Progress Towards Radiative Corrections in Nuclear Electroweak Matrix Elements through the Renormalization Group

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Testing the Standard Model in Charged-Weak Decays
INT
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In collaboration with Immo C. Reis

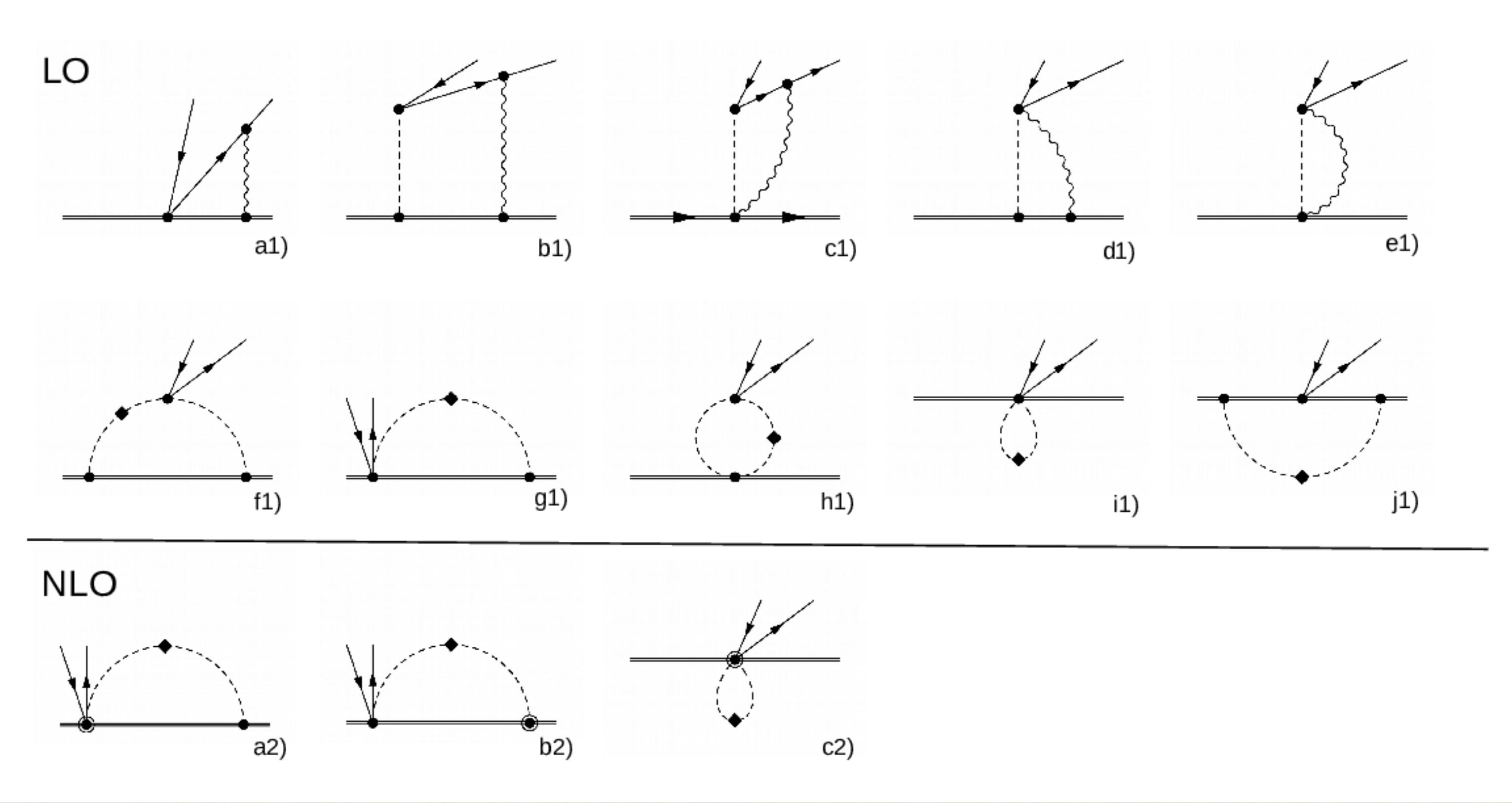
Few Body Syst. 65, 79 (2024)

