Dijet production in DIS CGC @ NLO

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Based on

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Outline

- Part I : The Color Glass Condensate
  an EFT for high parton densities
  sources & fields , Wilson lines ,
  non-linear RG evolution , the saturation scale

- Part II : Azimuthal particle correlation :
  a window to saturation
  Dijet production in DIS @ LO & NLO
  The back-to-back limit : CGC / TMD correspondence
  Joint small-x & Sudakov resummation
  Complete TMD factorization @ 1-loop
  Predictions for the Electron-Ion Collider

The Color Glass Condensate

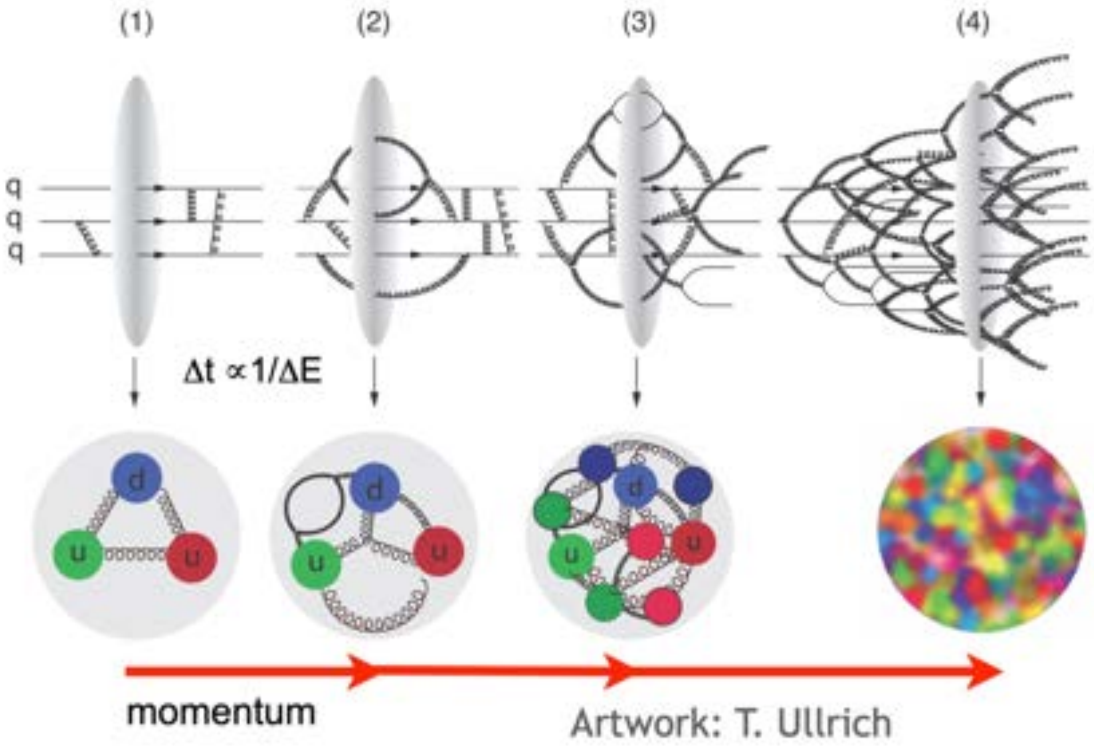
Colored Glass from Duke Chapel

The Color Glass Condensate

(4)

McLerran, Venugopalan (PRD 1994)

Sources vs fields



gluon proliferate @ small-x

⇓

large occupation number

strong fields

⇓

gluon density saturates

Separation of degrees of freedom:

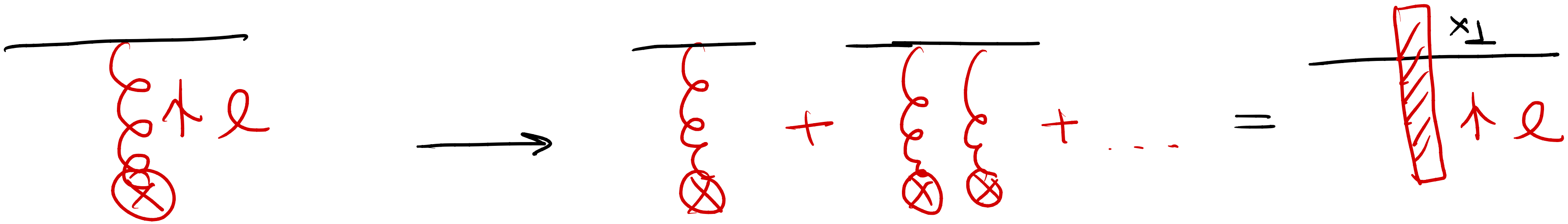
* large-x partons act as stochastic classical color sources ρ^a
* small-x partons dynamical gauge field A generated by ρ.

The Color Glass Condensate

Wilson lines

Fast moving parton propagating on background field

the "shockwave"



$ig \not{A}(l)$

in the eikonal approx
resum all scatterings

In eikonal approx

$$A^+ \gg A_\perp \gg A^-$$

$$\partial_+ A^+ = 0$$

$$A^+ \propto \delta(x^-)$$

$$2\pi\,\delta(l^-)\,\gamma^- \int e^{-i l_\perp \cdot x_\perp}\, V(x_\perp)$$

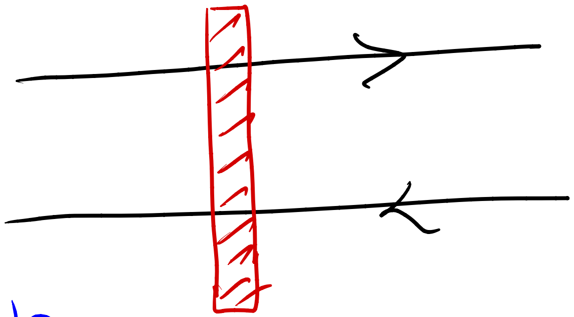
Wilson line:

$$V(x_\perp) = \underline{P}\; e^{i\int dx^- A^+(x^-, x_\perp)}$$

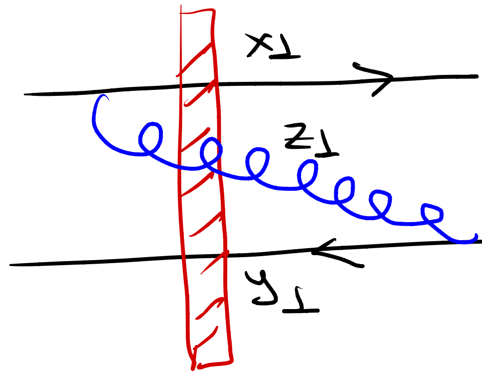
color rotation!

