Renormalization of the gravitational formfactors, a QED example

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Based on

Rodini et al., JHEP 09 (2020) 067

Metz et al., PLB 820 (2021) 136501

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Outline

Renormalization procedure for multiple local operators

Scheme dependence and scheme choice

QED form factors: forward and off-forward cases

What I will NOT talk about

Different mass decompositions
Interpretation of the operators

These will be covered in A. Metz and C. Lorcé seminars

Definition of the Energy-Momentum Tensor

Lagrangian renormalization is understood

$$T^{\mu\nu} = \mathcal{O}_1 + \frac{\mathcal{O}_2}{4} + \mathcal{O}_3$$

Hatta et al., JHEP 12 (2018) 008

Tanaka, JHEP 01 (2019) 120

$$\mathcal{O}_1 = -F^{\mu\alpha}F^{\nu}{}_{\alpha} \qquad \mathcal{O}_2 = g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$

$$\mathcal{O}_3 = \frac{i}{4}\,\bar{\psi}\,\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\,\psi \qquad \mathcal{O}_4 = g^{\mu\nu}m\bar{\psi}\psi$$

Renormalize fields, coupling and masses is not enough.

We must also renormalize the operators:

$$\mathcal{O}_{1,R} = Z_T\mathcal{O}_1 + Z_M\mathcal{O}_2 + Z_L\mathcal{O}_3 + Z_S\mathcal{O}_4$$
$$\mathcal{O}_{2,R} = Z_F\mathcal{O}_2 + Z_C\mathcal{O}_4$$
$$\mathcal{O}_{3,R} = Z_\psi\mathcal{O}_3 + Z_K\mathcal{O}_4 + Z_Q\mathcal{O}_1 + Z_B\mathcal{O}_2$$
$$\mathcal{O}_{4,R} = \mathcal{O}_4$$

The total EMT is not affected by the additional renormalization

$$T_R^{\mu\nu} = T^{\mu\nu}$$

$$T^{\mu}{}_{\mu} = (T_R)^{\mu}{}_{\mu} = (T^{\mu}{}_{\mu})_R = (1+\gamma_m)(m\bar{\psi}\psi)_R + \frac{\beta}{2e}(F^{\alpha\beta}F_{\alpha\beta})_R$$

However, in general, trace and renormalization do not commute

$$\mathrm{Tr}[O_R^{\mu\nu}] \neq (\mathrm{Tr}[O^{\mu\nu}])_R$$

In particular $\quad T_{e,R}^{\mu\nu} = \mathcal{O}_{3,R} \qquad T_{\gamma,R}^{\mu\nu} = \mathcal{O}_{1,R} + \frac{\mathcal{O}_{2,R}}{4}$

$$(T_{e,R})^{\mu}{}_{\mu} = (1+y)\left(m\bar{\psi}\psi\right)_R + x\left(F^{\alpha\beta}F_{\alpha\beta}\right)_R$$

$$(T_{\gamma,R})^{\mu}{}_{\mu} = (\gamma_m - y)\left(m\bar{\psi}\psi\right)_R + \left(\frac{\beta}{2e} - x\right)\left(F^{\alpha\beta}F_{\alpha\beta}\right)_R$$

Hatta et al., JHEP 12 (2018) 008

Tanaka, JHEP 01 (2019) 120

How to fix the counterterms (1)

Two main goals

1) Exploiting as much as possible known results

One renormalizes only the operators that ‘lives’ in separate representation of the 4-dimensional Lorentz group

2) Construct operators with a specific physical meaning

One renormalizes operators with unspecified Lorentz representation

The two methods are COMPLETELY equivalent

From 1) one can derive 2), up to finite multiplicative renormalization!

How to fix the counterterms (2)

$Z_{F,C}$ Are known from R. Tarrach, Nucl. Phys. B 196 (1982) 45

$Z_{L,T,Q,\psi}$ Are given by the evolution equations for the second moment of the flavor-singlet unpolarized parton distributions. Tanaka, JHEP 01 (2019) 120

$$\mathrm{AMF}(\mathrm{R}\,[e]) = Z_1 \mathrm{AMF}(e) + Z_2 \mathrm{AMF}(\gamma),$$
$$\mathrm{AMF}(\mathrm{R}\,[\gamma]) = Z_3 \mathrm{AMF}(\gamma) + Z_4 \mathrm{AMF}(e)$$

Average Moment Fractions (AMF) from PDFs

$$\mathrm{R}\,[\mathrm{AMF}(e)] = Z_\psi \mathrm{AMF}(e) + Z_Q \mathrm{AMF}(\gamma),$$
$$\mathrm{R}\,[\mathrm{AMF}(\gamma)] = Z_T \mathrm{AMF}(\gamma) + Z_L \mathrm{AMF}(e)$$

Average Moment Fractions (AMF) from EMT (++ components)

$$Z_\psi^{[\epsilon]} = Z_1^{[\epsilon]} \quad Z_Q^{[\epsilon]} = Z_2^{[\epsilon]}$$
$$Z_T^{[\epsilon]} = Z_3^{[\epsilon]} \quad Z_L^{[\epsilon]} = Z_4^{[\epsilon]}$$

The counterterms have the same divergent part

How to fix the counterterms (3)

The other counterterms are not independent!

$$Z_T + Z_Q = 1 \qquad Z_L + Z_\psi = 1$$

$$Z_M + Z_B + \frac{Z_F}{4} = \frac{1}{4} \qquad Z_S + Z_K + \frac{Z_C}{4} = 0$$

$$Z_M = \frac{Z_T}{d} - \frac{Z_F}{d}\left(1 - \frac{\beta}{2g} + x\right)$$

$$Z_S = -\frac{Z_L}{d} - \frac{Z_C}{d}\left(1 - \frac{\beta}{2g} + x\right) - \frac{y - \gamma_m}{d}$$

$$Z_B = \frac{Z_Q}{d} + \frac{x}{d} Z_F$$

$$Z_K = -\frac{Z_\psi}{d} + \frac{x}{d} Z_C + \frac{1 + y}{d}$$

Hatta et al., JHEP 12 (2018) 008

$$\widetilde{\mathcal{O}}_{1,R} = \mathcal{O}_{1,R} + \frac{1}{4}\left(1 - \frac{\beta}{2e} + x\right)\mathcal{O}_{2,R} + \frac{y-\gamma_m}{4}\mathcal{O}_{4,R},$$
$$\widetilde{\mathcal{O}}_{3,R} = \mathcal{O}_{3,R} - \frac{x}{4}\mathcal{O}_{2,R} - \frac{1+y}{4}\mathcal{O}_{4,R}$$

Tanaka, JHEP 01 (2019) 120

Hatta et al., JHEP 12 (2018) 008

Fixing x and y it corresponds to choose a scheme

$$(T_{e,R})^{\mu}{}_{\mu} = (1+y)\left(m\bar{\psi}\psi\right)_R + x\left(F^{\alpha\beta}F_{\alpha\beta}\right)_R$$
$$(T_{\gamma,R})^{\mu}{}_{\mu} = (\gamma_m - y)\left(m\bar{\psi}\psi\right)_R + \left(\frac{\beta}{2e} - x\right)\left(F^{\alpha\beta}F_{\alpha\beta}\right)_R$$