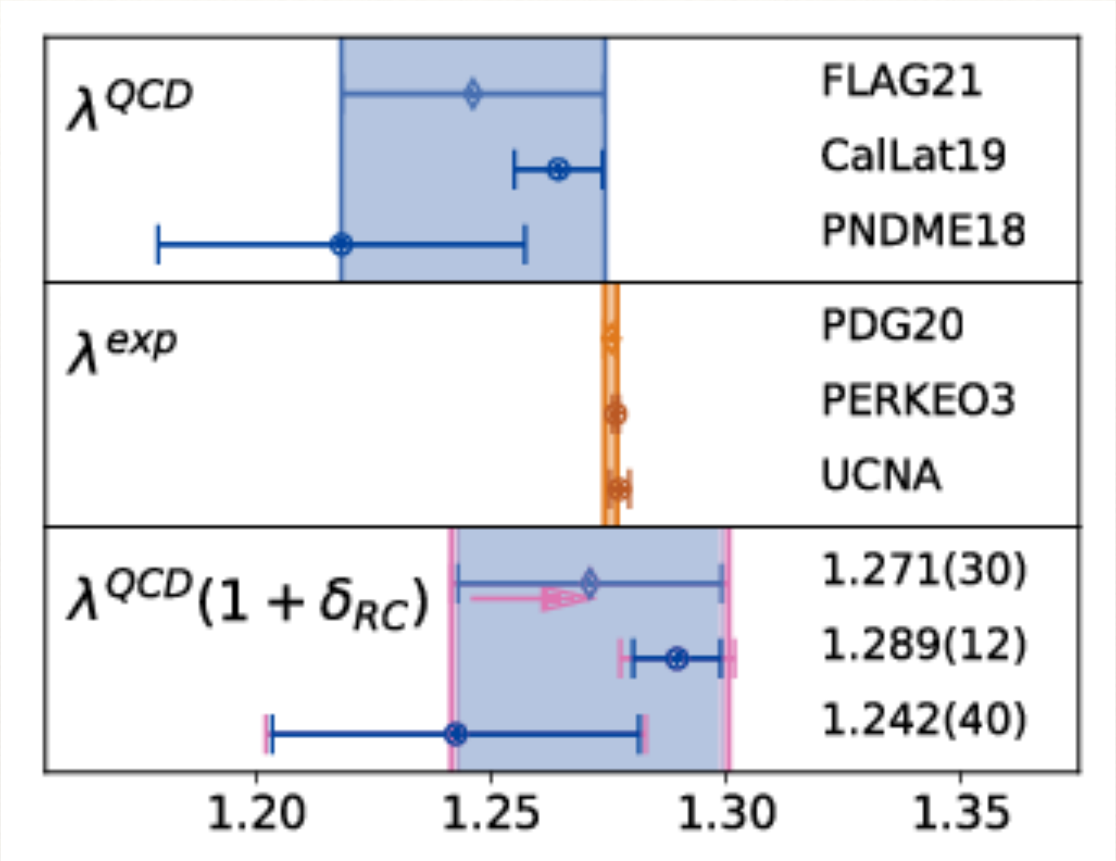
PRC 111, 064011 (2025)

In preparation

INT 20R-2b: Beyond the Standard Model Physics with Nucleons and Nuclei

- Electromagnetic corrections to g_A at the % level





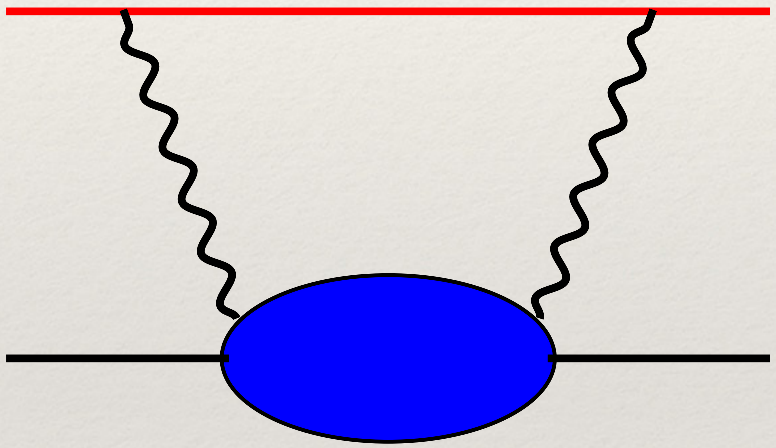
Cirigliano et al. PRL 129

- What would happen in few-body systems?

