Extracting the σ resonance from first-principles QCD

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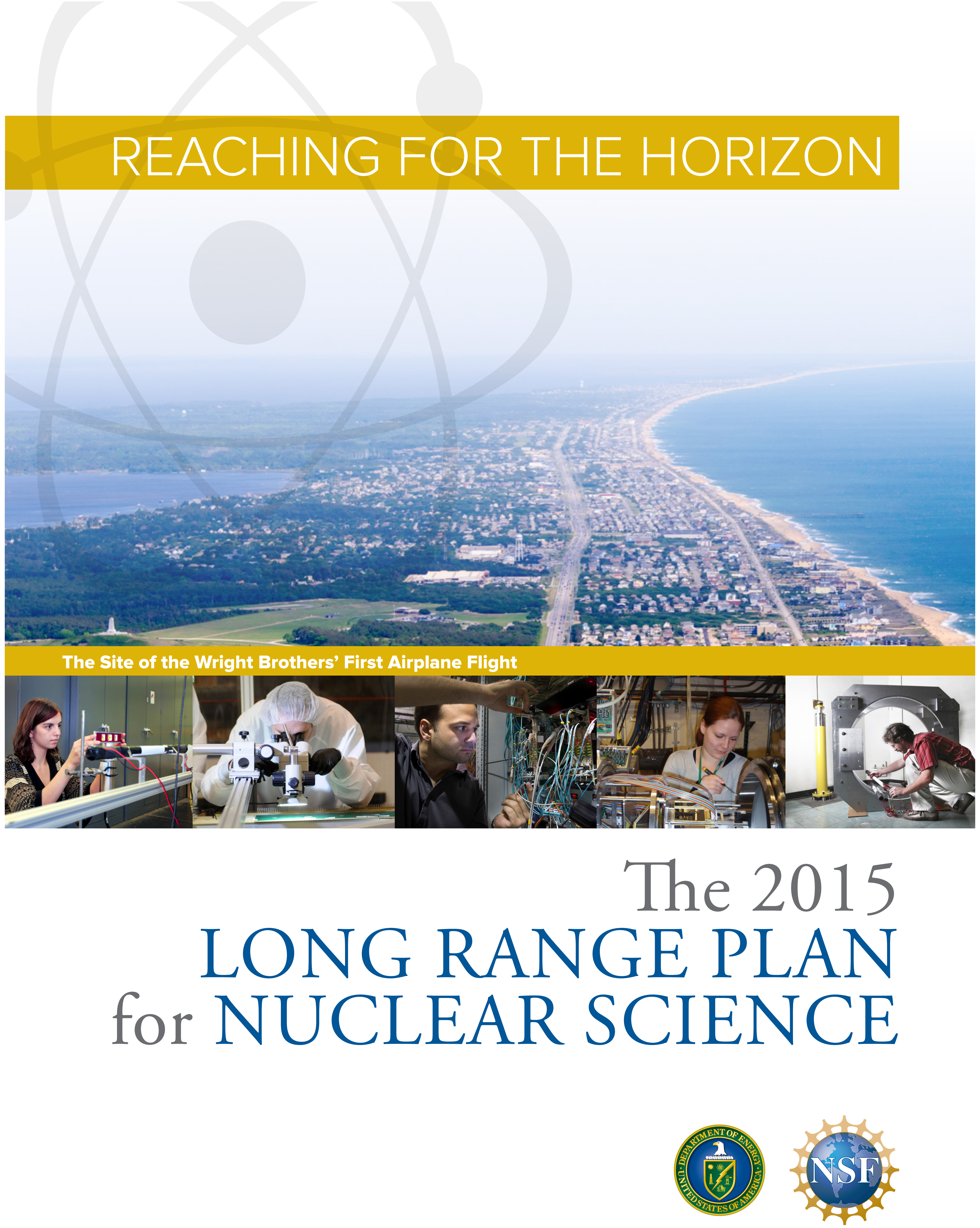
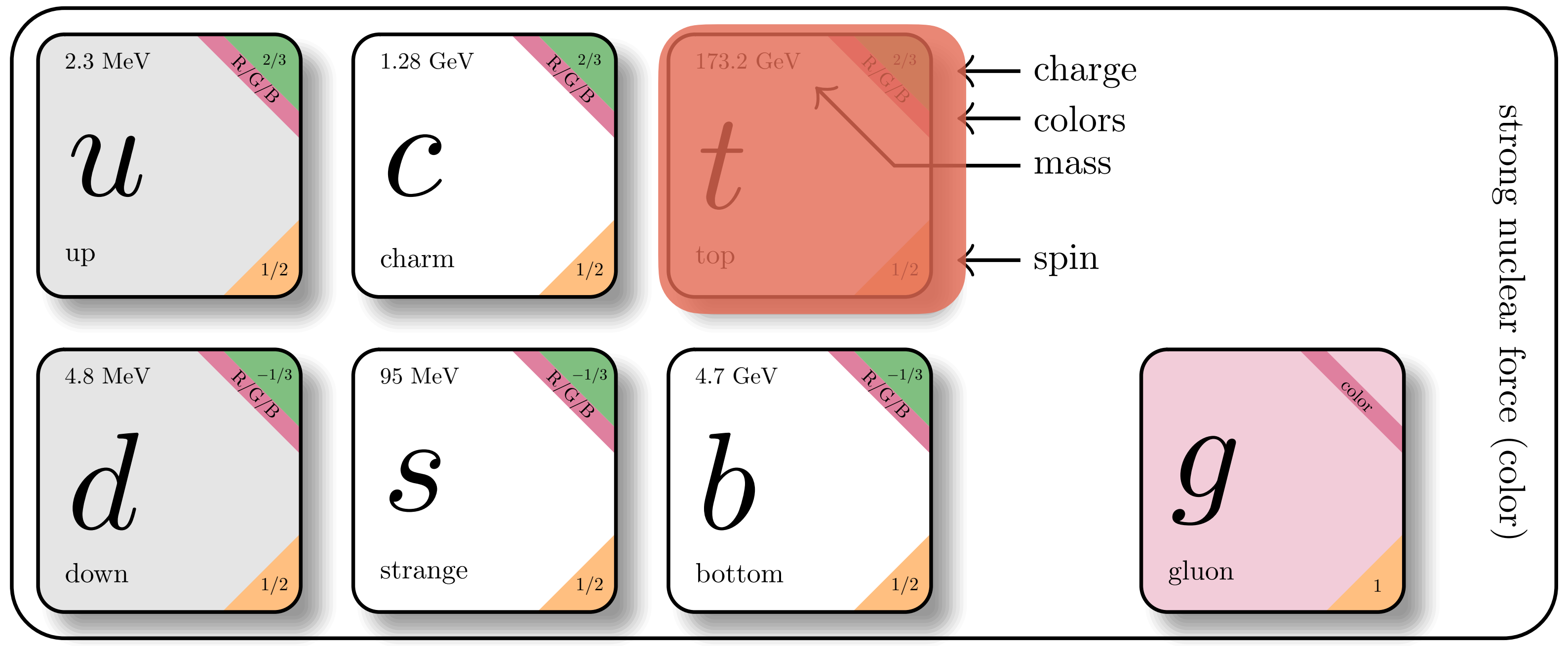
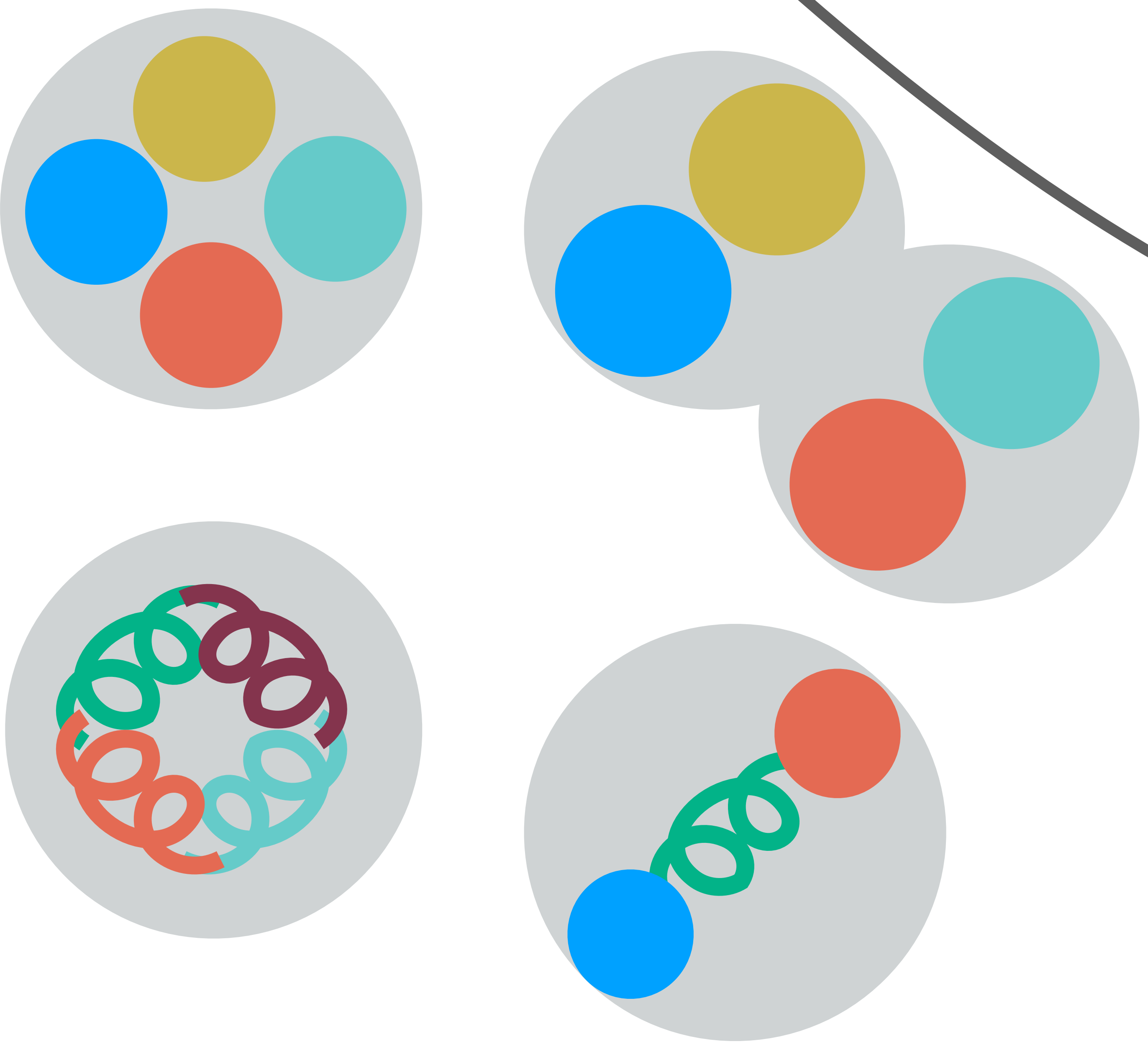
hadspec

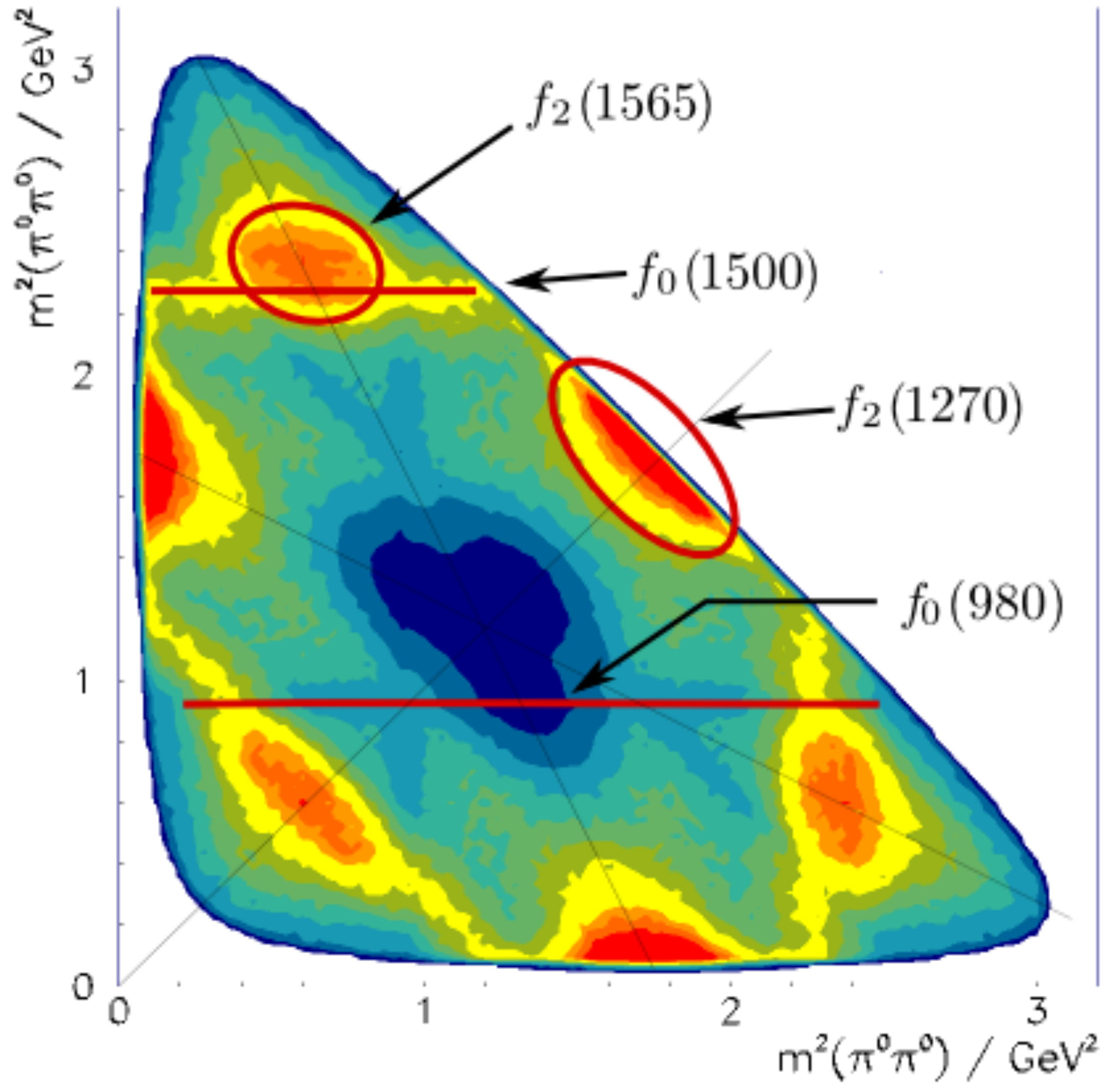


"... hadron spectroscopy illuminates the QCD interaction that binds quarks."

QCD

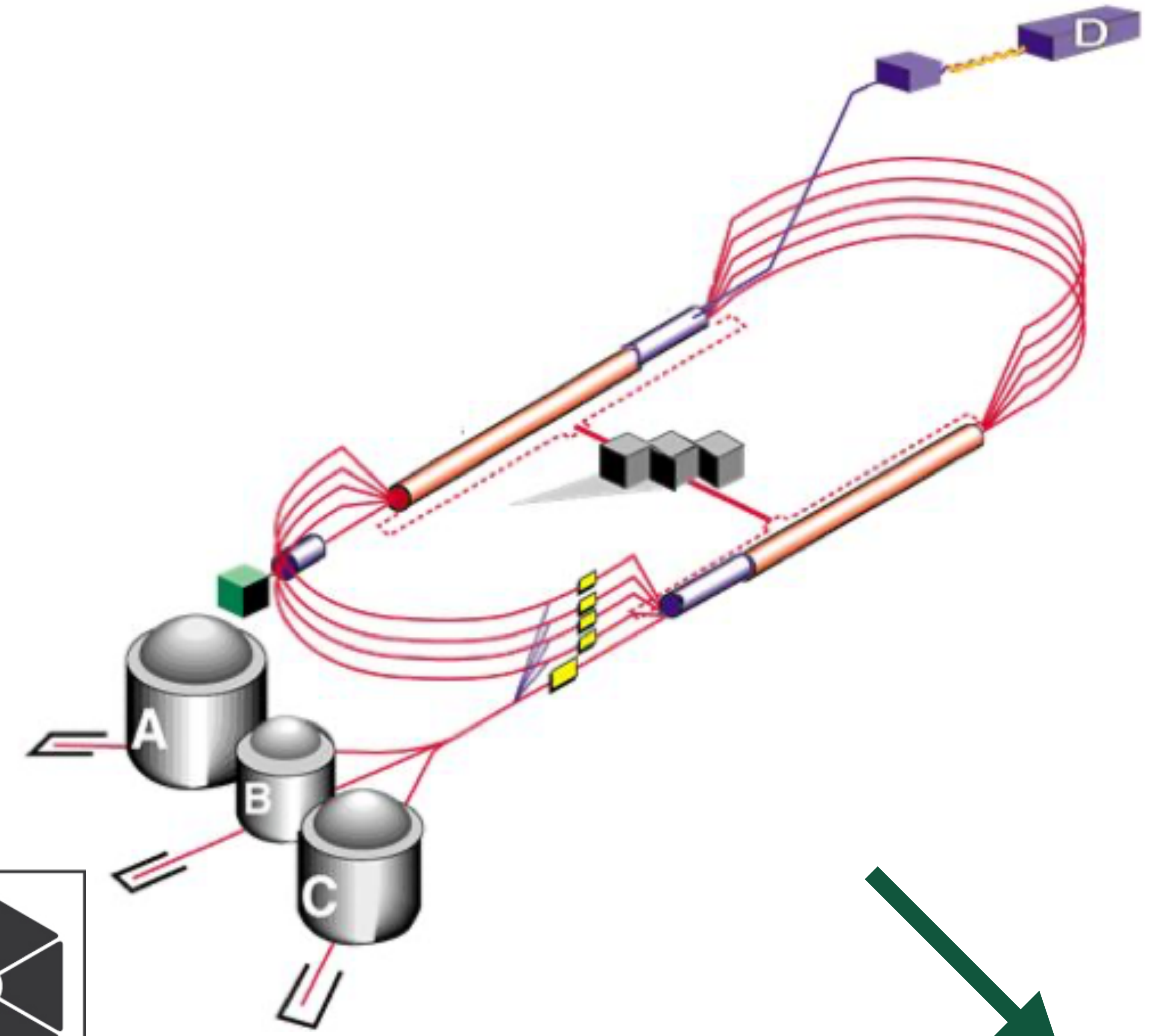


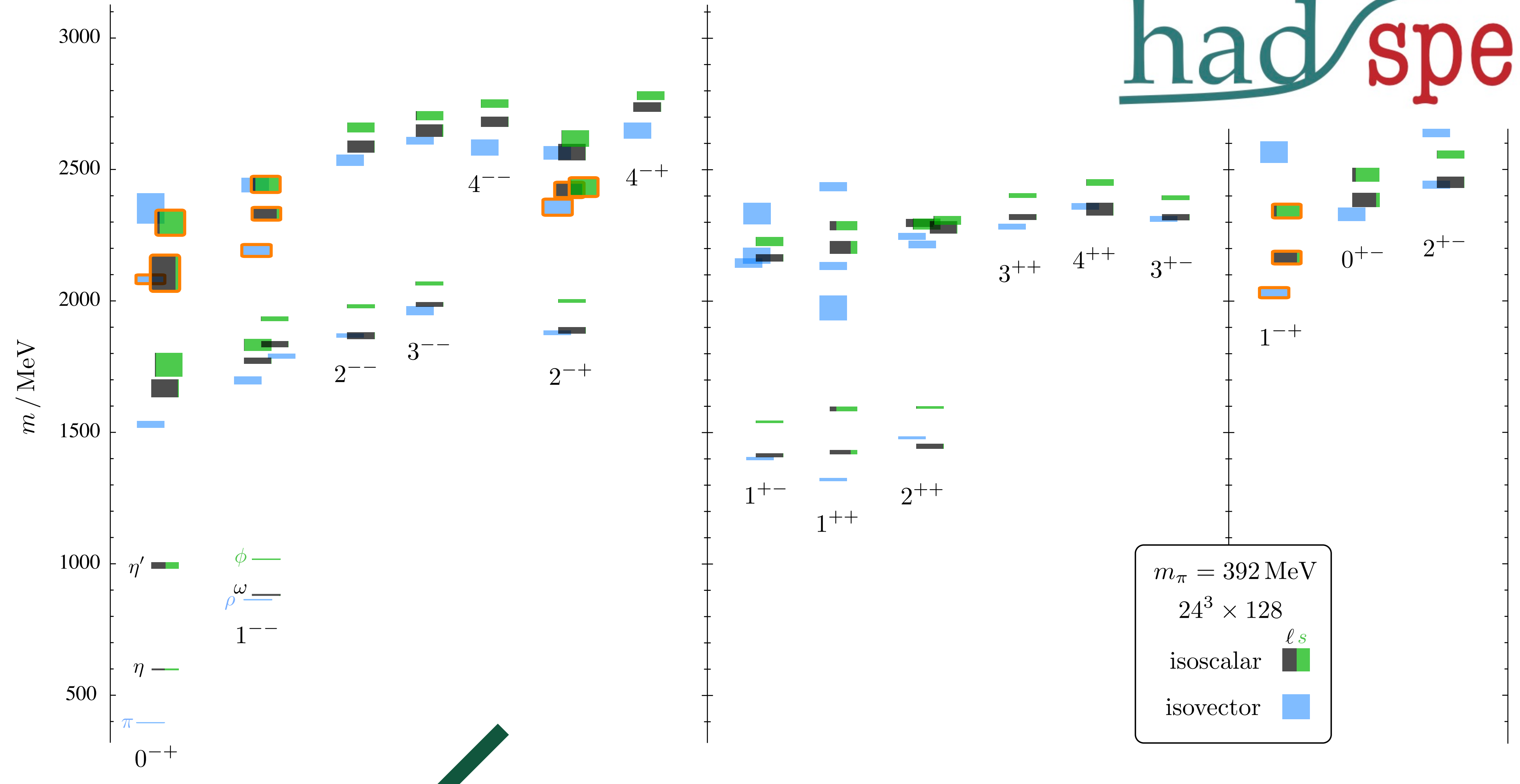




Understanding the QCD spectrum

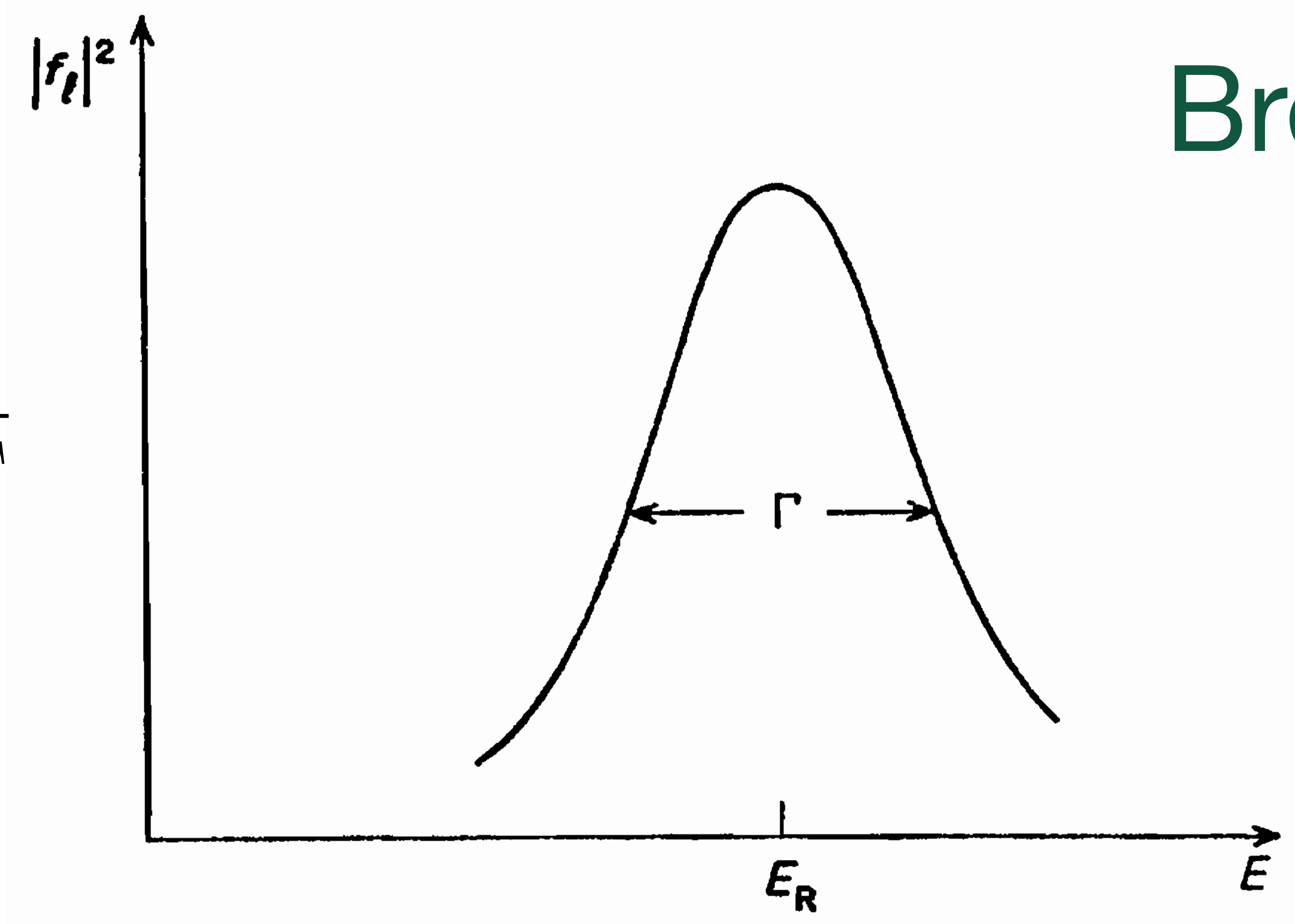
- Determine the spectrum





Very simple

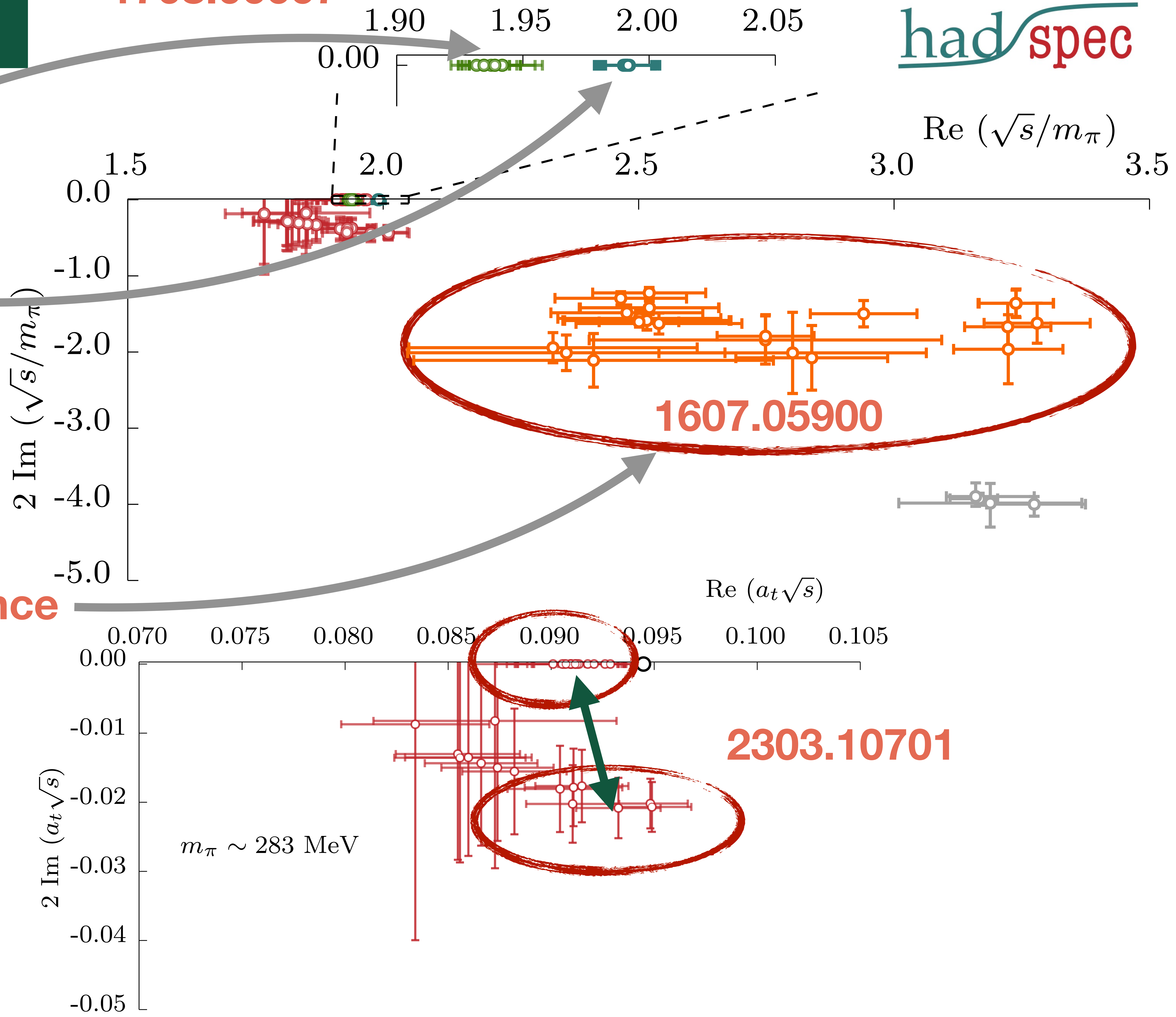
$$t_\ell(s) \simeq \frac{-M\Gamma}{M^2 - s - iM\Gamma}$$



Breit-Wigner

Light Scalars: the σ

- $m_\pi \sim 391\, MeV \rightarrow$ **BS**
- $m_\pi \sim 330\, MeV \rightarrow$ **BS**
- ? $m_\pi \sim 239\, MeV \rightarrow$ **Broad resonance**
- ? $m_\pi \sim 283\, MeV \rightarrow$ **??**



