Finite-Temperature Effects in Multi-Messenger Astrophysics

INT - Seattle - 14th June 2026

# CONVEXITY OF THE EQUATION OF STATE IN MERGER SIMULATIONS AND GRAVITATIONAL WAVES

**G. RIVIECCIO**, A. RIOS, P. CERDÀ-DURÀN, M. RUIZ, J. A. FONT

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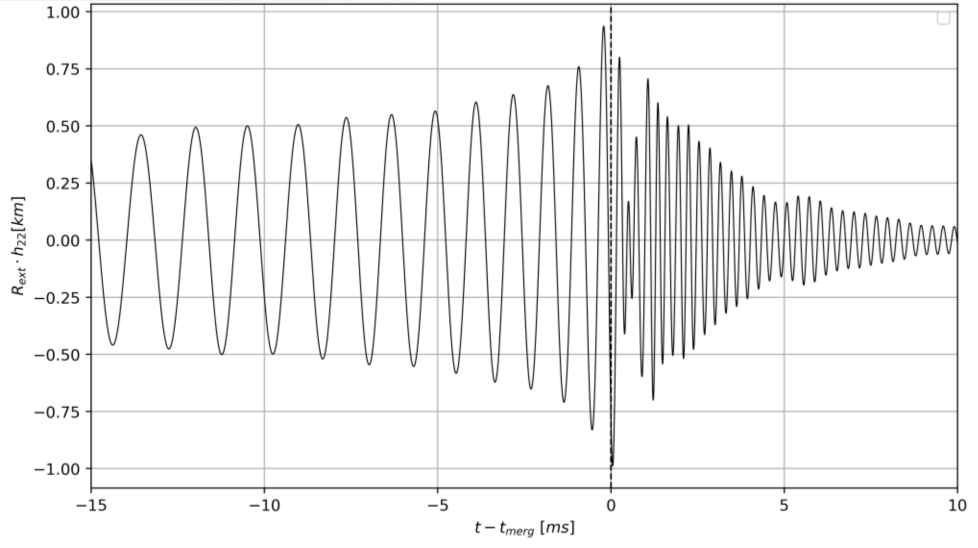
# Focus of the talk

**GR**, Guerra, Ruiz & Font (2024)
Gravitational wave imprints of non-convex dynamics in binary neutron star mergers
Phys. Rev. D 109, 064032

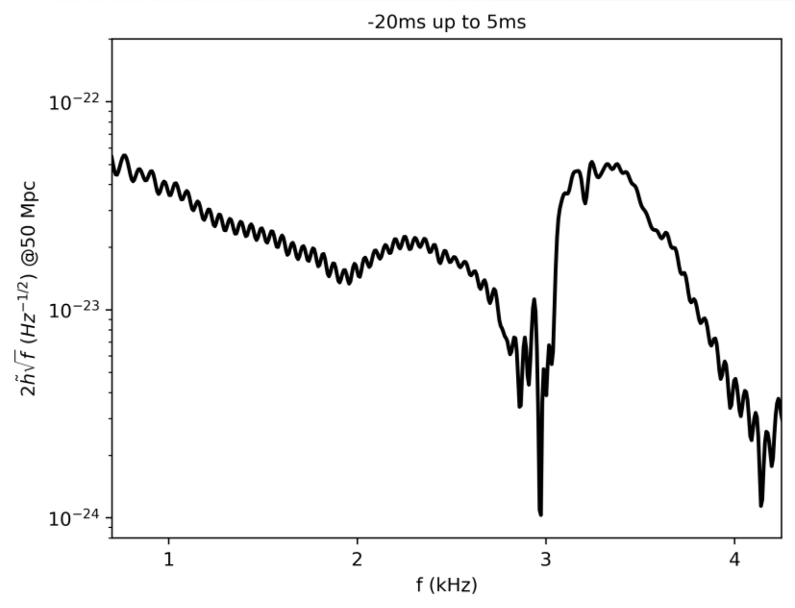
**GR**, Rios, Cerdà-Duràn, Ruiz & Font (2026)
Convexity of parametrized phase transition and gravitational wave signature
Still loading ....

# BINARY NEUTRON STARS (BNS) MERGER

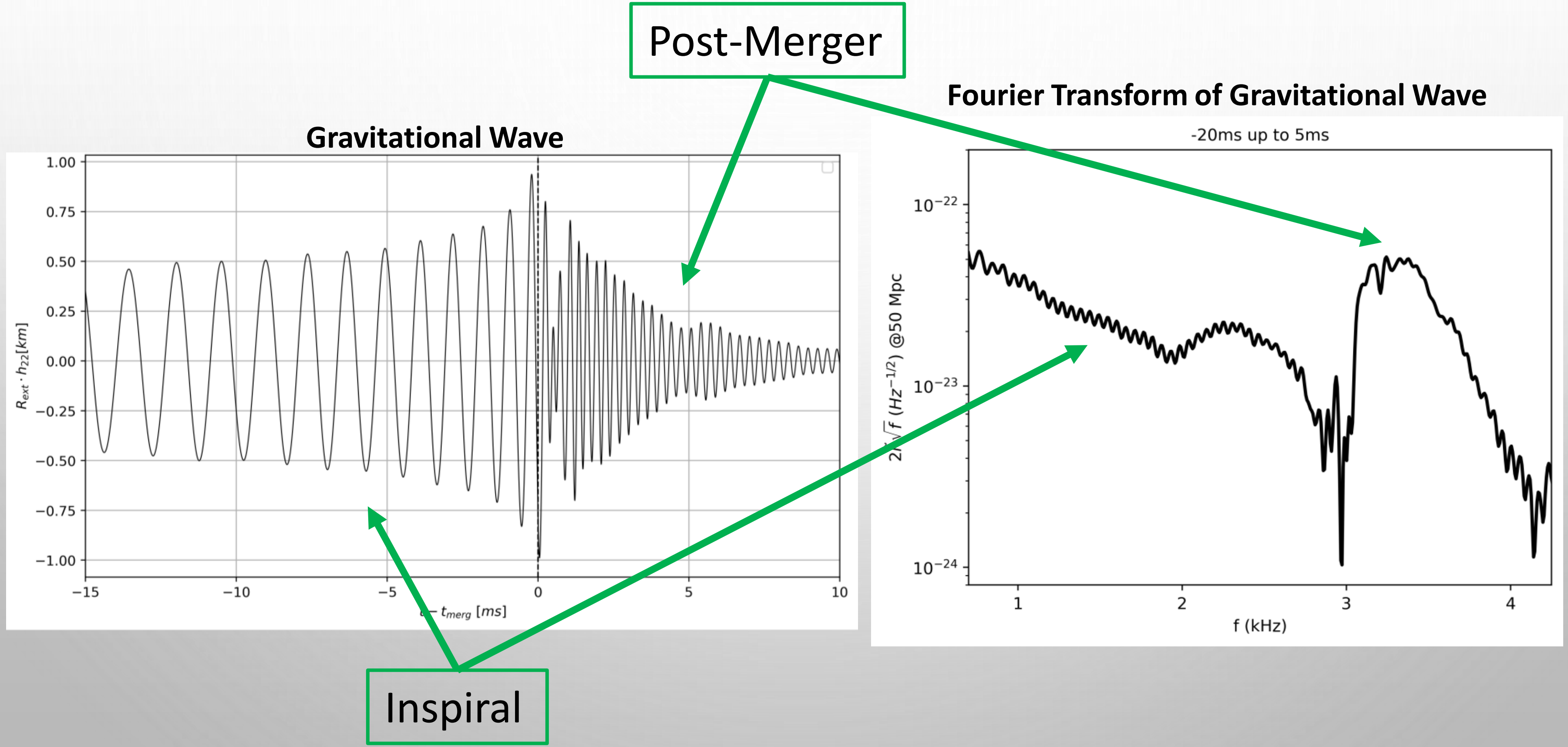
**Gravitational Wave**



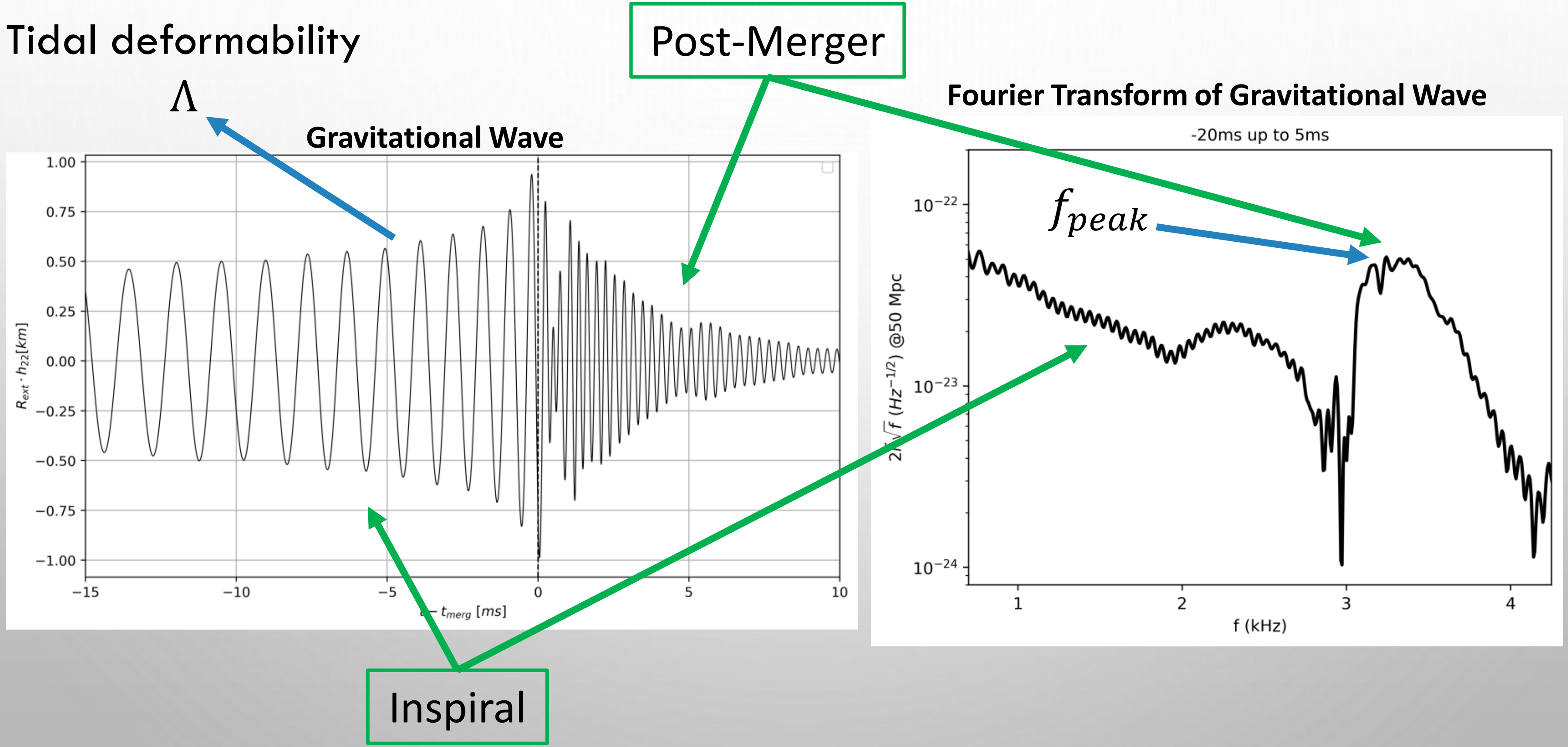

**Fourier Transform of Gravitational Wave**



# BINARY NEUTRON STARS (BNS) MERGER



# BINARY NEUTRON STARS (BNS) MERGER



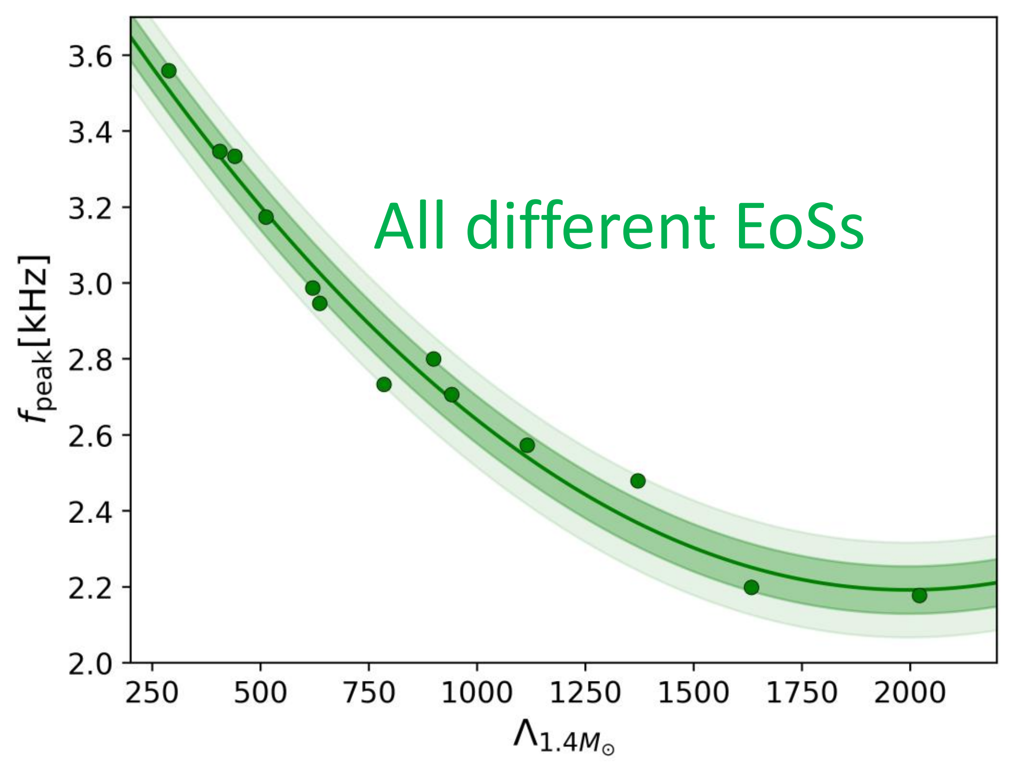
# Quasi-Universal Relation

$\Lambda$ ≔ Tidal deformability
(Measured pre-merger)