MS-like schemes   Hatta et al., JHEP 12 (2018) 008   Tanaka, JHEP 01 (2019) 120

D1 $x = 0 \quad y = \gamma_m$ Metz et al., Phys. Rev. D102 (2020) 114042

D2 $x = 0 \quad y = 0$ Rodini et al., JHEP 09 (2020) 067

Alternative construction of MSbar scheme   Metz et al., Phys. Rev. D102 (2020) 114042

MS-like schemes

Tanaka, JHEP 01 (2019) 120

Impose vanishing finite contributions to the derived counterterms

$$Z_X = \delta_{X,T} + \delta_{X,\psi} + \delta_{X,F} + \frac{a_X}{\epsilon} + \frac{b_X}{\epsilon^2} + \frac{c_X}{\epsilon^3} + \dots$$

From the definition of $Z_{M,S}$

$$\frac{1}{32}\left[(8 + 4a_T + 2b_T + c_T + \dots) - \left(1 + x - \frac{\beta}{2e}\right)(8 + 4a_F + 2b_F + c_F + \dots)\right] = 0$$

$$\frac{1}{32}\left[-(4a_L + 2b_L + c_L + \dots) - \left(1 + x - \frac{\beta}{2e}\right)(4a_C + 2b_C + c_C + \dots) + 8(\gamma_m - y)\right] = 0$$

One way to think about Msbar is to consider
MS scheme evolved at a higher scale
Or…

Variant of MSbar

Metz et al., Phys. Rev. D102 (2020) 114042

MSbar counterterms with finite contributions, which are uniquely determined by the divergent part

$$Z\Big|_{\overline{\mathrm{MS}}} = (1,0) + \alpha\,\frac{\bar{a}_1}{\epsilon}\,S_\epsilon + \alpha^2\left(\frac{\bar{b}_2}{\epsilon^2} + \frac{\bar{b}_1}{\epsilon}\right)S_\epsilon^2 + \alpha^3\left(\frac{\bar{c}_3}{\epsilon^3} + \frac{\bar{c}_2}{\epsilon^2} + \frac{\bar{c}_1}{\epsilon}\right)S_\epsilon^3$$

$$S_\epsilon\Big|_{\overline{\mathrm{MS}}_1} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}$$

$$Z_B = \frac{Z_Q}{d} + \frac{x}{d}Z_F$$

$$S_\epsilon\Big|_{\overline{\mathrm{MS}}_2} = (4\pi e^{-\gamma_E})^\epsilon$$

$$x = x_1\alpha + x_2\alpha^2 + \ldots$$

What we find (in MSbar 1)

$$O(\alpha_s) : \frac{1}{8}\Big(\bar{a}_{1,Q} + 2\bar{a}_{1,Q}\,(\log(4\pi) - \gamma_E) + 2x_1\Big)$$

What we want

$$O(\alpha_s) : \frac{1}{4}\,\bar{a}_{1,Q}(\log(4\pi) - \gamma_E)$$

EMT matrix elements

Ji, Phys. Rev. Lett. 78 (1997) 610

$$\langle \mathcal{O} \rangle = \langle P|\mathcal{O}|P\rangle$$

$$\langle e\left(p'\right), s'|\, T^{\mu\nu}_{i,R}\, |e\left(p\right), s\rangle = \left\langle e\left(P+\frac{\Delta}{2}\right), s'\middle| T^{\mu\nu}_{i,R} \middle| e\left(P-\frac{\Delta}{2}\right), s\right\rangle$$

$$= \bar{u}'\left(A_i(\Delta^2)\frac{P^\mu P^\nu}{m} + J_i(\Delta^2)\frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2m}\right.$$

$$\left. + D_i(\Delta^2)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4m} + m\bar{C}_i(\Delta^2)g^{\mu\nu} + C_i(\Delta^2)P^{[\mu}\gamma^{\nu]}\right)u$$