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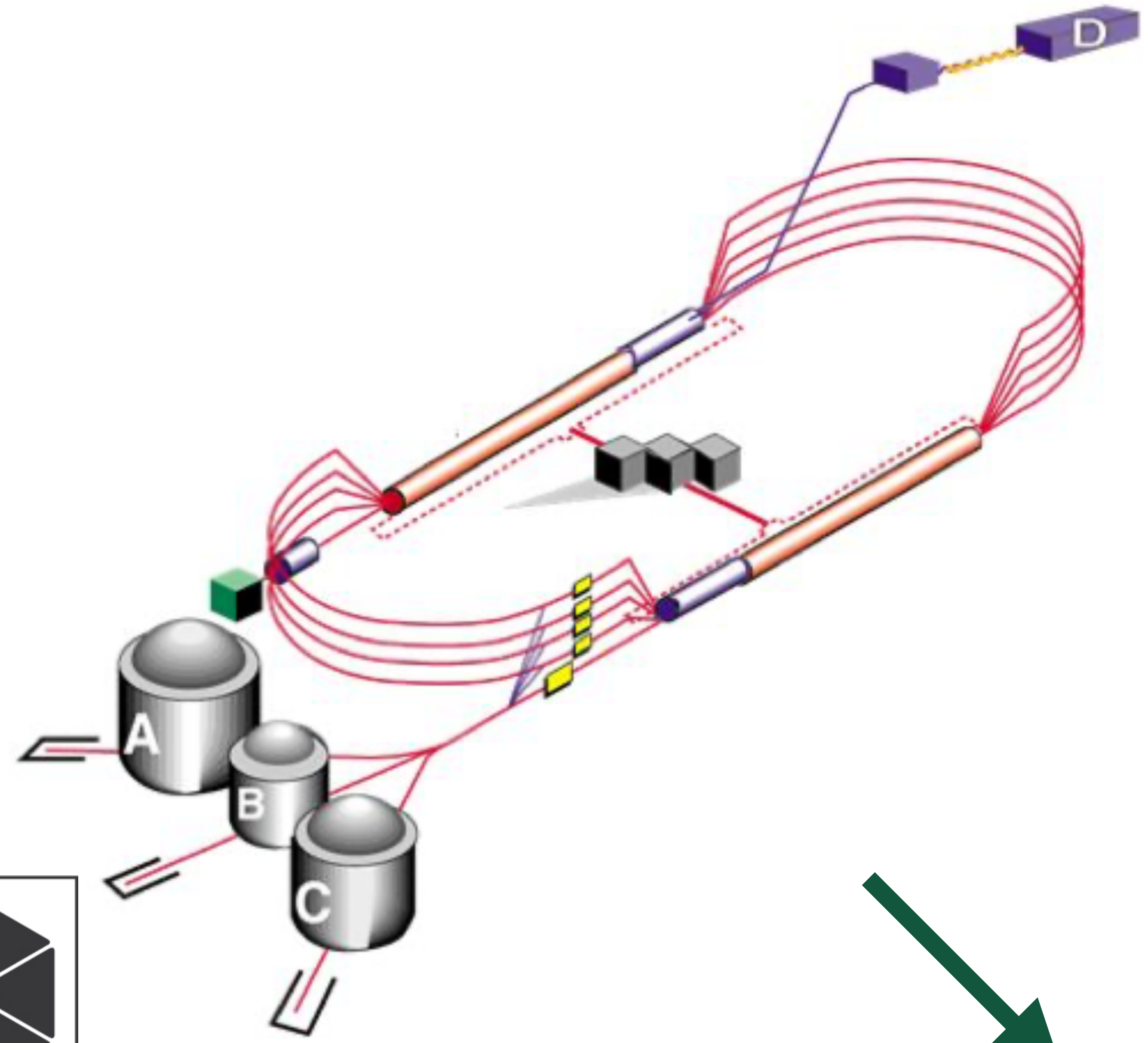
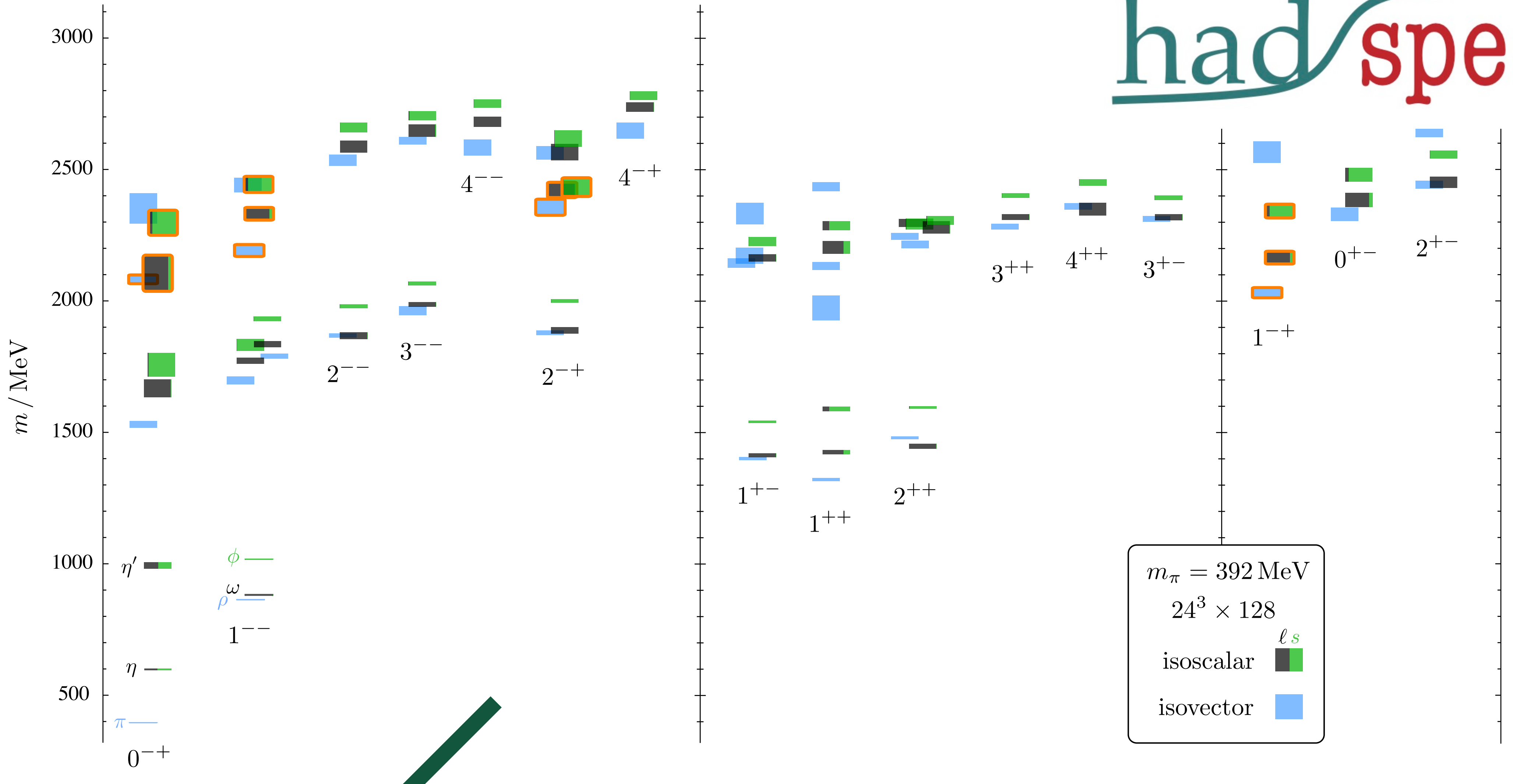


CEBAF Large Acceptance Spectrometer



Amplitude analyses

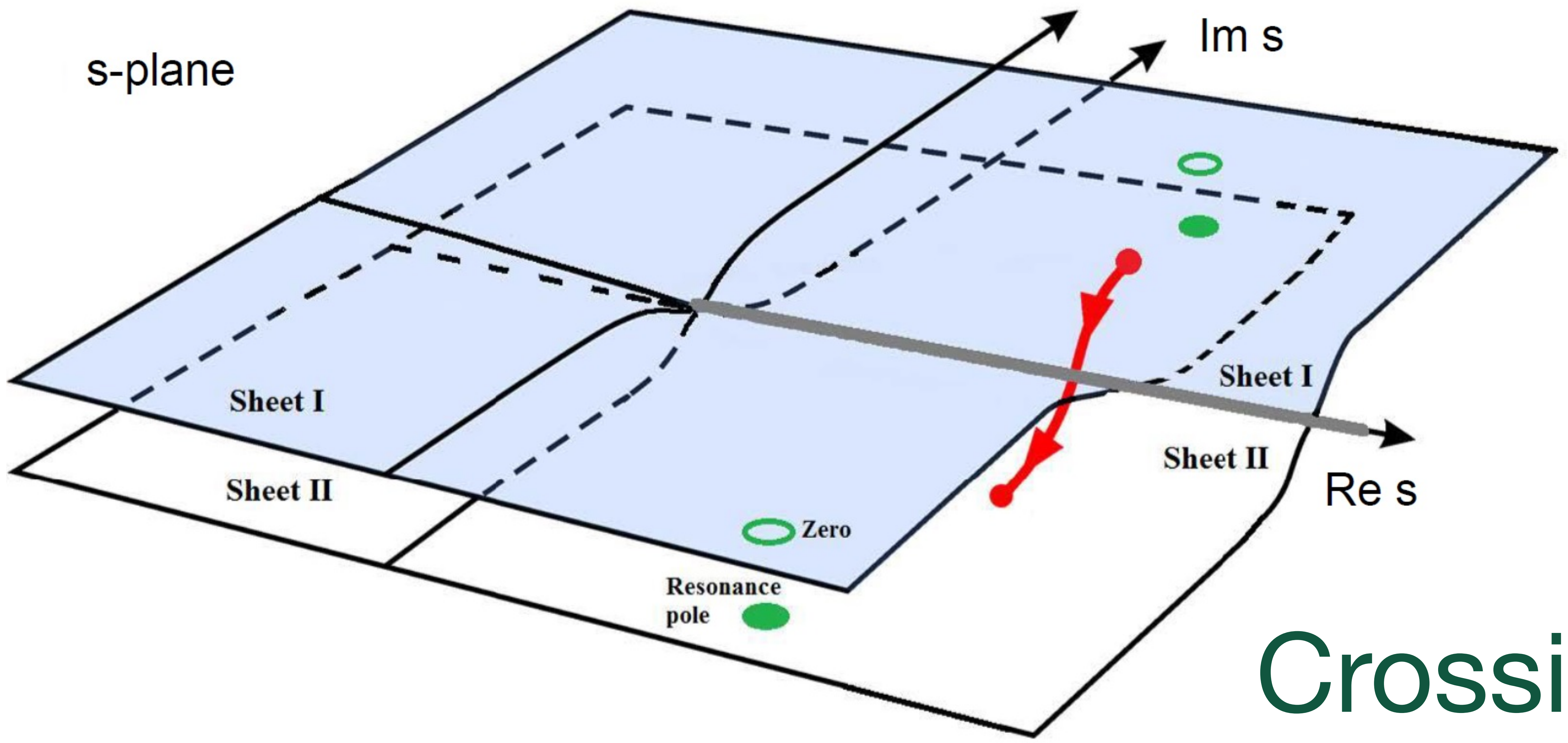
- Unitarity
- Analiticity
- Crossing

Observables

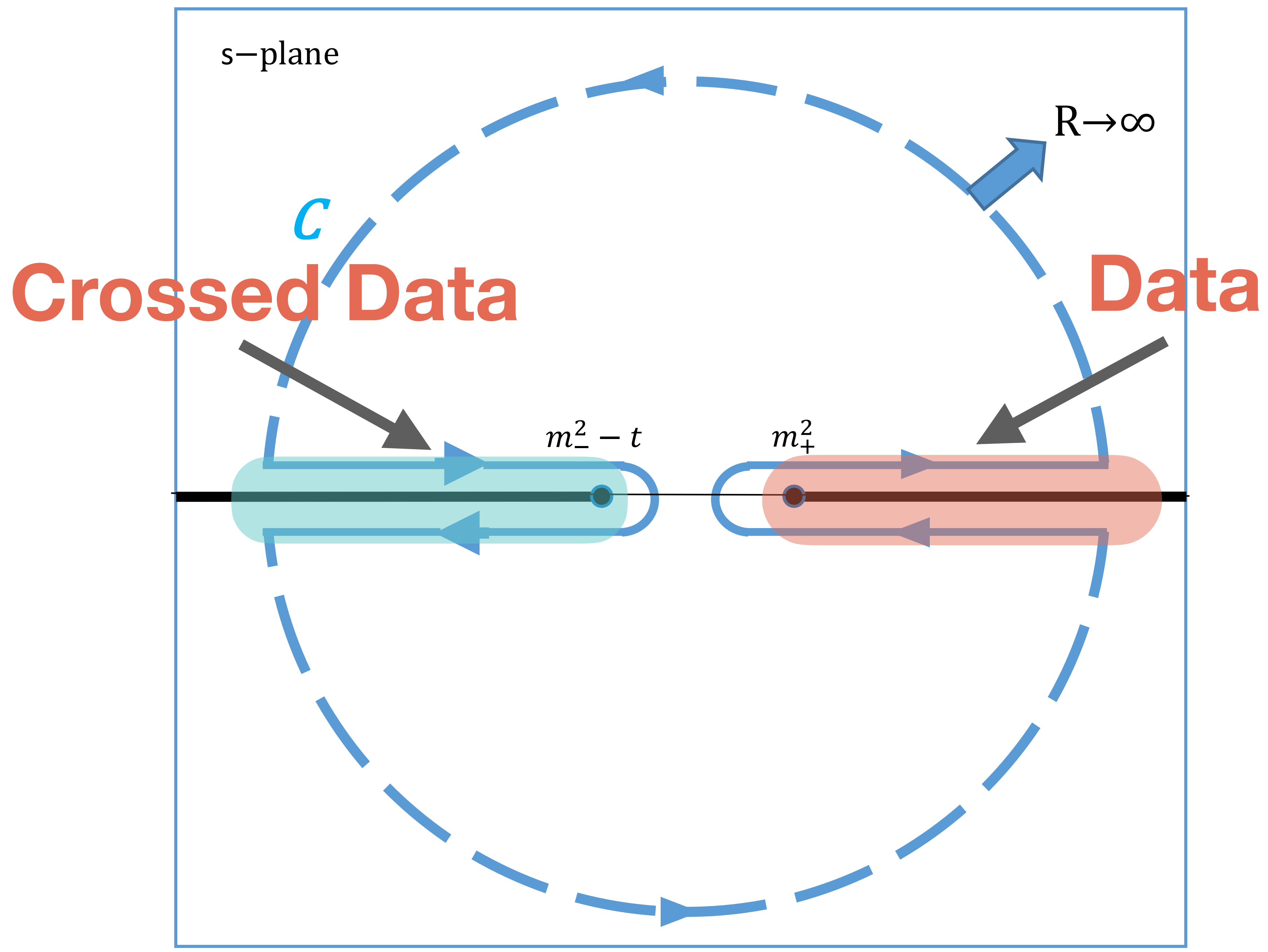
First principles

Unitarity

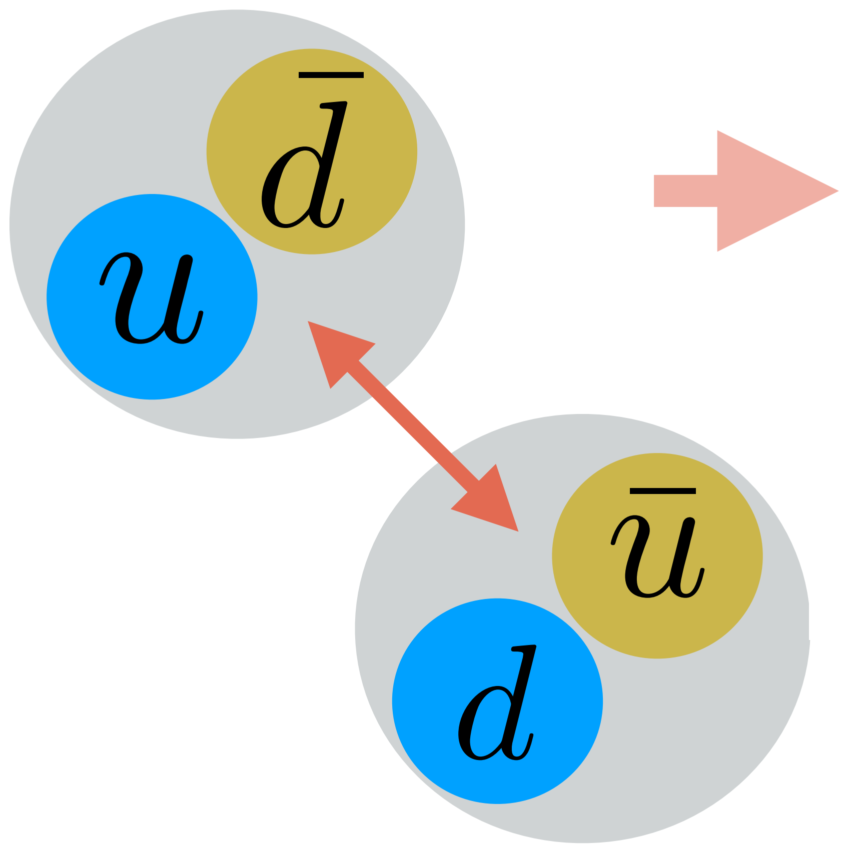
$$SS^\dagger = \mathbb{I}$$

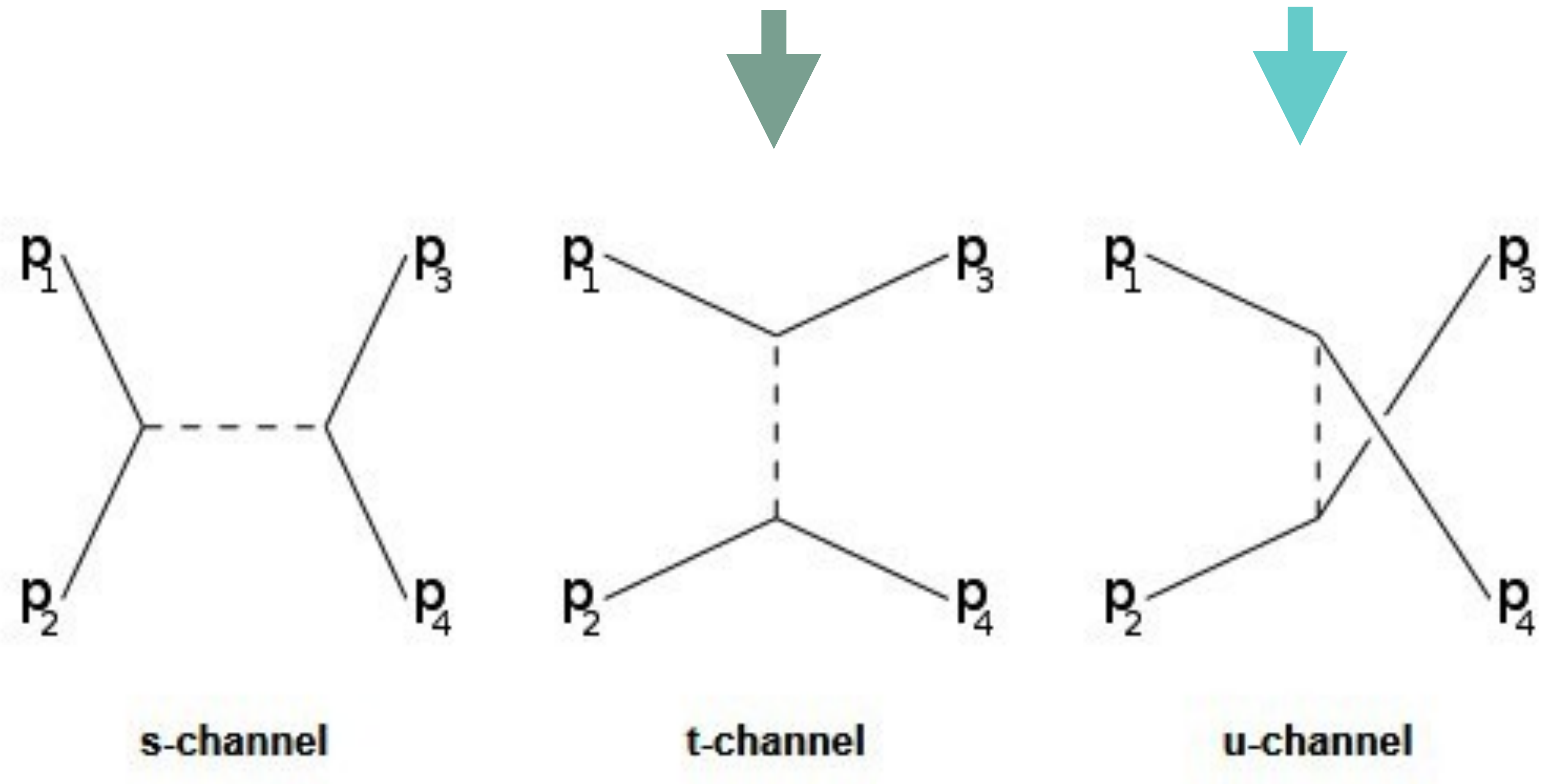


Causality ↔ Analiticity



Crossing

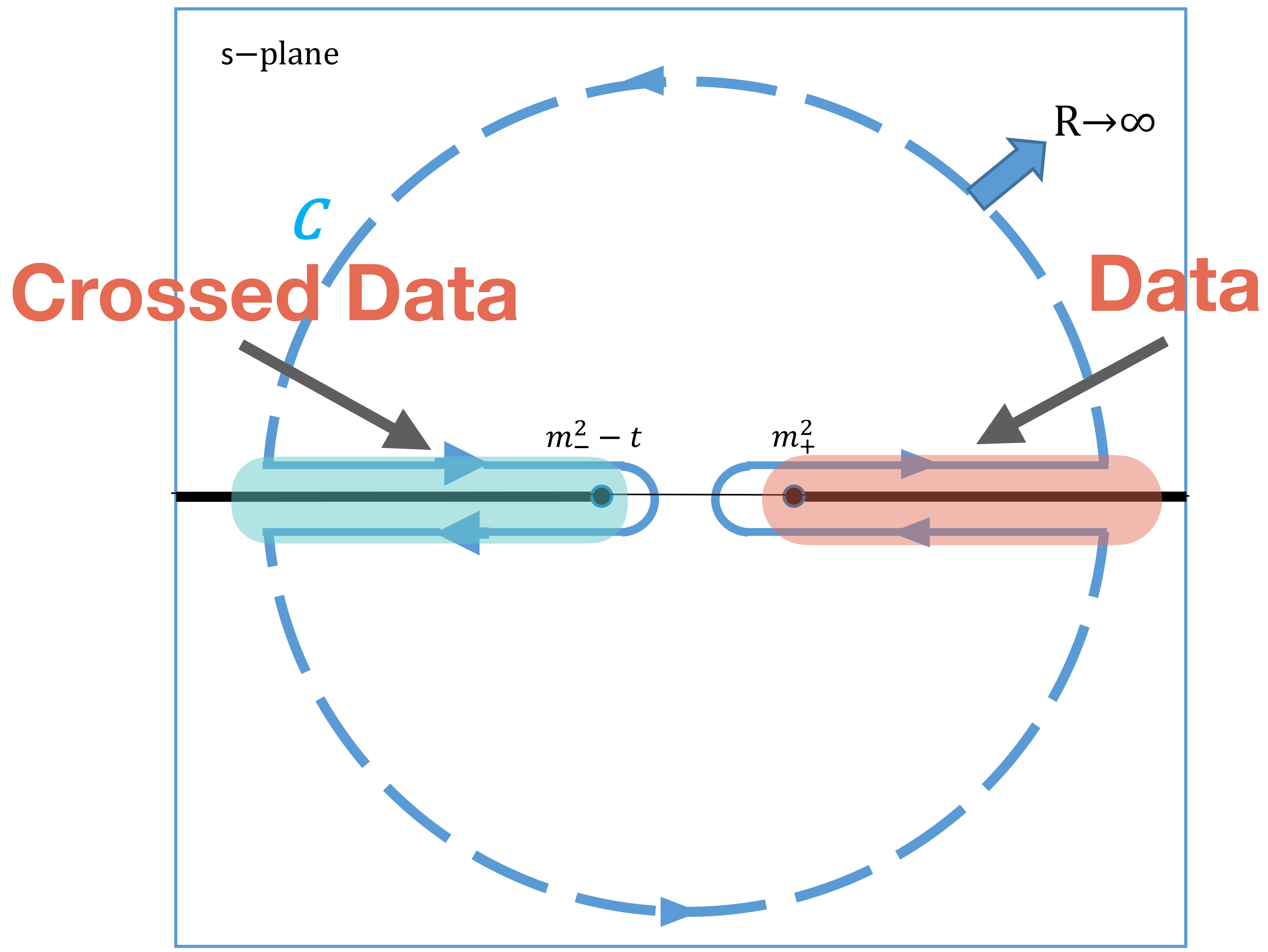




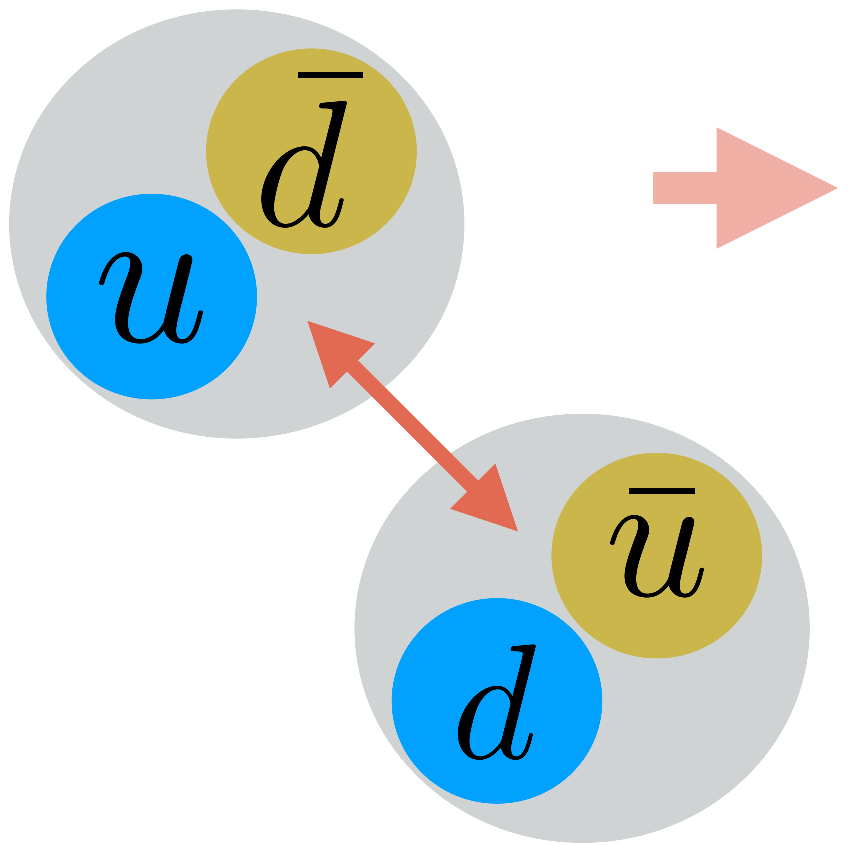
First principles

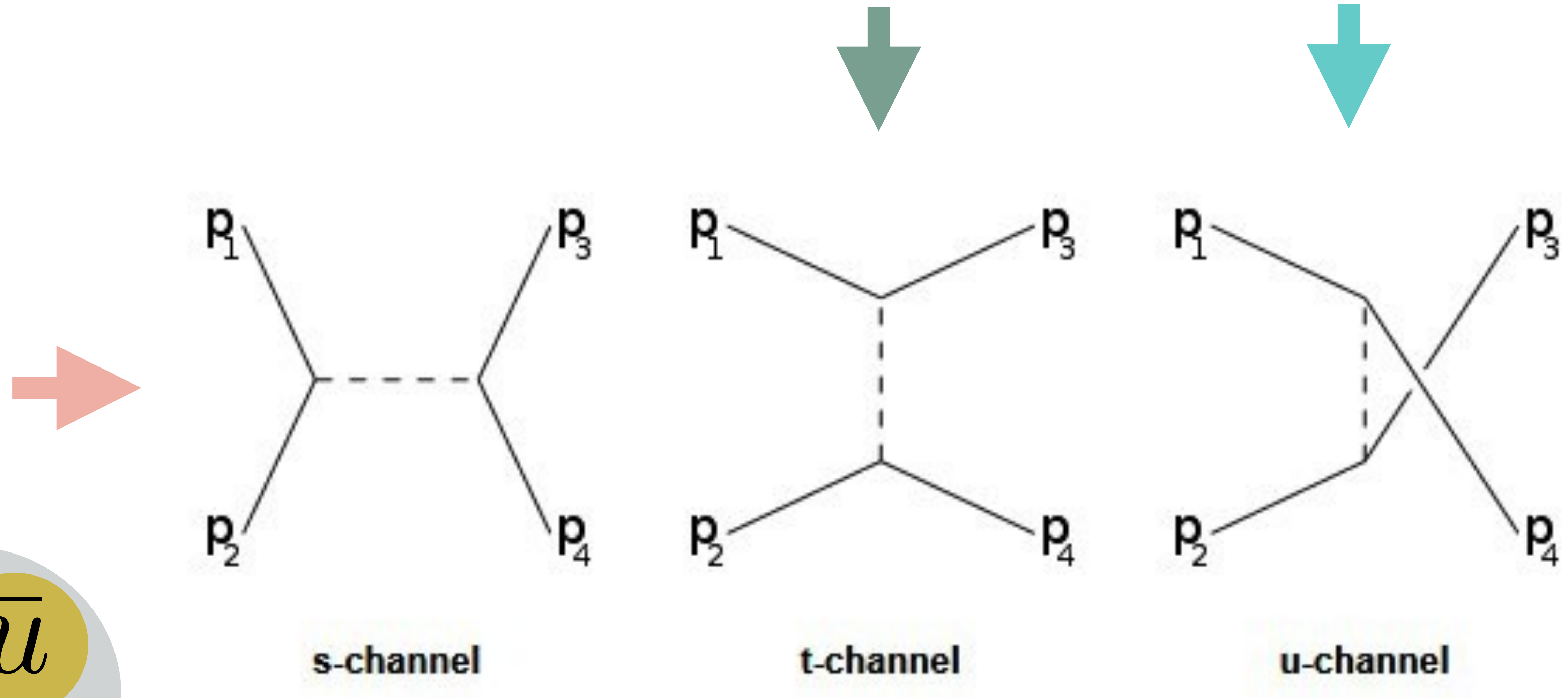
$$t(z) = \oint_C \frac{t(z')}{z'-z} dz'$$

Causality ↔ Analiticity



Crossing



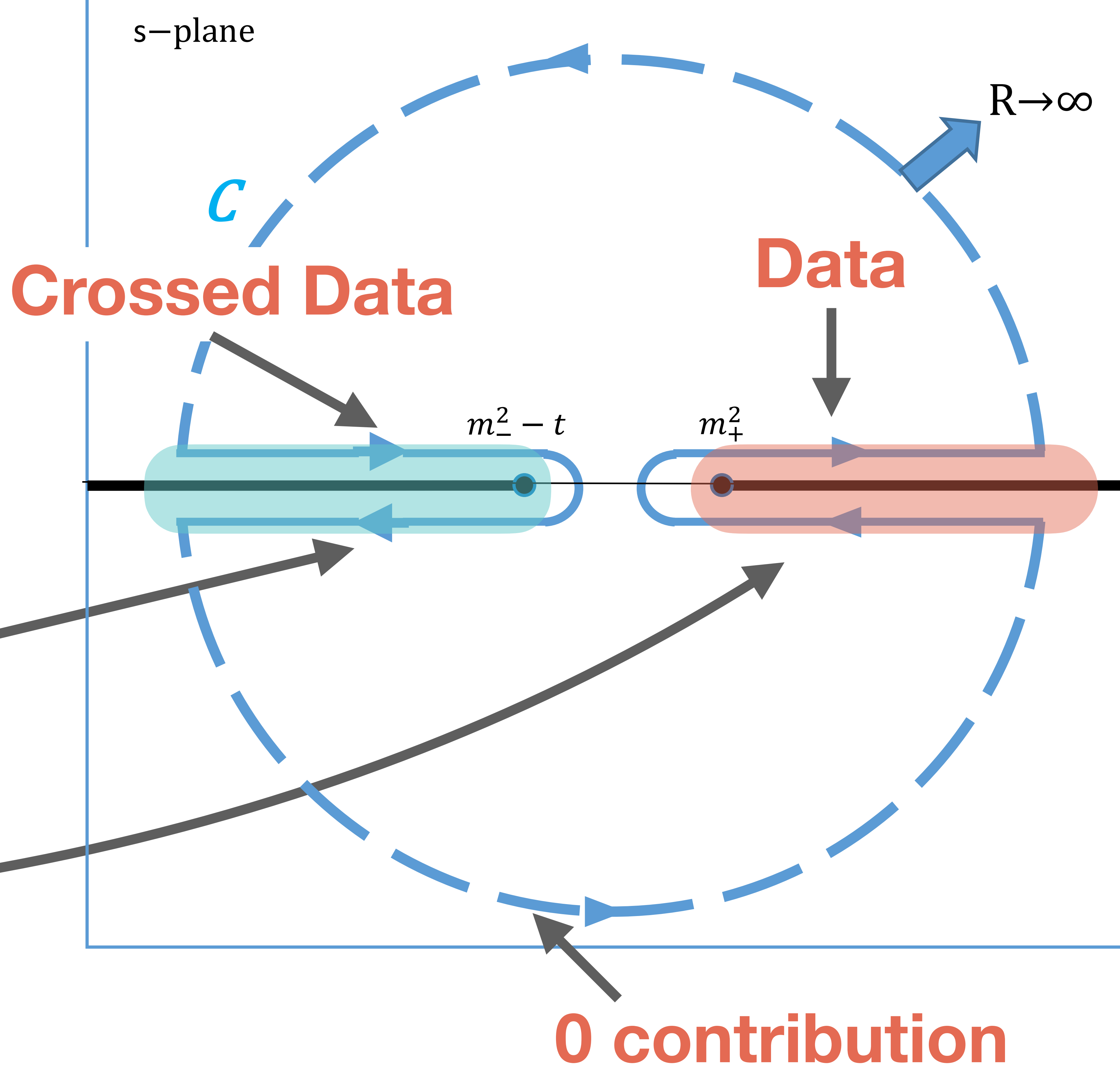


Dispersion relations

Cauchy

$$t(z) = \oint_C \frac{t(z')}{z'-z} dz'$$

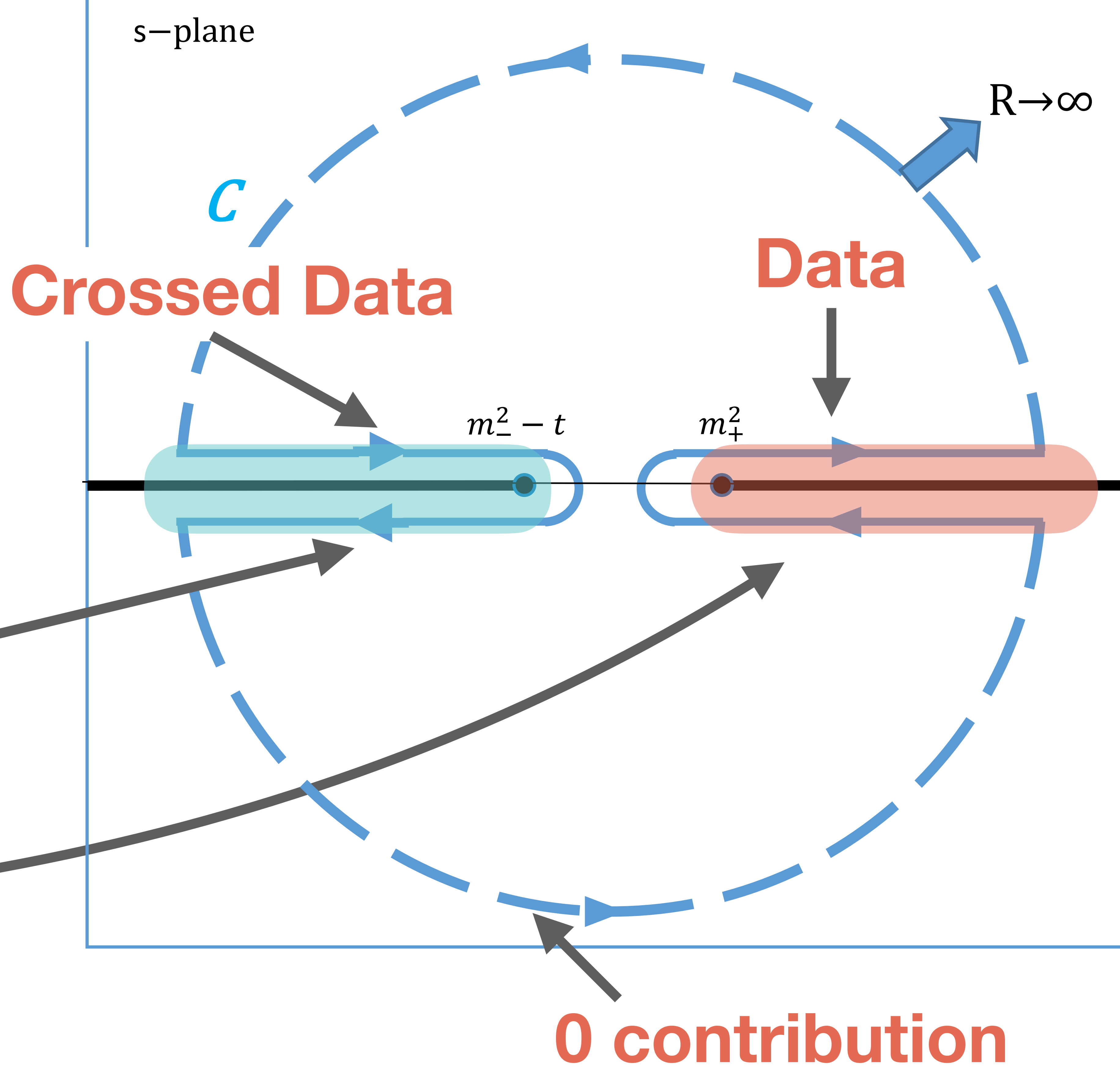
$$T^I(s,t) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}\, T^I(s',t)}{s'-s} + LHC$$



