Fundamental QCD in Nuclei: Diquark-based SRC, Hidden-Color, Glue balls & Other SU(3) predictions

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Electron-Ion Collider Foundation: Quantum Chromodynamics

"The electron beams at the EIC, and the knowledge the collisions of electrons with ions will reveal about the arrangement and interactions of quarks and gluons, ***will help us understand the force that holds these fundamental building blocks – and nearly all visible matter – together.***"

- *Doon Gibbs, Director of BNL*

"From RHIC to the EIC: Taking Our Exploration of Matter to the Next Frontier"

August 2022

Electron-Ion Collider

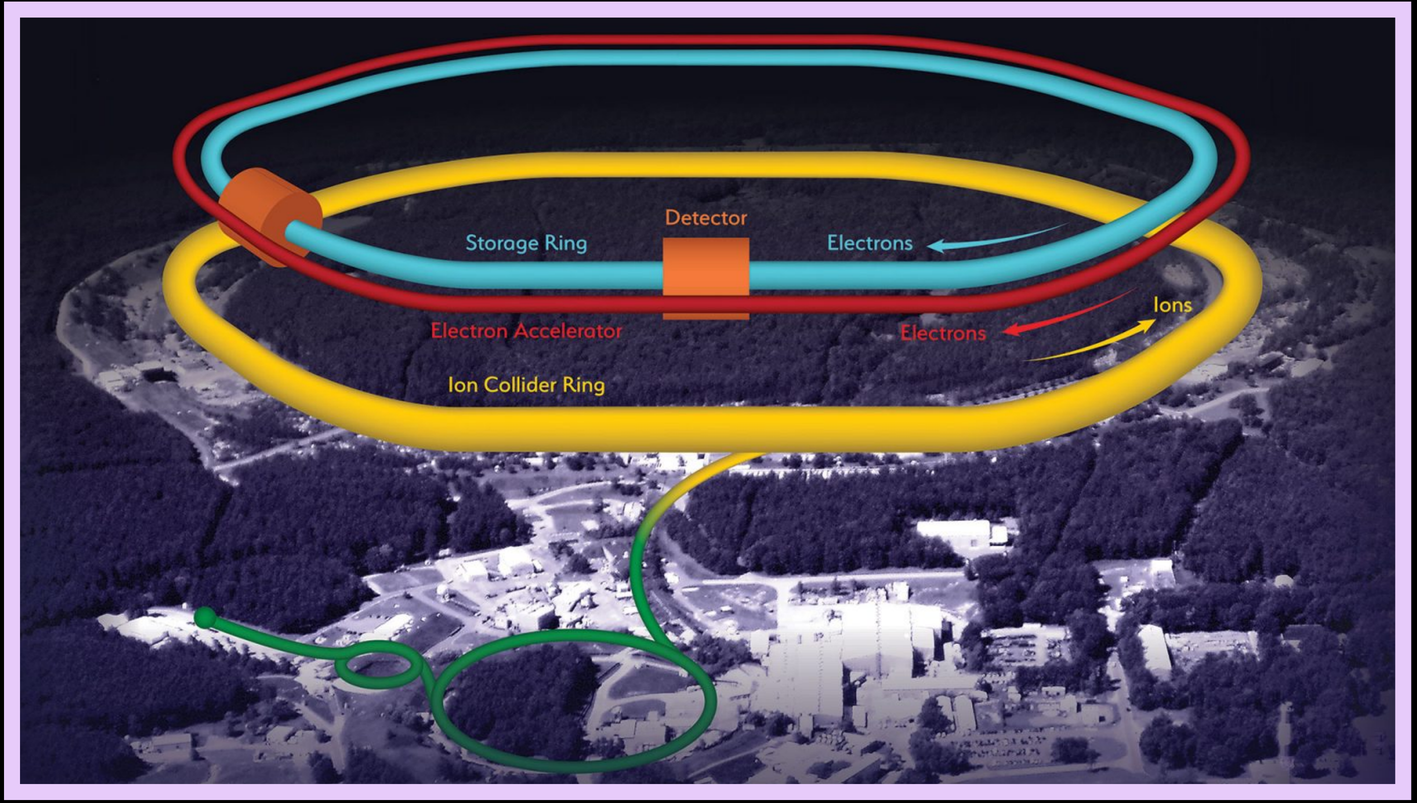


Image credit: Brookhaven National Lab

Quantum Chromodynamics: Nuclear & Particle

- Particle = Fundamental QCD: quarks, gluons, color charged objects
- Nuclear = Effective QCD: nucleons, nuclei
- Hadronic physics (~sub-nuclear): baryons, mesons
- Scale separation assumption between nuclear and particle physics
- Boundary between scales of extreme interest. Especially scale separation assumption breakdown!

Electron-Ion Collider

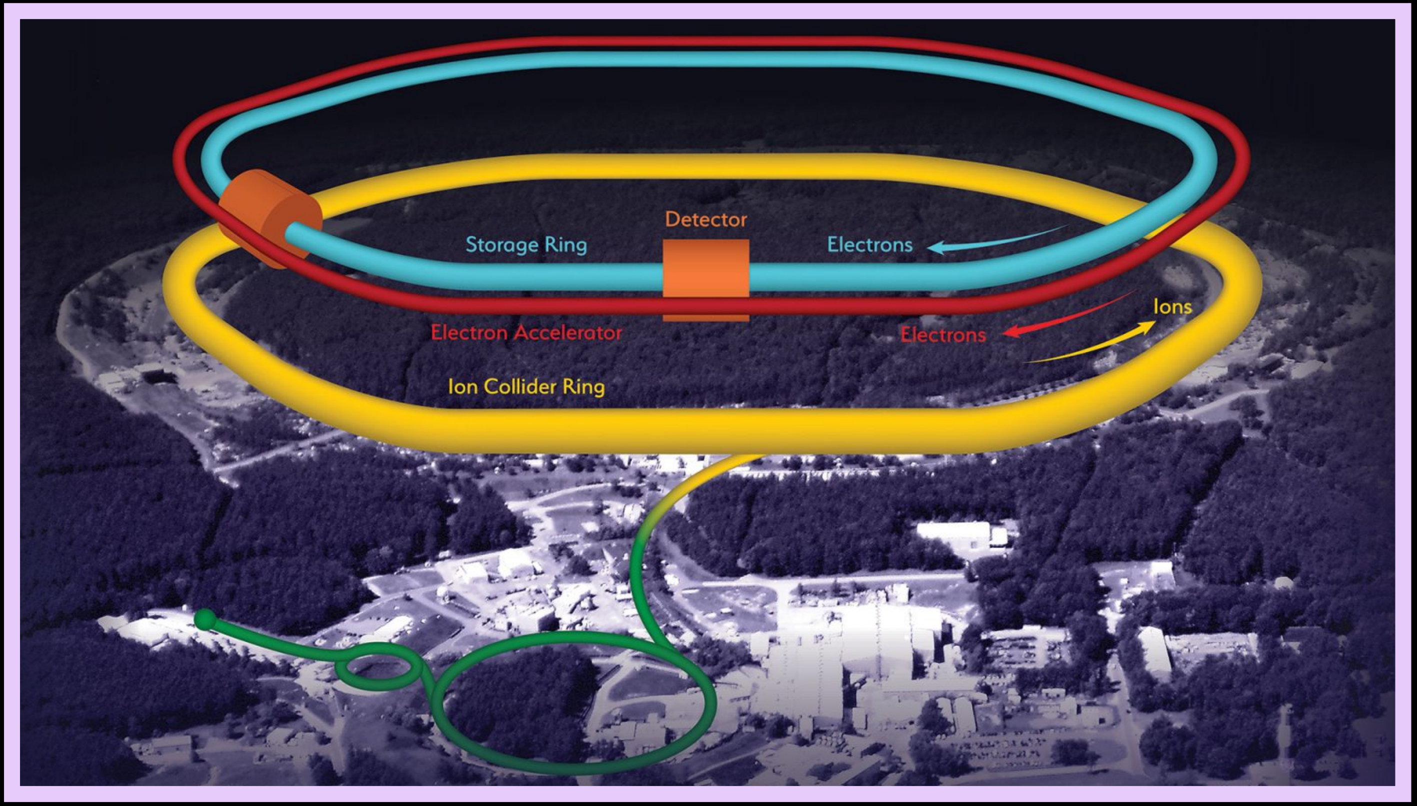
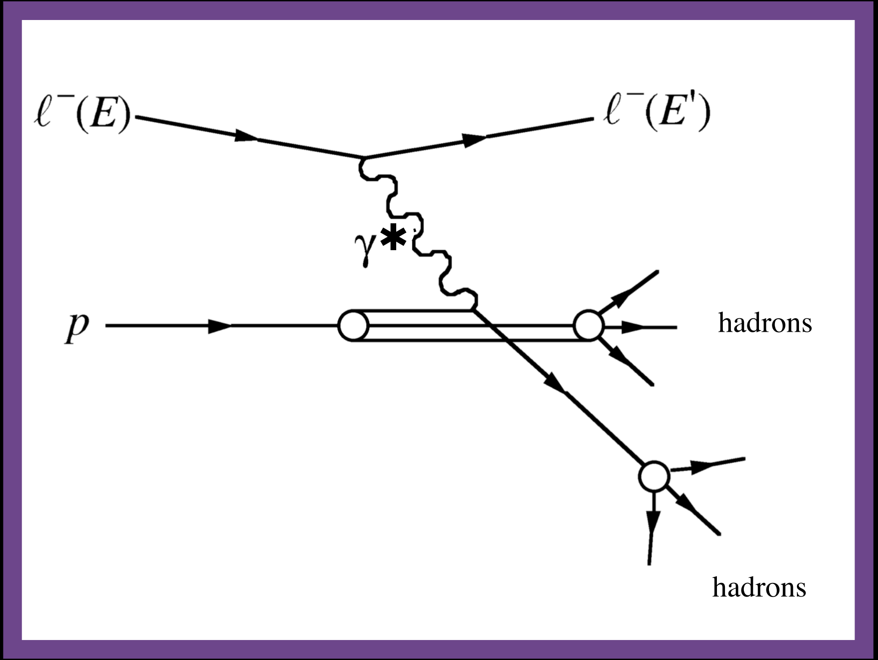


Image credit: Brookhaven National Lab

Bridge from QCD to nuclear: EMC effect

- Deep inelastic scattering (DIS) experiments
- Lepton scatters from target, exchanging virtual photon with 4-momentum q^2 given by: $Q^2 \equiv -q^2 = 2EE'(1-\cos\theta)$
- γ^* strikes quark: The fraction of nucleon momentum carried by the struck quark is known via the Bjorken scaling variable $x_B = \dfrac{Q^2}{2M_p\nu}$ where $\nu = E - E'$, M_p=mass of proton, lepton mass neglected



Adapted from Nuclear & Particle Physics by B.R. Martin, 2003

Differential cross section for DIS:

$$\boxed{\frac{d\sigma}{dxdy}\left(e^-p \to e^-X\right) = \sum_f x\, e_f^2 \left[\, q_f(x) + \bar{q}_{\bar{f}}(x)\right] \cdot \frac{2\pi\alpha^2 s}{Q^4}\left(1 + (1-y)^2\right)}$$

where $y = \dfrac{\nu}{E}$ is the fraction of ℓ^- energy transferred to the target. $F_2(x)$ is the **nucleon structure function**, defined as:

$$\boxed{F_2(x_B) \equiv \sum_f x_B\, e_f^2 \left(\; q_f(x_B) + \; \bar{q}_{\bar{f}}(x_B)\right)}$$

in terms of quark distribution functions $q_f(x)$: probability to find a quark with momentum $x_i \in [x, x+dx]$.

EMC effect: Distortion of nuclear structure functions

$$\text{Plotting ratio of } F_2(x_B) \equiv \sum_f x_B\, e_f^2 \left(q_f(x_B) + \bar{q}_{\bar{f}}(x_B)\right) \text{ vs. } x_B$$

- Predicted $F_2(x_B)$ ratio in complete disagreement with theory
- Why should quark behavior - confined in nucleons at QCD energy scales ~200 MeV - be so affected when nucleons embedded in nuclei, BE $\gtrsim$ 2.2 MeV?
- Mystery has not been solved to this day.

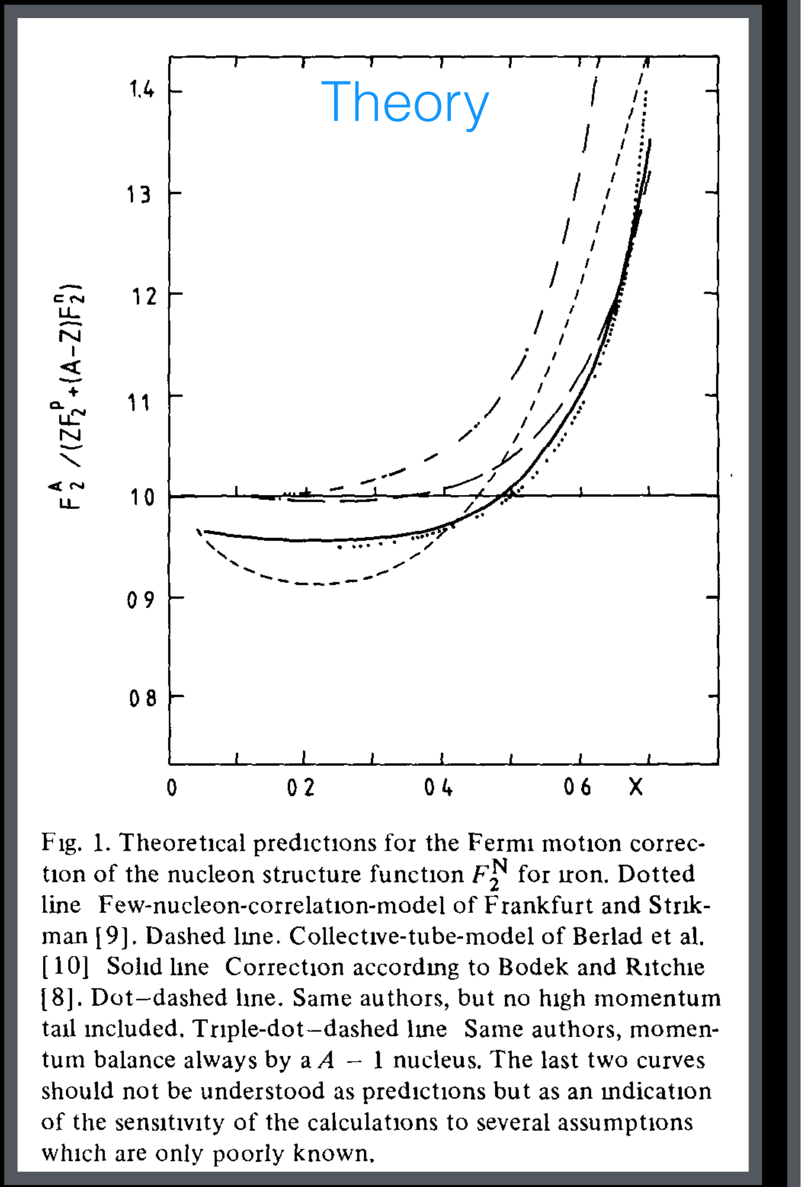


Fig. 1. Theoretical predictions for the Fermi motion correction of the nucleon structure function F_2^N for iron. Dotted line Few-nucleon-correlation-model of Frankfurt and Strikman [9]. Dashed line. Collective-tube-model of Berlad et al. [10] Solid line Correction according to Bodek and Ritchie [8]. Dot–dashed line. Same authors, but no high momentum tail included. Triple-dot–dashed line Same authors, momentum balance always by a $A-1$ nucleus. The last two curves should not be understood as predictions but as an indication of the sensitivity of the calculations to several assumptions which are only poorly known.

