# Overview:
# Numerical Methods for Neutrino Quantum Kinetics

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THE UNIVERSITY OF TENNESSEE KNOXVILLE

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries
PHYSICS FRONTIER CENTER

NSF

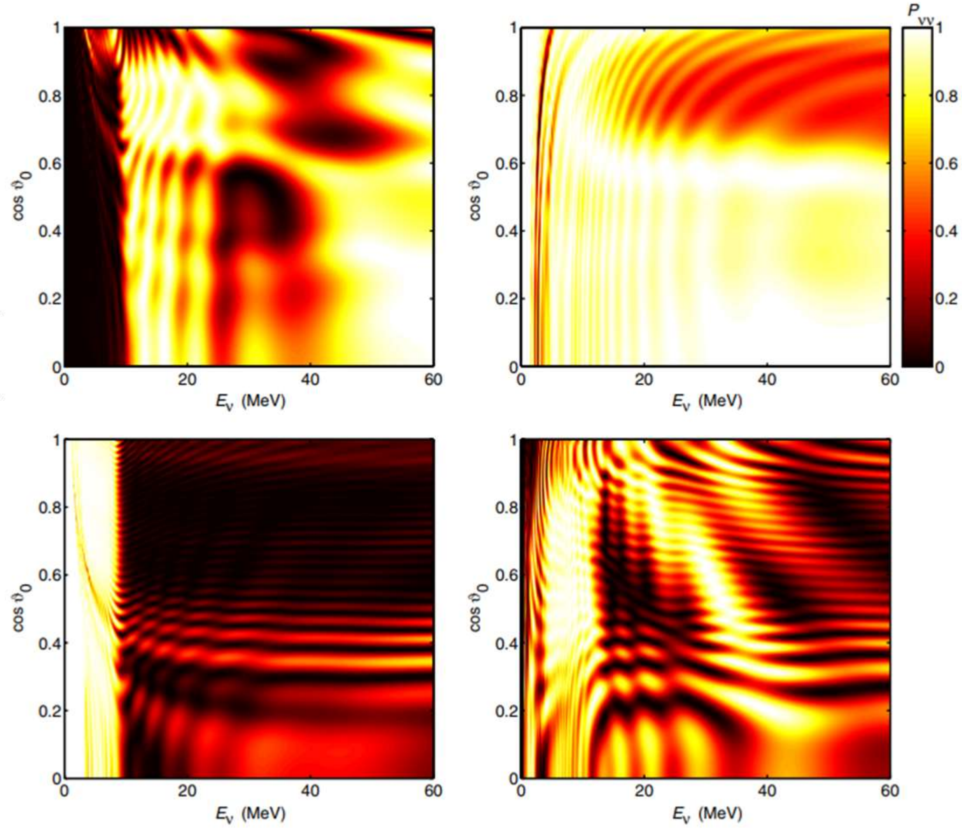
# The Problem

- Neutrino transport is the dominant cost of state-of-the-art simulations of core-collapse supernovae and neutron star mergers

- Neutrino flavor transformation modifies amount of heating, amount of mass ejection, and composition of ejecta

- Neutrino flavor transformation occurs on smaller length/time scales than transport

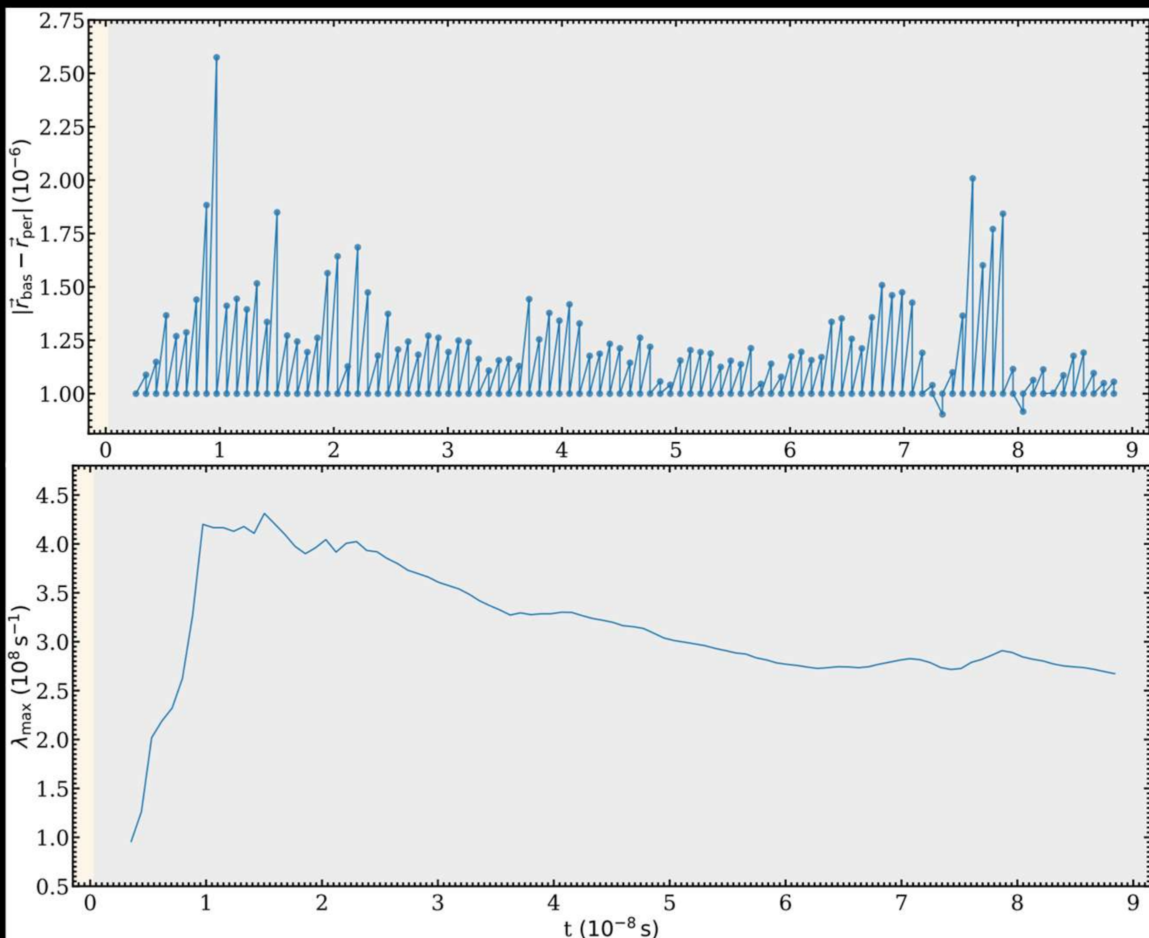
# The results are sensitive to resolution




Duan et al. (2006)

- High-resolution 3D NSM simulations: **12.5 meters**
  Kiuchi et al (2023)
- High-resolution 2D flavor transformation: **3 m**
  Nagakura (2023)
- Estimated required resolution: **0.0003 m**

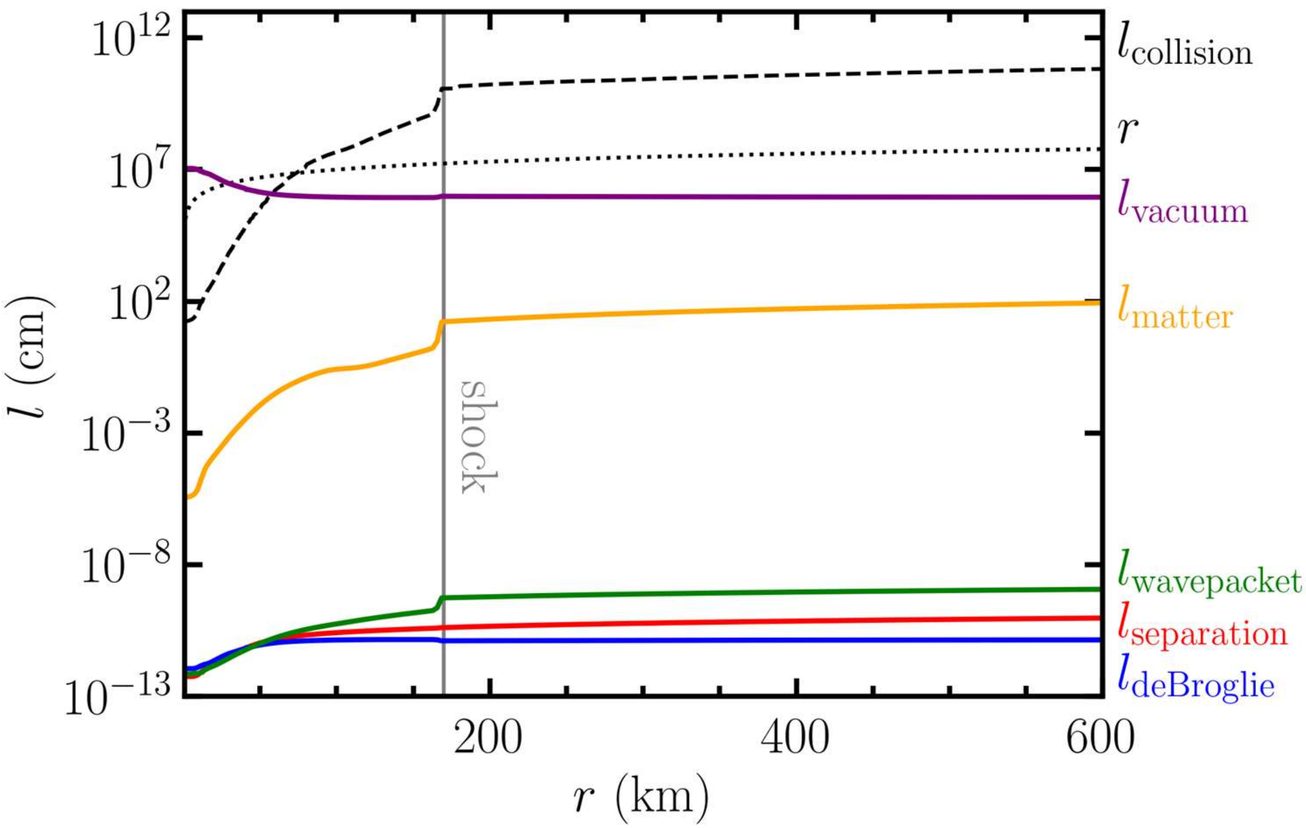
# One does not simply resolve the FFI.

