

Nuclear shapes from self-consistent mean-field and beyond approaches: deformation of $A=96$ isobars

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INT Program 23-1a

Intersection of nuclear structure and high-energy nuclear collisions

Seattle February 1st, 2023



1. Introduction

2. Self-consistent mean-field: HFB

3. Beyond self-consistent mean-field: symmetry restoration and configuration mixing

4. PGCM for ^{96}Ru and ^{96}Zr

3.1. Triaxial quadrupole deformation

3.2. Axial quadrupole-octupole deformation

3.3. What is missing?

Introduction

- The **intrinsic shape** of the nuclear states is **not a direct observable**, but...
- Intrinsic nuclear shapes can be inferred from the experimental data (energies and electromagnetic moments and transitions) by comparison with the predictions given by geometrical (simple) models.

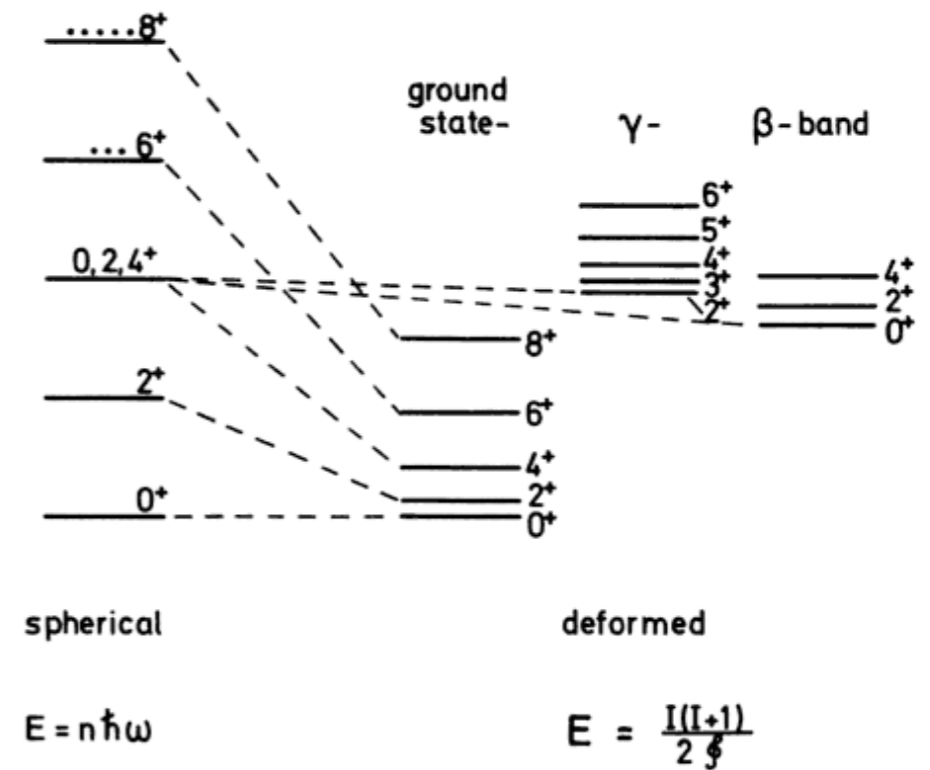
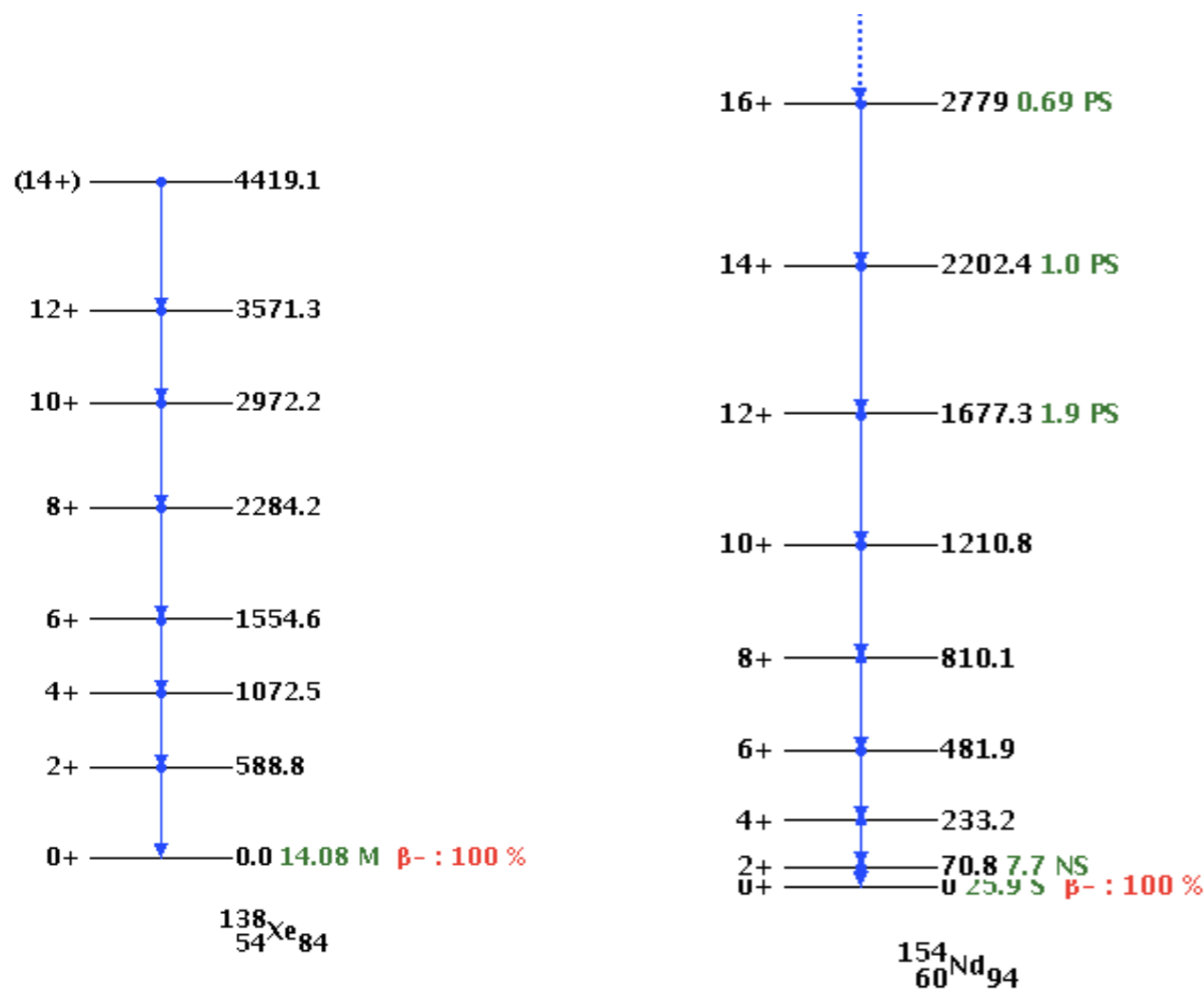


Figure 1.12. Schematic level schemes of spherical and deformed nuclei. (From [SDG 76].)

P. Ring and P. Schuck, *The Nuclear Many-Body Problem*

Introduction

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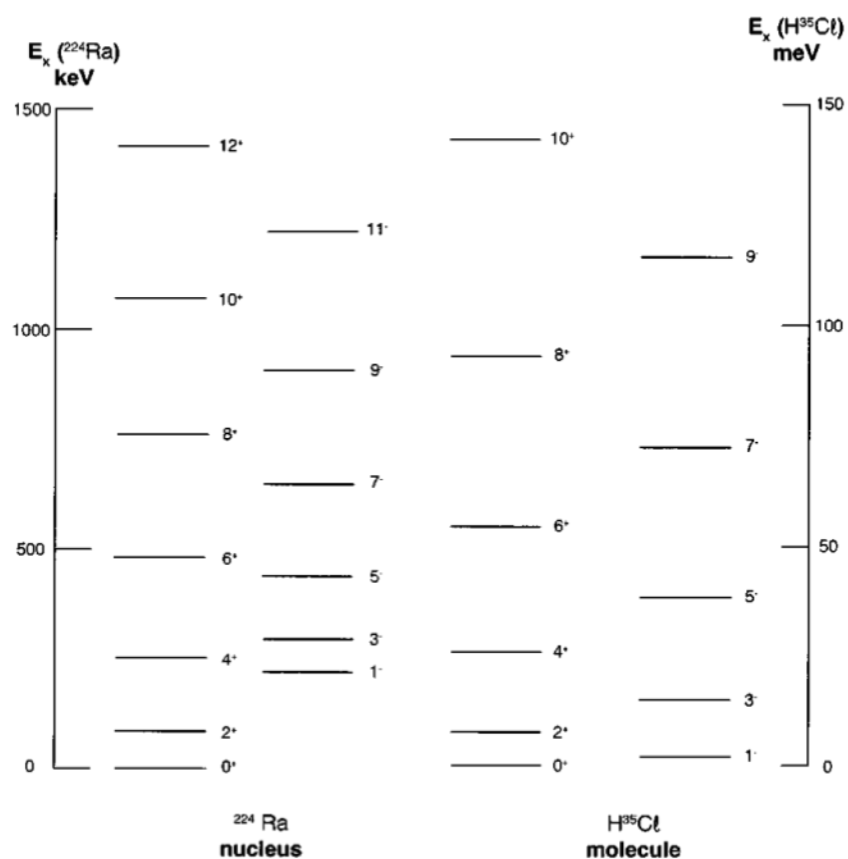
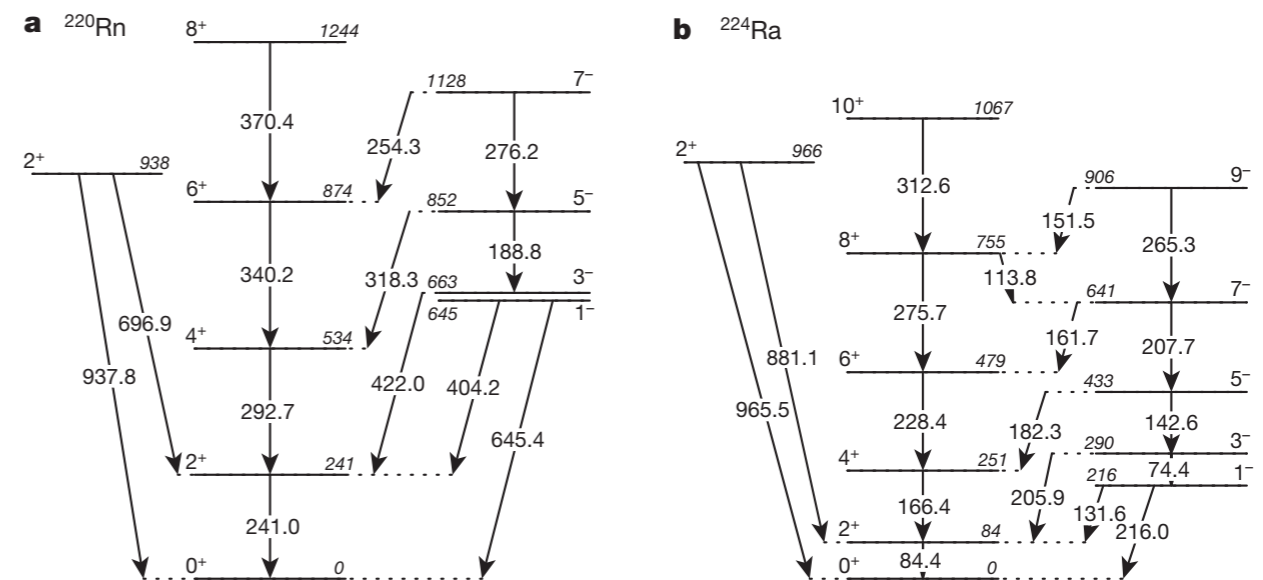


FIG. 1. The low-lying rotational spectra of ^{224}Ra , compared with that of the H^{35}Cl molecule. The spectrum of ^{224}Ra is taken from Poynter *et al.* (1989a). The rotational constants for the H^{35}Cl molecule are taken from Landolt-Börnstein (1974).

Positive and negative parity interleaved bands as rotational states of octupole shapes



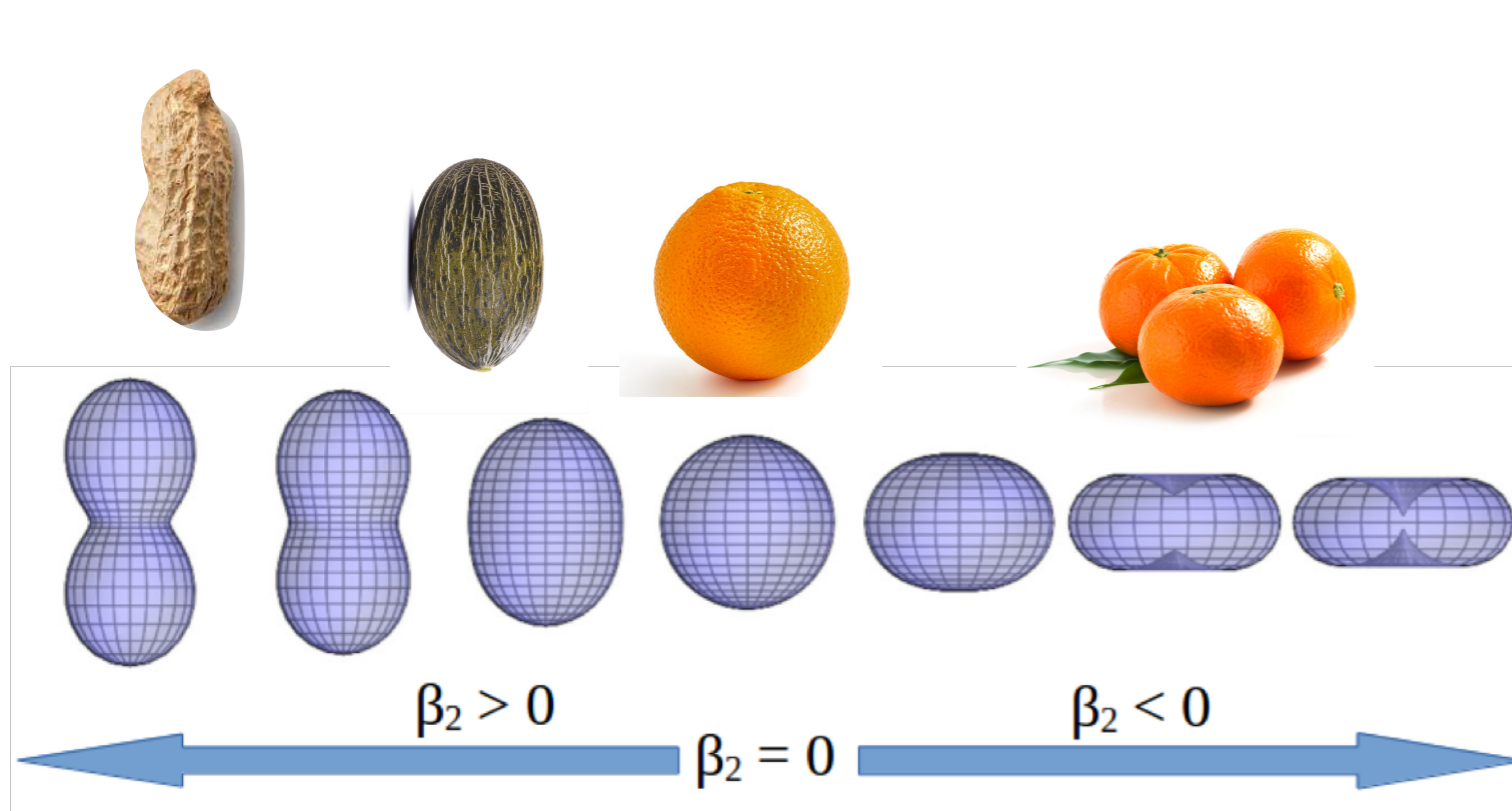
L.. P. Gaffney et al., Nature 497, 199 (2013)

P. A. Butler, W. Nazarewicz, Rev. Mod. Phys. 68, 349 (1996)

Introduction

- **Collective models** are based on the parametrization of the nuclear radius with a multipole expansion

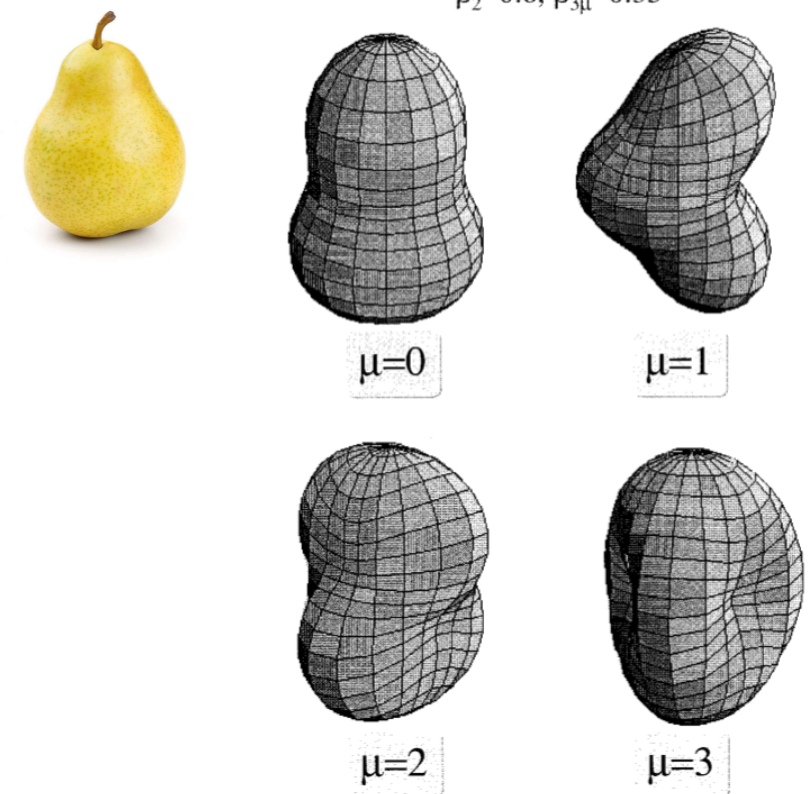
$$R(\Omega) = c(\alpha)R_0 \left[1 + \sum_{\lambda=2}^{\lambda_{\max}} \sum_{\mu=-\lambda}^{+\lambda} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\Omega) \right]$$