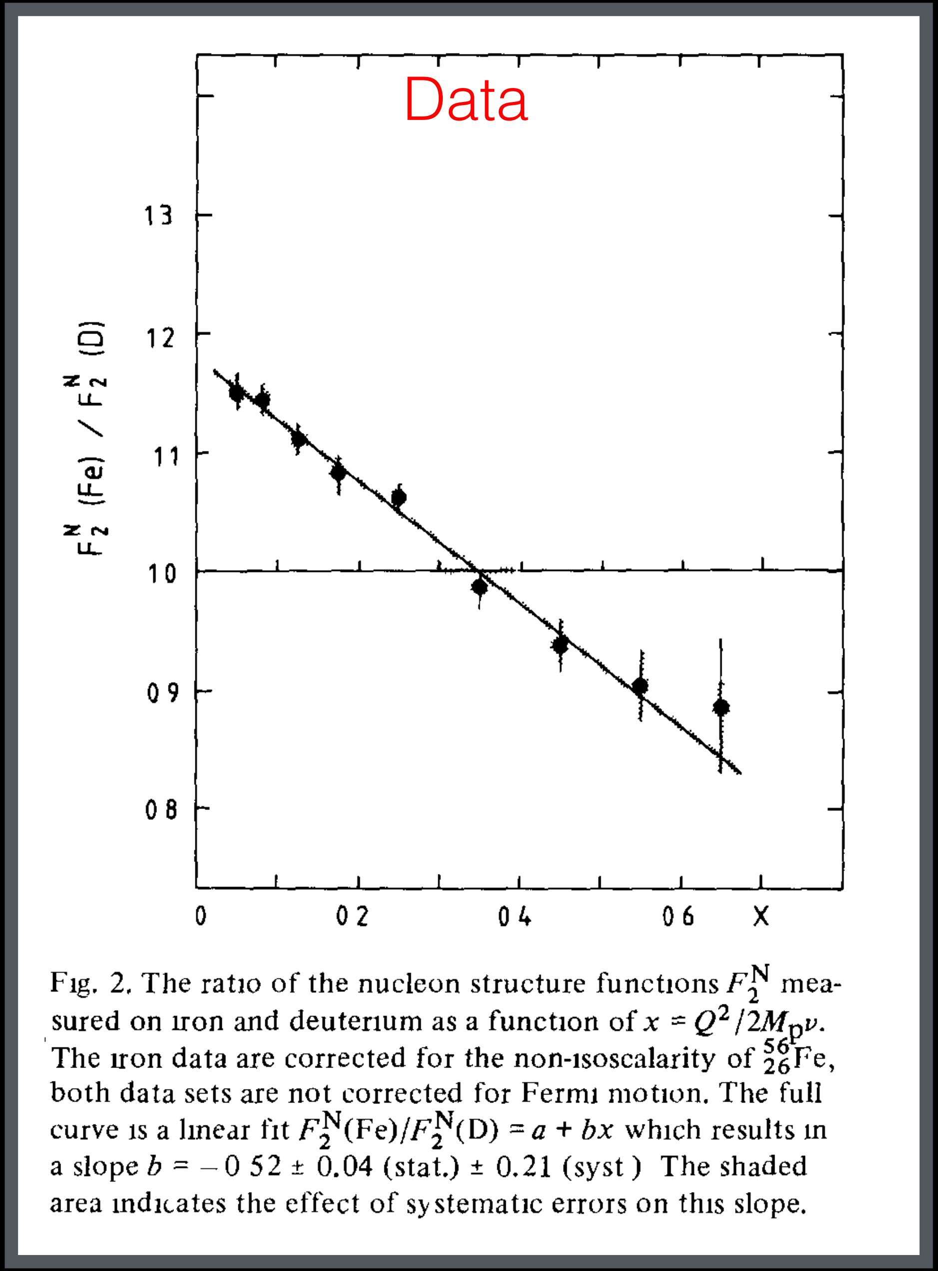


Fig. 2. The ratio of the nucleon structure functions F_2^N measured on iron and deuterium as a function of $x = Q^2/2M_p\nu$. The iron data are corrected for the non-isoscalarity of $^{56}_{26}$Fe, both data sets are not corrected for Fermi motion. The full curve is a linear fit $F_2^N(\mathrm{Fe})/F_2^N(\mathrm{D}) = a + bx$ which results in a slope $b = -0\ 52 \pm 0.04$ (stat.) ± 0.21 (syst) The shaded area indicates the effect of systematic errors on this slope.

"THE RATIO OF THE NUCLEON STRUCTURE FUNCTIONS F_2^N FOR IRON AND DEUTERIUM "
The European Muon Collaboration, J.J. AUBERT et al. 1983

EMC effect experiments & explanations

POSSIBLE EXPLANATIONS

- Mean field effects involving the whole nucleus
- Local effects, *e.g.*, 2-nucleon correlations

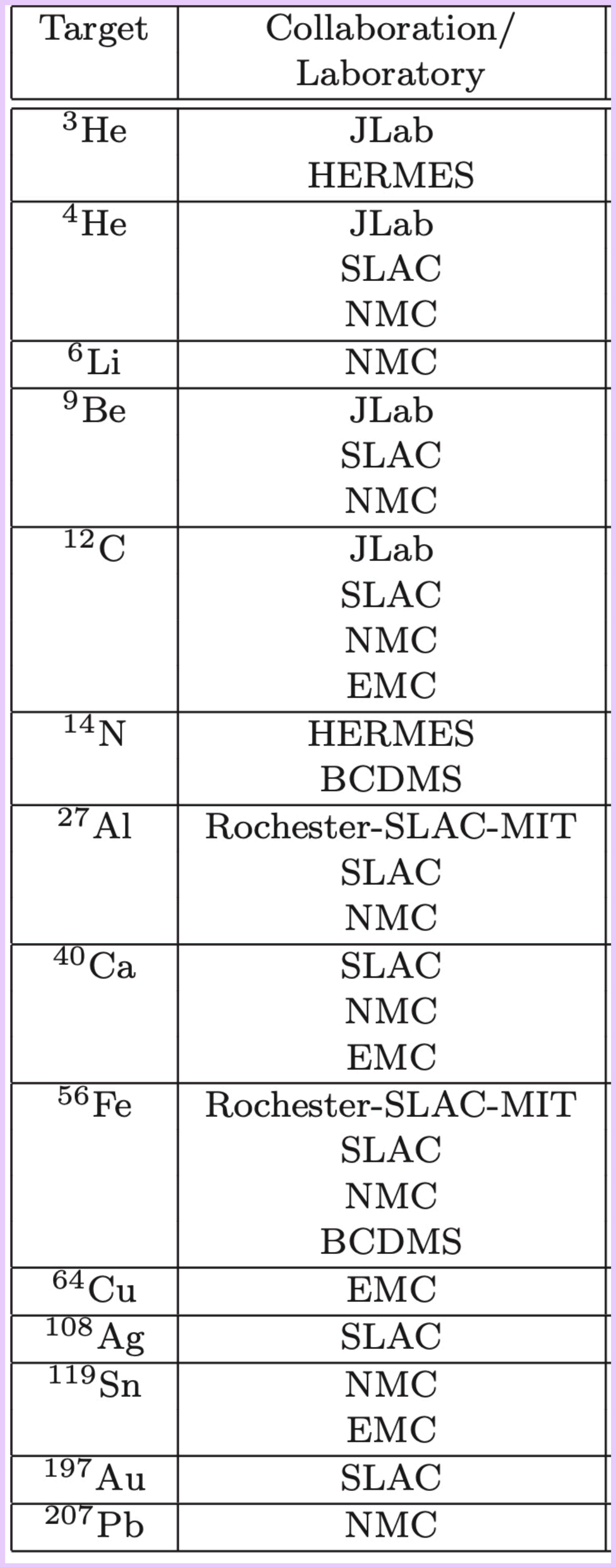
Simple mean field effects inconsistent with the EMC effect in light nuclei - MC of $^9\mathrm{Be} \Longrightarrow$clustering
Seely *et al.*, 2009.

“This one new bit of information has reinvigorated the experimental and theoretical efforts to pin down the underlying cause of the EMC effect.” *Malace et al., 2014*

Short-range N-N correlated pairs (SRC) may cause EMC effect (first suggested in *Ciofi & Liuti 1990, 1991)*.
Neutron-proton pairs later found to dominate SRC (CLAS collaboration, EP & others)

New model: **Diquark formation** proposed to create short-range correlations (SRC), modifying quark behavior in the NN pair

DOZENS OF EXPERIMENTS CONFIRM EMC EFFECT

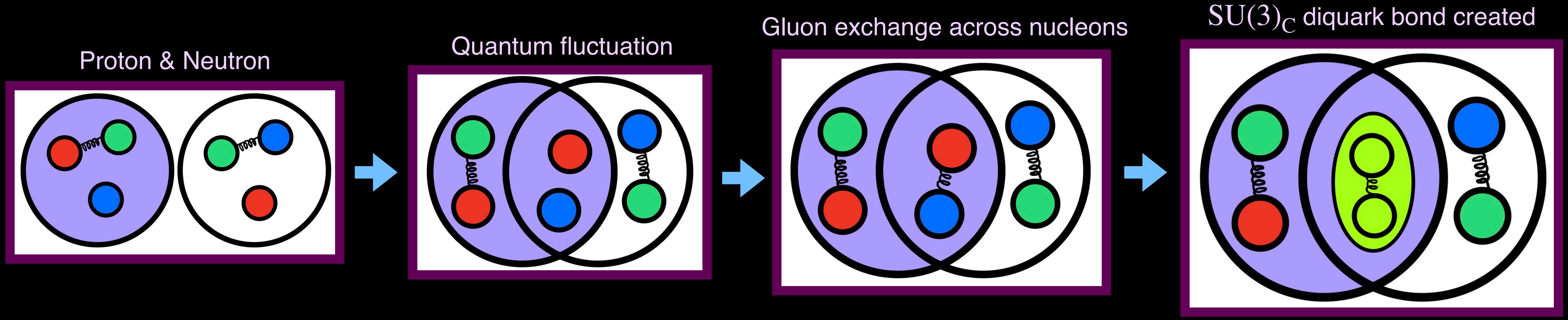


Target	Collaboration/ Laboratory
^{3}He	JLab HERMES
^{4}He	JLab SLAC NMC
^{6}Li	NMC
^{9}Be	JLab SLAC NMC
^{12}C	JLab SLAC NMC EMC
^{14}N	HERMES BCDMS
^{27}Al	Rochester-SLAC-MIT SLAC NMC
^{40}Ca	SLAC NMC EMC
^{56}Fe	Rochester-SLAC-MIT SLAC NMC BCDMS
^{64}Cu	EMC
^{108}Ag	SLAC
^{119}Sn	NMC EMC
^{197}Au	SLAC
^{207}Pb	NMC

Malace, Gaskell, Higinbotham & Cloet, Int.J.Mod.Phys.E 23 (2014)

Overview: Fundamental QCD dynamics in NN pairs

New model: **Diquark formation** proposed to create short-range correlations (SRC), modifying quark behavior in the NN pair



Short-range QCD potentials act on distance scales < 1 fm. Strong NN overlap can bring valence quarks within range.

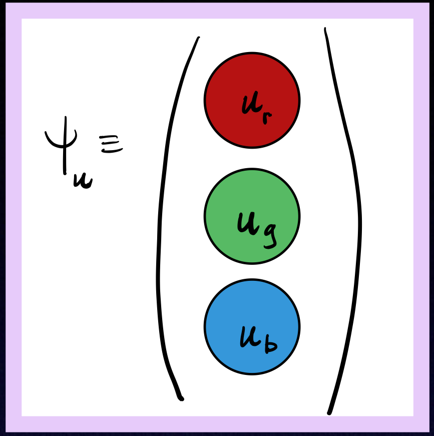
What is a diquark?

- Strong force described by special unitary group $\mathrm{SU}(3)_{\mathrm{C}}$, local symmetry of the strong interaction $\equiv$ QCD

- QCD $\Longrightarrow$ Diquark creation: Quark-quark bond with single gluon exchange **&** group theory transformation into a fundamentally different object:

$$3_C \otimes 3_C \to \bar{3}_C$$

Quark in the fundamental rep of $\mathrm{SU}(3)_{\mathrm{C}}$, the triplet aka 3_{C} :



Diquark wavefunction in the antifundamental rep of $\mathrm{SU}(3)_{\mathrm{C}}$, the antitriplet aka $\bar{3}_{\mathrm{C}}$:

$$\psi_a^{[ud]} = \frac{1}{\sqrt{2}} \epsilon_{abc} \left(u_\uparrow^b d_\downarrow^c - d_\uparrow^b u_\downarrow^c \right)$$

Like quarks and gluons, diquarks carry color charge. They cannot be seen directly due to color confinement. Only 1_C (red+green+blue or red-antired etc.) directly detected.