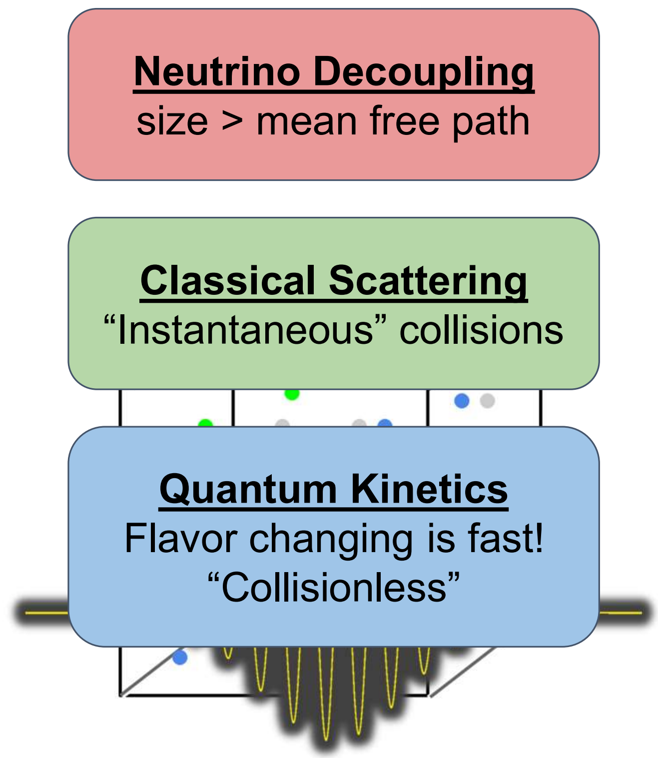




Erick Urquilla Orellana
(Licenciatura student @ U. El Salvador)

# How hard could it be?




(1e7)^4 = 1e28

# Theory of Neutrino Quantum Kinetics

$$\frac{\partial f_{ab}}{\partial t} + c\mathbf{\Omega} \cdot \nabla f_{ab} = \mathcal{C}_{ab} - \frac{i}{\hbar}[\mathcal{H}, f]_{ab}$$

Transport — Collision — Flavor

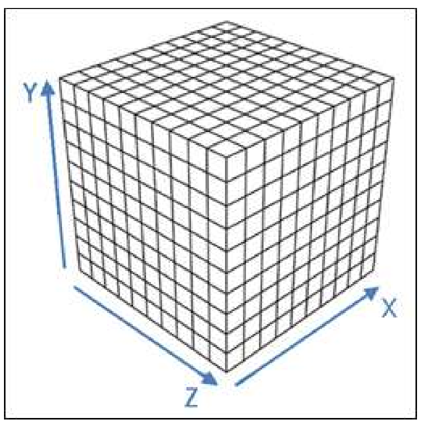
Vlasenko+ (2014)

Volpe (2015)

Blaschke & Cirigliano (2016)

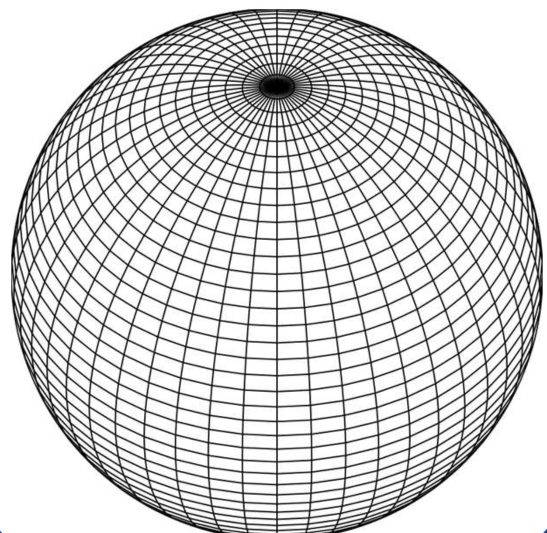
$f_{ab} =$

Position (3D)



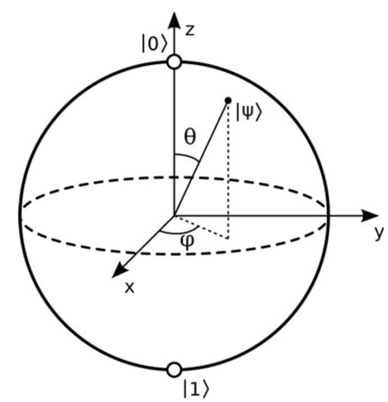

x

Momentum (3D)



x

Flavor
(3x3 matrix)



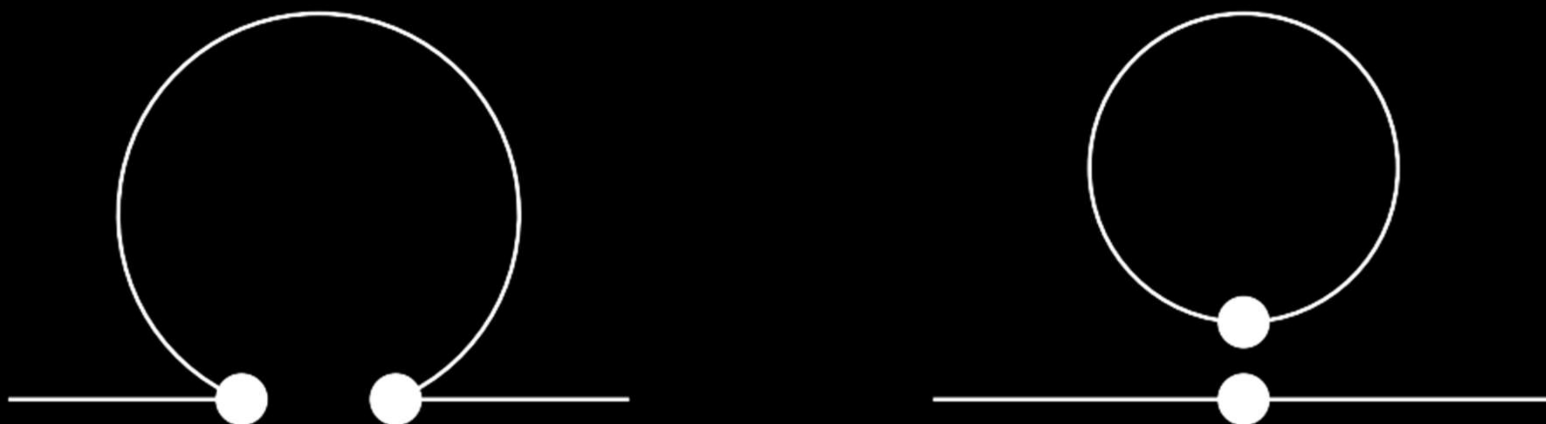
$$\frac{\partial f_{ab}}{\partial t} + c\mathbf{\Omega} \cdot \nabla f_{ab} = \mathcal{C}_{ab} - \boxed{\frac{i}{\hbar}[\mathcal{H}, f]_{ab}}$$

**Flavor**

Vlasenko+ (2014)

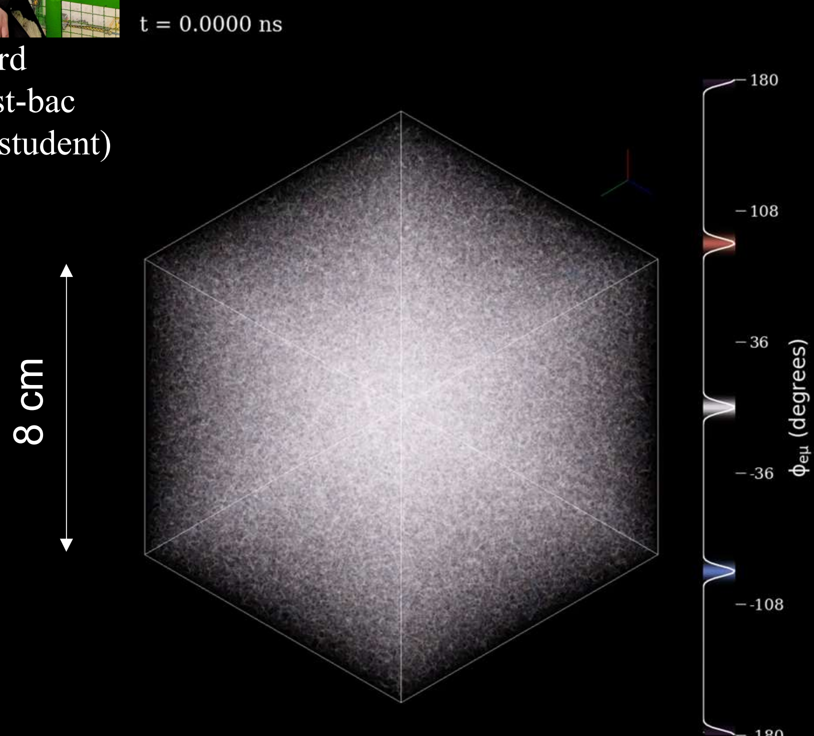
Volpe (2015)

Blaschke & Cirigliano (2016)

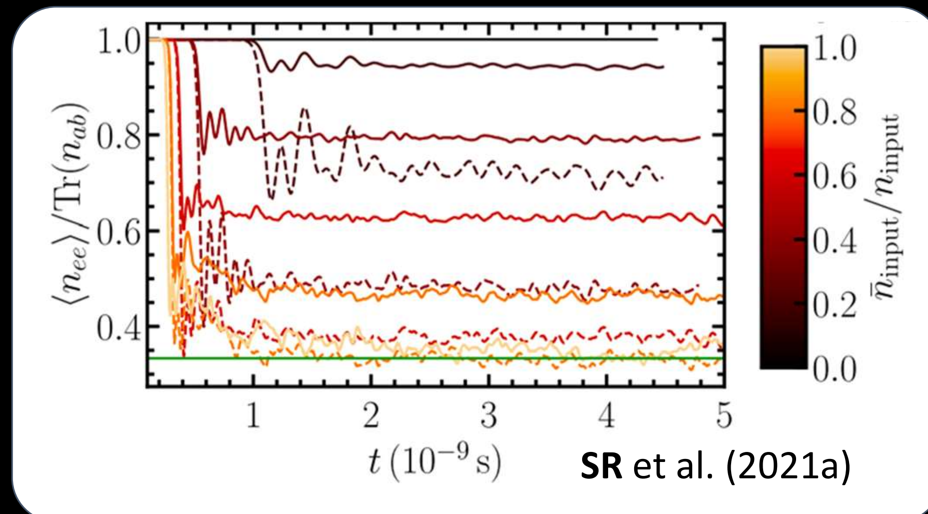


# What does the FFI look like?

Nicole Ford
(Berkeley post-bac
→ McGill grad student)




**SR** et al. (2021b)




Amount of flavor transformation depends on the angular distribution.