The Color Glass Condensate

non-linear RG evolution



energetic parton radiate $\longrightarrow$



+ other diagrams

dipole

$$\mathrm{Tr}\left[V(x_\perp)V^\dagger(y_\perp)\right]$$

$$\mathrm{Tr}\left[V(x_\perp)t^a V^\dagger(y_\perp)t^b\, U_{ab}(z_\perp)\right]$$

large N_c

double dipole

$$\mathrm{Tr}\left[V(x_\perp)V^\dagger(z_\perp)\right]\mathrm{Tr}\left[V(z_\perp)V^\dagger(y_\perp)\right]$$

- leads to non-linear RG Balitsky-Kovchegov (BK) / Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (JIMWLK) eqs.

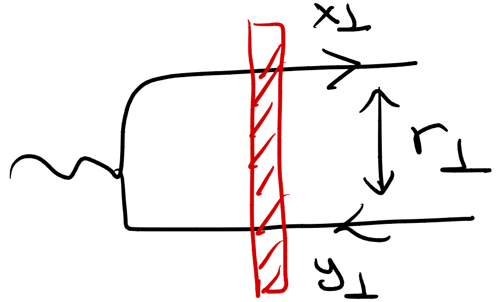
resum large energy logs

Known @ NLL

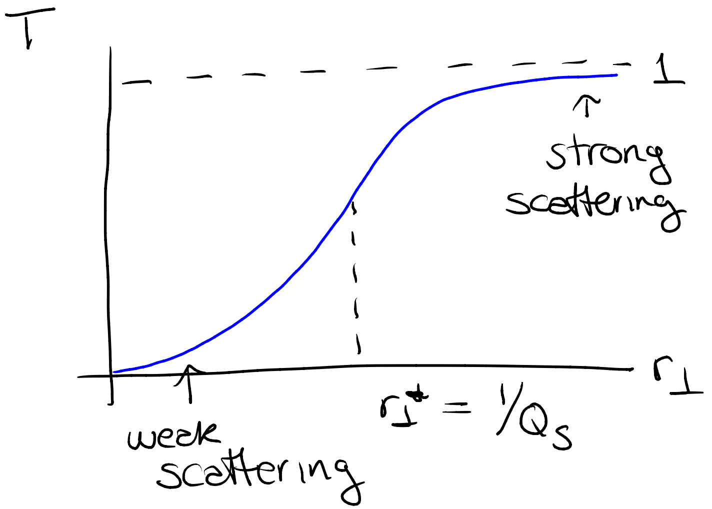
The Color Glass Condensate

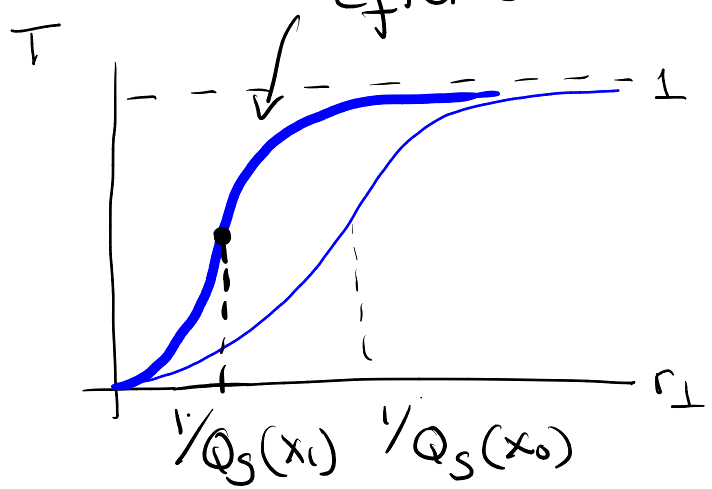
the saturation scale

$$T = 1 - \frac{1}{N_c}\mathrm{Tr}\left[V(x_\perp)V^\dagger(y_\perp)\right]$$



probability that dipole interacts with "shockwave"