Dispersion relations

Cauchy

$$t(z) = \oint_C \frac{t(z')}{z'-z} dz'$$



$$T^I(s,t) = \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\operatorname{Im} T^I(s',t)}{s'-s} + LHC$$

Crossing

$$T^I(s,t) = \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \left(\frac{\operatorname{Im} T^I(s',t)}{(s'-s)} + \frac{\sum C_{su}^{I\,I'} \operatorname{Im} T^{I'}(s',t)}{(s'-u)} \right)$$

Partial wave dispersion relations

$$T^I(s,t) = 32\pi \sum_\ell (2\ell+1) P_\ell(\cos\theta) t_\ell^I(s)$$

Fit → In

DR → Out

$$\tilde{t}_\ell^I(s) = \tau_\ell^I(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^\infty ds' \, K_{\ell\ell'}^{II'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

Example, ROY eqs. $I = \ell = 0$

$$\tilde{t}_0^0(s) = a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)\left(s - 4m_\pi^2\right) + \sum_{I',\ell'} \int_{4m_\pi^2}^\infty ds' \, K_{0\ell'}^{0I'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

Fit → In

DR → Out

$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K^{II'}_{\ell\ell'}(s',s) \, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

"Model-independent"

Algebraic functions

$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K^{II'}_{\ell\ell'}(s',s) \, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

All Isospins and partial waves

Fit → In

DR → Out

$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K^{II'}_{\ell\ell'}(s',s) \, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

"Model-independent"

- [x] Obtain your DRs ⟷ Crossing+analyticity
- [ ] Use all PWs available ⟷ Necessary Input
- [ ] Make *Fit → In* *DR → Out* compatible ⟷ Unitarity
- [ ] Make *DR → Out* and data compatible ⟷ Lattice QCD data description

$I = 2\ \pi\pi$

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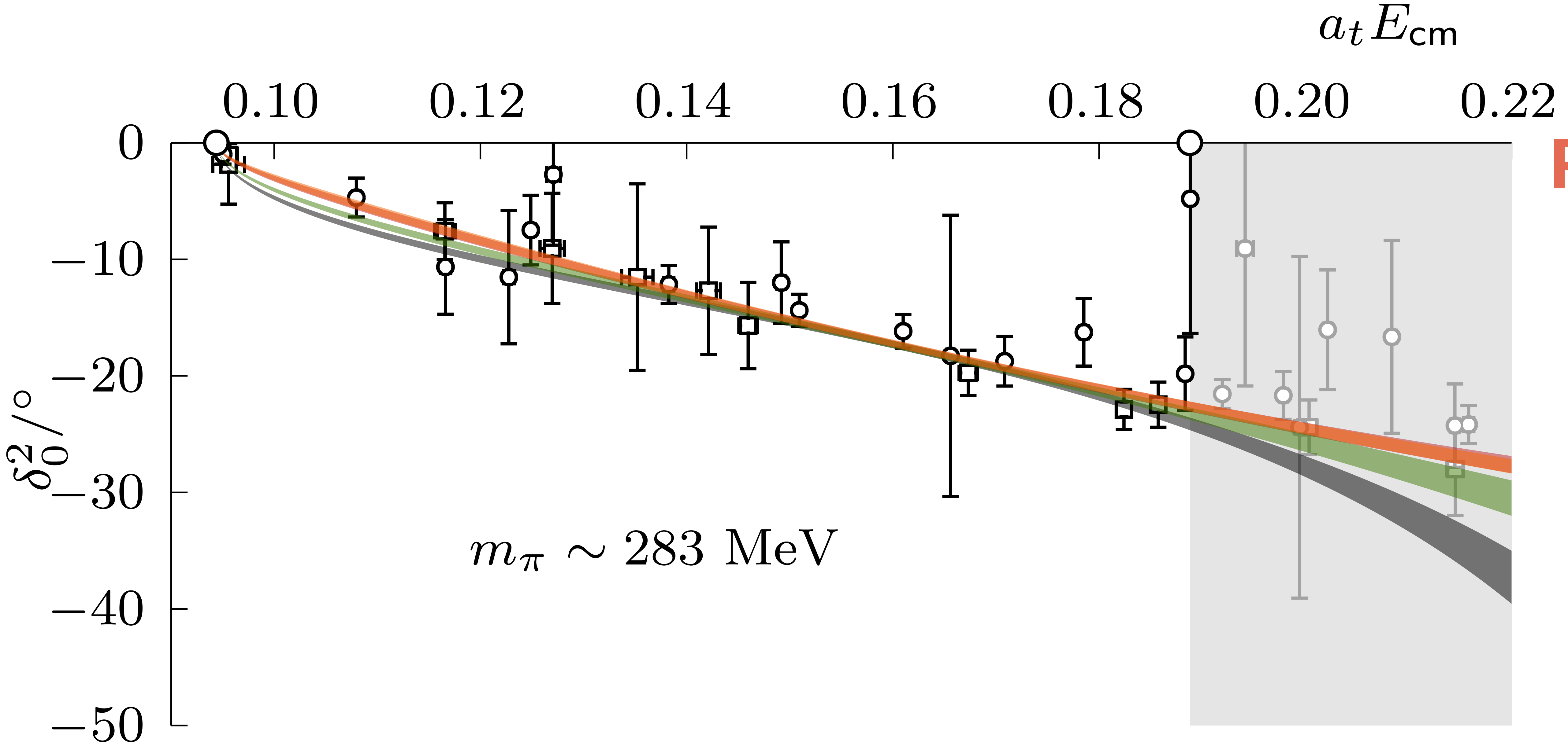
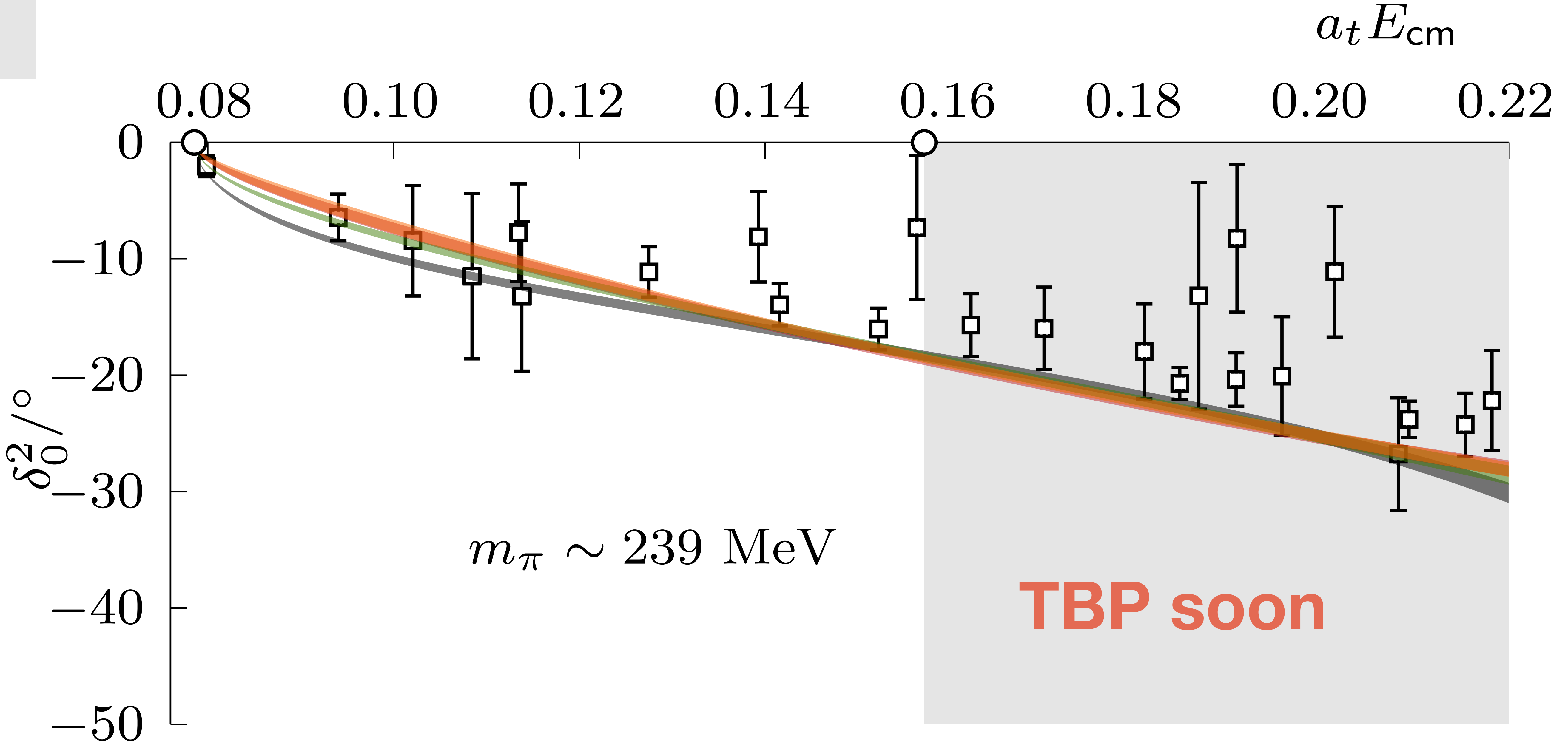


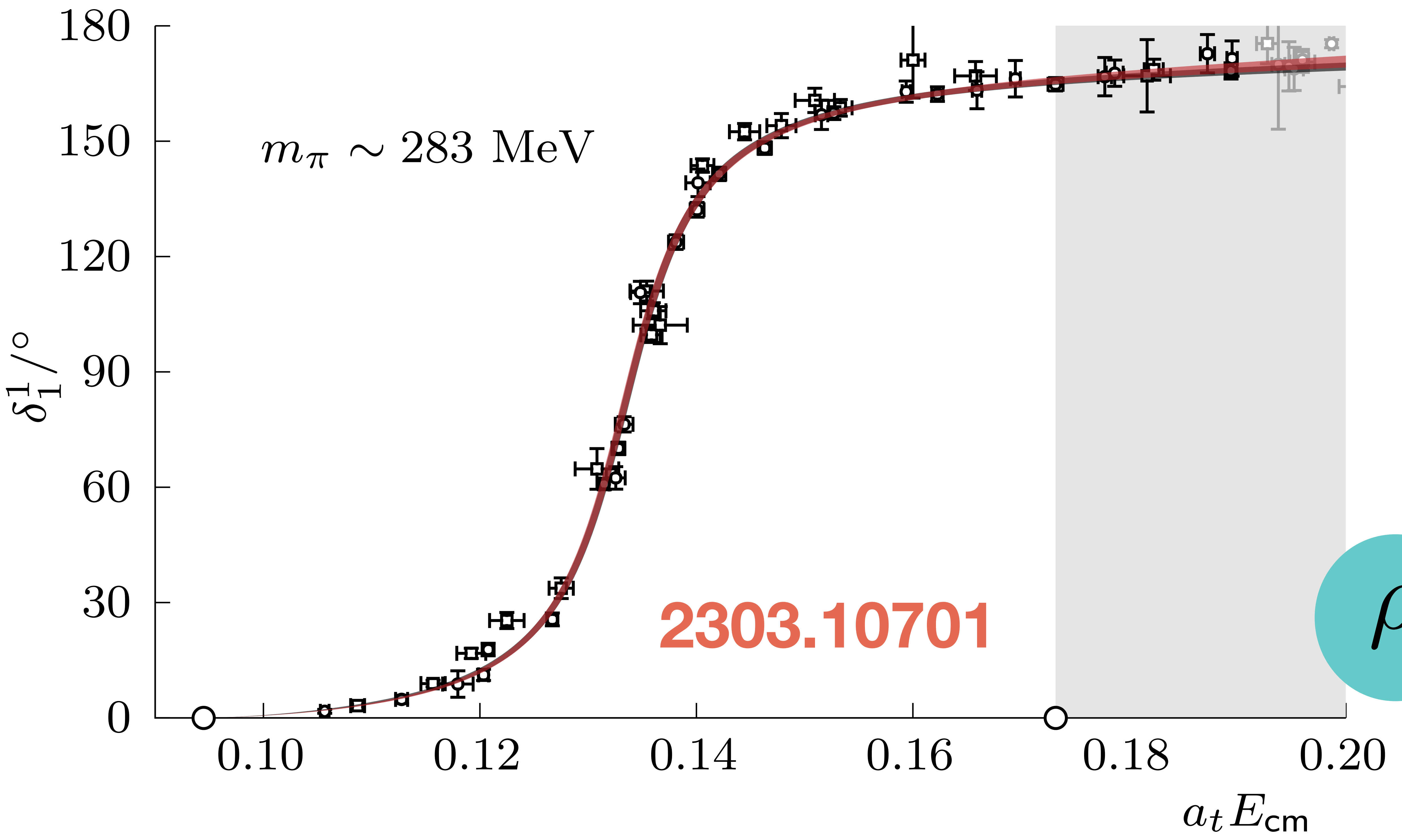
Percent error for $\delta(s)$

10+ parameterizations

Systematic spread at threshold



$I = 1\ \pi\pi$



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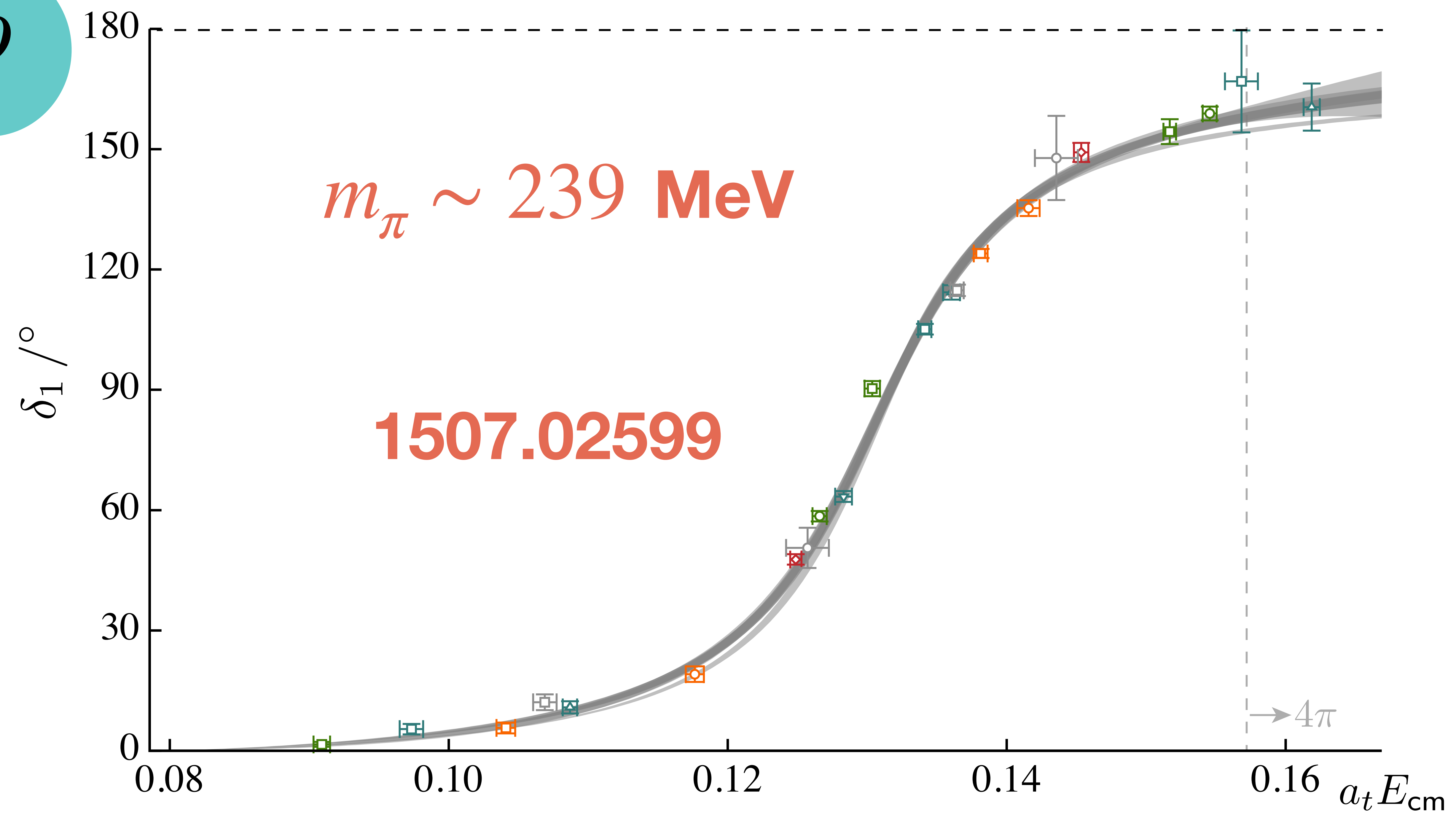
ρ

Percent error for $\delta(s)$



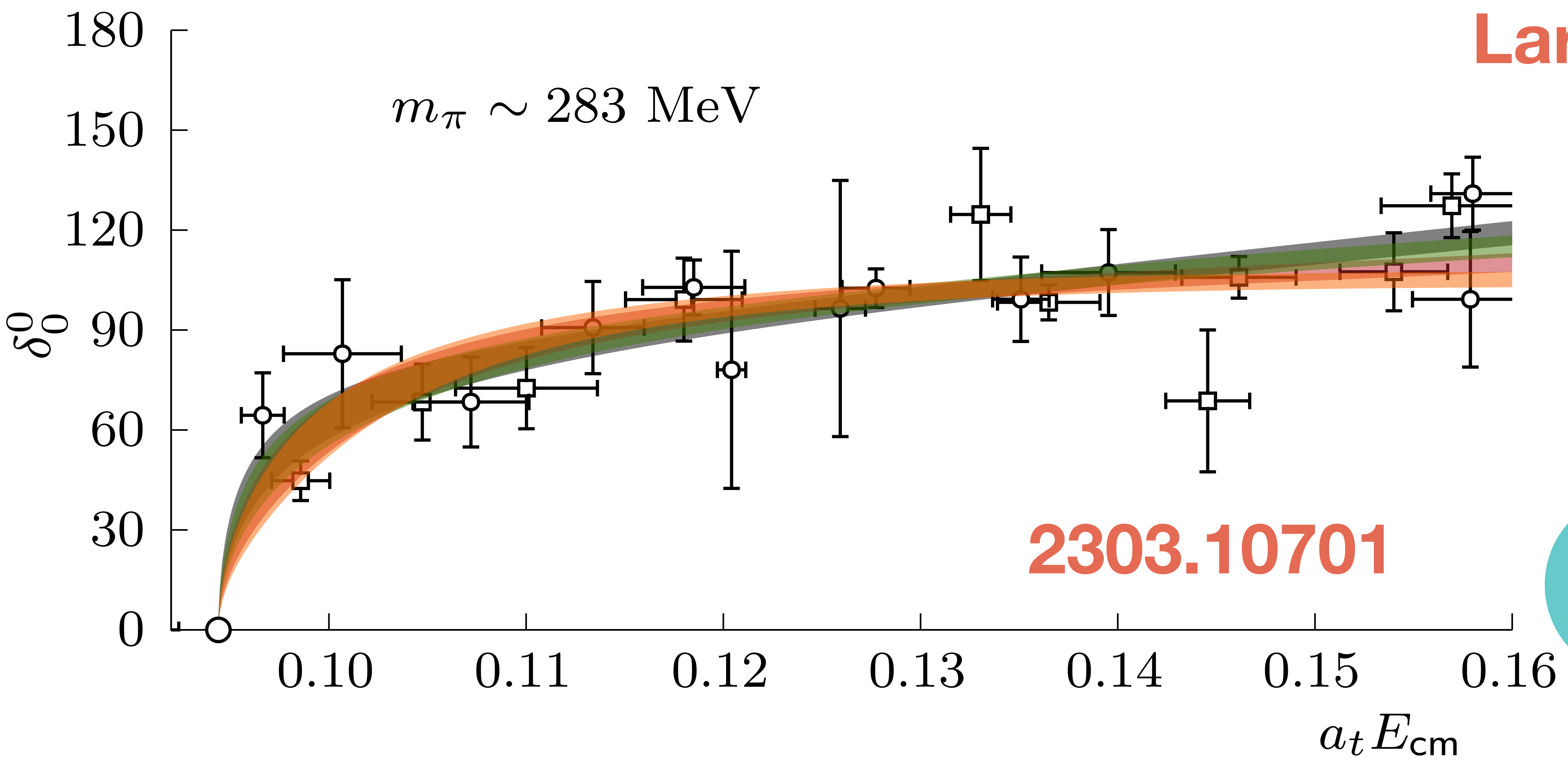
$m_\pi \sim 239$ **MeV**

1507.02599

Around 10 parameterizations

Very consistent amplitude fits

$I = 0\ \pi\pi$



Large derivative at threshold

$\delta(s) = \pi/2$ is far from threshold

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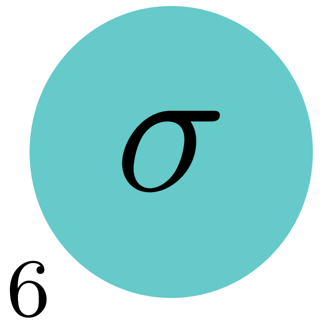
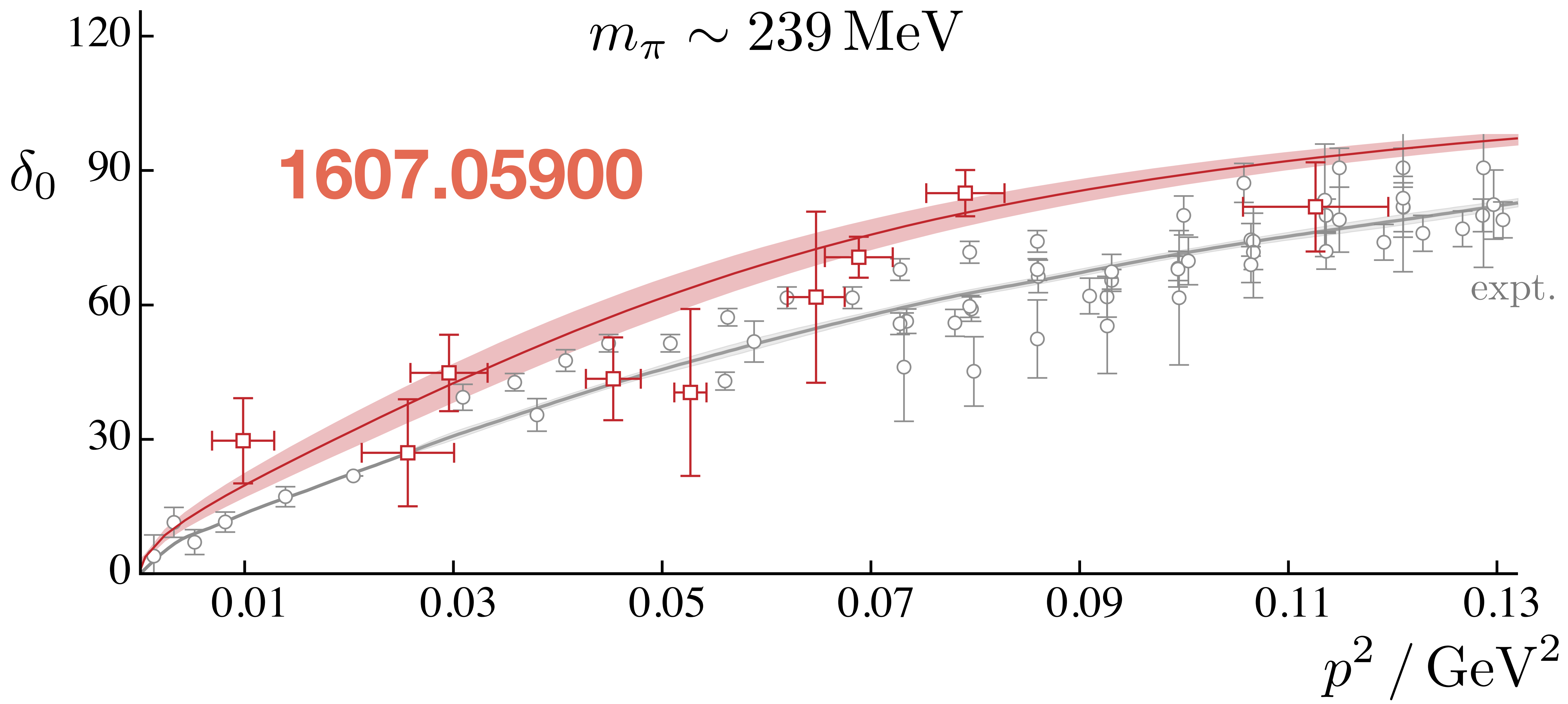




1607.05900

Over 20 parameterizations

Smaller derivative at threshold

Permutations

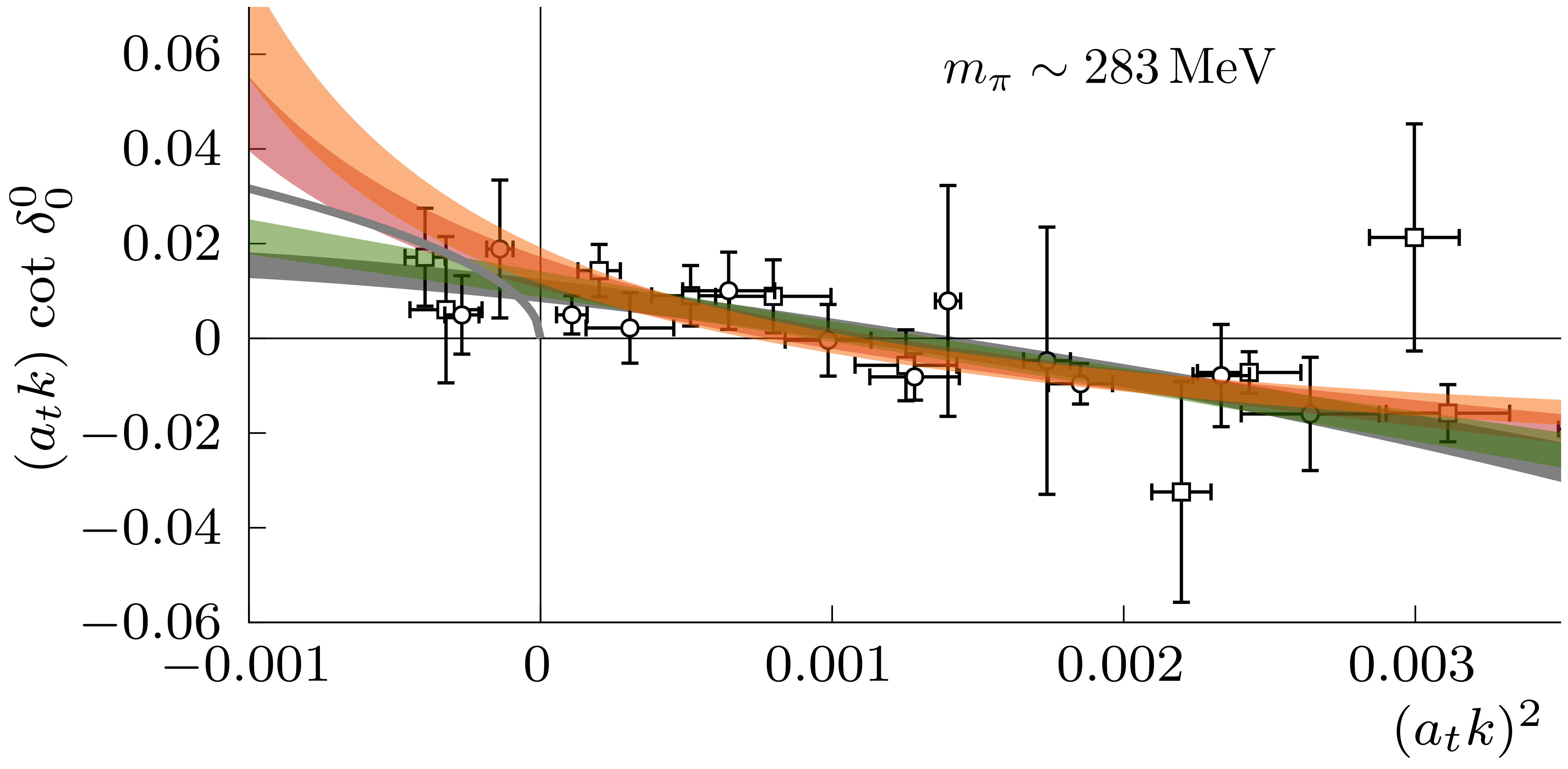
$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds'\, K_{\ell\ell'}^{II'}(s',s)\, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

For ℓ_{max} partial waves

$$N_I \ell_{max} N_{params} \sim 10^5$$

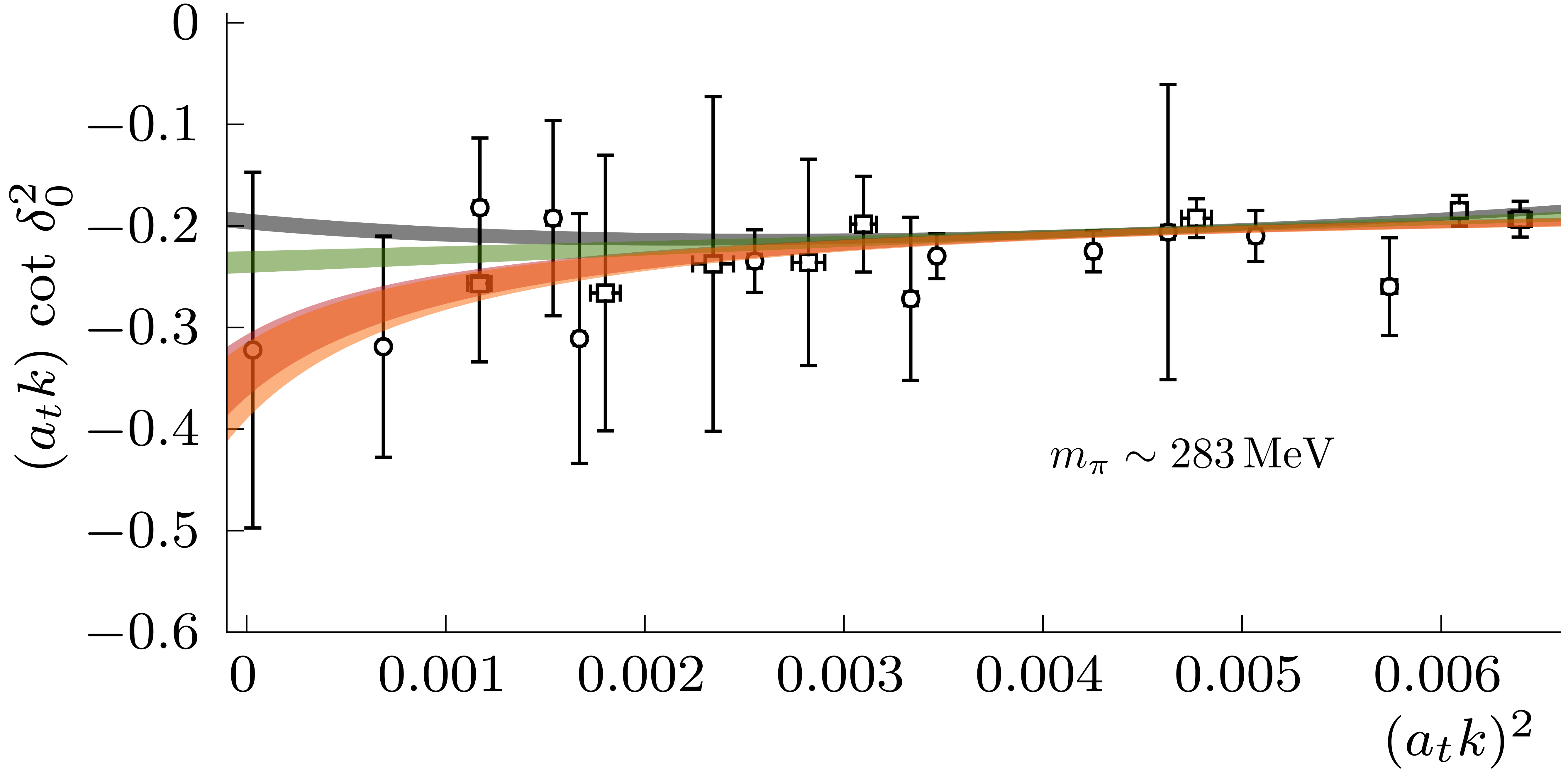
$k \cot \delta(s)$

Only S-waves really matter



$$k \cot \delta_0^I(s) \sim 1/a_0^I$$

Large spreads at threshold



Similar for $m_\pi \sim 239$ MeV

ROY/GKPY highly dependent on scatt. Lengths

$Fit \to In$

$DR \to Out$

$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^\infty ds' \, K^{II'}_{\ell\ell'}(s',s) \, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