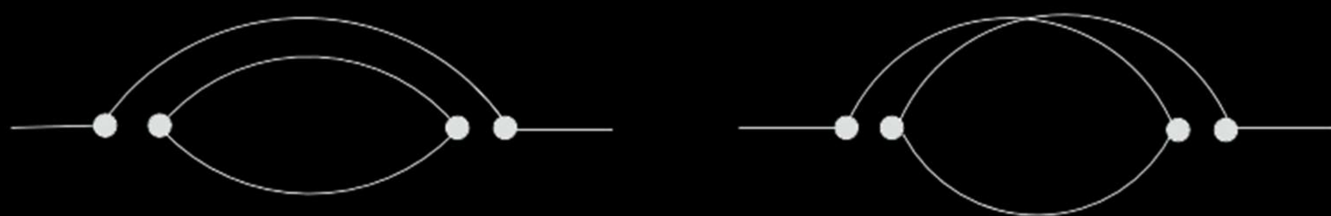
$$\frac{\partial f_{ab}}{\partial t} + c\mathbf{\Omega} \cdot \nabla f_{ab} = \boxed{\mathcal{C}_{ab}} - \frac{i}{\hbar}[\mathcal{H}, f]_{ab}$$

Vlasenko+ (2014)

Volpe (2015)

Blaschke & Cirigliano (2016)

"The Supernova Problem"



Neutrino Transport Reviews

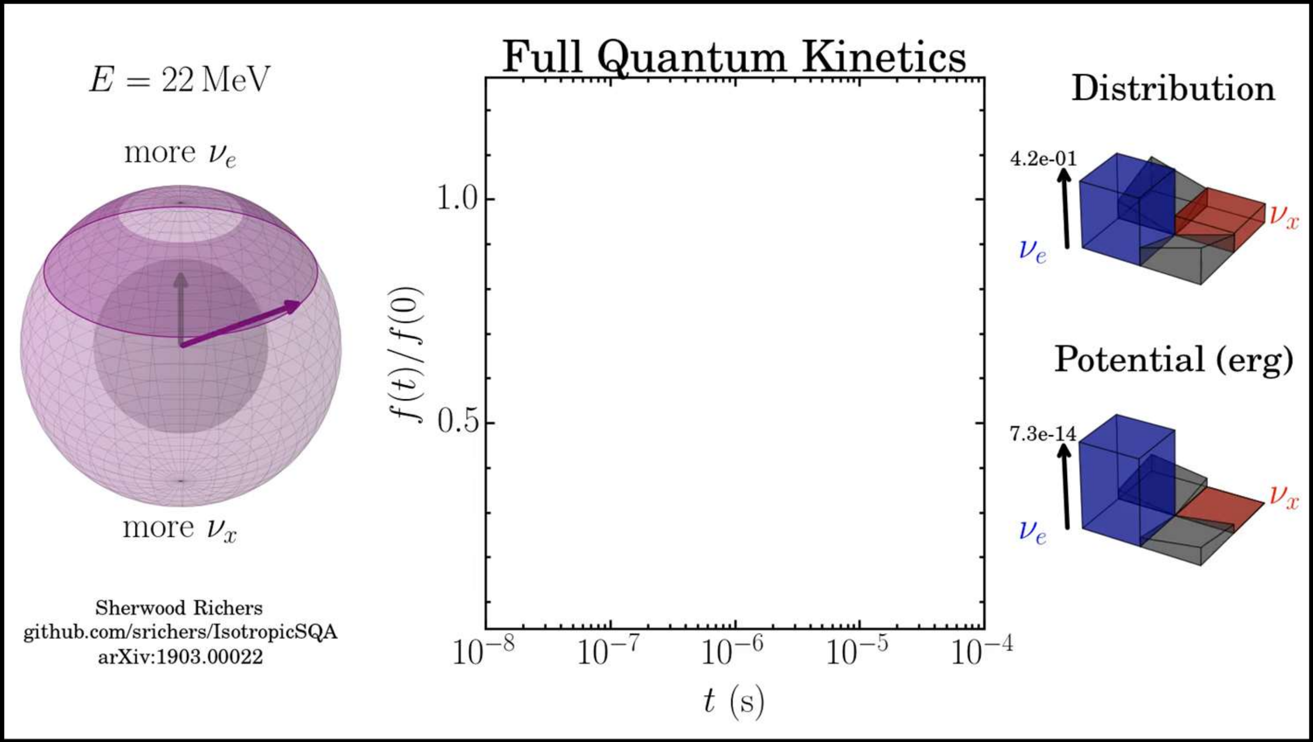
Bruenn (1985)
Burrows, Reddy, Thompson (2007)
Mezzacappa (2022)

Combining with one-loop effects

Cherry (2012)
Vlasenko (2017)
Vlasenko & McLaughlin (2018)
SR et al. (2019)
Shalgar & Tamborra (2020, 2022)
Johns (2021)
Martin et al. (2021)
Sasaki et al. (2021)
Nagakura (2022)
Hansen et al. (2022)
Johns & Xiong (2022)
Kato & Nagakura (2022)
Padilla-Gay et al. (2022)
Kato, Nagakura, & Zaizen (2023)
Lin & Duan (2023)
Xiong et al. (2023)

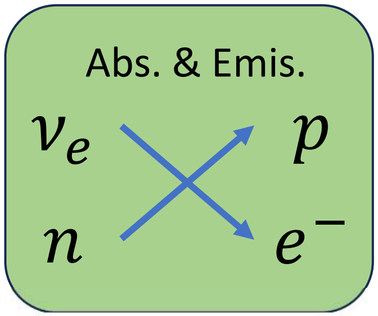
$$\frac{\partial f_{ab}}{\partial t} + c\boldsymbol{\Omega} \cdot \nabla f_{ab} = \mathcal{C}_{ab} - \frac{i}{\hbar}[\mathcal{H}, f]_{ab}$$

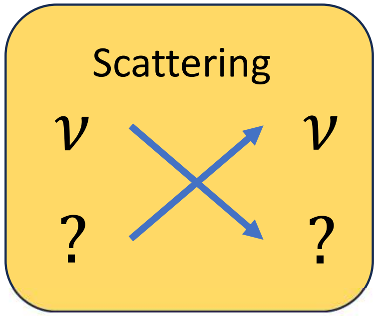
**Oscillations** and **collisions** are not generally separable



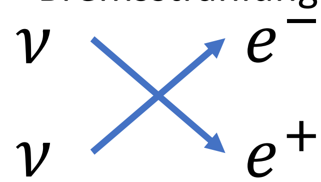

Richers+ (2019)

# Collisional Processes

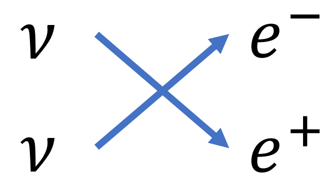



Pair Annihilation
Bremsstrahlung




4-neutrino
Processes



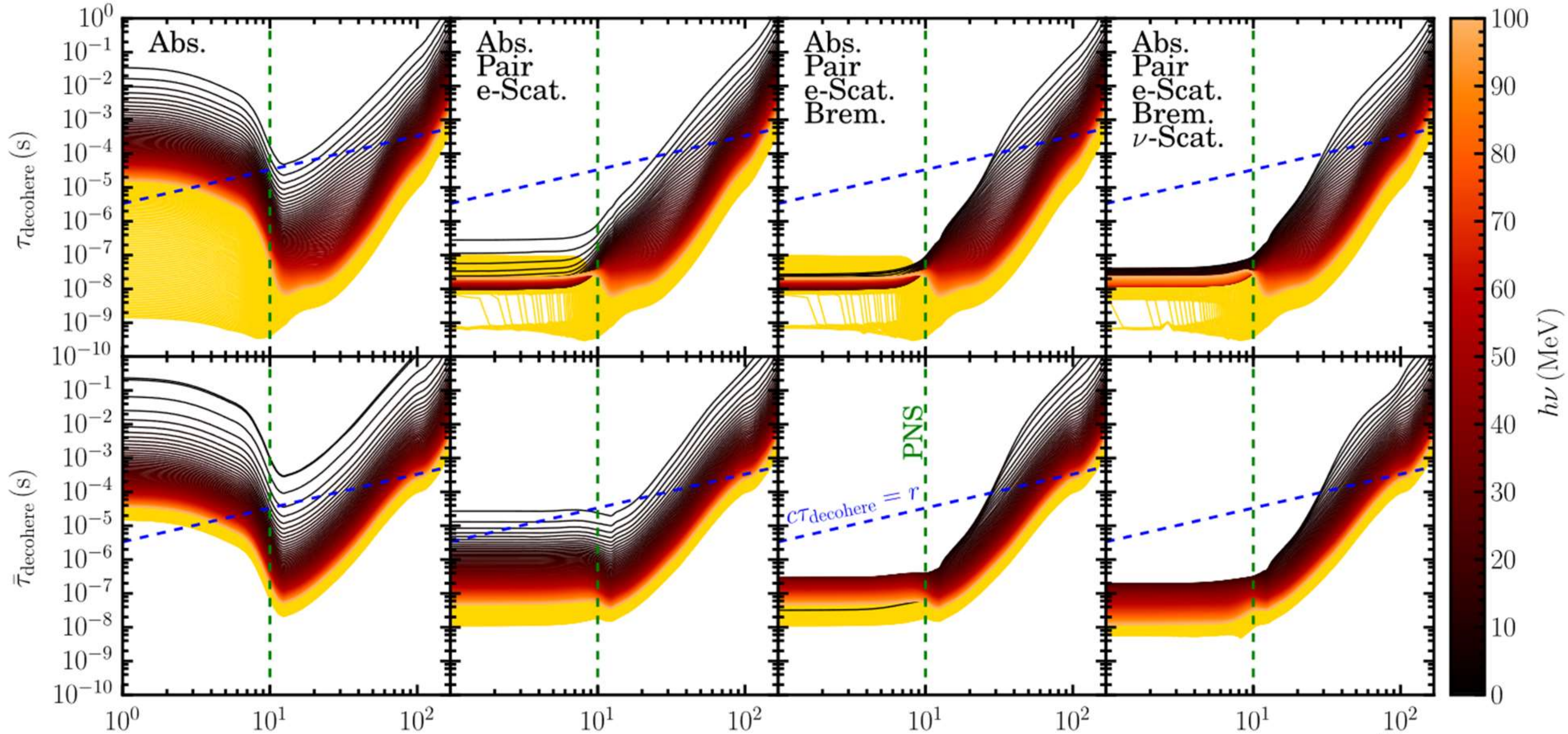

$$\Pi^+_{ab} = \int \frac{d^3\nu_1{}'}{c^4} \langle R \rangle^+_{ab} f'_{1ab},$$

$$\Pi^-_{ab} = \int \frac{d^3\nu_1{}'}{c^4} \langle R \rangle^-_{ab} (\delta_{ab} - f'_{1ab}),$$

$$R^+_{(\nu_a)} = \sum_c (1 + \delta_{ac}) \int \frac{d^3\nu_2{}'}{c^3} \frac{d^3\nu_3{}'}{c^3} \times r_{(p_1+p_3 \to p+p_2)} (1 - f'_{2cc}) f'_{3cc},$$

$$R^-_{(\nu_a)} = \sum_c (1 + \delta_{ac}) \int \frac{d^3\nu_2{}'}{c^3} \frac{d^3\nu_3{}'}{c^3} \times r_{(p+p_2 \to p_1+p_3)} f'_{2cc} (1 - f'_{3cc})$$