$Q_s(x_1) > Q_s(x_0)$

Q_s boundary between weak & strong scattering

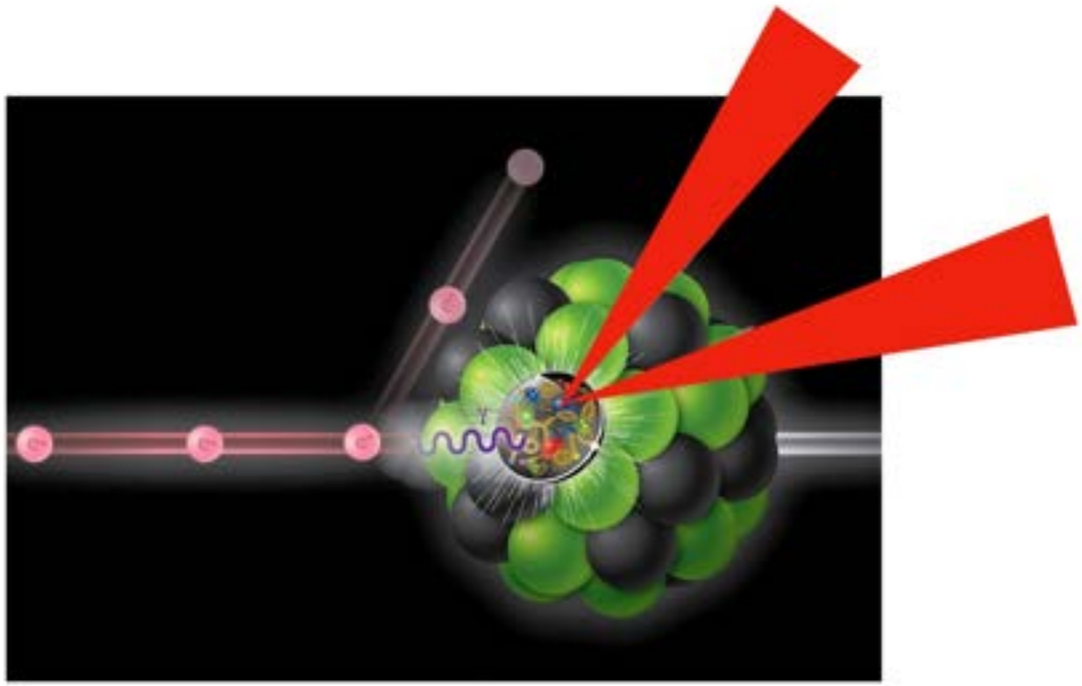
Q_s grows with energy / smaller x

$Q_s^2 \propto x^{-\lambda} A^{1/3}$

nuclear dependence

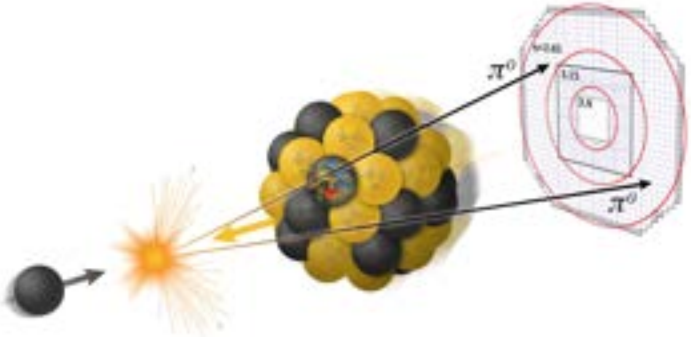
Azimuthal particle correlations

a window to saturation



Which observables are most sensitive to energy & nuclear size dependent saturation scale?

Hints of gluon saturation @ RHIC



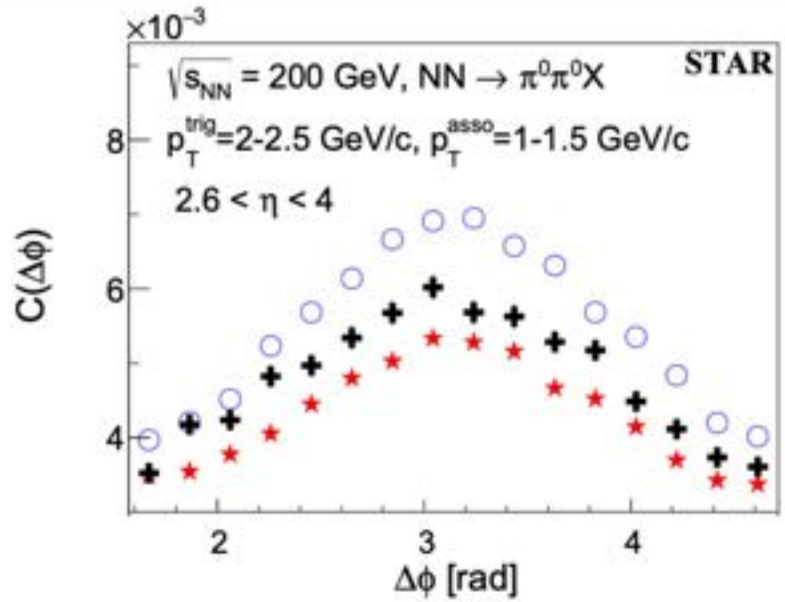
$y_1, y_2 >> 1$

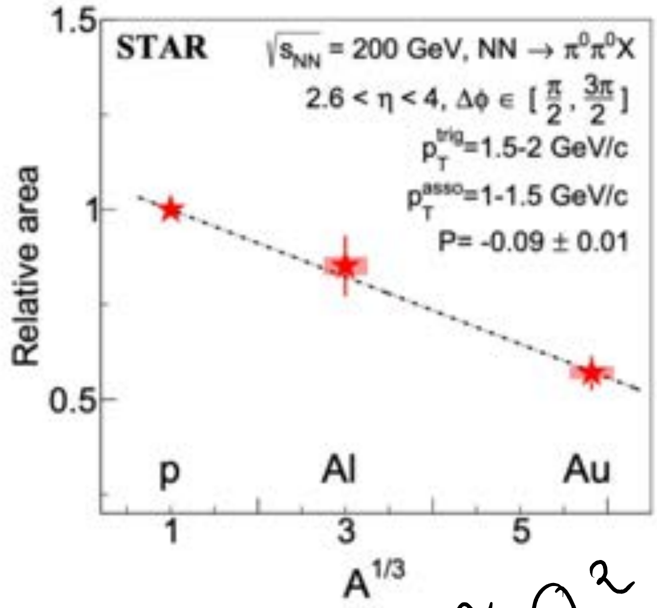
$$x_1 = 1/\sqrt{s}\,(k_{1\perp} e^{y_1} + k_{2\perp} e^{y_2}) \sim 1$$

$$x_2 = 1/\sqrt{s}\,(k_{1\perp} e^{-y_1} + k_{2\perp} e^{-y_2}) \ll 1$$

$$k_{1\perp} + k_{2\perp} \sim Q_s$$ ← larger for nuclei

↳ broadening & suppression





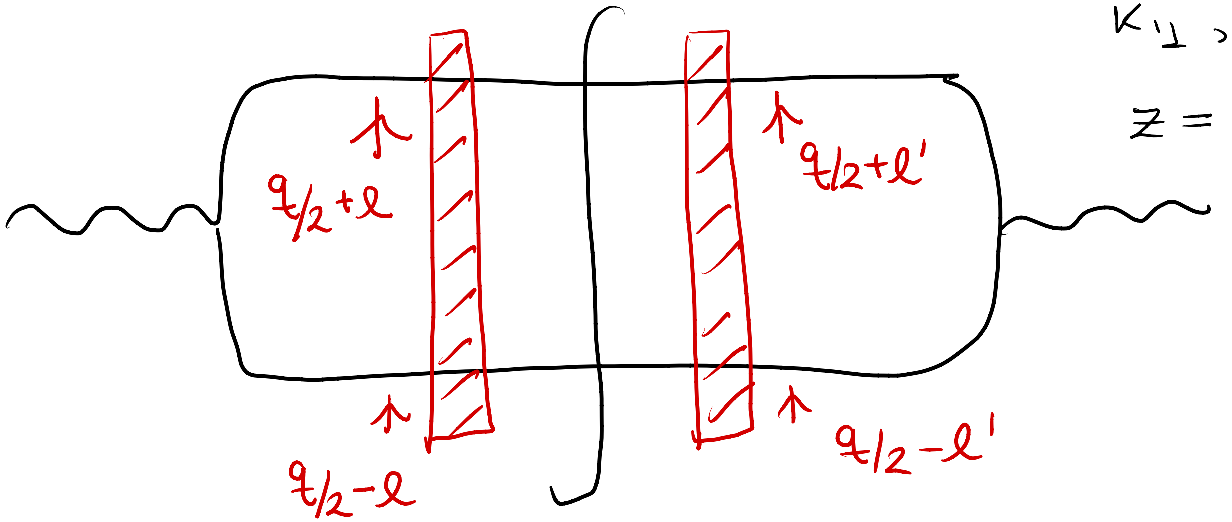
$\sim Q_s^2$

STAR Collaboration PRL 2022

Dijet production in DIS @ LO

Jalilian-Marian, Gelis (2003)