Model-independent

- [x] Obtain your DRs ⟷ Crossing+analyticity
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- [ ] Make $Fit \to In$ $DR \to Out$ compatible ⟷ Unitarity
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Make $Fit \to In$ $DR \to Out$ compatible ⟷ Unitarity

$$\left[d^2\right]^I_\ell \equiv \sum_{i=1}^{N_{\rm smpl}} \left(\frac{\mathrm{Re}\,\tilde{t}^I_\ell(s_i) - \mathrm{Re}\,t^I_\ell(s_i)}{\Delta\left[\mathrm{Re}\,\tilde{t}^I_\ell(s_i) - \mathrm{Re}\,t^I_\ell(s_i)\right]} \right)^2$$

Make $DR \to Out$ and data compatible ⟷ Lattice QCD data description

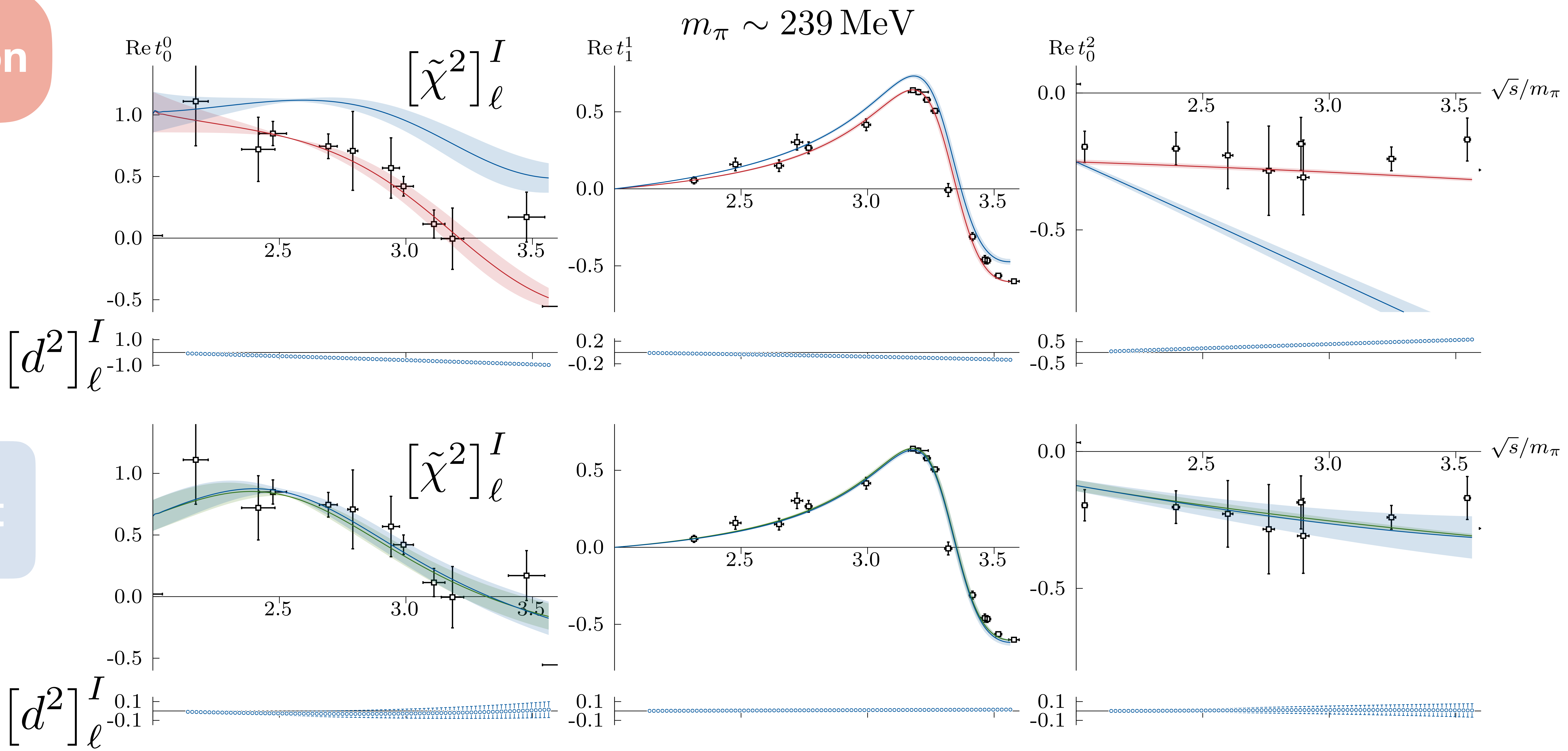
$$\left[\tilde{\chi}^2\right]^I_\ell \equiv \sum_{i,j=1}^{N_{\rm lat}} \left(\frac{\mathfrak{f}_i - \mathrm{Re}\,\tilde{t}^I_\ell(s_i)}{\Delta_i} \right) \mathrm{corr}\left(\mathfrak{f}_i, \mathfrak{f}_j\right) \left(\frac{\mathfrak{f}_j - \mathrm{Re}\,\tilde{t}^I_\ell(s_j)}{\Delta_j} \right)$$

Tests: good vs bad

Bad fit combination

Dispersive output

Good fit combination



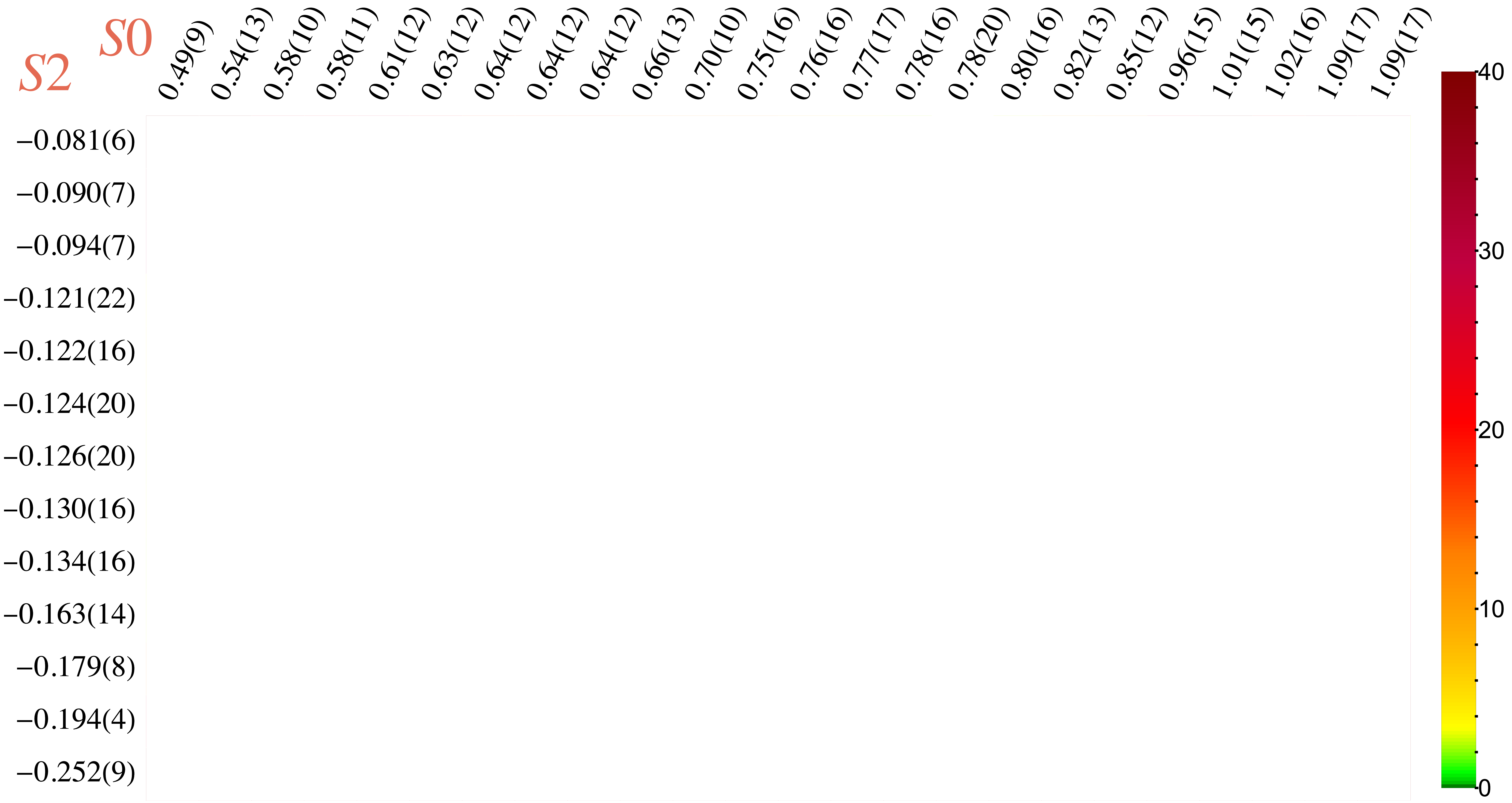
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

ROY

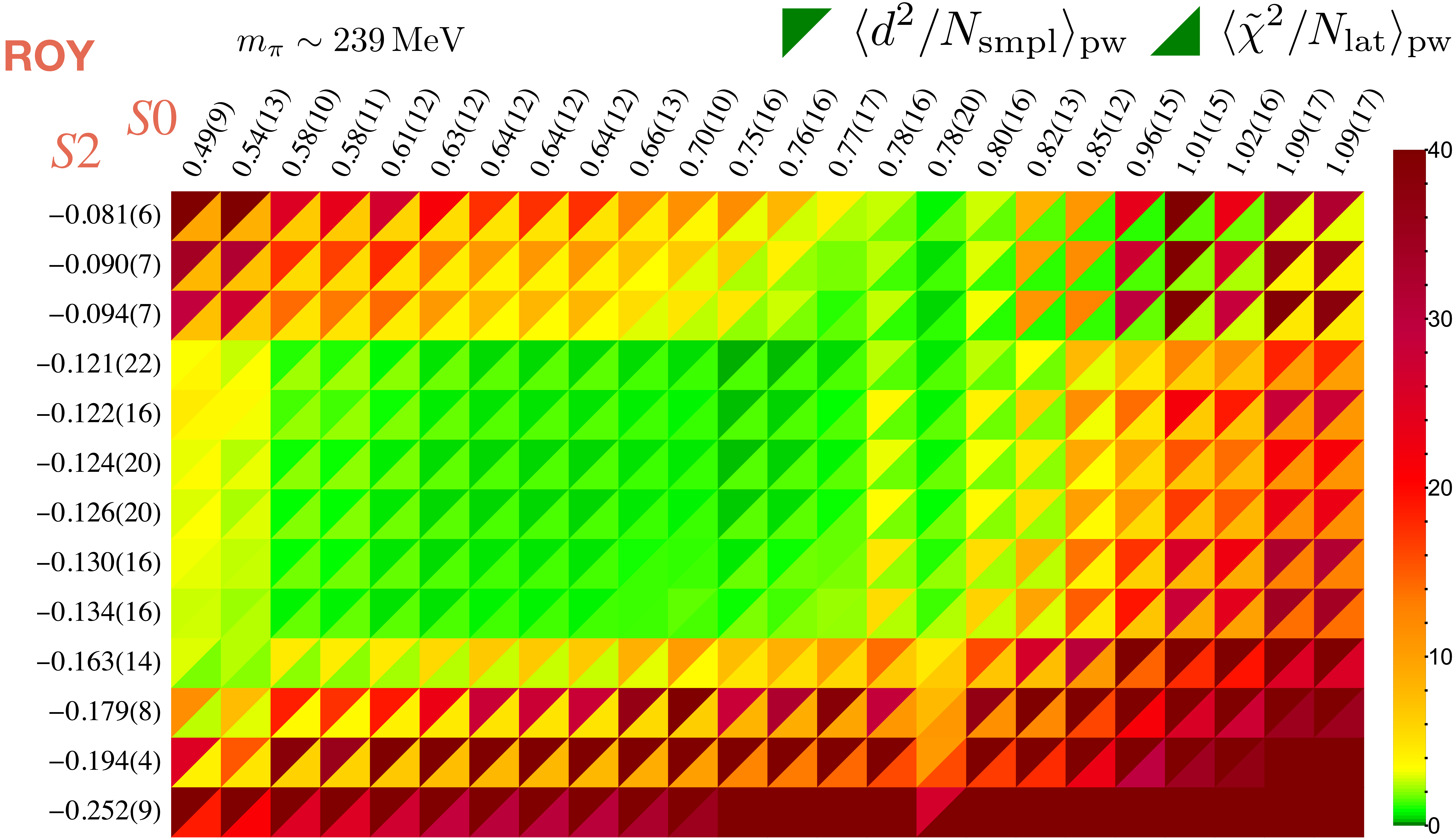
$m_\pi \sim 239\,\text{MeV}$

$\langle d^2/N_{\text{smpl}}\rangle_{\text{pw}}$ $\langle \tilde{\chi}^2/N_{\text{lat}}\rangle_{\text{pw}}$



Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$



Scattering plane

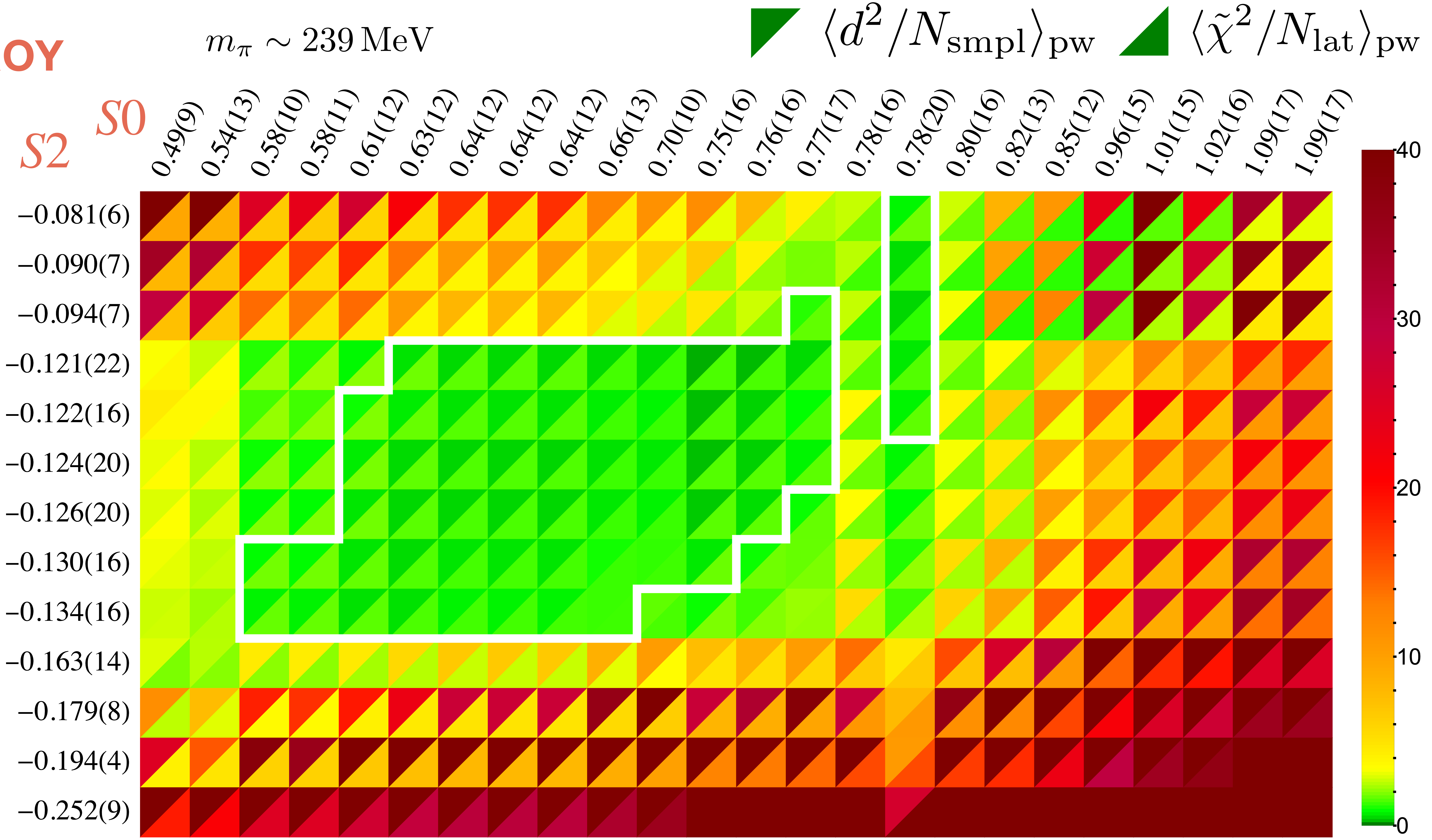
$N_I \ell_{\max} N_{\rm params} \sim 300-400$

$m_\pi \sim 239\,{\rm MeV}$

ROY

White

$d^2/N_{\rm smpl} < 3, \quad \tilde{\chi}^2/N_{\rm lat} < 3$



Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

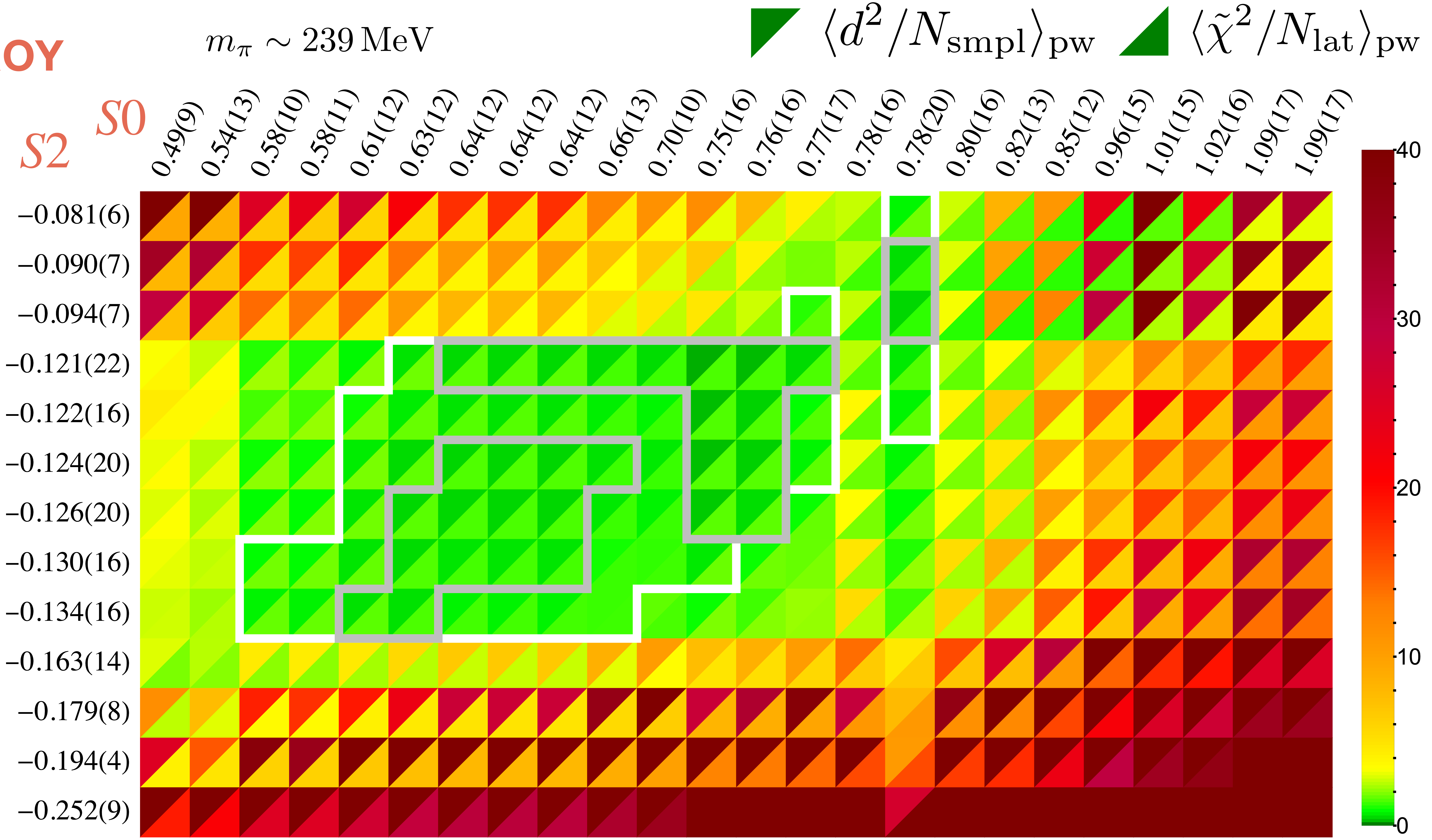
ROY

White

$d^2/N_{\text{smpl}} < 3, \quad \tilde{\chi}^2/N_{\text{lat}} < 3$

Light gray

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 3$



Scattering plane

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$d^2/N_{\text{smpl}} < 3, \quad \tilde{\chi}^2/N_{\text{lat}} < 3$

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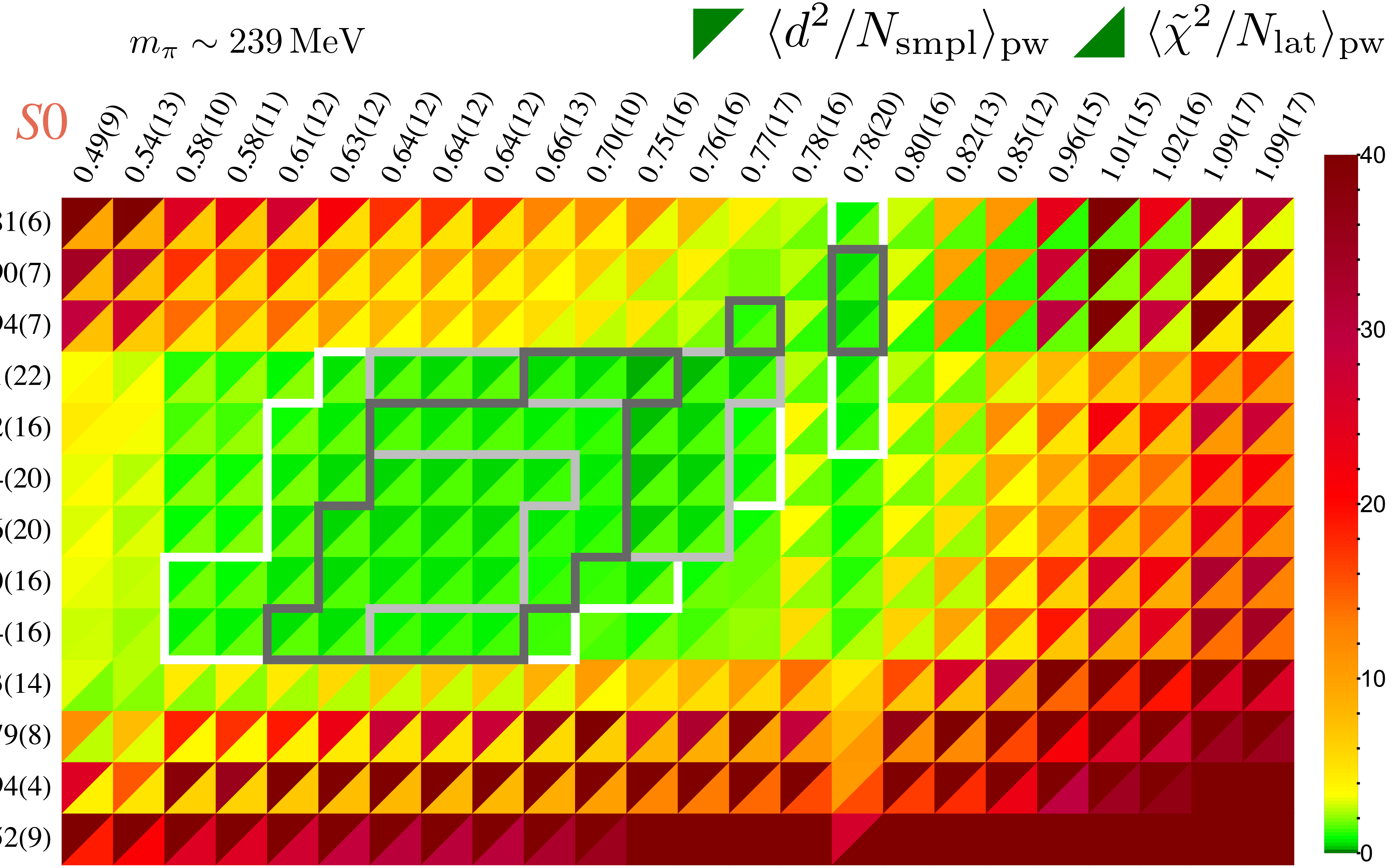
$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 3$

Dark gray

$d^2/N_{\text{smpl}} < 3, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

ROY

White

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Light gray

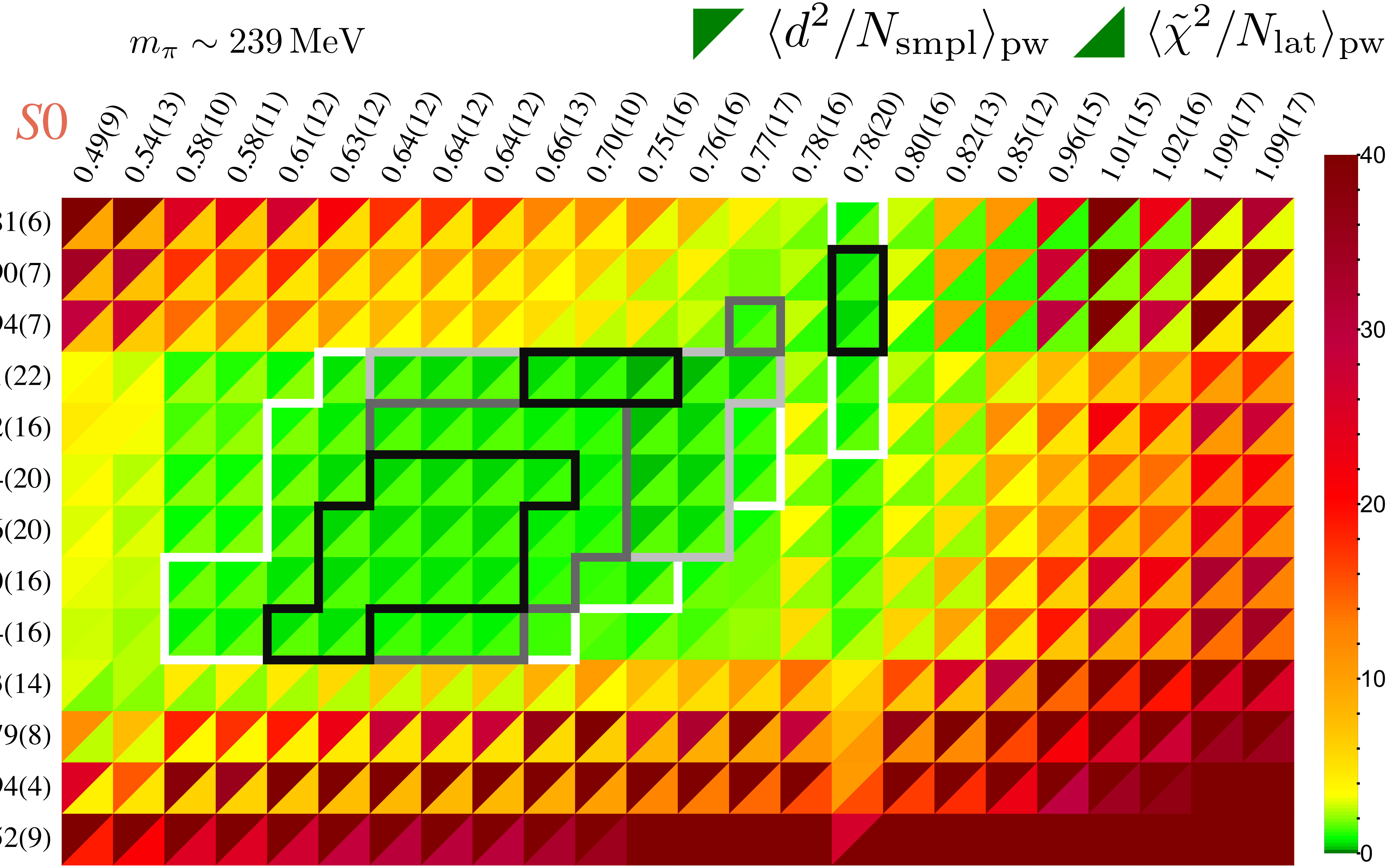
$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 3$

Dark gray

$d^2/N_{\text{smpl}} < 3, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$

Black

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



Scattering plane

$N_I \ell_{\max} N_{\text{params}} \;\sim 300-400$

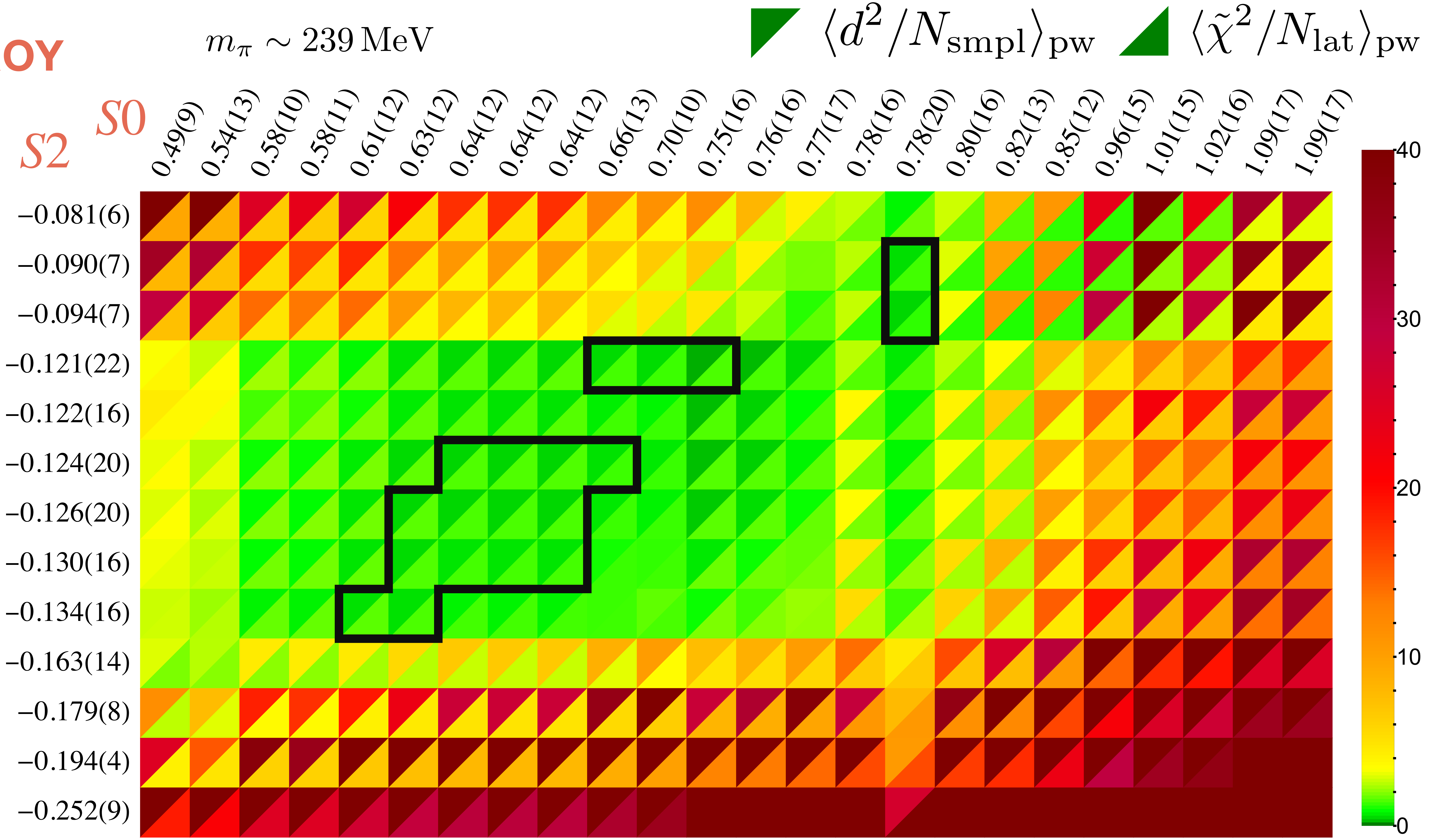
$m_\pi \sim 239\,\text{MeV}$

Black **ROY**

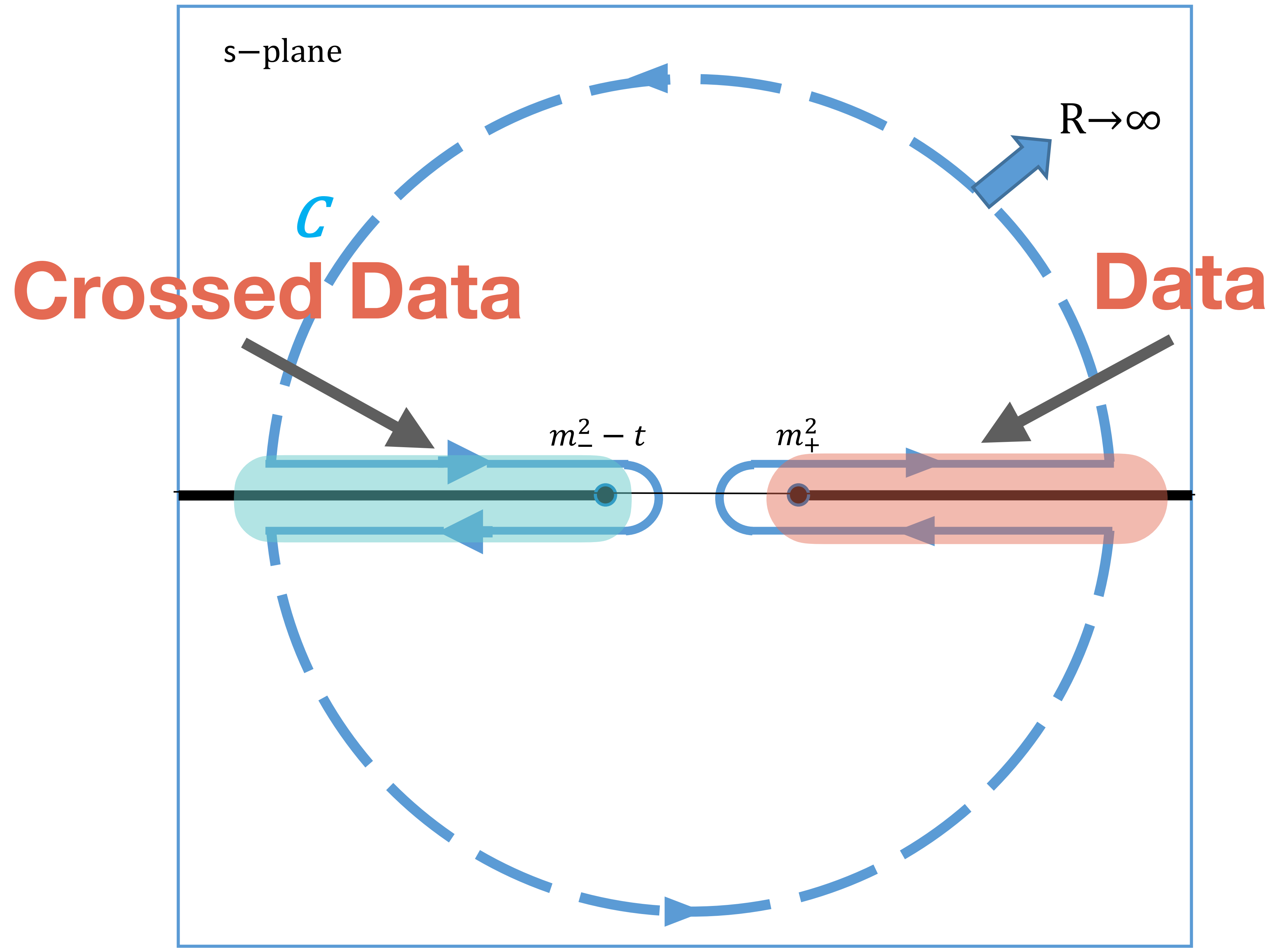
$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$