axial quadrupole shapes

Quadrupole-octupole shapes

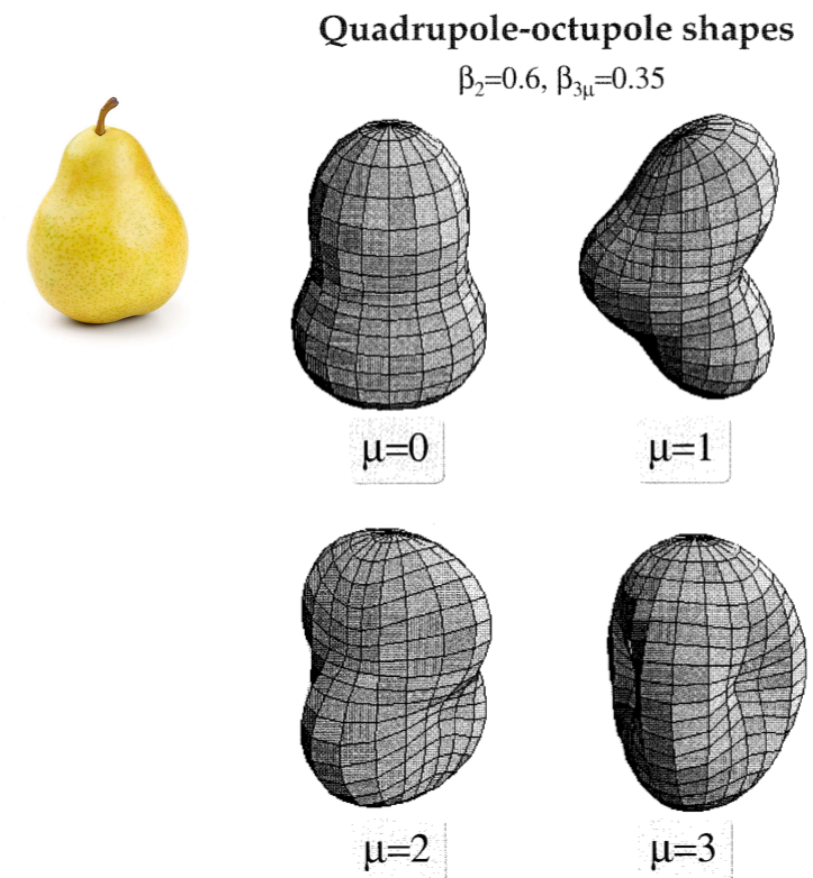
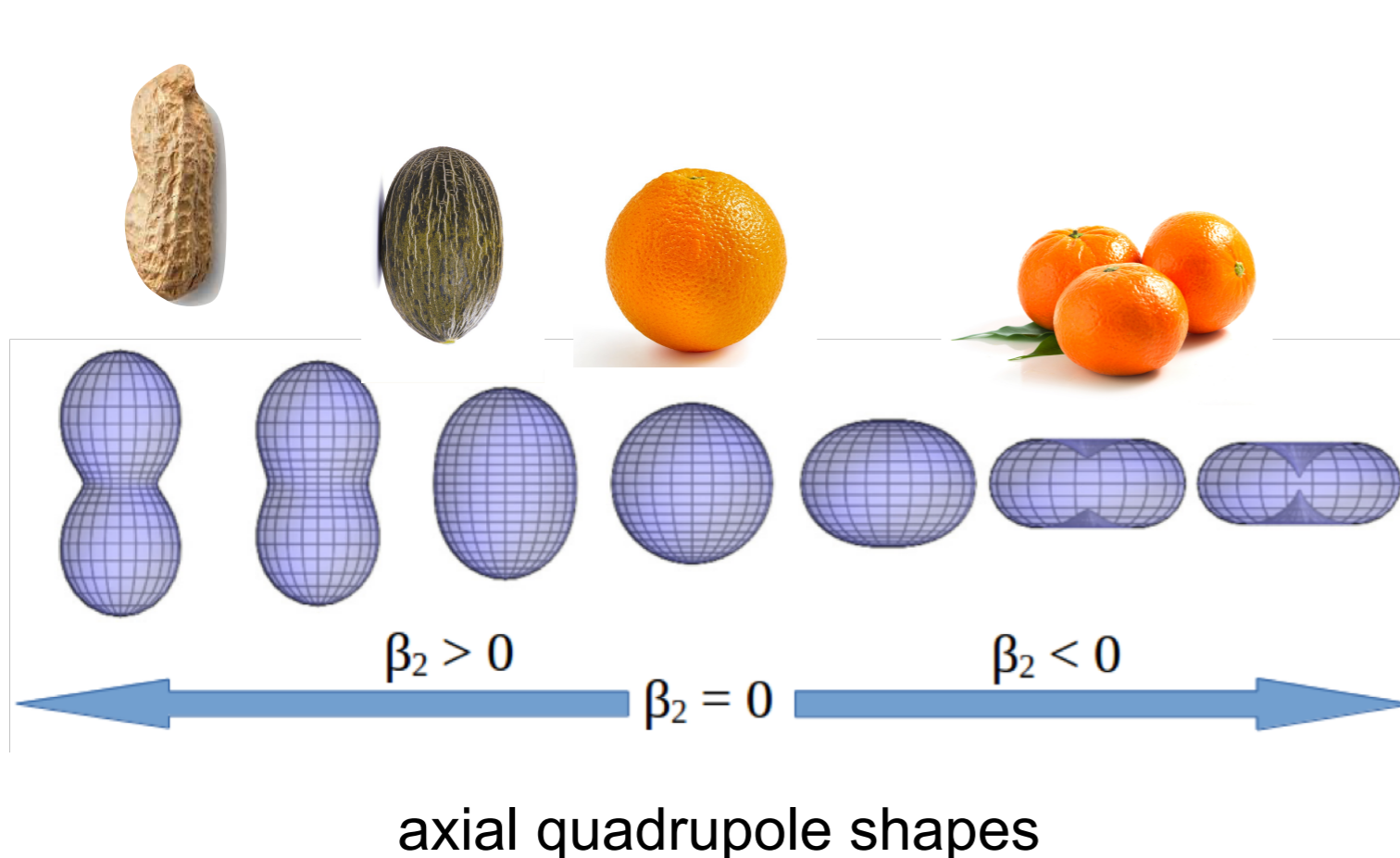
$$\beta_2=0.6, \beta_{3\mu}=0.35$$



P. A. Butler, W. Nazarewicz, Rev. Mod. Phys. 68, 349 (1996)

Introduction

Can we provide a **microscopic** description of these collective phenomena?



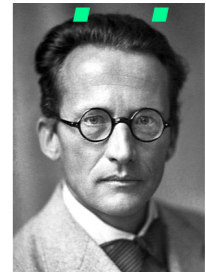
P. A. Butler, W. Nazarewicz, Rev. Mod. Phys. 68, 349 (1996)

Introduction

Let us assume that *we know* the nuclear interaction. Exact nuclear wave functions and energies cannot be obtained in general because of:

- the exploding dimensionality of the many-body Hilbert space
- the huge amount of two-, three- (eventually, N -) body matrix elements that are impossible to store

$$\hat{H}|\Psi_n\rangle = E_n|\Psi_n\rangle$$



Some of the most widely used *solutions* to attack these problems:

- Valence-space (Shell Model) calculations** with phenomenological (or normal-ordered, SRG evolved) two-body Hamiltonians
- Approximate methods (variational)** with phenomenological interactions (or energy density functionals)

Introduction

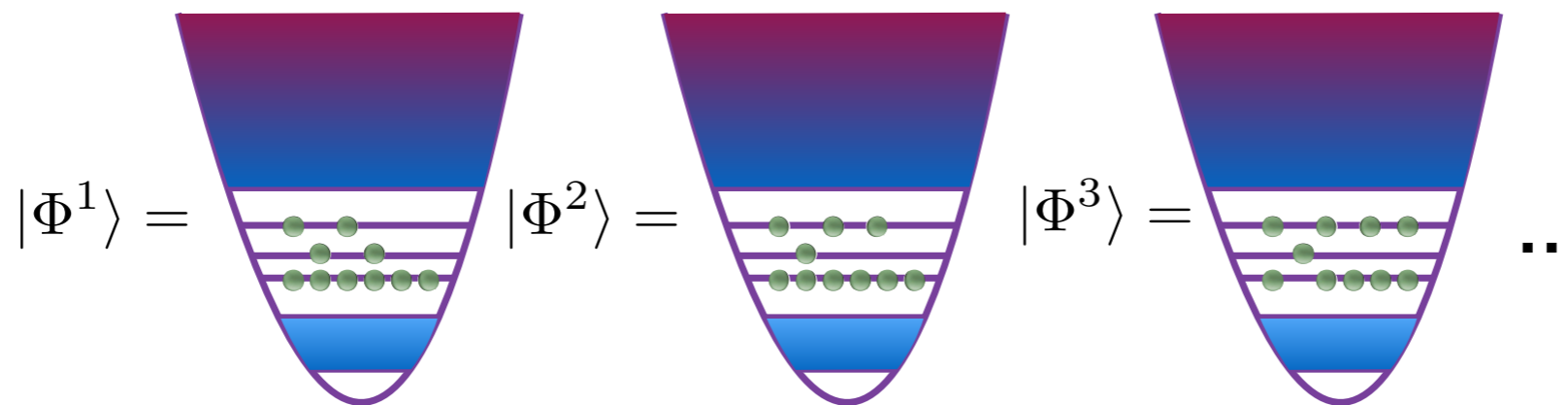
- Valence-space (Shell Model) calculations with phenomenological (or normal-ordered, SRG evolved) two-body Hamiltonians

Full diagonalization of an *adapted* Hamiltonian within a valence space

$$\hat{H}_{v.s.} |\Psi_{v.s.}^n\rangle = E_n |\Psi_{v.s.}^n\rangle$$

Nuclear wave functions are linear combinations of Slater determinants written in terms of occupations of spherical orbits

$$|\Psi_{v.s.}^n\rangle = \sum_{k \in v.s.} C_k^n |\Phi^k\rangle$$



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Some of the most widely used *solutions* to attack these problems:

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- **Approximate methods (variational)** with phenomenological interactions (or energy density functionals)

Introduction

- Approximate methods (variational) with phenomenological interactions (or energy density functionals)



Variational space of trial wave functions

Hamiltonian machine



Variational approach to the exact solution

Introduction

- Approximate methods (variational) with phenomenological interactions (or energy density functionals)

Variational spaces



Full Variational space of trial wave functions

Mean field approach

Hartree-Fock

HF-Bogoliubov

Beyond mean field approach

Configuration Mixing

Self-consistent mean-field: HFB



We use the variational method to find an approximate solution to the many-body problem defined by the Hamiltonian (non-relativistic, two-body):

$$\hat{H} = \sum_{ab} t_{ab} c_a^\dagger c_b + \frac{1}{4} \sum_{abcd} \bar{v}_{abcd} c_a^\dagger c_b^\dagger c_d c_c$$

where $c_a^\dagger |-\rangle \equiv |a\rangle$; $\langle \vec{r}st|a\rangle = \Phi_a(\vec{r}) \chi_{1/2, m_{s_a}} \chi_{1/2, m_{t_a}}$ are single-particle states

$$\{c_a, c_b\} = \{c_a^\dagger, c_b^\dagger\} = 0; \{c_a^\dagger, c_b\} = \delta_{ab} \quad (\text{Fermions})$$

$t_{ab} = \langle a|\hat{T}|b\rangle$ one-body single-particle matrix elements of the kinetic energy

$\bar{v}_{abcd} = \langle ab|\hat{V}|cd\rangle - \langle ab|\hat{V}|dc\rangle$ two-body matrix elements of the interaction

Self-consistent mean-field: HFB

We use the variational method to find an approximate solution to the many-body problem defined by the Hamiltonian (non-relativistic, two-body):

$$\hat{H} = \sum_{ab} t_{ab} c_a^\dagger c_b + \frac{1}{4} \sum_{abcd} \bar{v}_{abcd} c_a^\dagger c_b^\dagger c_d c_c$$

We define a set of generalized product-like wave functions $\{|\Phi\rangle\}$ which are vacua of certain quasiparticle operators β^\dagger, β

$$|\Phi\rangle = \prod_q \beta_q |-\rangle$$

The quasiparticle operators are defined as (**HFB transformation**):

$$\beta_k^\dagger = \sum_l U_{lk} c_l^\dagger + V_{lk} c_l$$

variational parameters

$$\text{with } \{\beta_k, \beta_l\} = \{\beta_k^\dagger, \beta_l^\dagger\} = 0; \{\beta_k^\dagger, \beta_l\} = \delta_{kl}$$

$$\text{and } W = \begin{pmatrix} U & V^* \\ V & U^* \end{pmatrix}; W^\dagger W = \mathbb{I}$$

Self-consistent mean-field: HFB



Hartree-Fock-Bogoliubov equations

We want to find the parameters (U, V), or equivalently, the one-body density matrix and pairing tensor that minimize the HFB energy.

However, the HFB transformation mixes creation and annihilation single-particle operators. Hence, the particle number symmetry can be broken by the HFB wave function

$$\beta_k^\dagger = \sum_l U_{lk} c_l^\dagger + V_{lk} c_l$$

Therefore, instead of minimizing the HFB energy, constraints on the number of particles are introduced through Lagrange multipliers:

$$\delta E'_{\text{HFB}} [|\Phi\rangle] = 0 \quad \text{with} \quad E'_{\text{HFB}} [|\Phi\rangle] = \langle \Phi | \hat{H} - \lambda_N \hat{N} - \lambda_Z \hat{Z} | \Phi \rangle$$

- $\beta_k |\Phi\rangle = 0$ trial wave functions are quasiparticle vacua
- $\lambda_N \rightarrow \langle \Phi | \hat{N} | \Phi \rangle = N$ Lagrange multiplier for neutrons
- $\lambda_Z \rightarrow \langle \Phi | \hat{Z} | \Phi \rangle = Z$ Lagrange multiplier for protons

Constrained Hartree-Fock-Bogoliubov equations

We can generalize the introduction of constraints in the HFB calculations to study not only one single solution but an energy landscape

$$\beta_k^\dagger = \sum_l U_{lk} c_l^\dagger + V_{lk} c_l$$

Therefore, we now minimize:

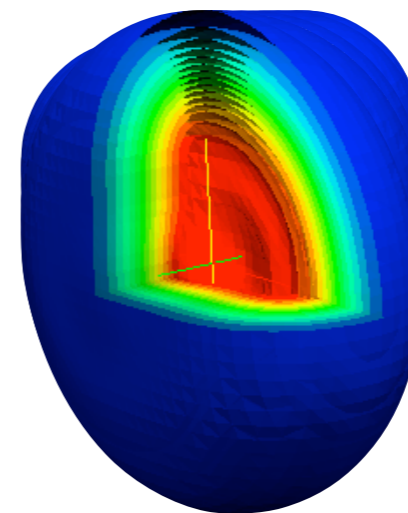
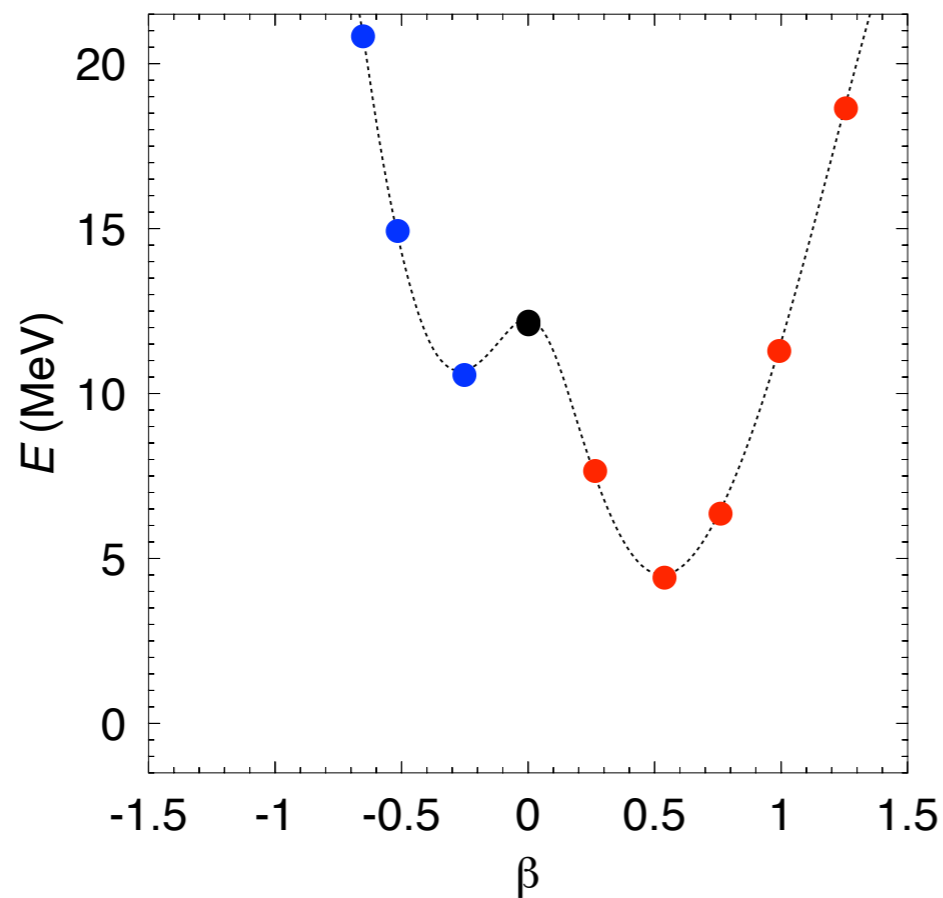
$$\delta E'_{\text{HFB}} [|\Phi(\vec{q})\rangle] = 0 \quad \text{with} \quad E'_{\text{HFB}} [|\Phi(\vec{q})\rangle] = \langle \Phi(\vec{q}) | \hat{H} - \lambda_N \hat{N} - \lambda_Z \hat{Z} - \vec{\lambda}_{\vec{q}} \cdot \hat{\vec{Q}} | \Phi(\vec{q}) \rangle$$

- $\beta_k(\vec{q}) | \Phi(\vec{q}) \rangle = 0$ trial wave functions are quasiparticle vacua
- $\lambda_N \rightarrow \langle \Phi(\vec{q}) | \hat{N} | \Phi(\vec{q}) \rangle = N$ Lagrange multiplier for neutrons
- $\lambda_Z \rightarrow \langle \Phi(\vec{q}) | \hat{Z} | \Phi(\vec{q}) \rangle = Z$ Lagrange multiplier for protons
- $\vec{\lambda}_{\vec{q}} \rightarrow \langle \Phi(\vec{q}) | \hat{\vec{Q}} | \Phi(\vec{q}) \rangle = \vec{q}$ Lagrange multipliers for collective coordinates