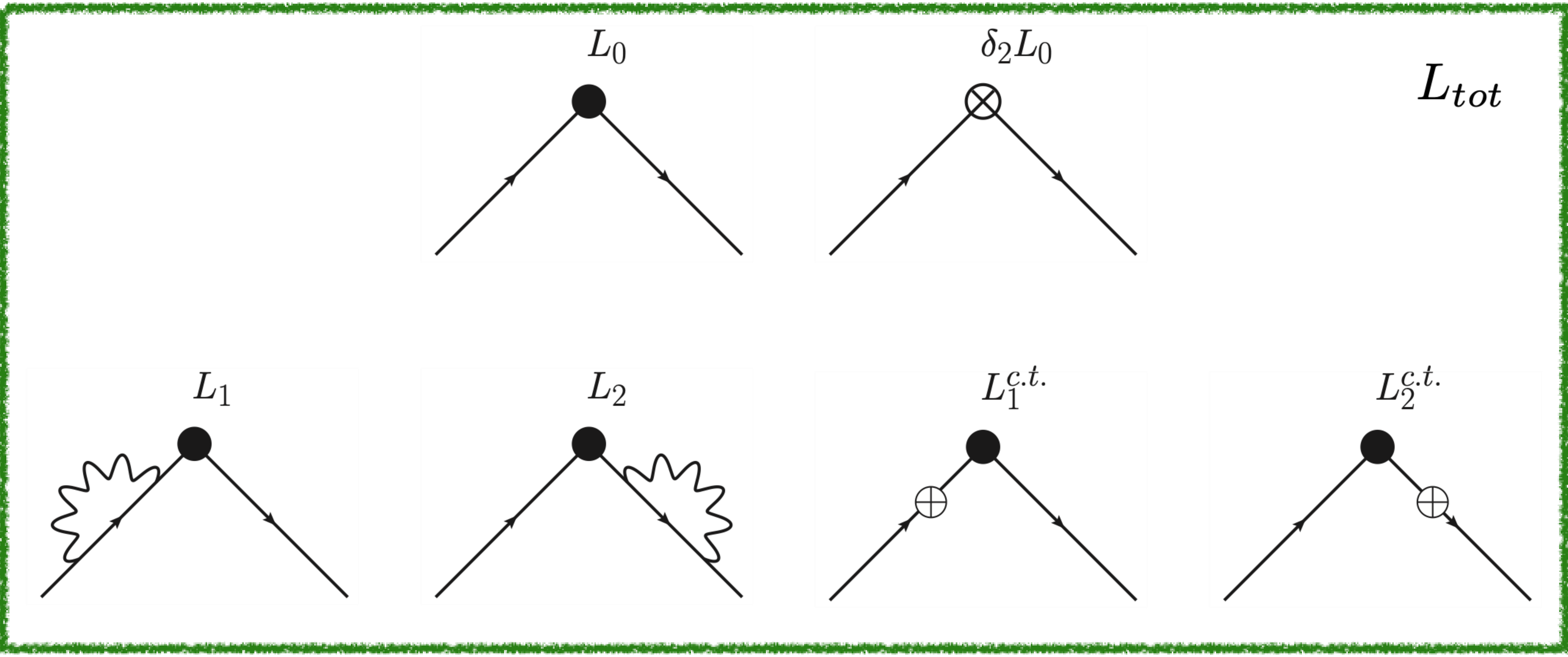
Two form factors for the forward limit

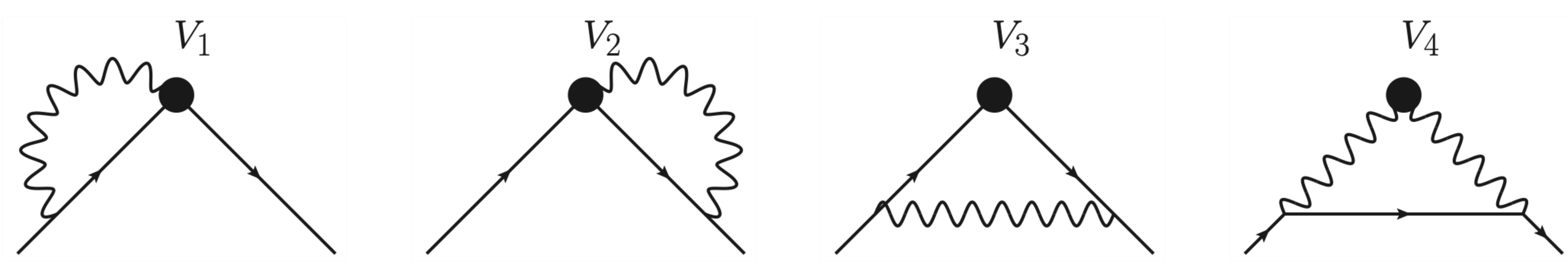
$$\langle\, T^{\mu\nu}_{i,R}\,\rangle = 2P^\mu P^\nu A_i(0) + 2M^2 g^{\mu\nu}\bar{C}_i(0)$$

$$i = e, \gamma$$

$$A_e(0) + A_\gamma(0) = 1 \qquad \bar{C}_e(0) + \bar{C}_\gamma(0) = 0$$

How to obtain the form factors (QED)





QED

$$Z_T + Z_Q = 1$$
$$Z_L + Z_\psi = 1$$
$$Z_M + Z_B + \frac{Z_F}{4} = \frac{1}{4}$$
$$Z_S + Z_K + \frac{Z_C}{4} = 0$$

$$x = 0 \qquad y = \begin{cases} \frac{\alpha}{3\pi} & \overline{\mathrm{MS}} \\ \frac{3\alpha}{2\pi} & \mathrm{D}_1 \\ 0 & \mathrm{D}_2 \end{cases}$$

$$Z_F = 1 + \frac{\beta}{e}\Delta_{\mathrm{UV}} \qquad Z_C = 2\gamma_m \Delta_{\mathrm{UV}}$$

$$Z_T = 1 \qquad Z_Q = 0 \qquad Z_\psi = 1 + \frac{2\alpha}{3\pi}\Delta_{\mathrm{UV}}$$

$$Z_L = -\frac{2\alpha}{3\pi}\Delta_{\mathrm{UV}} \qquad Z_M = -\frac{\alpha}{12\pi}\Delta_{\mathrm{UV}} \qquad Z_B = 0$$

$$Z_S = -\frac{7\alpha}{12\pi}\Delta_{\mathrm{UV}} \begin{cases} +0 & \overline{\mathrm{MS}} \\ -\frac{7\alpha}{24\pi} & \mathrm{D}_1 \\ +\frac{\alpha}{12\pi} & \mathrm{D}_2 \end{cases} \qquad Z_K = -\frac{\alpha}{6\pi}\Delta_{\mathrm{UV}} \begin{cases} +0 & \overline{\mathrm{MS}} \\ +\frac{7\alpha}{24\pi} & \mathrm{D}_1 \\ -\frac{\alpha}{12\pi} & \mathrm{D}_2 \end{cases}$$

$$\Delta_{\mathrm{UV}} = \frac{1}{\varepsilon} + \log(4\pi) - \gamma_{\mathrm{E}}$$

Rodini et al., JHEP 09 (2020) 067

How to obtain the form factors: QED (1/2)

1-loop calculation!

$$\mathcal{L} = \log\left(\frac{\mu^2}{m^2}\right)$$

$$L_{tot,R}\,(\Delta = 0) = 2P^\mu P^\nu \left(1 + \frac{\alpha}{\pi\varepsilon_I} - \frac{\alpha}{\pi} - \frac{\alpha\mathcal{L}}{4\pi}\right)$$

$$(V_1 + V_2)_R\,(\Delta = 0) = 2P^\mu P^\nu \left(-\frac{\alpha\mathcal{L}}{2\pi} - \frac{3\alpha}{2\pi}\right) - 2m^2 g^{\mu\nu} \times \begin{cases} \left(\frac{\alpha\mathcal{L}}{4\pi} + \frac{\alpha}{4\pi}\right) & \overline{\mathrm{MS}} \\ \left(\frac{\alpha\mathcal{L}}{4\pi} - \frac{\alpha}{24\pi}\right) & \mathrm{D}_1 \\ \left(\frac{\alpha\mathcal{L}}{4\pi} + \frac{\alpha}{3\pi}\right) & \mathrm{D}_2 \end{cases}$$

$$V_{3,R}\,(\Delta = 0) = 2P^\mu P^\nu \left(-\frac{\alpha}{\pi\varepsilon_I} + \frac{14\alpha}{9\pi} + \frac{\alpha\mathcal{L}}{12\pi}\right) + 2m^2 g^{\mu\nu} \left(\frac{5\alpha\mathcal{L}}{12} + \frac{7\alpha}{36\pi}\right)$$

$$V_{4,R}\,(\Delta = 0) = 2P^\mu P^\nu \left(\frac{2\alpha\mathcal{L}}{3\pi} + \frac{17\alpha}{18\pi}\right) + 2m^2 g^{\mu\nu} \times \begin{cases} \left(-\frac{\alpha\mathcal{L}}{6\pi} + \frac{\alpha}{18\pi}\right) & \overline{\mathrm{MS}} \\ \left(-\frac{\alpha\mathcal{L}}{6\pi} - \frac{17\alpha}{72\pi}\right) & \mathrm{D}_1 \\ \left(-\frac{\alpha\mathcal{L}}{6\pi} + \frac{5\alpha}{36\pi}\right) & \mathrm{D}_2 \end{cases}$$

Rodini et al., JHEP 09 (2020) 067

How to obtain the form factors: QED (2/2)

$$\mathcal{L} = \log\left(\frac{\mu^2}{m^2}\right)$$

$$A_e^R(0) = 1 - \frac{2\alpha\mathcal{L}}{3\pi} - \frac{17}{18}\frac{\alpha}{\pi} \qquad A_\gamma^R(0) = \frac{2\alpha\mathcal{L}}{3\pi} + \frac{17}{18}\frac{\alpha}{\pi}$$

$$\bar{C}_e^R(0) = \frac{\alpha\mathcal{L}}{6\pi}\begin{cases} -\frac{\alpha}{18\pi} & \overline{\mathrm{MS}} \\ +\frac{17\alpha}{72\pi} & \mathrm{D}_1 \\ -\frac{5\alpha}{36\pi} & \mathrm{D}_2 \end{cases} \qquad \bar{C}_\gamma^R(0) = -\frac{\alpha\mathcal{L}}{6\pi}\begin{cases} +\frac{\alpha}{18\pi} & \overline{\mathrm{MS}} \\ -\frac{17}{72}\frac{\alpha}{\pi} & \mathrm{D}_1 \\ +\frac{5\alpha}{36\pi} & \mathrm{D}_2 \end{cases}$$

Rodini et al., JHEP 09 (2020) 067

The D(t) form factor, why to bother?