$f_{peak}$ ≔Characteristic Frequency post-merger

We can find a relation that is EoS-independent (almost)

Useful to constrain the EoS

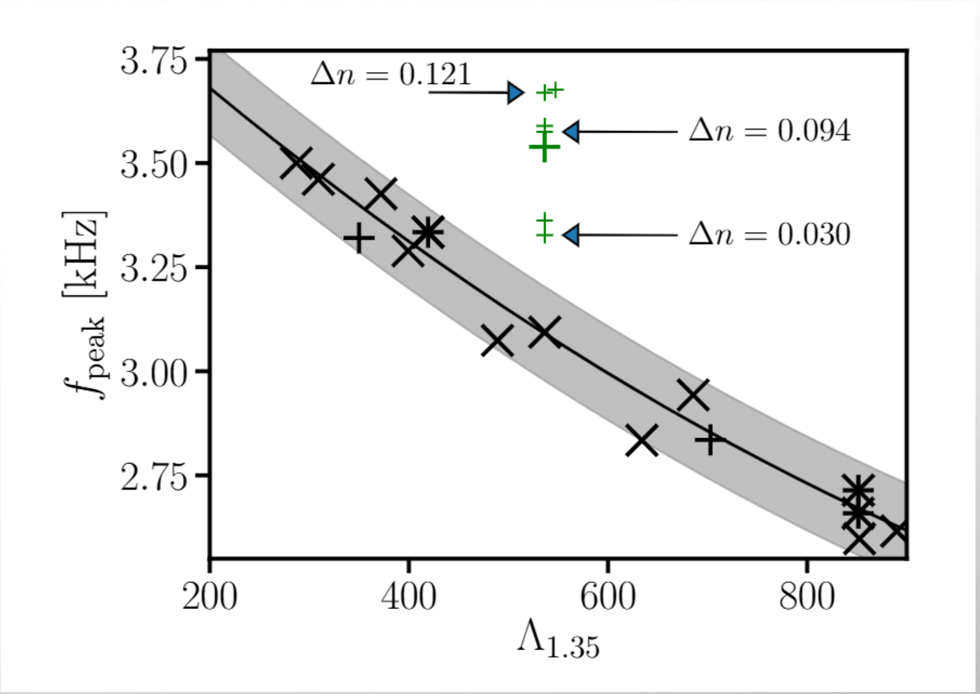



G. Rivieccio et al. (2024) Phys. Rev. D **109**, 064032

# Quasi-Universal Relation including Phase Transition

First order Phase transition softens the EoS during the merger

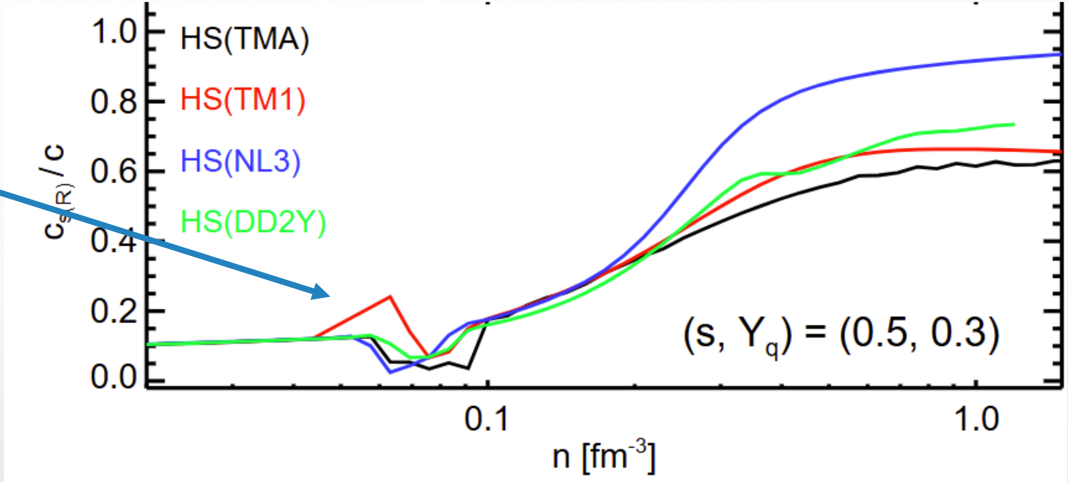
<ins>Shift in $f_{peak}$ is informative of a possible phase transition</ins>




Bauswein et al. 2019
Blacker et al. 2023,2025

# Sound speed is non-monotonic

$c_s^2 = \left.\frac{\partial P}{\partial \epsilon}\right|_s$ is non-monotonic




Aloy et al. (2019)

Because of the drop
of the sound speed:

Phase Transition are in general
**Nonconvex**

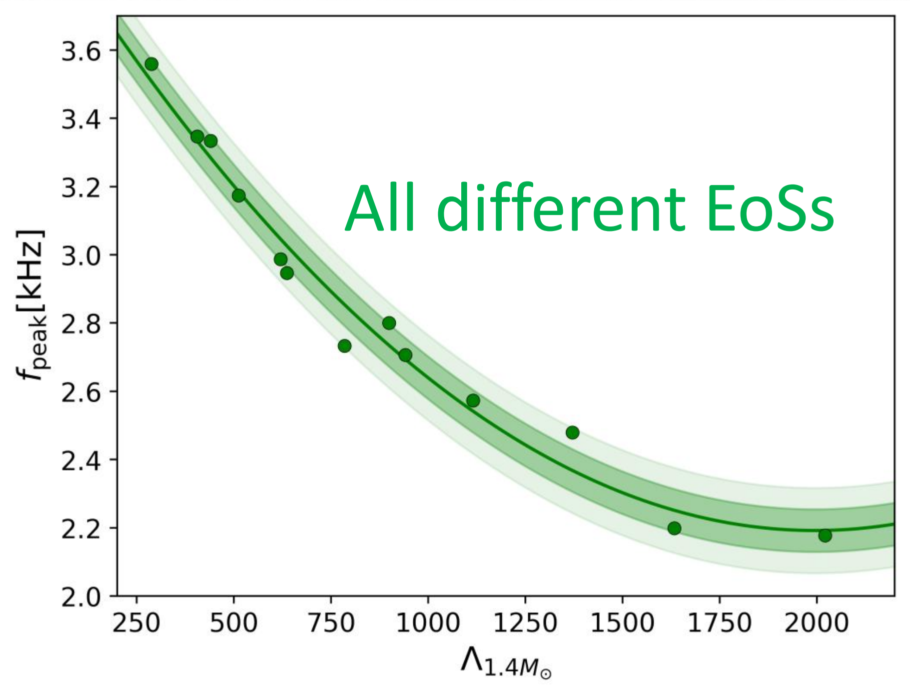
R Menikoff and B. Plohr Rev. Mod. Phys. 61, 75

# Quasi-Universal Relation

$\Lambda :=$ Tidal deformability
(Measured pre-merger)

$f_{peak}$ :=Characteristic Frequency post-merger

EoS-independent (almost)

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How does nonconvexity affect this relation?

# Wave structure - Fundamental Derivative

$$G \equiv -\frac{1}{2} V \frac{\left.\frac{\partial^2 P}{\partial V^2}\right|_s}{\left.\frac{\partial P}{\partial V}\right|_s}$$ Fundamental Derivative

Depends only on EoS

Menikoff and Plohr (1986), Ibañez et al. (2013)

# Wave structure - Fundamental Derivative

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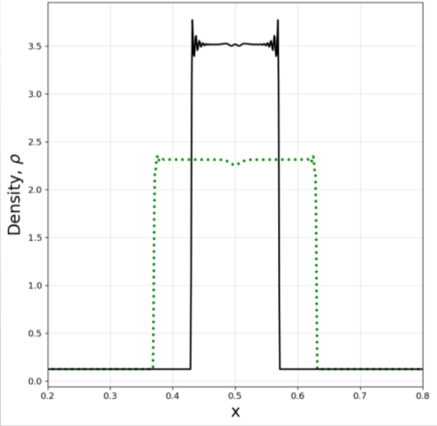
Menikoff and Plohr (1986), Ibañez et al. (2013)

$G > 0$ convex

Standard wave propagation

Colliding Slabs



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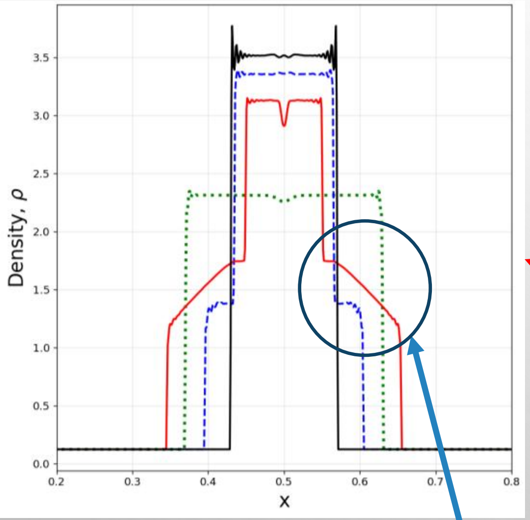
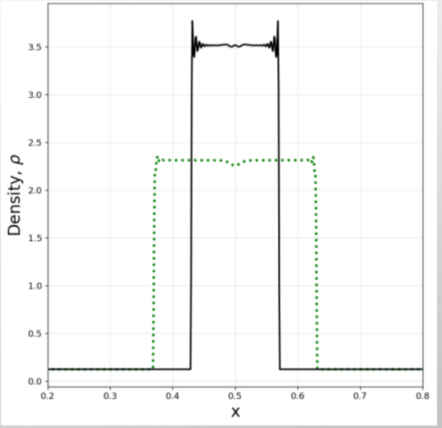



$G > 0$ **convex**

Standard wave propagation

$G < 0$ **nonconvex!**

Colliding Slabs




Rarefaction waves
(Lower density but entropy increase)

Composite waves
Rarefaction wave attached to a shock front

Split waves
(e.g. Phase Transition)