Self-consistent mean-field: HFB

Constrained Hartree-Fock-Bogoliubov equations

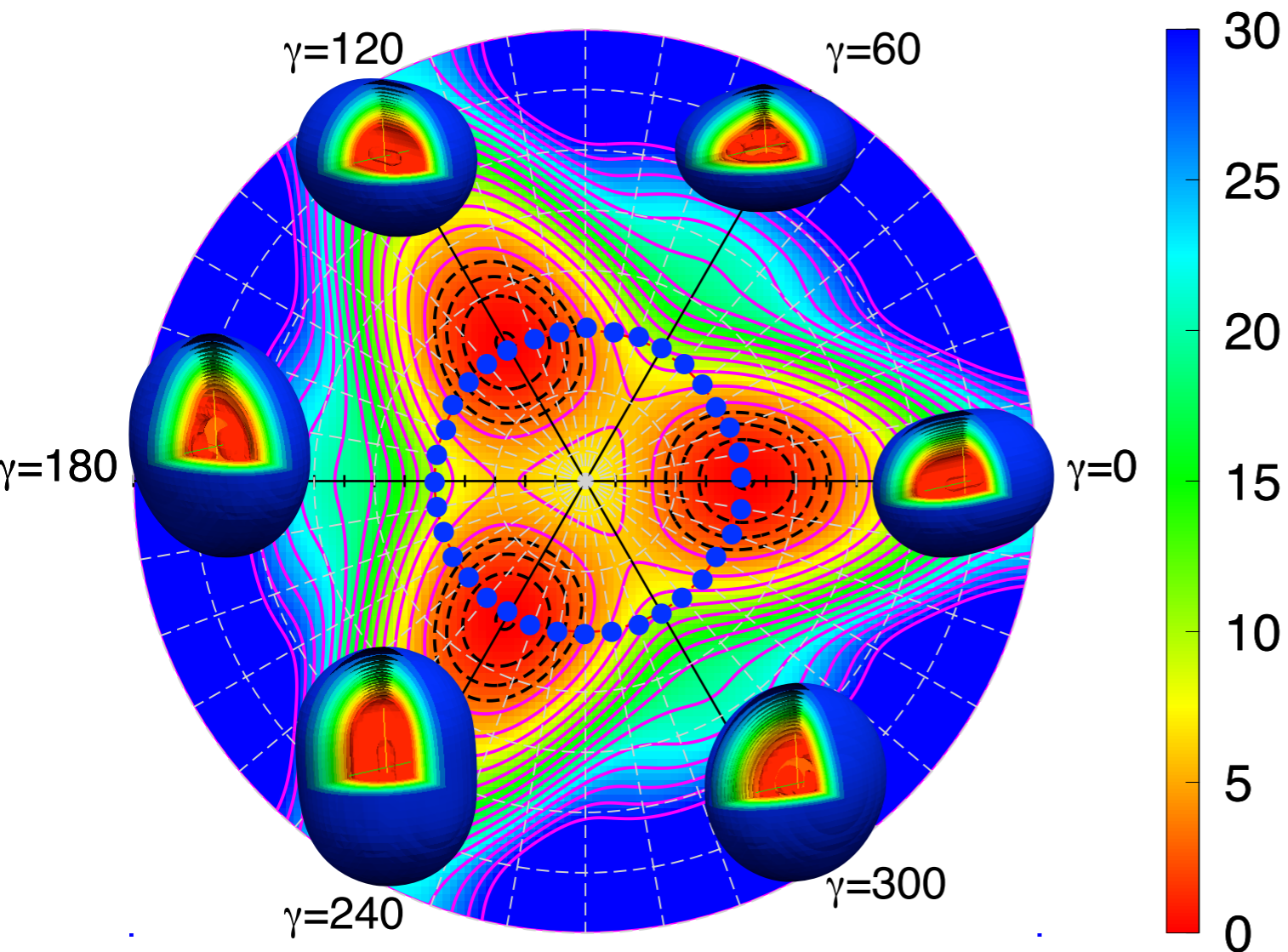
Example: ^{24}Mg



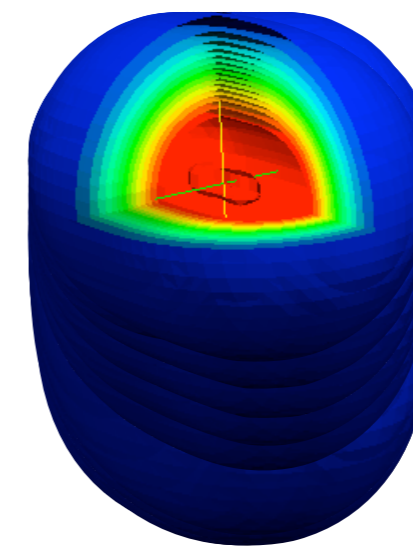
Self-consistent mean-field: HFB

Constrained Hartree-Fock-Bogoliubov equations

Example: ^{24}Mg

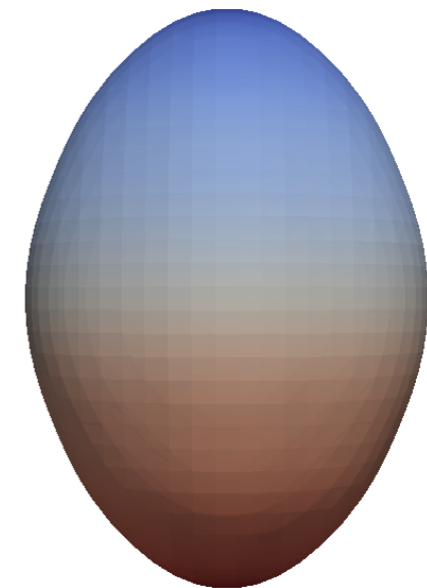
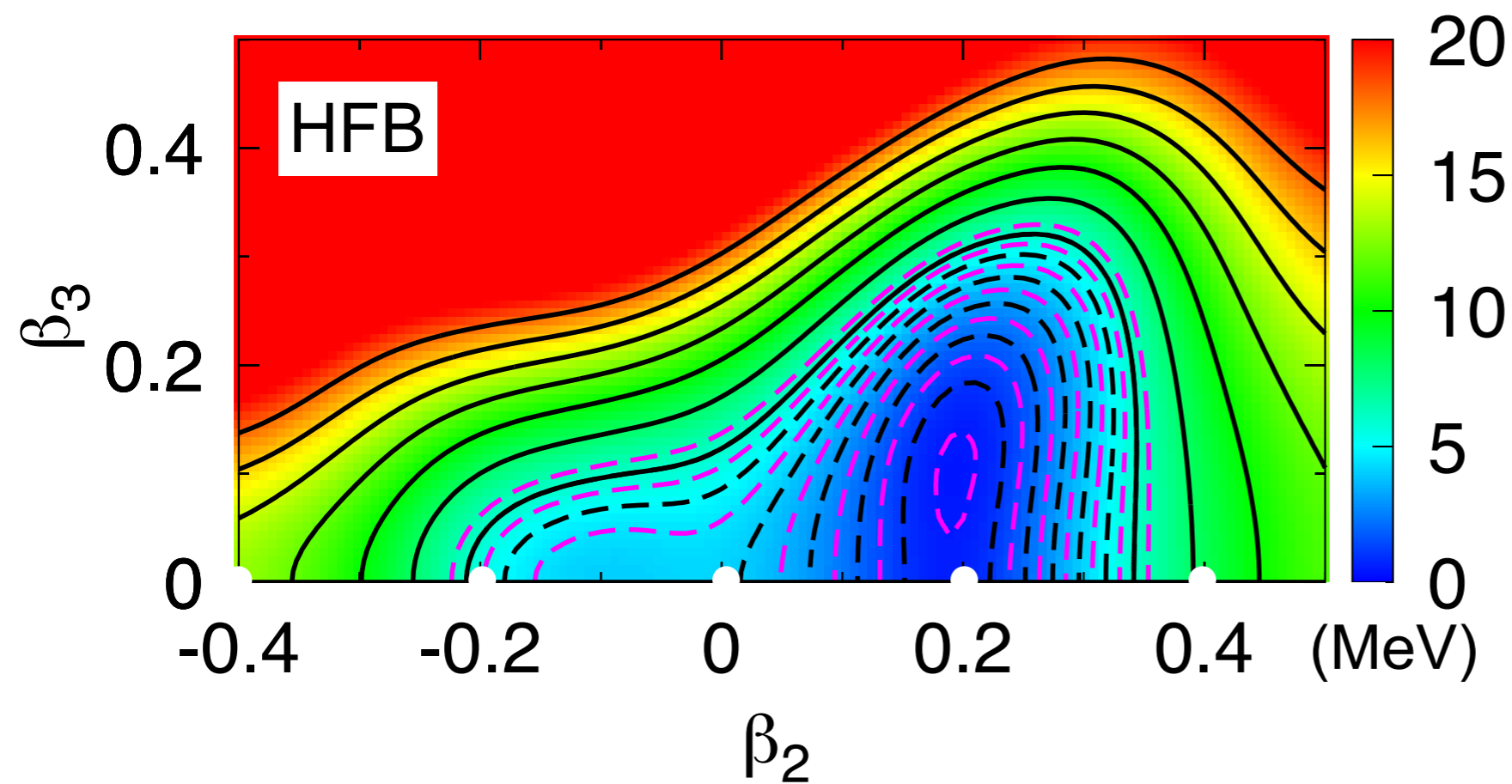


- Symmetry corresponding to the different orientation of the axes
- All configurations are included between $\gamma \in [0^\circ, 60^\circ]$



Self-consistent mean-field: HFB

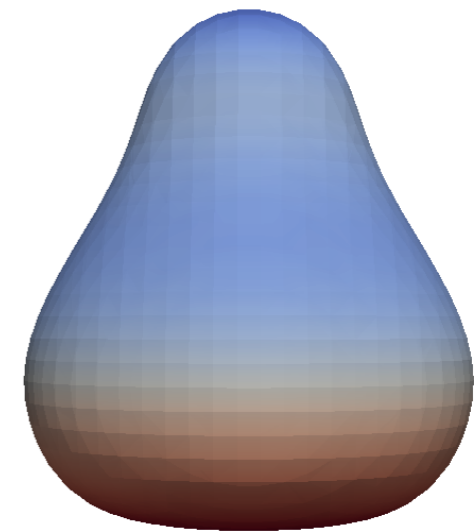
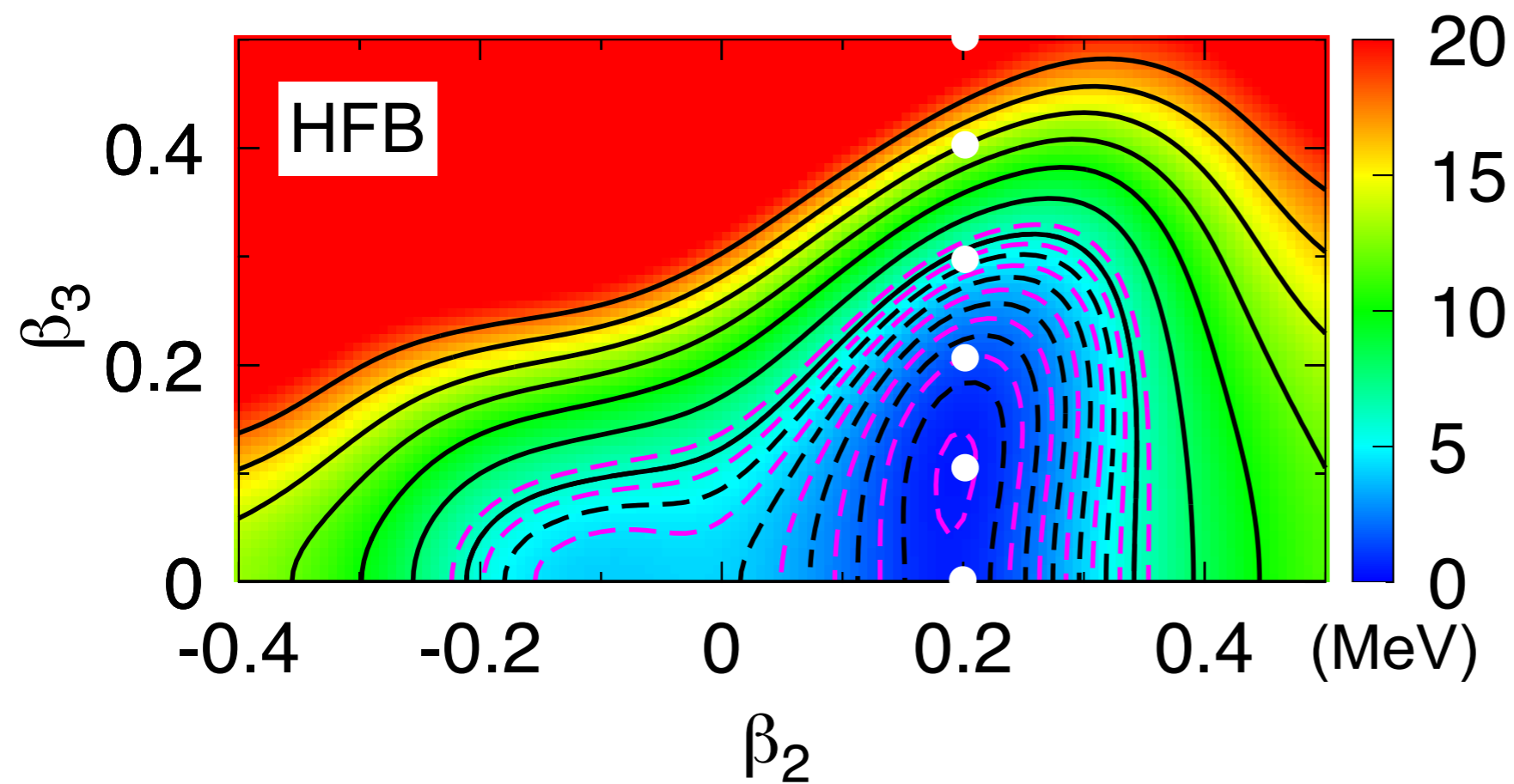
Example: ^{144}Ba axial calculations



R. Bernard, L. M. Robledo, T. R. R., PRC (2016)

Self-consistent mean-field: HFB

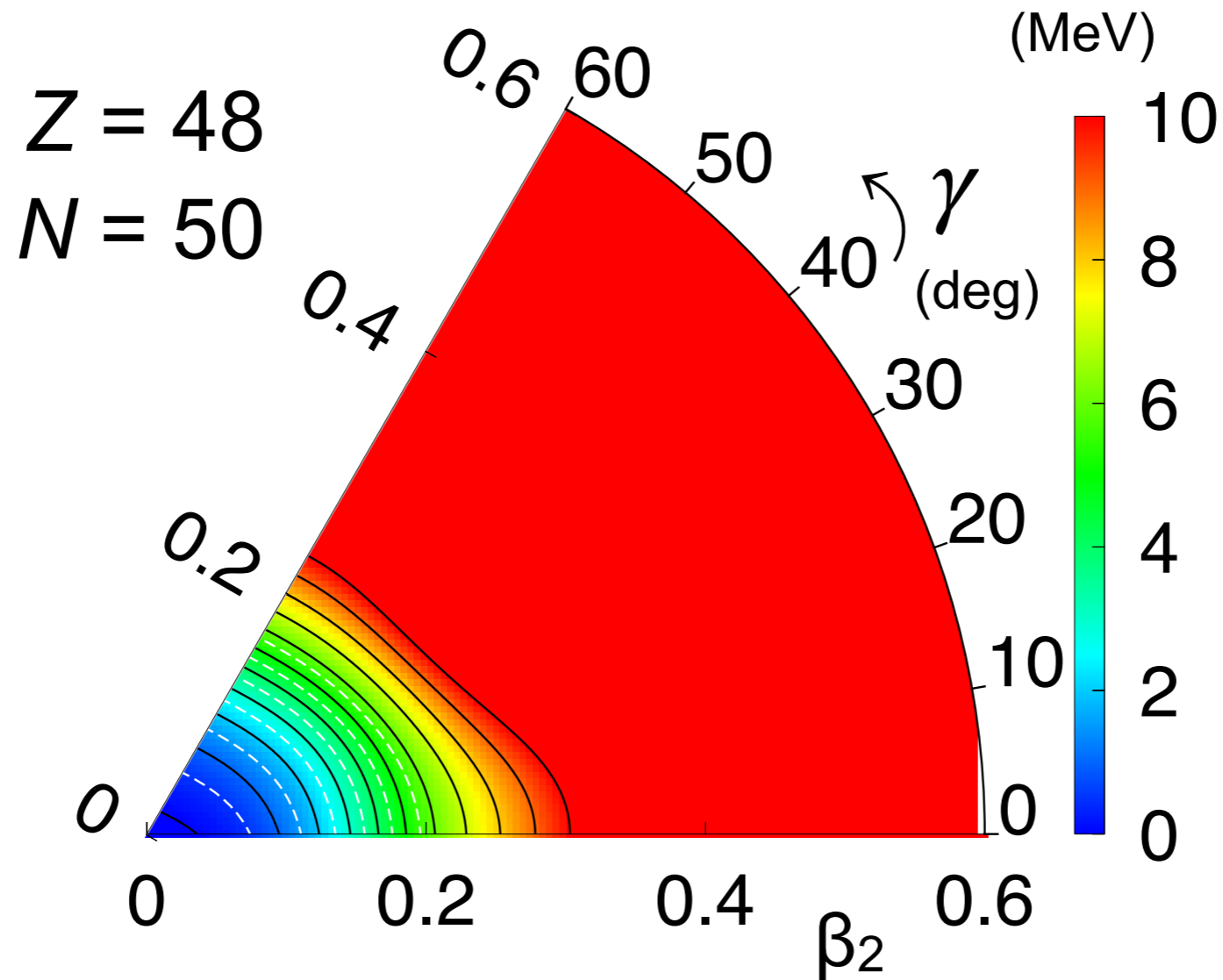
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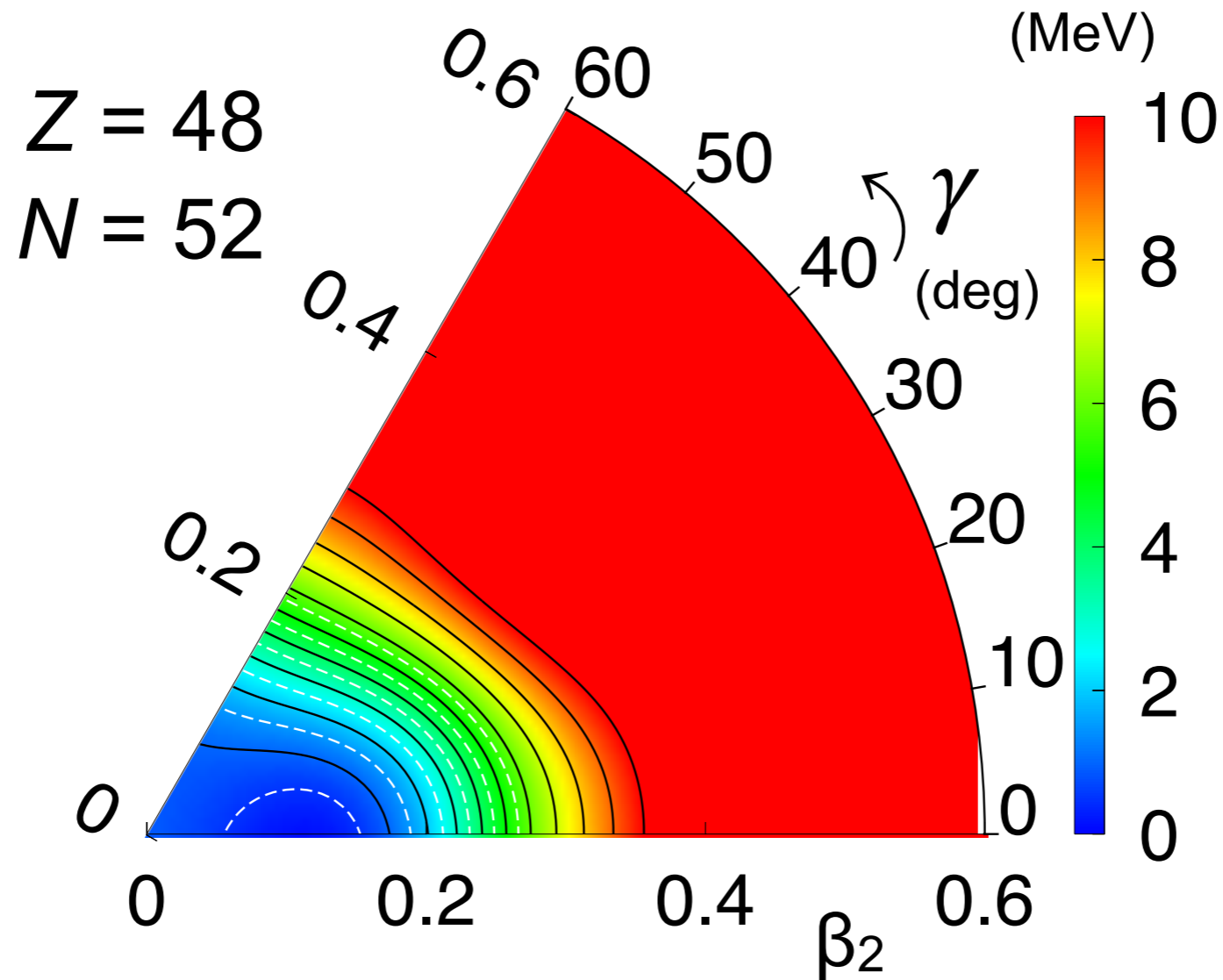
PN-VAP energy surfaces

<p>48 Cd 321.077 767 +2 112.411 5.3x10^{-8%}</p> <p>Ag 961.78 2162 +1 107.8682 58x10^{-6%}</p>		<p>6.084 0</p>	<p>Cd98 9.2 s 0+</p>	<p>Cd100 49.1 s 0+ EC</p>	<p>Cd101 1.2 m (5/2+) EC</p>	<p>Cd102 5.5 m 0+ EC</p>	<p>Cd103 7.3 m (5/2+) EC</p>	<p>Cd104 57.7 m 0+ EC</p>	<p>Cd105 55.5 m (5/2+) EC</p>	<p>Cd106 1.25 s 0+ EC</p>	<p>Cd107 6.50 h (5/2+) EC</p>	<p>Cd108 0.89 s 0+ EC</p>	<p>Cd109 462.6 d (5/2+) EC</p>	<p>Cd110 12.49 m 0+ EC</p>	<p>Cd111 12.80 m 1/2+ EC</p>	<p>Cd112 24.13 m 0+ EC</p>	<p>Cd113 9.3E+15 y 1/2+ EC</p>	<p>Cd114 28.73 m 0+ EC</p>	<p>Cd115 53.46 h (1/2+) EC</p>	<p>Cd116 7.49 m 0+ EC</p>	<p>Cd117 2.49 h (1/2+) EC</p>	<p>Cd118 50.3 m 0+ EC</p>	<p>Cd119 2.69 m (3/2+) EC</p>	<p>Cd120 50.80 s 0+ EC</p>	<p>Cd121 13.5 s (3/2+) EC</p>	<p>Cd122 5.24 s 0+ EC</p>	<p>Cd123 2.10 s (3/2+) EC</p>	<p>Cd124 1.25 s 0+ EC</p>	<p>Cd125 0.65 s (3/2+) EC</p>	<p>Cd126 0.506 s 0+ EC</p>	<p>Cd127 0.37 s (3/2+) EC</p>	<p>Cd128 0.34 s 0+ EC</p>	<p>Cd129 0.27 s (3/2+) EC</p>	<p>Cd130 0.20 s 0+ EC</p>
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PN-VAP energy surfaces