It is the least known of the global properties of particles

For non-vanishing momentum transfer
D(t) is related to the pressure and shear distributions

D(0) can be used to define an energy radius
and a mechanical radius of the system

D(0) is linked to the stability of the system

Polyakov, Schweitzer, IJMPA 33 (2018) 1830025

Metz et al., PLB 820 (2021) 136501

Varma, Schweitzer, PRD 102 (2020) 014047

Lorcé et al., EPJC 79 (2019) 89

Maybe massive photons…

Metz et al., PLB 820 (2021) 136501

We will limit ourselves to the D(t) form factor

$$\langle e(p',s')|\, T_i^{\mu\nu}\, |e(p,s)\rangle = \bar{u}(p',s')\bigg(A_i(t)\,\frac{P^\mu P^\nu}{m_e} + J_i(t)\,\frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2m_e} + D_i(t)\,\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4m_e} + m_e\,\bar{C}_i(t)\,g^{\mu\nu}\bigg)u(p,s)$$

How can the EMT for QED incorporate a massive photon?

$$T^{\mu\nu} = T^{\mu\nu}_{\text{QED}} + m_\gamma^2\left(A^\mu A^\nu - \frac{g^{\mu\nu}}{2}A^2\right) + T^{\mu\nu}_{\text{extra}}$$

Many options:

Stueckelberg Lagrangian, Higgs mechanism…

We used the Higgs model for a massive photon in the R_1 gauge

D(t) for an electron in QED

For the D(t) form factor at one loop
we do not need to worry about renormalization

$$D_i(\tau^2, \lambda^2) = \int_0^1 dx \int_0^{1-x} dy \, \frac{f_i(x,y)}{\tau^2 + a_i(x,y,\lambda^2)}$$

$$\tau^2 = \frac{-t}{m_e^2} > 0 \qquad \lambda = \frac{m_\gamma}{m_e}$$

$$f_e(x,y) = \frac{\alpha}{\pi} \frac{(x-2)(1-x-2y)^2}{y(1-x-y)} \qquad a_e(x,y,\lambda^2) = \frac{(1-x)^2 + x\lambda^2}{y(1-x-y)}$$

$$f_\gamma(x,y) = \frac{\alpha}{\pi} \frac{1-x-(1+x)(1-x-2y)^2}{y(1-x-y)} \qquad a_\gamma(x,y,\lambda^2) = \frac{x^2 + (1-x)\lambda^2}{y(1-x-y)}$$

Polyakov, Schweitzer, IJMPA 33 (2018) 1830025

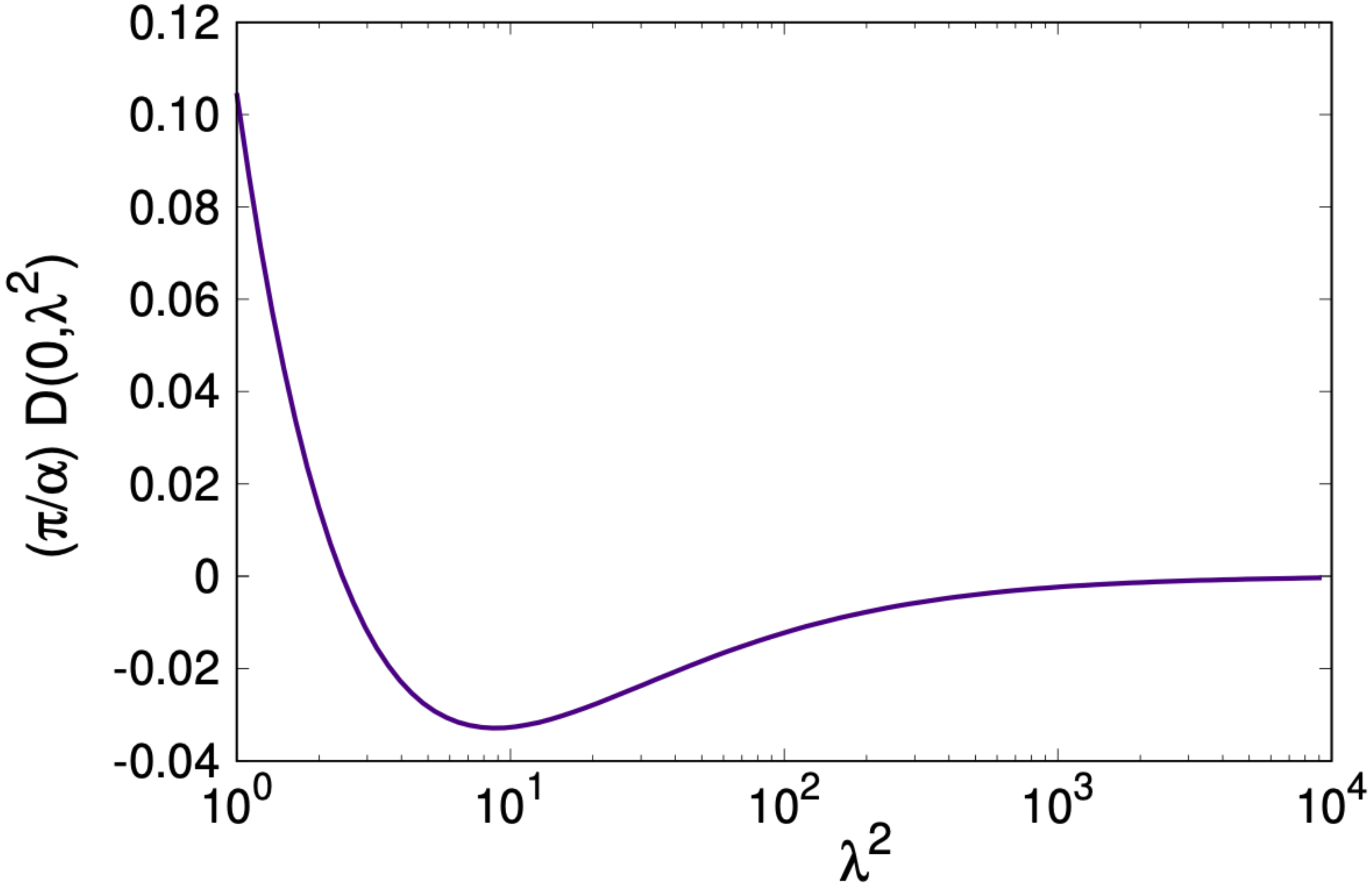
Metz et al., PLB 820 (2021) 136501

Varma, Schweitzer, PRD 102 (2020) 014047

Lorcé et al., EPJC 79 (2019) 89

Limits

$$\tau^2 \to 0$$



$$D_\gamma(\tau^2 \ll 1, \lambda^2 = 0) \simeq \frac{\alpha\pi}{4\sqrt{\tau^2}}$$