Therefore there is no direct evidence for diquarks. Work in progress for diquark detection experimental proposals (*e.g.*, diquark jets from DIS increase Λ production)

Strong indirect evidence exists (baryon mass splittings, Regge slopes).

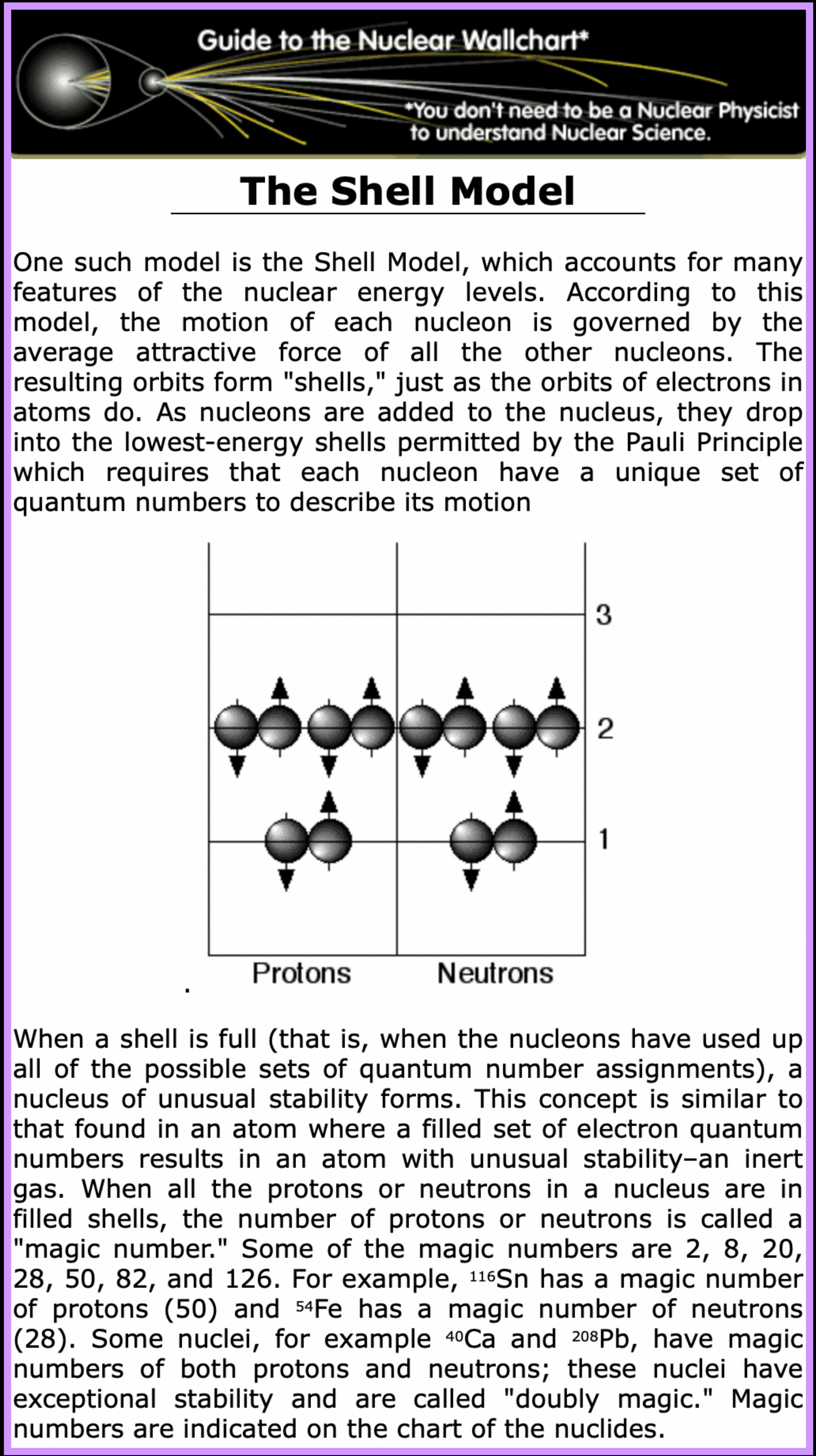
What are diquark-induced short-range correlations (SRC)?

First define SRC: Short-range correlated nucleon-nucleon pairs

- Nuclei consist of protons and neutrons ~80% of which are organized into shells
- Nuclear shell model organizes neutrons and protons into shells obeying the Pauli principle, just like electron shells in atoms

~20% of nucleons are in short-range correlated pairs - not shells

- SRC have very high relative momentum - nearly all nucleons above the Fermi momentum of the nucleus, $k_F \sim 250\ \mathrm{MeV}/c$, are in SRC



Guide to the Nuclear Wallchart*

*You don't need to be a Nuclear Physicist to understand Nuclear Science.

The Shell Model

One such model is the Shell Model, which accounts for many features of the nuclear energy levels. According to this model, the motion of each nucleon is governed by the average attractive force of all the other nucleons. The resulting orbits form "shells," just as the orbits of electrons in atoms do. As nucleons are added to the nucleus, they drop into the lowest-energy shells permitted by the Pauli Principle which requires that each nucleon have a unique set of quantum numbers to describe its motion

When a shell is full (that is, when the nucleons have used up all of the possible sets of quantum number assignments), a nucleus of unusual stability forms. This concept is similar to that found in an atom where a filled set of electron quantum numbers results in an atom with unusual stability–an inert gas. When all the protons or neutrons in a nucleus are in filled shells, the number of protons or neutrons is called a "magic number." Some of the magic numbers are 2, 8, 20, 28, 50, 82, and 126. For example, ^{116}Sn has a magic number of protons (50) and ^{54}Fe has a magic number of neutrons (28). Some nuclei, for example ^{40}Ca and ^{208}Pb, have magic numbers of both protons and neutrons; these nuclei have exceptional stability and are called "doubly magic." Magic numbers are indicated on the chart of the nuclides.

www2.lbl.gov/abc/wallchart/chapters/06/1.html

Diquark-induced SRC

What causes the "short-range" part of short-range NN correlations?

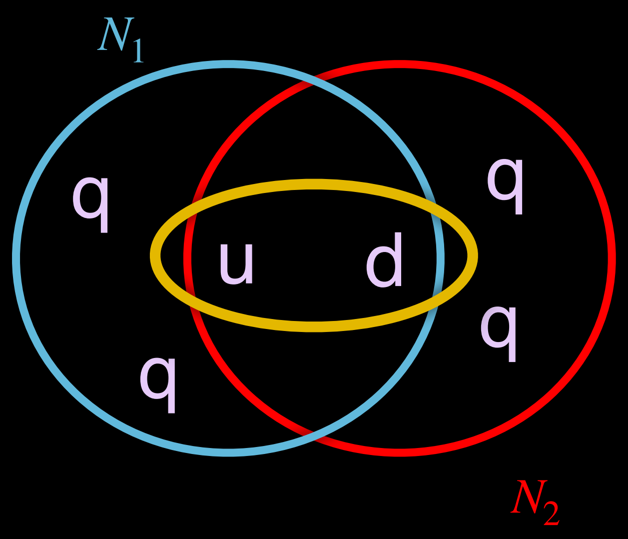
- Quantum fluctuations in separation distance between 2 nucleons
- Quantum fluctuations in relative momentum between 2 nucleons

How short is the range between the NN pair?

- SRC have relative momenta greater than the Fermi momentum, $k_F \sim 250 \text{ MeV/c}$
- Translates to a center-to-center separation distance of $d_{\text{NN}} \sim 0.79 \text{ fm}$
- Radius of proton $r_p \sim 0.84 \text{ fm}$
- Very large wavefunction overlap between SRC nucleons!

What causes the "correlation" in SRC?

- Diquark forms across nucleons
- Valence quarks from different nucleons "fall into" short-range quark-quark potential
- Highly energetically favorable $[ud]$ diquark created



Why spin-0 $[ud]$ diquark formation?

There are 4 options for diquarks created out of valence quarks in the proton and neutron:

- Spin-0, Isospin-0 $[ud]$
- Spin-1, Isospin-1 (ud)
- Spin-1, Isospin-1 (uu)
- Spin-1, Isospin-1 (dd)

The scalar $[ud]$ is lower in mass by nearly 200 MeV.

What about a spin-0, isospin-1 $[ud]'$? Doesn't work due to spin-statistics constraints on the diquark wave function:

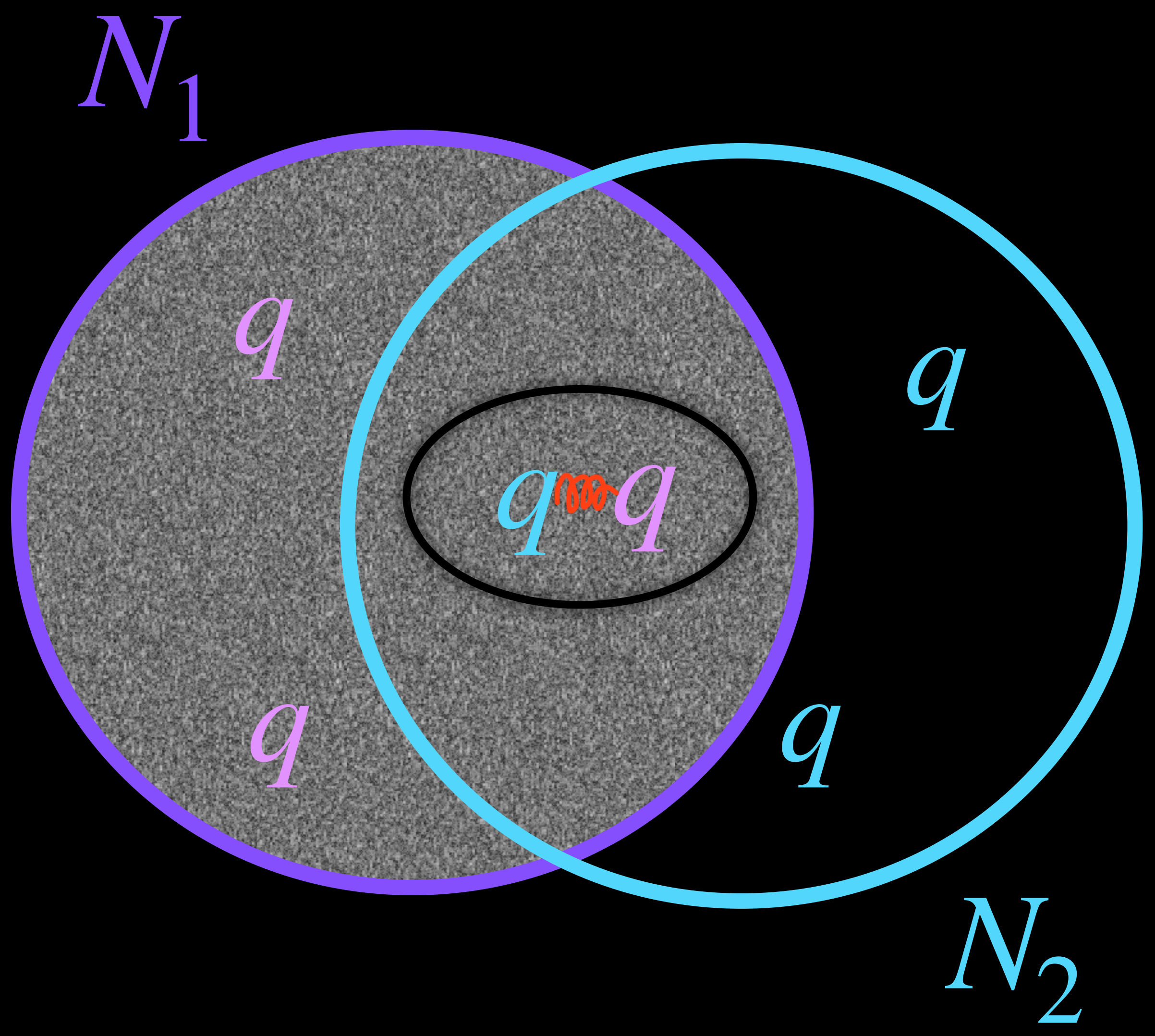
$$\Psi_{[ud]'} \propto \underset{\substack{\uparrow \\ \text{Antisymmetric}}}{\psi_{\text{color}}}\ \psi_{\text{spin}}\ \psi_{\text{iso}}\ \underset{\substack{\uparrow \\ \text{Symmetric, L=0}}}{\psi_{\text{space}}}$$

What are the requirements for a diquark to form in the nuclear environment?