$q^\mu = (-Q^2/2q^-, q^-, 0_\perp)$



$k_{1\perp}, k_{2\perp}$ transverse momenta of $q, \bar{q}$

$z = k_1^-/q^-$, $1-z = k_2^-/q^-$ light-cone fractions

$$q_\perp = k_{1\perp} + k_{2\perp}$$

$$P_\perp \simeq \frac{1}{2}(k_{1\perp} - k_{2\perp})$$

$\gamma^* + A \to q\bar{q} + X$

$$d\sigma \sim \mathcal{H}(Q, P_\perp; l_\perp, l_\perp') \otimes_{l, l'} \mathcal{G}(q_\perp; l_\perp, l_\perp')$$

perturbatively calculable

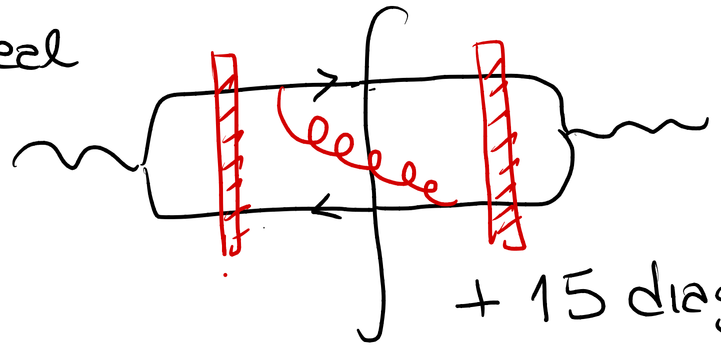
saturation scale implicit

distribution built from Wilson lines (dipole & quadrupole)

Dijet production in DIS @ NLO

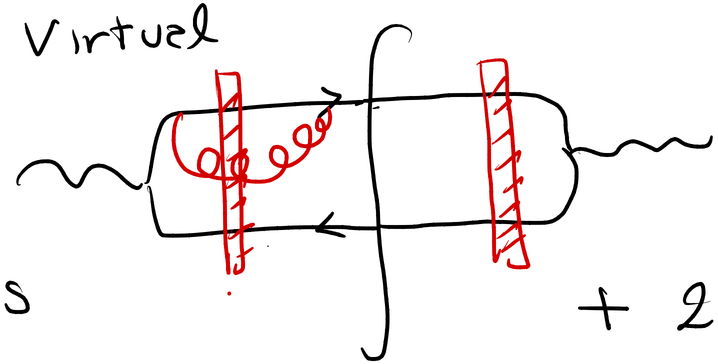
Caucal, Salazar, Venugopalan (JHEP 2021)

Real



$+ 15$ diagrams

Virtual



$+ 23$ diagrams

Regularization scheme

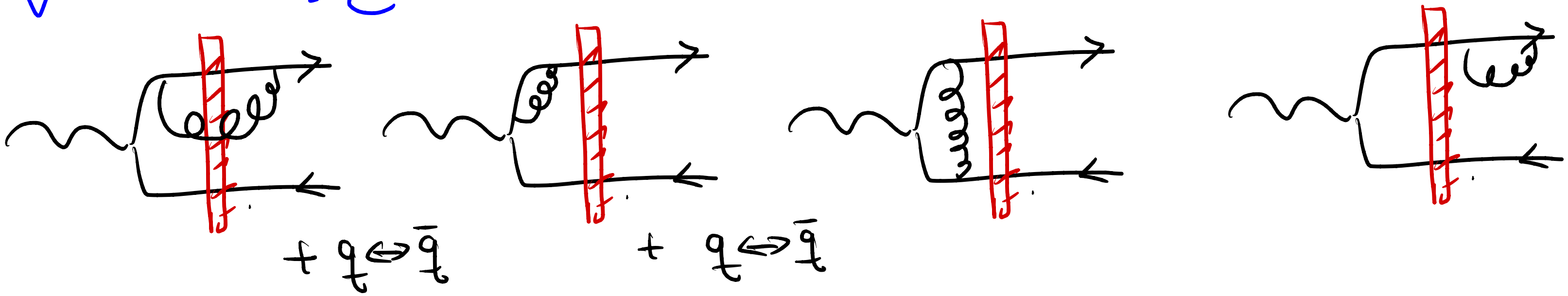
$$\int \frac{d^4 k_g}{(2\pi)^4} \rightarrow \underbrace{\int_{z_0 q^-} \frac{dk_g^-}{2\pi}}_{\text{cut-off}} \underbrace{\int \frac{dk_{g\perp}^{2-\varepsilon}}{(2\pi)^{2-\varepsilon}}}_{\text{dim-reg}} \int \frac{dk_g^+}{2\pi}$$

& Jet algorithm small cone or anti $k_\perp$

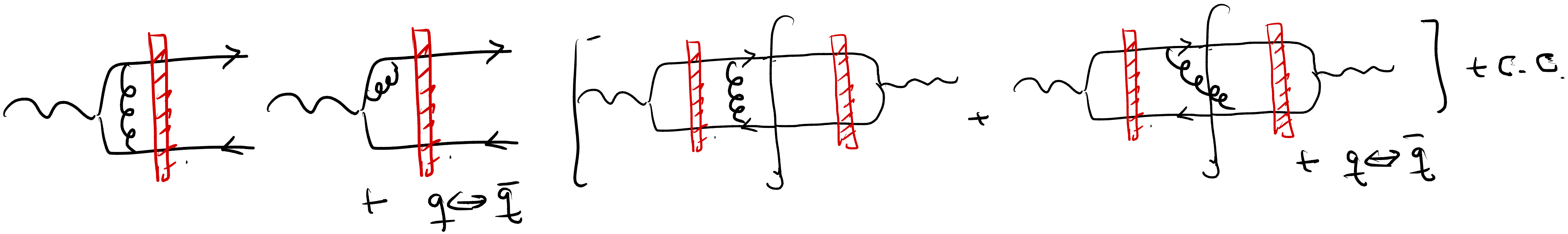
Divergences

- $\left.\begin{array}{l} k_{g\perp} \rightarrow \infty \\ r_{xz_\perp} \rightarrow 0 \end{array}\right\} \Rightarrow 1/\varepsilon$ UV
- $k_g \parallel k_1$ & $k_2 \Rightarrow 1/\varepsilon$ collinear
- $k_{g\perp} \rightarrow 0$ & $k_g^- \rightarrow 0$ $\Rightarrow \ln^2(1/z_0)$ soft
- $k_g^- \rightarrow 0$ $\quad \ln(1/z_0)$ "rapidity" absorbed into JIMWLK

