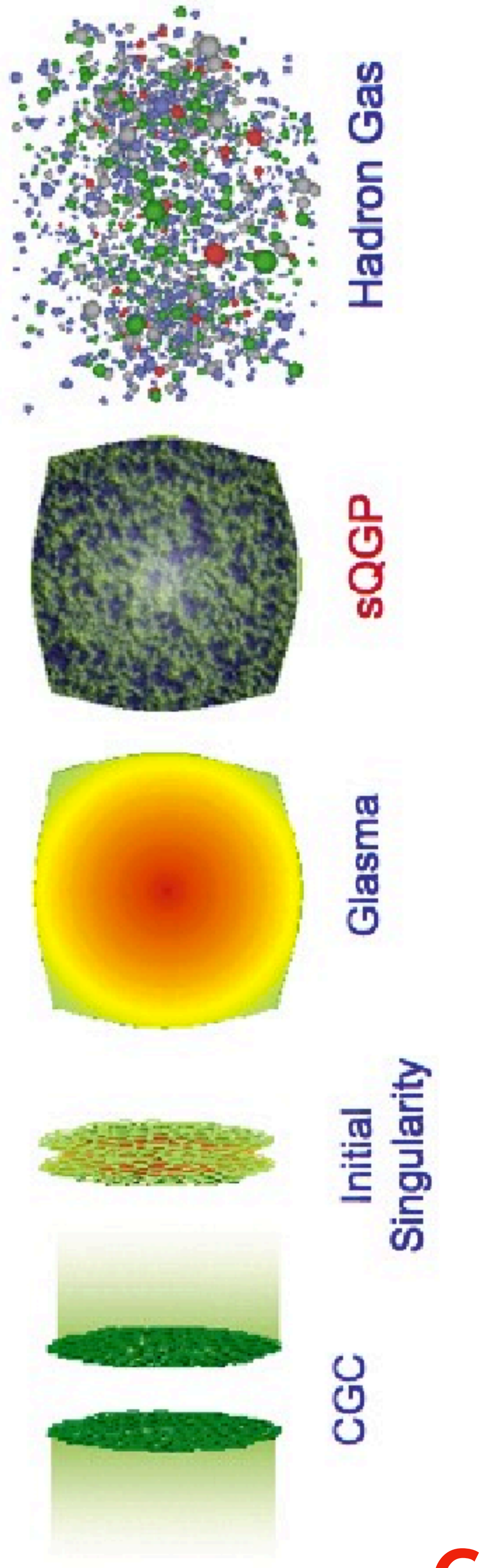
- The Color Glass Condensate is one framework that provides a potential unifying description of different observables and across different colliding systems