Much fewer combinations

Much smaller spread

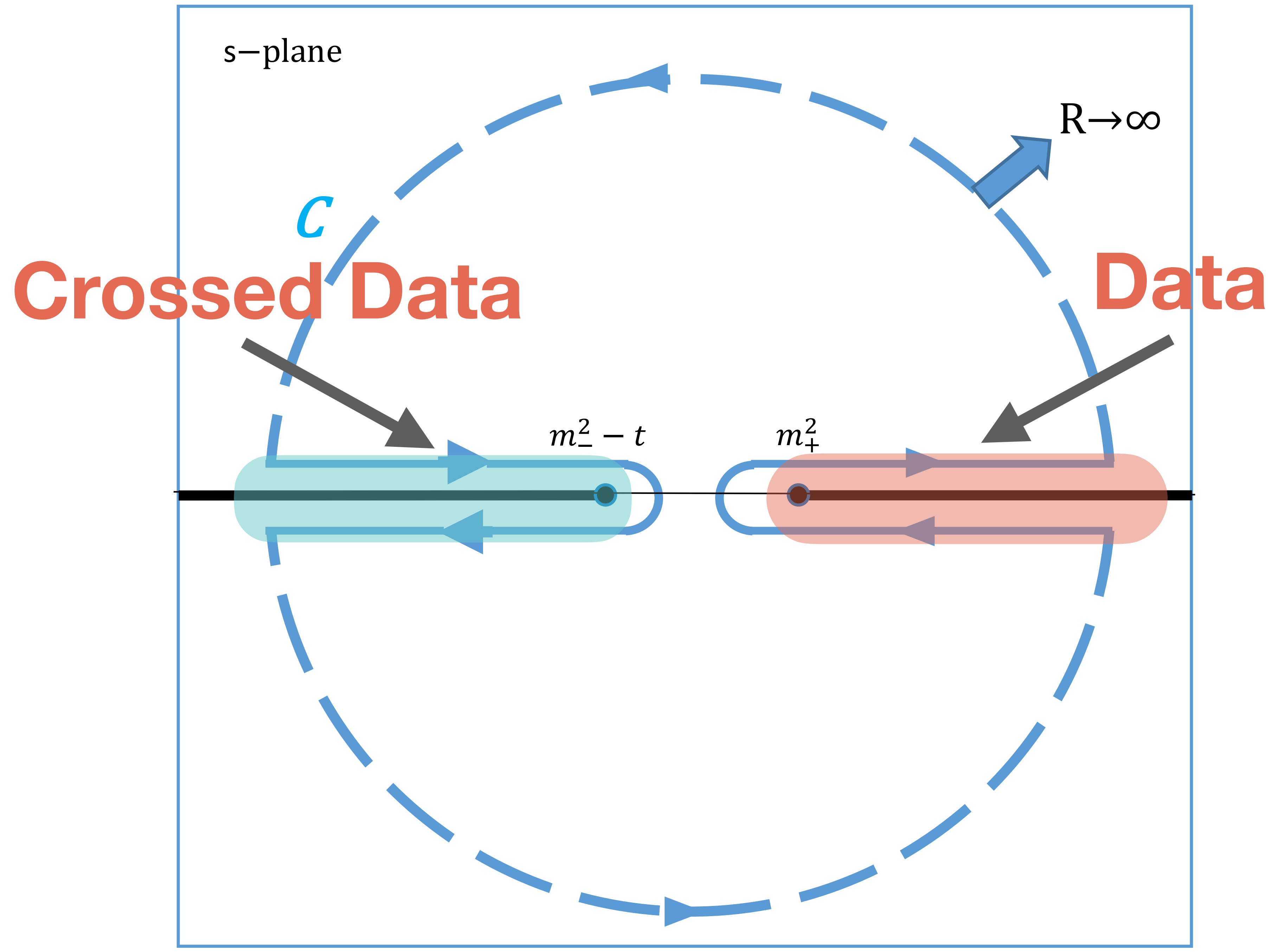


Outside the physical region



Both sides are good now

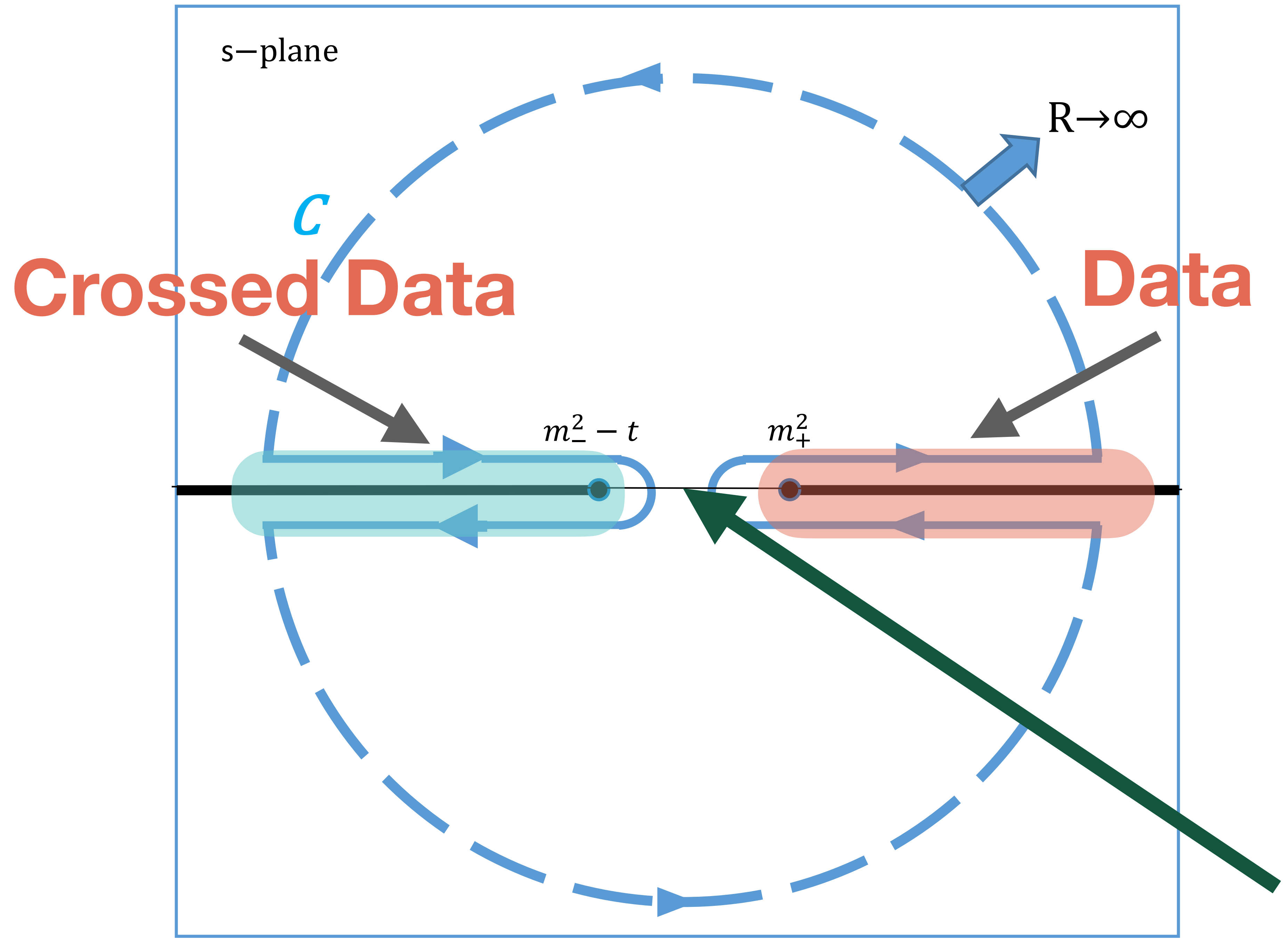
Outside the physical region



Both sides are good now

What happens everywhere else??

Outside the physical region

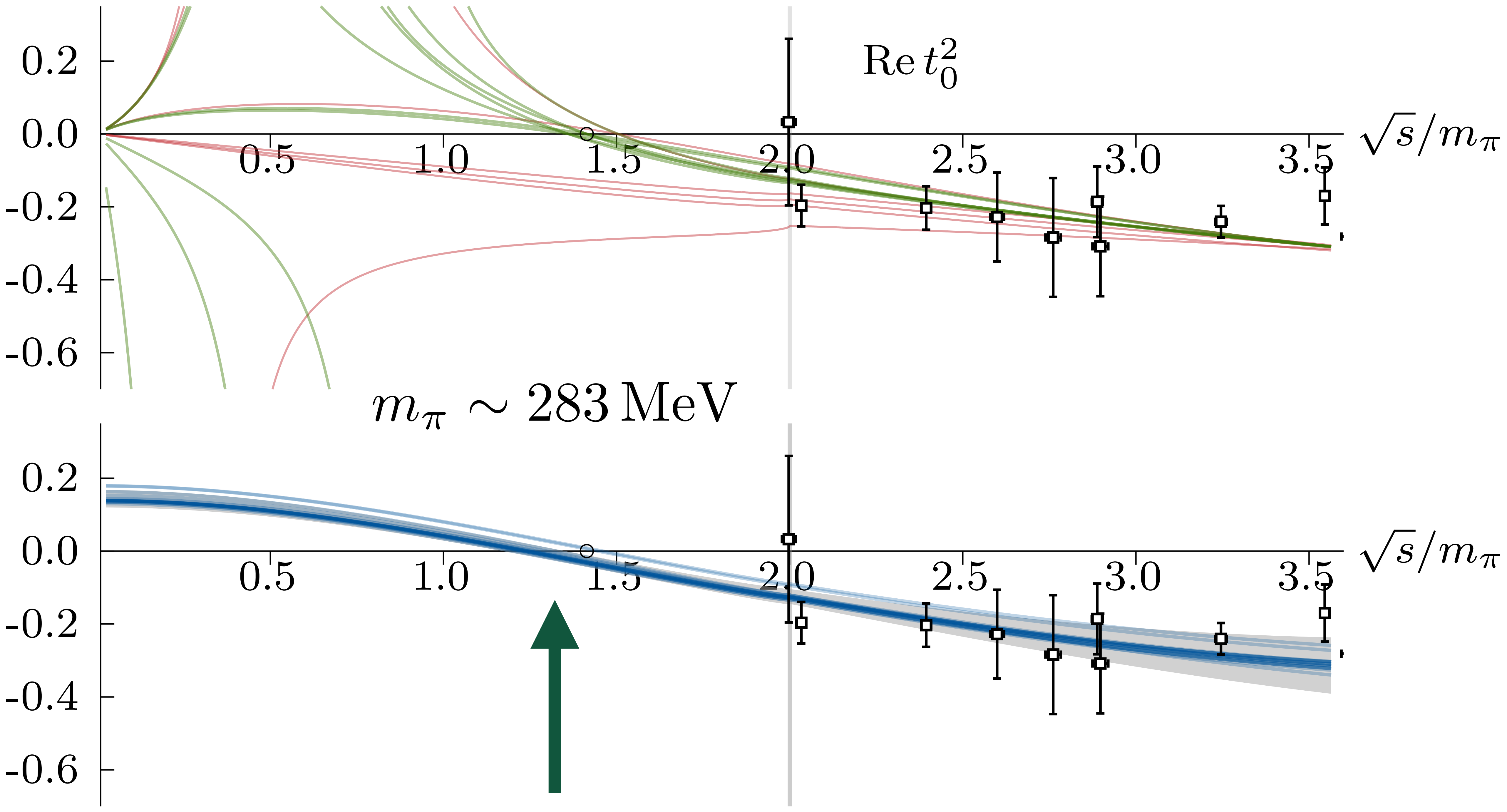


Both sides are good now

What happens everywhere else??

What happens here??

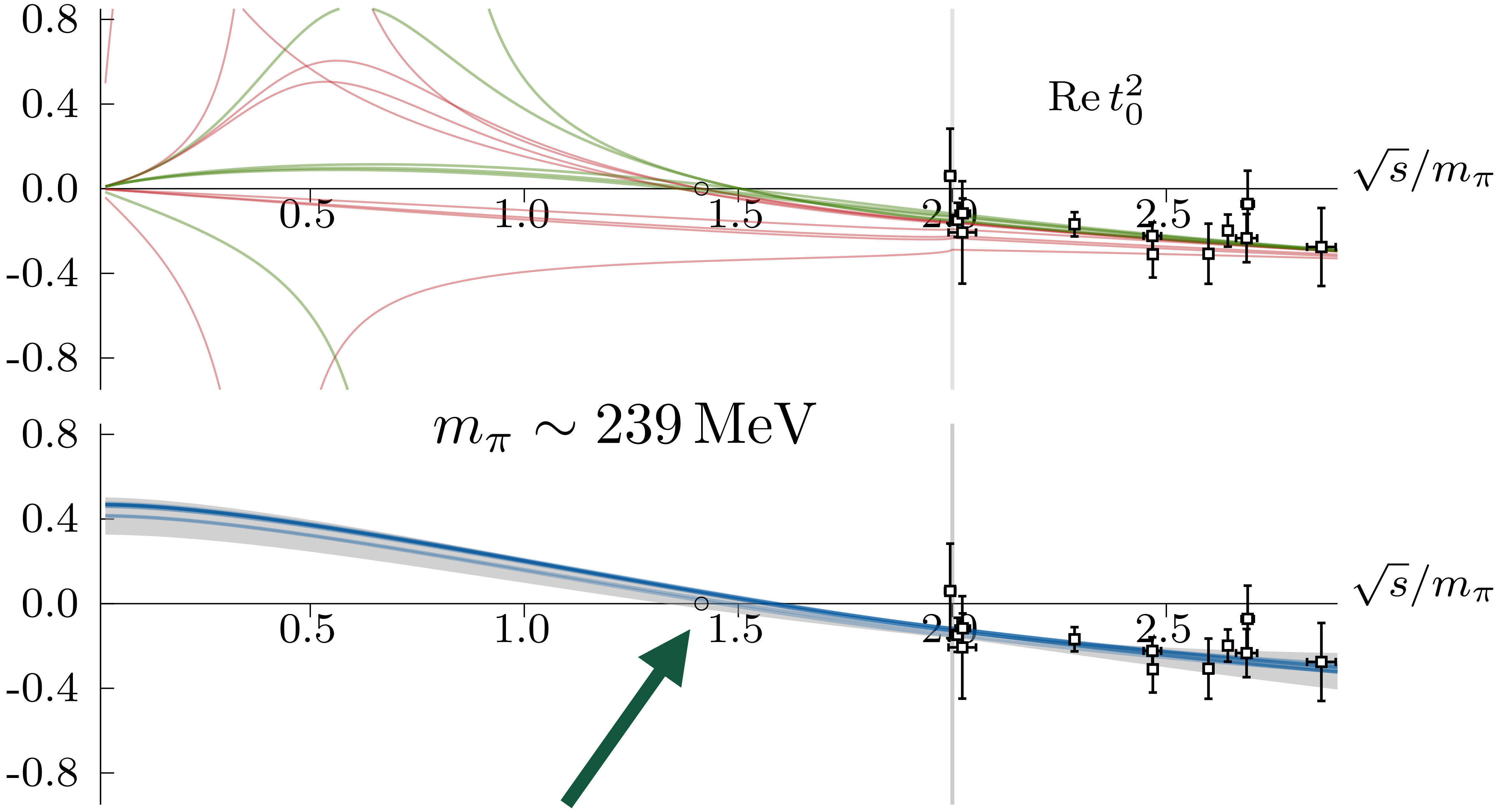
Sub-threshold



Adler zero

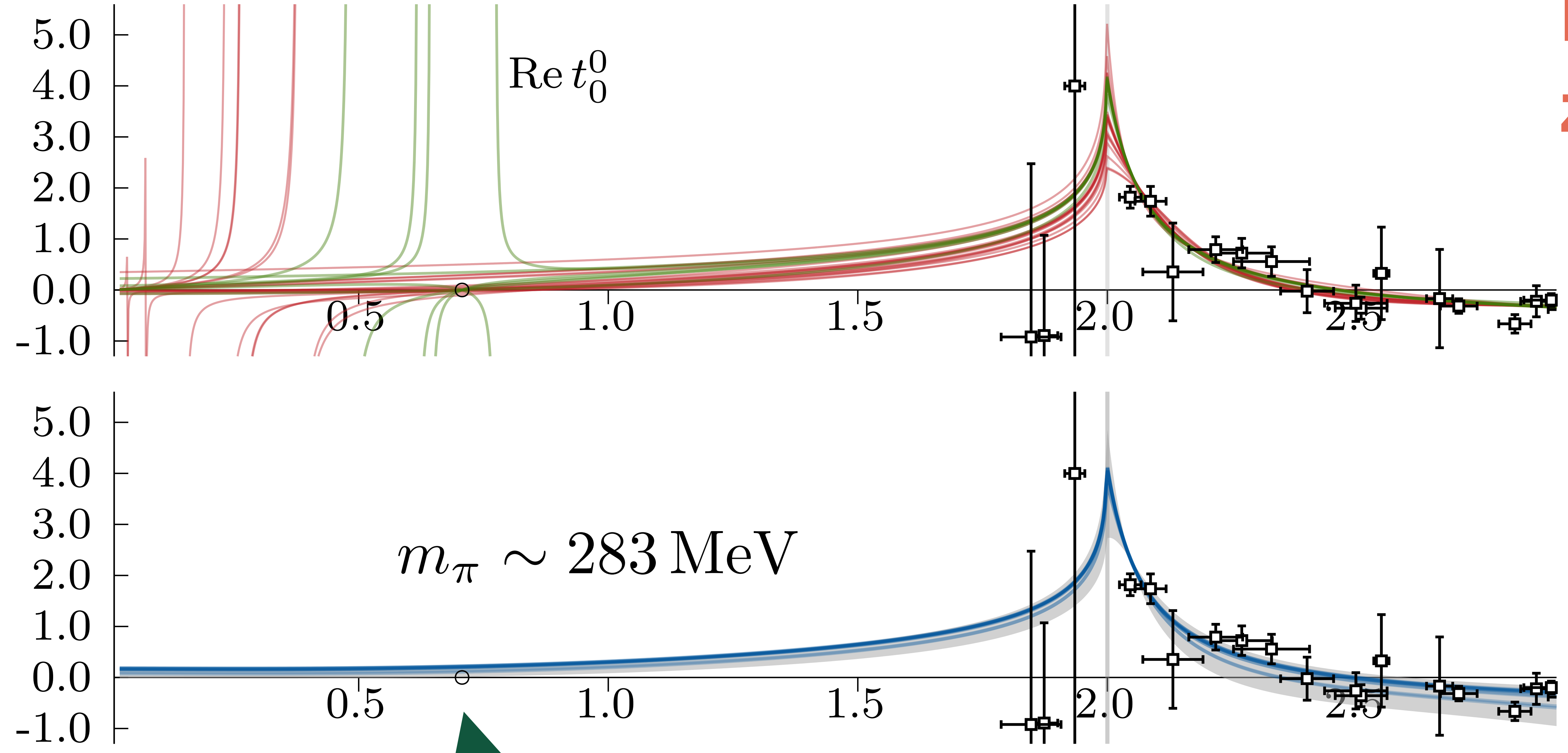
Even “bad” DRs produce Adler zeroes for I=2

Very “stable” for $I = 2\ \pi\pi$



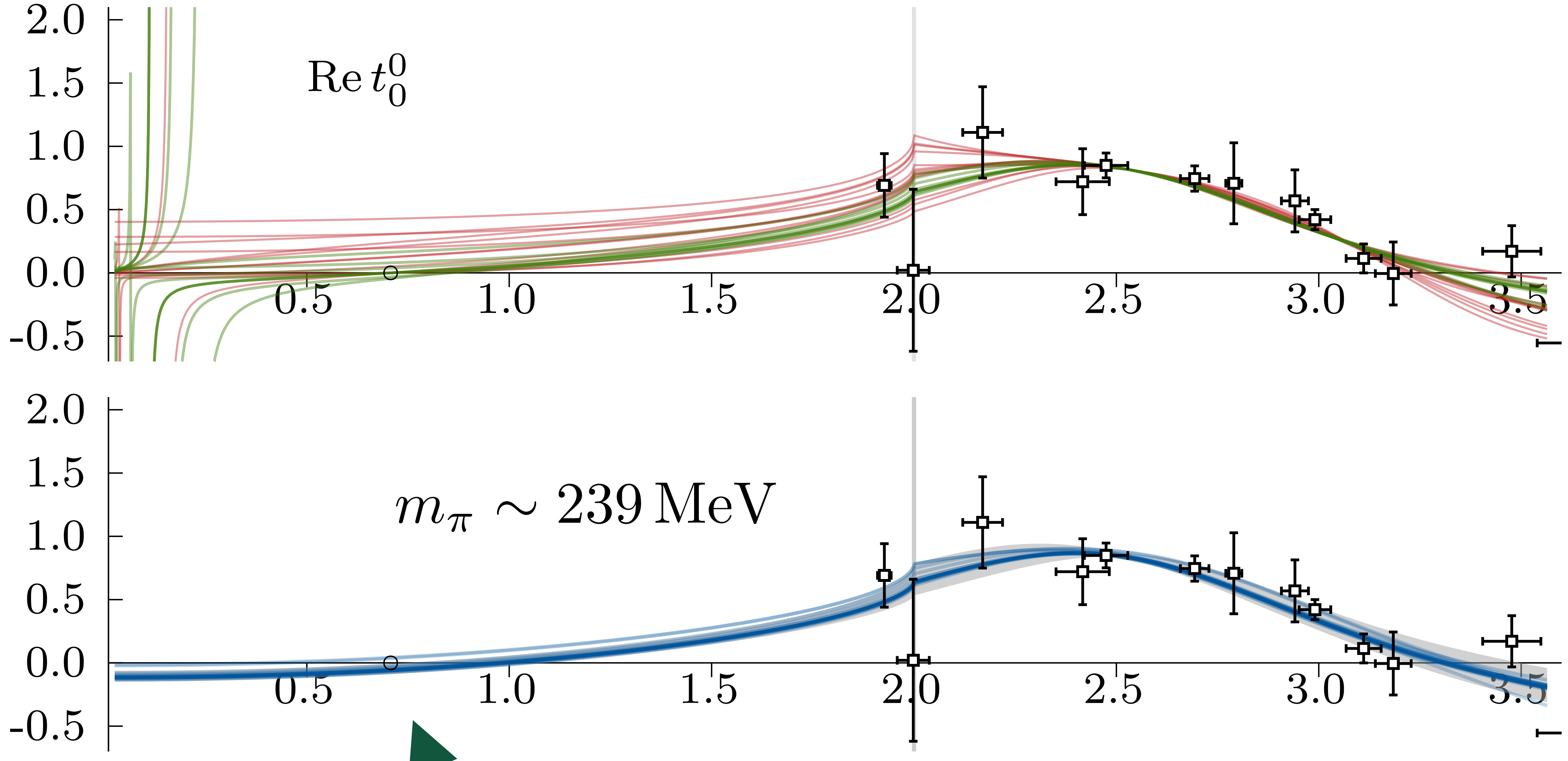
Adler zero

Sub-threshold



NO Adler zero

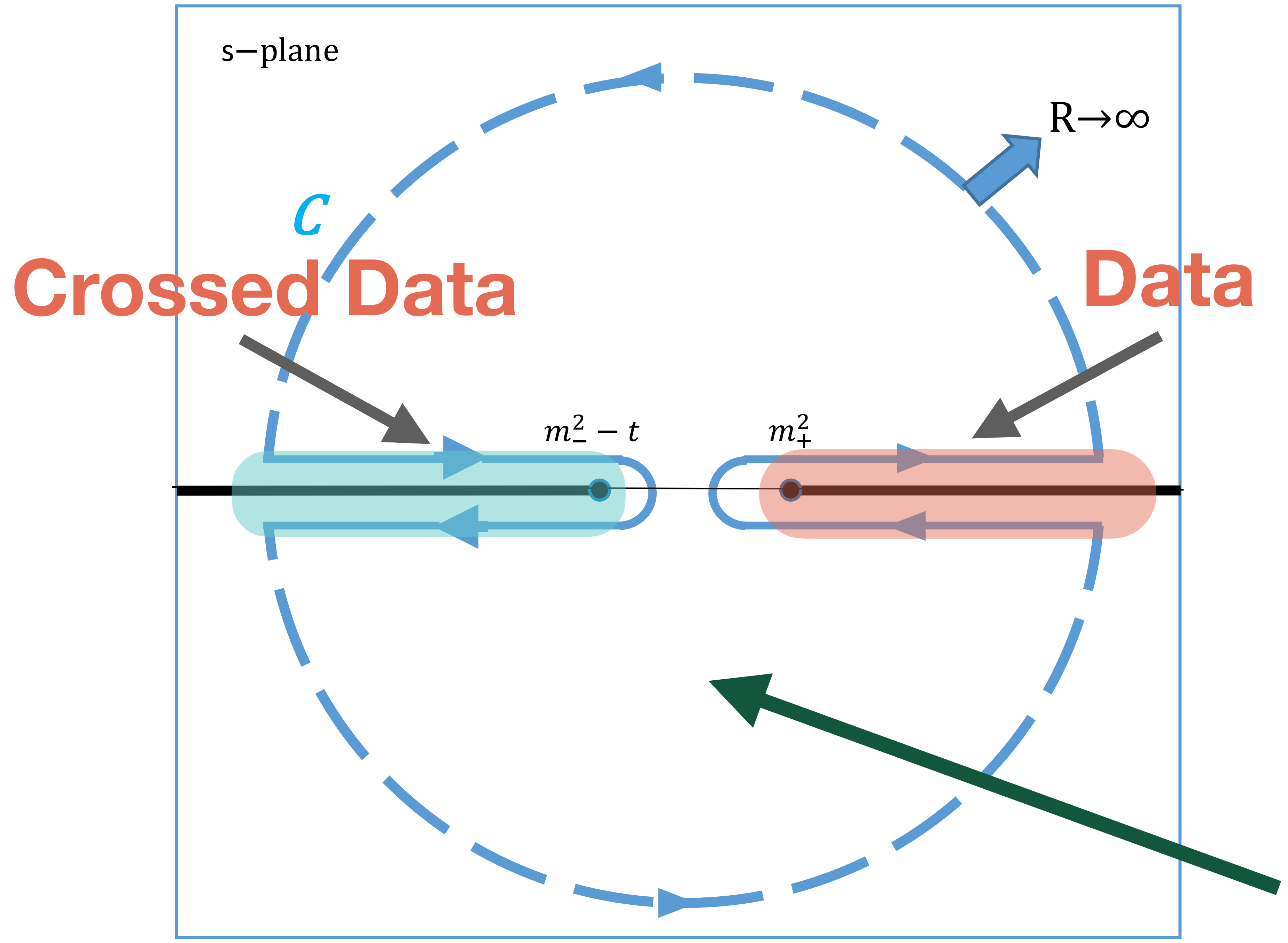
No good DR produces an $I=0$ $\pi\pi$ Adler zero for the heavier mass



Adler zero

All good DRs produce an $I=0$ $\pi\pi$ Adler zero for the lighter mass

Outside the physical region

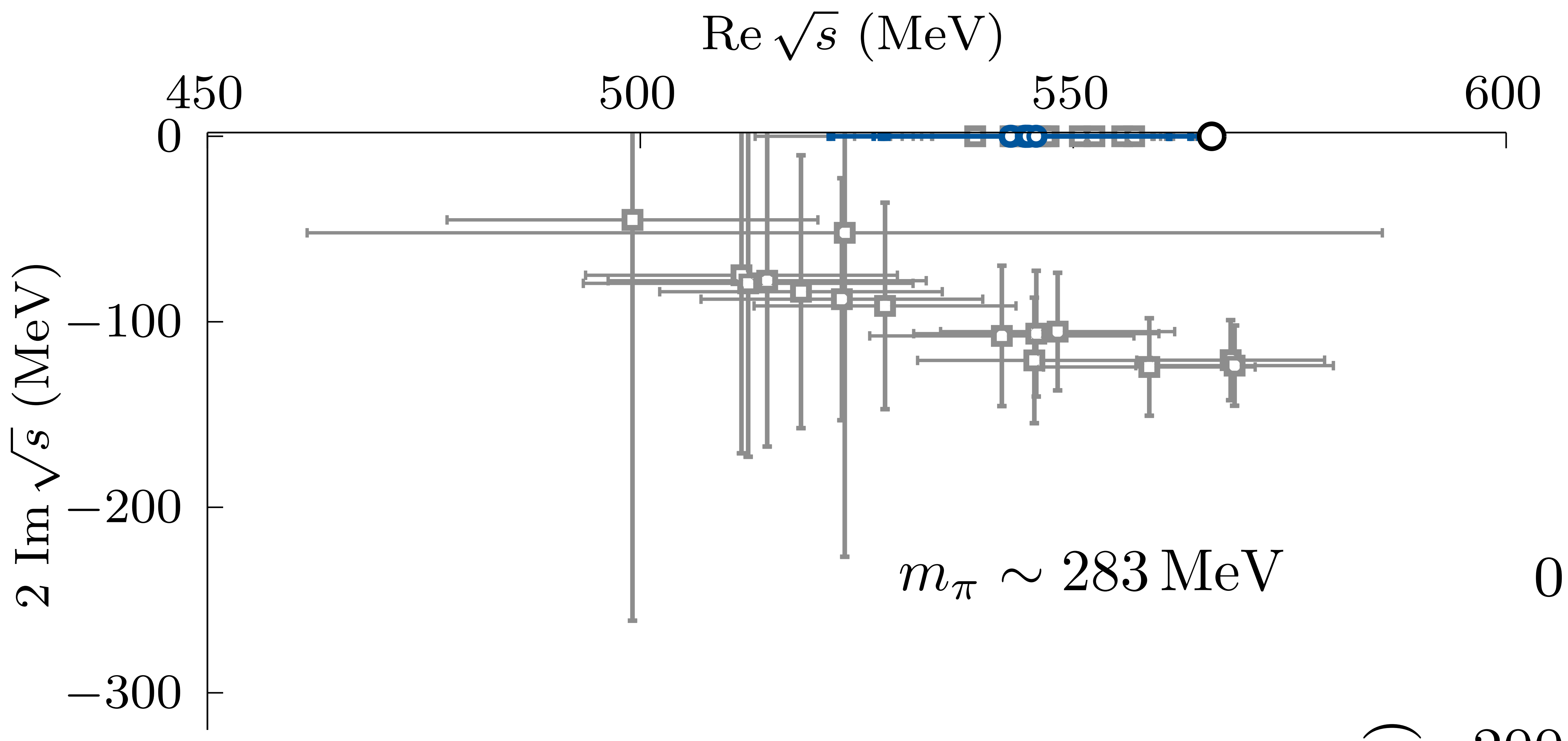


Both sides are good now

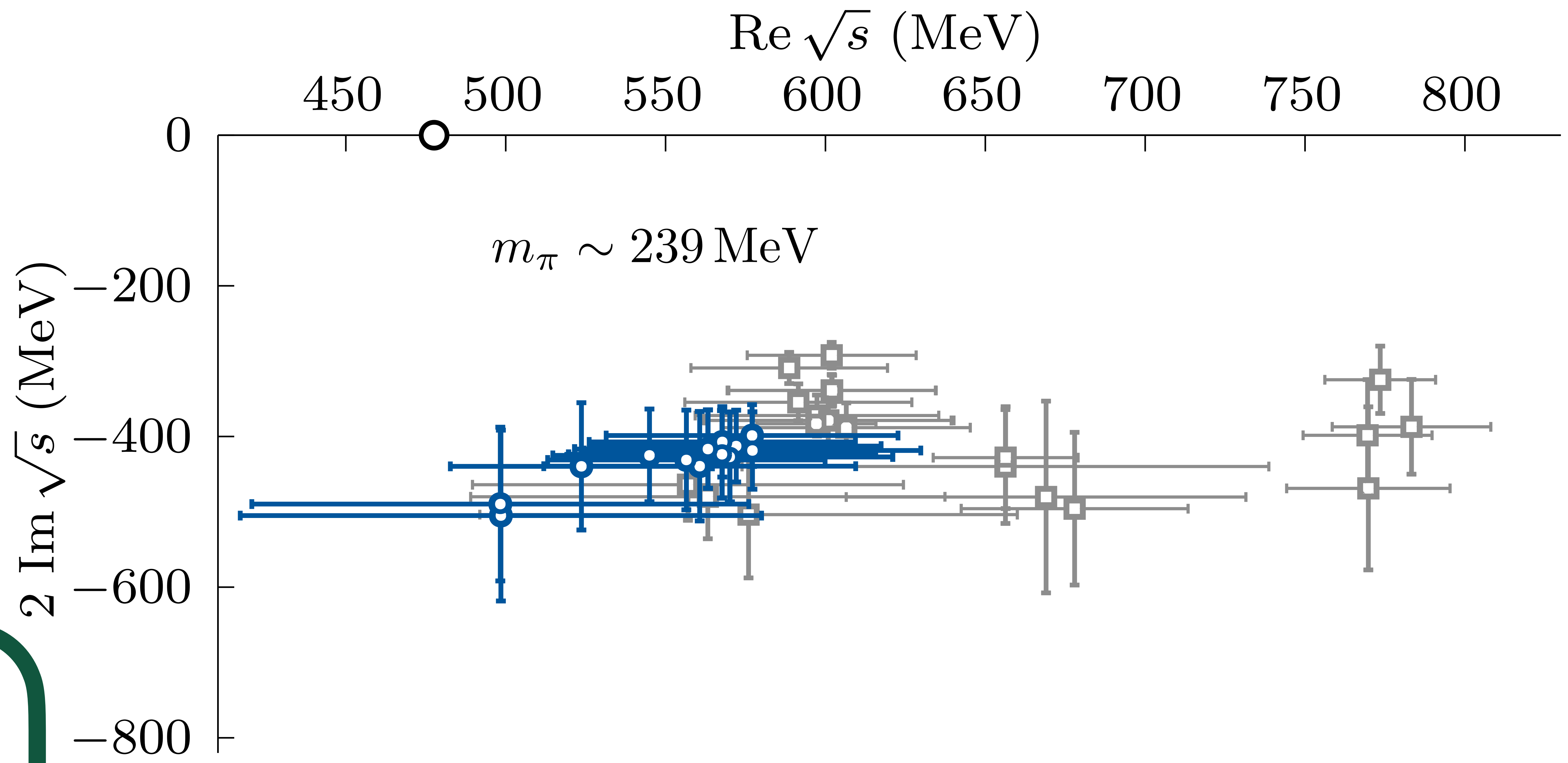
What happens everywhere else??