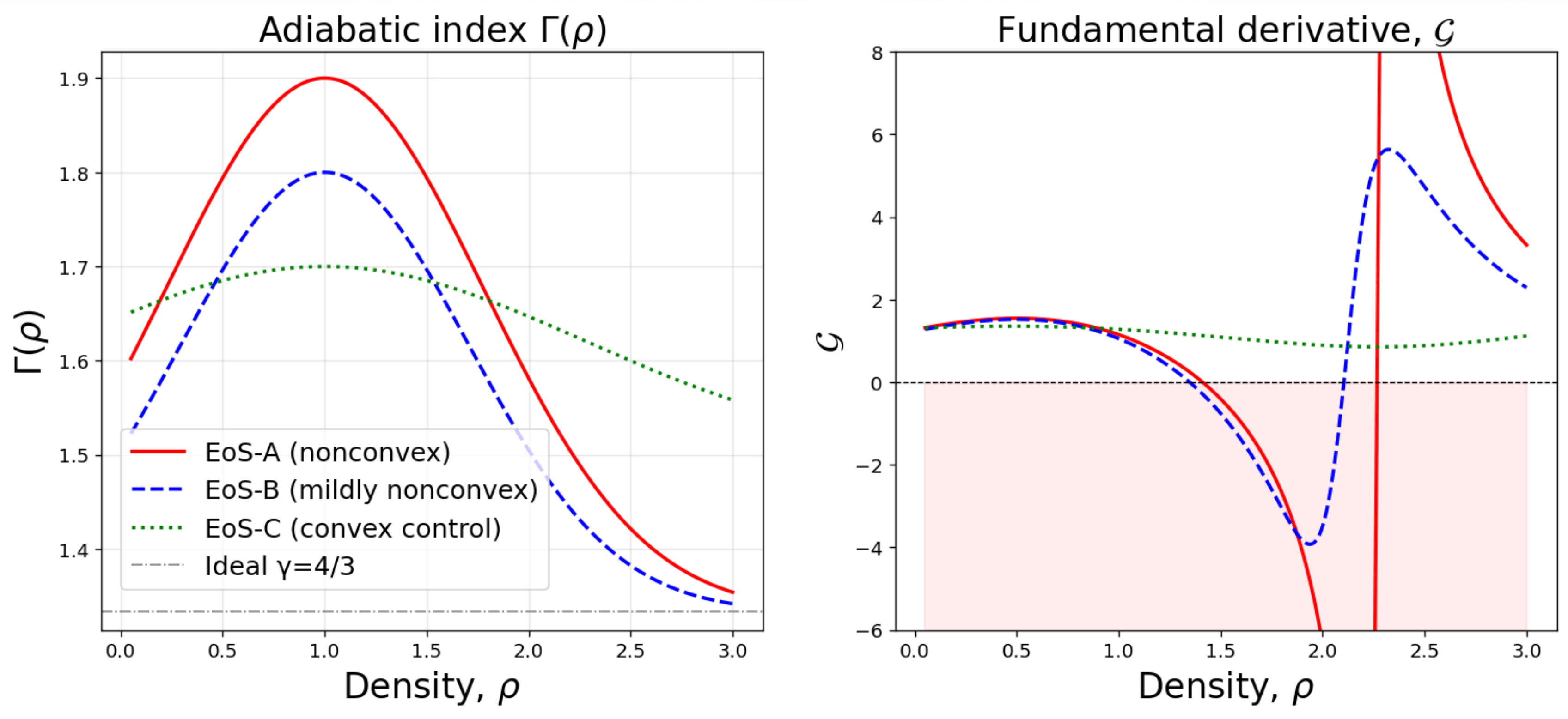
**ASSUMING A Γ-LAW EOS**

$$P = (\Gamma - 1)\,\rho\epsilon$$

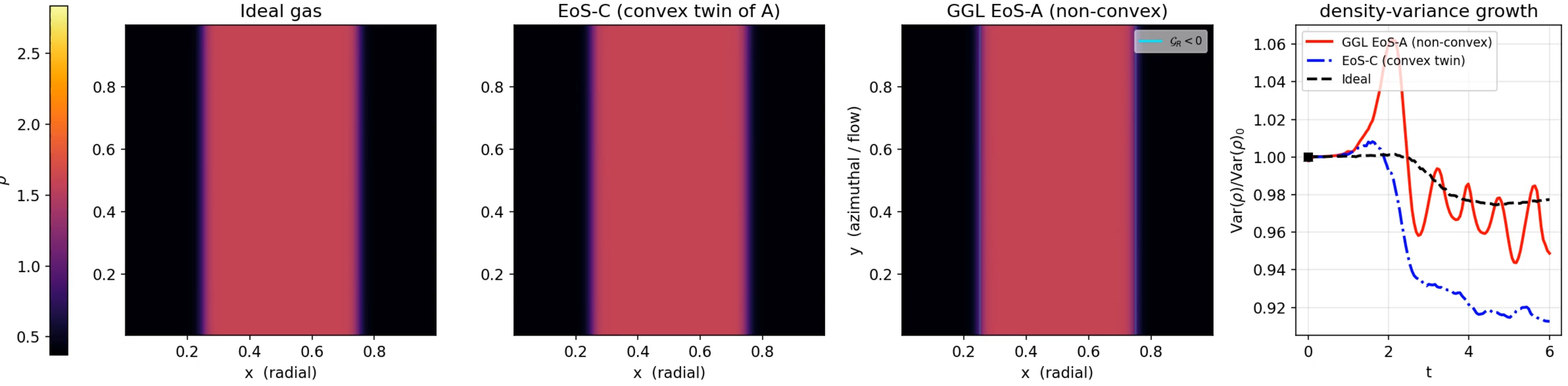
Ideal gas : $\Gamma = const$

GGL : $\Gamma = \Gamma_0 + (\Gamma_1 - \Gamma_0)e^{\frac{(\rho-\rho_1)^2}{2\sigma^2}}$

(Gaussian Gamma Law)



# 2D EXAMPLE : KEVIN-HELMHOLTZ INSTABILITY




GR et al. (2026) in prep.

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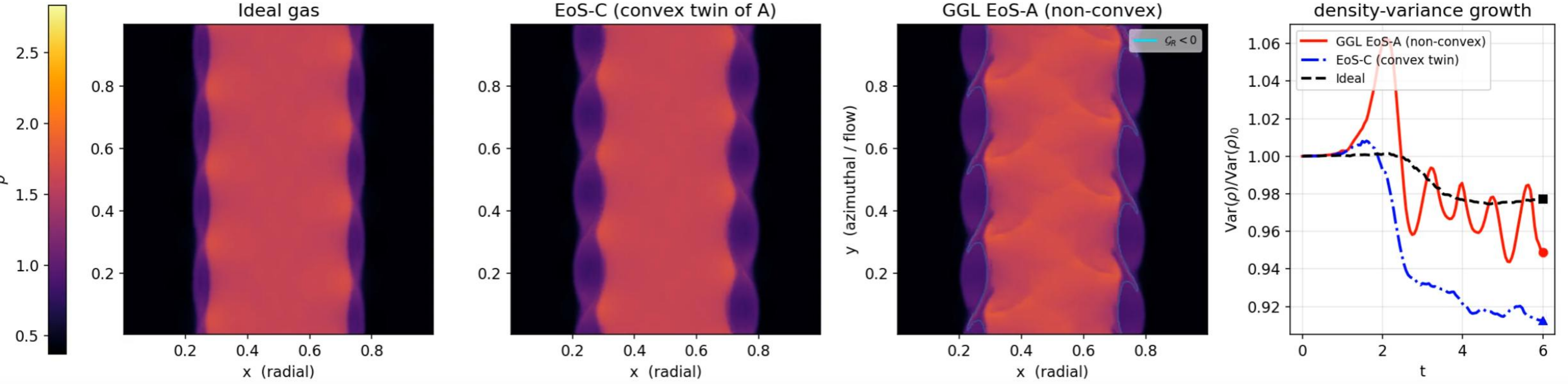
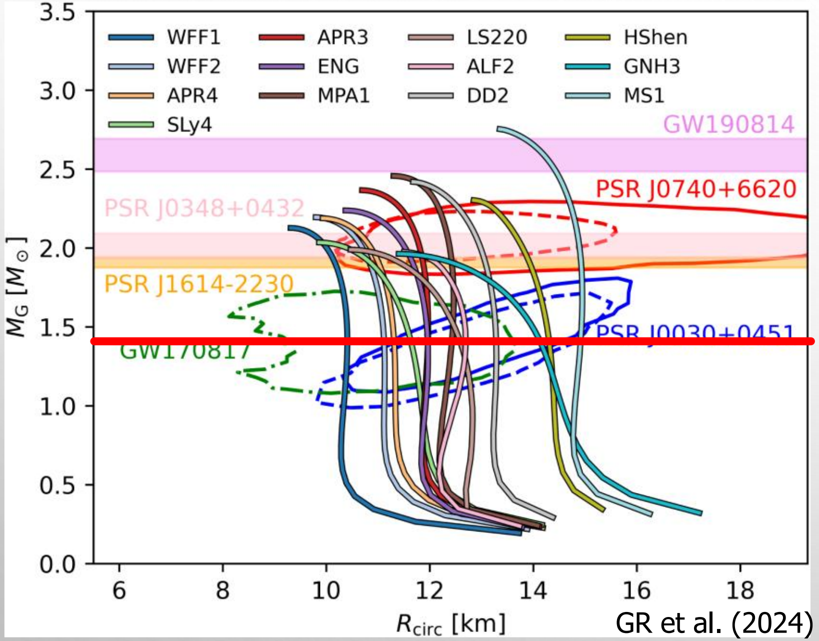



GR et al. (2026) in prep.

# WHAT ABOUT BNS SIMULATIONS?

Take initial Configurations at $1.4\ M_\odot$




$1.4\ M_\odot$

GR et al. (2024)

LORENE

https://gitlab.in2p3.fr/lorene/Lorene

Run GRHD simulations:

Einstein Toolkit + IllinoisGRMHD

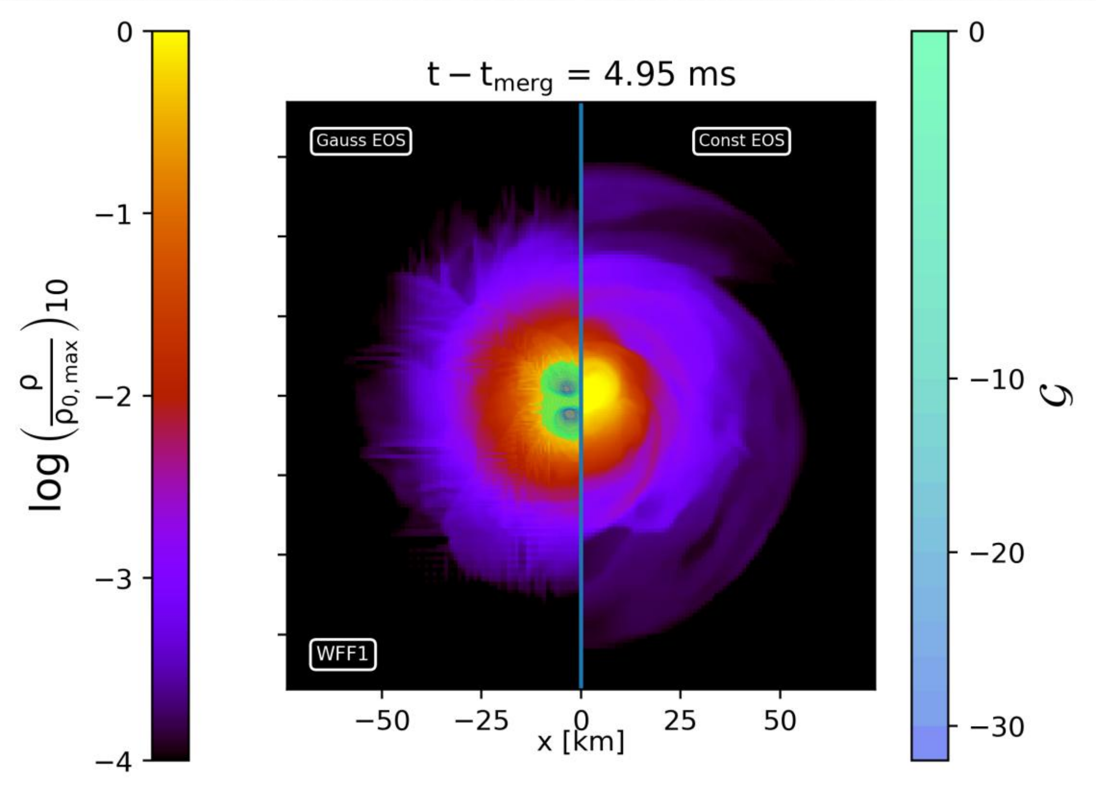
www.einsteintoolkit.org

Piecewise Politrope + Γ-law

$$P = k\rho^n + (\Gamma_{th} - 1)\epsilon n$$

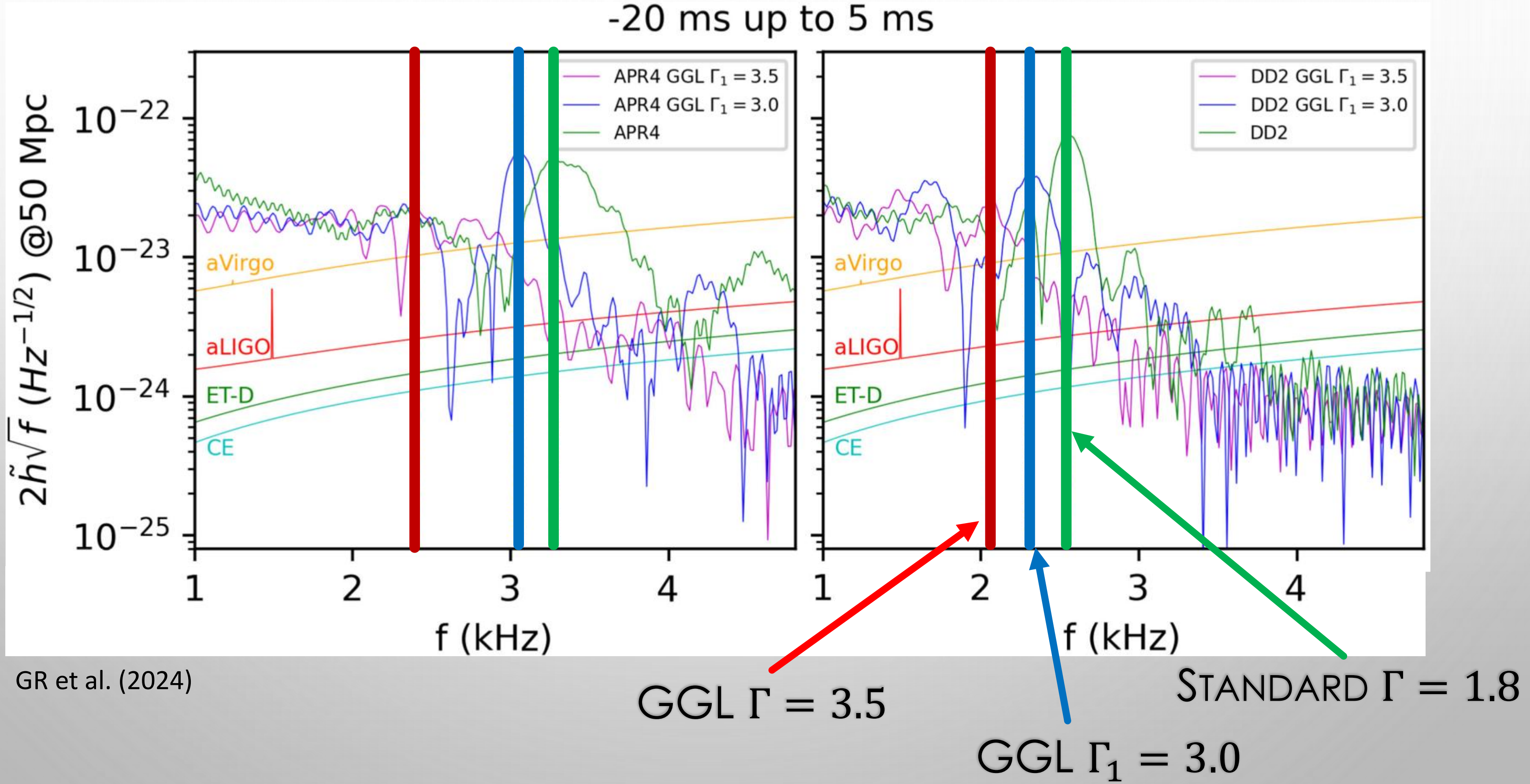
$\Gamma_{th} = 1.8$ and $\Gamma_{th} = GGL$

# RESULTS: NONCONVEX CORE AND NO SPIRAL ARM




GR et al. (2024)

# RESULTS: FREQUENCY PEAK SHIFT WITH STRONGER NONCONVEXITY

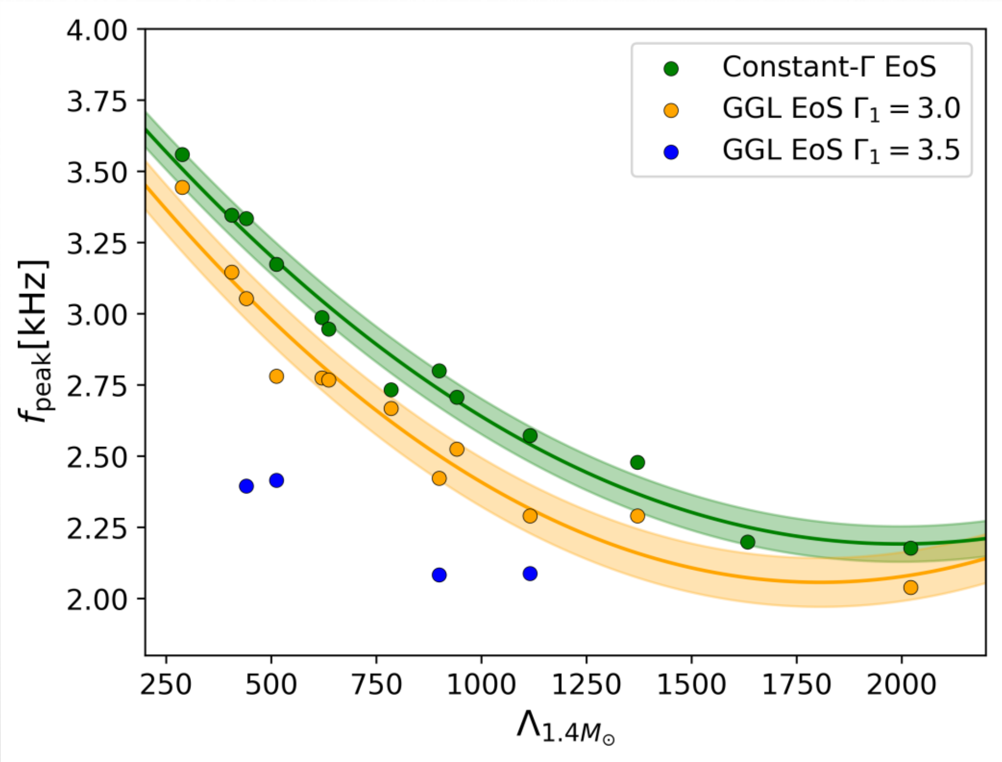



GR et al. (2024)

# QUASI-UNIVERSAL RELATION

Shifted Quasi-Universal relation!

Stronger outliers for more nonconvexity



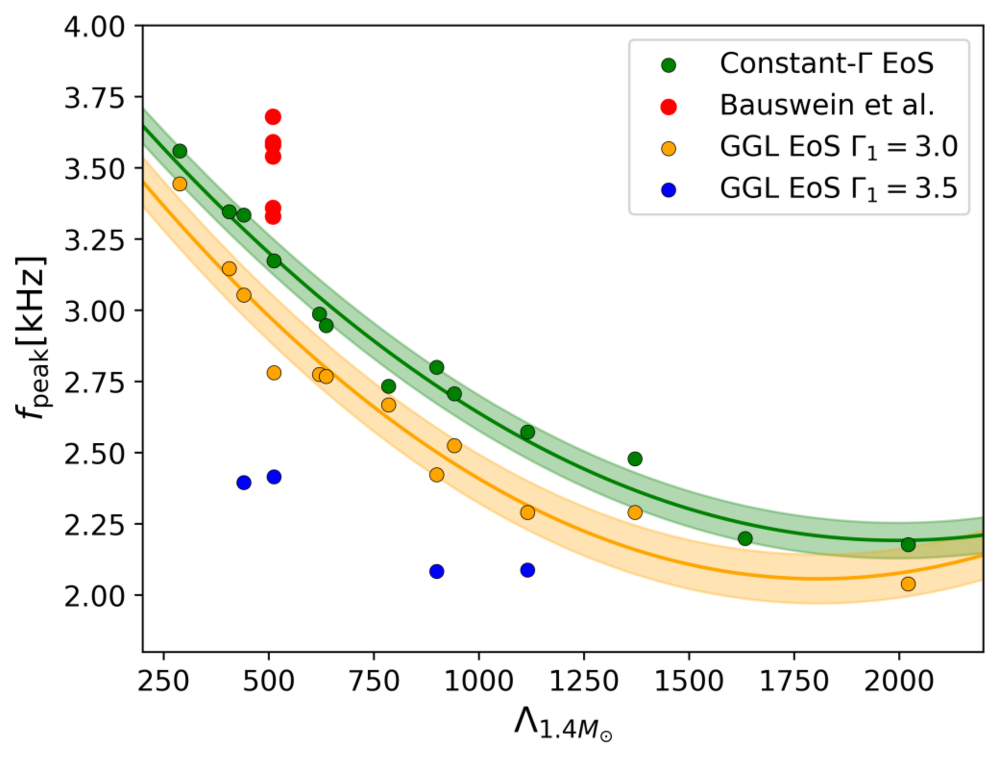

G. Rivieccio et al. (2024) Phys. Rev. D **109**, 064032

# QUASI-UNIVERSAL RELATION

Shifted Quasi-Universal relation!

Stronger outliers for more nonconvexity

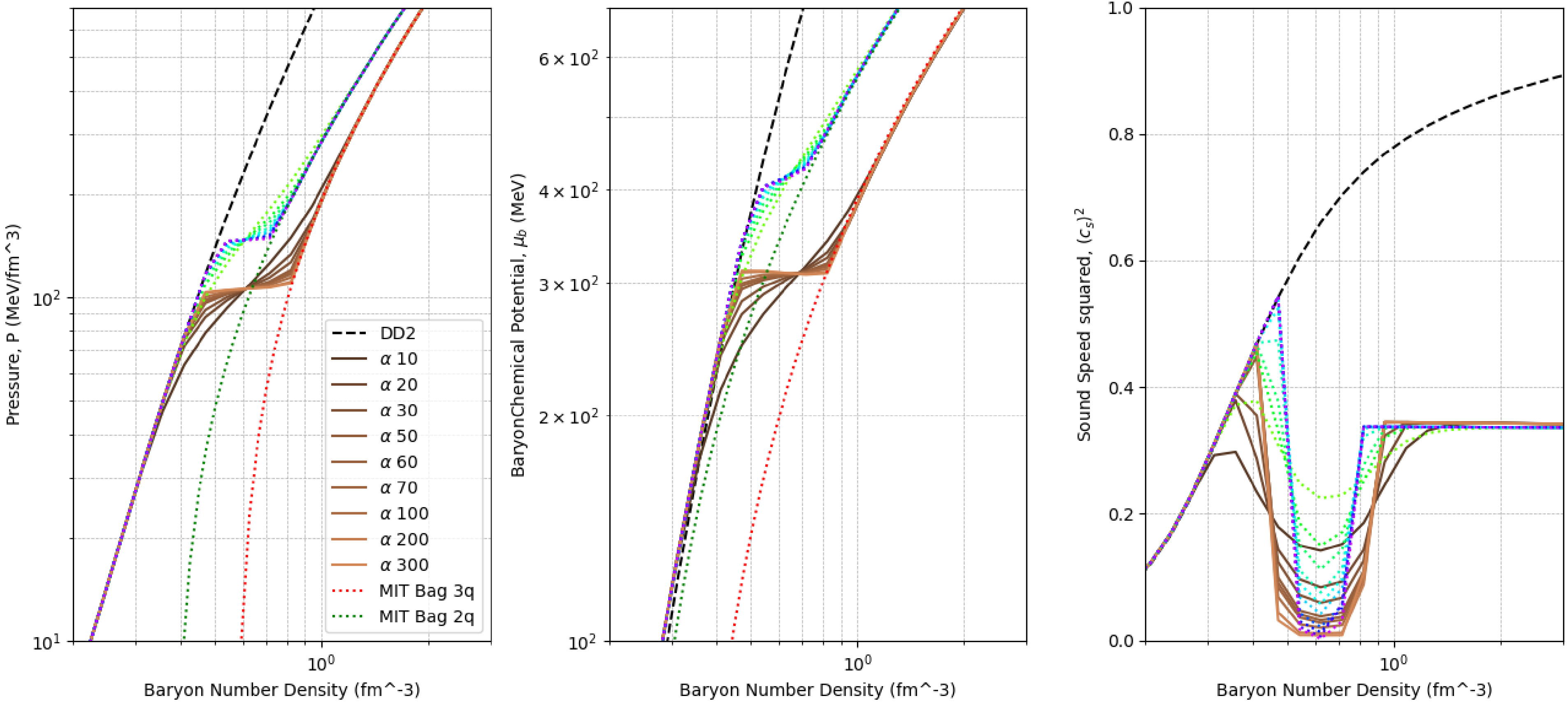
GGL EoS gives **lower frequencies** while phase transition has higher frequencies instead




G. Rivieccio et al. (2024) Phys. Rev. D **109**, 064032

We need to have a phase transition with controlled convexity!