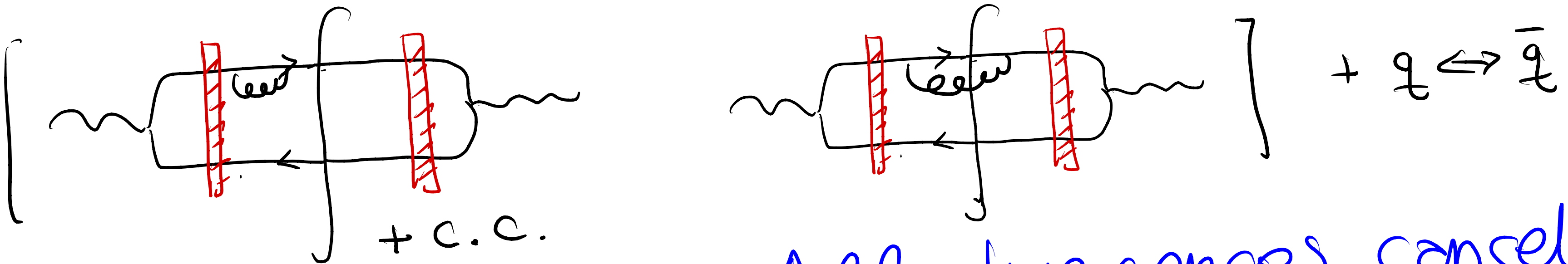
UV $1/\varepsilon$



Soft $\ln^2(1/z_0)$



Collinear $1/\varepsilon$



All divergences cancel!
except for $\ln(1/z_0)$

Rapidity divergence & impact factor

$$d\sigma_{NLO} = \int_{z_0} \frac{dz_g}{z_g}\, d\tilde{\sigma}_{NLO}(z_g)$$

$$d\sigma_{NLO} = \int_{z_0}^{z_f} \frac{dz_g}{z_g}\, d\tilde{\sigma}_{NLO}(0) + \underbrace{\int_0^{z} \frac{dz_g}{z_g} \left[d\tilde{\sigma}_{NLO} - d\tilde{\sigma}_{NLO}(0)\,\Theta(z_f - z_g) \right]}_{\text{Impact factor} \equiv d\sigma_{IF} \text{ finite!}}$$

JIMWLK factorization

$$d\tilde{\sigma}_{NLO}(0) = H_{LL}\, d\sigma_{LO} = \mathcal{H} \otimes H_{LL}\, \mathcal{G}$$

$$y \equiv \ln(z/z_0), \quad \frac{\partial \mathcal{G}}{\partial y} = H_{LL}\, \mathcal{G} \leftarrow$$

JIMWLK evolution of LO color correlator

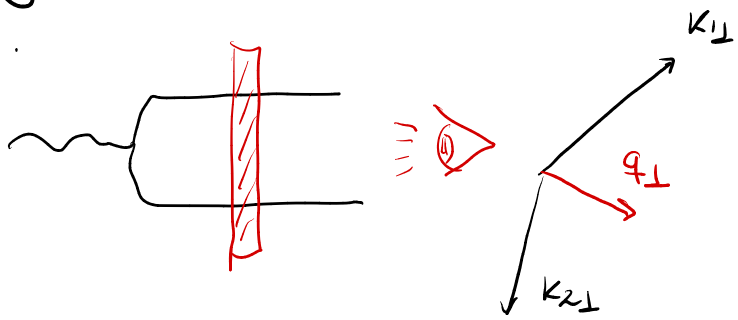
Review of back-to-back limit

Dominguez, Marquet, Xiao & Yuan (PRD 2011)

proved small-x TMD factorization @ LO

In the correlation limit ($q_\perp , Q_s \ll P_\perp$) and high-energy limit ($P_\perp \ll \sqrt{s}$)

↳ forward jets but close to back-to-back in the transverse plane



$q_\perp = k_{1\perp} + k_{2\perp}$

$P_\perp \simeq \frac{1}{2}(k_{1\perp} - k_{2\perp})$

$\simeq k_{1\perp}$

$$d\sigma \propto H_{LO}(z, Q, P_\perp)\; G(Y, q_\perp) \leftarrow$$ "TMD" built from Wilson line correlators

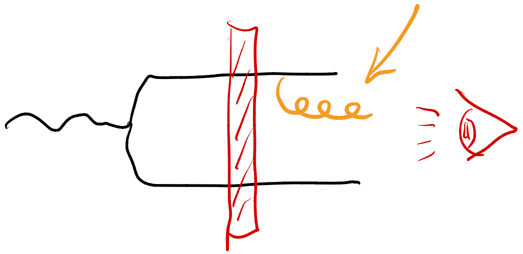
↳ saturation Q_s implicit

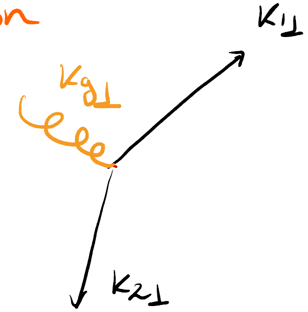
+ of phenomenology e.g.

Dihadron correlations EIC

Beyond LO: soft gluon resummation

Mueller, Xiao, Yuan (PRL 2012 & PRD 2013)





$k_{g\perp}$ generates imbalance

MXY postulated the joint resummation of small-x & double Sudakov logs

$$d\sigma \propto H_{LO}(z, Q, P_\perp) \int_{b_\perp} e^{-ib_\perp \cdot q_\perp} G(Y, b_\perp)\, e^{-S_{Sud}(P_\perp, b_\perp)}$$

G obeys small-x evolution BK-JIMWLK / DMMX eq.