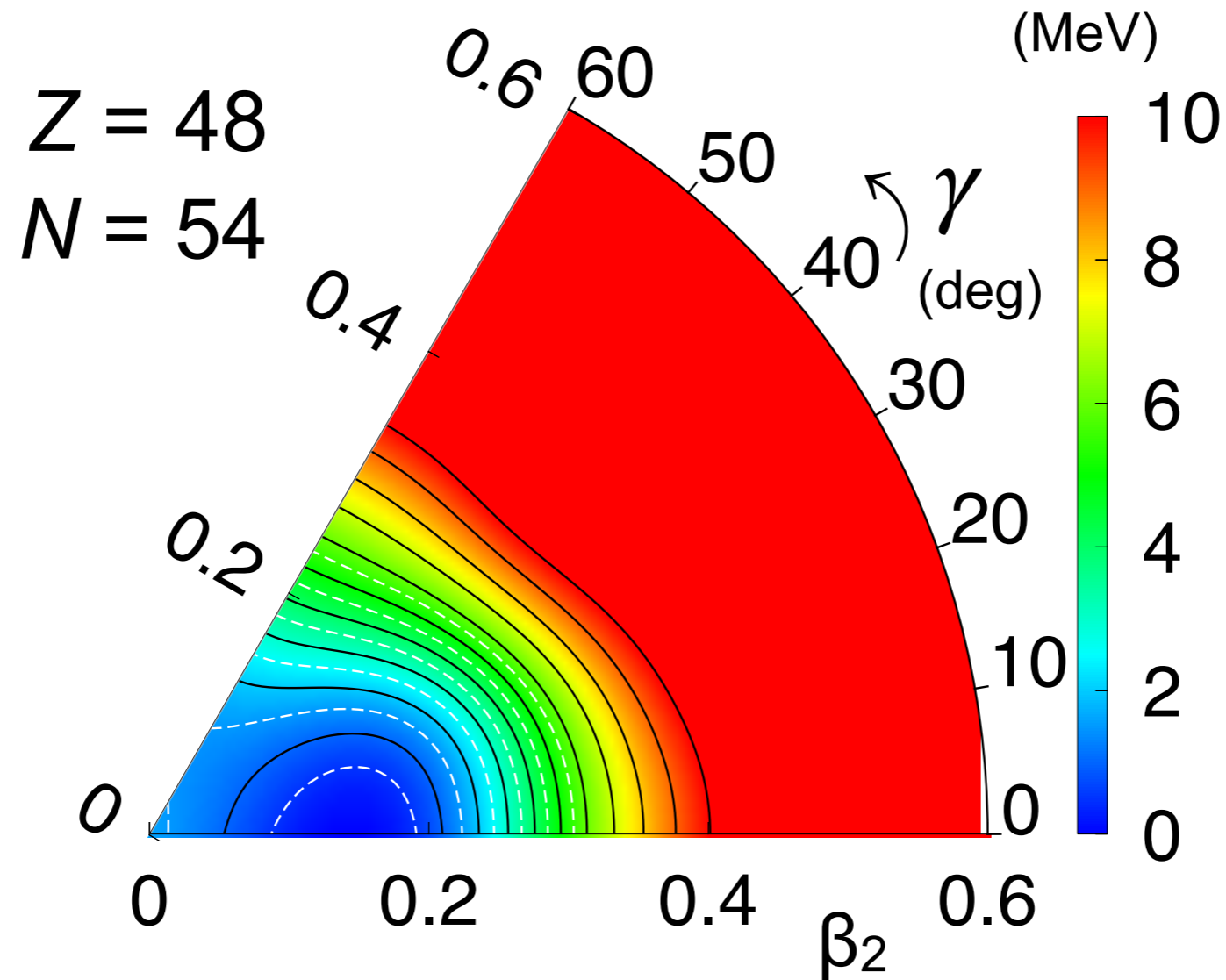
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		Cd100 49.1 s 0+																																	
		EC																																	
		Ag99																																	
48	Cd	Cd96	Cd97	Cd98	Cd100	Cd101	Cd102	Cd103	Cd104	Cd105	Cd106	Cd107	Cd108	Cd109	Cd110	Cd111	Cd112	Cd113	Cd114	Cd115	Cd116	Cd117	Cd118	Cd119	Cd120	Cd121	Cd122	Cd123	Cd124	Cd125	Cd126	Cd127	Cd128	Cd129	Cd130
		0+	3 s	9.2 s	49.1 s	0+	0+	7.3 m	57.7 m	55.5 m	1.25	6.50 h	0+	462.6 d	0+	1/2+	0+	9.3E+15 y	6.50 h	53.46 h	7.49	2.49 h	50.3 m	2.69 m	50.80 s	13.5 s	5.24 s	2.10 s	1.25 s	0.65 s	0.506 s	0.37 s	0.34 s	0.27 s	0.20 s
		ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC
Ag	Ag94	Ag95	Ag96	Ag97	Ag99	Ag100	Ag101	Ag102	Ag103	Ag104	Ag105	Ag106	Ag107	Ag108	Ag109	Ag110	Ag111	Ag112	Ag113	Ag114	Ag115	Ag116	Ag117	Ag118	Ag119	Ag120	Ag121	Ag122	Ag123	Ag124	Ag125	Ag126	Ag127		
	0+	2.0 s	5.1 s	19 s	0+	0+	12.9 m	12.9 m	65.7 m	69.2 m	4E29 d	2396 m	237 m	237 m	1/2-	24.6 s	735 d	3.330 h	257 h	3.6 s	20.0 m	288 m	72.8 s	376 s	2.1 s	123 s	0.78 s	0.48 s	0.509 s	0.172 s	166 ms	107 ms	109 ms		
	EC	ECp	ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC



PN-VAP energy surfaces

										EC		F										
										Cd102 5.5 m		0+										
										EC		F										
										Ag101		F										

48	Cd	321.077 767	Cd96	0+	6.0x10 ⁻⁸ %	Cd97	3 s	ECp	Cd98	9.2 s	0+	Cd99	16 s	(5/2+)	ECp,ECc...	Cd100	49.1 s	0+	EC	1104	7 m	0+	Cd105	55.5 m	5/2+	EC	Cd106	1.25	0+	Cd107	6.50 h	5/2+	EC	Cd108	0.89	0+	Cd109	462.6 d	5/2+	EC	Cd110	12.49	0+	Cd111	12.80	1/2+	*	Cd112	24.13	0+	Cd113	9.3E+15 y	1/2+	*	Cd114	28.73	0+	Cd115	53.46 h	1/2+	EC-β	Cd116	7.49	0+	Cd117	2.49 h	1/2+	β	Cd118	50.3 m	0+	β	Cd119	2.69 m	3/2+	β	Cd120	50.80 s	0+	β	Cd121	13.5 s	(3/2+)	β	Cd122	5.24 s	0+	β	Cd123	2.10 s	(3/2+)	β	Cd124	1.25 s	0+	β	Cd125	0.65 s	(3/2+)	β	Cd126	0.506 s	0+	β	Cd127	0.37 s	(3/2+)	β	Cd128	0.34 s	0+	β	Cd129	0.27 s	(3/2+)	β	Cd130	0.20 s	0+	β
47	Ag	961.78 2162	Ag94	10 ms	0+	5.3x10 ⁻⁸ %	Ag95	2.0 s	ECp	Ag96	5.1 s	(8+,9+)	Ag97	19 s	(9/2+)	EC	Ag98	46.7 s	(5+)	EC	Ag99	124 s	(9/2+)	EC	Ag104	69.2 m	5+	EC	Ag105	4E+29 d	1/2-	EC	Ag106	2396 m	1+	EC-β	Ag107	51.839	1/2-	EC-β	Ag108	237 m	1+	EC-β	Ag109	48.161	1/2-	EC-β	Ag110	24.6 s	1+	β	Ag111	735 d	1/2-	β	Ag112	3E+30 h	2(-)	β	Ag113	257 h	1/2-	β	Ag114	3.6 s	1+	β	Ag115	20.0 m	1/2-	β	Ag116	288 m	(2-)	β	Ag117	72.8 s	(1/2-)	β	Ag118	376 s	(1-)	β	Ag119	2.1 s	(7/2+)	β	Ag120	123 s	(3+)	β	Ag121	0.78 s	(7/2+)	β	Ag122	0.48 s	(3+)	β	Ag123	0.509 s	(7/2+)	β	Ag124	0.172 s	0+	β	Ag125	166 ms	0+	β	Ag126	107 ms	0+	β	Ag127	109 ms	0+	β

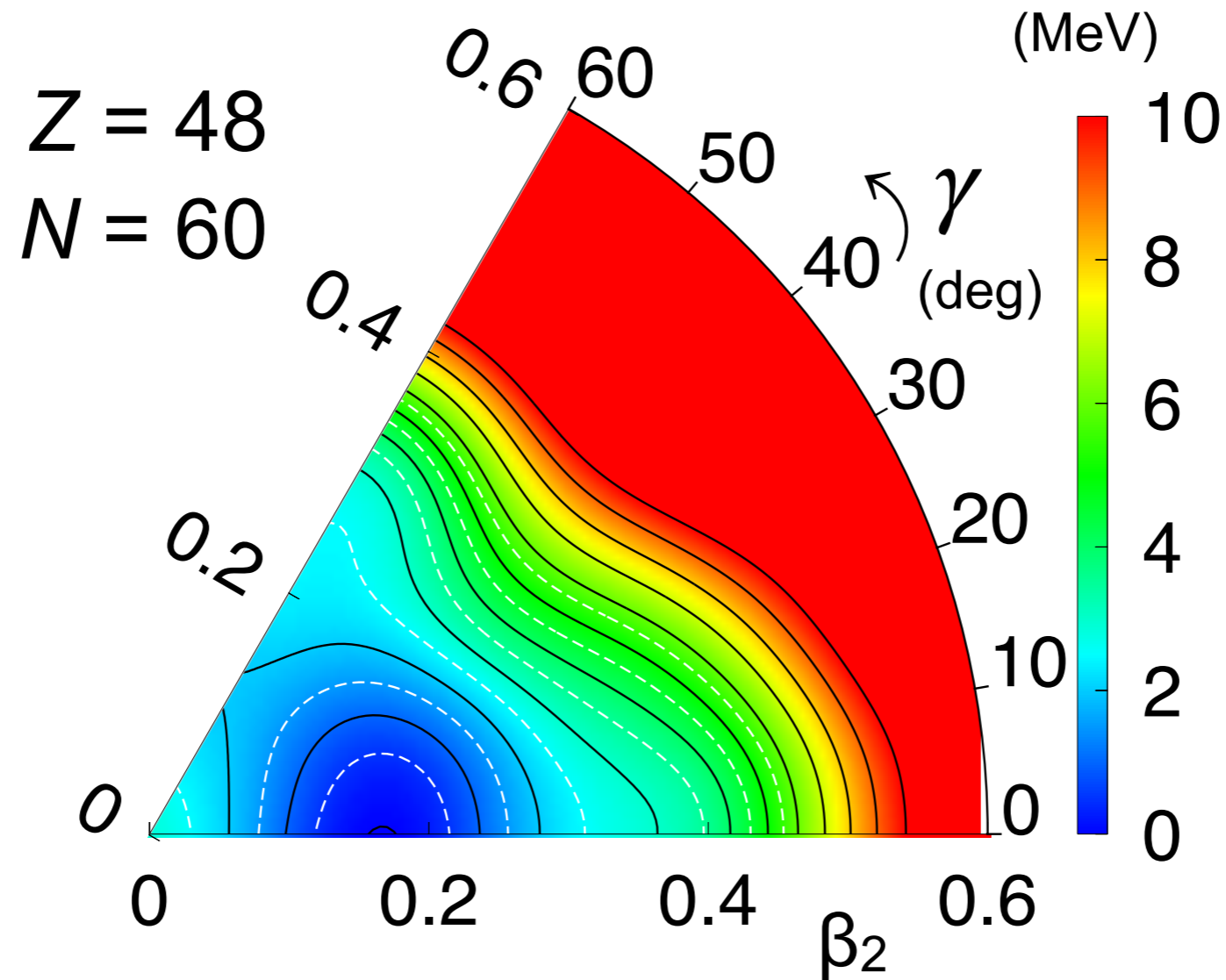


PN-VAP energy surfaces

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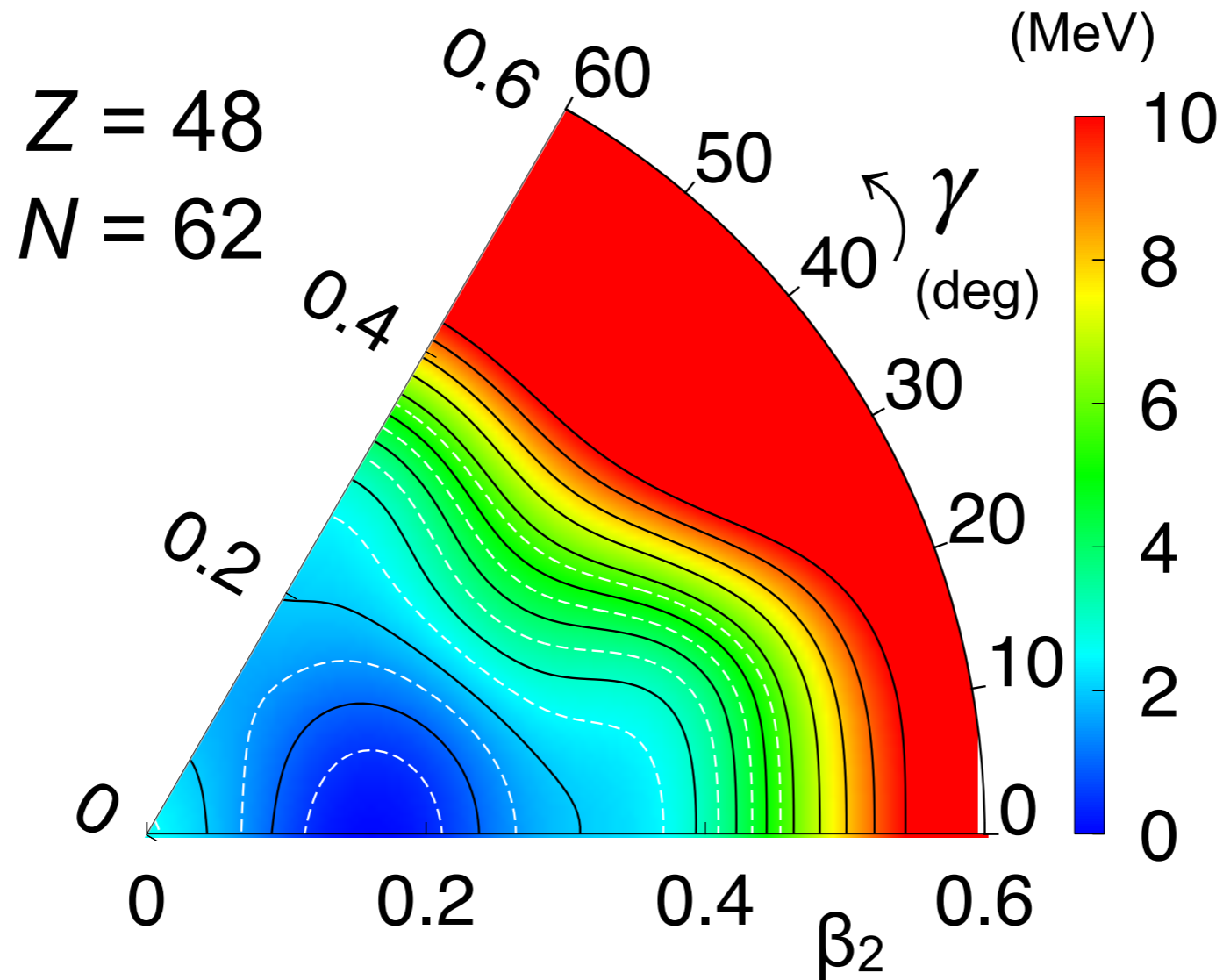
PN-VAP energy surfaces

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48	Cd	Cd96	Cd97	Cd98	Cd99	Cd100	Cd101	Cd102	Cd103	Cd104	Cd105	Cd106	Cd108	Cd111	Cd112	Cd113	Cd114	Cd115	Cd116	Cd117	Cd118	Cd119	Cd120	Cd121	Cd122	Cd123	Cd124	Cd125	Cd126	Cd127	Cd128	Cd129	Cd130
	321.077 767	0+	3 s	9.2 s	16 s	49.1 s	1.2 m	5.5 m	7.3 m	57.7 m	55.5 m	1.25	0+	1/2+	0+	1/2+	0+	1/2+	2.49 h	50.3 m	2.69 m	50.80 s	13.5 s	5.24 s	2.10 s	1.25 s	0.65 s	0.506 s	0.37 s	0.34 s	0.27 s	0.20 s	
	6.0x10 ⁻⁸ %	0+	ECp	EC	ECp,ECα...	EC	EC	EC	EC	EC	EC	EC	0+	1/2+	0+	1/2+	0+	1/2+	0+	0+	3/2+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	
	112.411 5.3x10 ⁻⁸ %	0+	ECp	EC	ECp,ECα...	EC	EC	EC	EC	EC	EC	EC	0+	1/2+	0+	1/2+	0+	1/2+	0+	0+	3/2+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	
Ag	961.78 2162	Ag94	Ag95	Ag96	Ag97	Ag98	Ag99	Ag100	Ag101	Ag102	Ag103	Ag104	Ag105	Ag110	Ag111	Ag112	Ag113	Ag114	Ag115	Ag116	Ag117	Ag118	Ag119	Ag120	Ag121	Ag122	Ag123	Ag124	Ag125	Ag126	Ag127		
	107.8682 58x10 ⁻⁸ %	0+	20 s	51 s	19 s	46.7 s	124 s	201 m	11.1 m	129 m	65.7 m	69.2 m	4129 d	24.6 s	735 d	3.30 h	537 h	36 s	20.0 m	288 m	72.8 s	376 s	2.1 s	123 s	0.78 s	0.48 s	0.509 s	0.172 s	166 ms	107 ms	109 ms		
	EC	ECp	ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC,β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻		



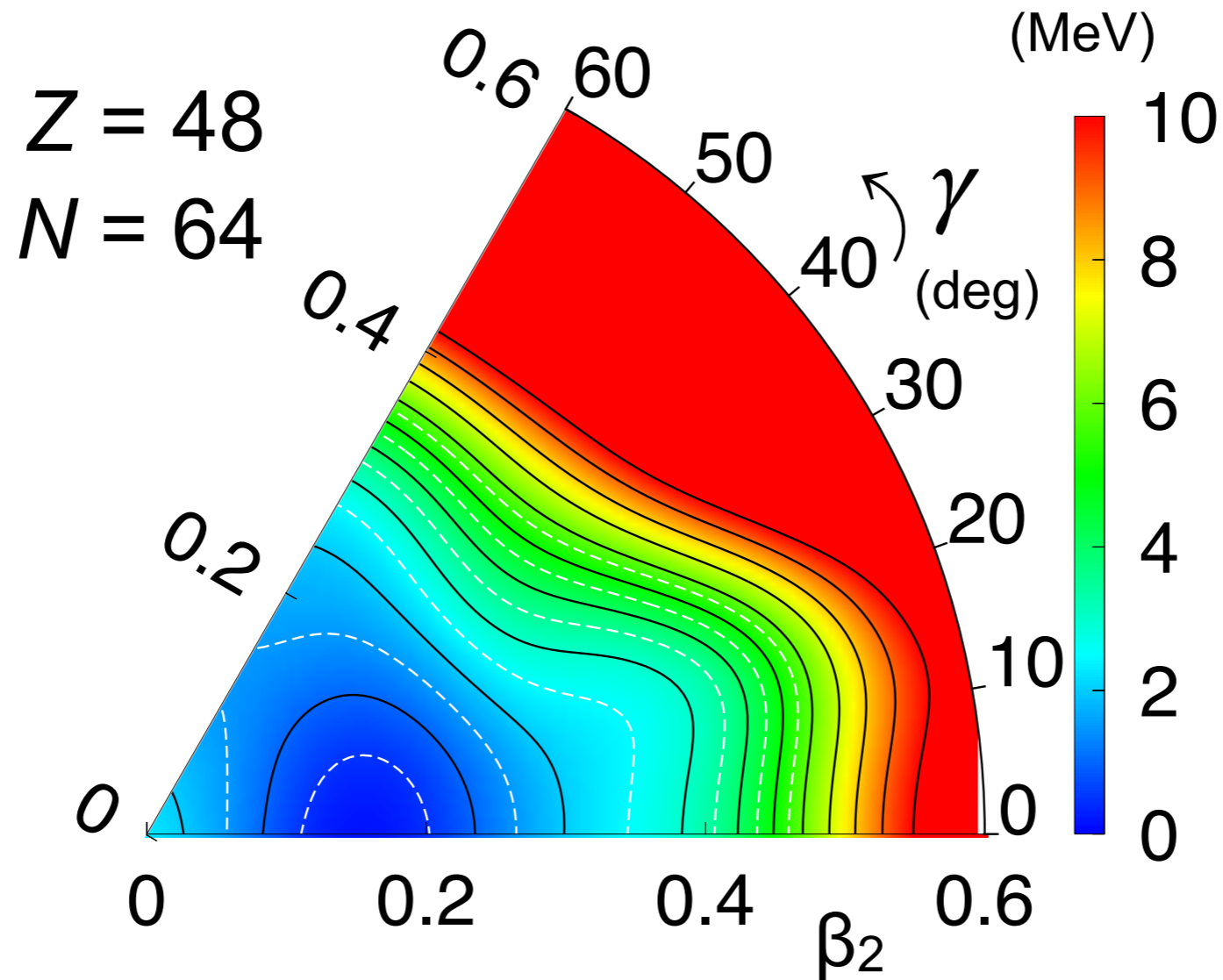
PN-VAP energy surfaces

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														0+																						
														12.49																						
														Ag100																						
48	Cd	Cd96	Cd97	Cd98	Cd99	Cd100	Cd101	Cd102	Cd103	Cd104	Cd105	Cd106	Cd107	Cd108	Cd109	Cd110	Cd111	Cd112	Cd113	Cd114	Cd115	Cd116	Cd117	Cd118	Cd119	Cd120	Cd121	Cd122	Cd123	Cd124	Cd125	Cd126	Cd127	Cd128	Cd129	Cd130
	321.077 767	0+	3 s	9.2 s	16 s	49.1 s	1.2 m	5.5 m	7.3 m	57.7 m	55.5 m	1.25 s	6.50 h	0+	0+	0+	0+	9.3E+15 y	0+	53.46 h	0+	2.49 h	50.3 m	2.69 m	50.80 s	13.5 s	5.24 s	2.10 s	1.25 s	0.65 s	0.506 s	0.37 s	0.34 s	0.27 s	0.20 s	
	112.411 5.3x10 ⁻⁸ %	ECp	EC	EC	ECp,ECα...	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻		
Ag	961.78 2162	Ag94	Ag95	Ag96	Ag97	Ag98	Ag99	Ag100	Ag101	Ag102	Ag103	Ag104	Ag105	Ag106	Ag107	Ag108	Ag109	Ag110	Ag111	Ag112	Ag113	Ag114	Ag115	Ag116	Ag117	Ag118	Ag119	Ag120	Ag121	Ag122	Ag123	Ag124	Ag125	Ag126	Ag127	
	107.8682 58x10 ⁻⁶ %	0+	20 s	51 s	19 s	124 s	201 m	11.1 m	11.1 m	12.9 m	65.7 m	69.2 m	41.29 d	2396 m	1+	1+	1+	2-	5 d	3.330 h	2.57 h	3.6 s	20.0 m	288 m	72.8 s	2.1 s	1.23 s	0.78 s	0.48 s	0.309 s	0.172 s	166 ms	107 ms	109 ms		
	EC	ECp	ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC,β ⁻	EC	EC	EC	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻	β ⁻		