Radiative Corrections in Nuclear Physics

Spectroscopy

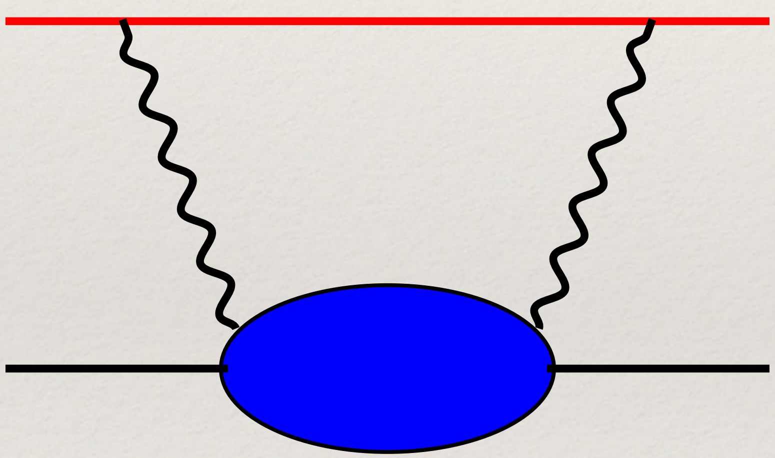
Lepton Scattering



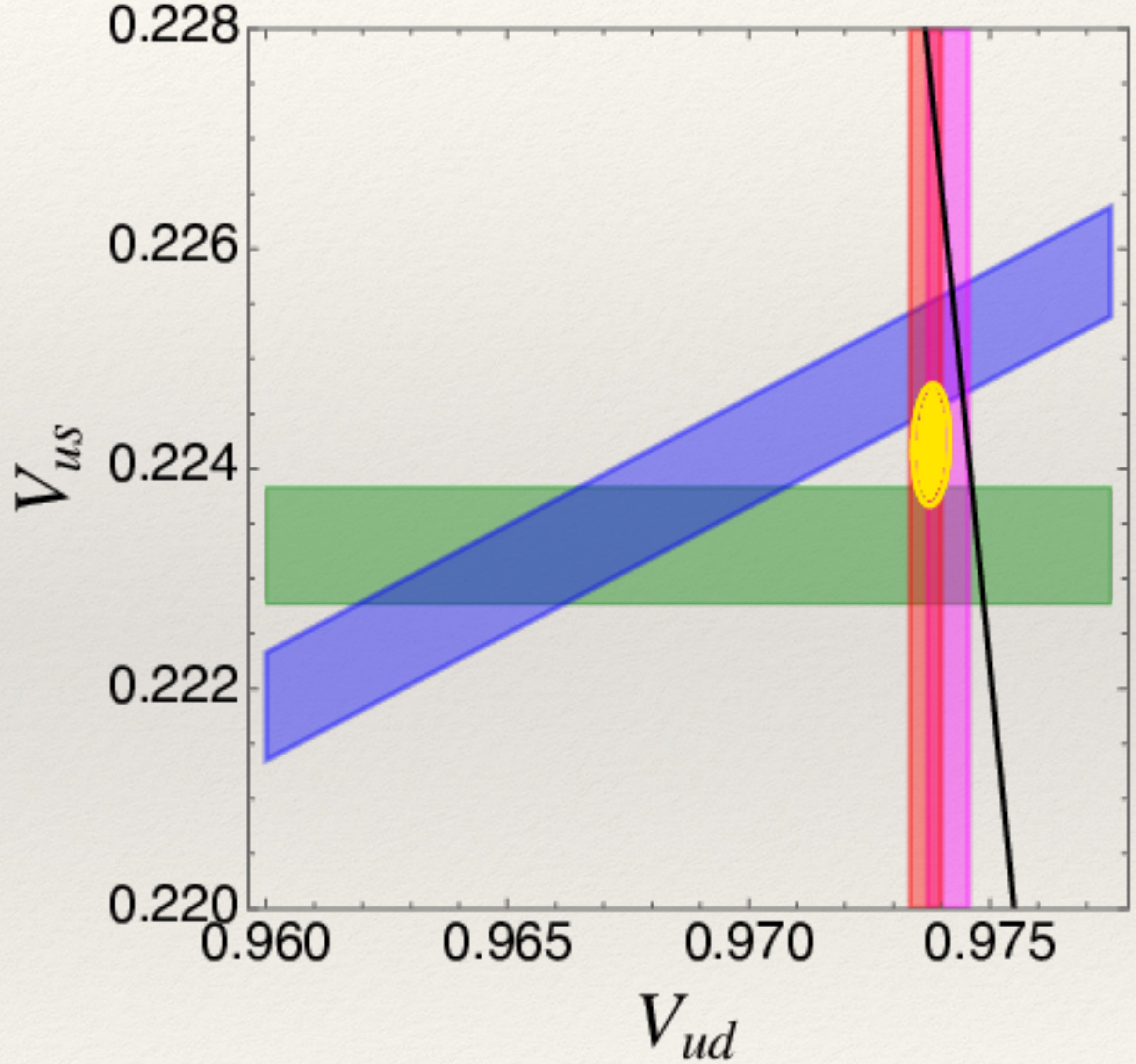
Beta Decay

Radiative Corrections in Nuclear Physics