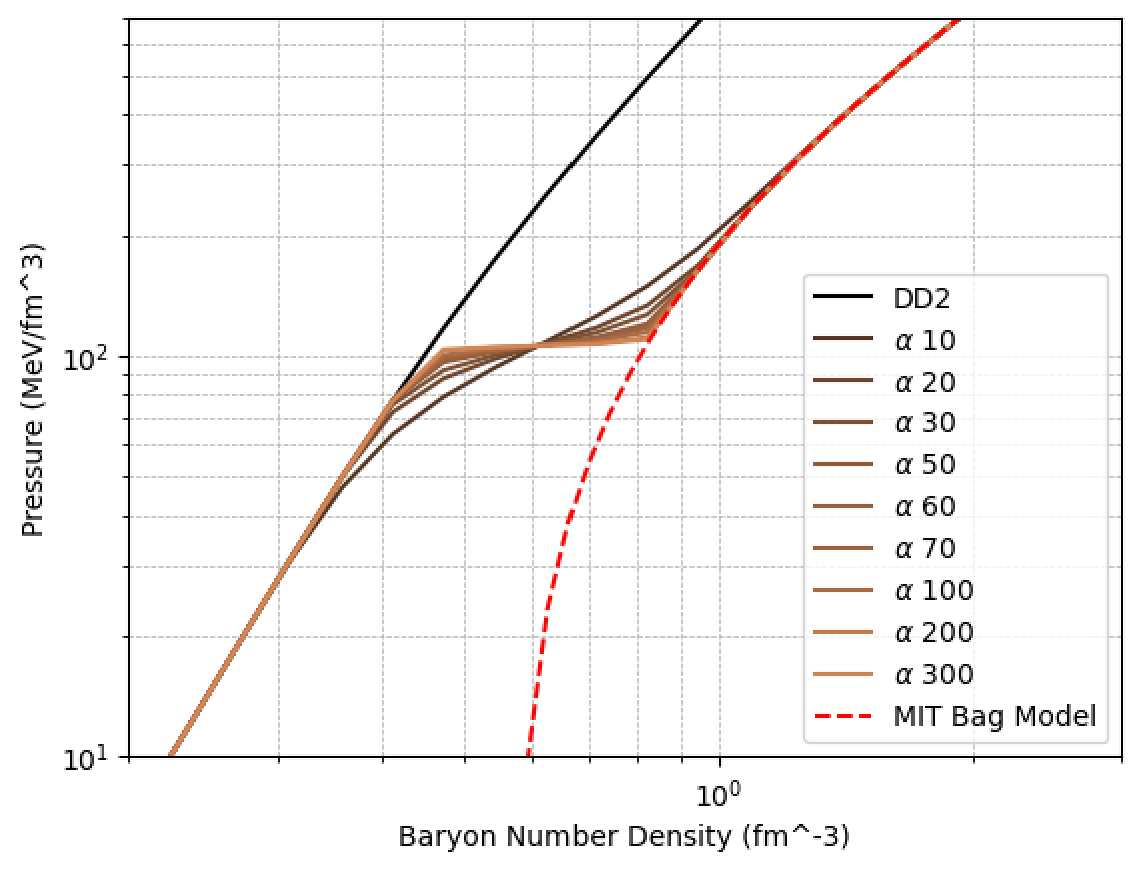
# PARAMETRISED PHASE TRANSITION




GR et al. (2026) in prep.

# PARAMETRISED PHASE TRANSITION

How do we build it?




GR et al. (2026) in prep.

# PARAMETRISED PHASE TRANSITION

How do we build it?

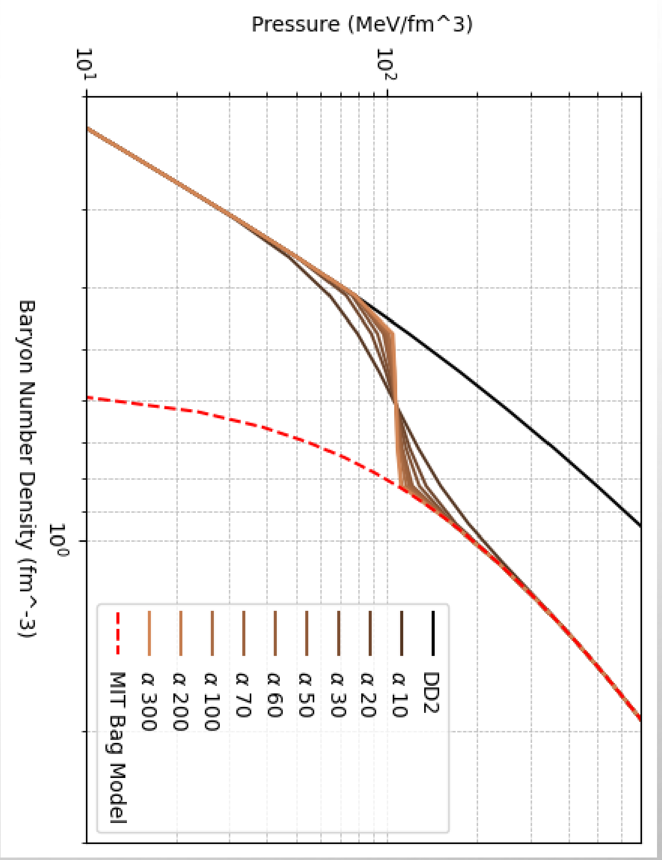
The inverted function is a step!
We apply a sigmoid.

$$n = n_{Nuc}\sigma + n_{Qrk}(1-\sigma)$$

where

$$\sigma = \frac{1}{1+e^{-\,\alpha(p-p_t)}}$$

$\boldsymbol{\alpha}$ controls the sharpness
$\boldsymbol{p_t}$ is the transition pressure




GR et al. (2026) in prep.

# PARAMETRISED PHASE TRANSITION

How do we build it?

We apply the conditions

Mechanical equilibrium:

$$p_{Nuc}(n_{on}) = p_{Qrk}(n_{off})$$

Chemical equilibrium:

$$\mu_{g,Nuc}(n_{on}) = \mu_{g,Qrk}(n_{off})$$

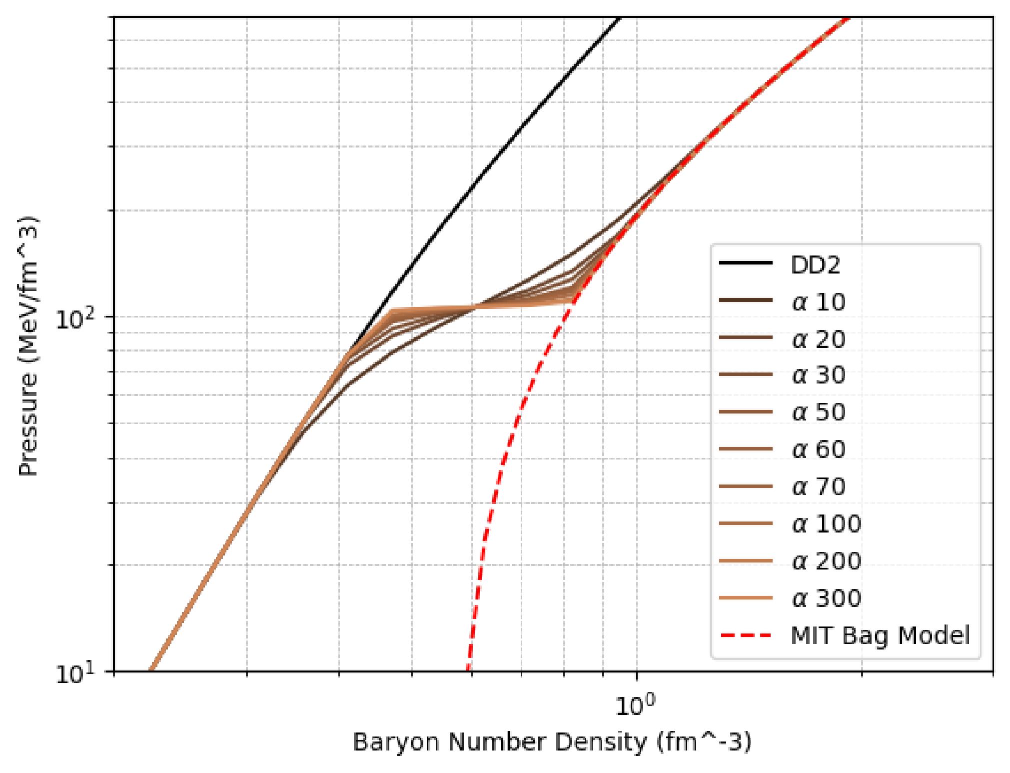
where

$$\mu_g = \mu_b + Y_q * \mu_q$$

or

$$\mu_g = \mu_b + Y_e * \mu_l$$

(if we do the transition with electrons)




GR et al. (2026) in prep.

# PARAMETRISED PHASE TRANSITION

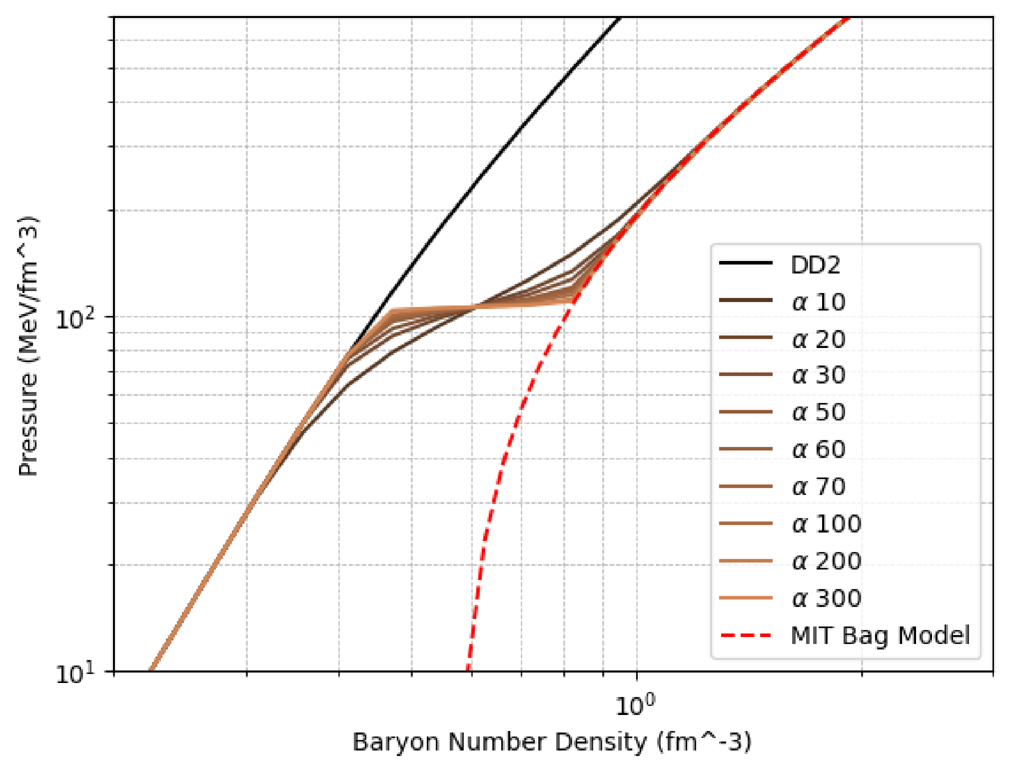
We have the pressure!

How do we get everything else?

- Invert $\mathrm{P} = n^2 \frac{\partial F}{\partial n}$
- From the free energy F we get everything else!

(Typel et al 2015)

To ensure thermodynamical concistency
**Piecewise quintic Hermite** polynomials

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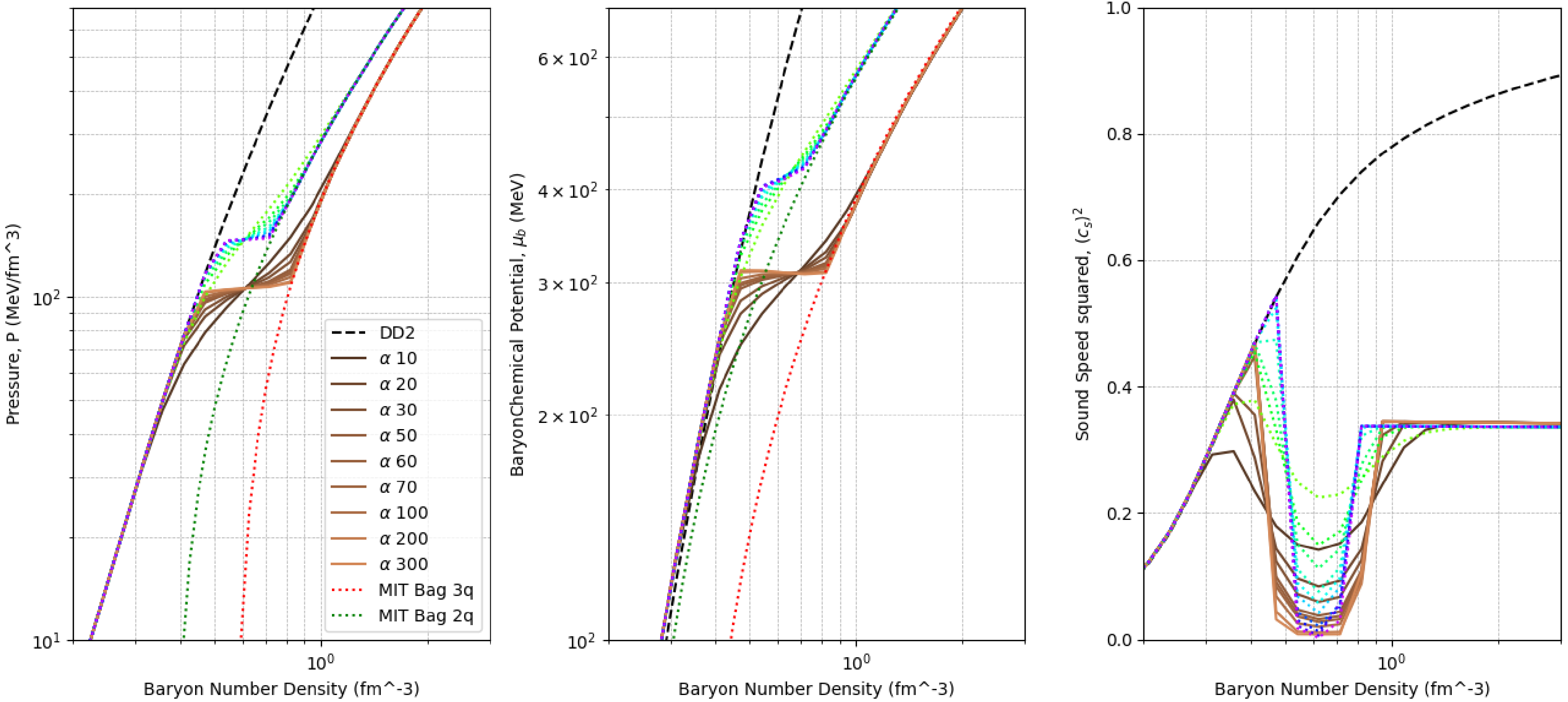
# PARAMETRISED PHASE TRANSITION




GR et al. (2026) in prep.