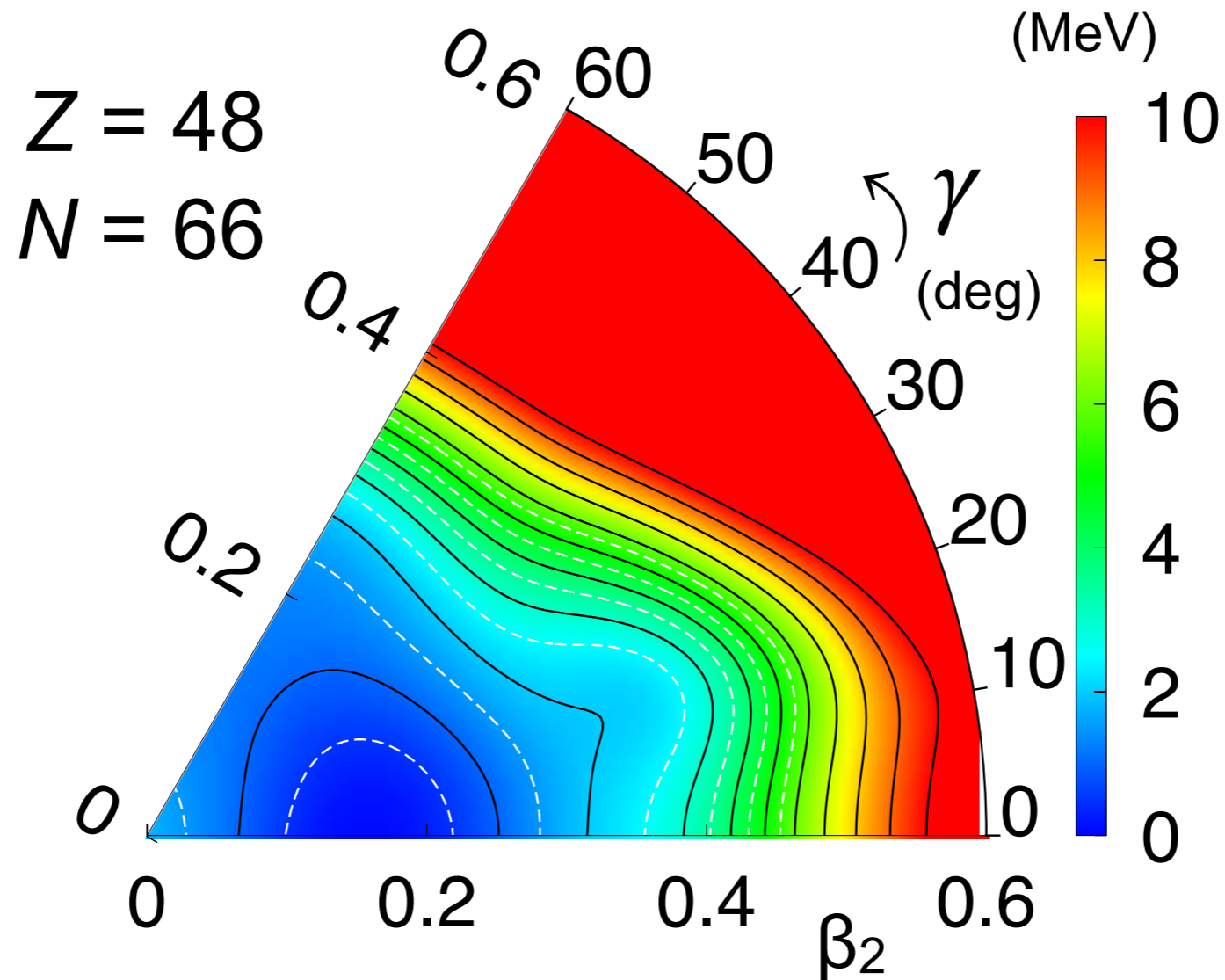
PN-VAP energy surfaces

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														43 Ag111																						
48	Cd	Cd96	Cd97	Cd98	Cd99	Cd100	Cd101	Cd102	Cd103	Cd104	Cd105	Cd106	Cd107	Cd108	Cd109	Cd110	14	Cd115	Cd116	Cd117	Cd118	Cd119	Cd120	Cd121	Cd122	Cd123	Cd124	Cd125	Cd126	Cd127	Cd128	Cd129	Cd130	82		
		0+	0+	0+	(5/2+)	0+	(5/2+)	0+	(5/2+)	0+	5/2+	0+	5/2+	0+	5/2+	0+	1/2+	1/2+	0+	0+	3/2+	0+	(3/2+)	0+	(3/2+)	0+	(3/2+)	0+	(3/2+)	0+	(3/2+)	0+	0+			
		ECp	EC	EC	ECp,EC...	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-			
		6.0x10 ⁻⁶ %										1.25				12.49	53.46 h	7.49	2.49 h	50.3 m	2.69 m	50.80 s	13.5 s	5.24 s	2.10 s	1.25 s	0.65 s	0.506 s	0.37 s	0.34 s	0.27 s	0.20 s				
		321.077	3 s	9.2 s	16 s	49.1 s	1.2 m	5.5 m	7.3 m	57.7 m	55.5 m	6.50 h	462.6 d	0+	462.6 d	12.49	53.46 h	7.49	2.49 h	50.3 m	2.69 m	50.80 s	13.5 s	5.24 s	2.10 s	1.25 s	0.65 s	0.506 s	0.37 s	0.34 s	0.27 s	0.20 s				
		112.411																																		
		5.3x10 ⁻⁸ %																																		
		EC	ECp	EC	ECp,EC...	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-		
		961.78	20 s	5.1 s	19 s	46.7 s	124 s	201 m	11.1 m	12.9 m	65.7 m	69.2 m	41.29 d	2396 m	237 m	48.161	13 h	3.6 s	20.0 m	288 m	72.8 s	376 s	2.1 s	1.23 s	0.78 s	0.48 s	0.309 s	0.172 s	166 ms	107 ms	109 ms					
		2162	10 ms	20 s	5.1 s	19 s	46.7 s	124 s	201 m	11.1 m	12.9 m	65.7 m	69.2 m	41.29 d	2396 m	237 m	48.161	13 h	3.6 s	20.0 m	288 m	72.8 s	376 s	2.1 s	1.23 s	0.78 s	0.48 s	0.309 s	0.172 s	166 ms	107 ms	109 ms				
		107.8682	0+	(8+,9+)	(9/2+)	(5+)	(9/2+)	(5+)	9/2+	5+	7/2+	5+	1/2-	1/2-	1+	1/2-	1/2-	1+	(2-)	(1/2-)	(1-)	(7/2+)	(3+)	(7/2+)	(3+)	(7/2+)	(7/2+)	(7/2+)	(3+)	(3+)	(7/2+)	(7/2+)	(7/2+)	(7/2+)		
		58x10 ⁻⁶ %	ECp	ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-	β-



PN-VAP energy surfaces

48																95.7	82															
Cd																Cd114	Cd															
0+																0+	0+															
28.73																28.73	28.73															
Ag113																Ag113	Ag															



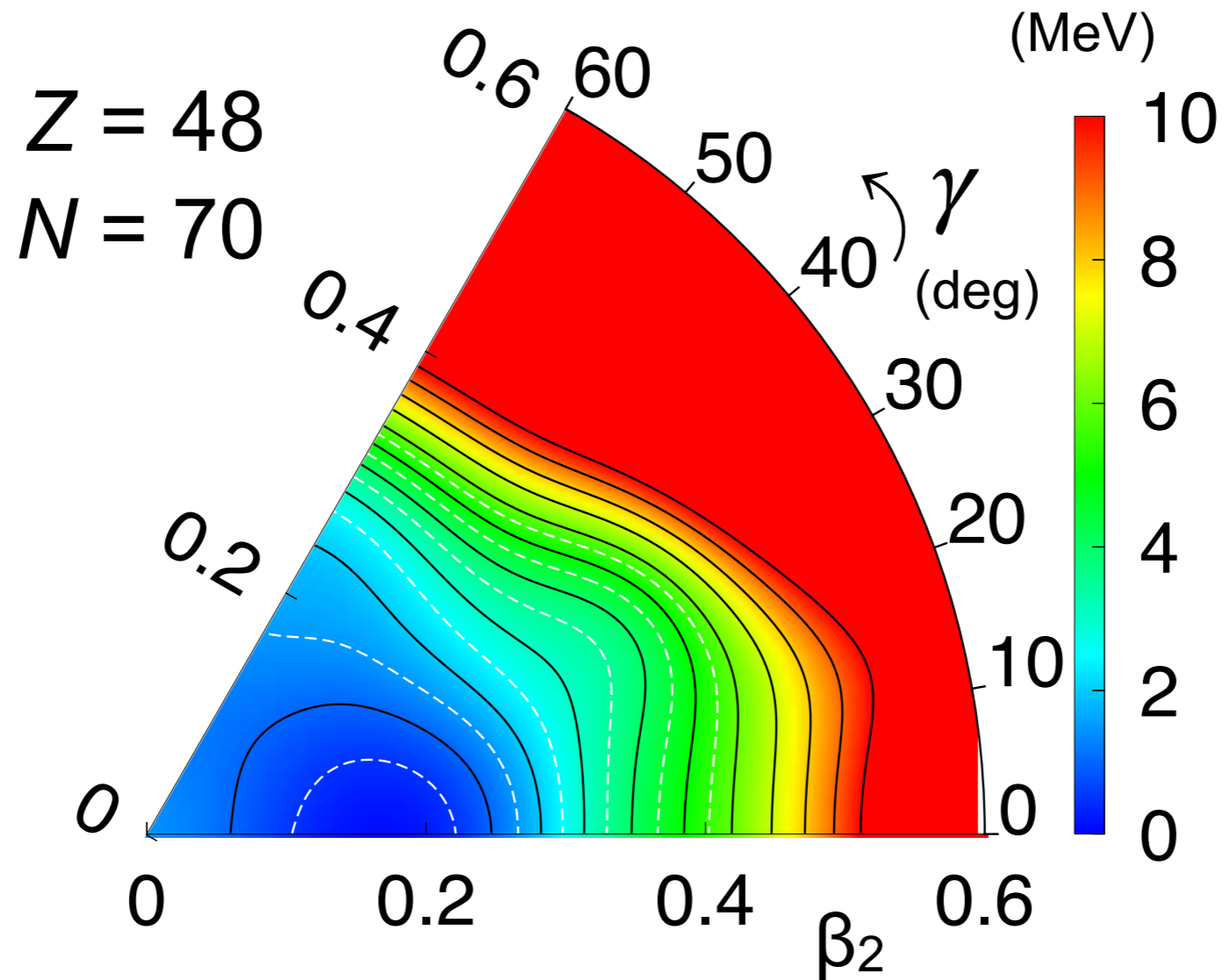
PN-VAP energy surfaces

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		Cd99																Cd116	Cd122										Cd130
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PN-VAP energy surfaces

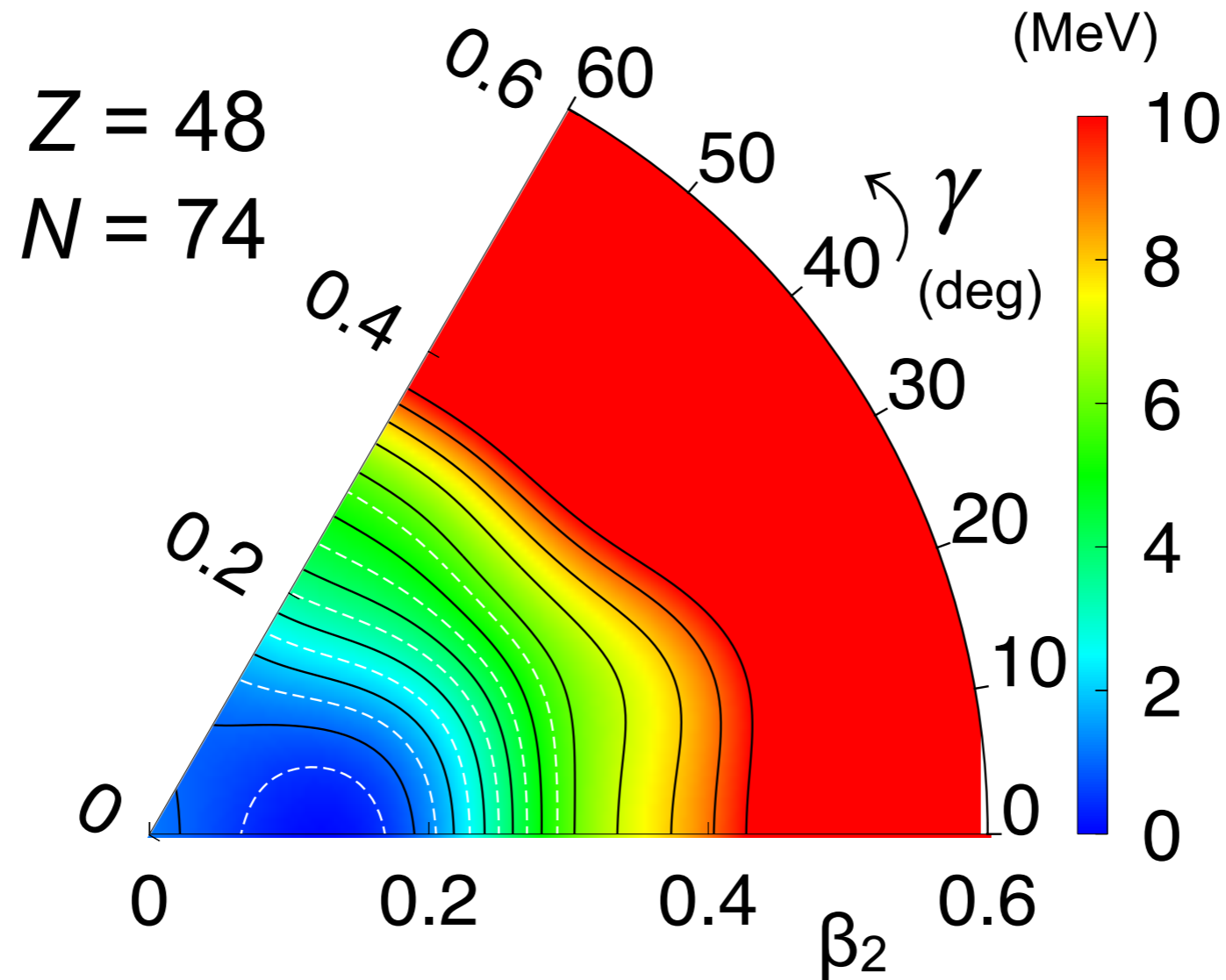
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																Cd118 50.3 m 0+										
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48	Cd	Cd96	Cd97	Cd98	Cd99	Cd100	Cd101	Cd102	Cd103	Cd104	Cd105	Cd106	Cd107	Cd108	Cd109	Cd110	Cd111	Cd112	Cd113	Cd114	Cd115	Cd116	Cd117	Cd118	Cd119	Cd120	Cd121	Cd122	Cd123	Cd124	Cd125	Cd126	Cd127	Cd128	Cd129	Cd130		
		0+	0+	0+	(5/2+)	0+	(5/2+)	0+	(5/2+)	0+	5/2+	0+	5/2+	0+	5/2+	0+	1/2+	0+	1/2+	0+	1/2+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	
		ECp	EC	EC	ECp,ECα	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	
		6.0x10 ⁻⁶ %																																				
		321.077	3 s	9.2 s	16 s	49.1 s	1.2 m	5.5 m	7.3 m	57.7 m	55.5 m	6.50 h	462.6 d	12.49	12.80	24.13	9.3E+15 y	28.73	53.46 h	7.49																		
		112.411																																				
		5.3x10 ⁻⁸ %																																				
		107.8682	10 ms	2.0 s	5.1 s	124 s	201 m	11.1 m	12.9 m	65.7 m	69.2 m	41.29 d	2396 m	237 m	237 m	24.6 s	3.330 h	257 h	3.6 s	26.0 m																		
		58x10 ⁻⁶ %																																				
		961.78	2162	19 s	46.7 s	124 s	201 m	11.1 m	12.9 m	65.7 m	69.2 m	41.29 d	2396 m	237 m	237 m	24.6 s	3.330 h	257 h	3.6 s	26.0 m																		
		EC	ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	



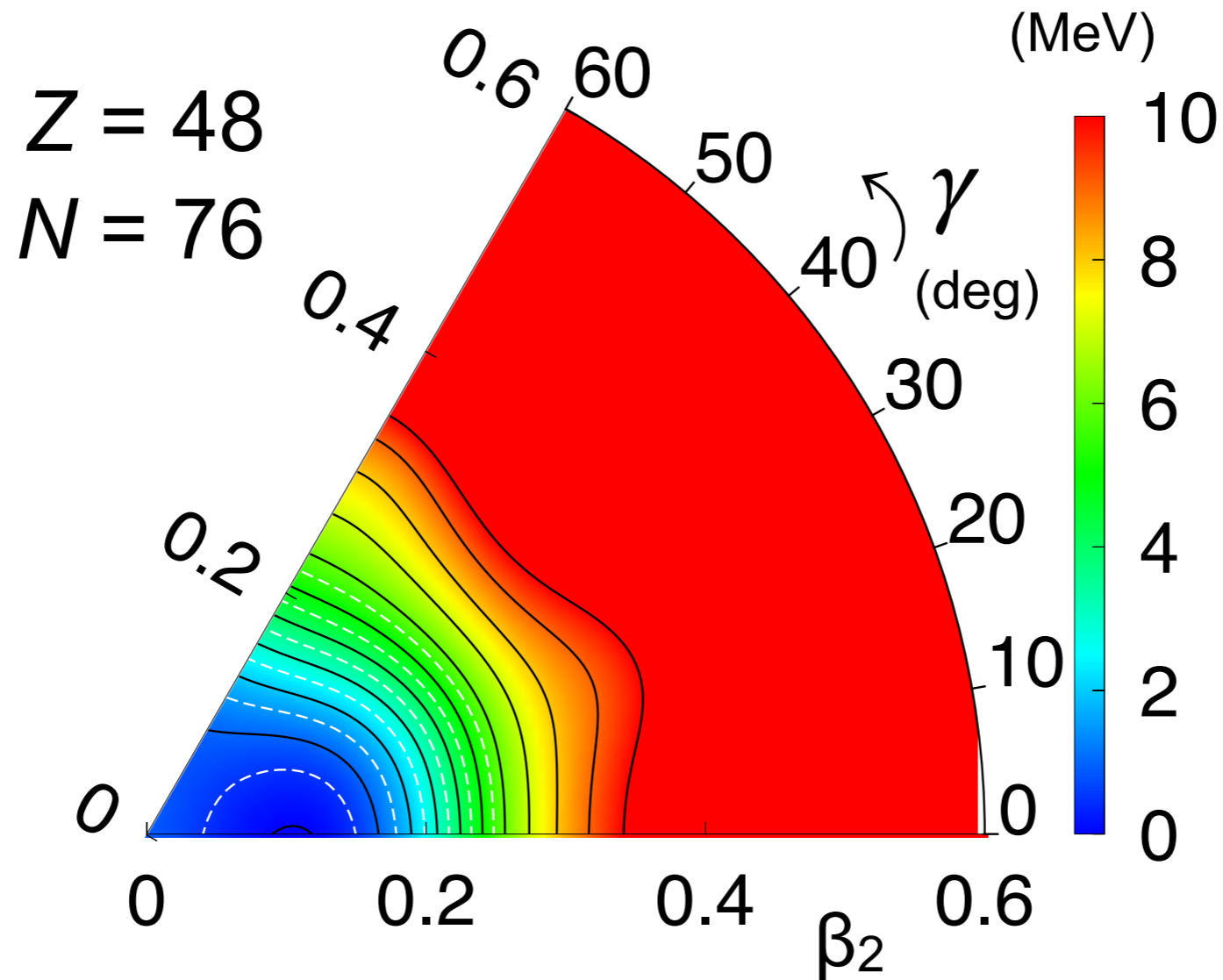
PN-VAP energy surfaces

48		Cd	Cd96	Cd97	Cd98	Cd99	Cd100	Cd101	Cd102	Cd103	Cd104	Cd105	Cd106	Cd107	Cd108	Cd109	Cd110	Cd111	Cd112	Cd113	Cd114	Cd115	Cd116	Cd117	Cd118	Cd119	Cd120	Cd121	Cd122	Cd123	Cd124	Cd125	Cd126	Cd127	Cd128	Cd129	Cd130	
Ag		Ag94	Ag95	Ag96	Ag97	Ag98	Ag99	Ag100	Ag101	Ag102	Ag103	Ag104	Ag105	Ag106	Ag107	Ag108	Ag109	Ag110	Ag111	Ag112	Ag113	Ag114	Ag115	Ag116	Ag117	Ag118	Ag119	Ag120	Ag121	Ag122	Ag123	Ag124	Ag125	Ag126	Ag127	Ag128	Ag129	Ag130
6.0x10 ⁻⁸ %		0+	3 s	9.2 s	16 s	49.1 s	1.2 m	5.5 m	7.3 m	57.7 m	55.5 m	5/2+	0+	6.50 h	462.6 d	5/2+	0+	1/2+	0+	9.3E+15 y	1/2+	0+	53.46 h	7.49	2.49 h	50.3 m	2.69 m	5.24 s	1.25 s	0.65 s	0.506 s	0.37 s	0.34 s	0.27 s	0.20 s			
112.411		ECp	EC	EC	ECp,ECα	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	
5.3x10 ⁻⁸ %		0+	ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	
107.8682		0+	0+	(8+,9+)	(9/2+)	(5+)	(9/2)+	(5)+	9/2+	5+	7/2+	5+	1/2-	1+	1/2-	1+	1/2-	1+	1/2-	2(-)	1/2-	1+	1/2-	2(-)	(1/2)-	(1)-	(7/2)+	0+	0+	(3/2+)	0+	(3/2+)	(3/2+)	0+	(3/2+)	0+	0+	
58x10 ⁻⁶ %		EC	ECp	ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC



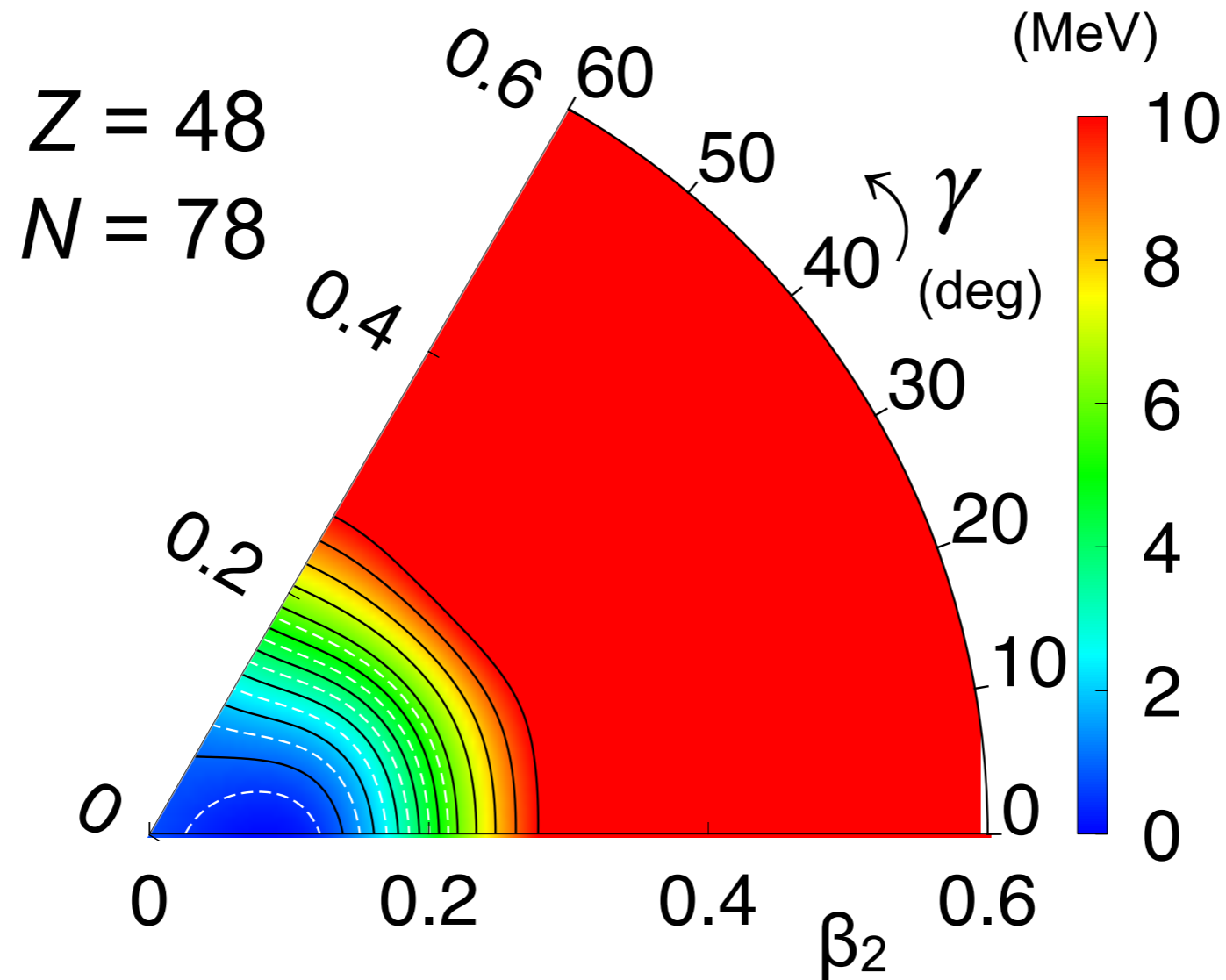
PN-VAP energy surfaces

48	Cd	Cd96	Cd97	Cd98	Cd99	Cd100	Cd101	Cd102	Cd103	Cd104	Cd105	Cd106	Cd107	Cd108	Cd109	Cd110	Cd111	Cd112	Cd113	Cd114	Cd115	Cd116	Cd117	Cd118	Cd119	Cd120	Cd121	Cd122	Cd123	Cd124	Cd125	Cd126	Cd127	Cd128	Cd129	Cd130
321.077 767	6.0x10 ⁻⁸ %	0+	3 s	9.2 s	16 s	49.1 s	1.2 m	5.5 m	7.3 m	57.7 m	55.5 m	0+	6.50 h	0+	462.6 d	0+	1/2+	0+	9.3E+15 y	0+	53.46 h	7.49	2.49 h	50.3 m	2.69 m	50.80 s	13.5 s	5.24 s	1.25 s	0+	0.37 s	0.34 s	0.27 s	0.20 s		
112.411 5.3x10 ⁻⁸ %	ECp	EC	EC	ECp,ECα	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	
961.78 2162	107.8682 5.8x10 ⁻⁶ %	0+	2.0 s	5.1 s	19 s	124 s	201 m	11.1 m	12.9 m	65.7 m	69.2 m	41.29 d	2396 m	237 m	1/2-	1/2-	1/2-	24.6 s	3.330 h	735 d	3.6 s	20.0 m	288 m	72.8 s	2.1 s	123 s	0.78 s	125 ms	107 ms	109 ms	0+	0+	0+	0+		
EC	EC	ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	



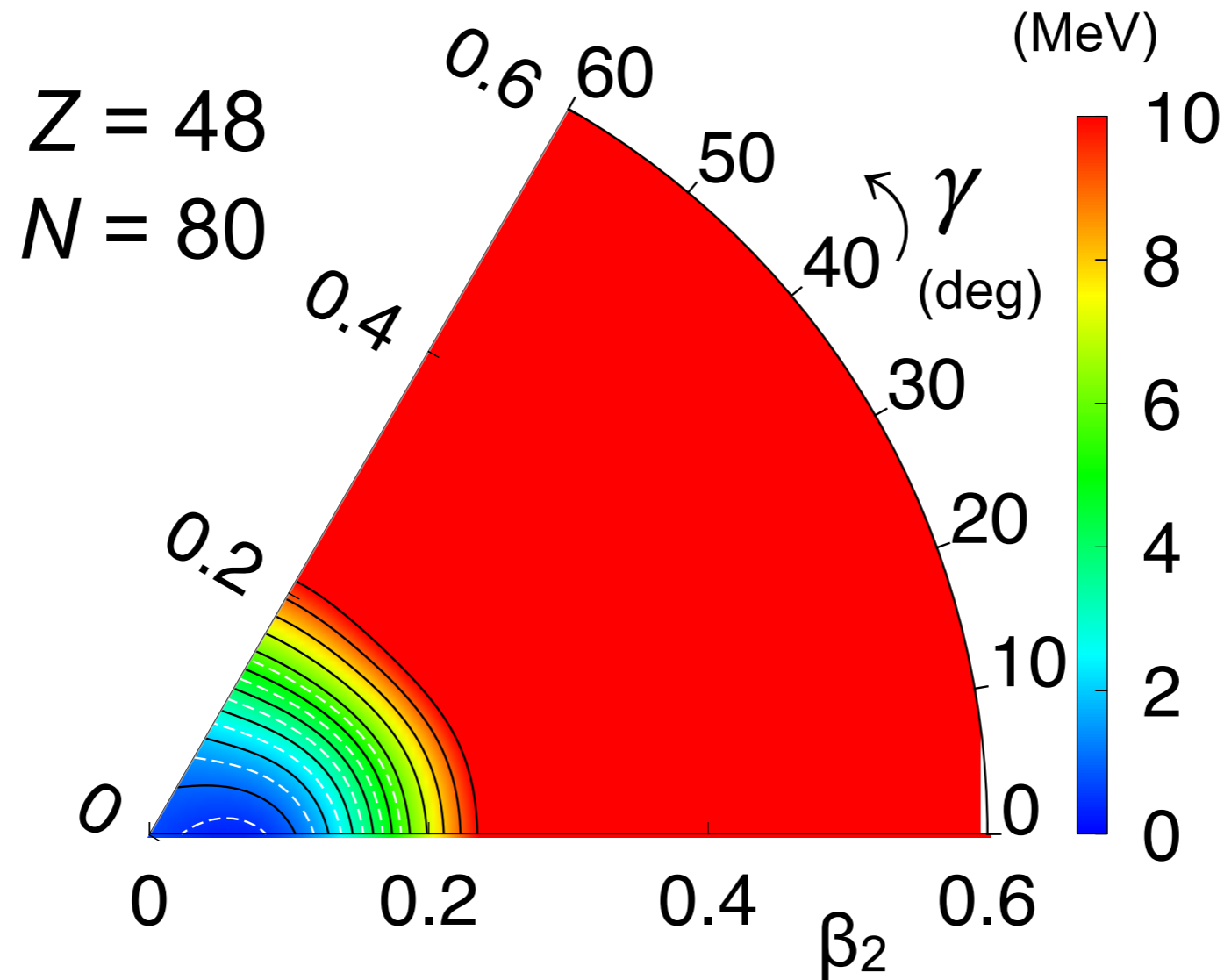
PN-VAP energy surfaces

48		Cd96		Cd97		Cd98		Cd99		Cd100		Cd101		Cd102		Cd103		Cd104		Cd105		Cd106		Cd107		Cd108		Cd109		Cd110		Cd111		Cd112		Cd113		Cd114		Cd115		Cd116		Cd117		Cd118		Cd119		Cd120		Cd121		Cd122		Cd123		Cd124		β ⁿ			
321.077 767		0+		3 s		9.2 s		16 s		49.1 s		1.2 m		5.5 m		7.3 m		57.7 m		55.5 m		1.25		6.50 h		462.6 d		462.6 d		12.49		12.80		24.43		53.46 h		7.49		2.49 h		50.3 m		50.80 s		13.5 s		5.24 s		2.10 s		1.25 s		0+		0.27 s		0.20 s		β ⁿ			
5.3x10 ⁻⁸ %		ECp		EC		ECp,ECα...		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		β ⁿ									
961.78 2162		Ag94		Ag95		Ag96		Ag97		Ag98		Ag99		Ag100		Ag101		Ag102		Ag103		Ag104		Ag105		Ag106		Ag107		Ag108		Ag109		Ag110		Ag111		Ag112		Ag113		Ag114		Ag115		Ag116		Ag117		Ag118		Ag119		Ag120		Ag121		Ag122		Ag123		β ⁿ	
112.411		0+		20 s		5.1 s		19 s		46.7 s		124 s		201 m		11.1 m		129 m		65.7 m		69.2 m		41.29 d		2396 m		237 m		24.6 s		31.30 h		257 h		3.6 s		20.0 m		288 m		72.8 s		2.1 s		1.23 s		0.78 s		0.48 s		0.309 s		β ⁿ									
5.3x10 ⁻⁸ %		ECp		ECp		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		β ⁿ									
107.8682		0+		(8+,9+)		(9/2+)		(5+)		(9/2)+		(5)+		9/2+		5+		7/2+		5+		1/2-		1+		1-		1+		1+		1+		1+		1+		1-		1-		1-		1-		1-		1-		β ⁿ													
58x10 ⁻⁶ %		EC		ECp		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		EC		β ⁿ													



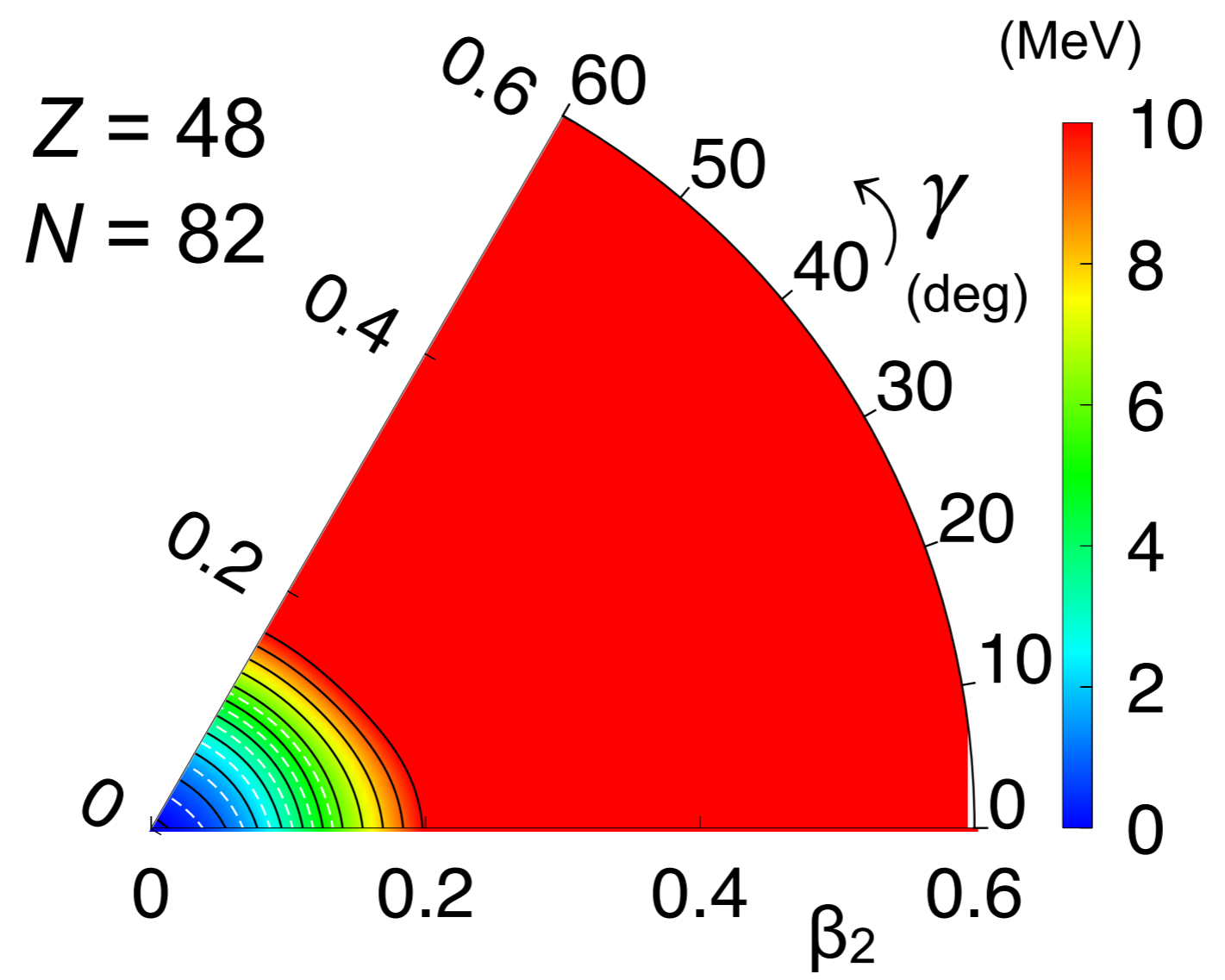
PN-VAP energy surfaces

	β-n																								β-									
	Cd128																								Cd130									
	0.34 s																								130									
	0+																								1+									
	β-																								β-									
	Ag127																																	
48	Cd	Cd96	Cd97	Cd98	Cd99	Cd100	Cd101	Cd102	Cd103	Cd104	Cd105	Cd106	Cd107	Cd108	Cd109	Cd110	Cd111	Cd112	Cd113	Cd114	Cd115	Cd116	Cd117	Cd118	Cd119	Cd120	Cd121	Cd122	Cd123	Cd124	Cd125	Cd126		
	321.077 767	0+	3 s	9.2 s	16 s	49.1 s	1.2 m	5.5 m	7.3 m	57.7 m	55.5 m	6.50 h	462.6 d	0+	462.6 d	57.2 h	12.49	12.80	24.43	53.46 h	7.49	2.49 h	50.3 m	2.69 m	50.80 s	13.5 s	5.24 s	2.10 s	1.25 s	0.65 s	0.506 s			
	112.411 5.3x10 ^{-8%}	0+	ECp	EC	ECp,ECα...	EC	EC	EC	EC	EC	EC	EC	EC	0+	1/2+	0+	1/2+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	
Ag	Ag94	Ag95	Ag96	Ag97	Ag98	Ag99	Ag100	Ag101	Ag102	Ag103	Ag104	Ag105	Ag106	Ag107	Ag108	Ag109	Ag110	Ag111	Ag112	Ag113	Ag114	Ag115	Ag116	Ag117	Ag118	Ag119	Ag120	Ag121	Ag122	Ag123	Ag124	Ag125	Ag126	
	961.78 2162	10 ms	20 s	51 s	19 s	124 s	201 m	11.1 m	12.9 m	65.7 m	69.2 m	41.29 d	2396 m	237 m	237 m	237 m	24.6 s	31.30 h	735 d	31.30 h	257 h	3.6 s	20.0 m	288 m	72.8 s	2.1 s	123 s	0.78 s	0.48 s	0.309 s	0.172 s	166 m		
	107.8682 58x10 ^{-6%}	0+	ECp	ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	