Beta Decay



Cirigliano et al. PLB 838

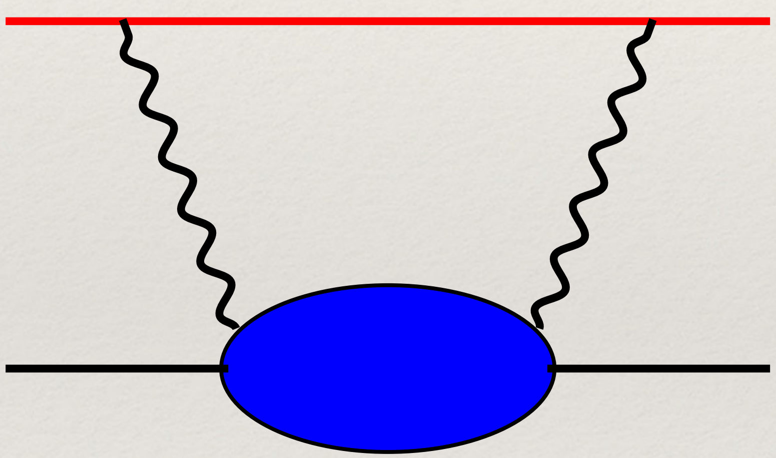


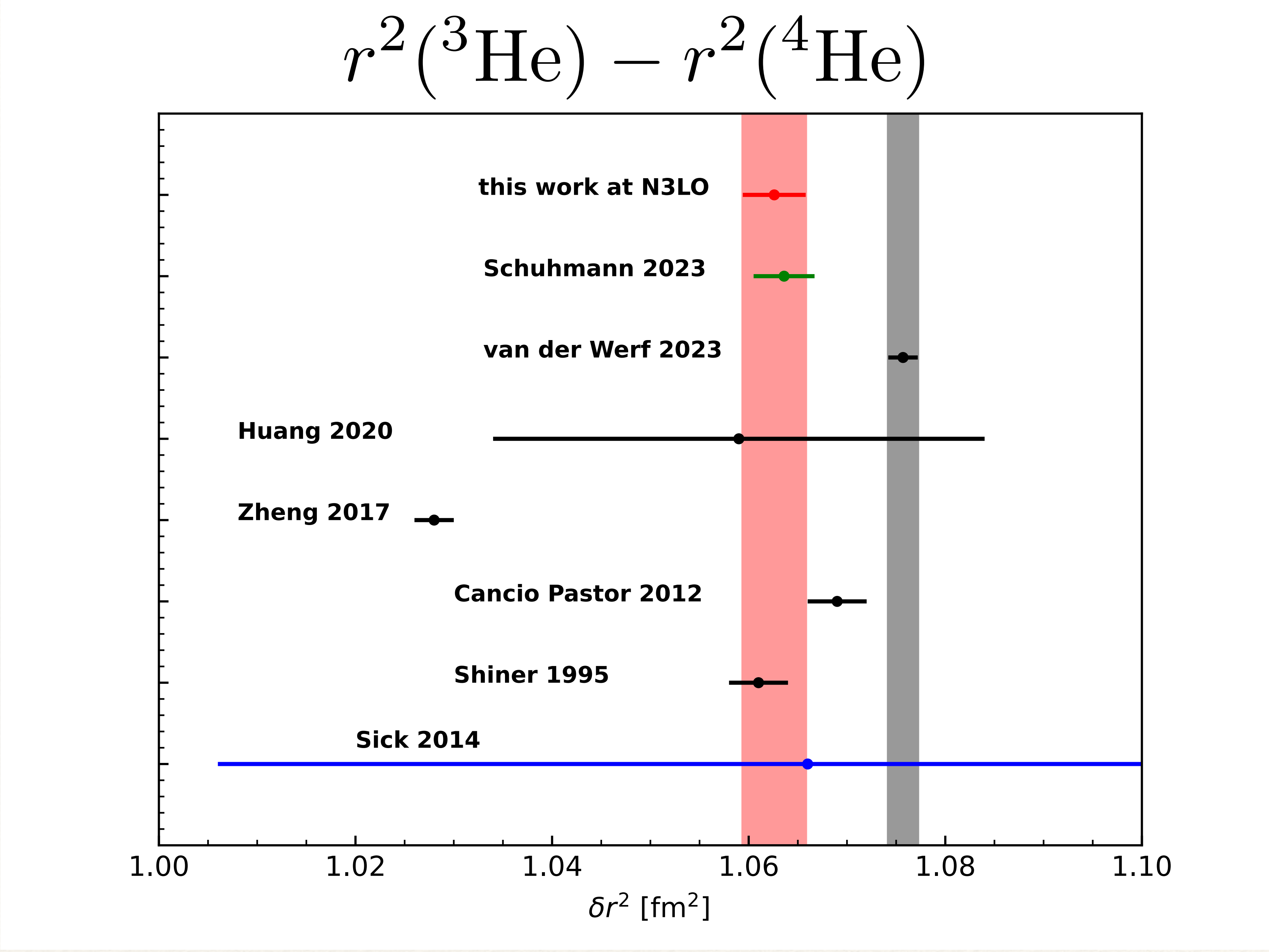
$K \to \pi l \nu$ $\quad$ $K \to \mu\nu/\pi \to \mu\nu$