Diquark formation across N-N pairs

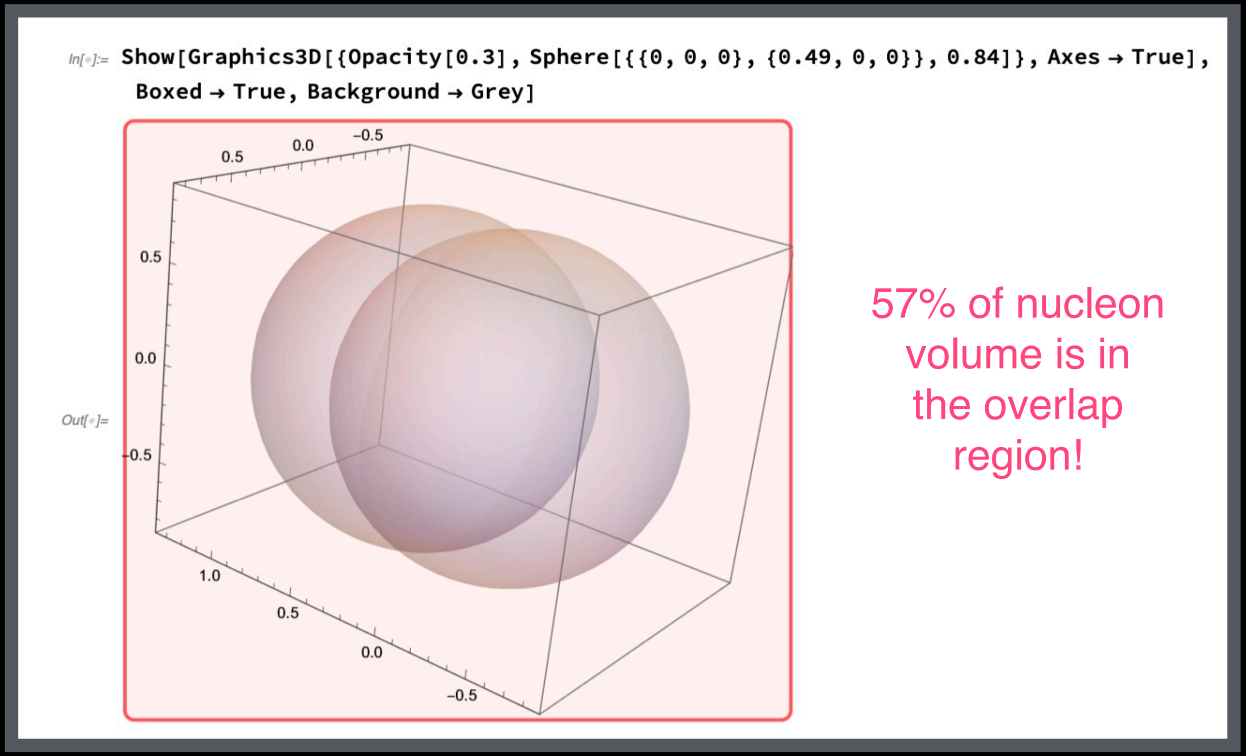
Requirements for diquark induced SRC:

1. Nucleon-Nucleon wavefunctions must STRONGLY overlap
2. Attractive short-range QCD potential between valence quarks
3. Significant binding energy for diquark to form (much stronger than nuclear binding energies - comparable to confinement scale)



1. SRC 3D-overlap for relative momenta 400 MeV/c & 800 MeV/c

- *SRC Plot 1*: According to the ^{12}C measurements from 2021 CLAS, NN tensor force dominates at 400 MeV/c relative momenta. Natural unit conversion gives 0.49 fm = 400 MeV/c.



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In[∘]:= Show[Graphics3D[{Opacity[0.3], Sphere[{{0, 0, 0}, {0.49, 0, 0}}, 0.84]}, Axes → True],
         Boxed → True, Background → Grey]
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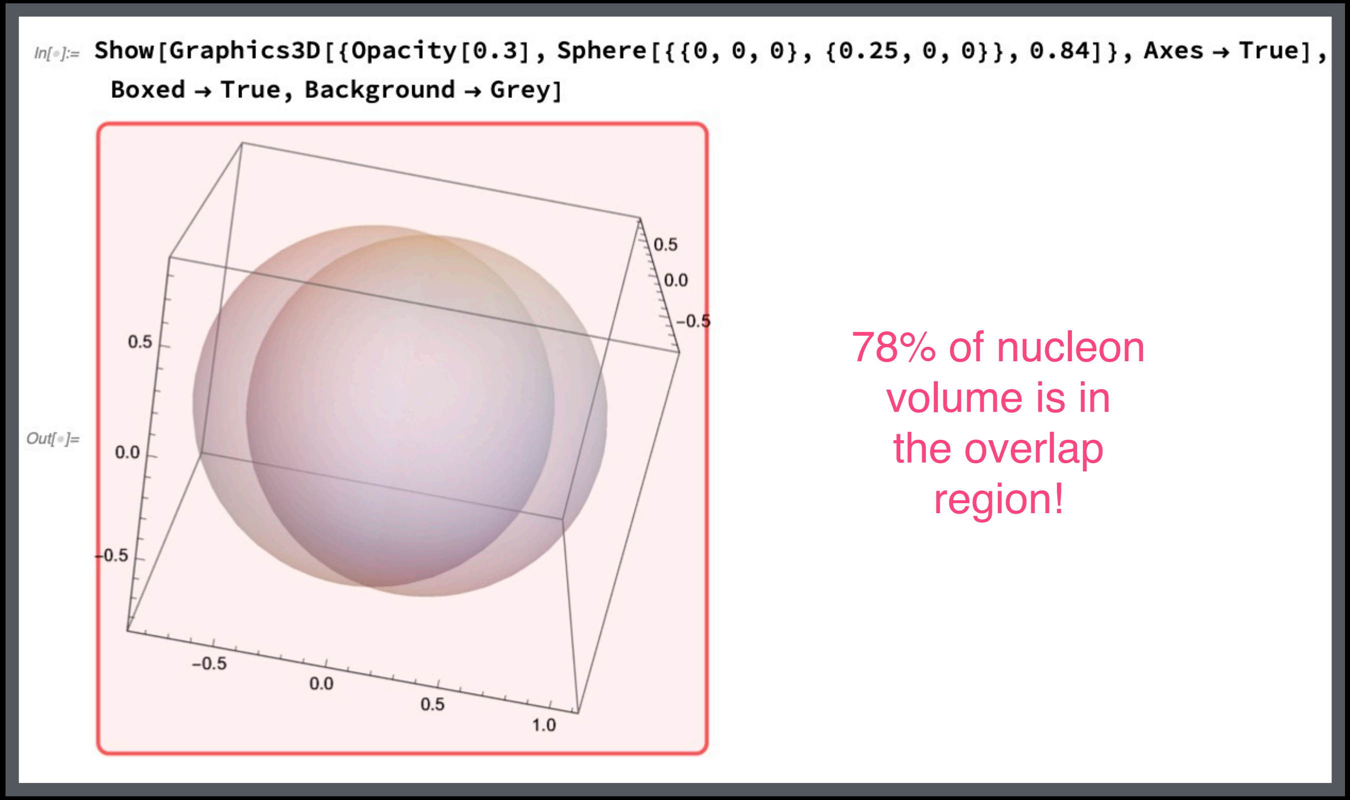
57% of nucleon volume is in the overlap region!

- *SRC Plot 2*: Tensor-scalar transition momenta - according to the ^{12}C measurements from 2021 CLAS, NN scalar force is in effect at 800 MeV/c relative momenta . Natural unit conversion gives 0.25 fm = 800 MeV/c .



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78% of nucleon volume is in the overlap region!

2. Quark-quark potential in QCD: $V(r)$ calculation

- The $\mathrm{SU}(3)_\mathrm{C}$ invariant QCD Lagragian:

$$\mathscr{L}_\mathrm{QCD} = -\frac{1}{4}F^{\mu\nu a}F^{a}_{\mu\nu} + \bar{\Psi}_f\left(i\gamma^\mu D_\mu - m\right)\Psi_f$$

where covariant derivative $D_\mu = \partial_\mu - ig_s A^a_\mu t^a$ acts on quark fields, t^a are the 3x3 traceless Hermitian matrices (e.g., the 8 Gell-Mann matrices), g_s the strong interaction coupling, $\alpha_s \equiv \frac{g_s^2}{4\pi}$

- QCD potential for states in representations R and R' is given by:

$$V(r) = \frac{g_s^2}{4\pi r} t^a_R \otimes t^a_{R'}$$

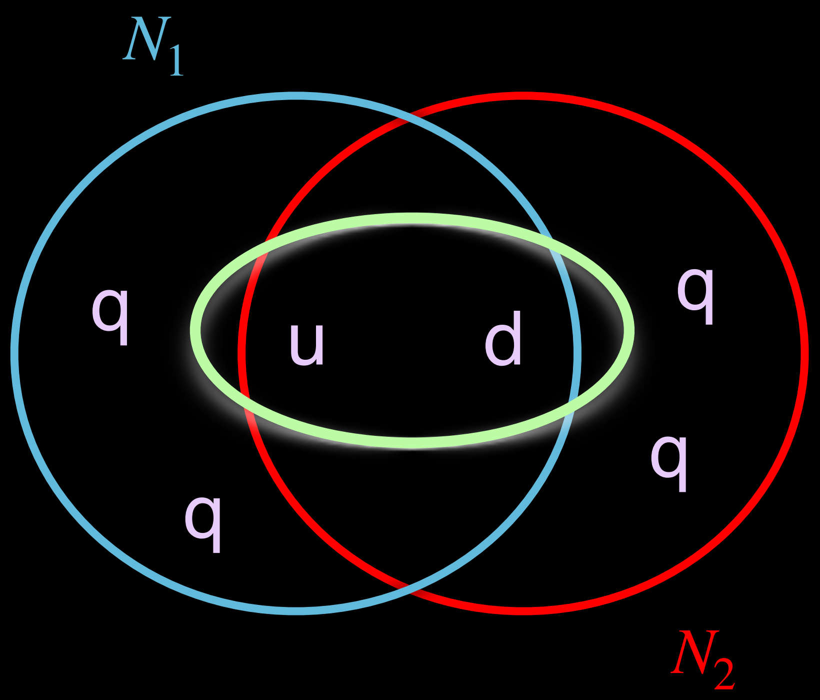
- To compute $V(r)$ for a $3_c \otimes 3_c \to \overline{3}_{c'}$, we use the definition of the scalar $C_2(R)$, $t^a_R\ t^a_R \equiv C_2(R)\ \mathbf{1}$, the *quadratic Casimir operator* (NB: R_f is the final state representation):

$$V(r) = \frac{g_s^2}{4\pi r}\cdot\frac{1}{2}\cdot\left(C_2\left(R_f\right) - C_2(R) - C_2(R')\right)$$

- Diquarks combine 2 fundamental representation quarks into an anti-fundamental, $3_\mathrm{C} \otimes 3_\mathrm{c} \to \overline{3}_\mathrm{C}$:

$$\boxed{V(r) = -\frac{2}{3}\frac{g_s^2}{4\pi r} \Longrightarrow \text{Diquark is bound!}}$$

Diquark induced N-N correlation:



Compare to color singlet attractive potential:

$$\boxed{q\bar{q}:\quad V(r) = -\frac{4}{3}\frac{g_s^2}{4\pi r}}$$

3. Diquark binding energy: Color hyperfine structure

Use Λ^0 baryon to find binding energy of $[ud]$:

$$\text{B.E.}_{[ud]} = m_u^b + m_d^b + m_s^b - M_{\Lambda^0}$$

Spin-spin interaction contributes to hadron mass;
QCD hyperfine interactions:

1. $M_{(\text{baryon})} = \sum_{i=1}^{3} m_i + a' \sum_{i<j} \left(\sigma_i \cdot \sigma_j\right) / m_i m_j$
2. $M_{(\text{meson})} = m_1 + m_2 + a\left(\sigma_1 \cdot \sigma_2\right) / m_1 m_2$

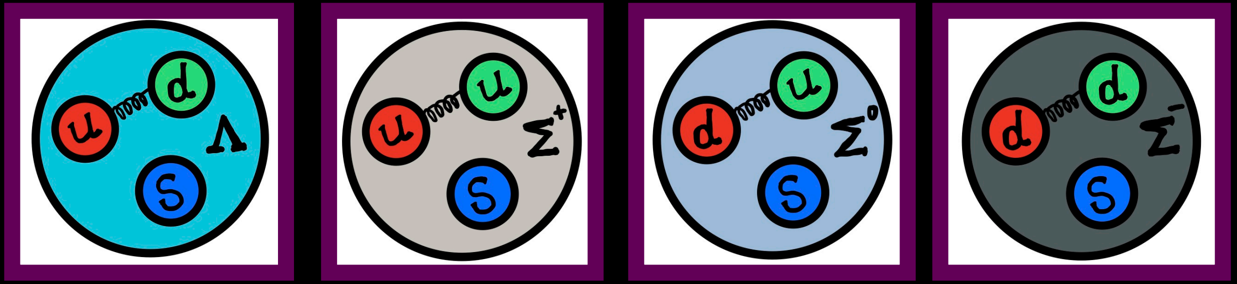
(de Rujula, Georgi & Glashow 1975, Gasiorowicz & Rosner 1981, Karliner & Rosner 2014)

Effective masses of light quarks are found using Eq.1 and fitting to measured baryon masses:

$$m_u^b = m_d^b \equiv m_q^b = 363\text{ MeV}, \;\; \mathrm{m}_\mathrm{s}^\mathrm{b} = 538\text{ MeV}$$

$$\text{B.E.}_{[ud]} = m_u^b + m_d^b + m_s^b - M_\Lambda = 148 \;\pm\; 9\text{ MeV}$$