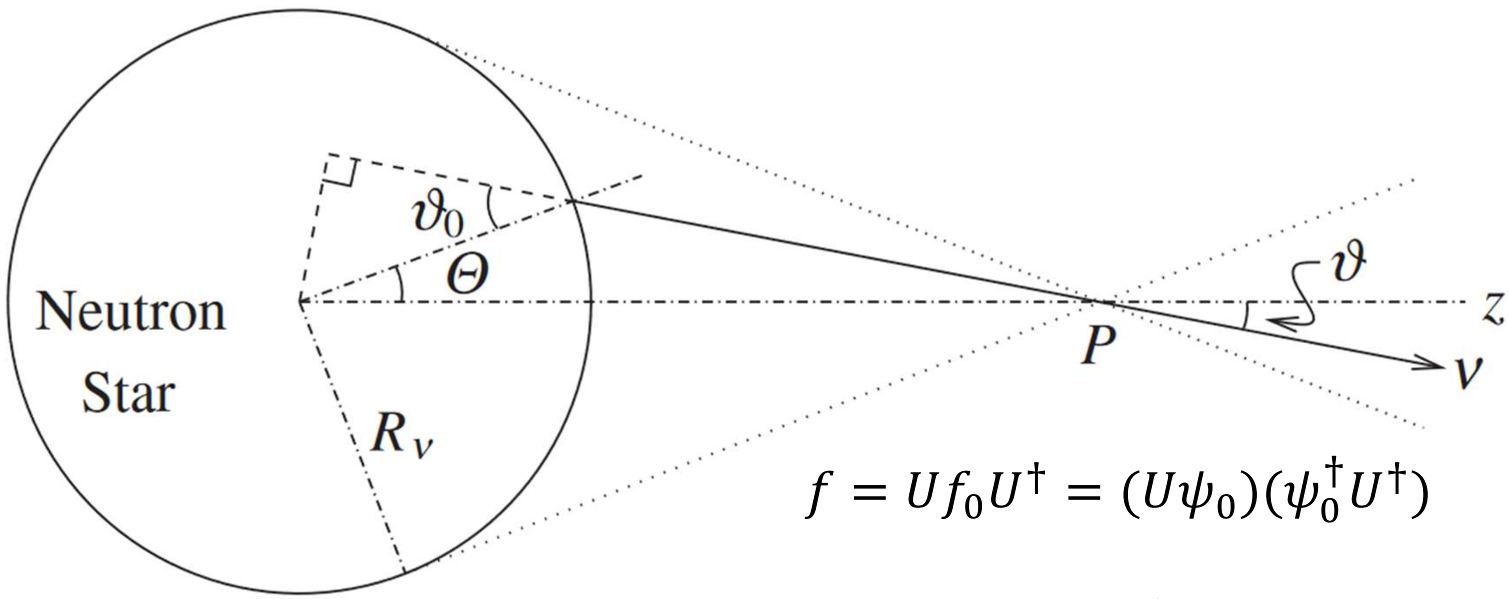
# Multiple collision processes matter




SR et al. (2019)

# Simulation of Neutrino Quantum Kinetics

# Bulb Model (Dirichlet boundary conditions)




Duan et al. (2006)

$$f = U f_0 U^\dagger = (U\psi_0)(\psi_0^\dagger U^\dagger)$$

Evolve U instead of f (unitary operator)

Evolve OUTWARD → 1+0 dimensional

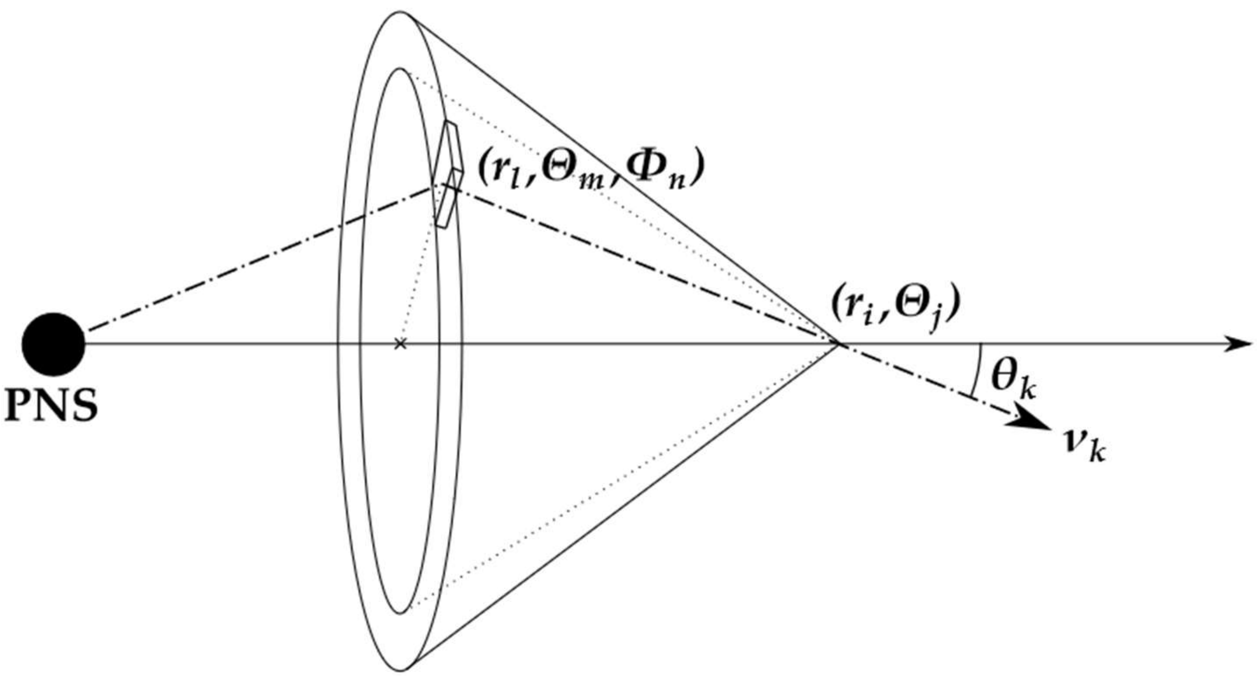
(Single-angle approximation)

→ Numerical demonstrations of collective oscillations, MNR, Halo effect

(see also Galais+2012, Malkus+2012, Tian+2017, many more)

# Bulb Model (Halo Effect)

Zaizen+ (2020)




Cirigliano+ (2018)
Zaizen+ (2020)
Sarikas+ (2012)
Cherry+ (2012, 2013)

$$\frac{\partial f_{ab}}{\partial t} + c\boldsymbol{\Omega} \cdot \nabla f_{ab} = \mathcal{C}_{ab} - \frac{i}{\hbar}[\mathcal{H}, f]_{ab}$$

Transport — Collision — Flavor

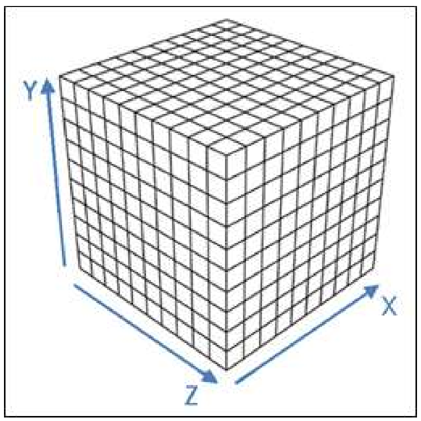
Vlasenko+ (2014)

Volpe (2015)

Blaschke & Cirigliano (2016)

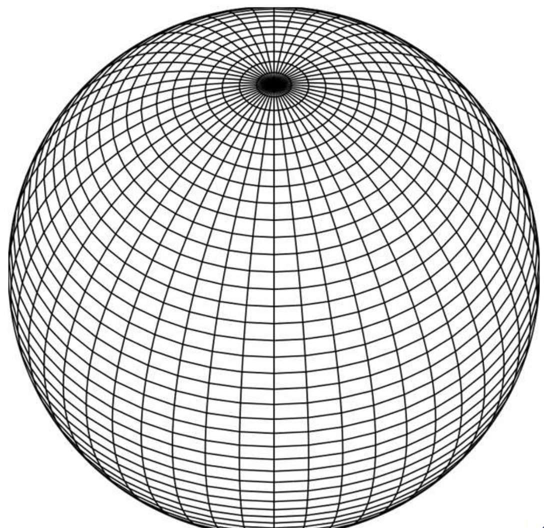
$f_{ab} =$

Position (3D)



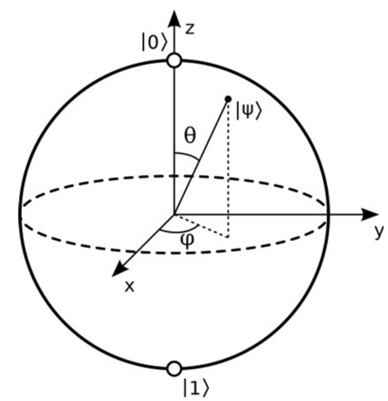

x

Momentum (3D)