DMMX: Dominguez, Mueller, Munier, Xiao (PLB 2011)

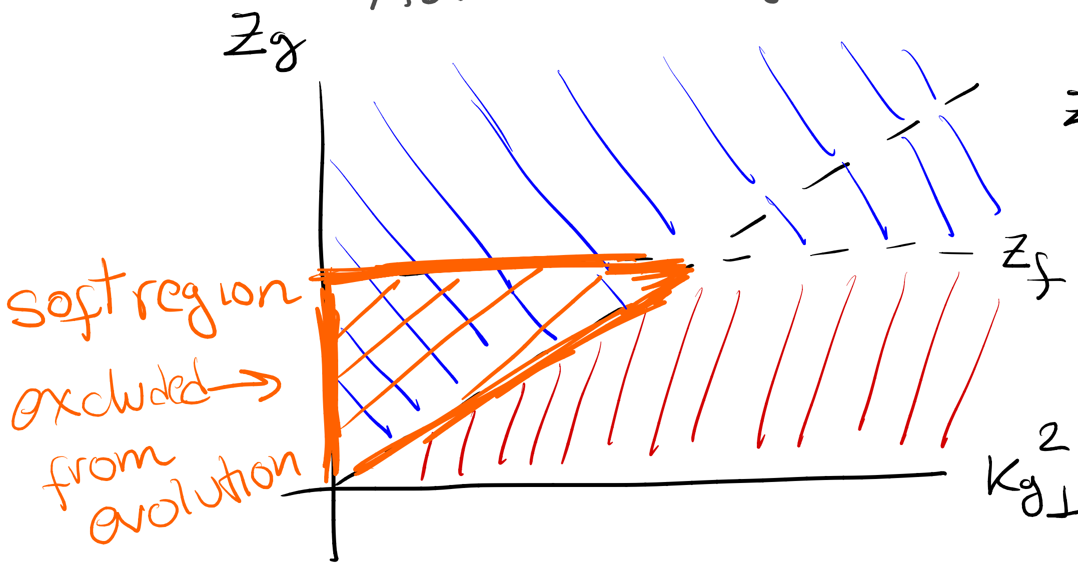
Sudakov form factor: double log

$$S_{Sud}(P_\perp, b_\perp) = \frac{\alpha_s N_c}{\pi} \int_{c_0^2/b_\perp^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \frac{1}{2} \ln\left(\frac{P_\perp^2}{\mu^2}\right)$$

"large logs incomplete cancellation between virtual & real emissions"

The need for the kinematic constraint

Caucal, Salazar, Schenke, Venugopalan (JHEP 2022)
Altinoluk, Beuf, Marquet, Taels (JHEP 2022) photoproduction



$$z_g = \frac{z_f}{Q_f^2} k_{g\perp}^2 \Longrightarrow z_g < \frac{z_f}{Q_f^2} k_{g\perp}^2$$

$$\Updownarrow$$

$$k_f^+ < k_g^+$$

order in lifetime

kinematic constraint

$$d\sigma_{NLO} = \int_{z_0}^{z_f} \frac{dz_g}{z_g} d\tilde{\sigma}_{NLO}(0)\, \Theta\left(\frac{z_f}{z_g Q_f^2} - r_<^2\right) \leftarrow (*) \qquad k_{g\perp}^2 \sim 1/r_<^2$$

$$+ \underbrace{\int_0^{z} \frac{dz_g}{z_g}\left[d\tilde{\sigma}_{NLO} - d\tilde{\sigma}_{NLO}(0)\, \Theta(z_f - z_g)\, \Theta\left(\frac{z_f}{z_g Q_f^2} - r_<^2\right)\right]}_{\text{"correct impact factor } d\sigma_{IF}\text{"}}$$

Small-x evolution in the back-to-back limit

$$\underbrace{\int_{z_0}^{z_f} \frac{dz_g}{z_g} H_{LL}\,\Theta\!\left(\frac{z_f}{z_g Q_f^2} - r_<^2\right)}\, G_{WW}$$

Kinematically consistent LL evolution eq for WW TMD

What kind of logs are we resumming?

$$z_0 < z_g < \frac{z_f}{Q_f^2 r_<^2} \longleftrightarrow \frac{1}{M_{dijet}^2 r_<^2}$$

What's z_0? Minimum light-cone fraction

$$z_0 = \frac{k_{g\,min}^-}{q^-} = \frac{k_{g\perp}^2}{2k_{g\,max}^+ q^-} = \frac{k_{g\perp}^2}{2P^+ q^-} \propto \frac{1}{s r_<^2}$$

Phase space

$$\frac{1}{r_<^2 s} < z_g < \frac{1}{M^2 r_<^2} \Longrightarrow \underbrace{\ln\left(s/M^2\right)}_{\text{small-}x\ \log}$$

Impact factor in the back-to-back limit

Caucal, Salazar, Schenke, Stebel, Venugopalan (JHEP 2023)

$$d\sigma_{IF} \to H_{LO+NLO}(z, Q, P_\perp, z_f) \int_{b_\perp} e^{-ib_\perp \cdot q_\perp} G_{WW}(b_\perp) S_{Sud}(P_\perp, b_\perp)$$

fully analytic absorbs finite pieces not log enhanced

Sudakov double & single logs

$$S_{Sud}(P_\perp, b_\perp) = -\int_{c_0^2/b_\perp^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s N_c}{\pi} \Big[\underbrace{\frac{1}{2}\ln\left(\frac{P_\perp^2}{\mu^2}\right) + \frac{C_F}{N_c}\ln\left(\frac{1}{z_1 z_2 R}\right) - \frac{\pi\beta_0}{N_c} + \ln\left(\frac{P_\perp^2 + \bar{Q}^2}{P_\perp^2}\right)}_{\text{Same as in CSS see e.g. HXYZ 2021}} \underbrace{-1}_{\text{discrepancy!}} \Big]$$

Small-x TMD factorization @ NLO

Combining evolution & impact factor & assuming Sudakov logs exponentiate

Sudakov factor

$$d\sigma \propto \underbrace{\mathcal{H}_{LO+NLO}(z, Q, P_\perp; z_f)}_{\text{hard function}} \int_{b_\perp} e^{-ib_\perp \cdot q_\perp} \underbrace{G_{WW}(Y_f, b_\perp)}_{\text{TMD/UGD}} \, e^{-S_{Sud}(P_\perp, b_\perp)}$$

TMD/UGD satisfies K.C DMMX eq in rapidity Y

$Y_0 = \ln(z_0)$
$\downarrow$
$Y_f = \ln(z_f)$

$$S_{Sud}(P_\perp, b_\perp) = -\int_{c_0^2/b_\perp^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s N_c}{\pi} \left[\underbrace{\frac{1}{2}\ln\left(\frac{P_\perp^2}{\mu^2}\right) + \frac{C_F}{N_c}\ln\left(\frac{1}{z_1 z_2 R}\right) - \frac{\pi \beta_0}{N_c} + \ln\left(\frac{P_\perp^2 + \bar{Q}^2}{P_\perp^2}\right)}_{\text{Same as in CSS}} - 1 \right]$$