$$D_\gamma(\tau^2 = 0, \lambda^2 \ll 1) \simeq \frac{\alpha}{3\lambda}$$

$$D_e(\tau^2 = 0, \lambda^2 = 0) = -\frac{5\alpha}{18\pi}$$

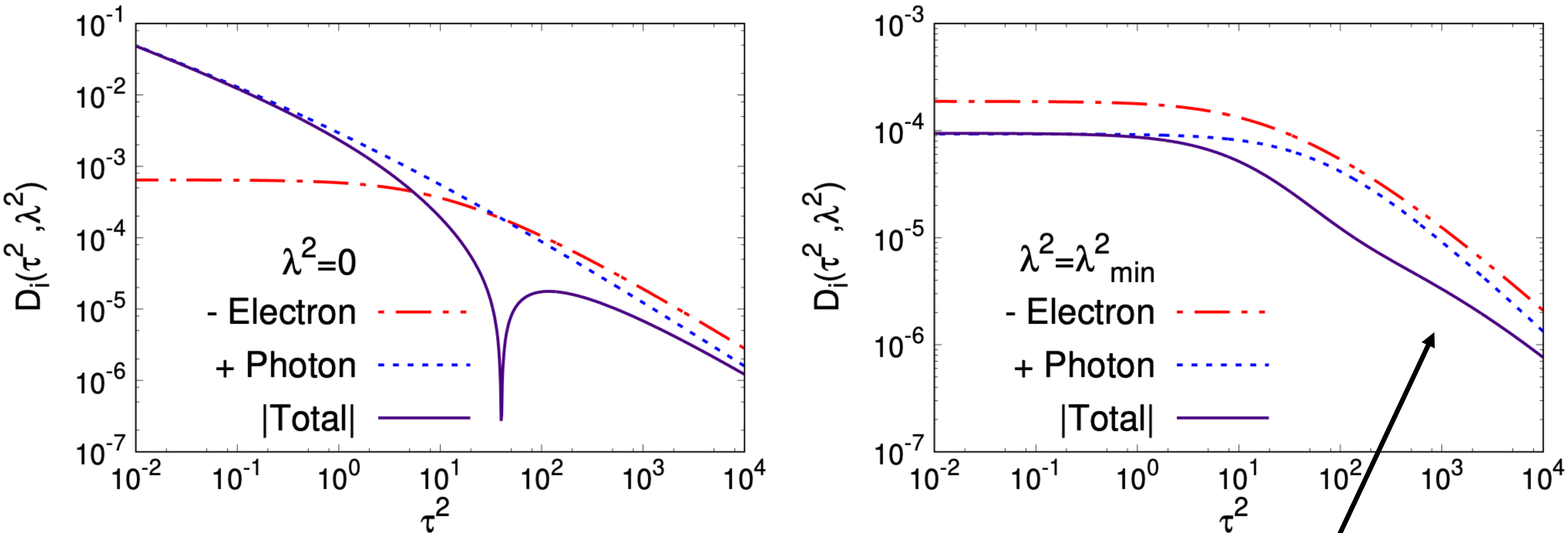
$$D_e(\tau^2 \gg 1, \lambda^2 = 0) = \frac{\alpha}{\pi}\frac{10 - 5\log\left(\tau^2\right)}{3\tau^2}$$

$$D_\gamma(\tau^2 \gg 1, \lambda^2 = 0) = \frac{\alpha}{\pi}\frac{2 + 2\log\left(\tau^2\right)}{3\tau^2}$$

Metz et al., PLB 820 (2021) 136501

Milton, PRD 15 (1977) 538

Berends, Gastmans, Annals Phys. 98 (1976) 225



Mechanical radius

$$\langle r^2(\lambda^2)\rangle_{\mathrm{mech}} = \frac{6\,D(0,\lambda^2)}{m_e^2\int_0^\infty d\tau^2 D(\tau^2,\lambda^2)}$$

$$\lambda^2 = 0$$

Due to the slow fall-off at large moment transfer one has

$$\langle r^2(\lambda^2\neq 0)\rangle_{\mathrm{mech}} = 0$$

Position space

$$\Delta^0 = 0$$

$$\hat{D}_i(\rho, \lambda^2) = \int \frac{d^3\boldsymbol{\tau}}{(2\pi)^3} e^{-i\boldsymbol{\tau}\cdot\boldsymbol{\rho}} D_i(\tau^2, \lambda^2) = \frac{1}{2\pi^2 \rho} \,\mathrm{FST}\left(\tau D_i(\tau^2, \lambda^2); \tau, \rho\right)$$

$$\hat{\bar{C}}_i(\rho, \lambda^2) = \int \frac{d^3\boldsymbol{\tau}}{(2\pi)^3} e^{-i\boldsymbol{\tau}\cdot\boldsymbol{\rho}} \bar{C}_i(\tau^2, \lambda^2)$$

$$\hat{D}_i(\rho, \lambda^2) = \int_0^1 dx \int_0^{1-x} dy \, \frac{f_i(x, y)}{4\pi\rho} \, e^{-\rho\sqrt{a_i(x,y,\lambda^2)}}$$

$$\hat{p}_i(\rho, \lambda^2) = \frac{p_i(\rho, \lambda^2)}{m_e^4} = \frac{1}{6\rho^2} \frac{d}{d\rho} \rho^2 \frac{d}{d\rho} \hat{D}_i(\rho, \lambda^2) - \hat{\bar{C}}_i(\rho, \lambda^2)$$

$$\hat{s}_i(\rho, \lambda^2) = \frac{s_i(\rho, \lambda^2)}{m_e^4} = -\frac{\rho}{4} \frac{d}{d\rho} \frac{1}{\rho} \frac{d}{d\rho} \hat{D}_i(\rho, \lambda^2)$$

Metz et al., PLB 820 (2021) 136501

Polyakov, Schweitzer, IJMPA 33 (2018) 1830025

Lorcé et al., EPJC 79 (2019) 89

Position space

$$\hat{\bar{C}}_i(\rho,\lambda^2) = \phi_i(\lambda^2)\,\frac{\delta'(\rho)}{\rho} \qquad\qquad \bar{C}_i(\tau^2,\lambda^2) = \text{constant}$$

Von Laue condition (which directly follows from EMT conservation)

$$\int_0^\infty d\rho\rho^2\hat{p}(\rho,\lambda^2) = \frac{1}{12\pi}\int_0^\infty d\tau\tau^3 D(\tau^2,\lambda^2)\delta'(\tau) = 0$$

$$\bar{C}_e(\tau^2,\lambda^2) + \bar{C}_\gamma(\tau^2,\lambda^2) = 0 \quad \forall\tau^2,\lambda^2$$

$$\int_0^\infty d\rho\rho^2\hat{p}_{i,D}(\rho,\lambda^2) = \frac{1}{12\pi}\int_0^\infty d\tau\tau^3 D_i(\tau^2,\lambda^2)\delta'(\tau) = 0$$

$$\bar{C}_i$$

Metz et al., PLB 820 (2021) 136501 | Polyakov, Schweitzer, IJMPA 33 (2018) 1830025 | Lorcé et al., EPJC 79 (2019) 89

Position space

$$\hat{p}_{i,\text{fin.}}(\rho,\lambda^2) = \int_0^1 dx \int_0^{1-x} dy\ e^{-\rho\sqrt{a_i(x,y,\lambda^2)}} f_i(x,y)\,\frac{a_i(x,y,\lambda^2)}{24\pi\rho}$$

$$\hat{p}_{i,D,\text{sing.}}(\rho) = \frac{\delta'(\rho)}{12\pi\rho}\int_0^1 dx \int_0^{1-x} dy\ f_i(x,y)$$

$$\hat{s}_{i,\text{fin.}}(\rho,\lambda^2) = -\int_0^1 dx \int_0^{1-x} dy\ e^{-\rho\sqrt{a_i(x,y,\lambda^2)}} f_i(x,y)\,\frac{3+3\sqrt{a_i(x,y,\lambda^2)}\rho + a_i(x,y,\lambda^2)\rho^2}{16\pi\rho^3}$$

$$\hat{s}_{i,\text{sing.}}(\rho) = \frac{3\delta(\rho)-\rho\delta'(\rho)}{8\pi\rho^2}\int_0^1 dx \int_0^{1-x} dy f_i(x,y)$$