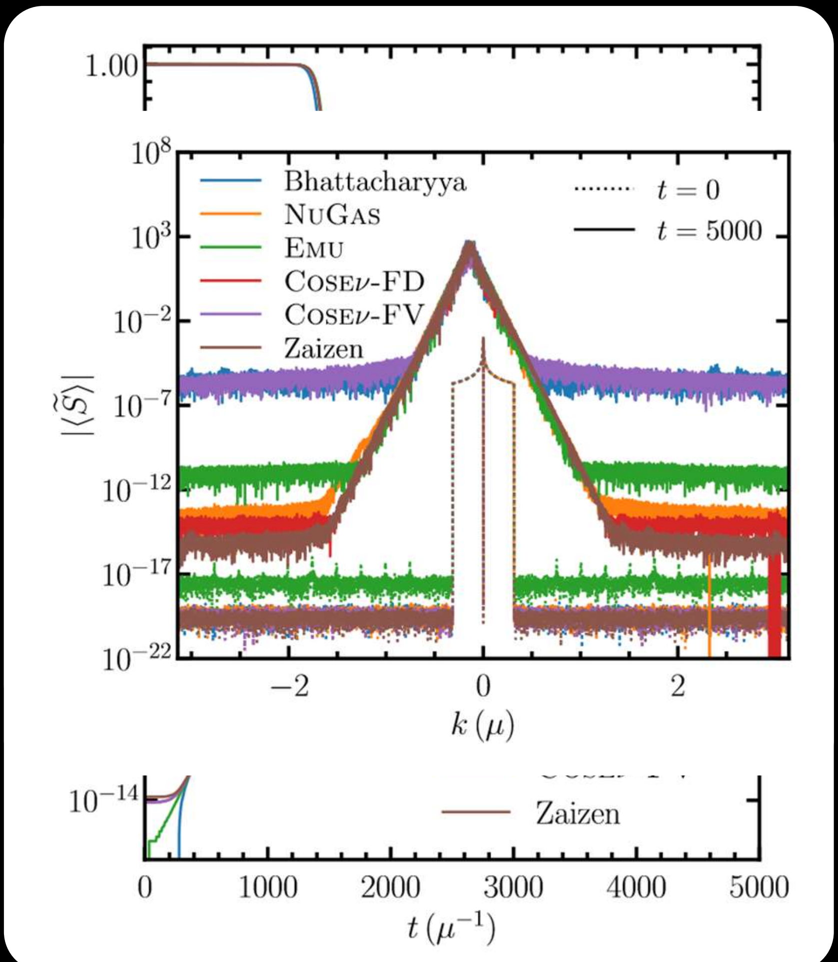


x

Flavor
(3x3 matrix)



# General Features of the FFI




SR+ (2022), following many other works

1. Exponential growth of perturbations
   Sawyer (2005), Dasgupta, Sen, Mirizzi, Morinaga, Padilla-Gay, Abbar, Xiong, Wu, Bhattacharyya, Zaizen, George, Duan, Sigl, Capozzi, Shalgar, Raffelt, Chakraborty, Kato ... [many contributions]

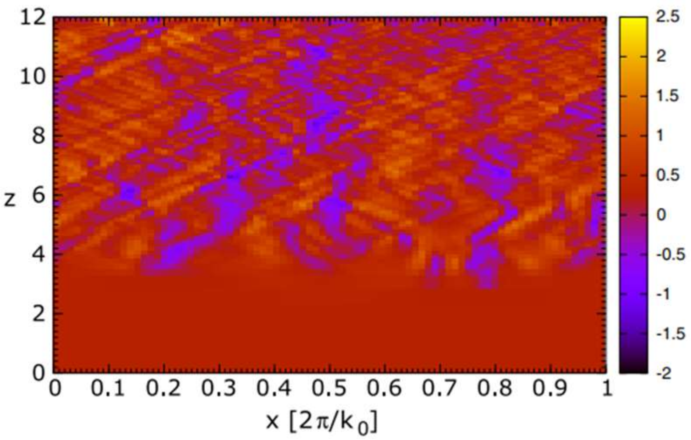
2. Complete mixing within "ELN Crossing", incomplete elsewhere to preserve lepton #
   Bhattacharyya & Dasgupta (2021)

3. Modes spreading to exponential distribution.
   **SR** et al. (2021)

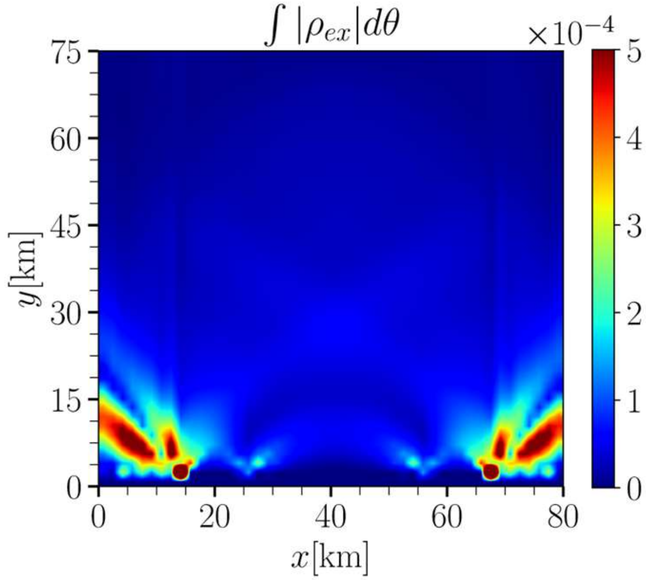
4. Coherent post-saturation flavor wave
   Duan et al. (2021)

5. Non-trivial interplay with collisions
   Padilla-Gay, Shalgar, Johns, Xiong, Sasaki, Sigl, Tamborra, Hansen, Martin, SR

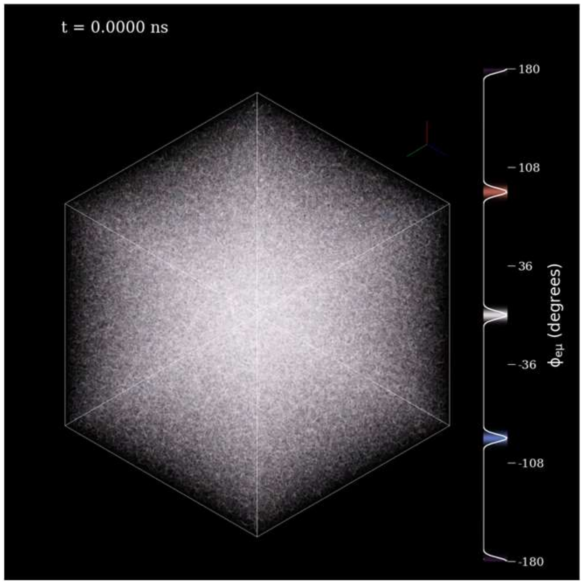
# Multiple dimensions allow broken symmetries




Mirizzi+ (2015)



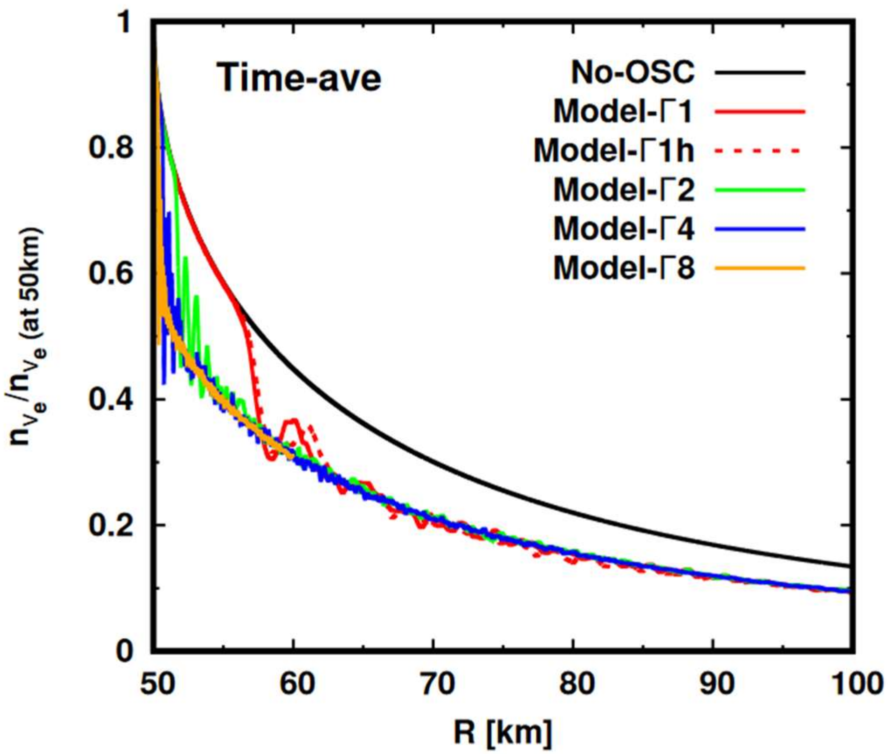

Padilla-Gay+ (2020)




SR et al. (2021b)

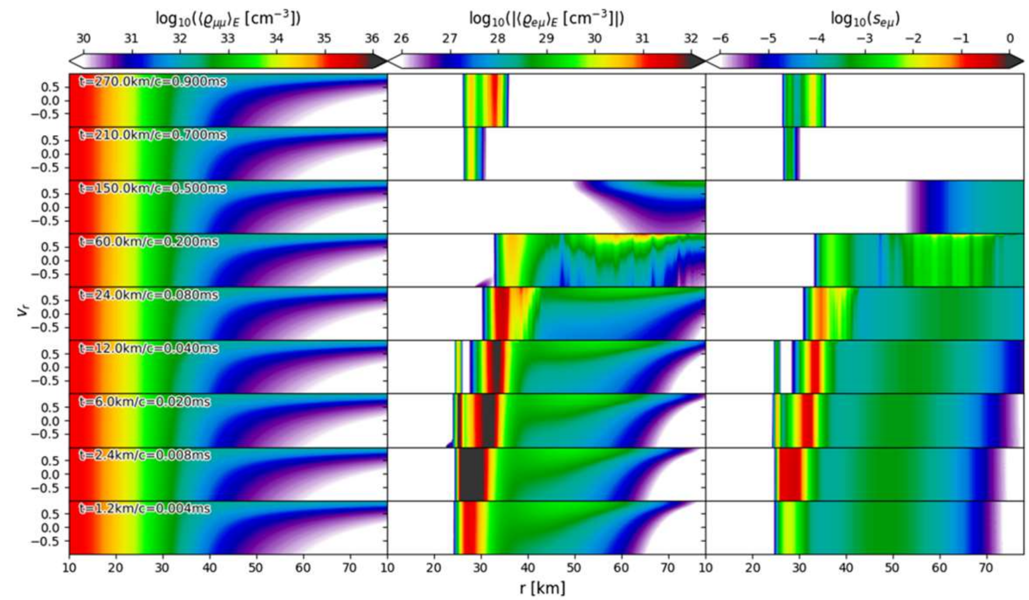
<u>Local</u> FFI in 3D is similar to well-constructed 1D model

# Reduced coupling enables global analysis




Nagakura & Zaizen (2022)

→ FFI can modify CCSN and NSM outcomes




Xiong+ (2023)

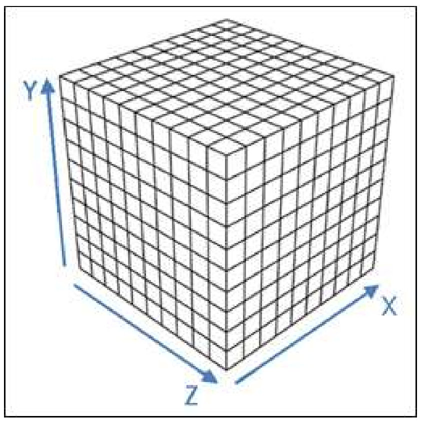
Collisional instability significantly modifies heavy lepton neutrinos in CCSNe

$$\frac{\partial f_{ab}}{\partial t} + c\Omega\,\partial_\alpha T^{\alpha\beta}_{ab} = \mathcal{C}_{ab} - \frac{i}{\hbar}\left[\mathcal{H}_{\alpha,ab}, T^{\alpha\beta}_{ab}\right]$$