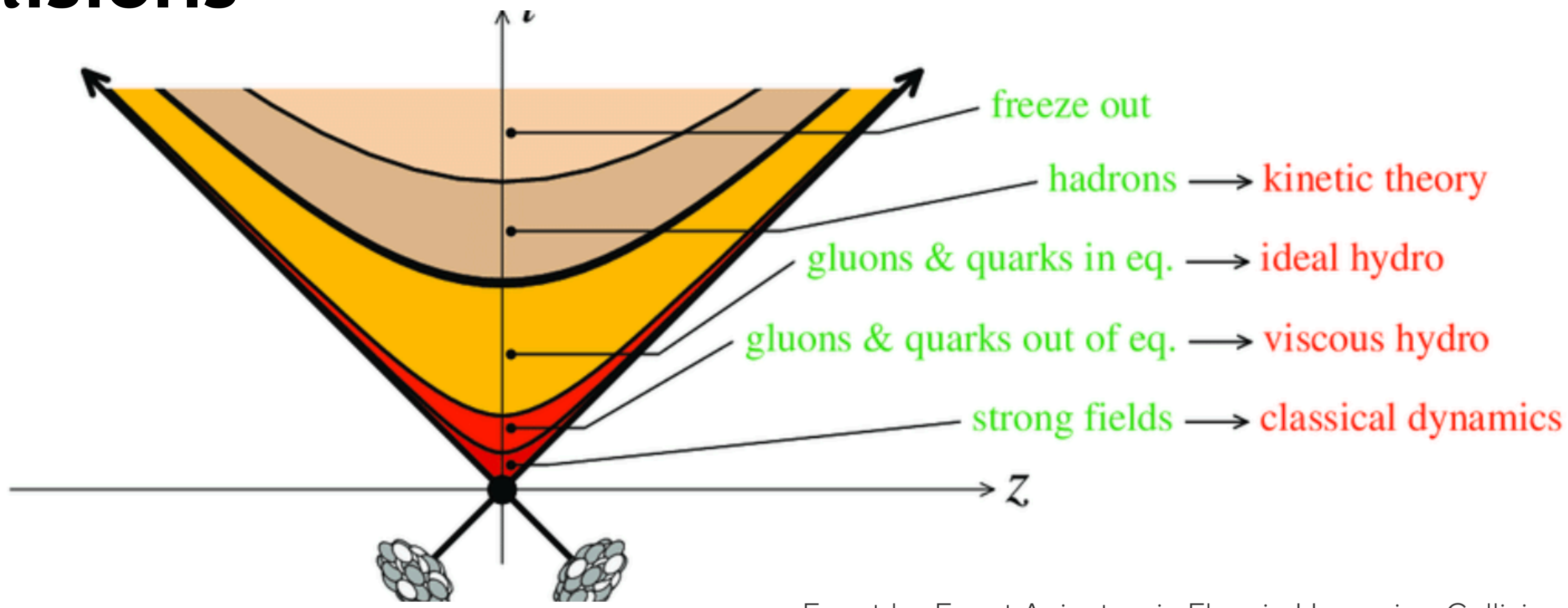
- Theory developments: precision, factorization/resummation, spin, beyond eikonal, initial conditions, novel observables, …

- Saturation leaves its signatures in inclusive, semi-inclusive and exclusive processes

Back-up slides

Heavy-ion collisions





IP-GLASMA (CGC) + MUSIC
(Hydrodynamic evolution)

Event-by-Event Anisotropic Flow in Heavy-ion Collisions from Combined Yang-Mills and Viscous Fluid Dynamics

Charles Gale, Sangyong Jeon, Björn Schenke, Prithwish Tribedy, and Raju Venugopalan
Phys. Rev. Lett. **110**, 012302 – Published 2 January 2013

Challenge: distinguish CGC initial state momentum anisotropies from final state interactions (couple to initial geometry) in observables

Novel proposals:
Giacalone, Schenke, Shen (PRL 2020)

For a comprehensive review see
Schenke Rept.Prog.Phys. 84 (2021

Interdisciplinary connections

- High-energy theory:

Scattering amplitudes in the Regge limit in N=4 SYM/ integrability

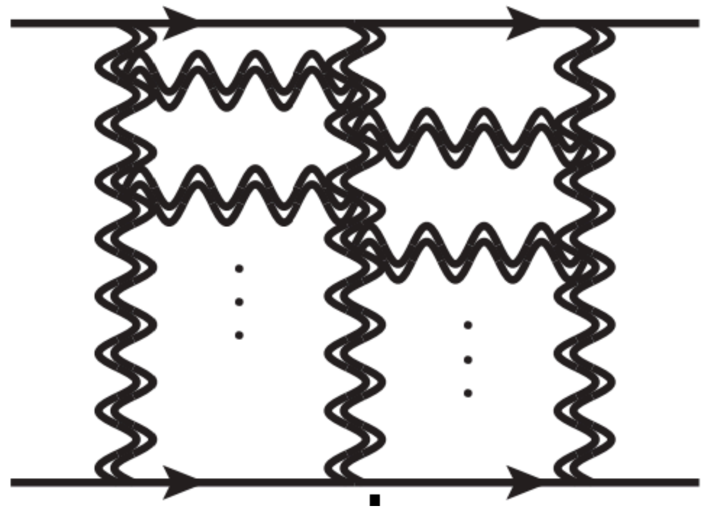


Figure from Simon Caron-Huot

- Condensed matter:

Correspondence between CGC and spin glass system

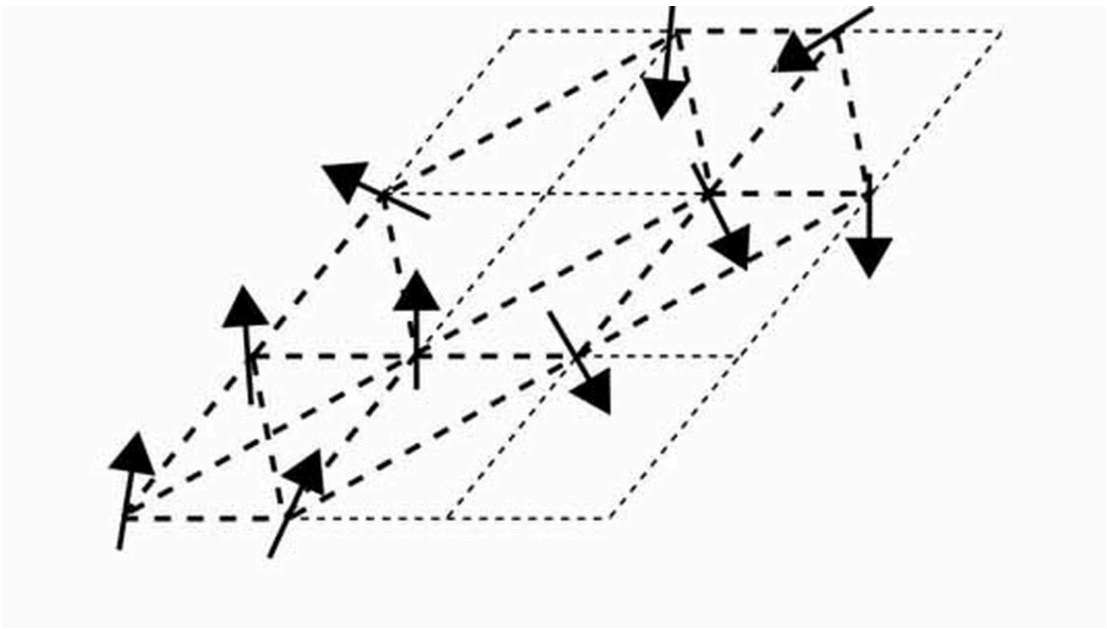
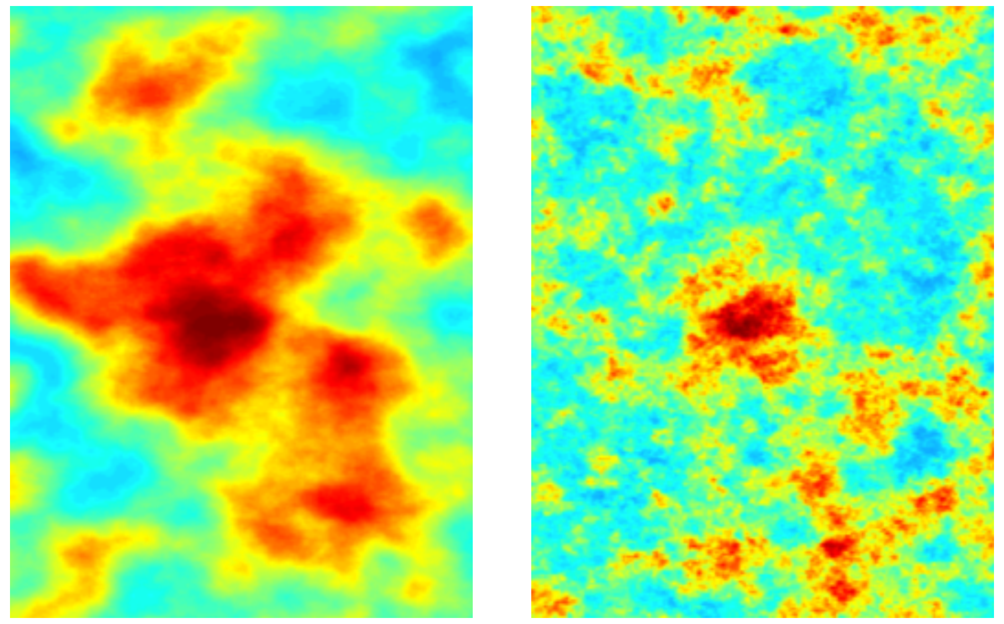


Figure from Santa Fe institute

- Statistical mechanics:

RGE evolution can be mapped to Langevin equation



- High-energy pheno:

High-energy neutrinos in cosmic rays

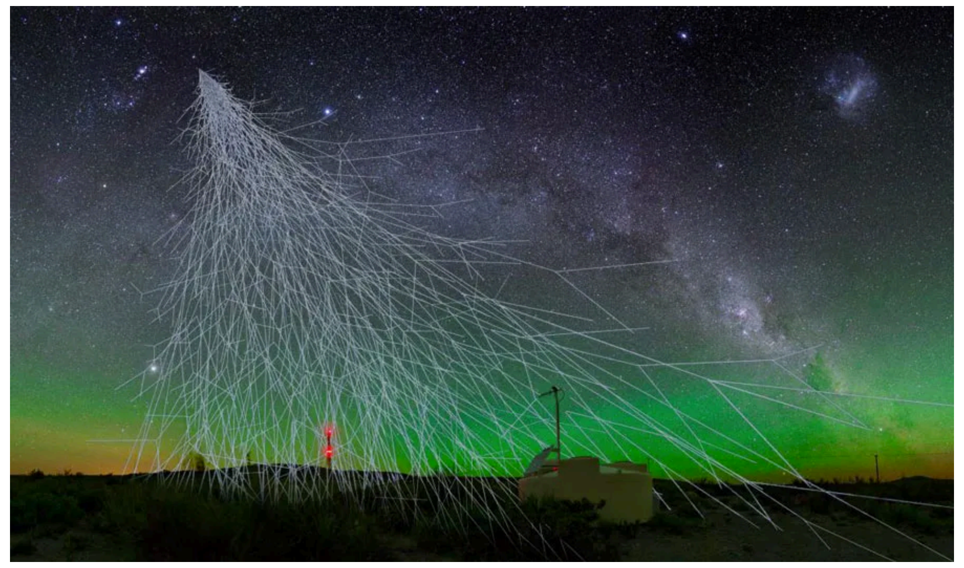


Figure from Astronomy Magazine

- Cosmology:

Gravitons also saturate, possible implications in the physics of black holes

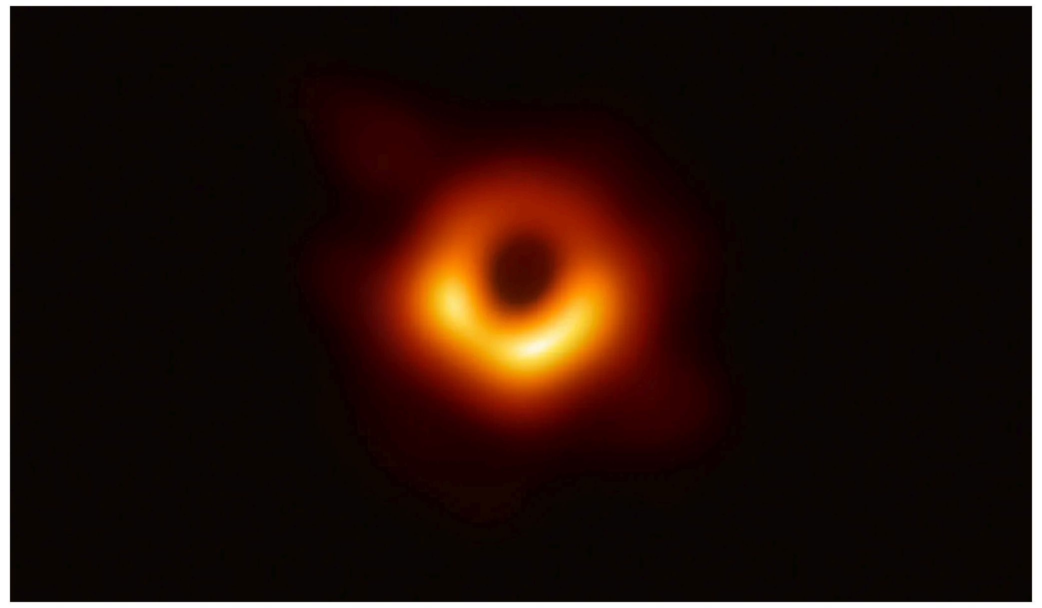


Figure from ETH collaboration