What happens here??

Dispersive σ

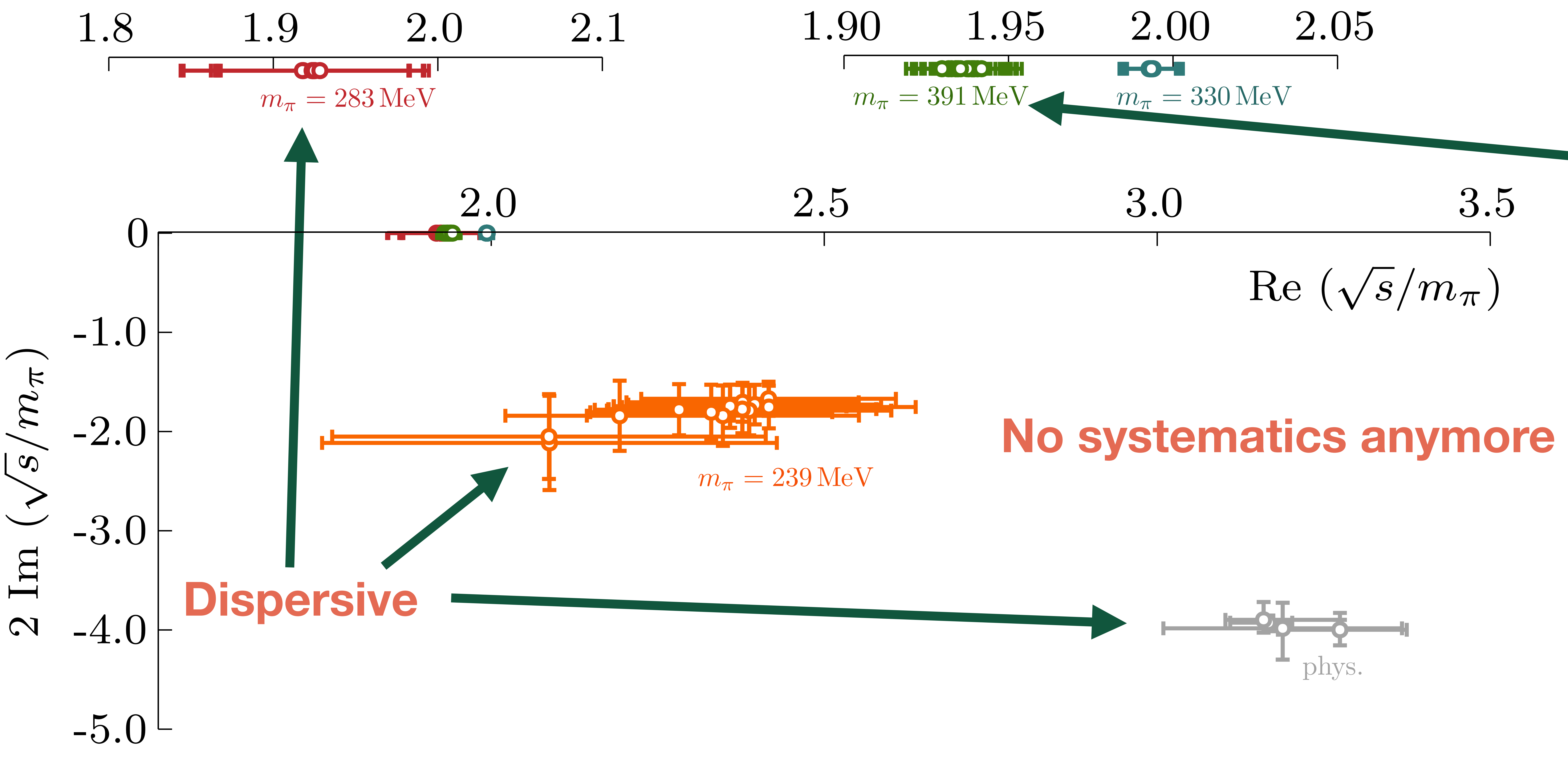


No tension anymore



We traded statistical uncertainty increase by large systematic reduction

Dispersive σ



Another dispersive approach

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We traded statistical uncertainty increase by large systematic reduction

Summary and outlook

hadspec

First-principles extraction of a broad resonance directly from QCD

Better constraints over scattering lengths

The lighter the π, the more relevant this approach is

Future

Include second, larger volume for the lighter pion mass

Extract the $f_0(980)$??

πK scattering ??

A “model-free” scalar octet ??

Extra slides

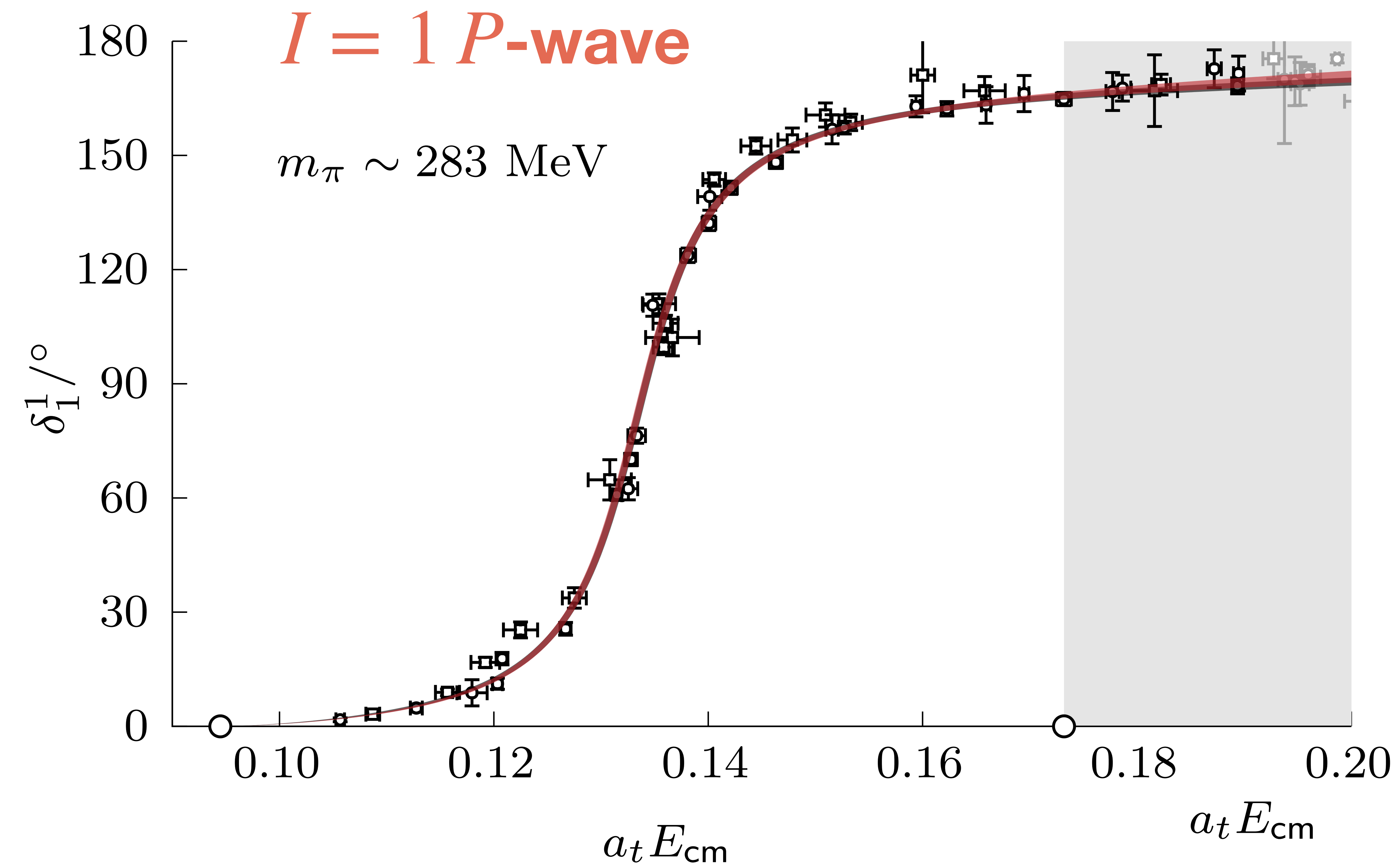
Permutations

$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K_{\ell\ell'}^{II'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

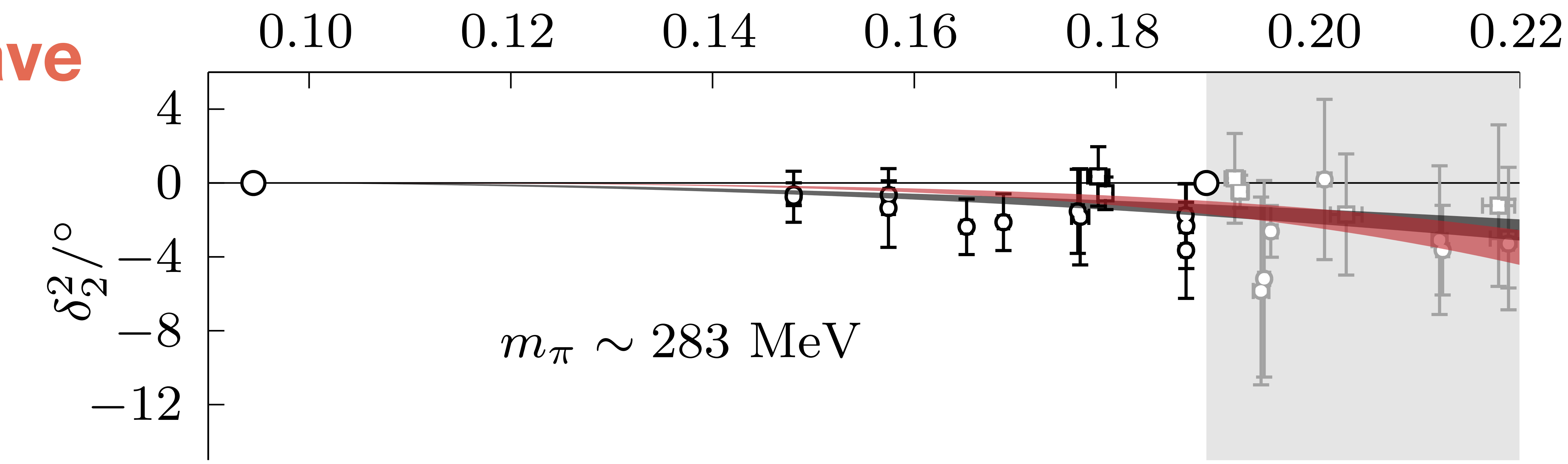
For ℓ_{max} partial waves

$$N_I \ell_{max} N_{params} \sim 10^5$$

We can fix most

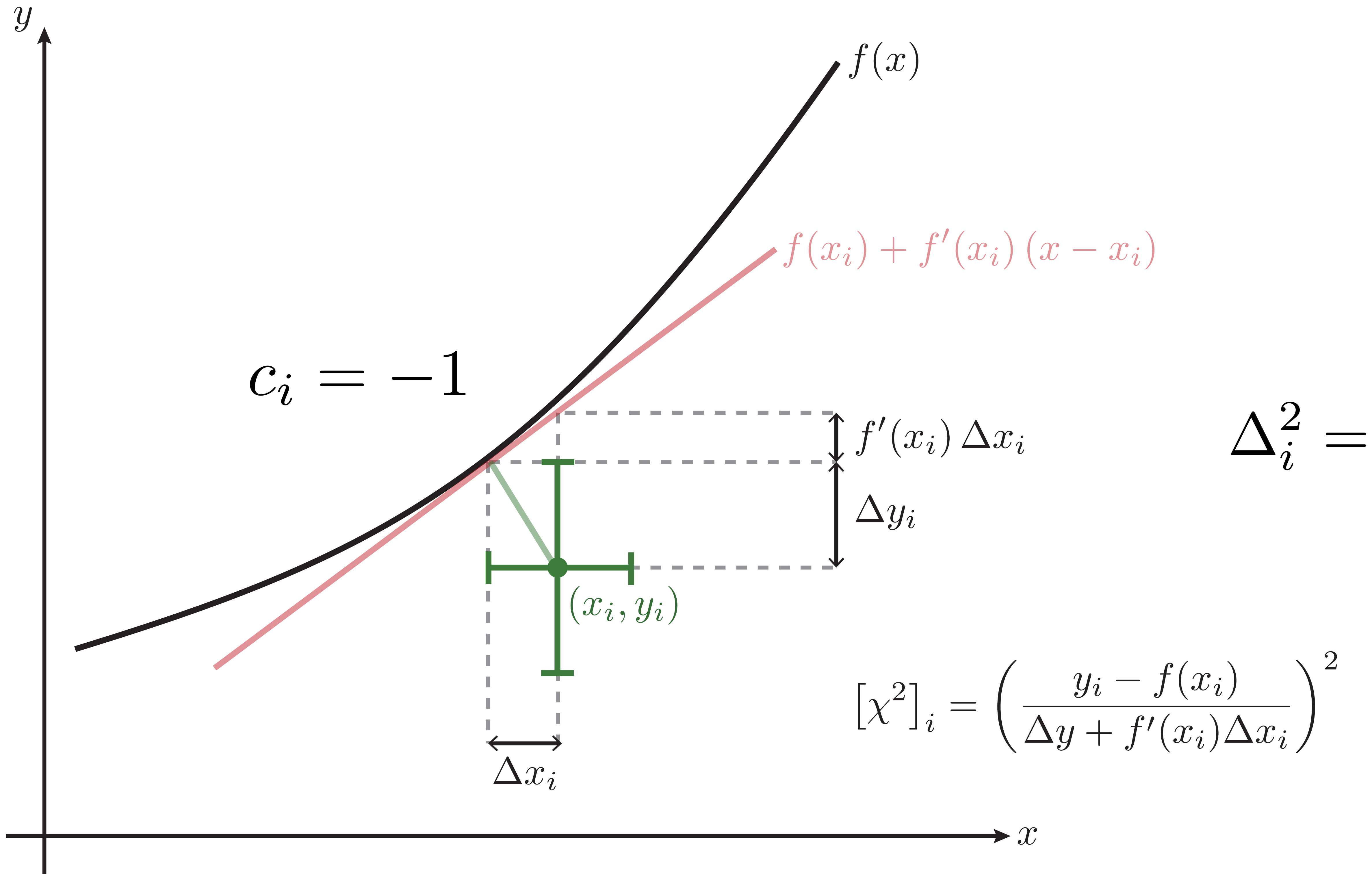


$I = 2\, D$-wave



Make $DR \to Out$ and data compatible ⟷ Lattice QCD data description

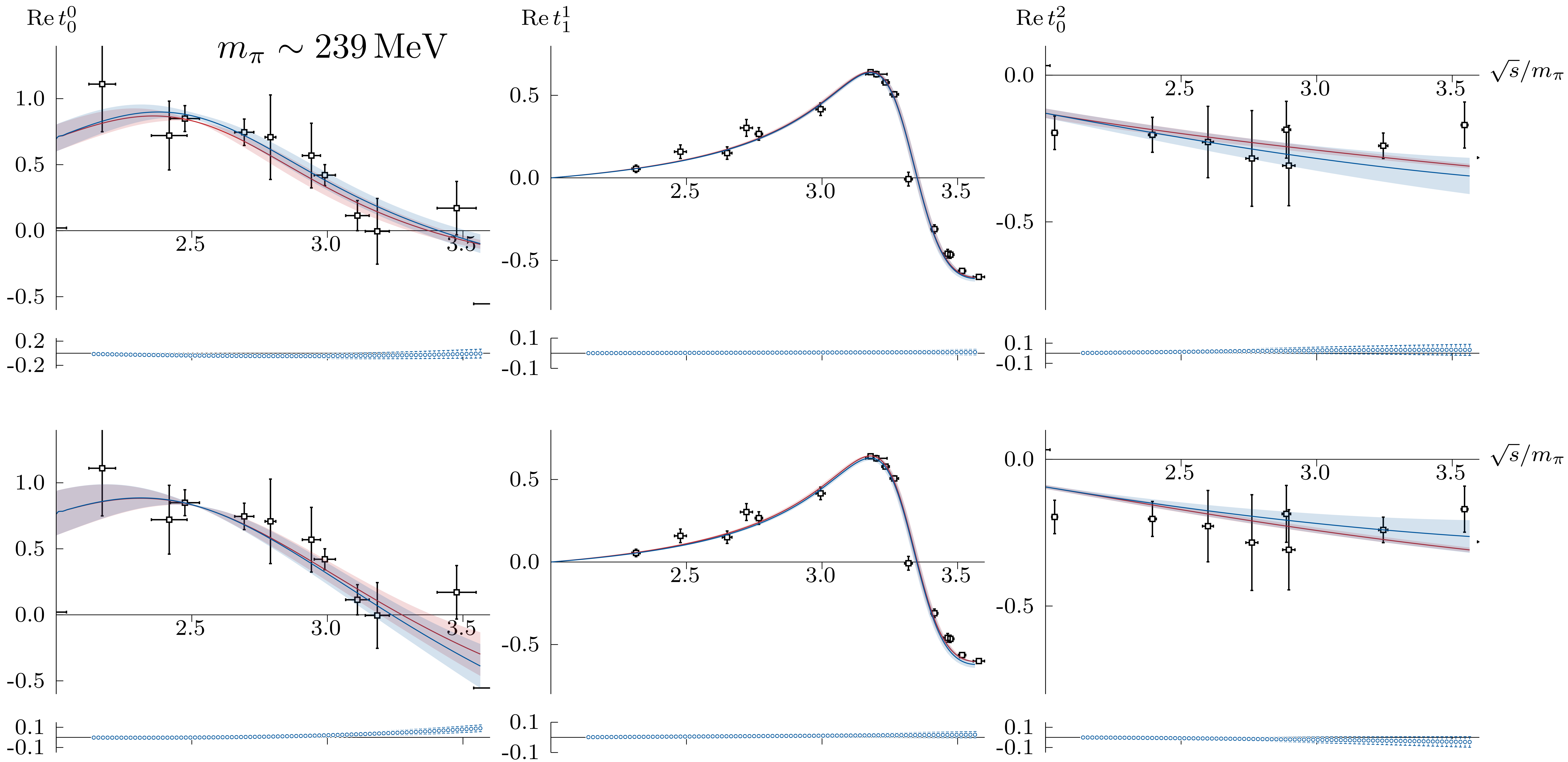
$$\left[\tilde{\chi}^2\right]_\ell^I \equiv \sum_{i,j=1}^{N_{\rm lat}} \left(\frac{\mathfrak{f}_i - \mathrm{Re}\,\tilde{t}_\ell^I\left(s_i\right)}{\Delta_i}\right) \mathrm{corr}\left(\mathfrak{f}_i, \mathfrak{f}_j\right) \left(\frac{\mathfrak{f}_j - \mathrm{Re}\,\tilde{t}_\ell^I\left(s_j\right)}{\Delta_j}\right)$$



$$\Delta_i^2 = \begin{pmatrix} \Delta\mathfrak{f}_i & \frac{d\tilde{f}_\ell^I\left(s_i\right)}{dE_i}\Delta E_i \end{pmatrix} \begin{pmatrix} 1 & -c_i \\ -c_i & 1 \end{pmatrix} \begin{pmatrix} \Delta\tilde{f}_i \\ \frac{d\tilde{f}_\ell^I\left(s_i\right)}{dE_i}\Delta E_i \end{pmatrix}$$

Ok but not great

Visually, they describe the data and fit, but they are not perfect



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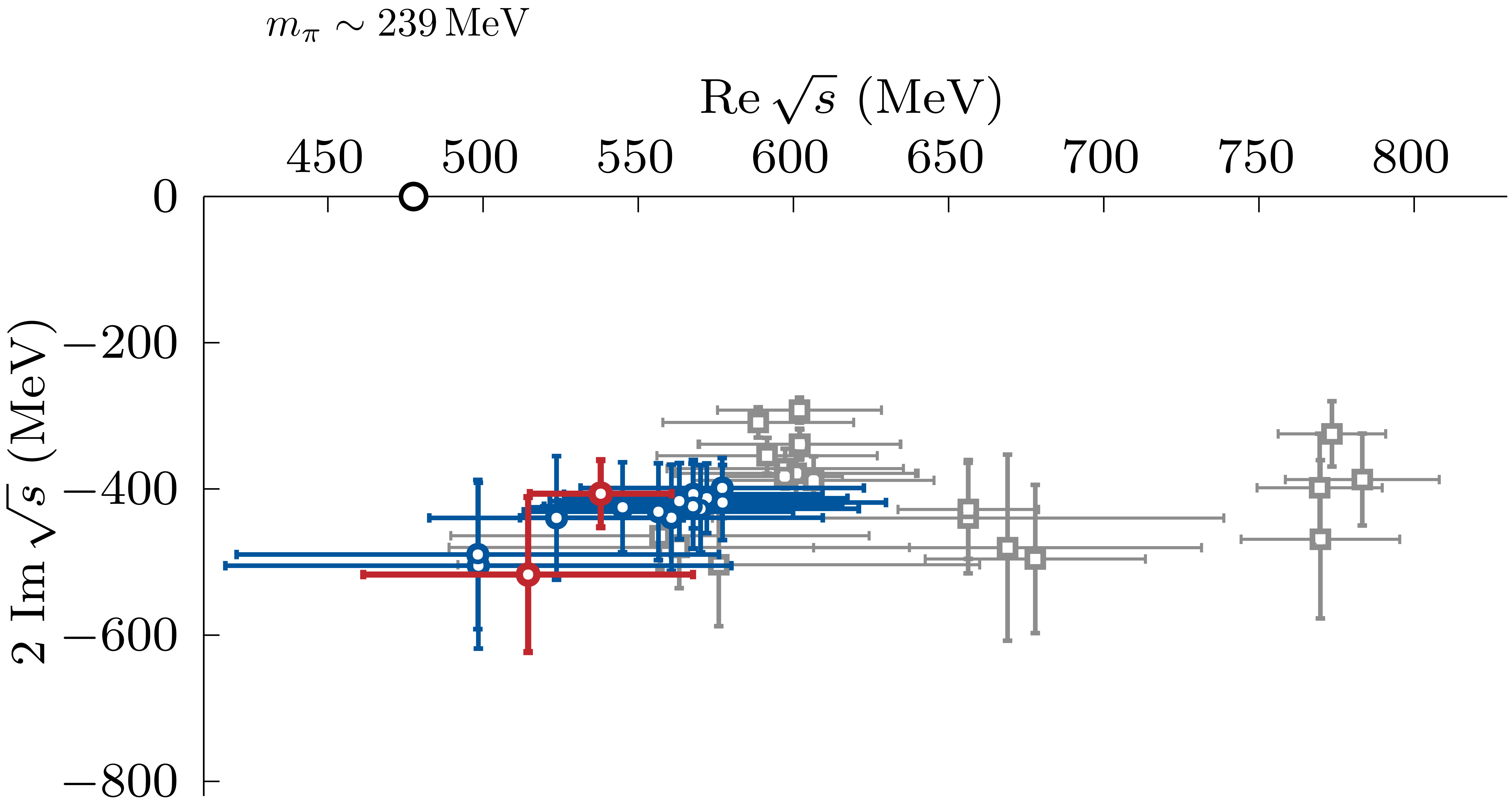
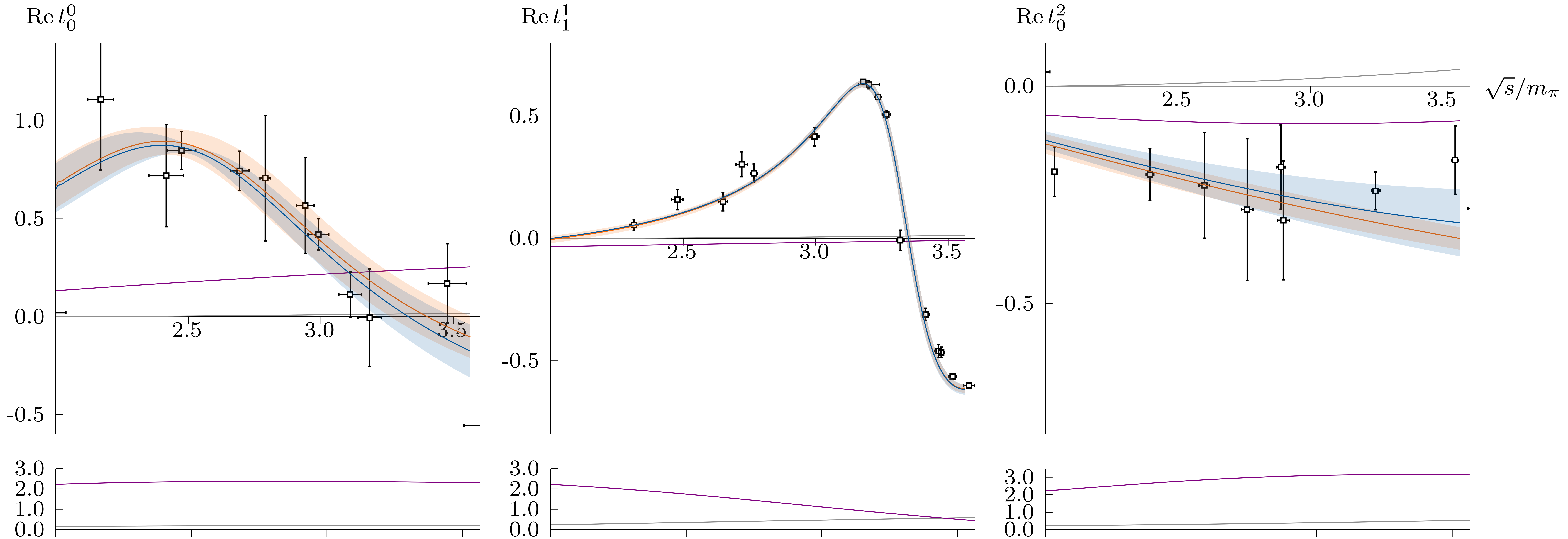


GKPY vs ROY

GKPY: Minimally subtracted → one less subtraction than ROY

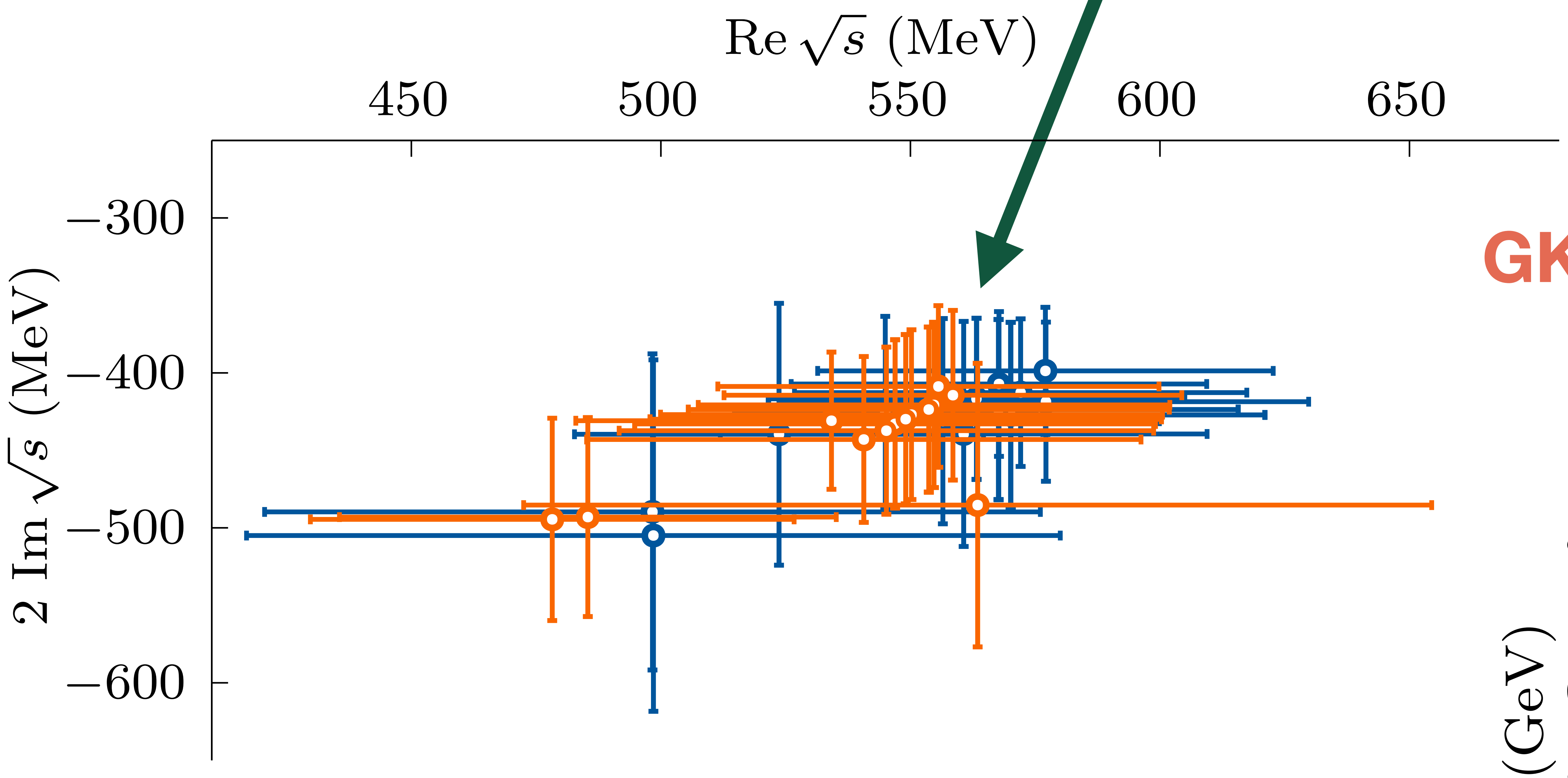
For our analysis, Regge contribution too large for d^2