Same as in CSS see eg. HXYZ 2021

discrepancy 1

Small-x TMD factorization @ NLO

From $Y_f = \ln(z_f/z_0)$ to $\eta_f = \ln(x_0/x_f)$ evolution

↳ evolution equation becomes non-local

$$d\sigma \propto \mathcal{H}_{LO+NLO}(z, Q, P_\perp) \int_{b_\perp} e^{-ib_\perp \cdot q_\perp} G_{WW}(\eta_f, b_\perp)\, e^{S_{Sud}(P_\perp, b_\perp)}$$

↑ satisfies K.C non-local DMMX eq

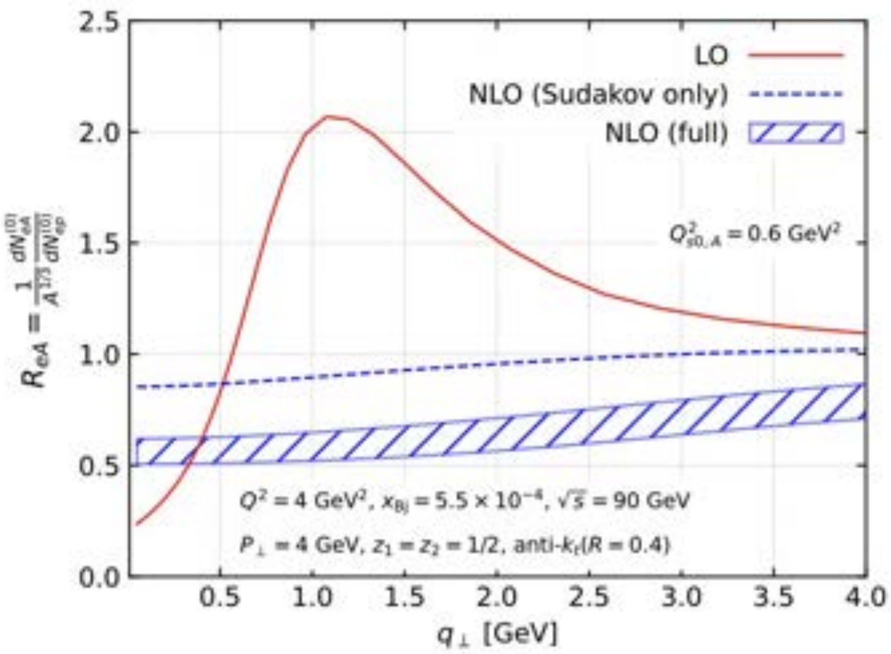
$\eta_0 = 0$ to $\eta_f = \ln(x_0/x_f)$

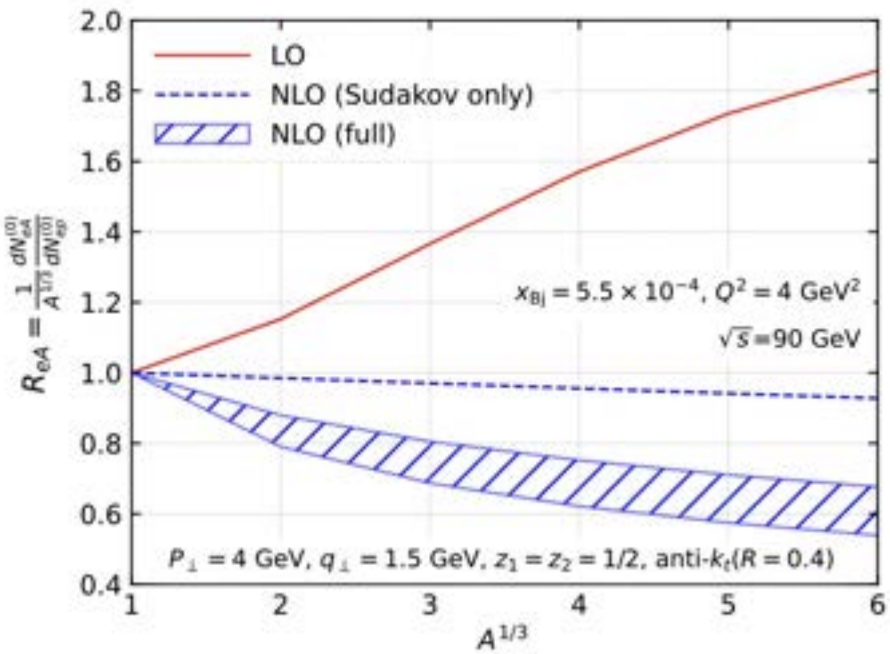
analogous to Dudoue et al (JHEP 2019)

$$S_{Sud}(P_\perp, b_\perp) = -\int_{c_0^2/b_\perp^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2) N_c}{\pi} \underbrace{\left[\frac{1}{2}\ln\left(P_\perp^2/\mu^2\right) + \frac{C_F}{N_c}\ln\left(\frac{1}{z_1 z_2 R}\right) - \frac{\pi\beta_0}{N_c} + \ln\left(\frac{P_\perp^2 + \bar{Q}^2}{P_\perp^2}\right)\right]}_{\text{Same as in CSS}}$$

see e.g. HXYZ 2021

Nuclear modification Ratio





$$R_{eA} = \frac{1}{A^{1/3}} \frac{dN_{eA}}{dN_{ep}}$$, dN min-bias is the yield

* LO result
  Shows suppression & Cronin peak (broadening)

* NLO (Sudakov only)
  Shows a slight suppression but close to unity

* Full NLO (Sud + small-x evol)
  Show significant A-dependent suppression!

Caucal, Salazar, Schenke, Stebel, Venugopalan (preprint)

Summary

- CGC is an effective theory to describe the physics of high parton density @ small-x
- Azimuthal correlation are a promising tool to search for signatures of gluon saturation
- We performed a complete NLO calculation for dijet production in DIS within the CGC
- We showed TMD factorization (1-loop) at small-x holds when jets are back-to-back
- Provided numerical predictions for EIC

Back up Slides

The missing "β_0" Sudakov single log

Simple obs : in the back-to-back limit one has.

$$\Xi \longrightarrow r_\perp^i r_\perp^j \; \alpha_s \, G_{WW}^{ij}(b_\perp)$$

but @ what scale μ_0^2 run α_s & G_{WW}?