neutron $\quad$ $0^+ \to 0^+$

Radiative Corrections in Nuclear Physics

Spectroscopy

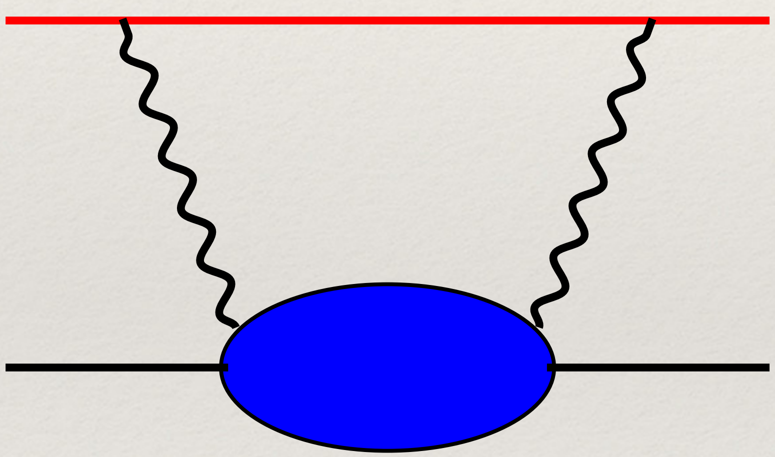


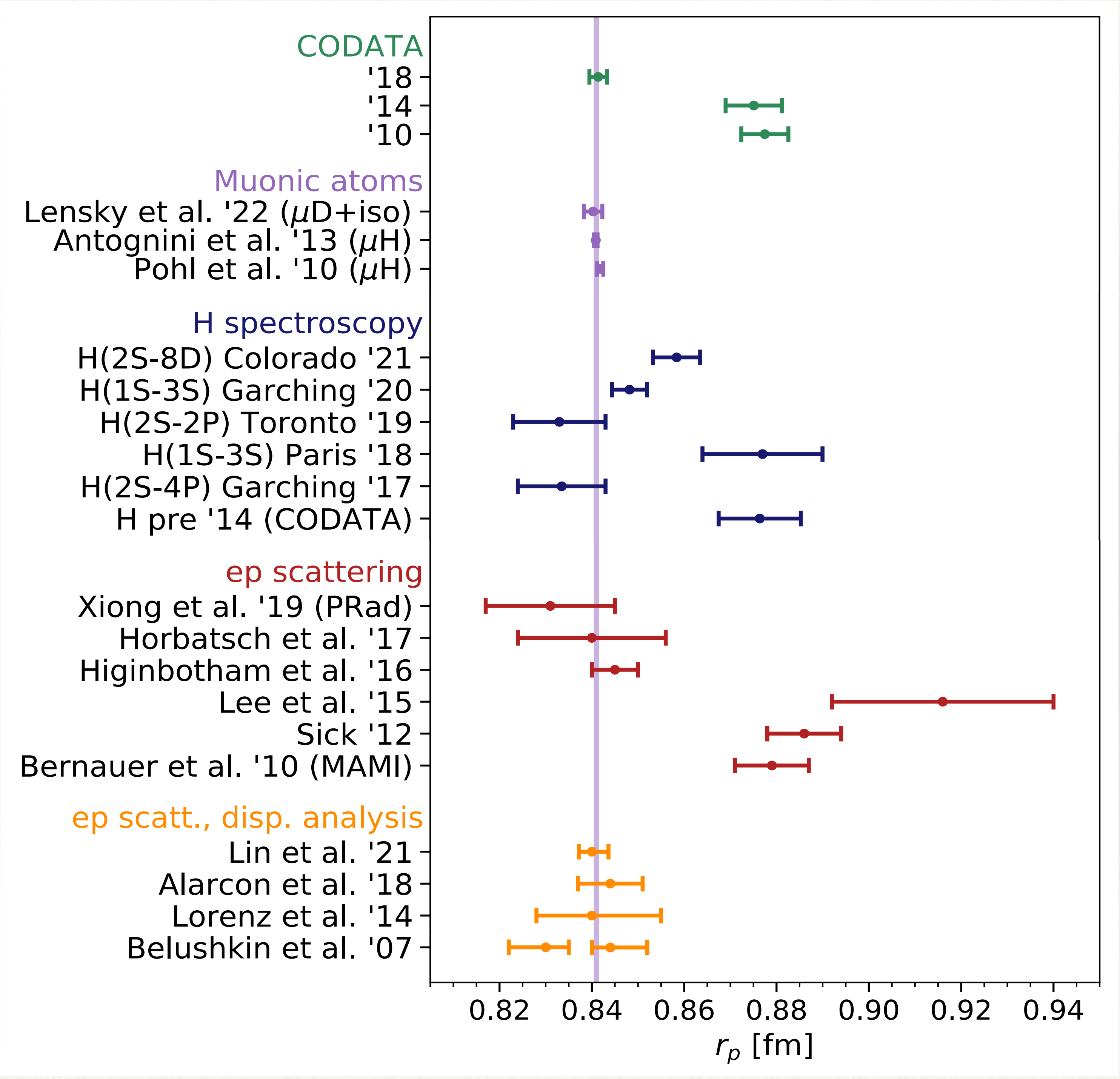


Li Muli, TRR, Bacca PRL 134

Radiative Corrections in Nuclear Physics

Lepton Scattering





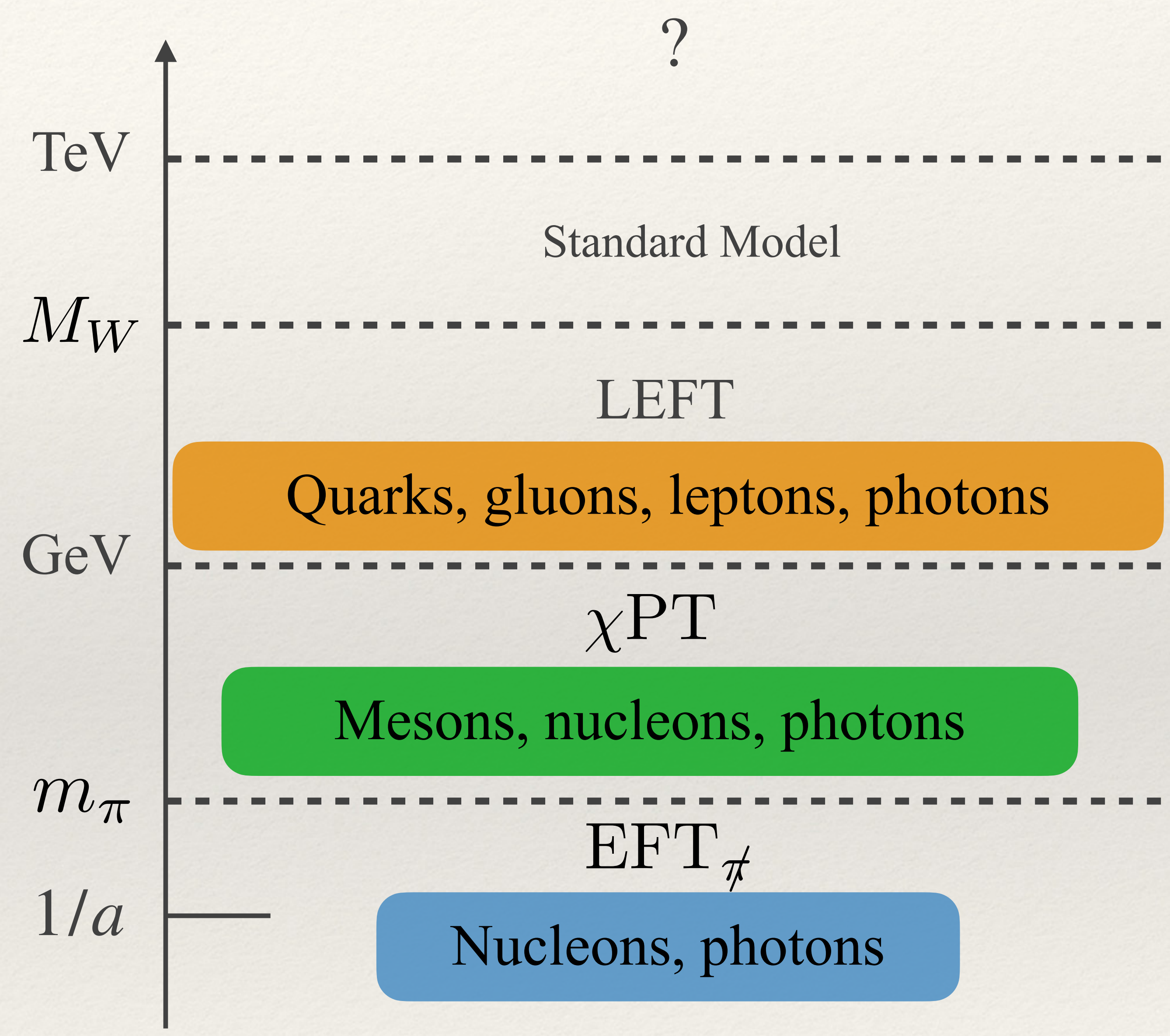
Antognini et al. ARNPS 72

Effective Field Theory Tower

Goal: Low energy, high precision, universal

Effective field theories should…

1. Have all particles that are near the mass-shell
2. Have a homogeneous power counting governed by a single ratio of scales
3. Be renormalization group invariance up to the order we are working
4. Preserve relevant symmetries (gauge, discrete, internal)
5. Make life easier



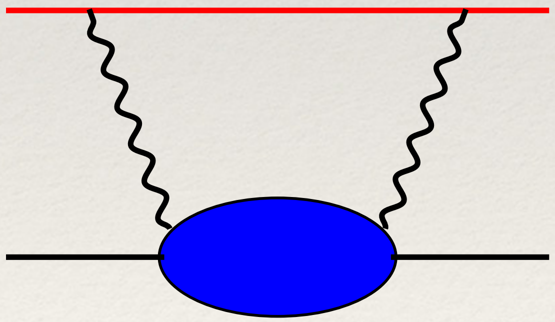
Nonrelativistic EFT with Virtual Photons

- Energy and momenta are different but correlated scales

$$v \sim p/M = \sqrt{E/M}$$

$$\text{potential region}: (E, \vec{p}) \sim (Mv^2, Mv)$$

- But our prototype diagram doesn't scale with a single power of *v*?