$m_\pi \sim 239\,\mathrm{MeV}$

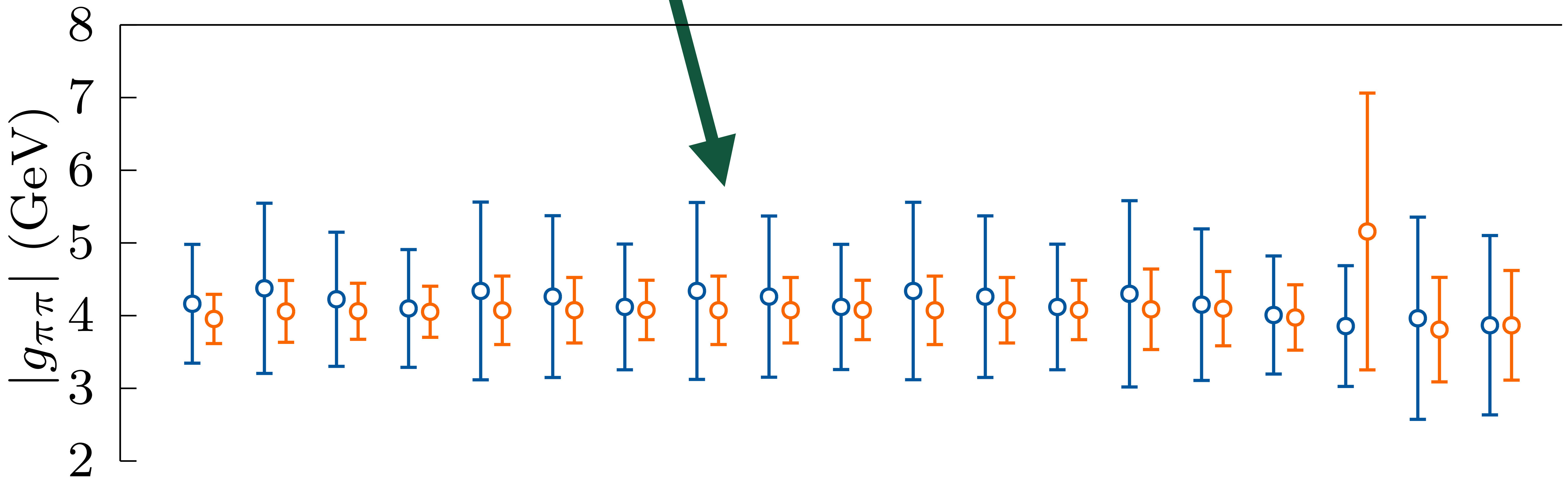


GKPY vs ROY

However, pole extraction is more accurate in most cases, particularly for the coupling



GKPY produces $\sim 40\,\%$ **uncertainty in most cases**



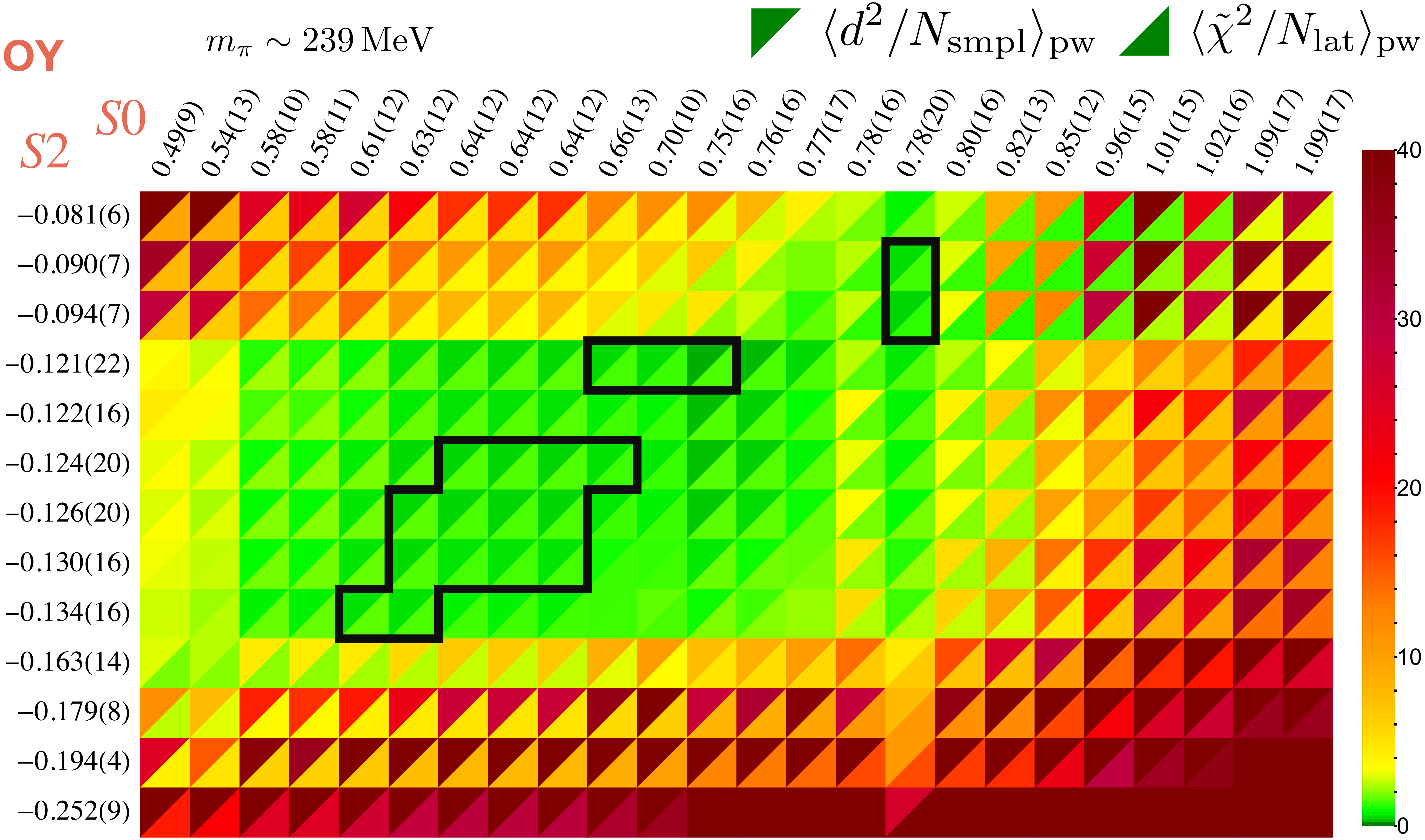
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

Black **ROY**

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



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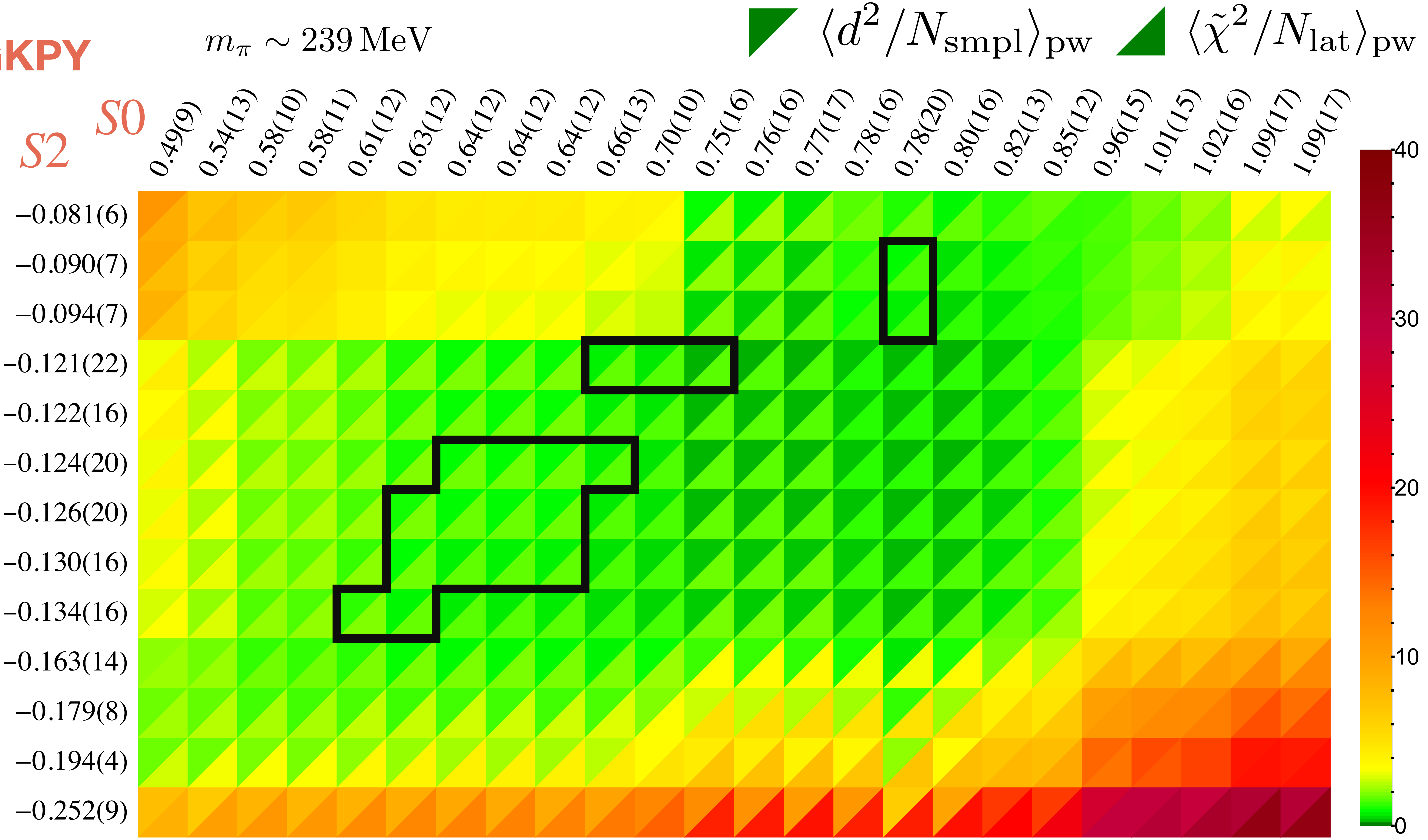


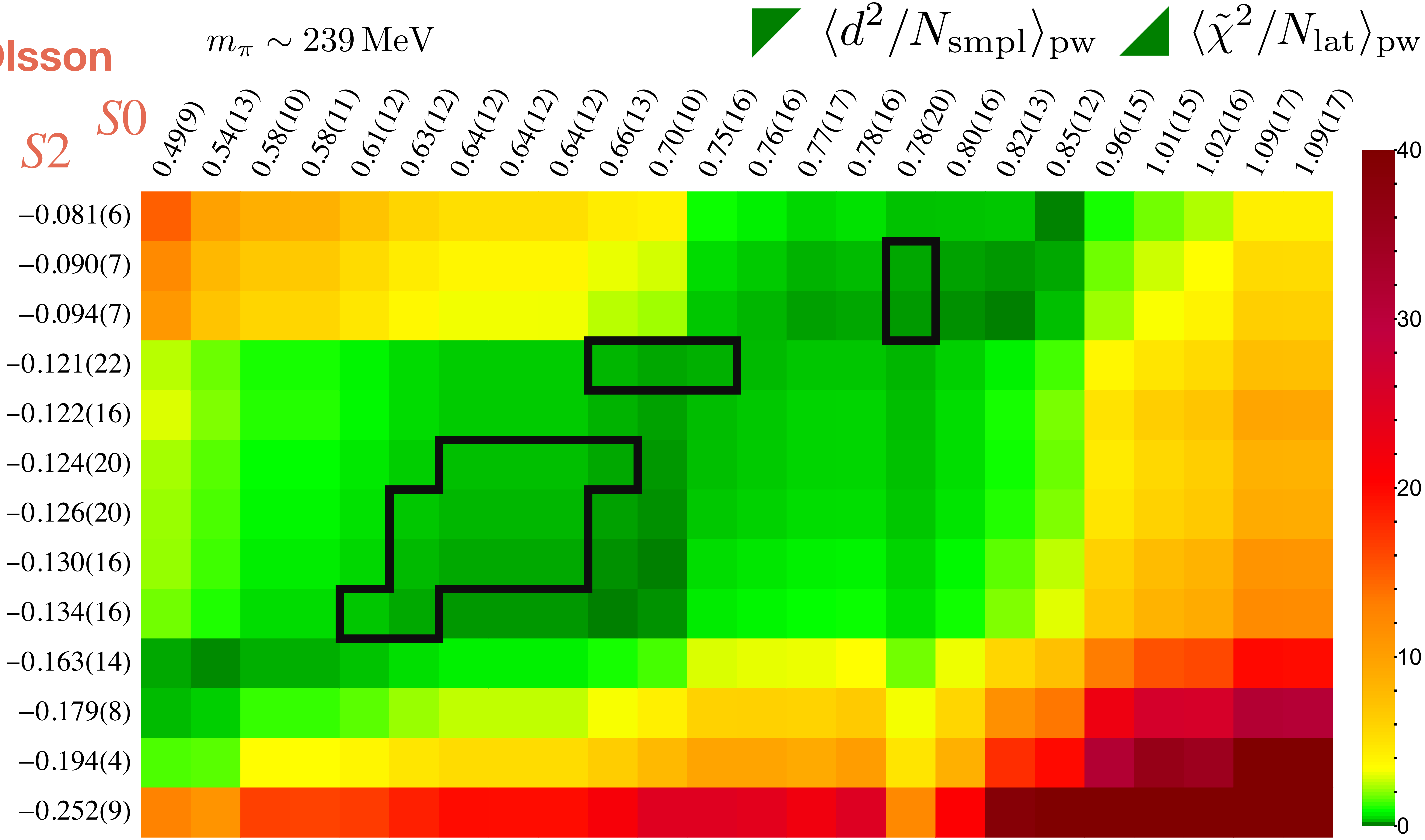
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

Black $d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$

Olsson



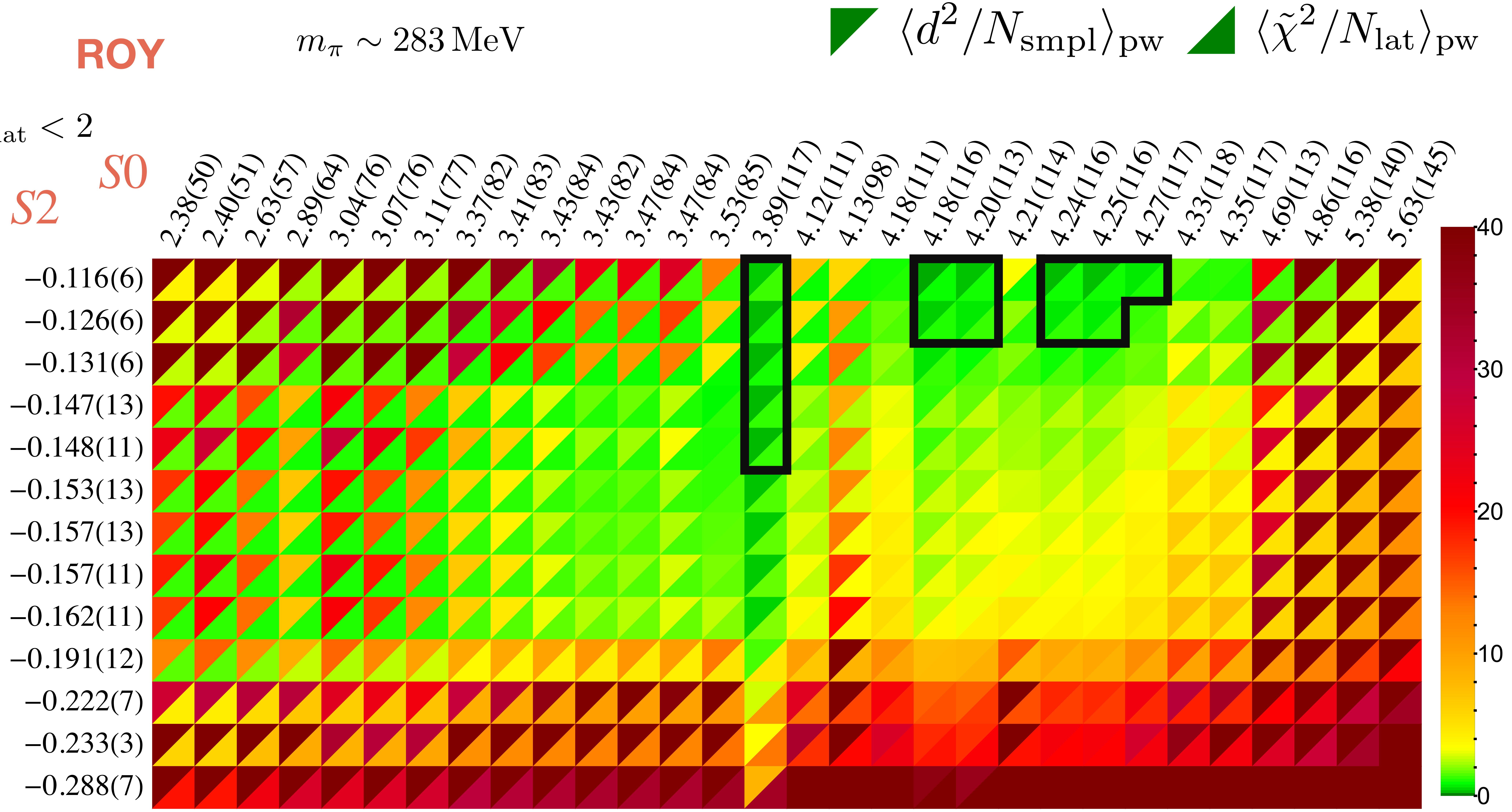
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 283\,\text{MeV}$

Black **ROY**

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$

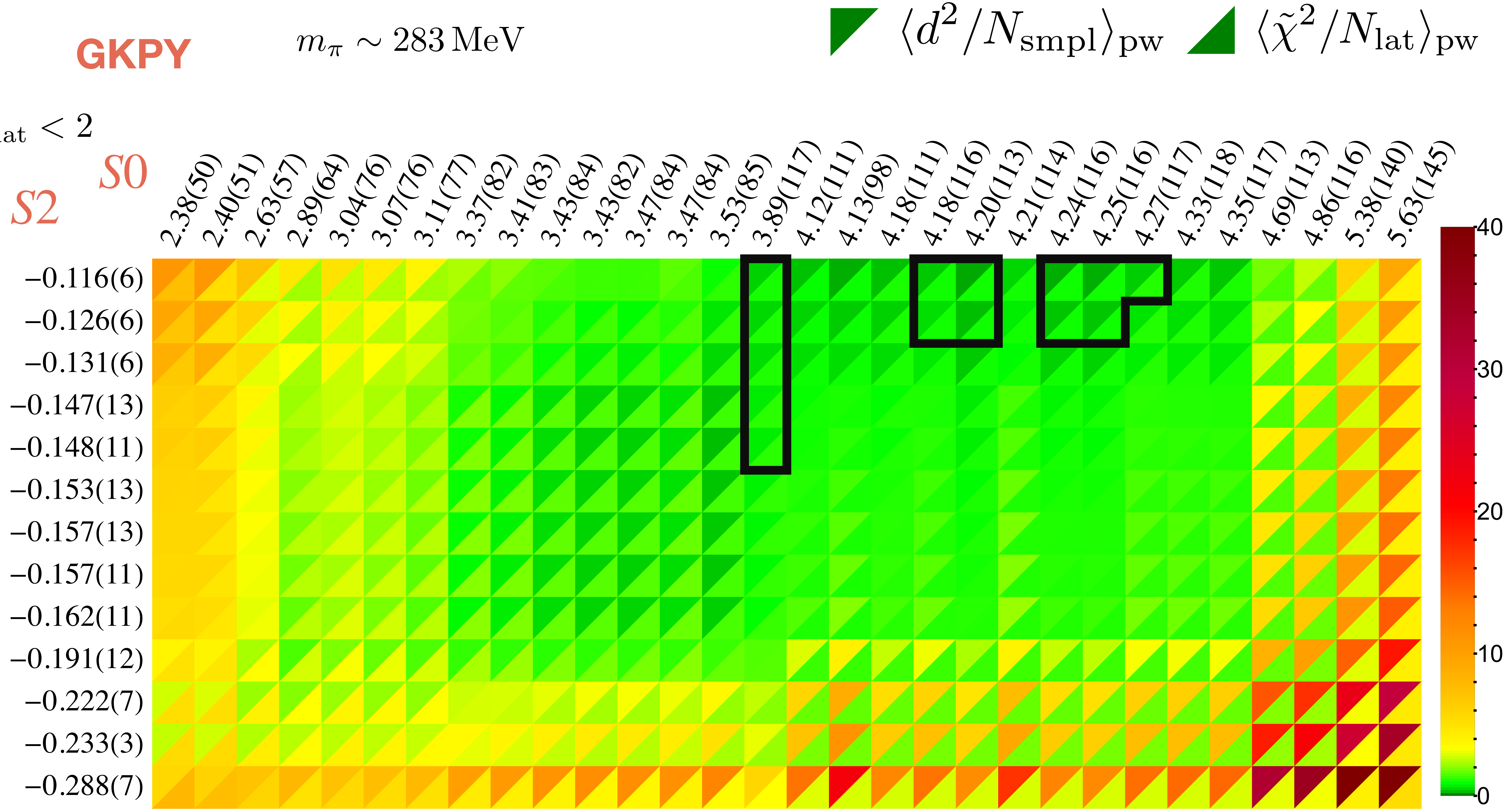


Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 283\,\text{MeV}$

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

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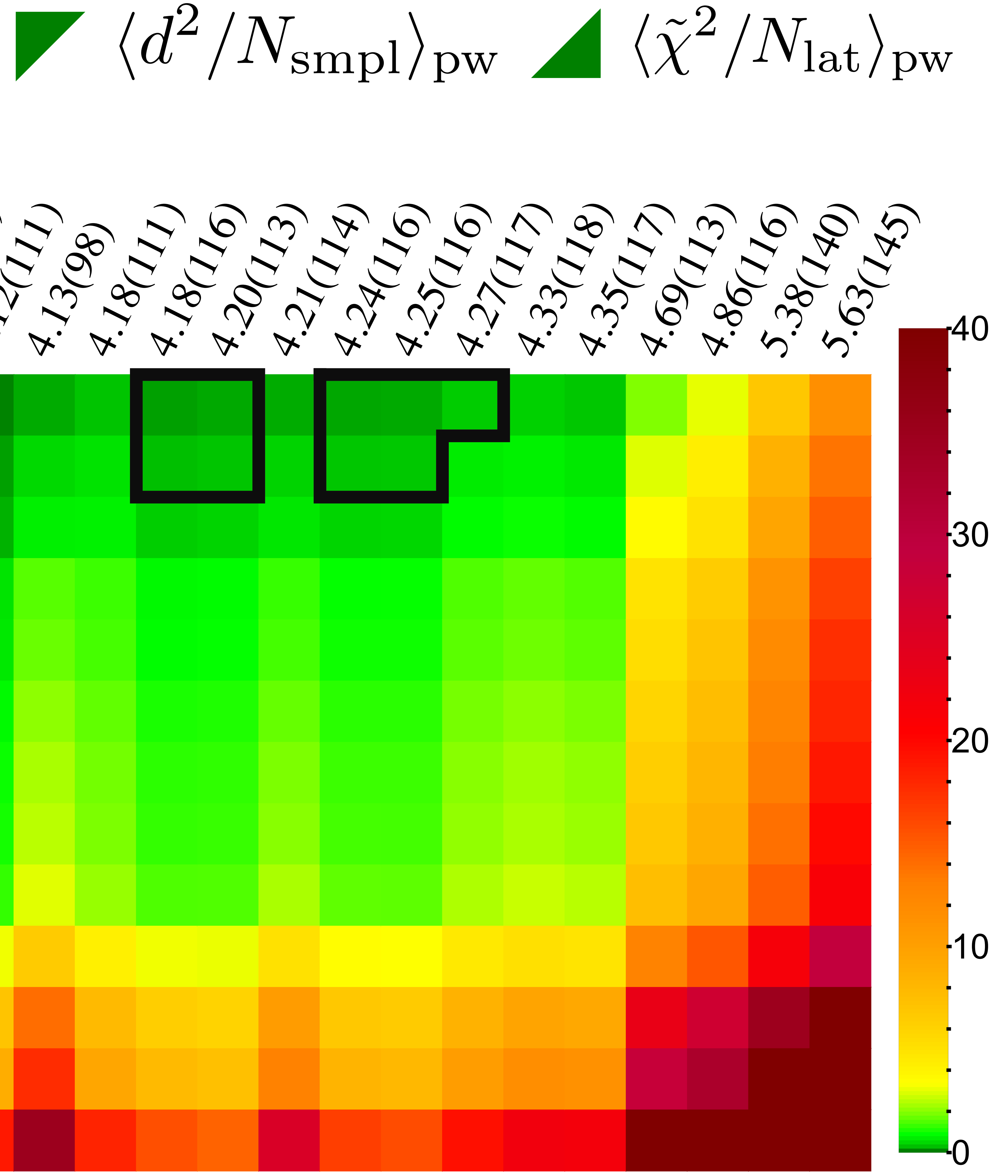


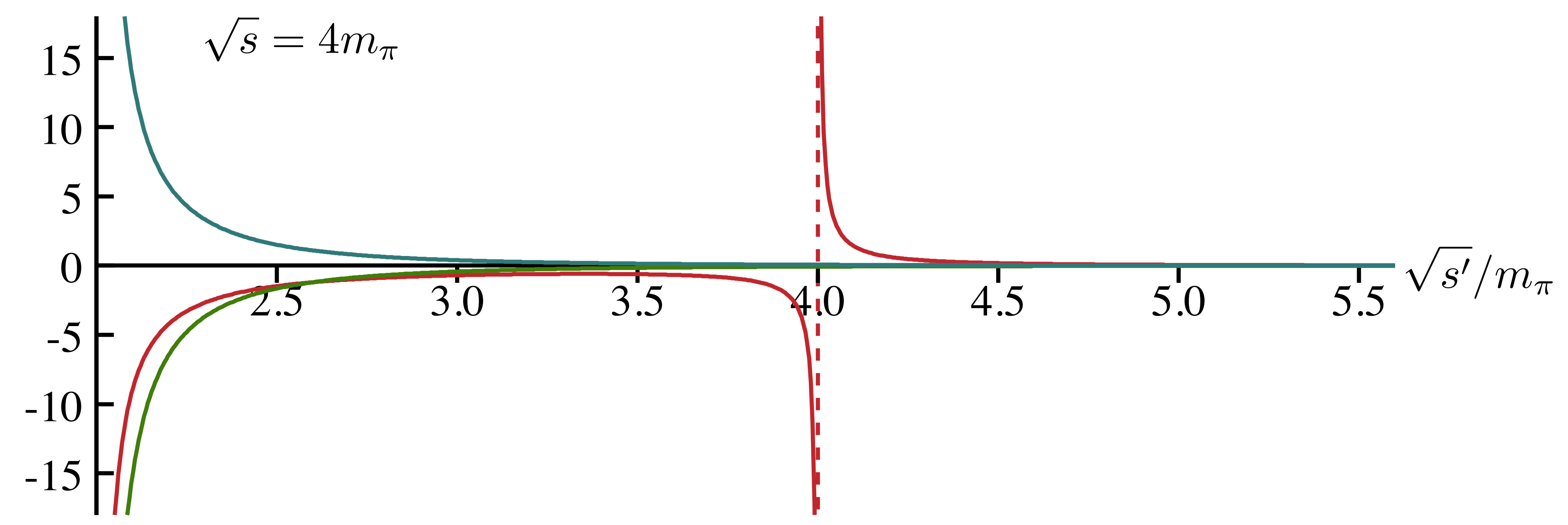
The good

Fit → In

DR → Out

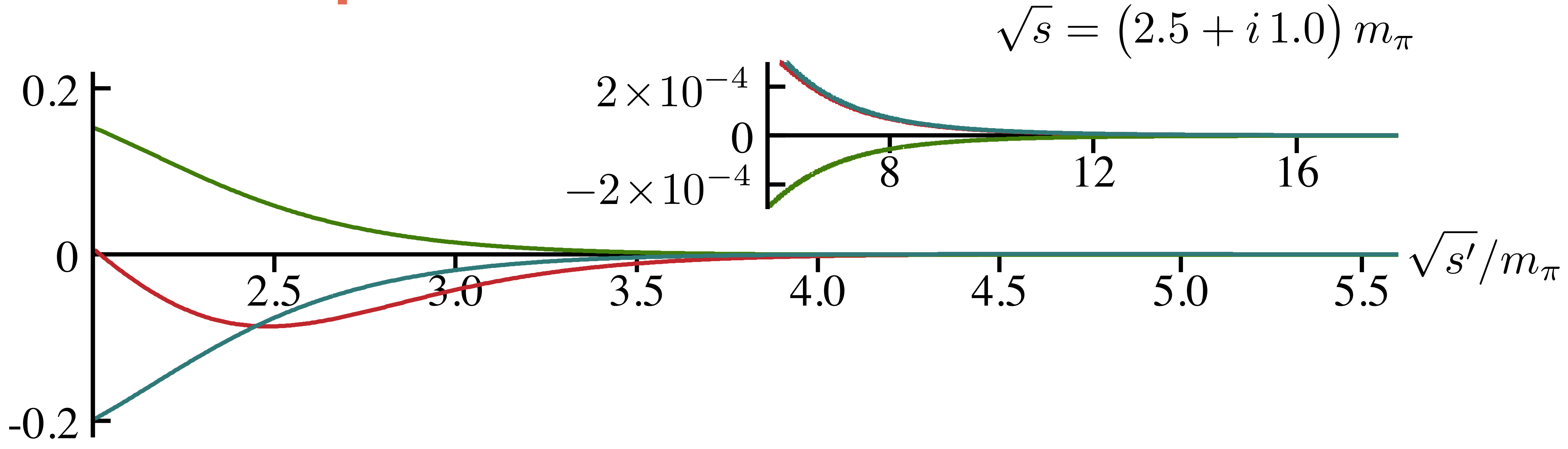
$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K^{II'}_{\ell\ell'}(s',s) \, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

Real s



Smeared over a large energy region

Complex s



An ϵ on the real axis → ϵ' in the complex plane

The bad

Not happening

$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K_{\ell\ell'}^{II'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

Partial waves

Extrapolated

$$\int_{4m_\pi^2}^{\infty} = \int_{4m_\pi^2}^{s_{max}} + \int_{s_{max}}^{\infty} = \int_{4m_\pi^2}^{s_{fit}} + \int_{s_{fit}}^{s_{max}} + \int_{s_{max}}^{\infty}$$

Regge

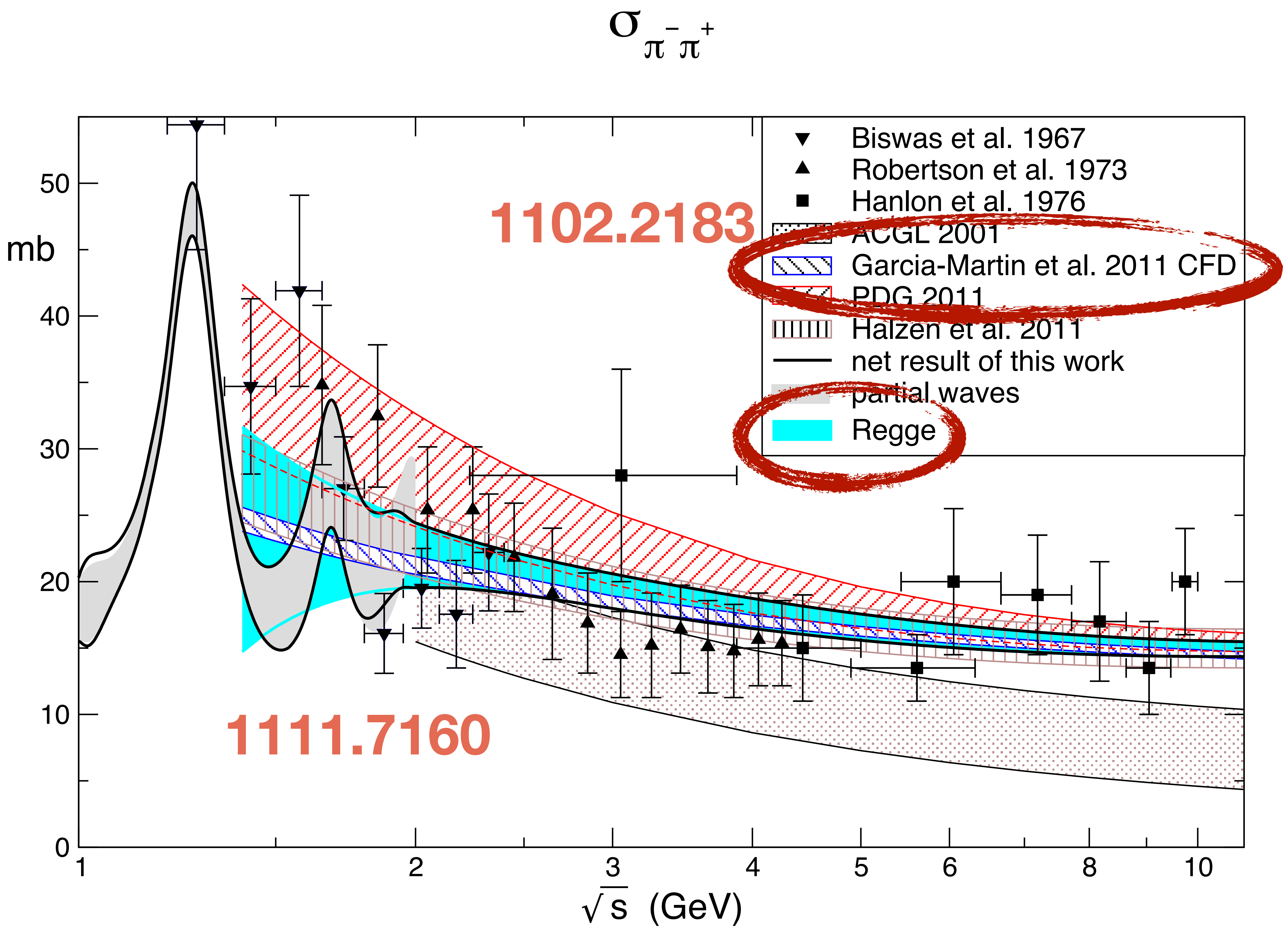
- Regge must be extrapolated from phys. m_π
- Regge is wrong below $a_t m_\pi \sim 0.22 - 0.25$