Relevant diquark-carrying baryons: Λ, Σ^+, Σ^0, Σ^-



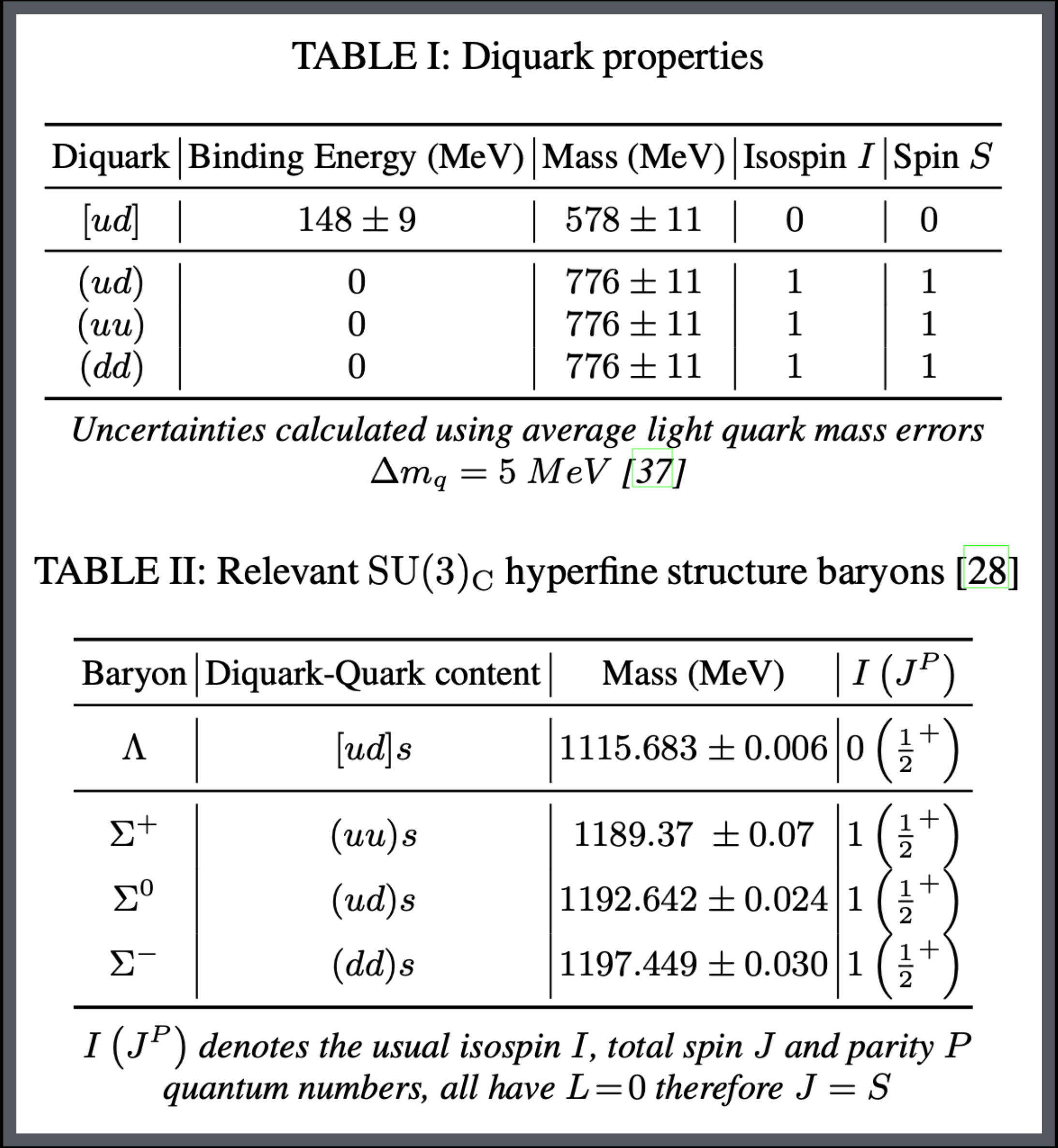


TABLE I: Diquark properties

Diquark	Binding Energy (MeV)	Mass (MeV)	Isospin I	Spin S
$[ud]$	148 ± 9	578 ± 11	0	0
(ud)	0	776 ± 11	1	1
(uu)	0	776 ± 11	1	1
(dd)	0	776 ± 11	1	1

Uncertainties calculated using average light quark mass errors $\Delta m_q = 5\ MeV$ [37]

TABLE II: Relevant $\mathrm{SU}(3)_\mathrm{C}$ hyperfine structure baryons [28]

Baryon	Diquark-Quark content	Mass (MeV)	$I\left(J^P\right)$
Λ	$[ud]s$	1115.683 ± 0.006	$0\left(\frac{1}{2}^+\right)$
Σ^+	$(uu)s$	1189.37 ± 0.07	$1\left(\frac{1}{2}^+\right)$
Σ^0	$(ud)s$	1192.642 ± 0.024	$1\left(\frac{1}{2}^+\right)$
Σ^-	$(dd)s$	1197.449 ± 0.030	$1\left(\frac{1}{2}^+\right)$

$I\left(J^P\right)$ *denotes the usual isospin* I*, total spin* J *and parity* P *quantum numbers, all have* $L=0$ *therefore* $J = S$

"Diquark Induced Short-Range Correlations & the EMC Effect," JRW, Nucl.Phys.A 2023

Diquark formation across N-N pairs

Requirements for diquark induced SRC:

1. Nucleon-Nucleon wavefunctions must STRONGLY overlap
2. Attractive short-range QCD potential between valence quarks
3. Significant binding energy for diquark to form (much stronger than nuclear binding energies - comparable to confinement scale)

✓

What are the implications of NN diquark formation?
Quark flavor dependence of low mass $[ud]$ affects np vs. pp SRC

Diquark formation prediction for A=3 SRC

$$\text{Nucleon wavefunction}: \quad |\mathrm{N}\rangle = \alpha\,|\mathrm{qqq}\rangle + \beta\,|\mathrm{q[qq]}\rangle$$

Scalar [ud] diquark formation for nucleons with 3-valence quark internal structure $|\mathrm{N}\rangle \propto |\mathrm{qqq}\rangle$:

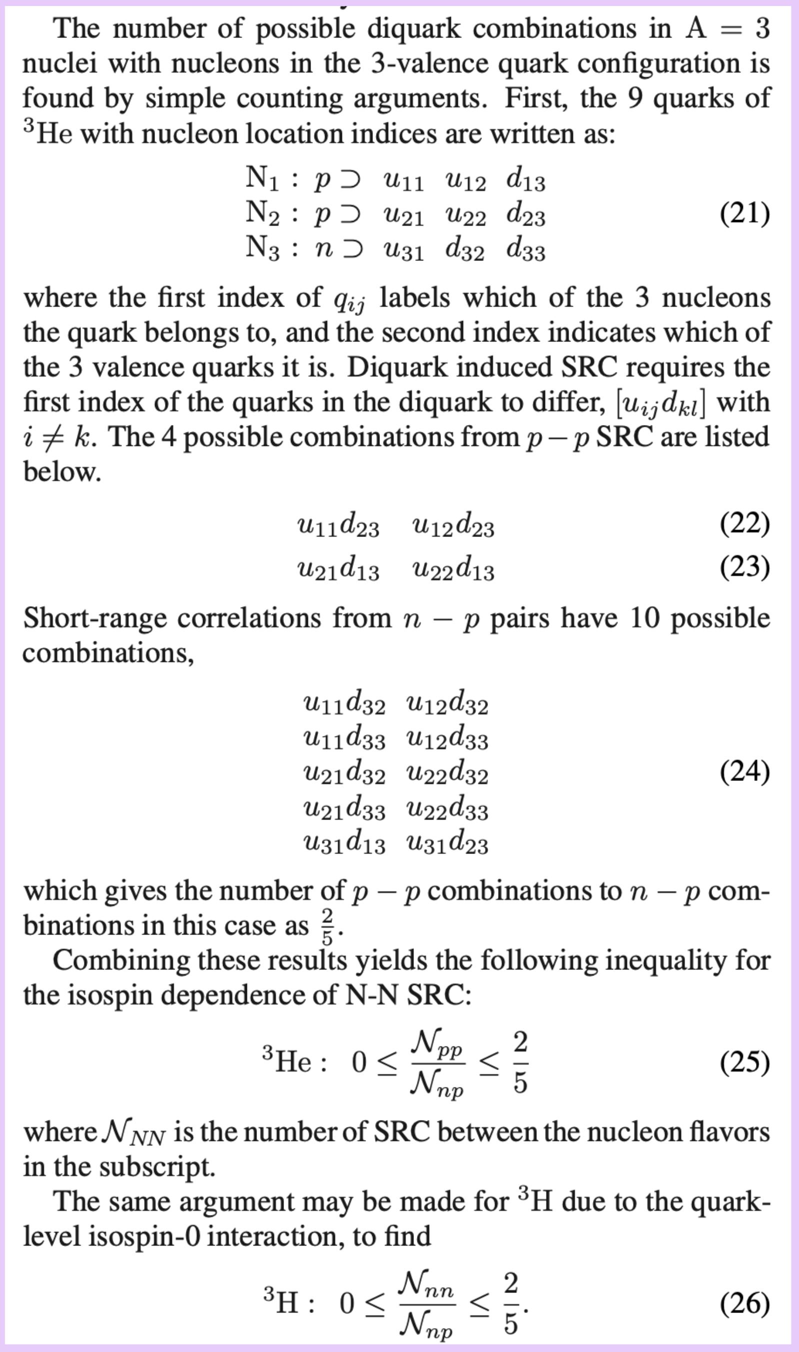
$$^3H: \quad 2n + p \to 4u,\, 5d \implies np \supset [ud] \text{ x10} \quad \implies nn \supset [ud] \text{ x4} \quad \implies 60\,\%\ \mathrm{np},\ 40\,\%\ \mathrm{nn}$$

$$^3He: \quad 2p + n \to 5u,\, 4d \implies np \supset [ud] \text{ x10} \quad \implies pp \supset [ud] \text{ x4} \quad \implies 60\,\%\ \mathrm{np},\ 40\,\%\ \mathrm{pp}$$

Scalar diquark formation for nucleons in quark-diquark internal configuration $|\mathrm{N}\rangle \propto |\mathrm{q[qq]}\rangle$:

$$^3H: \quad u\,[ud] \ + \ d\,[ud] \ + \ d\,[ud] \implies 100\,\%\ \mathrm{np}$$

$$^3He: \quad u\,[ud] \ + \ u\,[ud] \ + \ d\,[ud] \implies 100\,\%\ \mathrm{np}$$



The number of possible diquark combinations in A = 3 nuclei with nucleons in the 3-valence quark configuration is found by simple counting arguments. First, the 9 quarks of ^{3}He with nucleon location indices are written as:

$$\begin{aligned} \mathrm{N}_1 &: p \supset \ u_{11} \ u_{12} \ d_{13} \\ \mathrm{N}_2 &: p \supset \ u_{21} \ u_{22} \ d_{23} \\ \mathrm{N}_3 &: n \supset \ u_{31} \ d_{32} \ d_{33} \end{aligned} \tag{21}$$

where the first index of q_{ij} labels which of the 3 nucleons the quark belongs to, and the second index indicates which of the 3 valence quarks it is. Diquark induced SRC requires the first index of the quarks in the diquark to differ, $[u_{ij}d_{kl}]$ with $i \neq k$. The 4 possible combinations from $p-p$ SRC are listed below.

$$u_{11}d_{23} \quad u_{12}d_{23} \tag{22}$$

$$u_{21}d_{13} \quad u_{22}d_{13} \tag{23}$$

Short-range correlations from $n-p$ pairs have 10 possible combinations,

$$\begin{matrix} u_{11}d_{32} & u_{12}d_{32} \\ u_{11}d_{33} & u_{12}d_{33} \\ u_{21}d_{32} & u_{22}d_{32} \\ u_{21}d_{33} & u_{22}d_{33} \\ u_{31}d_{13} & u_{31}d_{23} \end{matrix} \tag{24}$$

which gives the number of $p-p$ combinations to $n-p$ combinations in this case as $\frac{2}{5}$.

Combining these results yields the following inequality for the isospin dependence of N-N SRC:

$$^3\mathrm{He}: \quad 0 \leq \frac{\mathcal{N}_{pp}}{\mathcal{N}_{np}} \leq \frac{2}{5} \tag{25}$$

where $\mathcal{N}_{NN}$ is the number of SRC between the nucleon flavors in the subscript.

The same argument may be made for ^{3}H due to the quark-level isospin-0 interaction, to find

$$^3\mathrm{H}: \quad 0 \leq \frac{\mathcal{N}_{nn}}{\mathcal{N}_{np}} \leq \frac{2}{5}. \tag{26}$$

JRW, Nuc.Phys.A 2023

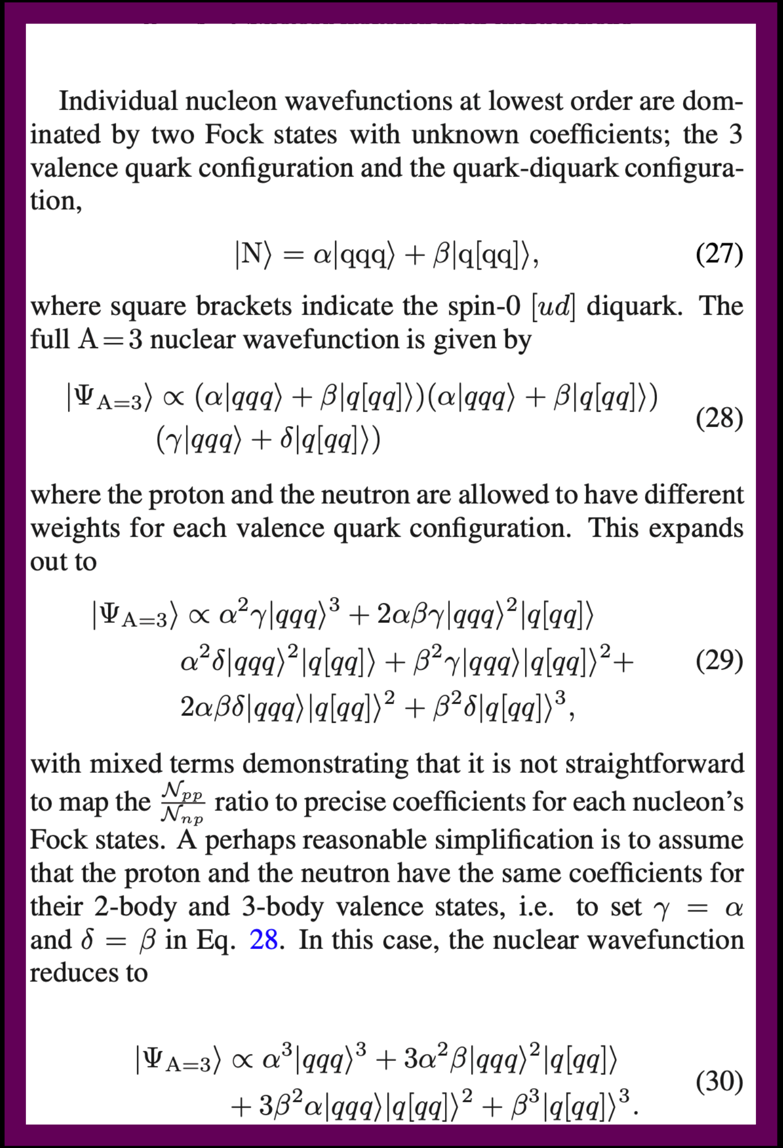
Combine into isospin dependent SRC ratio predictions :

$$^3He: \quad 0 \leq \frac{\mathcal{N}_{pp\ \mathrm{SRC}}}{\mathcal{N}_{np\ \mathrm{SRC}}} \leq \frac{2}{5}, \quad {}^3H: \quad 0 \leq \frac{\mathcal{N}_{nn\ \mathrm{SRC}}}{\mathcal{N}_{np\ \mathrm{SRC}}} \leq \frac{2}{5}, \quad Maximum\ 40\%!$$