# Parametrised Phase Transition

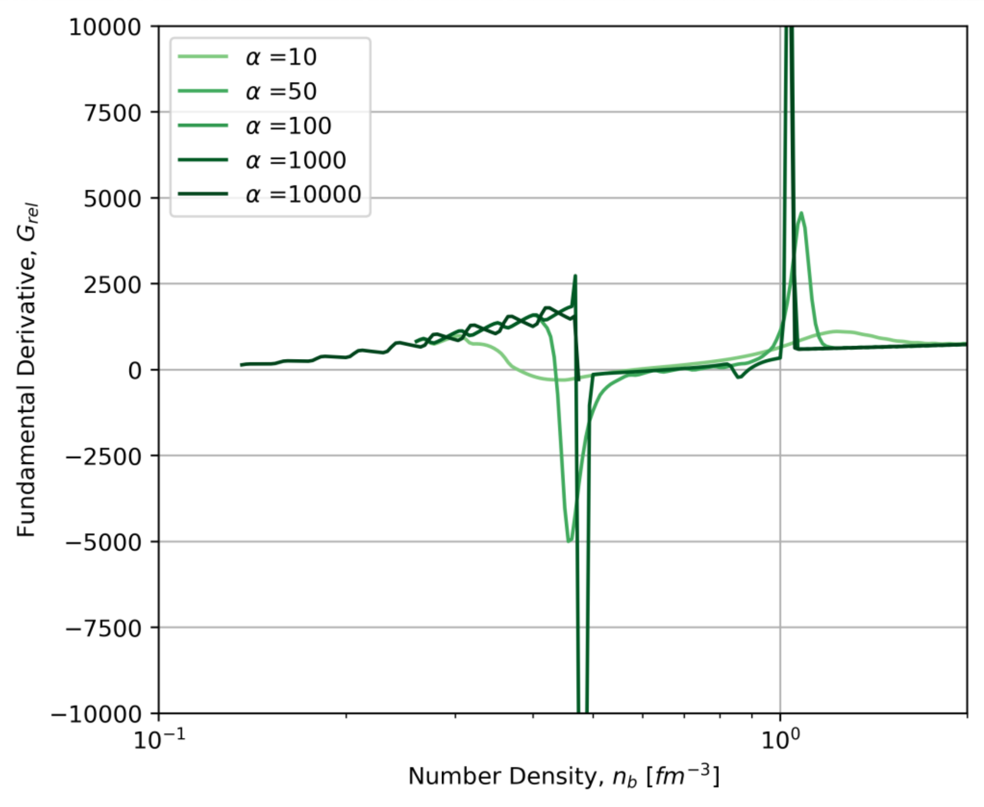
And the fundamental derivative?

We write

$$G = 1 + \frac{n_b}{2\ c_s^2} \frac{\partial c_{s,C}^2}{\partial n_b}\bigg|_s - \frac{3}{2} c_{s,R}^2$$

(C is Classical, R is Relativistic)

We will have a continous crossover with still non-convex matter.

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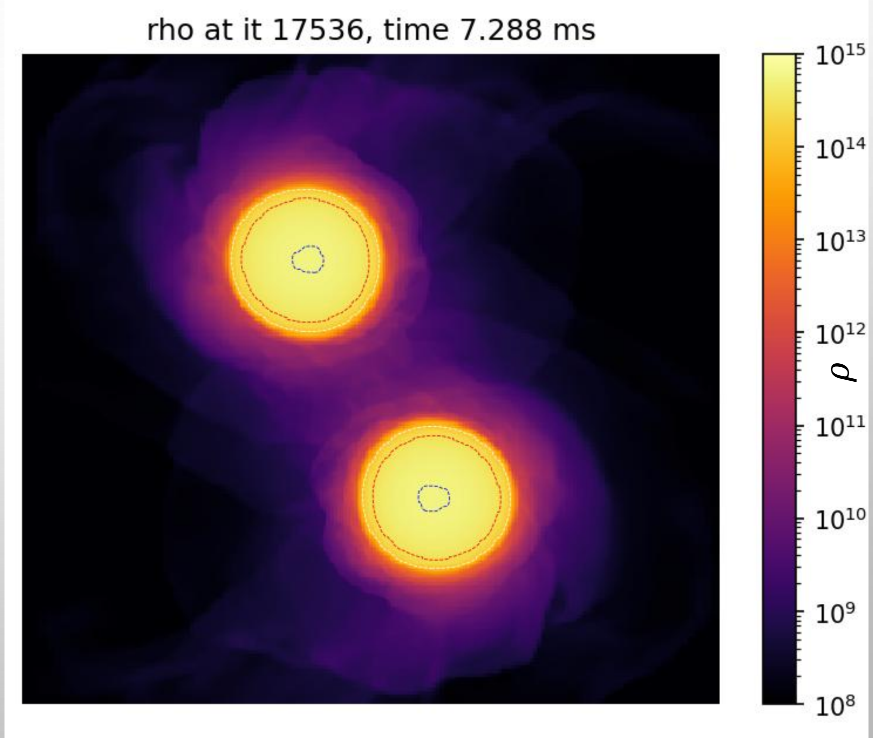
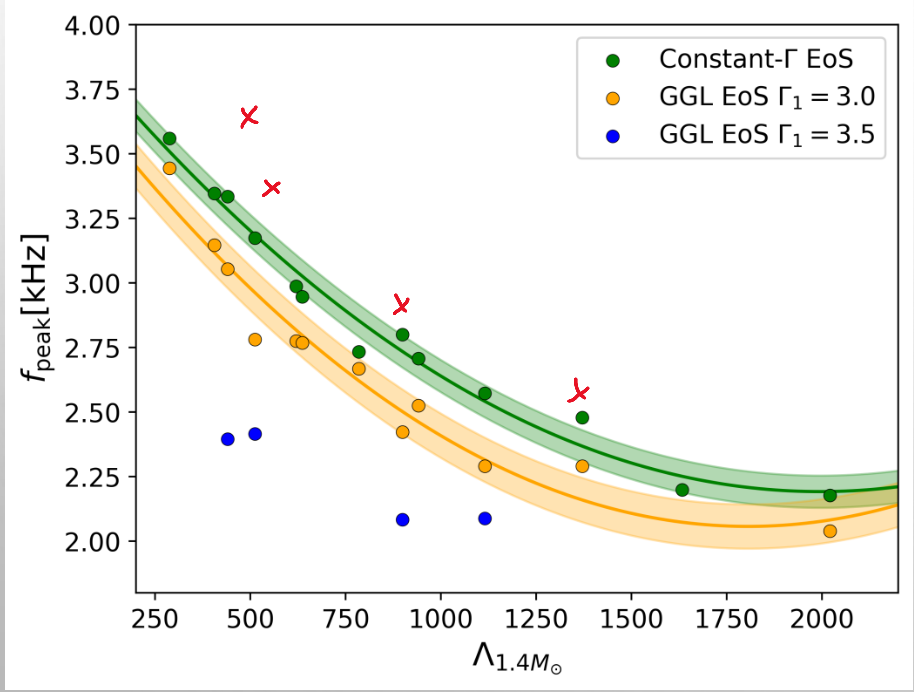
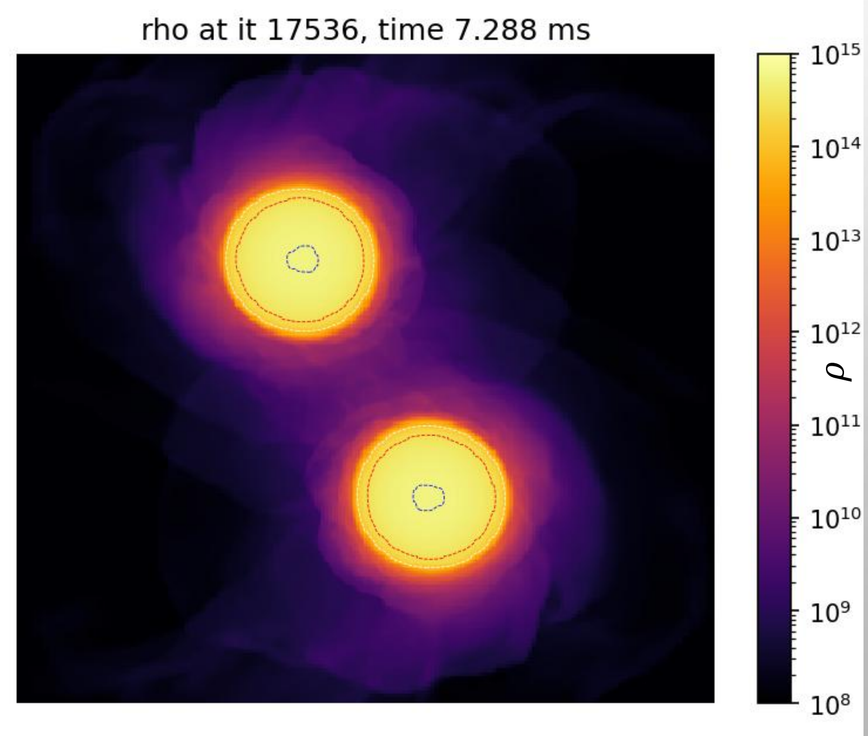
# Simulations are running!



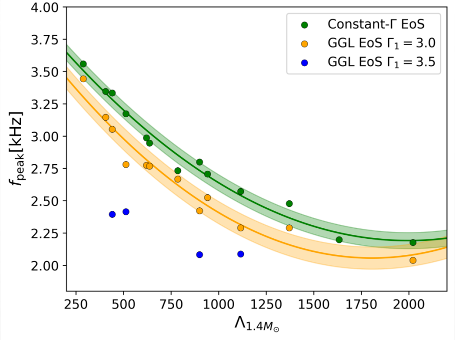
# Simulations are running!

We expect to populate the high frequency side with both nonconvex crossover and first order phase transition.