Regge

Regge must be extrapolated from phys. m_π

$\mathbb{P}$ → gluon exchanges → constant over m_q

ρ, f_2 → resonances, not constant → $\lambda \sim \Gamma/M$

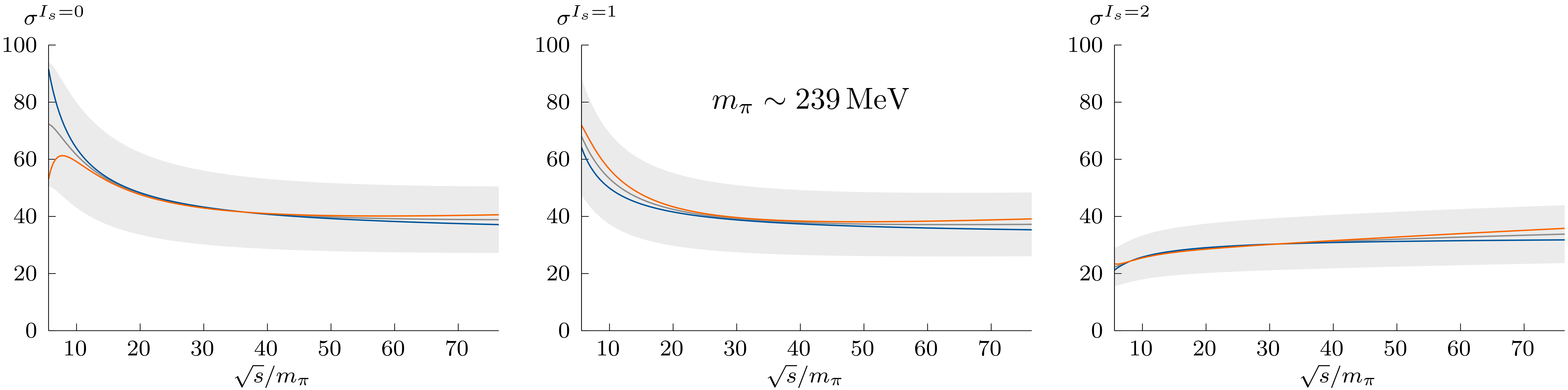


Our $F_{Regge} = \dfrac{F_{Regge1} + F_{Regge2}}{2}$

Big uncertainty $\Delta F_{Regge} = 0.3\, F_{Regge}$

Regge

Regge must be extrapolated from phys. m_π

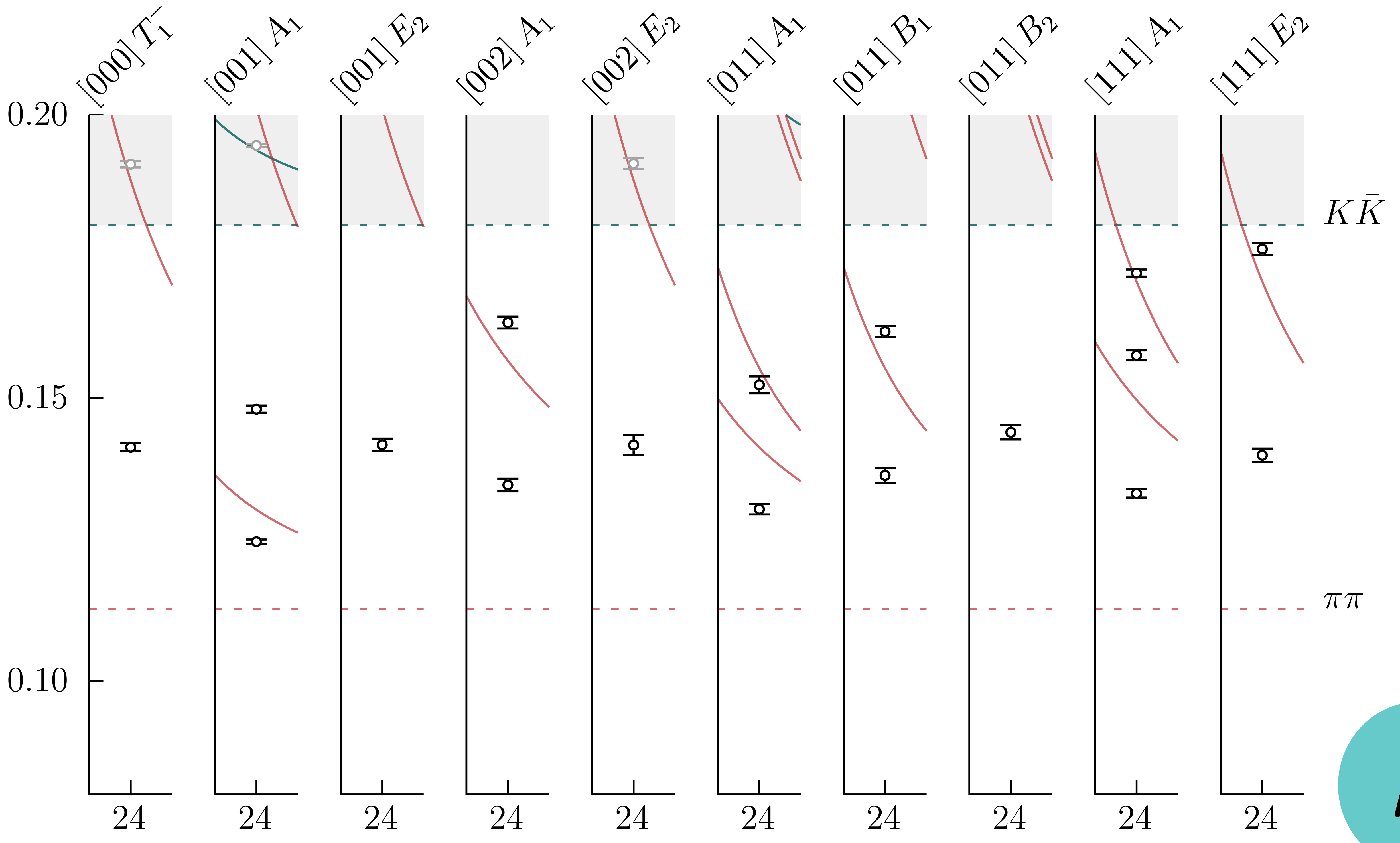


Our $F_{Regge} = \dfrac{F_{Regge1} + F_{Regge2}}{2}$

Big uncertainty $\Delta F_{Regge} = 0.3\, F_{Regge}$

$I = 1\ \pi\pi$

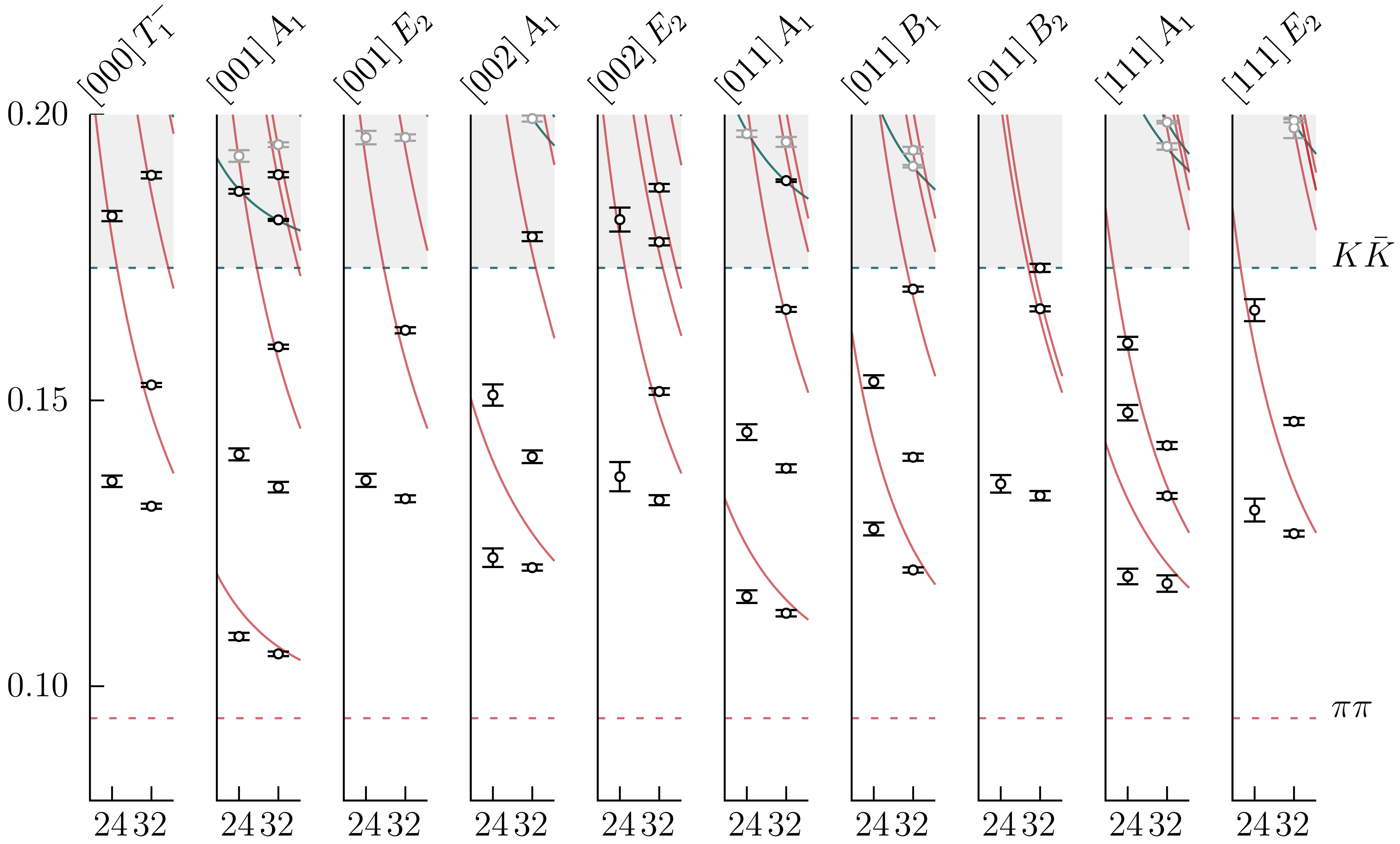
Similar spectrum to previous masses



$m_\pi \sim 330\,\text{MeV}$

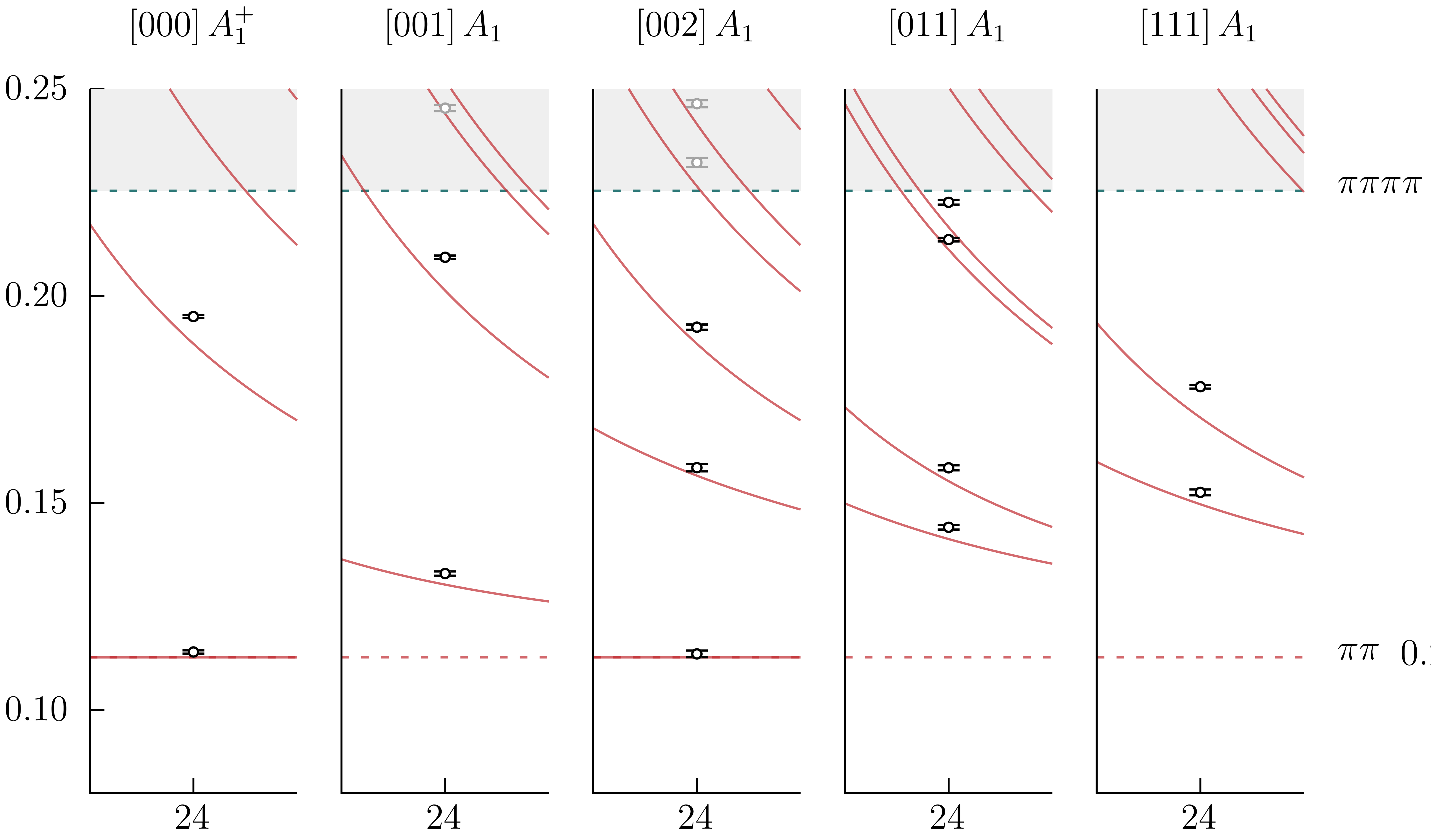
ρ

$m_\pi \sim 283\,\text{MeV}$



Allows us to study the ρ resonance m_q dependence

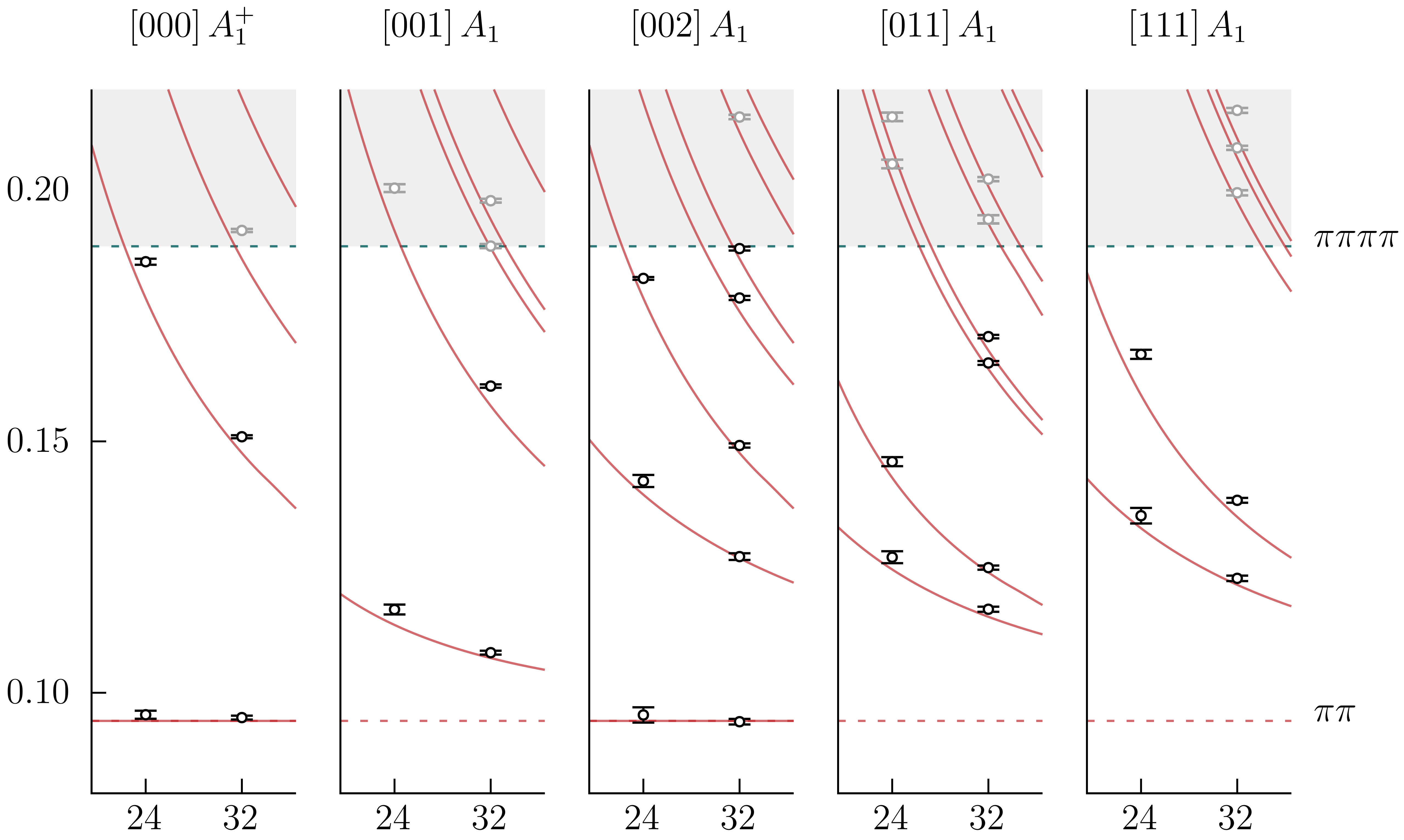
$I = 2\ \pi\pi$



$m_\pi \sim 330\,\text{MeV}$

Similar spectrum to previous masses

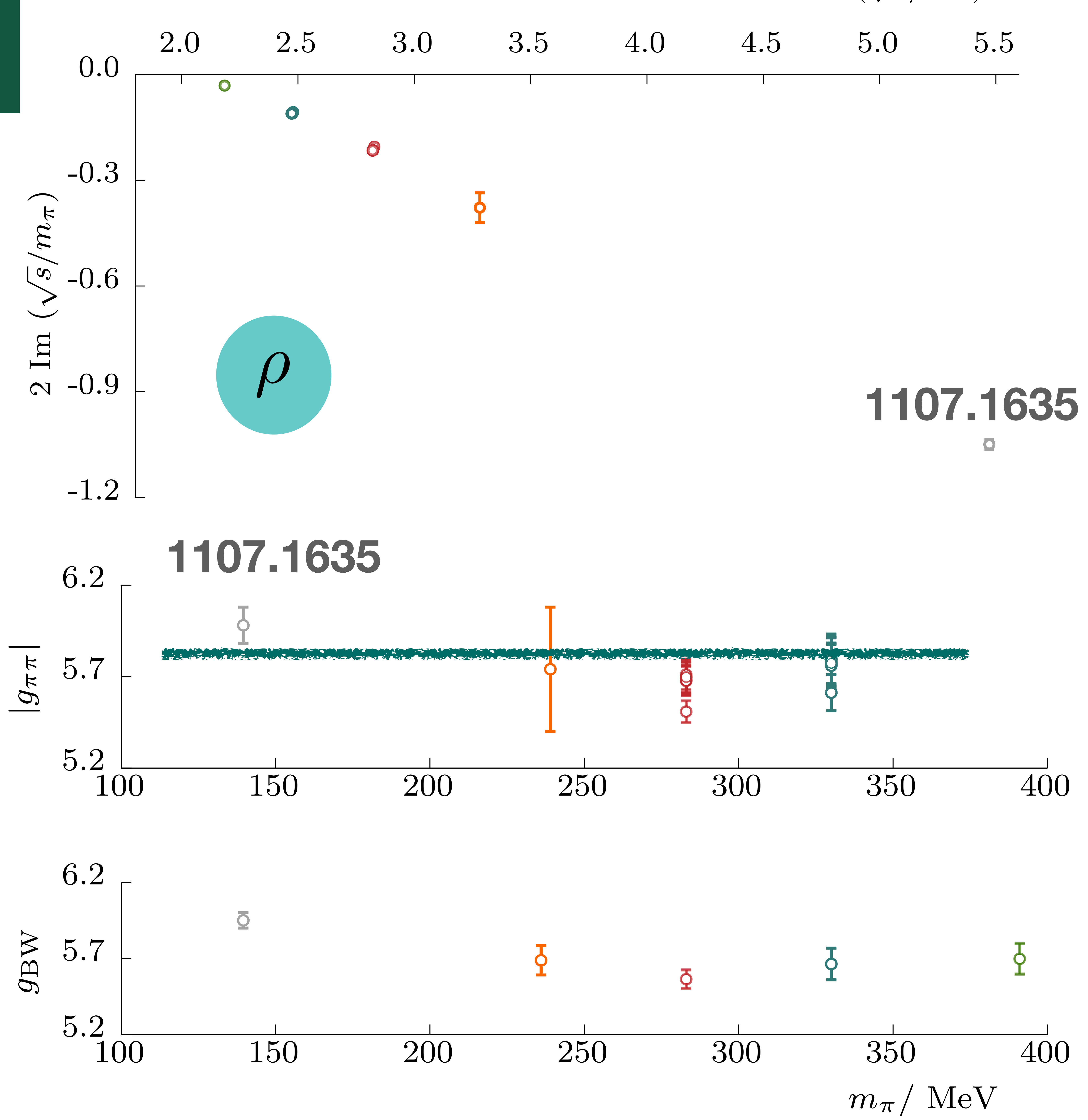
$m_\pi \sim 283\,\text{MeV}$



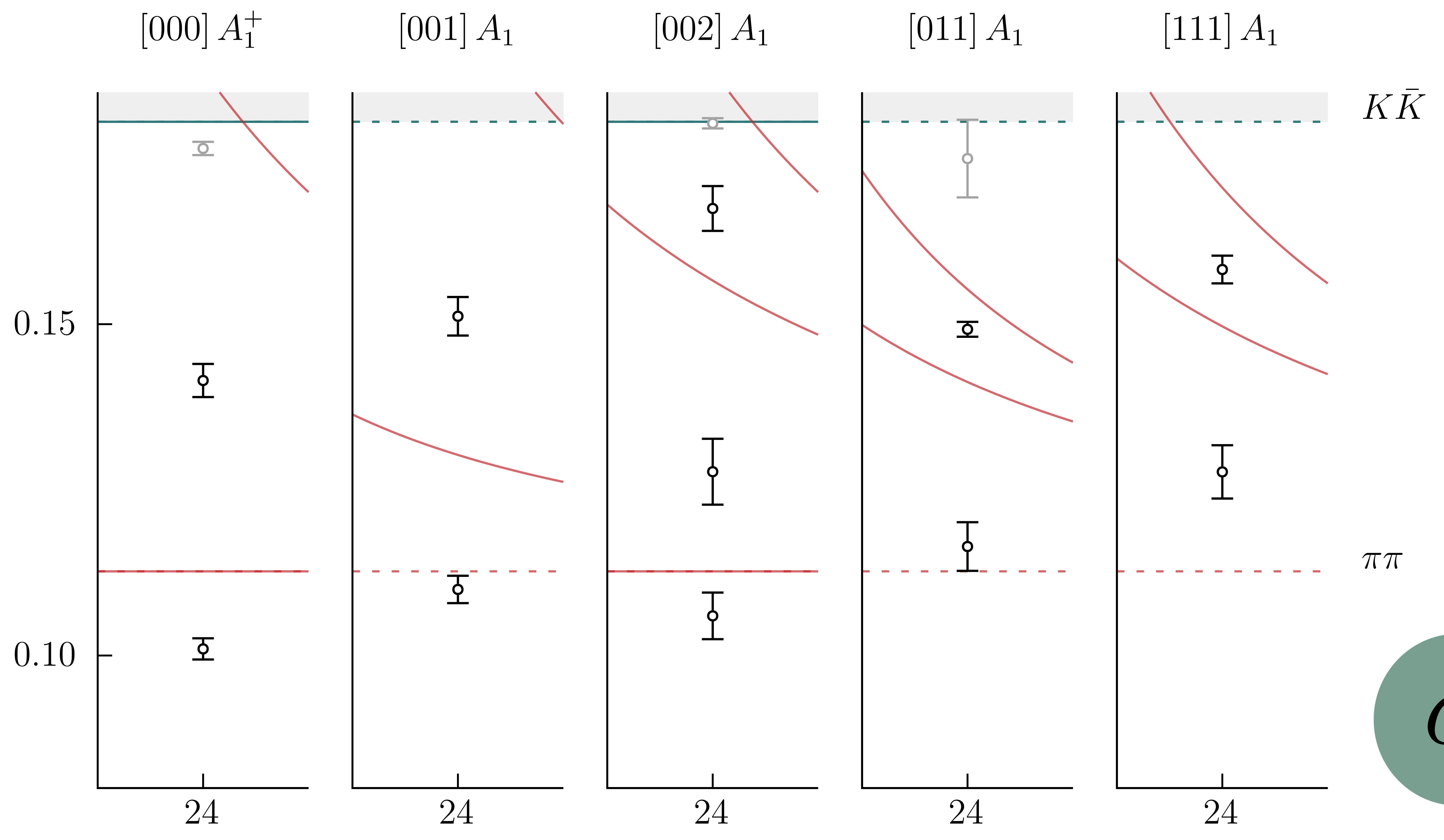
$I = 1\ \pi\pi$

Ordinary m_q dependence

g constant



$I = 0\ \pi\pi$

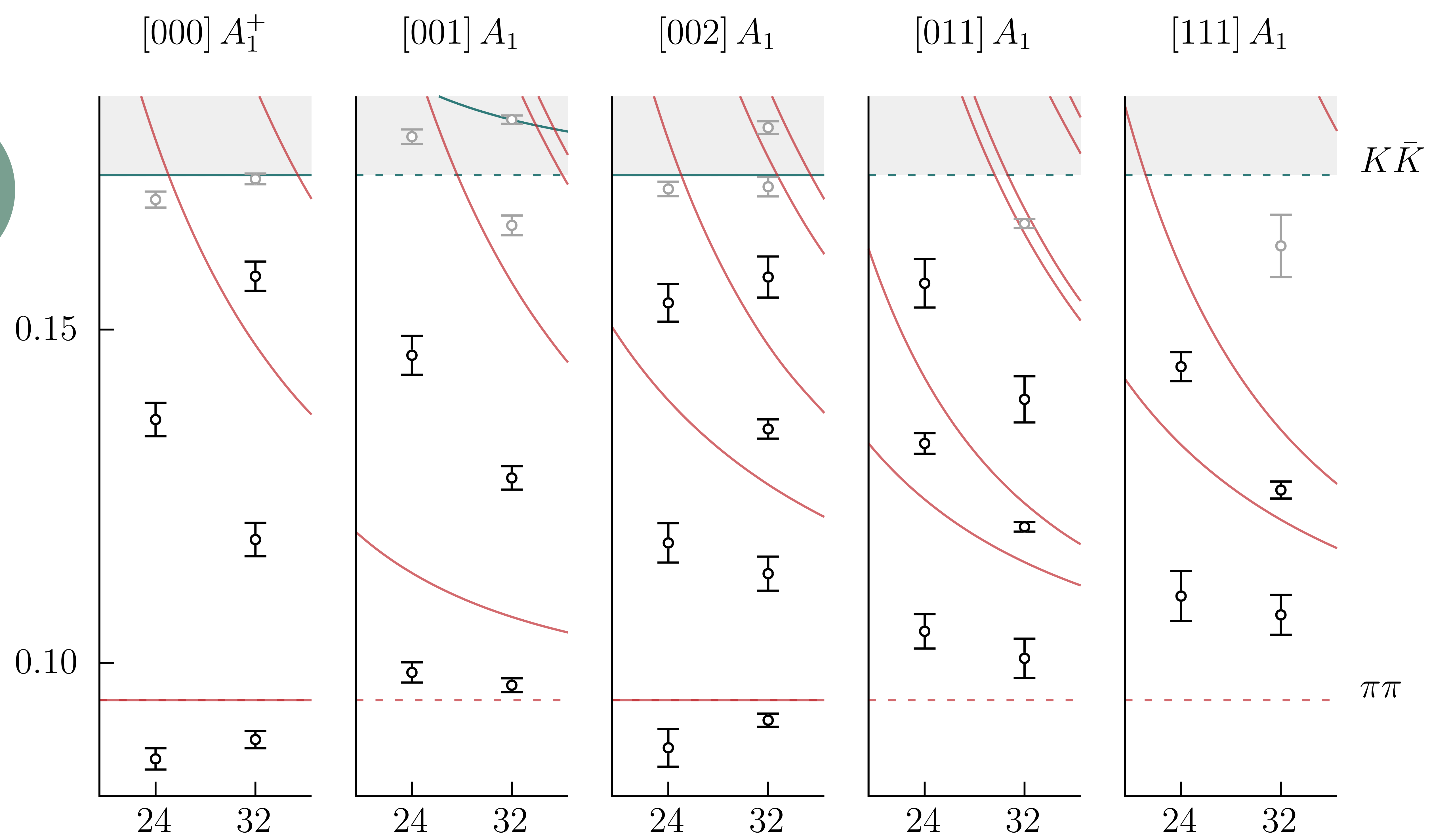


$m_\pi \sim 330\,\mathrm{MeV}$

Similar spectrum to previous masses

σ

$m_\pi \sim 283\,\mathrm{MeV}$



Over 60 "elastic" levels for I=0

$I = 0 \ \pi\pi$

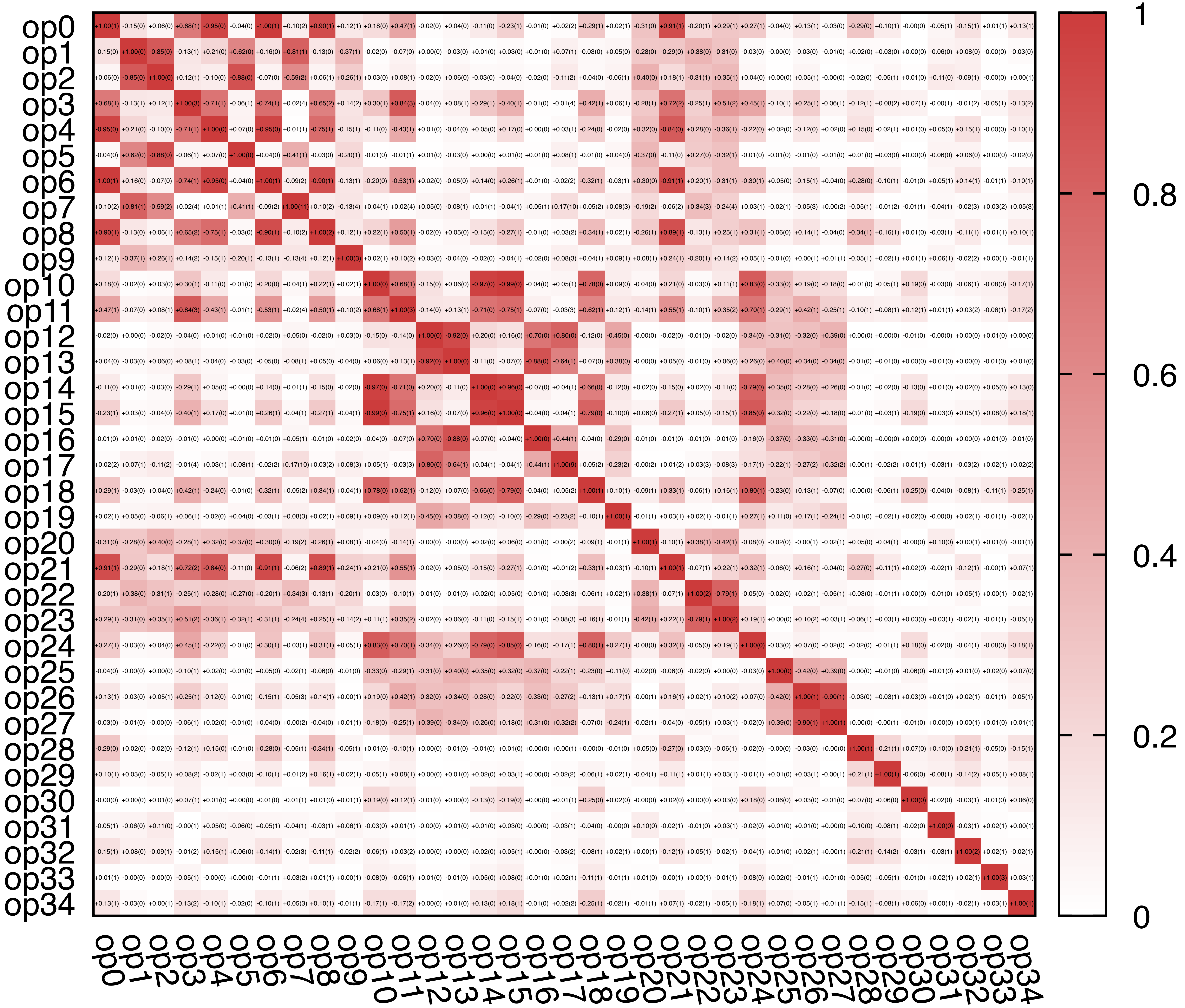
Many ops. for a good GEVP

Distillation → 0905.2160

Time src avg. correlations

Some highly correlated

More than a few relevant ops.



$I = 0\ \pi\pi$

Many fits for a good E_n

Many fits for diff.

- t_0
- t_{min}, t_{max}
- N_{exp}

Model averaging technique

2008.01069

2208.13755