Transport Collision Flavor

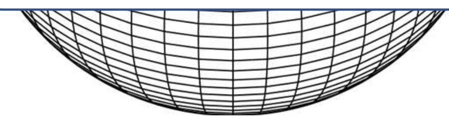
$f_{ab} =$

Position (3D)




x

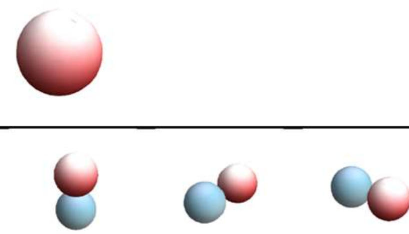
Momentum (3D)



Momentum (1D)

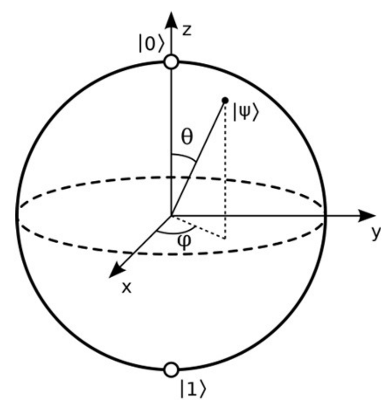
$(E)$

$(F^i)$

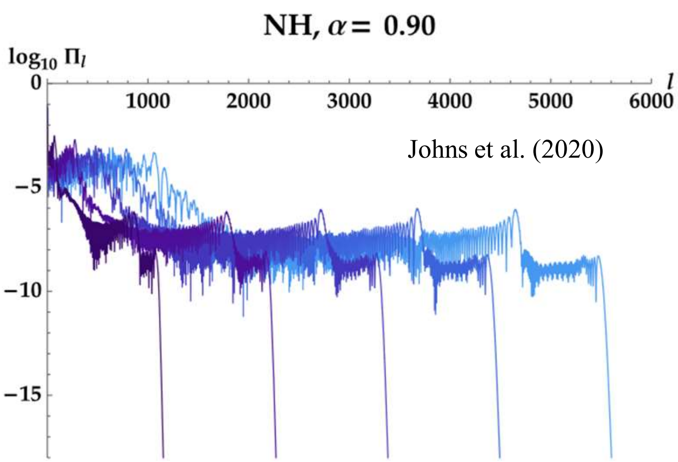


x

Flavor
(3x3 matrix)

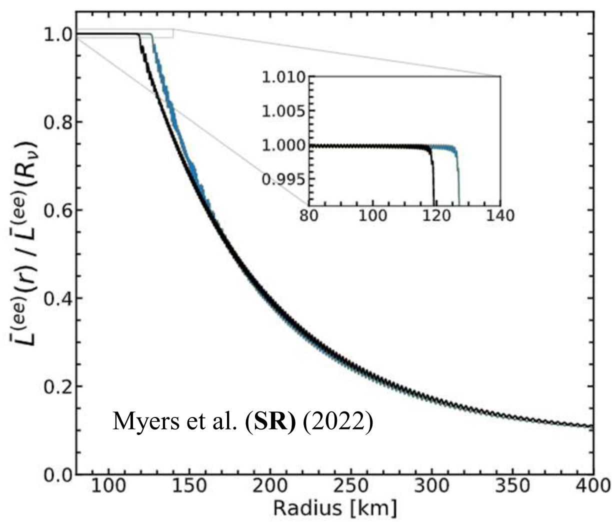


# Moments are fast, but face difficulties



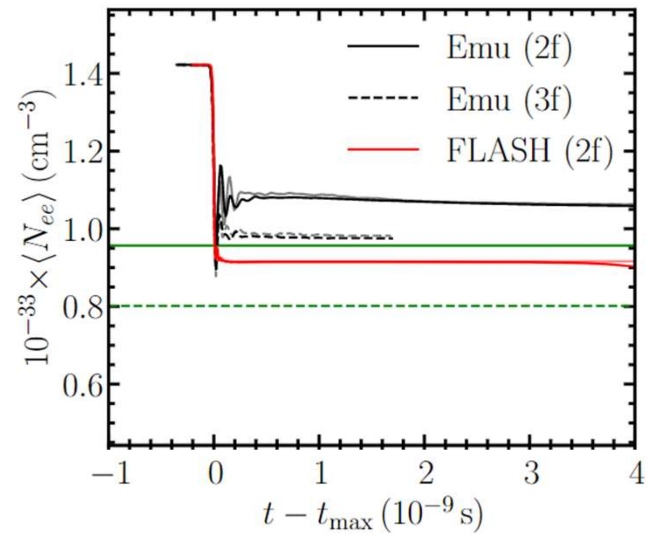

Moments with k=0 FFI

Power goes to high-l moments




Moments with bulb model

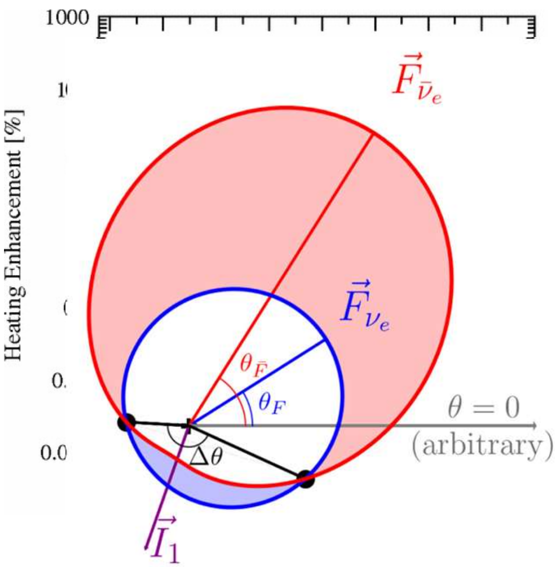
Reproduction of collective oscillations




Grohs, SR et al. (2022)

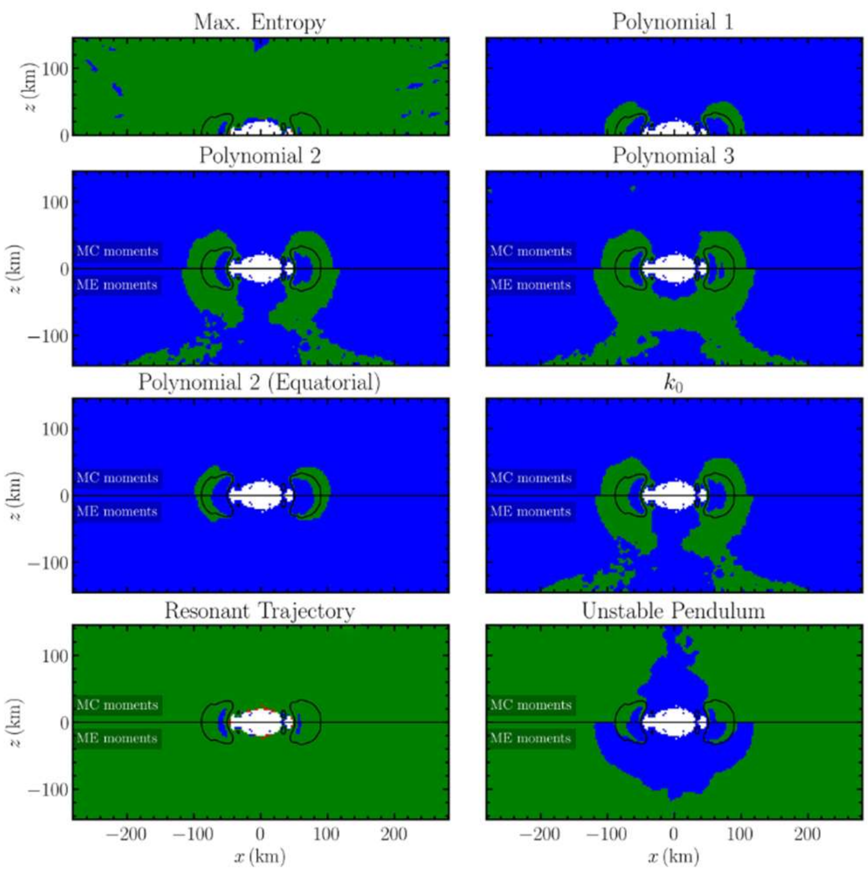
30x cheaper than Emu

# Post-processing simulations without flavor transformation




Collective oscillations don't change CCSN explosion

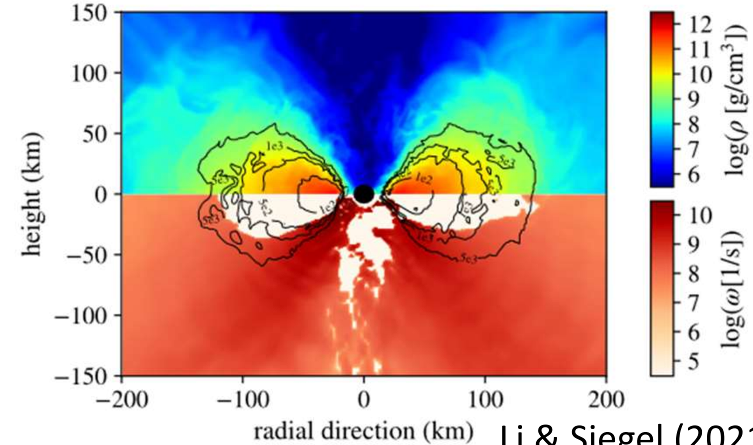
The FFI can! (Nagakura 2023)




Tamborra+(2017)
Wu+(2017)
George+(2020)
Abbar+(2020, 2021)
Morinaga+(2020)
Azari+(2019,2020)
Nagakura & Johns (2021)
Capozzi+(2021)