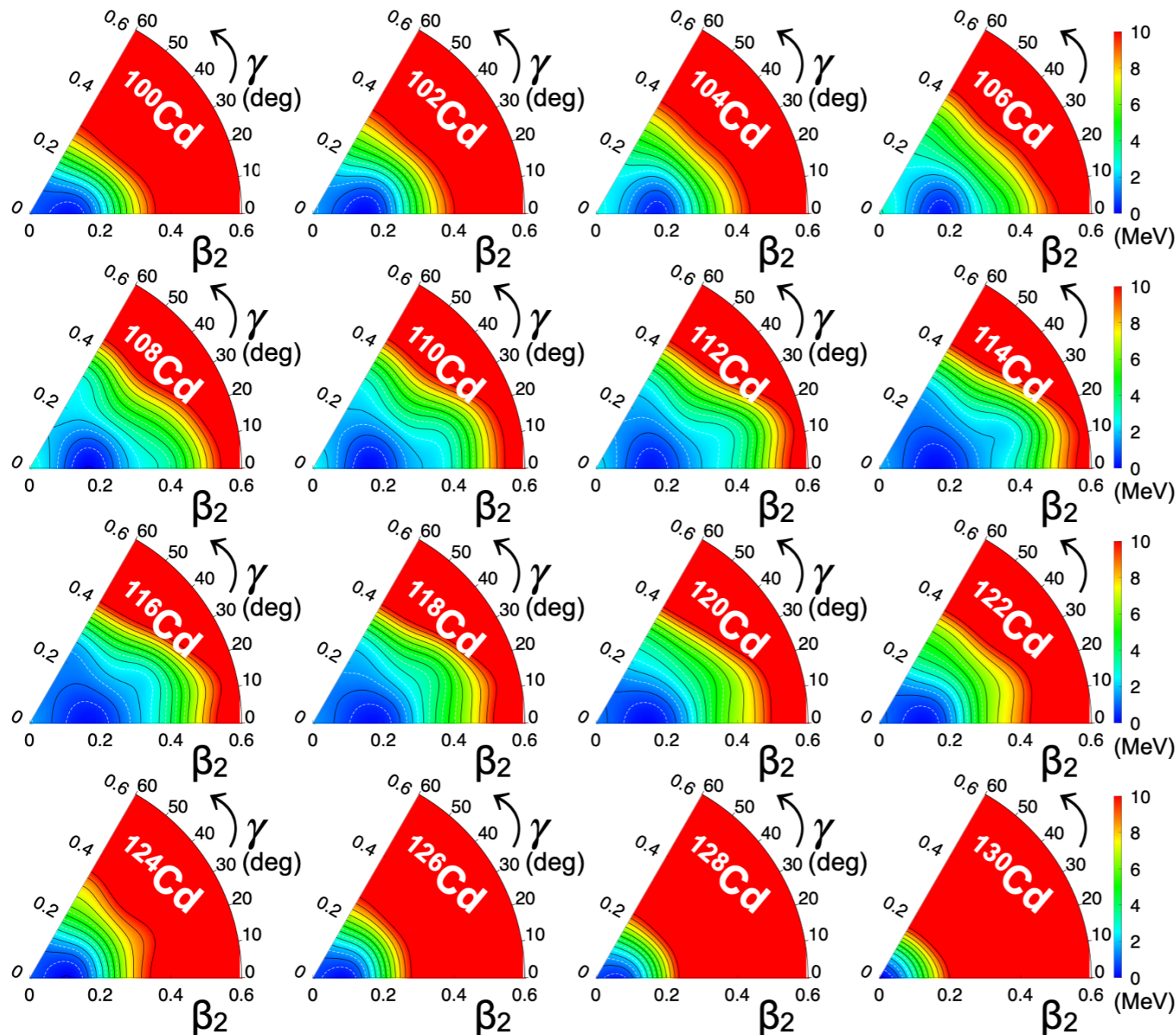
PN-VAP energy surfaces

48		Cd	Cd96	Cd97	Cd98	Cd99	Cd100	Cd101	Cd102	Cd103	Cd104	Cd105	Cd106	Cd107	Cd108	Cd109	Cd110	Cd111	Cd112	Cd113	Cd114	Cd115	Cd116	Cd117	Cd118	Cd119	Cd120	Cd121	Cd122	Cd123	Cd124	Cd125	Cd126	Cd127	Cd130
Ag		Ag94	Ag95	Ag96	Ag97	Ag98	Ag99	Ag100	Ag101	Ag102	Ag103	Ag104	Ag105	Ag106	Ag107	Ag108	Ag109	Ag110	Ag111	Ag112	Ag113	Ag114	Ag115	Ag116	Ag117	Ag118	Ag119	Ag120	Ag121	Ag122	Ag123	Ag124	Ag125	Ag126	
321.07 112.411 5.3x10 ⁻⁸ %		6.0x10 ⁻⁸ %	3 s	9.2 s	16 s	49.1 s	1.2 m	5.5 m	7.3 m	57.7 m	55.5 m	6.50 h	462.6 d	57.2 y	12.49	12.80	24.43	9.3E+15 y	6.50	53.46 h	7.49	2.49 h	50.3 m	2.69 m	50.80 s	13.5 s	5.24 s	2.10 s	1.25 s	0.65 s	0.506 s	0.37 s	0.20 s		
0+		0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	
EC		ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC
107.8682 58x10 ⁻⁶ %		10 ms	20 s	5.1 s	19 s	46.7 s	124 s	201 m	11.1 m	12.9 m	65.7 m	69.2 m	41.29 d	2396 m	237 m	20.0 m	24.6 s	3.30 h	735 d	3.30 h	537 h	4.6 s	20.0 m	288 m	72.8 s	2.1 s	1.23 s	0.48 s	0.309 s	0.172 s	0.172 s	166 ms	107 ms		
0+		0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+
EC		ECp	ECp	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC



PN-VAP energy surfaces

► Shape evolution in cadmium isotopes

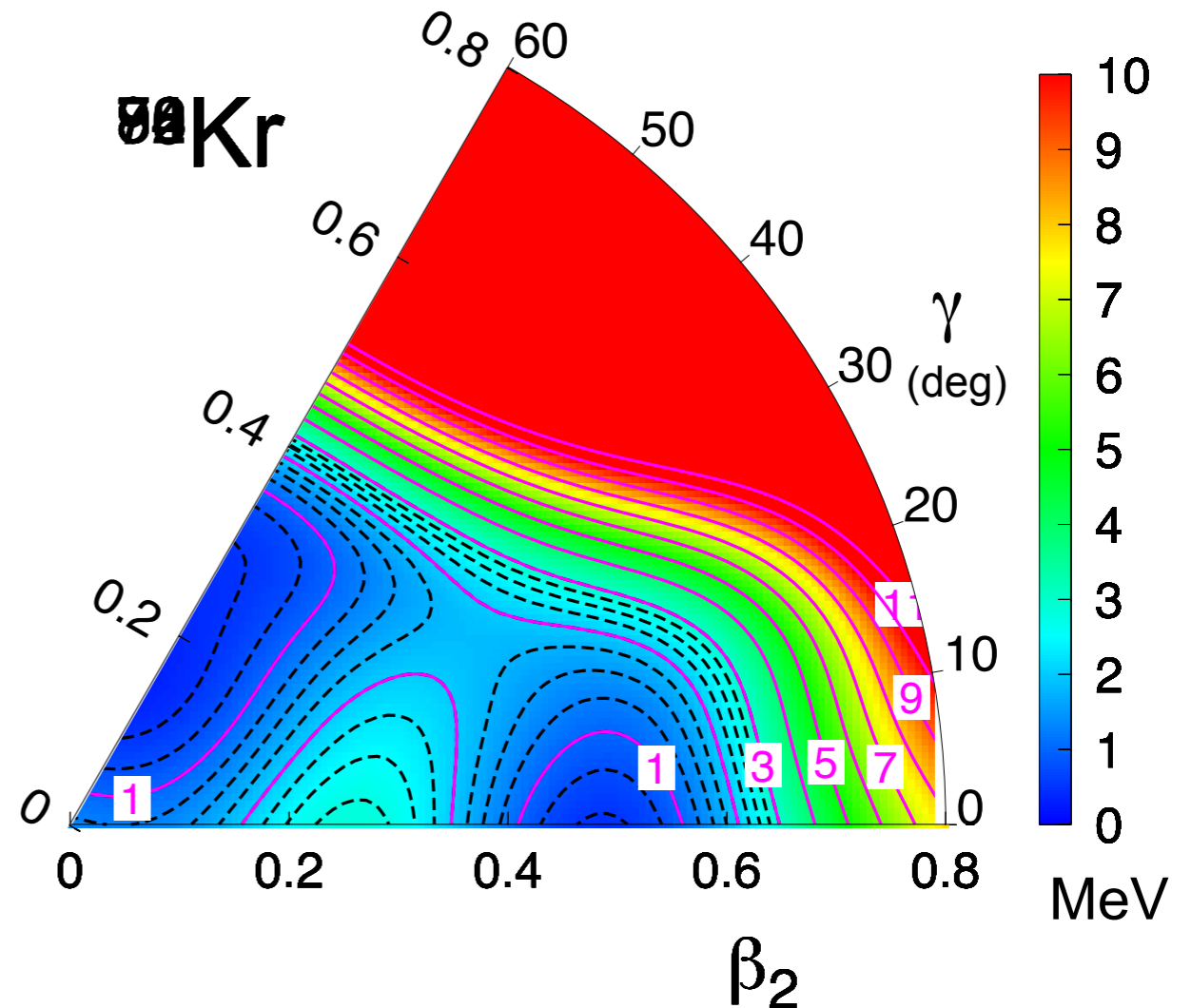


- ➔ Slightly prolate deformed minima are found along the whole isotopic chain.
- ➔ Deformation is larger (and almost constant) in the mid-shell and smaller when approaching to the magic neutron numbers ($N = 50, 82$).
- ➔ A depression at $\beta_2 \sim 0.35$, $\gamma \sim 20$ is found in $^{110-118}\text{Cd}$.

PN-VAP energy surfaces

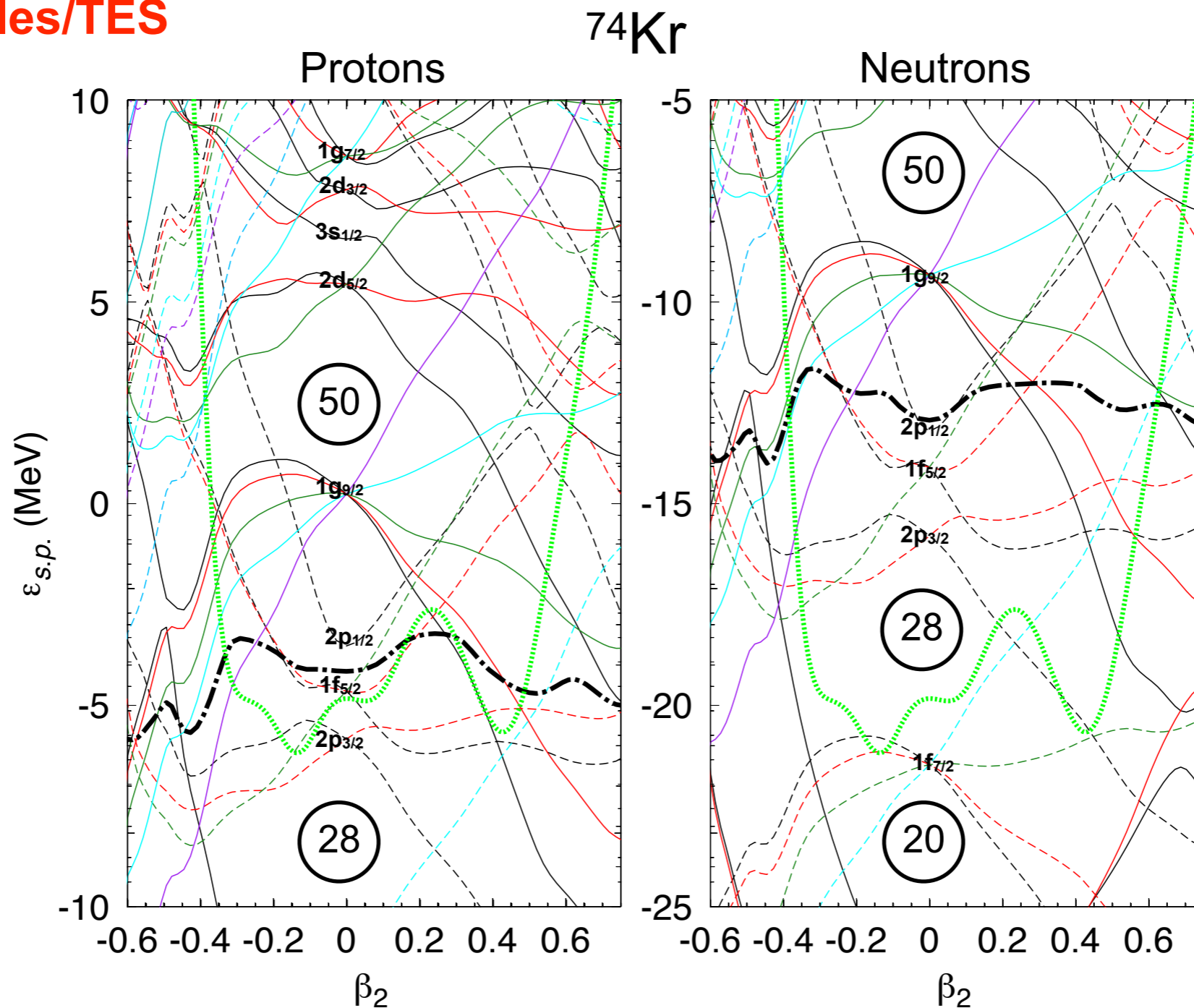
	Kr70	Kr72 17.2 s	Kr74 11.50 m	Kr76 14.8 h	Kr78	Kr80	Kr82	Kr84	Kr86	Kr88 2.84 h	Kr90 32.32 s	Kr92 1.840 s	Kr94 0.20 s	Kr96	Kr98
	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+	0+
		EC	EC	EC	0.35	2.25	11.6	57.0	17.3	β^-	β^-	β^-n	β^-n		

- ✓ Oblate shape in ^{70}Kr
- ✓ Two minima in $^{72-76}\text{Kr}$
- ✓ γ -softness in $^{78-80}\text{Kr}$
- ✓ Slightly prolate deformation in $^{82-84}\text{Kr}$
- ✓ Spherical semi-magic ^{86}Kr
- ✓ γ -softness in $^{88-92}\text{Kr}$
- ✓ Oblate shape in ^{94}Kr
- ✓ Oblate/prolate minima in $^{96-98}\text{Kr}$



Shell structure

Single particles/TES



Symmetry breaking

Let me be provocative: Nuclear shape comes as a non-observable degree of freedom (or an ‘artifact’) of the variational approximation at the mean-field level (one-body density approximation)

Remember:
$$\beta_k^\dagger = \sum_l U_{lk} c_l^\dagger + V_{lk} c_l$$

$$\delta E'_{\text{HF}} [|\Phi(\vec{q})\rangle] = 0 \quad \text{with} \quad E'_{\text{HF}} [|\Phi(\vec{q})\rangle] = \langle \Phi(\vec{q}) | \hat{H} - \lambda_N \hat{N} - \lambda_Z \hat{Z} - \vec{\lambda}_{\vec{q}} \cdot \hat{\vec{Q}} | \Phi(\vec{q}) \rangle$$

- $\beta_k(\vec{q}) | \Phi(\vec{q}) \rangle = 0$ trial wave functions are quasiparticle vacua
- $\lambda_N \rightarrow \langle \Phi(\vec{q}) | \hat{N} | \Phi(\vec{q}) \rangle = N$ Lagrange multiplier for neutrons
- $\lambda_Z \rightarrow \langle \Phi(\vec{q}) | \hat{Z} | \Phi(\vec{q}) \rangle = Z$ Lagrange multiplier for protons
- $\vec{\lambda}_{\vec{q}} \rightarrow \langle \Phi(\vec{q}) | \hat{\vec{Q}} | \Phi(\vec{q}) \rangle = \vec{q}$ Lagrange multipliers for collective coordinates

Beyond self-consistent mean-field

Symmetry restoration

- The Hamiltonian \hat{H} has certain symmetries

$$[\hat{H}, \hat{S}_i] = 0 \quad \text{with} \quad \hat{S}_i = (\hat{N}, \hat{Z}, \hat{J}^2, \hat{J}_z, \hat{\Pi}, \dots)$$

- However, the HFB wave function is not an eigenstate of the operators \hat{S}_i (in general)

$$\hat{S}_i |\Phi\rangle \neq c_{S_i} |\Phi\rangle$$

- A better approach to the actual eigenvalues/eigenstates of the Hamiltonian should recover the symmetries of the Hamiltonian **PROJECTION TECHNIQUES**

$$|\Phi\rangle \rightarrow \hat{P}^{S_i} |\Phi\rangle = |\Psi^{S_i}\rangle \quad \text{with} \quad \hat{S}_i |\Psi^{S_i}\rangle = c_{S_i} |\Psi^{S_i}\rangle$$

Projection operator

Beyond self-consistent mean-field

- **Examples of projection operators**

- Particle number:
$$P^N = \frac{1}{2\pi} \int_0^{2\pi} e^{i\varphi(\hat{N}-N)} d\varphi$$

- Parity:
$$P^\pi = \frac{1}{2} \left(\mathbb{I} - \pi \hat{\Pi} \right)$$

- Angular Momentum:
$$P_M^I = \sum_{K=-I}^{+I} g_K^I P_{MK}^I$$

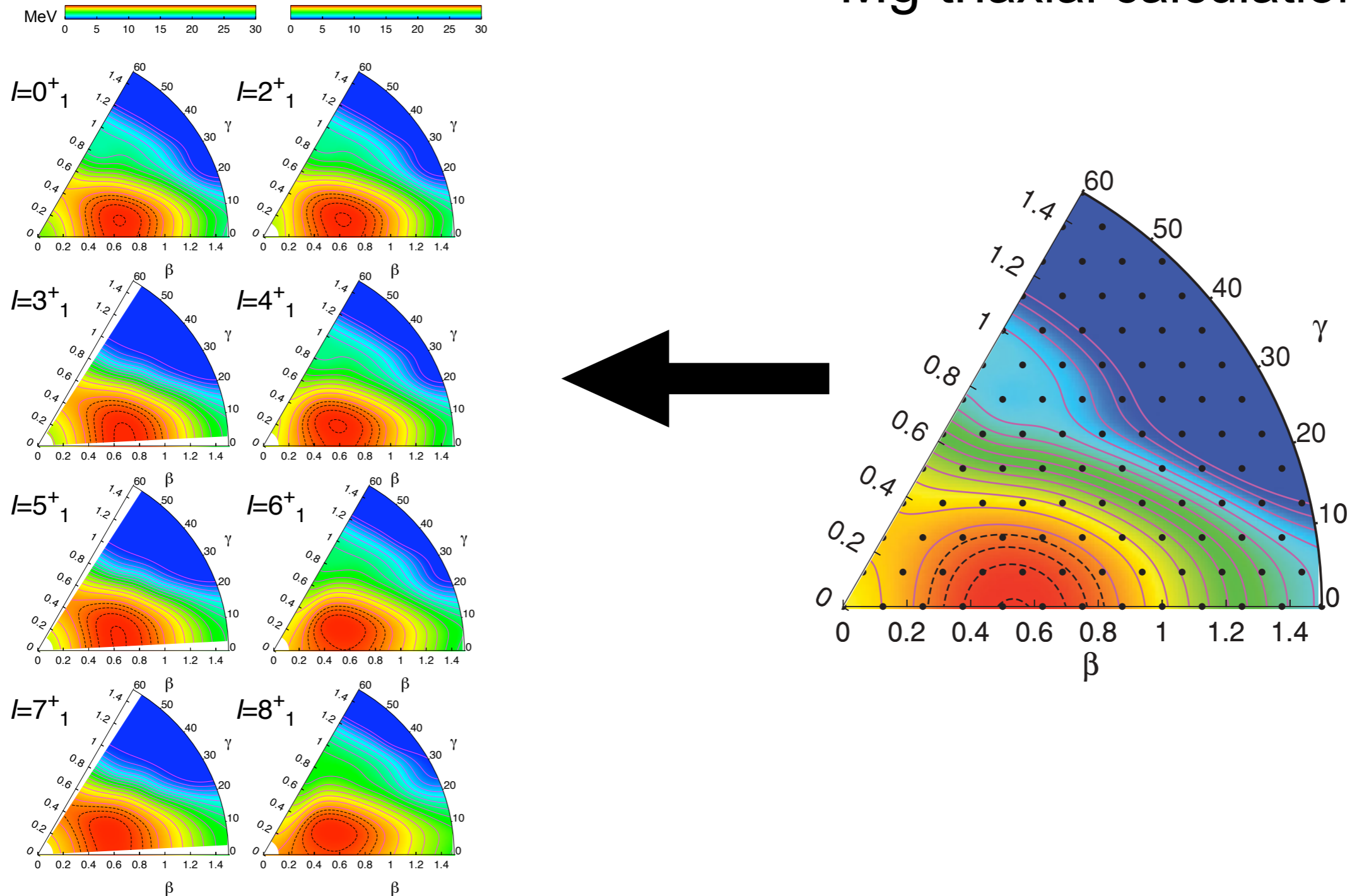
$$P_{MK}^I = \frac{2I+1}{8\pi^2} \int_0^{2\pi} d\alpha \int_0^\pi d\beta \sin \beta \int_0^{2\pi} d\gamma \mathcal{D}_{MK}^{I*}(\Omega) \hat{R}(\Omega) \quad \text{with } \Omega = (\alpha, \beta, \gamma) \text{ Euler angles}$$

$$\hat{R}(\Omega) = e^{-i\alpha \hat{J}_z} e^{-i\beta \hat{J}_y} e^{-i\gamma \hat{J}_z} \quad \text{Rotation operator}$$

$$\mathcal{D}_{MK}^I(\Omega) = \langle IM | \hat{R}(\Omega) | IK \rangle = e^{-i\gamma K} e^{-i\alpha M} d_{MK}^I(\beta) \quad \text{Wigner matrices}$$