In the large-distance limit we find

$$\hat{p}(\rho\to\infty,\lambda^2=0) \simeq \hat{p}_\gamma(\rho\to\infty,\lambda^2=0) = \frac{\alpha}{24\pi\rho^4} + \ldots$$

$$\hat{s}(\rho\to\infty,\lambda^2=0) \simeq \hat{s}_\gamma(\rho\to\infty,\lambda^2=0) = -\frac{\alpha}{4\pi\rho^4} + \ldots$$

In agreement with the known asymptotic limits

In agreement with the photon dominance in the low-t region

Metz et al., PLB 820 (2021) 136501

Donoghue et al., PLB 529 (2002) 132

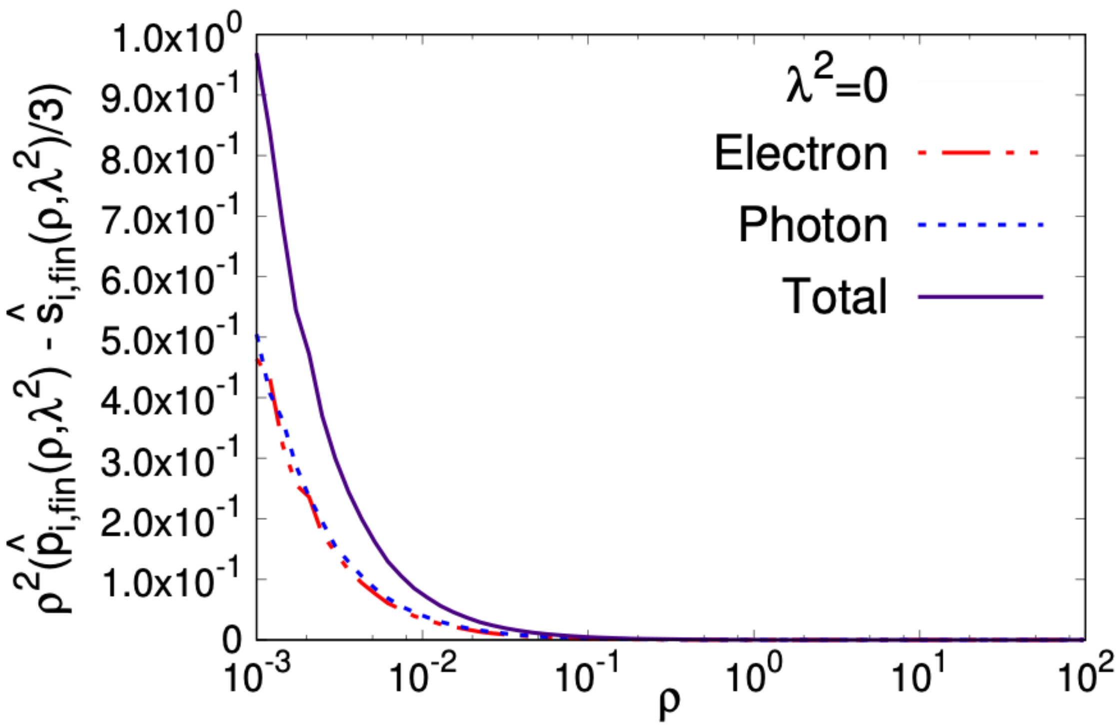
Force distributions

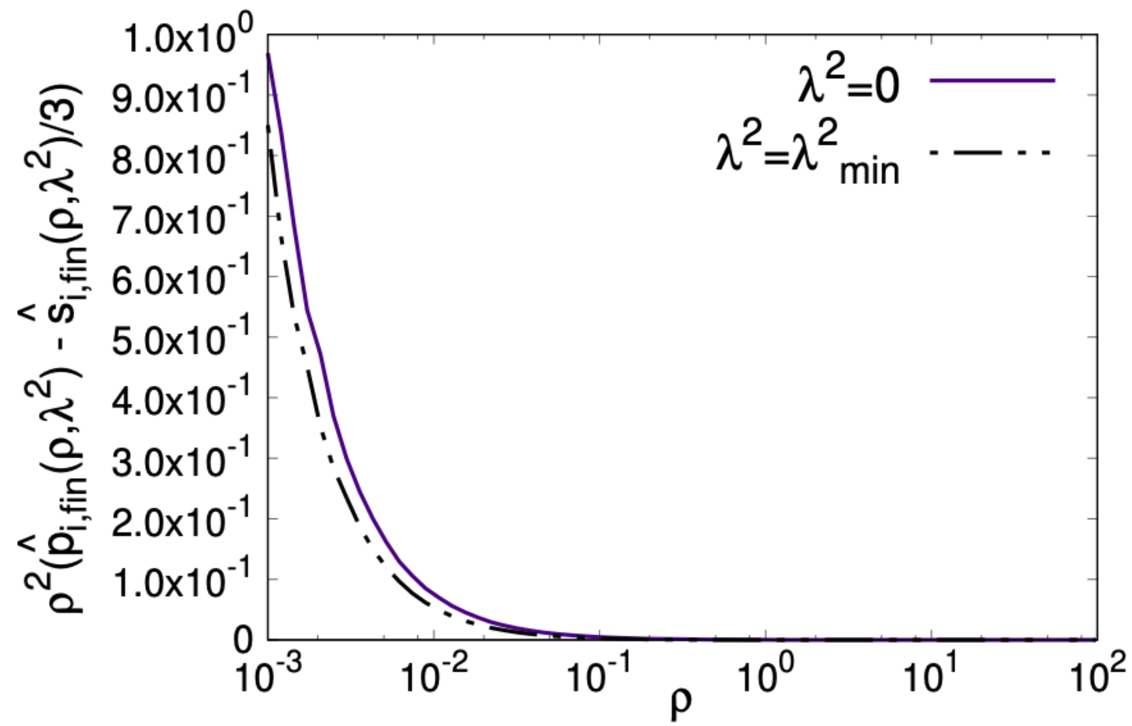
Forces per unit area
experienced by a spherical shell of radius ρ

$$\frac{dF_n}{dS_n} = \frac{2}{3}\hat{s}(\rho,\lambda^2) + \hat{p}(\rho,\lambda^2)$$