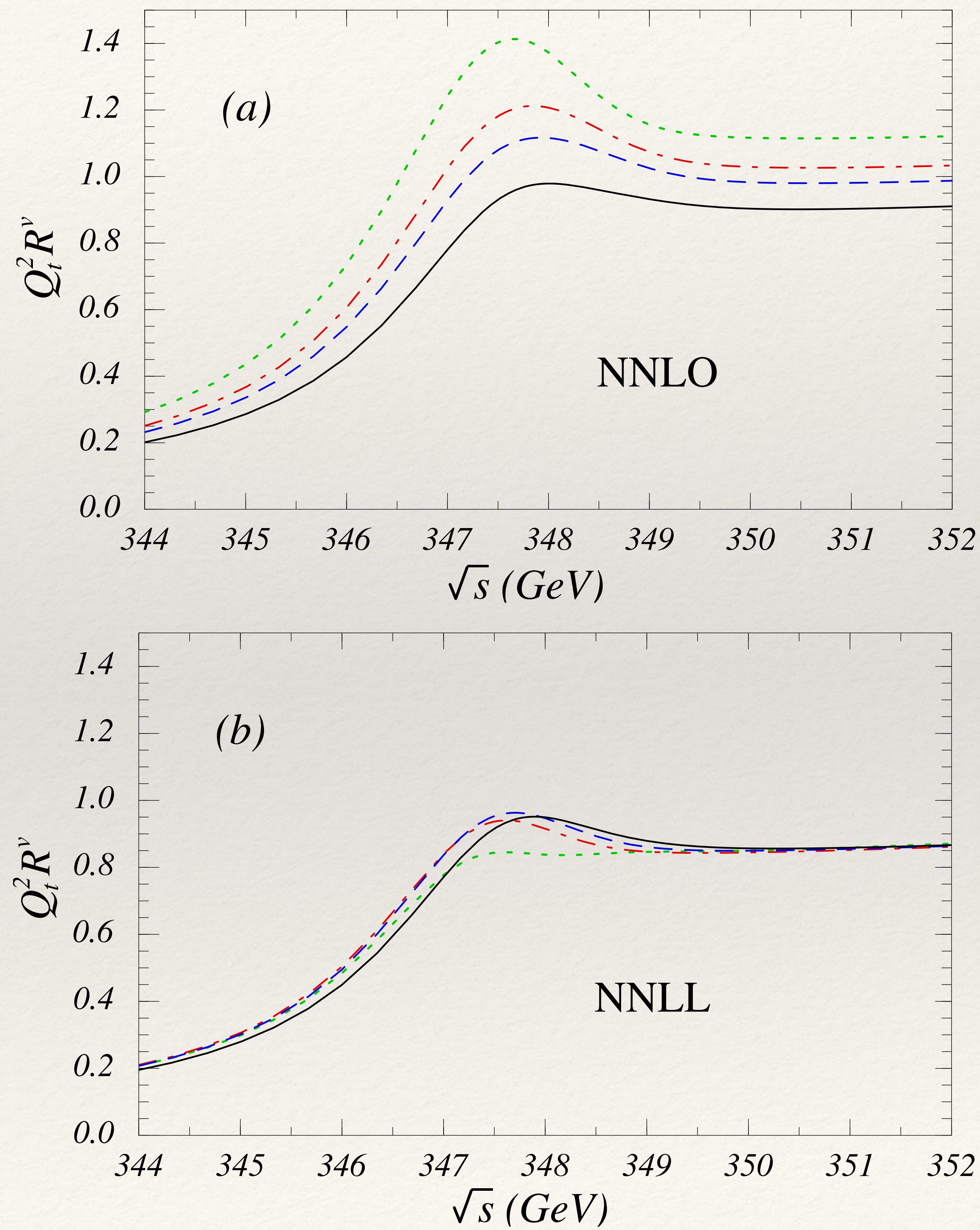


$$\supset v^{-3},\ v^{-2}$$

Caswell, Lepage, Labelle, Luke, Manohar, Rothstein, Savage, Grießhammer, Grinstein, Stewart, Hoang, Pineda, Soto, Brambilla, Vairo

Nucleons vs. $t\bar{t}$

NRQCD	NN EFT
Heavy potential quarks	Heavy potential nucleons
Massless gluons	Massless photons (light pions)
$p \sim m\alpha$ $E \sim m\alpha^2$	$p \sim 1/a$ $E \sim 1/M_N a^2$



Hoang et al. PRD 65

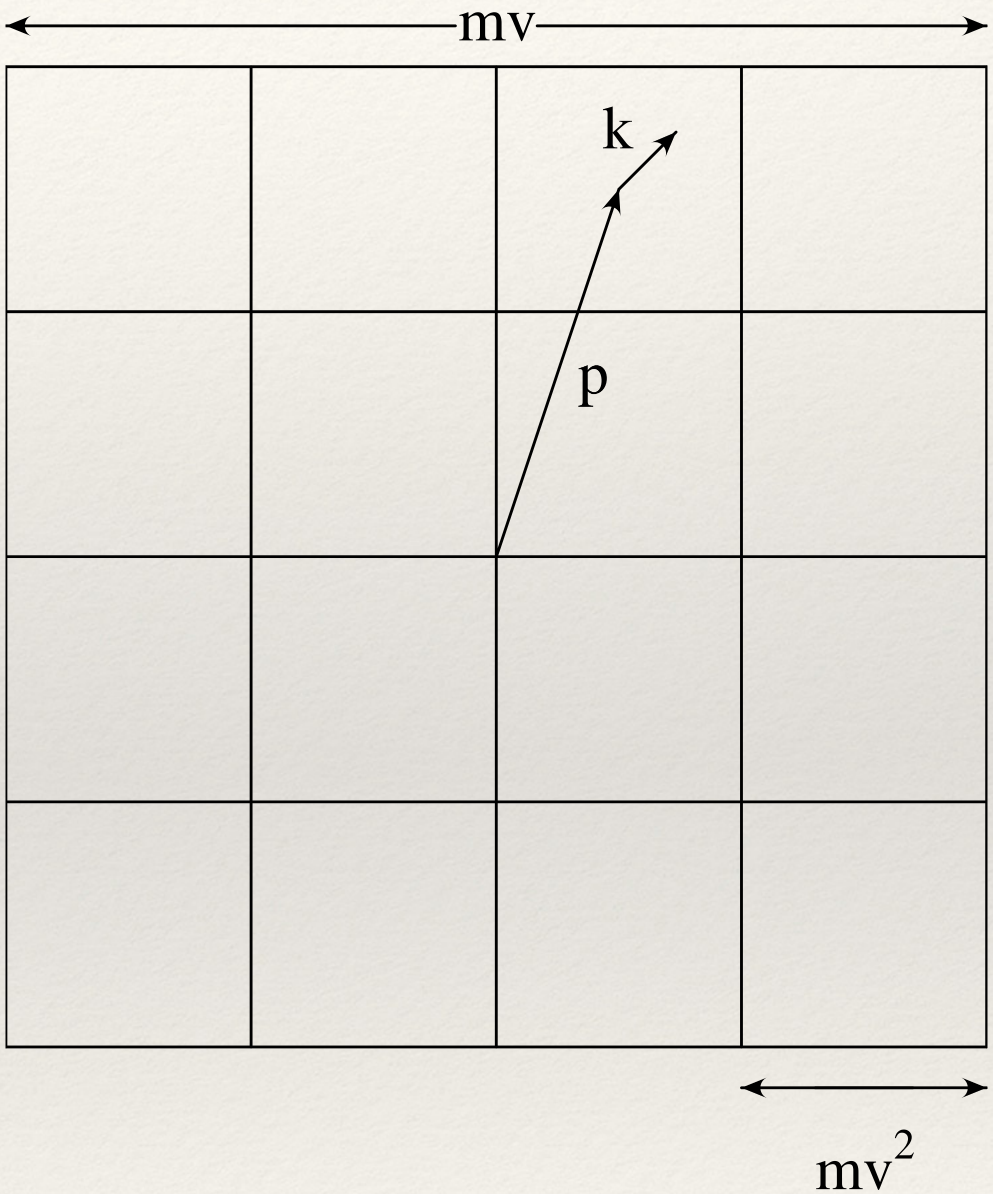
Nonrelativistic EFT with Virtual Photons