Let $\mu_0^2 = c_0^2/b_\perp^2$ ← intrinsic scale of TMD

exact

$$\alpha_s(\mu_0^2) \, G_{WW}(b_\perp, \mu_0^2)$$

but α_s typically runs @ hard scale $P_\perp$

$$= \alpha_s(P_\perp^2) \underbrace{e^{-\int_{\mu_0^2}^{P_\perp^2} d\mu^2/\mu^2 \, \beta_0(\alpha_s(\mu^2))}}_{\beta_0 \text{ Sudakov log!}} G_{WW}(b_\perp, \mu_0^2)$$

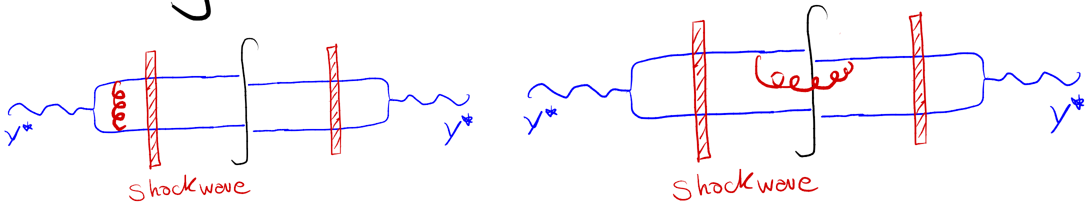
Factorization of finite pieces into WW

finite pieces:

(I) diagrams gluon does not interact with shock-wave give terms proportional to WW

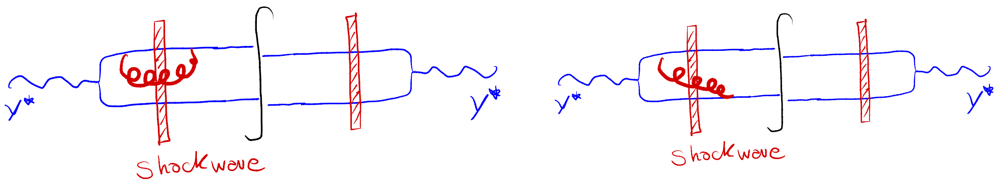
↳ trivially (as LO case)

e.g



⇒ same color correlator as LO

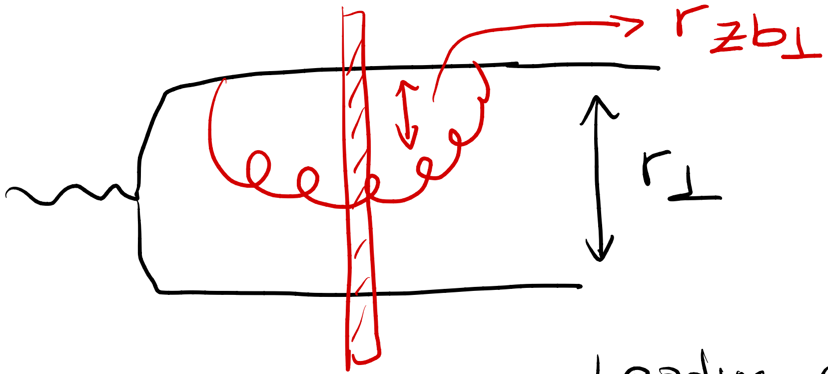
(II) diagrams gluon interacts with the shockwave ⇒ additional $z_\perp$ coordinat



⇒ more complex color correlators involving $z_\perp$ coordinate

usual correlation limit $r_\perp \ll b_\perp$ ($q\bar{q}$ size). What about $r_{zb\perp}$? (qg size)

Factorization of finite pieces into WW



$r_\perp \sim 1/P_\perp$ fourier transform

Explicit expansion in $r_{zb\perp}$ comes with powers $1/P_\perp$!

Leading contribution involves only WW distrib.

Physically, gluon radiation is dominated by hard emissions

$$1/r_{zb\perp} \sim k_{g\perp} \sim P_\perp$$

Real emission crossing SW does not contribute @ Leading Power (LP)

↳ All finite corrections @ LP can be absorbed by defining new NLO hard factor H_{NLO}

* By product : fixes $\frac{z_f}{Q_f^2} = \frac{e\, c_0^2\, z_1 z_2}{P_\perp^2 + \bar{Q}^2}$ in order to have convergent LP

consistent with our expectation $\propto 1/M^2_{dijet}$

↳ $S_f = 1 - \ln\left[\frac{P_\perp^2 + \bar{Q}^2}{P_\perp^2}\right]$

consistent with HXYZ 2021

Y to η evolution

Let's assume we can use the Gaussian approximation $\quad \mu_\perp^2 = \dfrac{M_{q\bar{q}}^2 + Q^2}{c_0^2 e}$

For BK our kinematic constraint implies $\quad r_<^2 \equiv \min(r_{zb'}^2, r_{zb}^2)$

$$\frac{\partial S(r_{bb'})}{\partial Y} = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 z_\perp}{(2\pi)} \Theta\left(-Y - \ln(r_<^2 \mu_\perp^2)\right) \frac{r_{bb'}^2}{r_{zb}^2 r_{zb'}^2} \left[S_Y(r_{zb}) S(r_{zb'}) - S_Y(r_{bb'}) \right]$$

Can be cast as

$$\frac{\partial \mathcal{S}(r_{bb'})}{\partial \eta} = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 z_\perp}{(2\pi)} \Theta\left(\eta - \ln\left(\frac{r_{bb'}^2}{r_<^2}\right)\right) \frac{r_{bb'}^2}{r_{zb}^2 r_{zb'}^2}$$

Same eq as Duclové, Iancu, Mueller, Soyez, Triantafyllopoulos (JHEP 2019) $\rightarrow$

$$\left[\mathcal{S}_{\eta - \ln(r_{bb'}^2/r_{zb}^2)}(r_{zb}) \, \mathcal{S}_{\eta - \ln(r_{bb'}^2/r_{zb'}^2)}(r_{zb'}) - \mathcal{S}_\eta(r_{bb'}) \right]$$

non-local equation

Relation between Y & η evolution: $\quad S_{Y_f} = \mathcal{S}_{\eta_f} \quad$ with $\quad \eta = Y_f + \ln(\mu_\perp^2 r_{bb'}^2) + \ln(1/x_f)$

Note when $Y_f = -\ln(r_{bb'}^2 \mu_\perp^2) \rightarrow \eta_f = \ln\left[\dfrac{x_0}{x_f}\right], \; x_f = \dfrac{\mu_\perp^2}{s} \leftarrow$ natural choice

$\ln(z_f)$

Will cancel Sudakov single log -1 !

Numerical Results