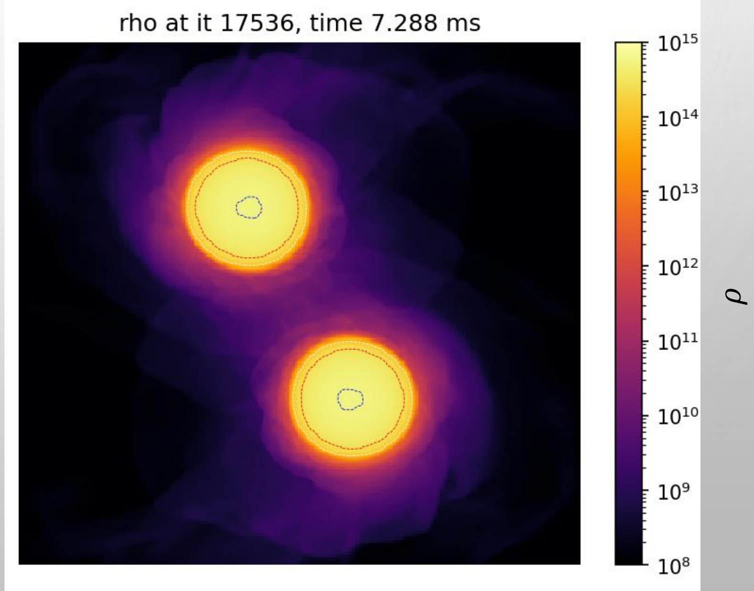


# Take away message!




- Convexity determines the wave structure and the evolution of the fluid
- Nonconvexity has an imprint in the gravitational wave!
- Phase transitions and crossovers are nonconvex
- Simulation are running to determine the definitive contribution of nonconvexity in BNS merger



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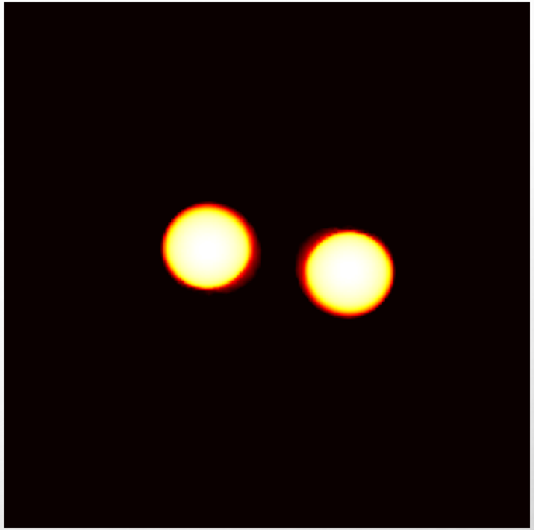

# THANKS!

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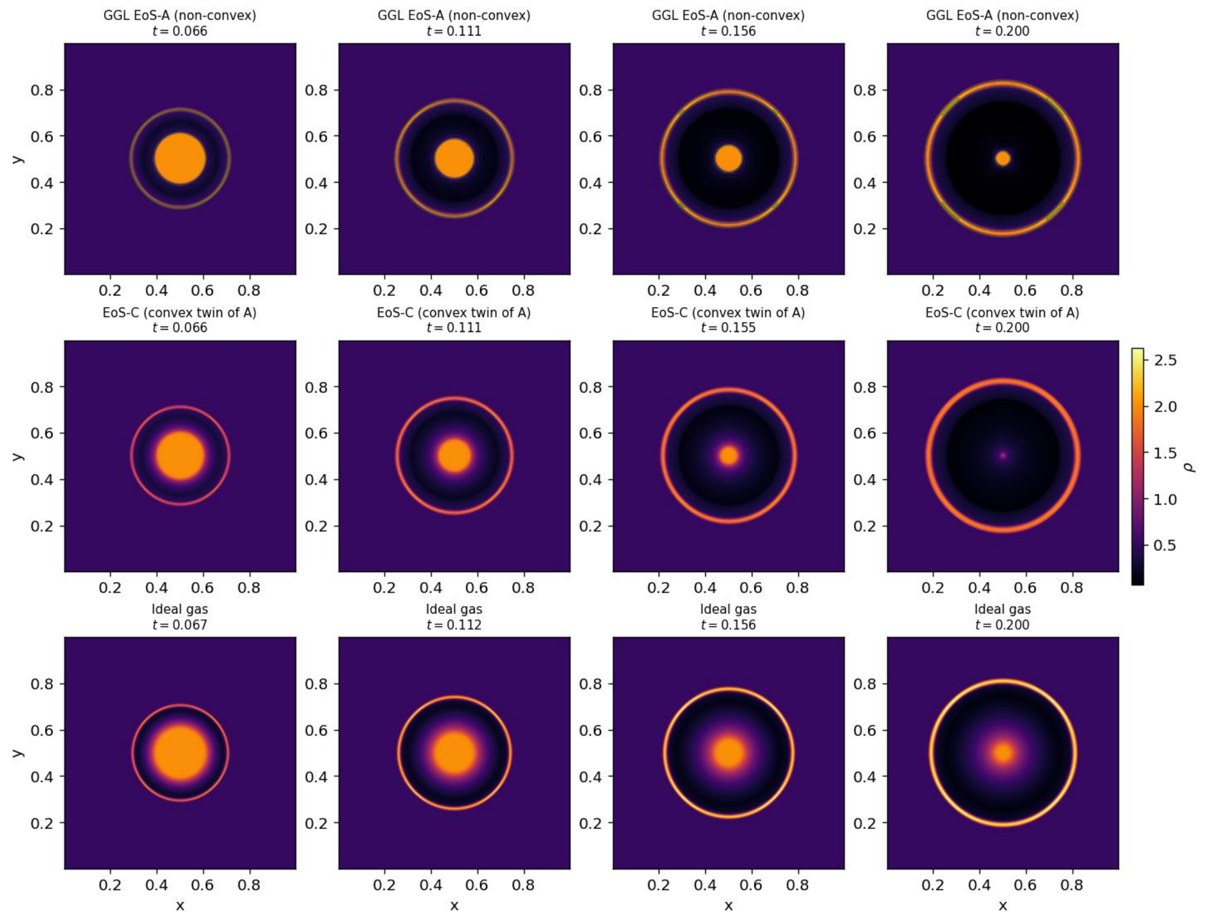
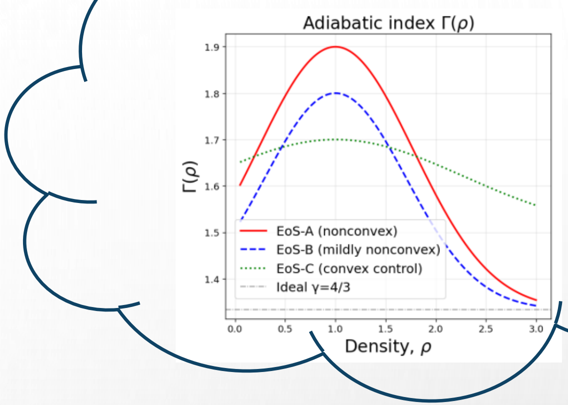
# AN 2D EXAMPLE : DENSE DISK DIFFUSING



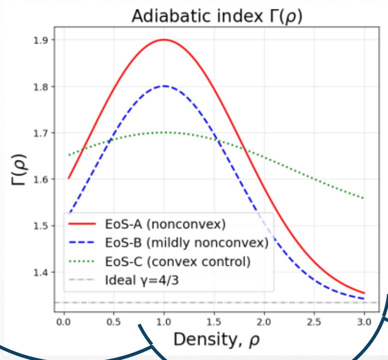

Convex EoS:

- Smooth edge of the disk

Nonconvex EoSs:

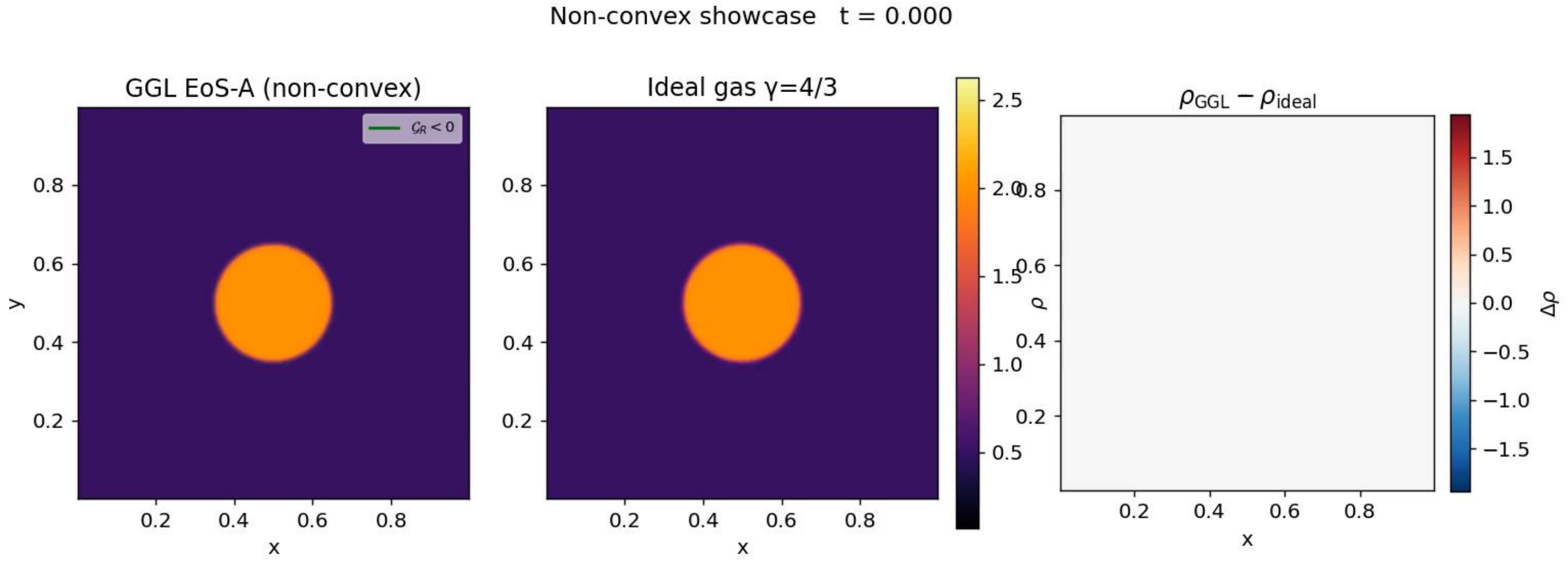
- Sharp edge, well defined disk

# AN 2D EXAMPLE : DENSE DISK DIFFUSING




Nonconvex EoS:
- Sharp edge of the
- Rarefaction wave

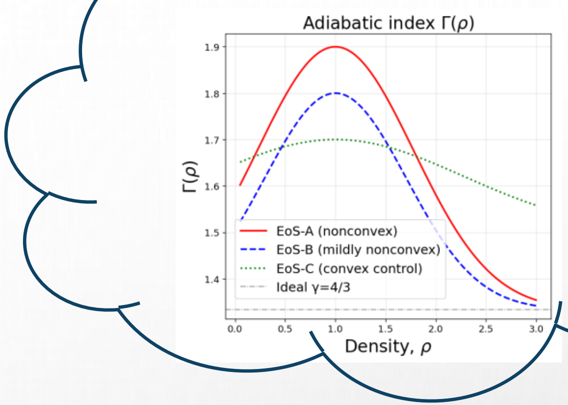
Convex EoS:
- Diffuse disk
- Stronger

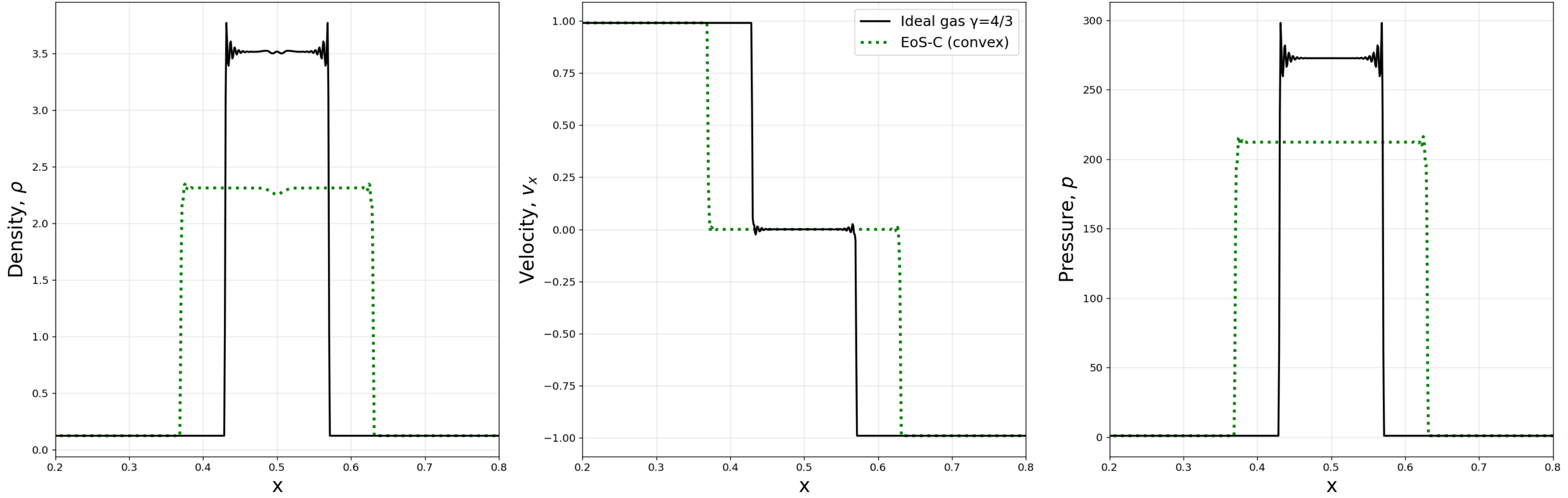


# An example : Colliding Slabs

Convex GGL → 

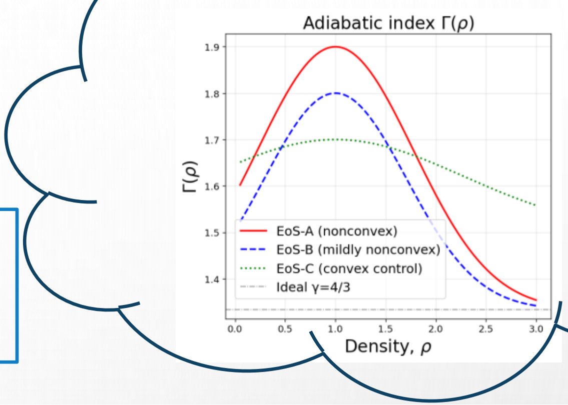
- Smaller density and pressure
- Same overall behaviour