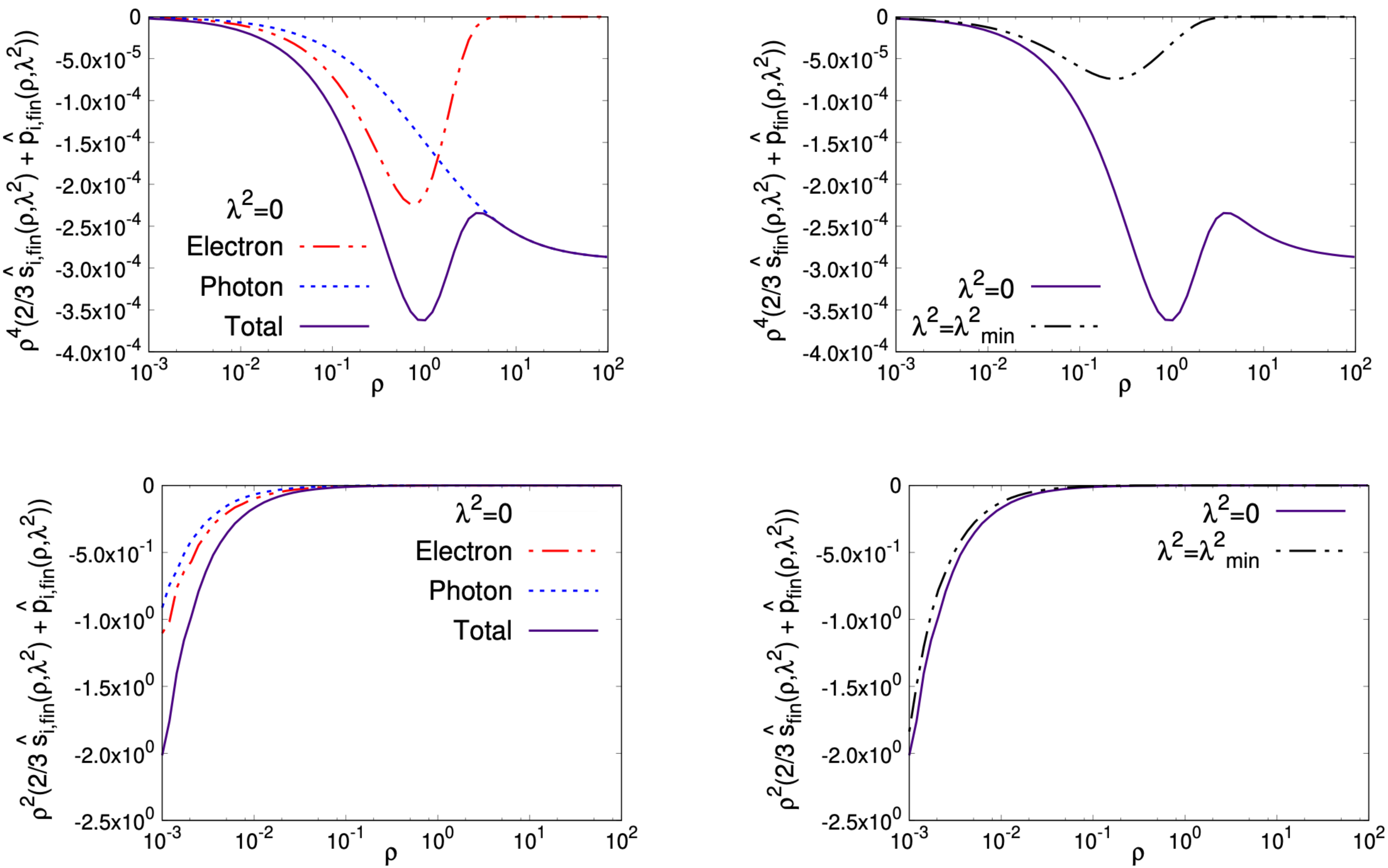
$$\frac{dF_t}{dS_t} = \hat{p}(\rho,\lambda^2) - \frac{1}{3}\hat{s}(\rho,\lambda^2)$$