Diquark formation induced SRC inequality comparison to data: JLab experiment E12-11-112 A=3 mirror nuclei results

Nature paper from JLab/LBNL: $\frac{\mathcal{N}_{\rm pp}}{\mathcal{N}_{\rm np}} = \frac{1}{4.23} \sim 0.24$

Shujie Li, John Arrington & collaborators, September 2022



Individual nucleon wavefunctions at lowest order are dominated by two Fock states with unknown coefficients; the 3 valence quark configuration and the quark-diquark configuration,

$$|\mathrm{N}\rangle = \alpha|\mathrm{qqq}\rangle + \beta|\mathrm{q[qq]}\rangle, \tag{27}$$

where square brackets indicate the spin-0 $[ud]$ diquark. The full A = 3 nuclear wavefunction is given by

$$\begin{aligned}|\Psi_{\mathrm{A=3}}\rangle \propto\ & (\alpha|qqq\rangle + \beta|q[qq]\rangle)(\alpha|qqq\rangle + \beta|q[qq]\rangle)\\ & (\gamma|qqq\rangle + \delta|q[qq]\rangle)\end{aligned} \tag{28}$$

where the proton and the neutron are allowed to have different weights for each valence quark configuration. This expands out to

$$\begin{aligned}|\Psi_{\mathrm{A=3}}\rangle \propto\ & \alpha^2\gamma|qqq\rangle^3 + 2\alpha\beta\gamma|qqq\rangle^2|q[qq]\rangle\\ & \alpha^2\delta|qqq\rangle^2|q[qq]\rangle + \beta^2\gamma|qqq\rangle|q[qq]\rangle^2 +\\ & 2\alpha\beta\delta|qqq\rangle|q[qq]\rangle^2 + \beta^2\delta|q[qq]\rangle^3,\end{aligned} \tag{29}$$

with mixed terms demonstrating that it is not straightforward to map the $\frac{\mathcal{N}_{pp}}{\mathcal{N}_{np}}$ ratio to precise coefficients for each nucleon's Fock states. A perhaps reasonable simplification is to assume that the proton and the neutron have the same coefficients for their 2-body and 3-body valence states, i.e. to set $\gamma = \alpha$ and $\delta = \beta$ in Eq. 28. In this case, the nuclear wavefunction reduces to

$$\begin{aligned}|\Psi_{\mathrm{A=3}}\rangle \propto\ & \alpha^3|qqq\rangle^3 + 3\alpha^2\beta|qqq\rangle^2|q[qq]\rangle\\ & + 3\beta^2\alpha|qqq\rangle|q[qq]\rangle^2 + \beta^3|q[qq]\rangle^3.\end{aligned} \tag{30}$$

JRW, Nuc.Phys.A 2023

Isospin dependent SRC ratio inequalities from diquark induced SRC :

$$^3He: \quad 0 \le \frac{\mathcal{N}_{pp\ \rm SRC}}{\mathcal{N}_{np\ \rm SRC}} \le 0.4$$

$$^3H: \quad 0 \le \frac{\mathcal{N}_{nn\ \rm SRC}}{\mathcal{N}_{np\ \rm SRC}} \le 0.4$$

$\Longrightarrow$ Nucleon wavefunction : $\alpha|\mathrm{qqq}\rangle + \beta|\mathrm{q[ud]}\rangle$ combination may have approximately equal coefficients, $\alpha \approx \beta$

⬇

2 Caveats: Non-zero probability that existing diquarks may be broken up if overlap sufficient - Nucleon wavefunction written to lowest order - corrections in the form of spin-1 diquarks will exist

Nuclear structure functions $F_2(x_B)$ and PDFs from the diquark model - PDF calculation a collaboration work in progress

Non-relativistic check: Diquark formation modification of F_2 from Fermi motion of quarks in SRC

Recall quark (parton) momentum distribution functions $q(x_B)$:

$$\boxed{F_2(x_B) \equiv \sum_f x_B\, e_f^2 \left(q_f(x_B) + \bar{q}_{\bar{f}}(x_B) \right)}$$

$$\text{Fermi energy :} \quad E_F = \frac{p_F^2}{2m}$$

$$\text{Fermi momentum :} \quad p_F = \sqrt{2mE_F} \propto m^{\frac{1}{2}}$$

- Diquarks lower the mass of the system
- Effective masses of quarks in nucleons: $m_u = m_d = 363 \text{ MeV}$
- $[ud]$ diquark mass: $m_{[ud]} = 578 \text{ MeV}$
- Therefore each quark loses 75 MeV and its Fermi momentum is depleted:

$$m_{\text{final}} = \sqrt{m_q - \frac{BE}{2}} \implies p_{\text{final}} < p_i$$

$$\text{Momentum ratio of quark in diquark to free quark :}$$

$$\frac{p_{\text{final}}}{p_{\text{initial}}} = \sqrt{\frac{m_f}{m_i}} \approx 0.89$$

Hidden color state in $^4\mathrm{He}$ nuclear wavefunction:
12-quark color-singlet HEXADIQUARK proposed in the core of all A>3 nuclei

What are hidden color states?

- Rigorous prediction of $\mathrm{SU}(3)_C$ based QCD
- Color-singlets with quantum numbers that match nuclei
- Nucleus = bag of color singlets
- Hidden-color = 1 color singlet
- Example: Hexadiquark hidden-color state in ${}^4\mathrm{He}$

QCD states within the nuclear wavefunction:

$$|{}^4\mathrm{He}\rangle = C_{nnpp}\left|\boxed{(u[ud])_{1_C}}\boxed{(d[ud])_{1_C}}\boxed{(u[ud])_{1_C}}\boxed{(d[ud])_{1_C}}\right\rangle +$$

$$C_{\mathrm{HdQ}}\left|\boxed{(([ud][ud])_{\bar{\mathbf{6}}_C}([ud][ud])_{\bar{\mathbf{6}}_C}([ud][ud])_{\bar{\mathbf{6}}_C})_{1_C}}\right\rangle + \ldots$$

JRW, S.J.Brodsky, G. de Teramond, I.Schmidt, F.Goldhaber, Nuc. Phys. A 2021

Hidden-color research spans four+ decades:

Brodsky, Ji & Lepage, PRL 1983

Brodsky & Chertok, "The Asymptotic Form-Factors of Hadrons and Nuclei and the Continuity of Particle and Nuclear Dynamics" PRD 1976

M. Harvey, "Effective nuclear forces in the quark model with Delta and hidden color channel coupling" Nuc. Phys. A 1981

G.A.Miller "Pionic and Hidden-Color, Six-Quark Contributions to the Deuteron b1 Structure Function" Phys. Rev. C 2014

Hidden-color states in the nuclear wavefunction

- Hidden-color states in the deuteron: How much do they contribute to ψ_D?
- Probed with superfast quark studies at JLab (*Arrington, Sargsian, et al.*)
- Catching the hidden-color tiger by the tail
- ${}^4\mathrm{He}$ proposed to have larger hidden-color component than ${}^2\mathrm{H}$ but same question: $C_{\mathrm{HdQ}} = ?$

$$|\mathrm{D}\rangle = C_{np}\left|(d[ud])_{1_C}(u[ud])_{1_C}\right\rangle + C_{\mathrm{HC}_1}\left|(ud\ ud\ ud)_{1_C}\right\rangle + C_{\mathrm{HC}_2}\left|(uu\ dd\ ud)_{1_C}\right\rangle + \ldots$$

- *Building hidden-color states requires Fermi statistics upon quark exchange, Bose statistics upon diquark exchange.*
- *Spin-statistics constrains the other components of the wavefunction, often requires nonzero L & higher spin states $\Longrightarrow$ higher mass, less contribution to wavefunction (small coefficient C)*

Glue-balls & Hidden-Glue

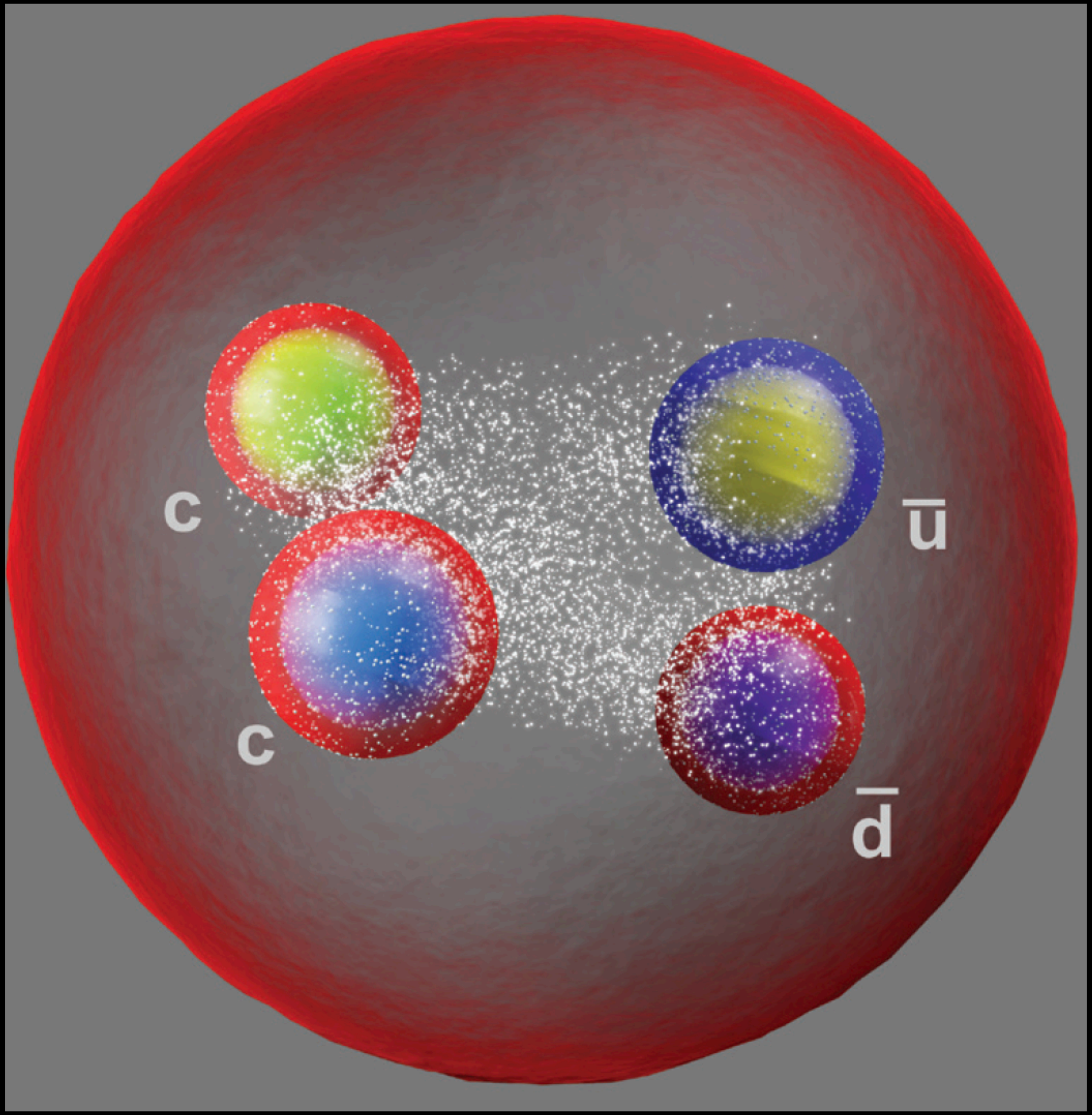
- Glueballs are color singlets
- 2-gluon example:
- Group theory rules of SU(3) predict $8_C \otimes 8_C = 1_C \oplus 8_C$
- Ground state 1_C proposed as the Pomeron
- Prediction of glueball Pomeron: No L odd excitations

$$\eta = C_{q\bar{q}} \, | \frac{(u\bar{u} + d\bar{d} - 2s\bar{s})}{\sqrt{6}} \rangle + C_{gg} \, | g^i_j g^j_i \rangle + \ldots$$

- *Glueball wavefunction build very simple - few quantum numbers (vacuum quantum numbers for ground state J=0)*
- *Components of glueball wavefunctions: Color component (Symmetric) & Spatial component*
- *Spin-statistics constrains spatial component to be symmetric* $\implies L$ *odd forbidden*

Tetraquarks & Pentaquarks

Hidden-color in plain sight!