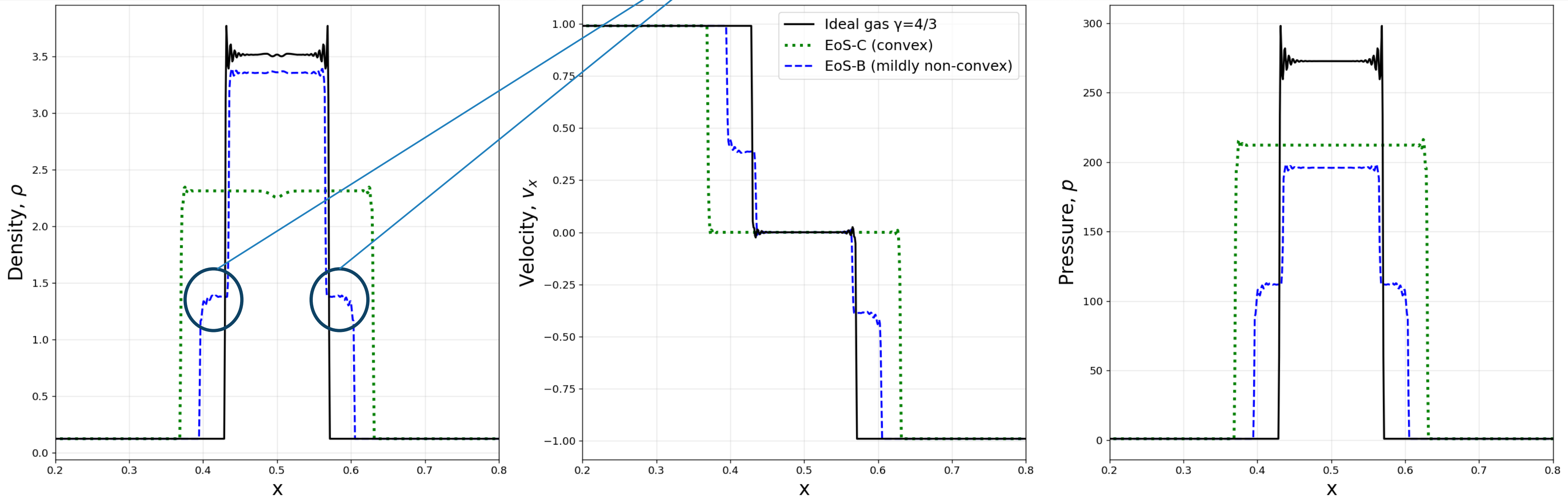


# An example : Colliding Slabs

EoS-B Mildly nonconvex GGL — **Rarefaction wave!**

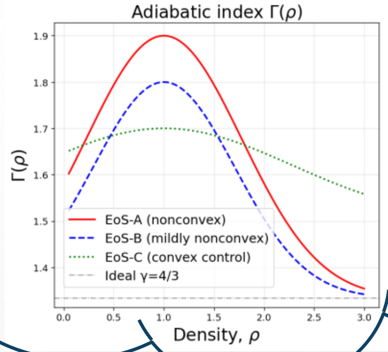


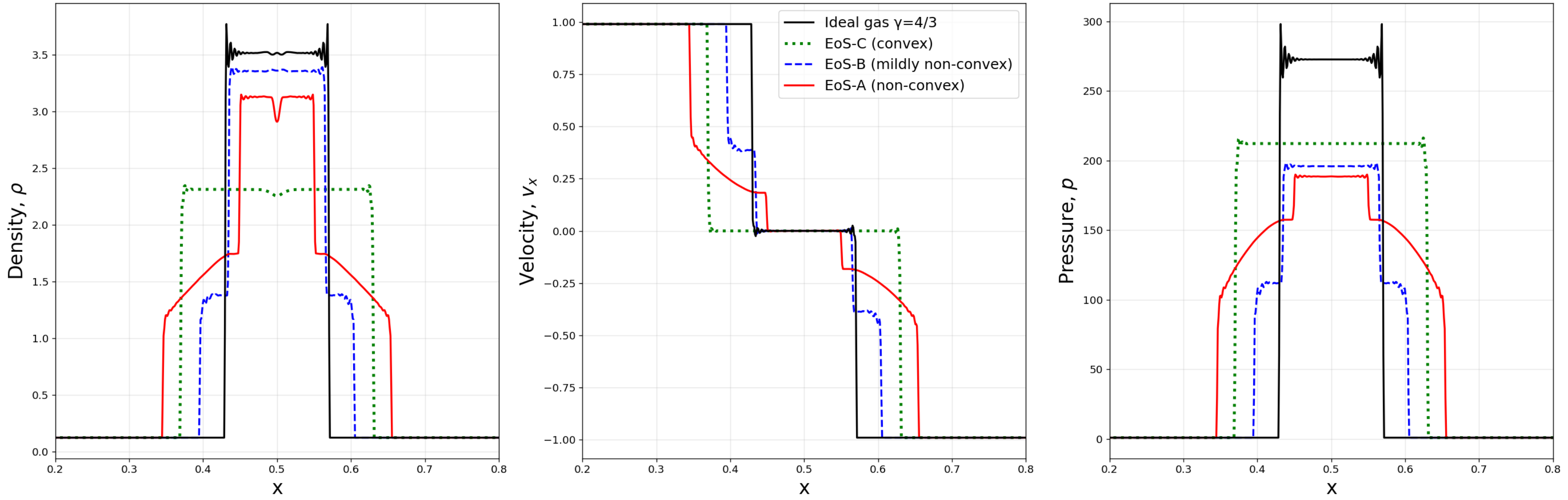



# An example : Colliding Slabs

EoS-A Nonconvex GGL — **Composite wave!**

Rarefaction wave attached to shock front

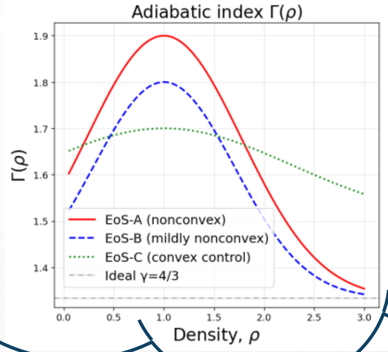


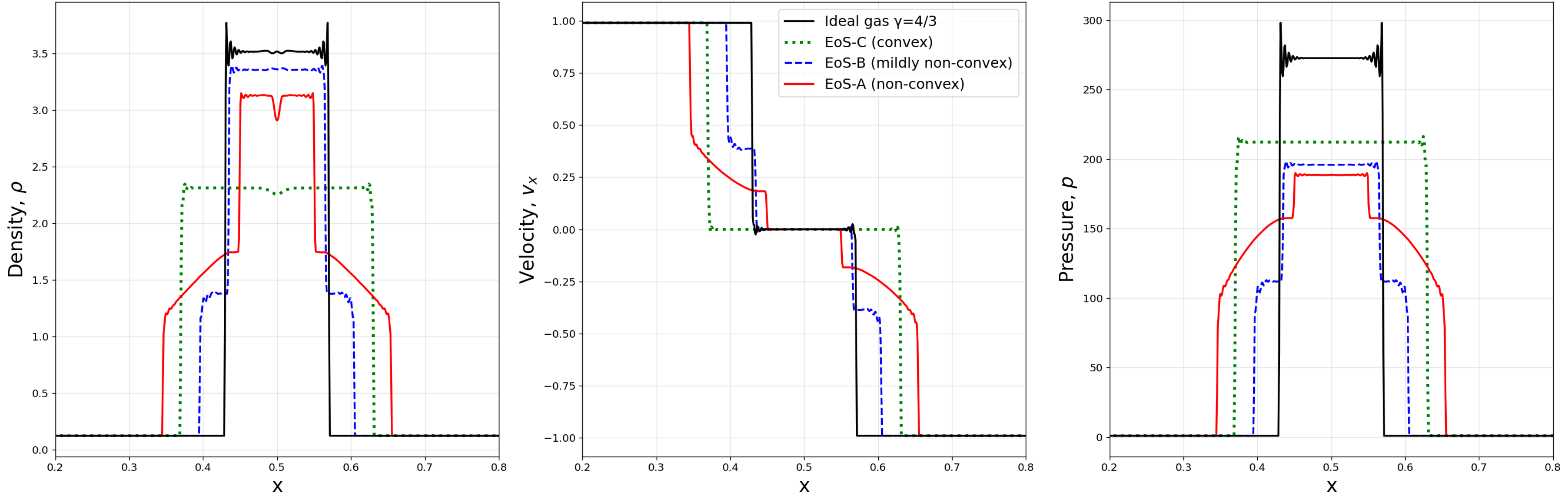



# An example : Colliding Slabs

- Convex EoS: Simple wave structure

- Nonconvex EoSs: Composite waves and higher $\rho$ for similar $P$



# Wave structure

The evolution equation for a compressible fluid

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\partial \boldsymbol{f}(\boldsymbol{u})}{\partial x} = 0$$

The jacobian $\boldsymbol{f}' = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}$ has eigenvalue $\lambda_k$ and eigenvector $\boldsymbol{r}_k$

Nonlinearity factor: $\nu_\mu = \nabla_{\mathbf{u}} \lambda_k(\boldsymbol{u}) \boldsymbol{r}_k(\boldsymbol{u})$

$$\nu_\mu \propto G \equiv -\frac{1}{2} V \frac{\left.\frac{\partial^2 P}{\partial V^2}\right|_s}{\left.\frac{\partial P}{\partial V}\right|_s}$$ Fundamental Derivative

Lax (1973), Menikoff and Plohr (1986)

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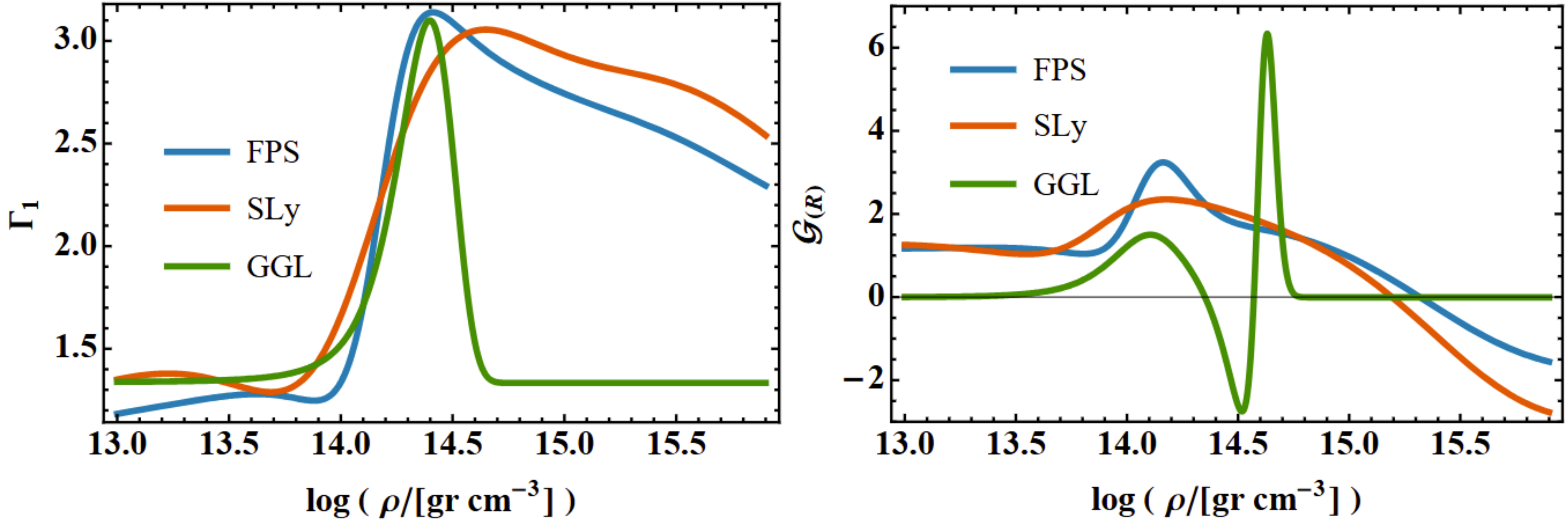
$$P = (\Gamma - 1)\,\rho\epsilon$$

Ideal gas : $\Gamma = const$

GGL : $\Gamma = \Gamma_0 + (\Gamma_1 - \Gamma_0)e^{\frac{(\rho-\rho_1)^2}{2\sigma^2}}$

(Gaussian Gamma Law)




Aloy et al. (2019)