Normal force

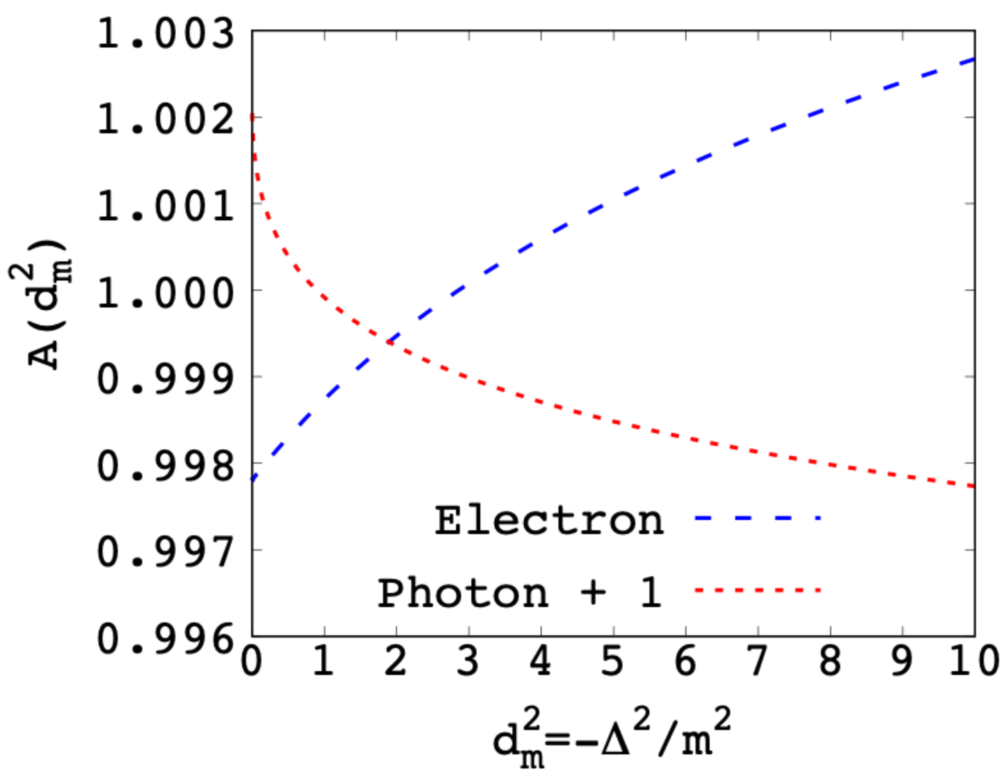


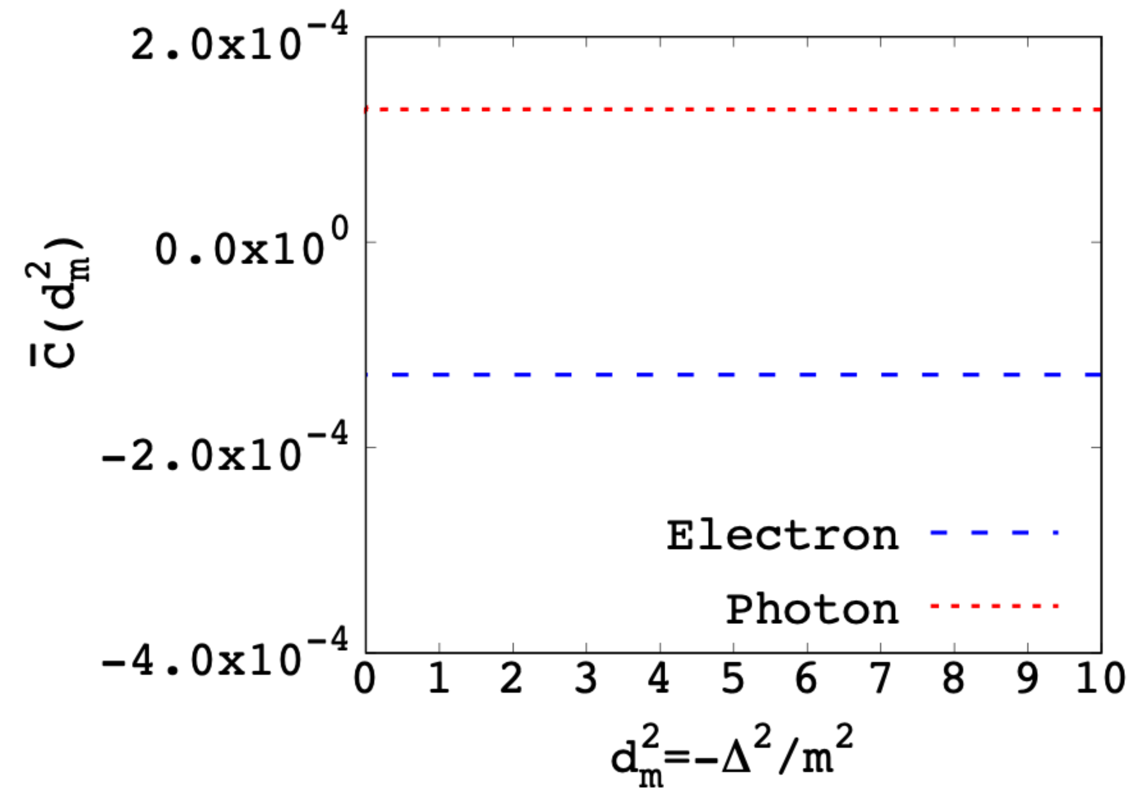


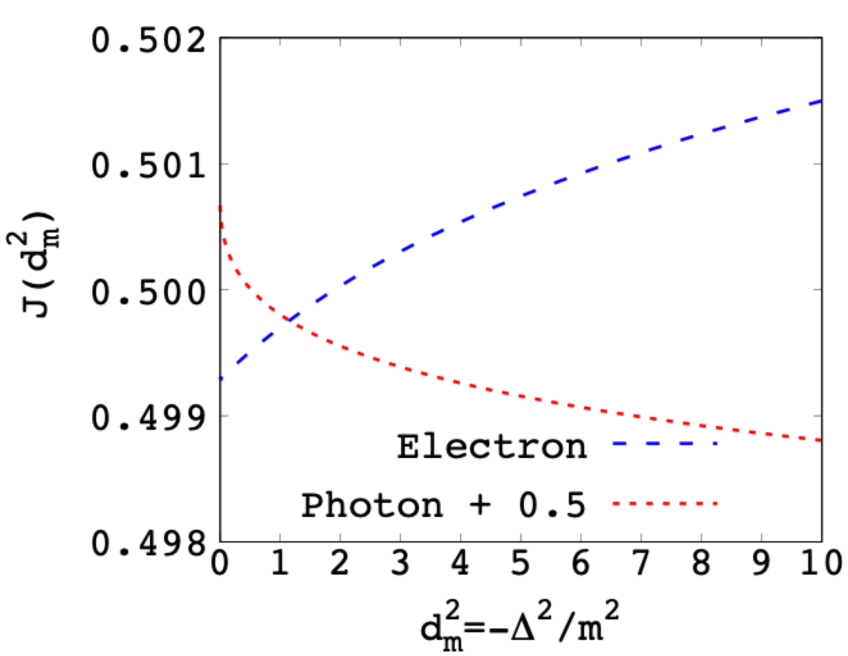
Tangential force

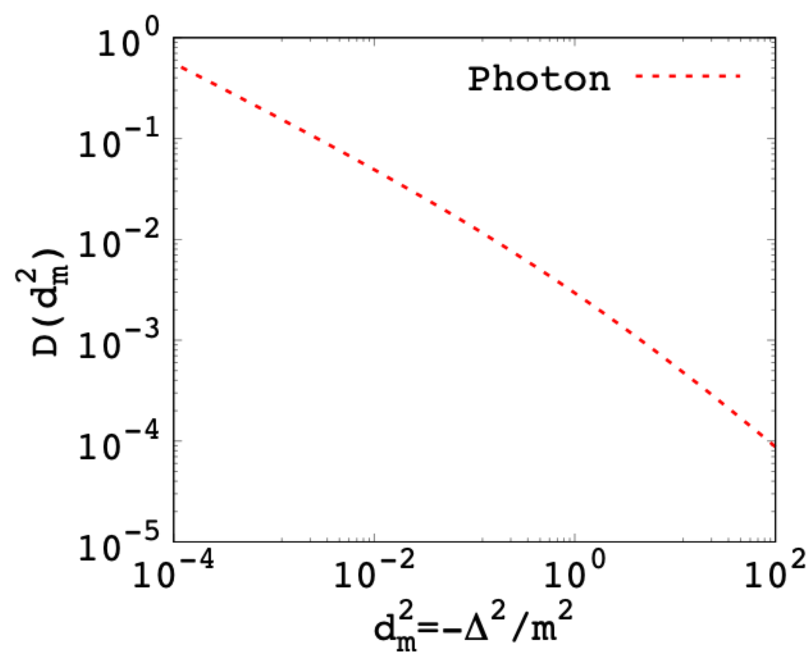


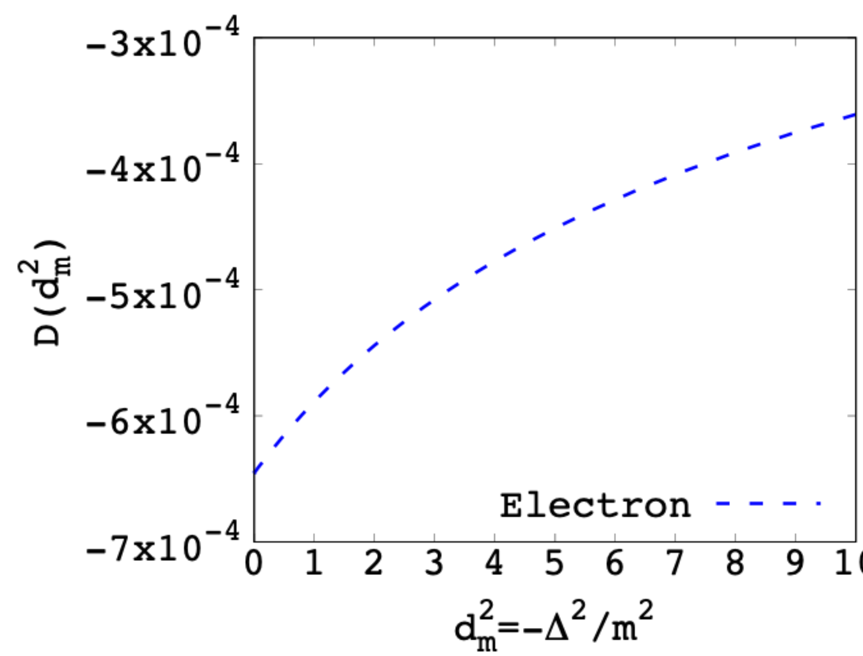
Off-forward form factors: a quick peek











Summary

Renormalization of multiple local operators

Renormalization of the form factors (scheme dependence)

1-loop QED calculation as simple example

Conclusions

The EMT form factors are the fundamental building blocks
For studying the physics of mass and pressure-shear distributions

Scheme dependence prevents a unique interpretation
of the terms of the decompositions

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