We can quickly detect instability in NSMs and CCSNe

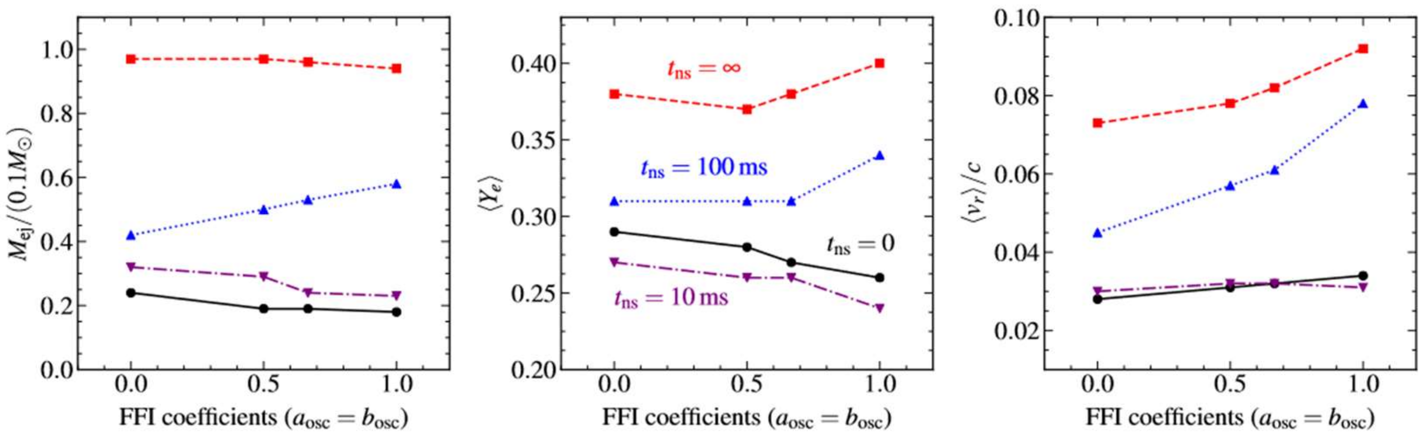
# Effective models probe sensitivity



Li & Siegel (2021)

Li & Sigel 2021: [k=0 dispersion] FFI → more neutron-rich outflow
Just+2022: [Polynomial] FFI → modest neutron-rich enhancement
Fernández+2022: [Opt. depth] FFI → Long-lived HMNS can reverse neutron enhancement

Ehring+2023: [Density cutoff] FFI can help (low-mass) or hinder (high-mass) CCSN explosion



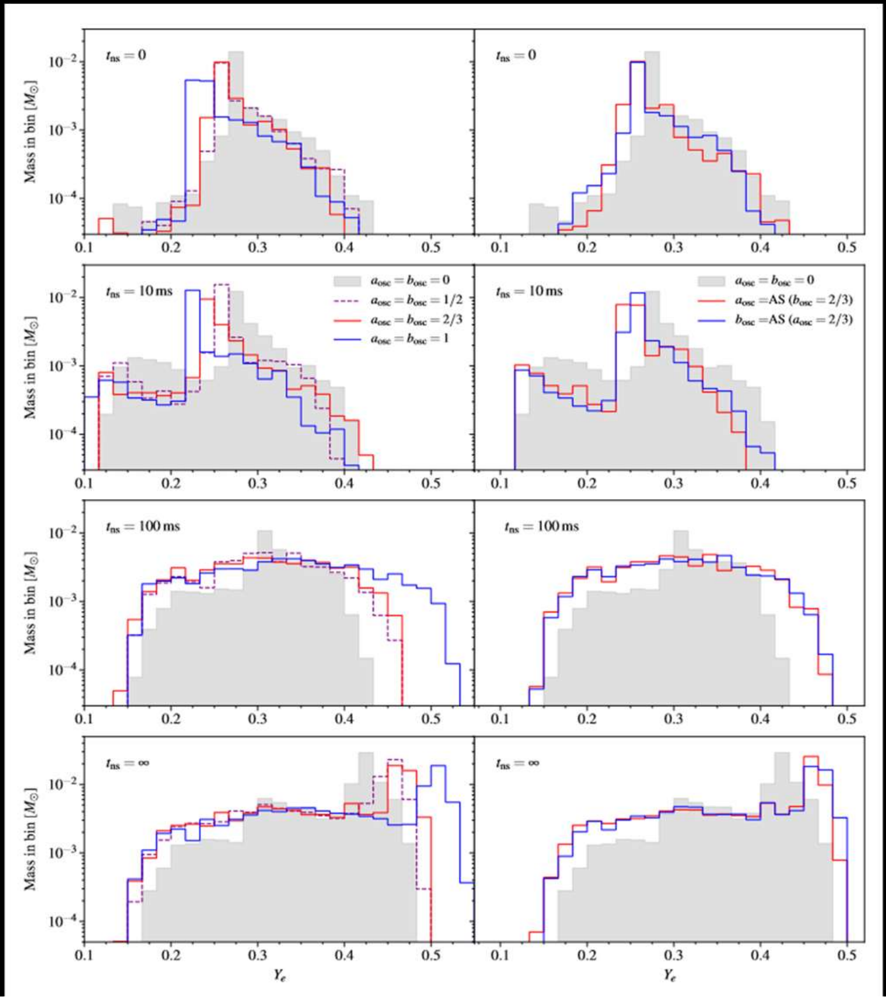
Fernandez, SR et al (2022)

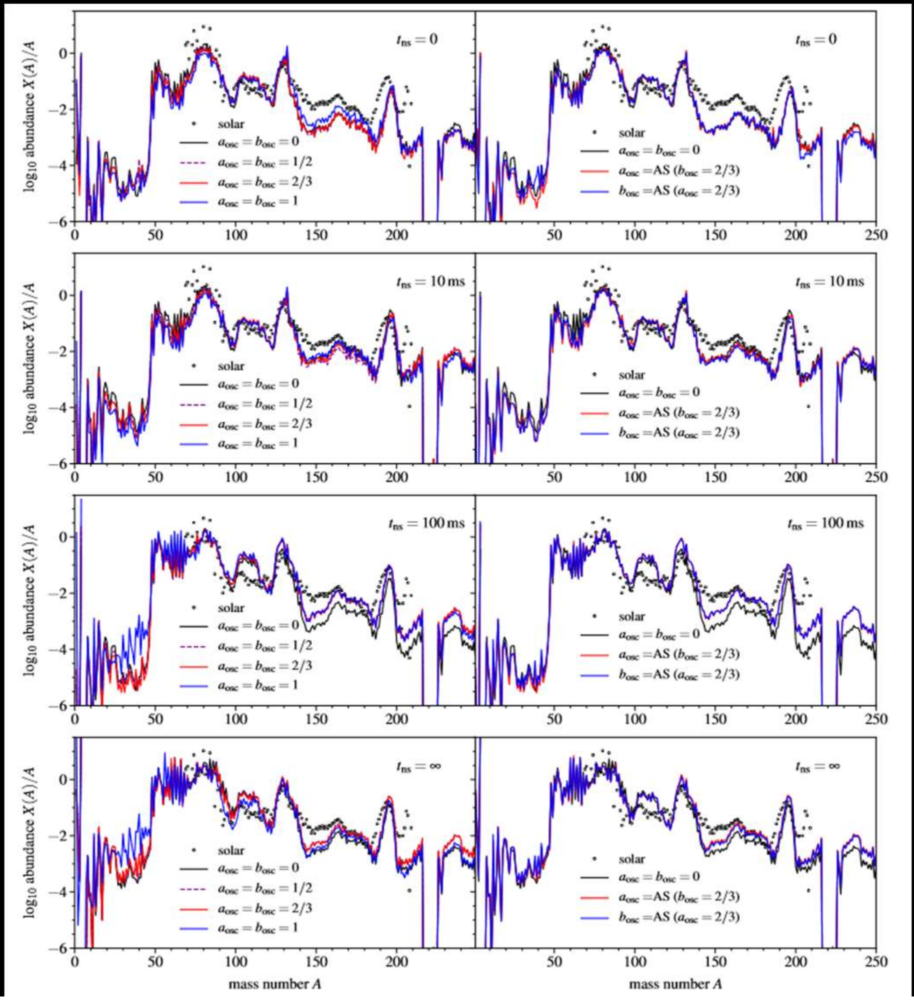
Multi-angle effective models

Bhattacharyya & Dasgupta (2022)

Zaizen & Nagakura (2023)

# Expect FFI to have a moderate impact on outflows





Fernandez, SR, et al. (2022)

# Replacing Simulation with Machine Learning

Invariance
- Rotation (Lorentz)
- $\nu_i \leftrightarrow \nu_j$
- $\nu_i \leftrightarrow \bar{\nu}_i$

Conservation
- $\sum_i F^\alpha_{\nu_i}$ and $\sum_i F^\alpha_{\bar{\nu}_i}$
- $\sum_i (F^t_{\nu_i} - F^t_{\bar{\nu}_i})$

Other
- Do <u>exactly nothing</u> when stable
- $\tilde{F}$ <u>must be stable</u>
- Flux factor <1
- Positive density

$$\begin{pmatrix} F^\alpha_{\nu_e} \\ F^\alpha_{\nu_\mu} \\ F^\alpha_{\nu_\tau} \end{pmatrix} \longrightarrow \blacksquare \longrightarrow \begin{pmatrix} \tilde{F}^\alpha_{\nu_e} \\ \tilde{F}^\alpha_{\nu_\mu} \\ \tilde{F}^\alpha_{\nu_\tau} \end{pmatrix}$$

$4N_F$ quantities
[Full system: 24]

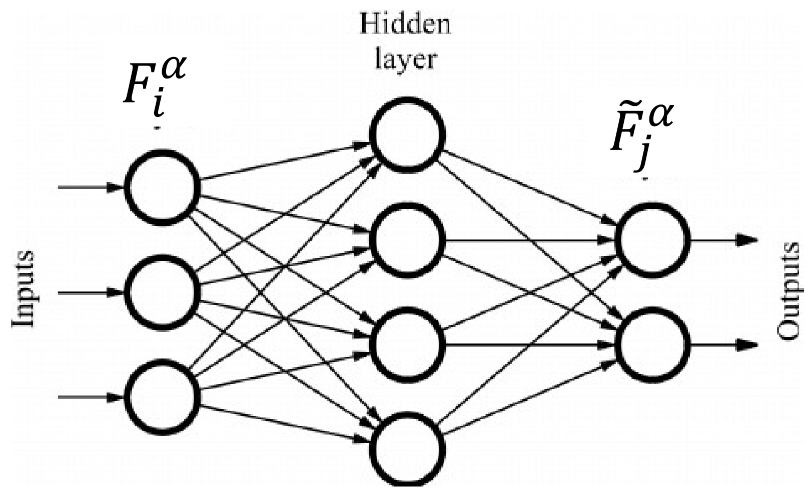
$4N_F$ quantities
[Full system: 24]

# Replacing Simulation with Machine Learning

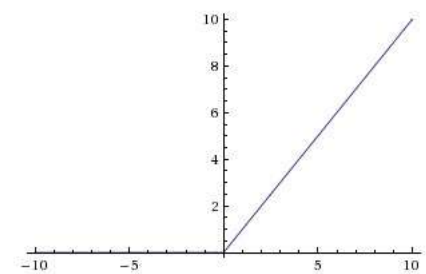
Invariance
- Rotation (Lorentz)
- $\nu_i \leftrightarrow \nu_j$
- $\nu_i \leftrightarrow \bar{\nu}_i$