Combination of multiple color singlets vs. one color singlet - identity of tetra- and penta-quarks an open question in QCD

Compact configuration of diquarks?

$$cc\bar{u}\bar{d}$$

Or molecular mesonic state?

$$c\bar{u} + c\bar{d}$$

Hexadiquark (HdQ) color singlet in A≥4 nuclei

JRW, S.J.Brodsky, G. de Teramond, I.Schmidt, F.Goldhaber, Nuc. Phys. A 2021

- $^4\mathrm{He}$ nucleus to contain a new 12 quark color singlet state
- 6 scalar diquarks strongly bound together to form a color singlet - low mass, S-wave, isosinglet spin singlet states only
- Proposed as cause of $^4\mathrm{He}$ anomalously large binding energy, ~28 MeV

$$\left|\psi_{\mathrm{HdQ}}\right\rangle \propto |[ud][ud][ud][ud][ud][ud]\rangle$$

$$\psi_{[ud]} \equiv \frac{1}{\sqrt{2}}\epsilon_{abc}\left(u^b \uparrow d^c \downarrow - d^b \uparrow u^c \downarrow\right)$$

Quark indices a, b, c = 1, 2, 3 are color indices in the fundamental $\mathrm{SU}(3)_C$ *representation.*

Diquarks are in the anti-fundamental representation: $3_C \otimes 3_c \rightarrow \bar{3}_C$

Hexadiquark (HdQ) color singlet in A≥4 nuclei

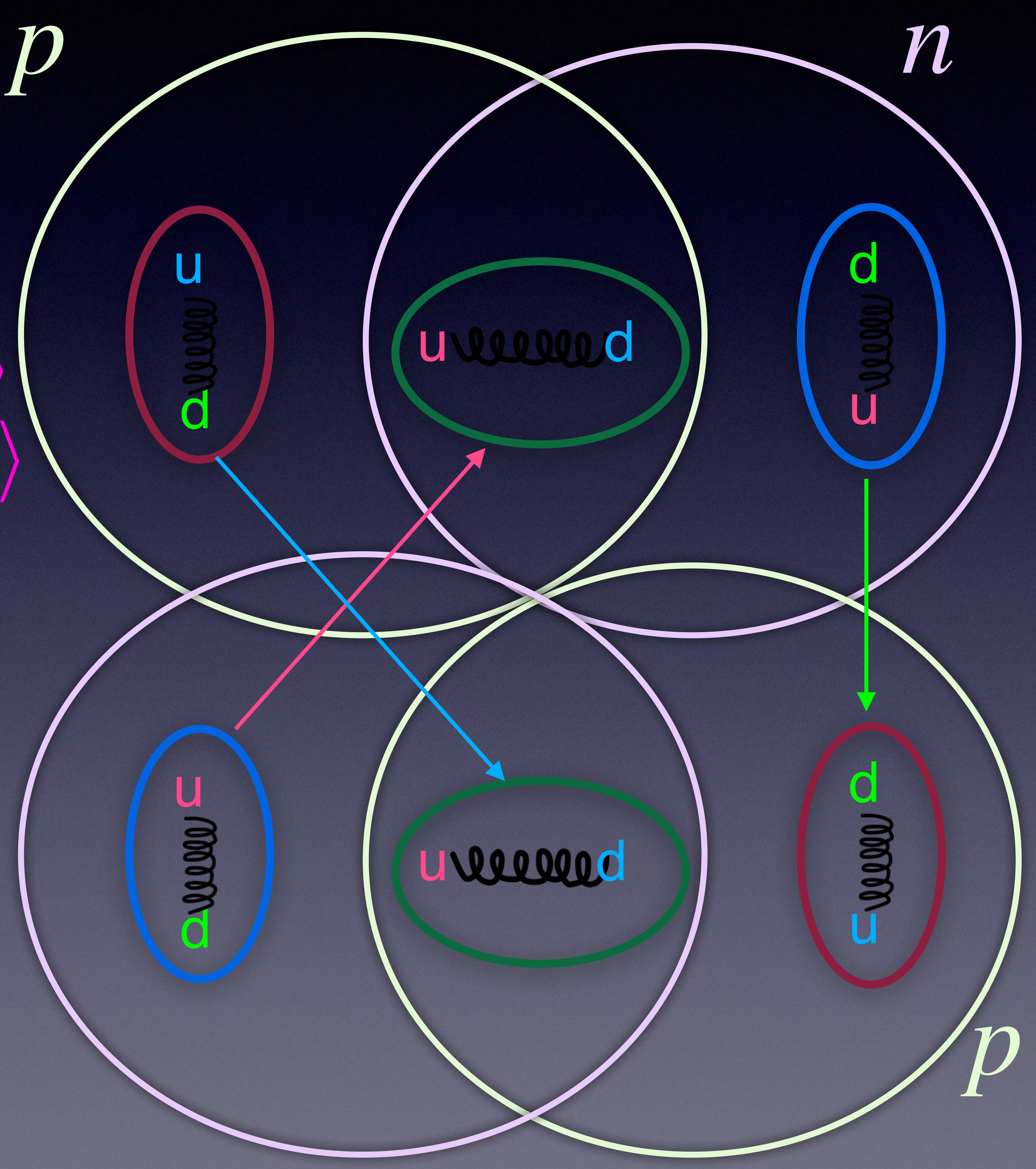
JRW, S.J.Brodsky, G. de Teramond, I.Schmidt, F.Goldhaber, arXiv:2004.14659, Nuc. Phys A 2020

- $^4\mathrm{He}$ nuclear wavefunction a linear combination of nnpp and HdQ with unknown coefficients

$$|\alpha\rangle = C_{pnpn}\left|(u[ud])_{1_C}(d[ud])_{1_C}(u[ud])_{1_C}(d[ud])_{1_C}\right\rangle$$
$$+\ C_{\mathrm{HdQ}}\left|(([ud][ud])_{\bar{\mathbf{6}}_C}([ud][ud])_{\bar{\mathbf{6}}_C}([ud][ud])_{\bar{\mathbf{6}}_C})_{1_C}\right\rangle$$

- n-p dominance of SRC required by the HdQ model

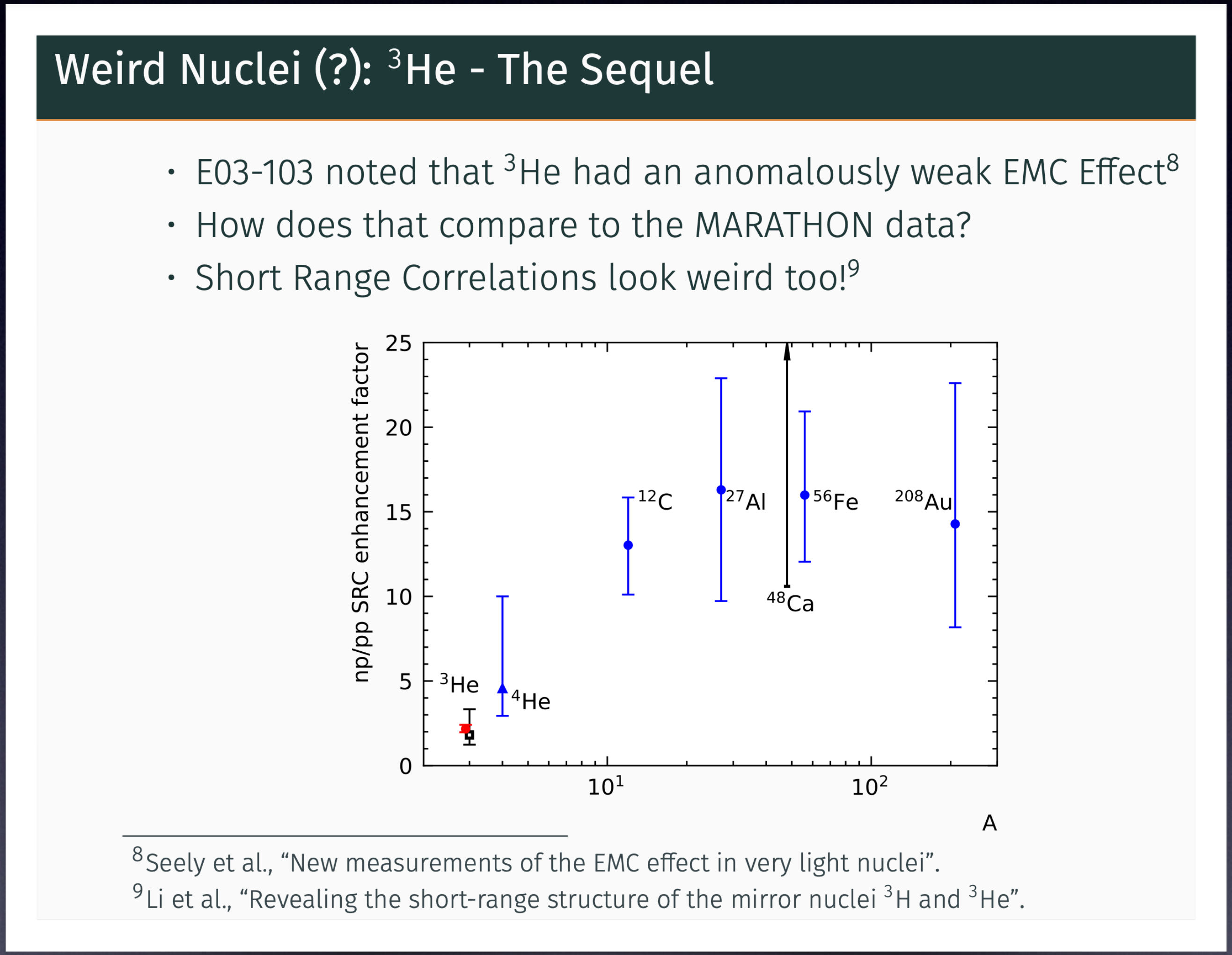
- New hadronic excitations predicted due to 6_C bonds between diquarks



Hidden-color, Diquarks & the EMC Effect: Hexadiquark hidden-color state in A≥4 nuclei and Diquarks across nucleons in A=3?

- Hexadiquark in $^4\mathrm{He}$ nuclear wavefunction proposed as cause of EMC effect in A≥4
- Diquark based SRC proposed as cause of EMC effect in A=3
- $\Longrightarrow$ different behavior for A=3 nuclei
- MARATHON A=3 mirror nuclei experiment: “Light nuclei are weird” (*T.Hague*) - do not follow the SRC/EMC behavior - not enough np, not enough EMC
- *HdQ NB: New hadronic excitations predicted due to 6_C bonds between diquarks - X17 solution, to be measured at JLab, PAC50 approved*

V.Kubarovsky, JRW, S.Brodsky, 2206.14441



Weird Nuclei (?): ^{3}He - The Sequel

- E03-103 noted that ^{3}He had an anomalously weak EMC Effect[8]
- How does that compare to the MARATHON data?
- Short Range Correlations look weird too![9]

---

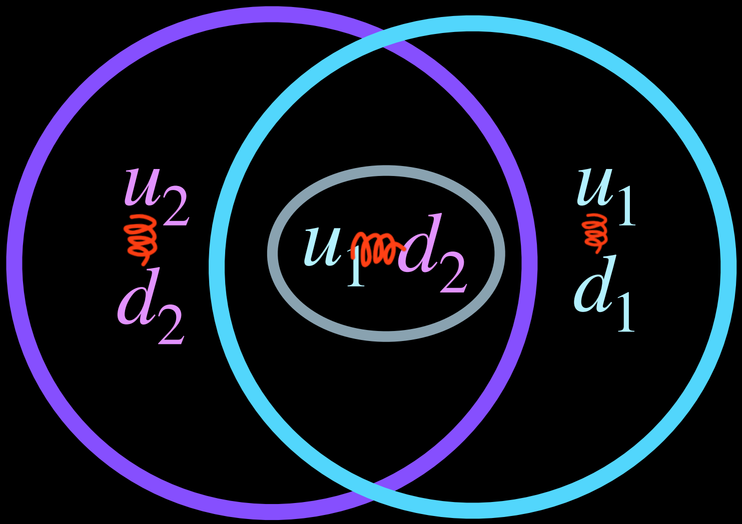
[8] Seely et al., “New measurements of the EMC effect in very light nuclei”.

[9] Li et al., “Revealing the short-range structure of the mirror nuclei ^{3}H and ^{3}He”.

- slide 32 from Tyler Hague, Hadron Ion Tea seminar at Berkeley Lab
https://www.youtube.com/watch?v=nj2mtR3DCzk

Summary: Quantum Chromodynamics in the EIC Era

- Diquark formation proposed to cause short-range correlated nucleon pairs & EMC effect in A=3 (NN)
- Hidden color state within $^4\mathrm{He}$ nucleus proposed as cause of EMC effect in A=4 and larger nuclei (4N)
- Diquarks proposed as cause of SRC and EMC effect in A=3
- Tentative evidence for diquark created SRC from measured $\frac{\mathcal{N}_{pp\ \mathrm{SRC}}}{\mathcal{N}_{np\ \mathrm{SRC}}}$ from MARATHON @JLab
- Future: F_2 calculations, strength of 3N & 4N correlations
- Future Aim: Calculate ~20% SRC from QCD - requires 3D imaging of nucleons - EIC Detectors I & II



np SRC pair

Fin

Jennifer Rittenhouse West
Berkeley Lab & EIC Center @JLab
Intersection of Nuclear Structure & High-energy Nuclear Collisions @INT
23 February 2023

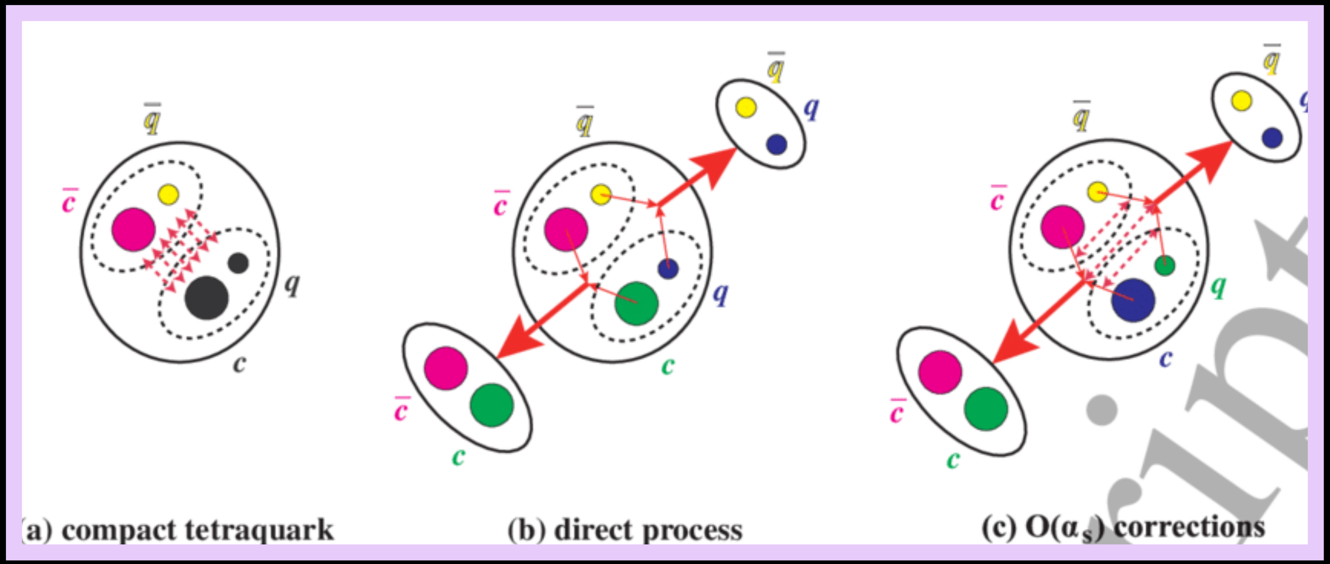
back up slides

EIC Physics: Testing Predictions of QCD

Rigorous predictions of the $\mathrm{SU(3)_C}$ basis of QCD
- 6 examples:

1. Diquarks (quark-quark bound state)
2. Tetraquarks (diquark-diquark bound state)
3. Hidden-color states in nuclear wavefunctions
4. Glueballs (color-singlet combinations of gluons)
5. “Hidden-glue” states in mesonic wavefunctions
6. Color transparency (collisions that cause hadrons to become point-like and therefore color neutral, exiting the nucleus as if it was transparent)

Double-diquark bound state: Tetraquarks



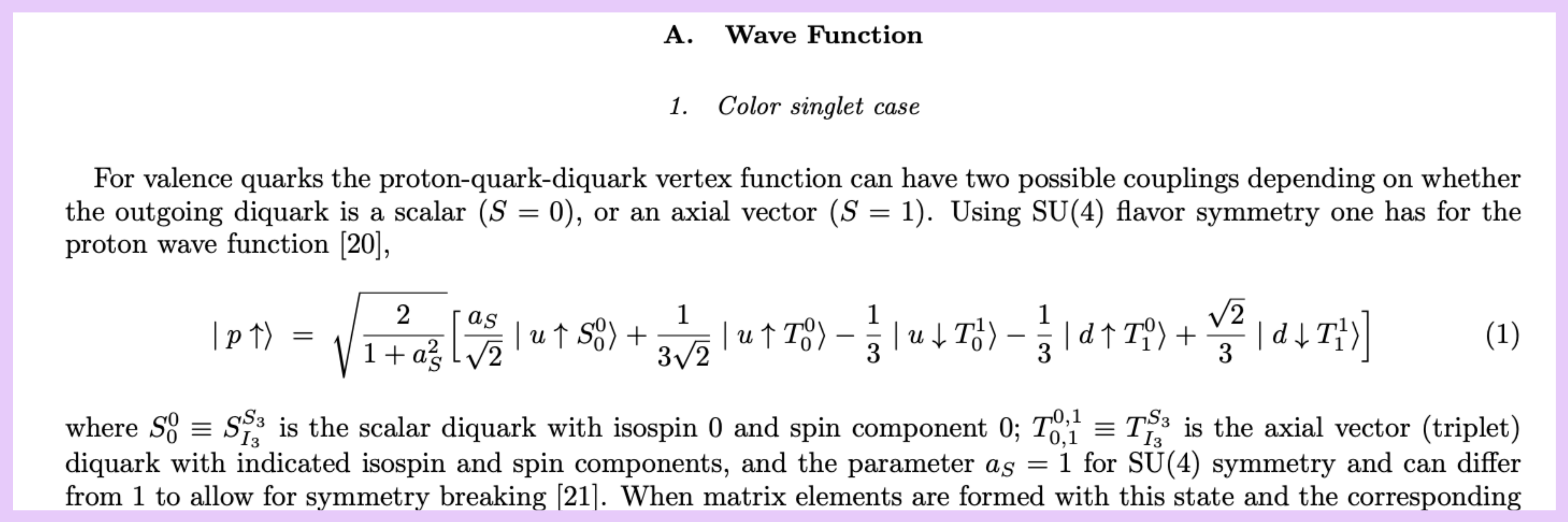
a) compact tetraquark (b) direct process (c) $O(\alpha_s)$ corrections

Image credit: Hua-Xing Chen (陈华星) 2020, “Decay Properties of the Zc(3900) through the Fierz rearrangement”

Proton wavefunction - tensor, scalar pieces

Mapping between nuclear and fundamental QCD
language necessary - work in progress with S.Liuti

From our draft:



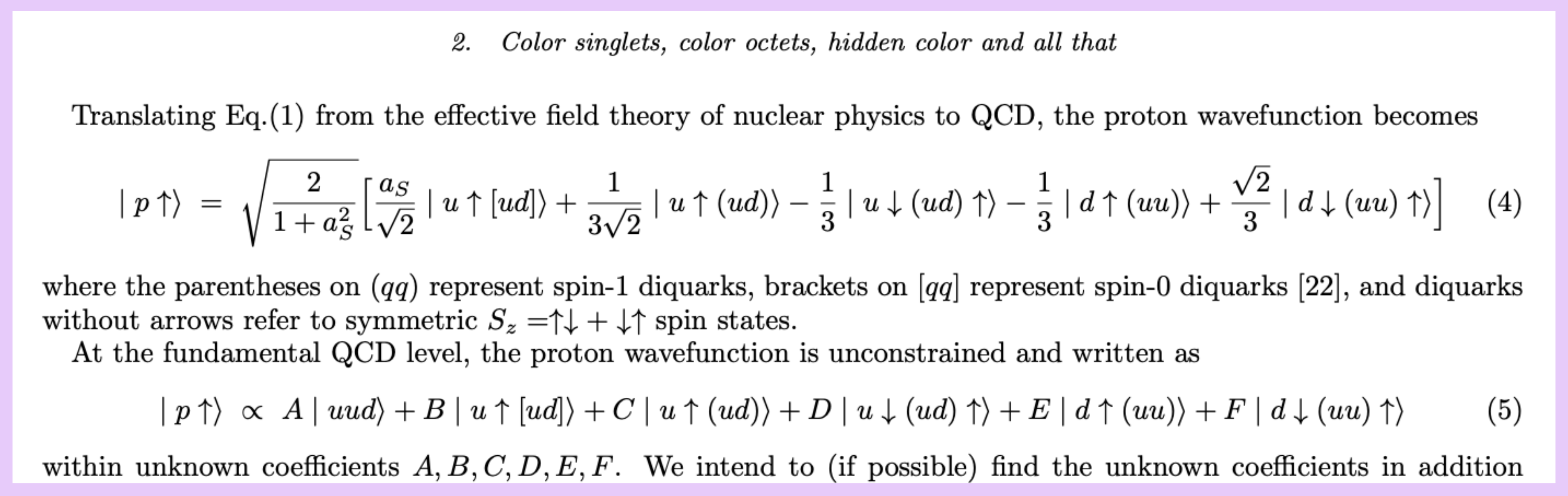
A. Wave Function

1. *Color singlet case*

For valence quarks the proton-quark-diquark vertex function can have two possible couplings depending on whether the outgoing diquark is a scalar ($S = 0$), or an axial vector ($S = 1$). Using SU(4) flavor symmetry one has for the proton wave function [20],

$$|\,p\uparrow\rangle \;=\; \sqrt{\frac{2}{1+a_S^2}}\Big[\frac{a_S}{\sqrt{2}}\,|\,u\uparrow S_0^0\rangle + \frac{1}{3\sqrt{2}}\,|\,u\uparrow T_0^0\rangle - \frac{1}{3}\,|\,u\downarrow T_0^1\rangle - \frac{1}{3}\,|\,d\uparrow T_1^0\rangle + \frac{\sqrt{2}}{3}\,|\,d\downarrow T_1^1\rangle\Big] \tag{1}$$

where $S_0^0 \equiv S_{I_3}^{S_3}$ is the scalar diquark with isospin 0 and spin component 0; $T_{0,1}^{0,1} \equiv T_{I_3}^{S_3}$ is the axial vector (triplet) diquark with indicated isospin and spin components, and the parameter $a_S = 1$ for SU(4) symmetry and can differ from 1 to allow for symmetry breaking [21]. When matrix elements are formed with this state and the corresponding



2. *Color singlets, color octets, hidden color and all that*

Translating Eq.(1) from the effective field theory of nuclear physics to QCD, the proton wavefunction becomes

$$|\,p\uparrow\rangle \;=\; \sqrt{\frac{2}{1+a_S^2}}\Big[\frac{a_S}{\sqrt{2}}\,|\,u\uparrow[ud]\rangle + \frac{1}{3\sqrt{2}}\,|\,u\uparrow(ud)\rangle - \frac{1}{3}\,|\,u\downarrow(ud)\uparrow\rangle - \frac{1}{3}\,|\,d\uparrow(uu)\rangle + \frac{\sqrt{2}}{3}\,|\,d\downarrow(uu)\uparrow\rangle\Big] \tag{4}$$

where the parentheses on (qq) represent spin-1 diquarks, brackets on $[qq]$ represent spin-0 diquarks [22], and diquarks without arrows refer to symmetric $S_z = \uparrow\downarrow + \downarrow\uparrow$ spin states.

At the fundamental QCD level, the proton wavefunction is unconstrained and written as

$$|\,p\uparrow\rangle \;\propto\; A\,|\,uud\rangle + B\,|\,u\uparrow[ud]\rangle + C\,|\,u\uparrow(ud)\rangle + D\,|\,u\downarrow(ud)\uparrow\rangle + E\,|\,d\uparrow(uu)\rangle + F\,|\,d\downarrow(uu)\uparrow\rangle \tag{5}$$

within unknown coefficients A, B, C, D, E, F. We intend to (if possible) find the unknown coefficients in addition

QCD in nuclei: SU(3) ingredients

- Fundamental representation: Triplet, aka 3_C, the quark
- Anti-fundamental representation: Anti-triplet, aka $\bar{3}_C$, the antiquark
- Adjoint representation: Octet, aka 8_C, the gluon
- Combination rules:
- $3 \otimes \bar{3} \rightarrow 1_C$
- $8_C \otimes 8_C = 8_c \oplus 1_C$

SU(3) is the mathematical basis of QCD. Rules predict physical particles:

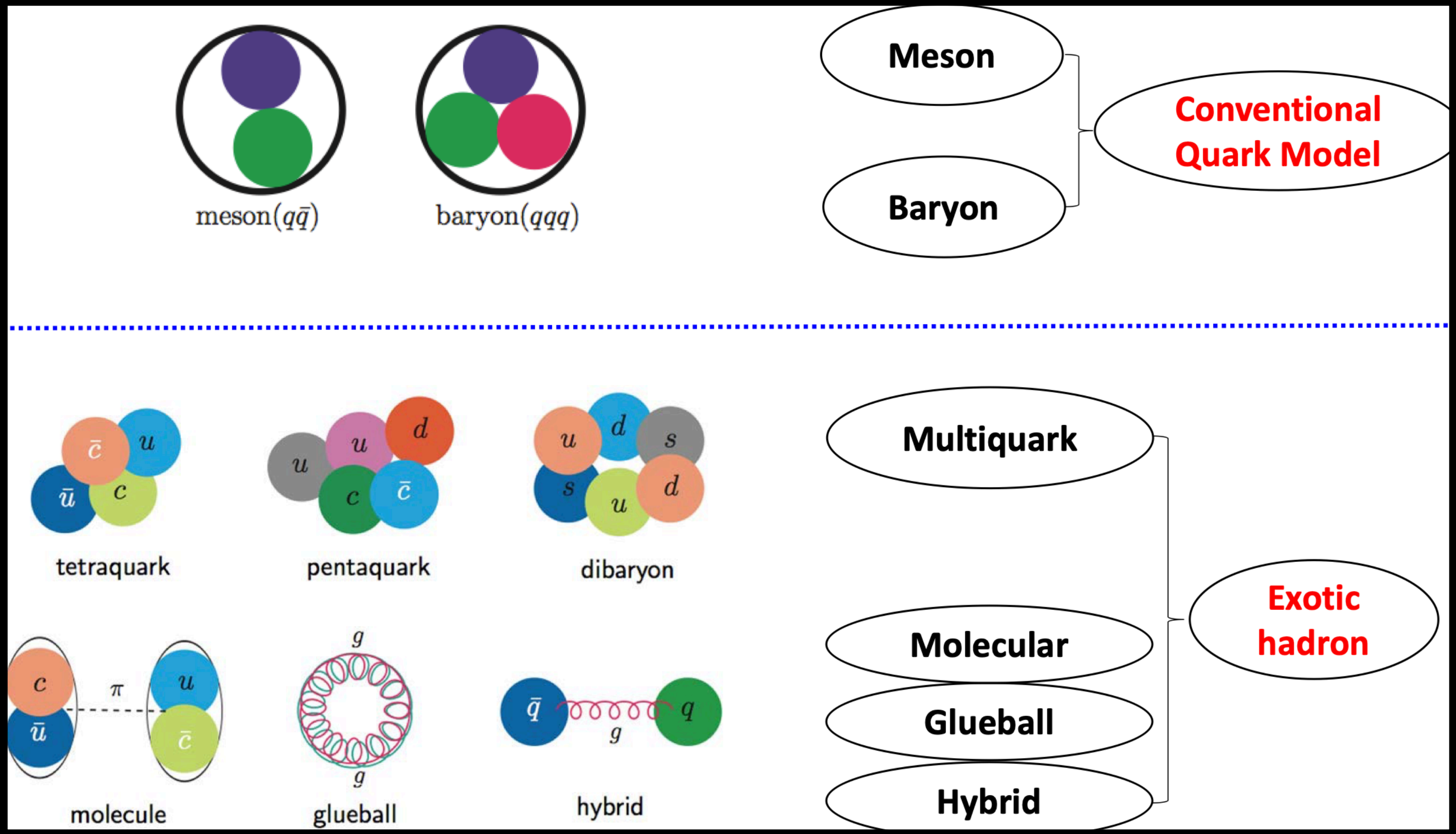


Image credit: Hua-Xing Chen