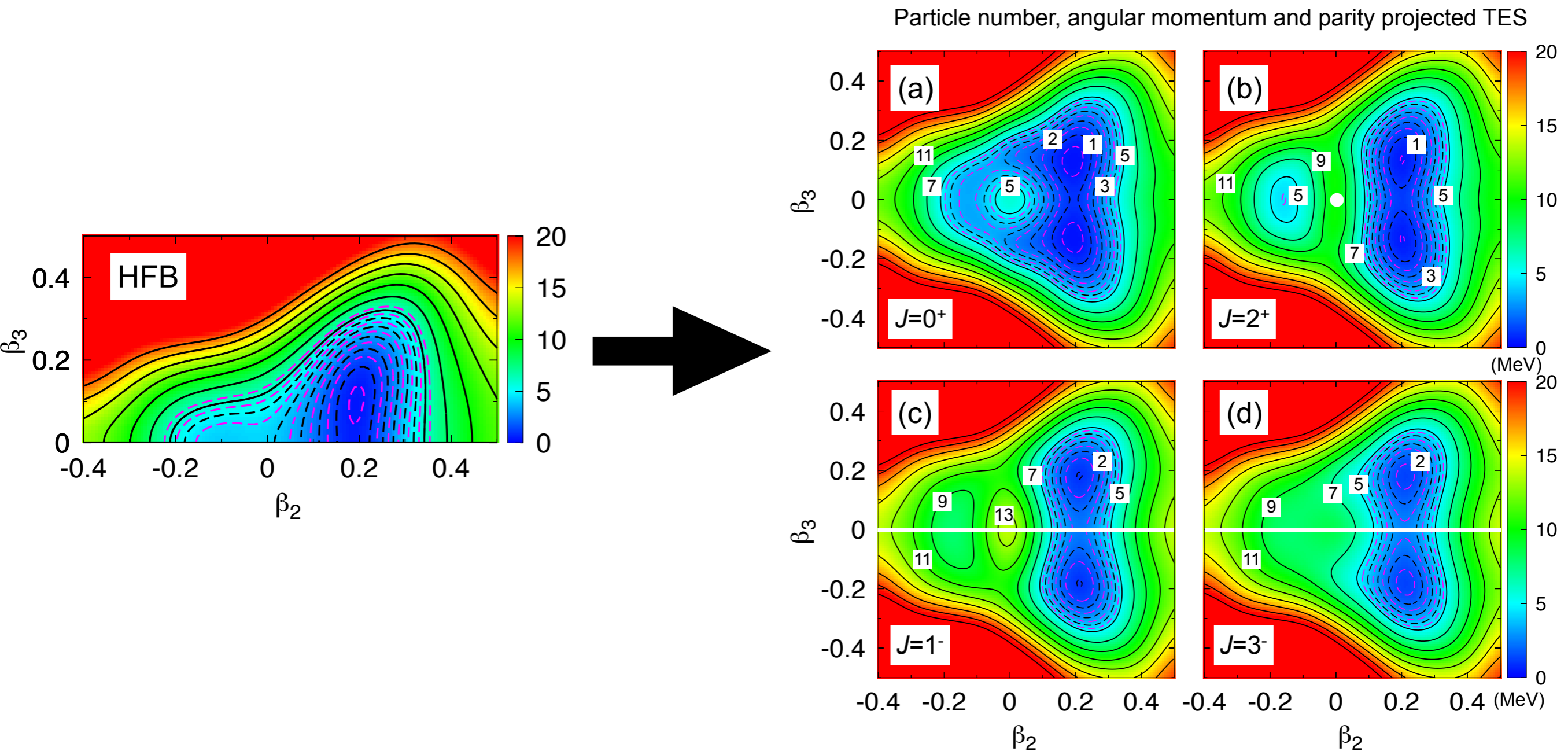
Beyond self-consistent mean-field

^{24}Mg triaxial calculations



Beyond self-consistent mean-field

^{144}Ba axial calculations



R. Bernard, L. M. Robledo, T. R. R., PRC (2016)

Configuration mixing: PGCM

Nuclear wave functions: Generator Coordinate Method (GCM) ansatz

$$|\Psi_{\sigma}^{JMNZ\pi}\rangle = \sum_{qK} f_{\sigma;qK}^{JMNZ\pi} P_M^J P^K^N P^Z P^{\pi} |\Phi(q)\rangle$$

$\Gamma \equiv (JMNZ\pi)$

linear combination

coefficients of the
linear combination

“basis” states

Configuration mixing: PGCM

Nuclear wave functions: Generator Coordinate Method (GCM) ansatz

$$|\Psi_{\sigma}^{JMNZ\pi}\rangle = \sum_{qK} f_{\sigma;qK}^{JMNZ\pi} P_{MK}^J P^N P^Z P^{\pi} |\Phi(q)\rangle$$

$\Gamma \equiv (JMNZ\pi)$

coefficients of the linear combination

The coefficients are obtained by minimizing the expectation value of the Hamiltonian (energy) with those coefficients as the variational parameters:

$$\sum_{q'K'} \left(\mathcal{H}_{qK,q'K'}^{\Gamma} - E_{\sigma}^{\Gamma} \mathcal{N}_{qK,q'K'}^{\Gamma} \right) f_{\sigma;q'K'}^{\Gamma} = 0$$

Hill-Wheeler-Griffin (HWG) equation

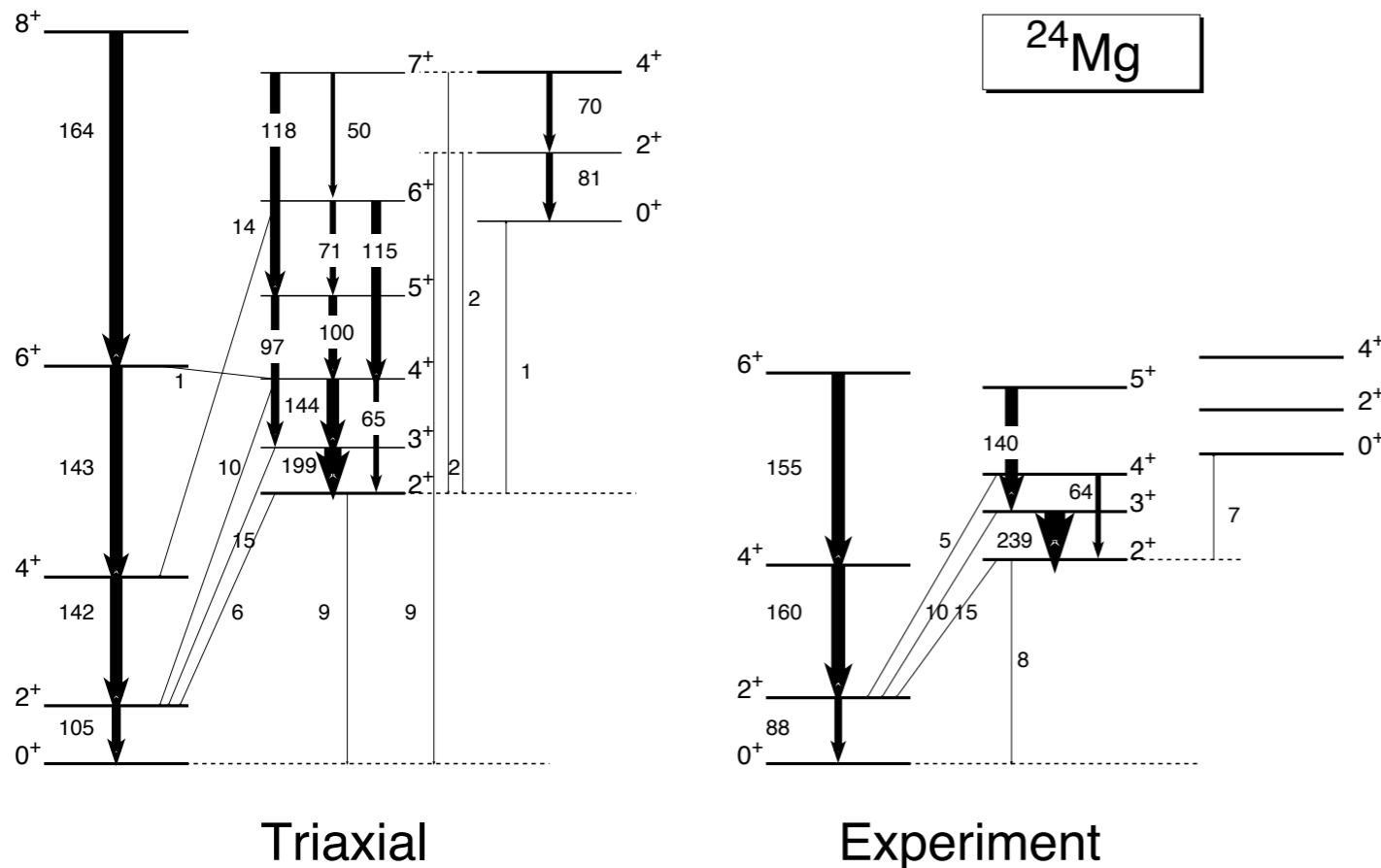
$$\mathcal{H}_{qK,q'K'}^{\Gamma} = \langle \Phi(q) | \hat{H} P_{KK'}^J P^N P^Z P^{\pi} | \Phi(q') \rangle,$$

$$\mathcal{N}_{qK;q'K'}^{\Gamma} = \langle \Phi(q) | P_{KK'}^J P^N P^Z P^{\pi} | \Phi(q') \rangle$$

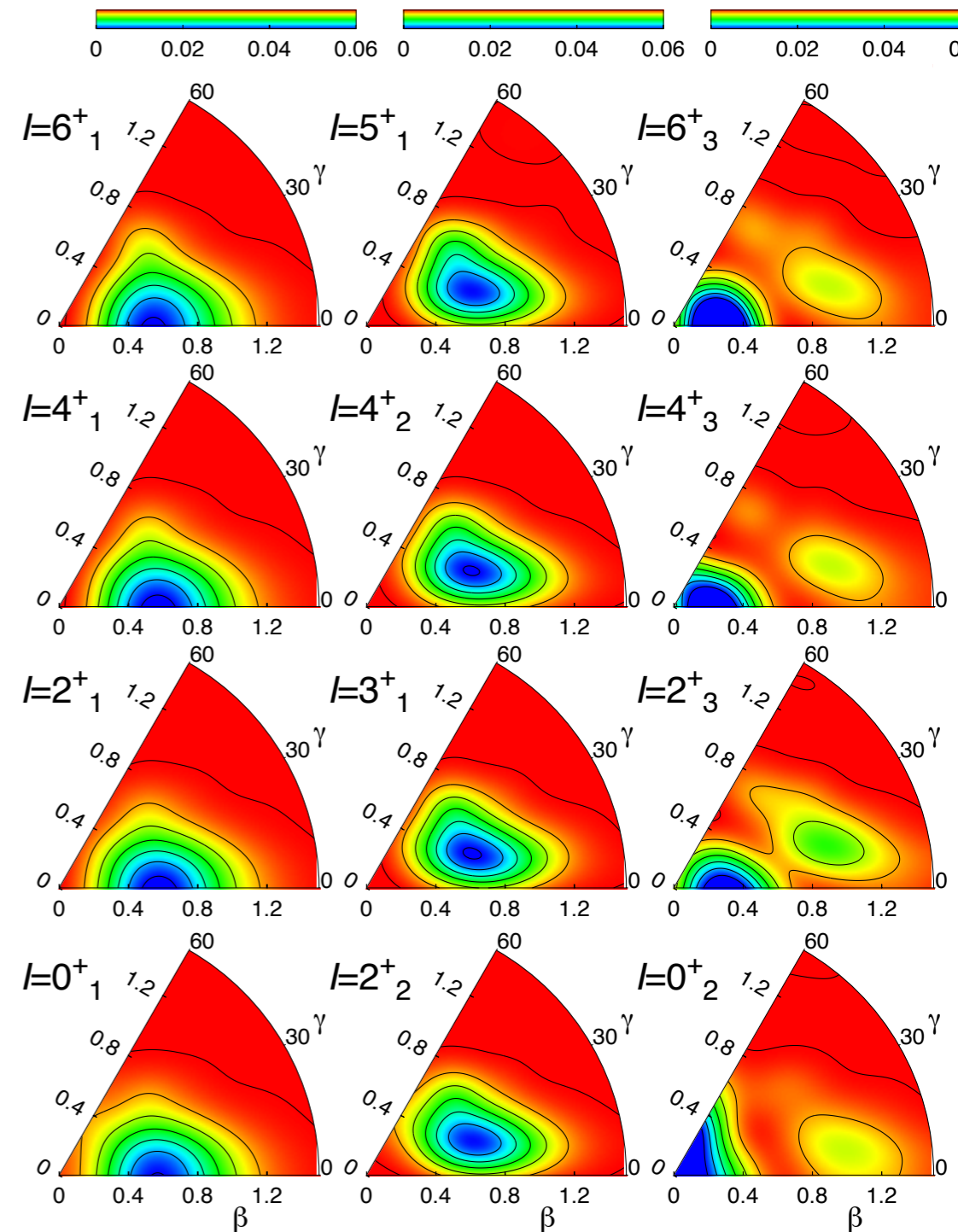
Hamiltonian and norm kernels

Configuration mixing: PGCM

^{24}Mg triaxial calculations



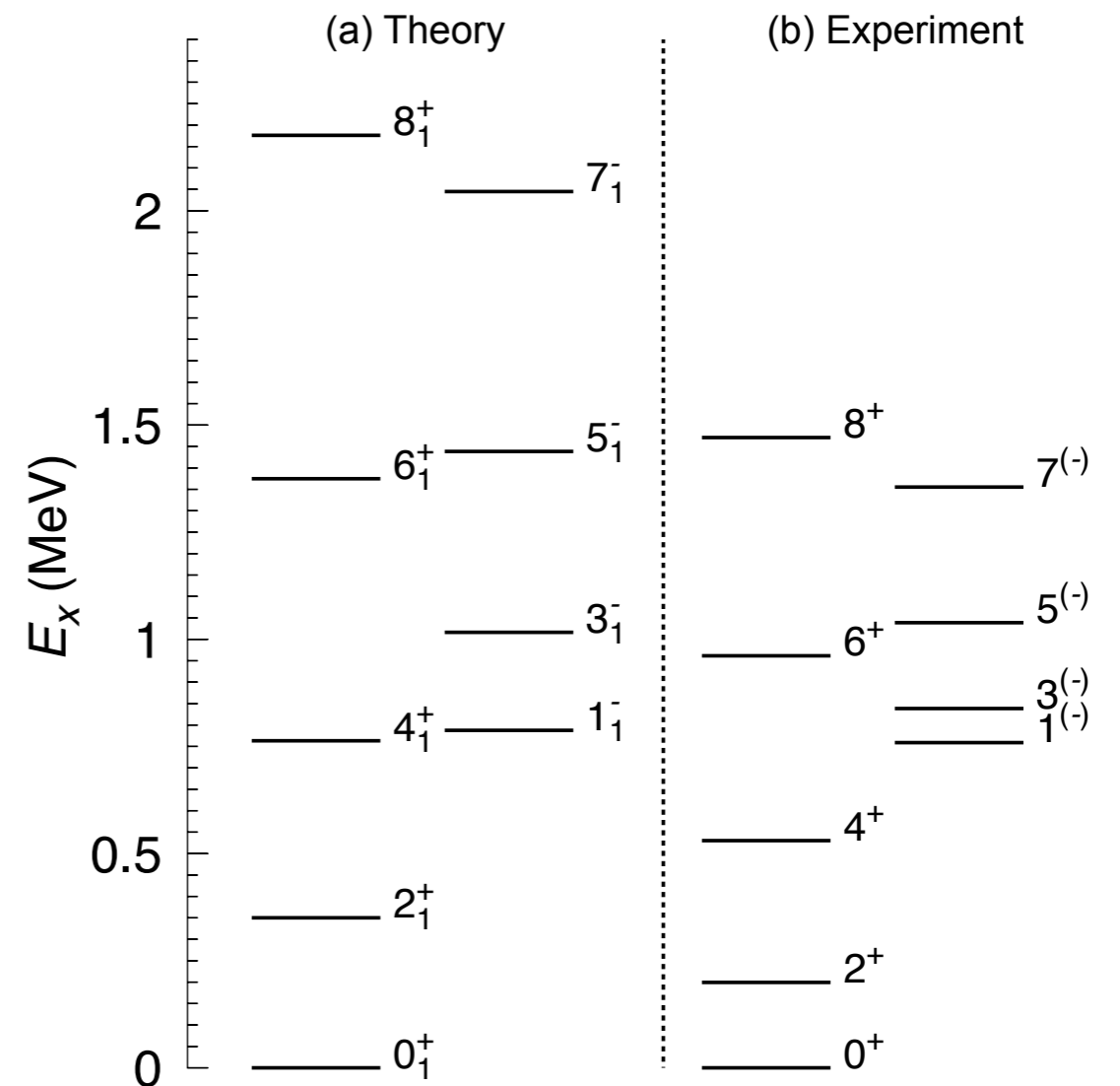
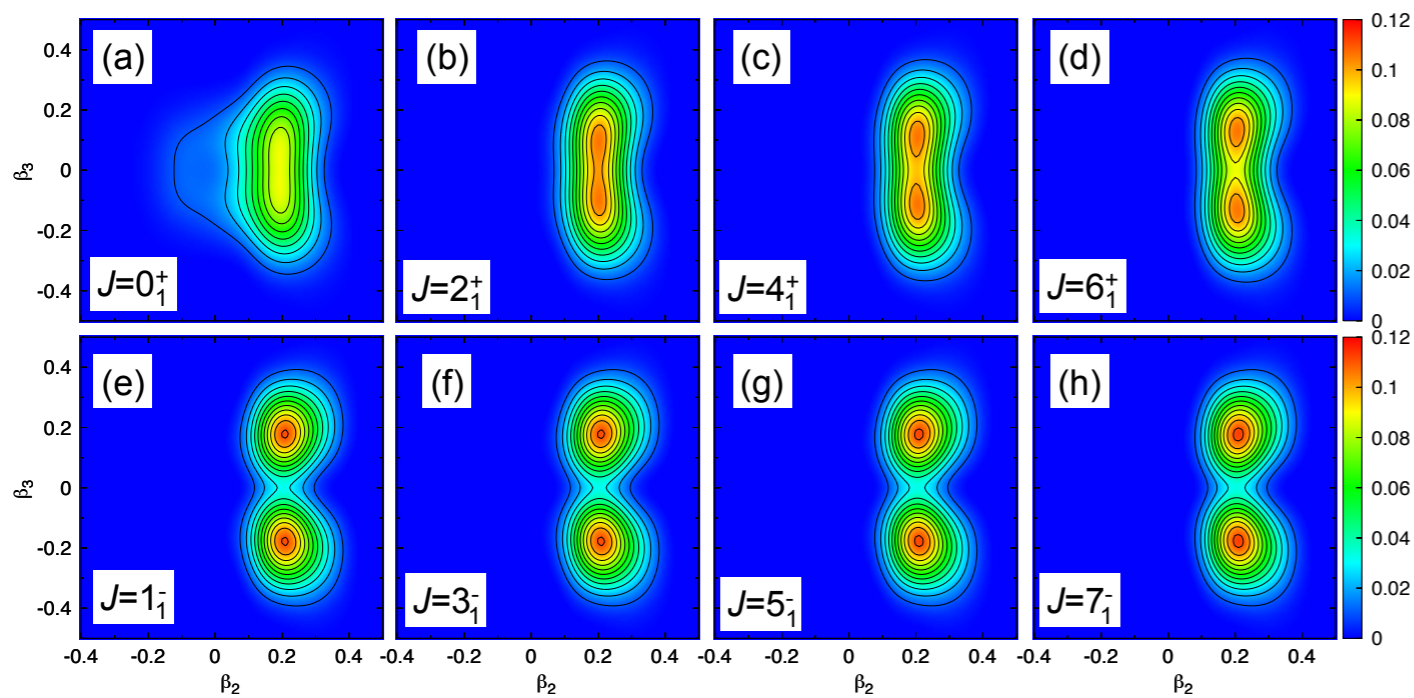
collective wave functions



Configuration mixing: PGCM

^{144}Ba axial calculations

collective wave functions



Configuration mixing: PGCM

- Nuclear wave functions wave functions: Generator Coordinate Method (GCM) ansatz

$$|\Psi_{\sigma}^{JMNZ\pi}\rangle = \sum_{qK} f_{\sigma;qK}^{JMNZ\pi} P_M^J P^K P^N P^Z P^{\pi} |\Phi(q)\rangle$$

$\Gamma \equiv (JMNZ\pi)$

Remark: we are (computational) limited to explore only certain degrees of freedom