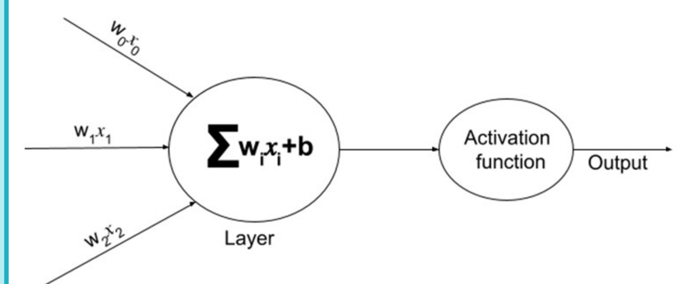
Conservation
- $\sum_i F^{\alpha}_{\nu_i}$ and $\sum_i F^{\alpha}_{\bar{\nu}_i}$
- $\sum_i (F^{t}_{\nu_i} - F^{t}_{\bar{\nu}_i})$

Other
- Flux factor <1
- Positive density



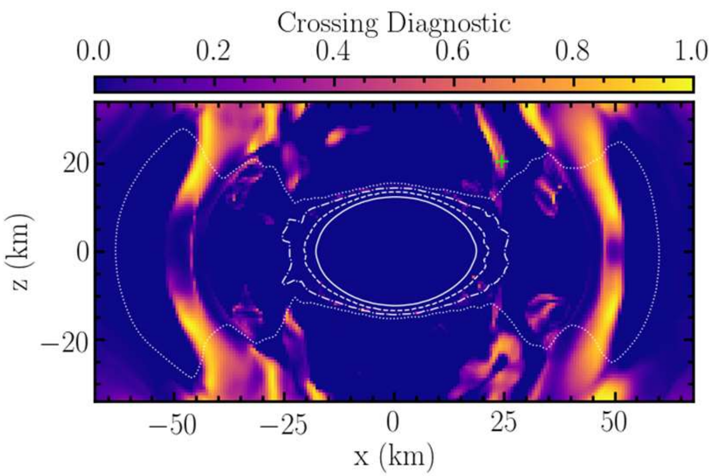

**Feed-forward NN**
- 300 samples (plus augmentation)
- 1 hidden layer, 32 wide
- ADAM optimizer
- 50% dropout probability
- Batch Normalization
- ReLU activation
- Training time: 1 minute

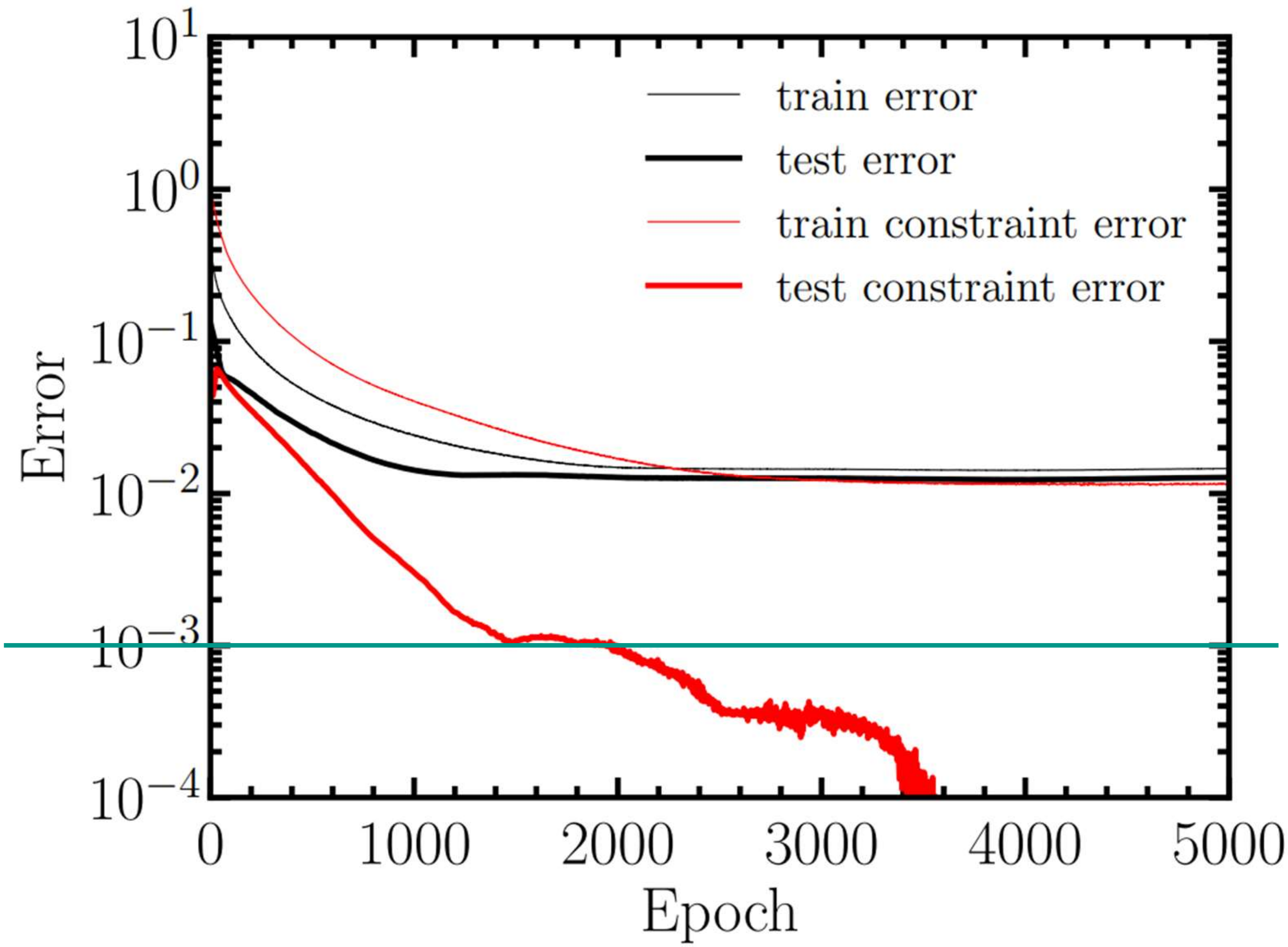


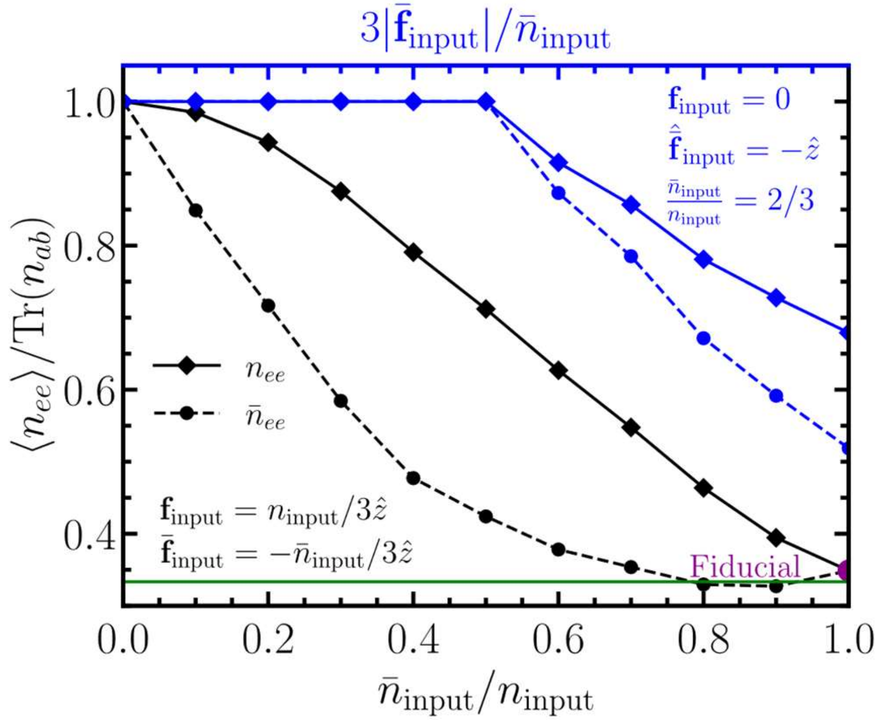
# Replacing Simulation with Machine Learning

<u>On average</u>:
- Percent-level accuracy
- Learned constraints

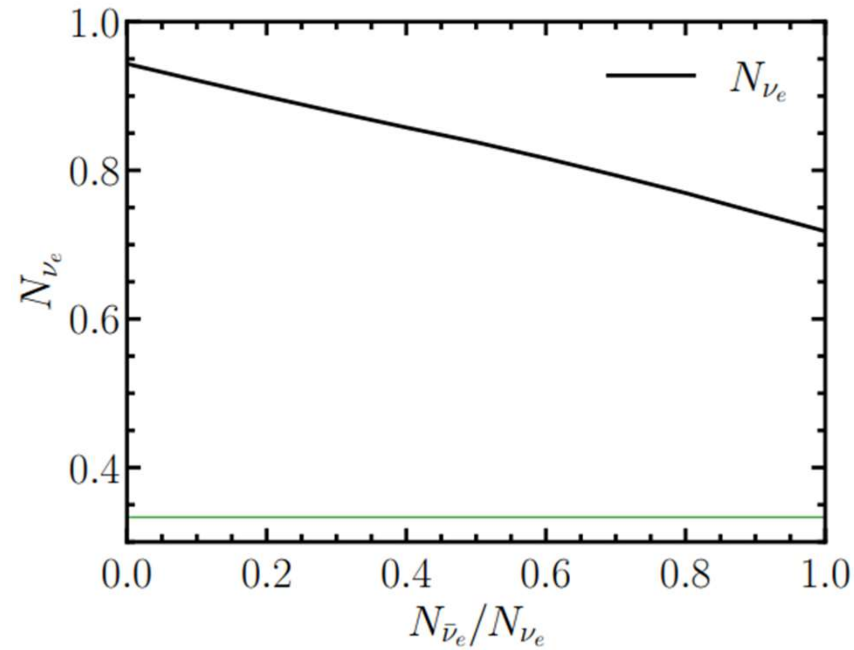


# A Warning about Parameter Space




SR et al. (2021) using Emu




Neural Network

Results generalize poorly outside the training data

# The Future

Better Effective Models

Larger exact calculations

More complete microphysics

Better approximate methods

Expect rapid development in coming years!

General adoption into dynamical models requires reliable predictions