- Energy and momenta are different but correlated scales

$$v \sim p/M = \sqrt{E/M}$$

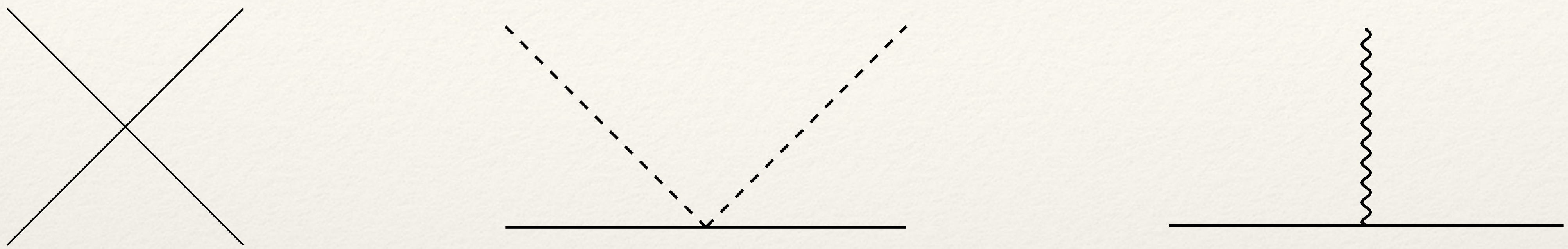
- Homogeneous power counting requires mode separation

$$\text{potential} \sim (Mv^2, Mv) : N_{\mathbf{p}}$$
$$\text{soft} \sim (Mv, Mv) : A_p$$
$$\text{ultrasoft} \sim (Mv^2, Mv^2) : A$$



Caswell, Lepage, Labelle, Luke, Manohar, Rothstein, Savage, Grießhammer, Grinstein, Stewart, Hoang, Pineda, Soto, Brambilla, Vairo

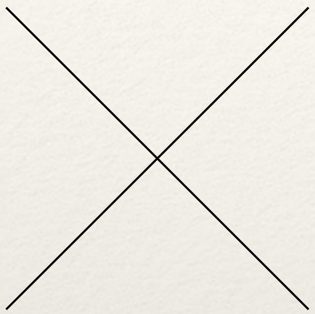
Velocity EFT Lagrangian



$$
\begin{aligned}
\mathcal{L} = & \sum_p N_p^\dagger \left(iD_0 - \frac{(p - iD)^2}{2M_N} \right) N_p - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
& + \sum_p \left| p^\mu A_p^\nu - p^\nu A_p^\mu \right|^2 - \sum_{p',p} V(p', p) \\
& - \frac{4\pi\alpha}{2M_N} \sum_{q,q',p,p'} A_{q'} \cdot A_q N_{p'}^\dagger Q N_p \\
& + \frac{e}{2M_N} \epsilon^{ijk} \left(\nabla^j A^k \right) \sum_p N_p^\dagger \sigma^i \left[\kappa_0 + \kappa_1 \tau^3 \right] N_p \,,
\end{aligned}
$$

TRR and Reis, FBS 65

Neutron-Proton Potential



$$V_{pn} = \sum_{v=-1} \sum_{p',p} V^{(v)}_{abcd}(p',p) p^\dagger_{p',a} p_{p,b} n^\dagger_{-p',c} n_{-p,d}$$

$$V^{(-1)}_{abcd} = C^{(S=1)}_{0,pn} P^{(1)}_{ab,cd} + C^{(S=0)}_{0,pn} P^{(0)}_{ab,cd}$$

$$V^{(0)}_{abcd} = \frac{1}{2} \left(p'^2 + p^2\right) \left[C^{(S=1)}_{2,pn} P^{(1)}_{ab,cd} + C^{(S=0)}_{2,pn} P^{(0)}_{ab,cd}\right]$$

$$V^{(1)}_{abcd} = \frac{1}{4} \left(p'^2 + p^2\right)^2 \left[C^{(S=1)}_{4,pn} P^{(1)}_{ab,cd} + C^{(S=0)}_{4,pn} P^{(0)}_{ab,cd}\right]$$

Kaplan, Savage, Wise NPB 478, PLB 424, NPB 534
van Kolck NPA 645

Proton-proton potential

$$V_{pp} = \sum_{v=-2} \sum_{\mathbf{p}',\mathbf{p}} V^{(v)}_{abcd}(\mathbf{p}', \mathbf{p}) p^\dagger_{\mathbf{p}',a} p^\dagger_{-\mathbf{p}',c} p_{-\mathbf{p}d} p_{\mathbf{p},b}$$

$$V^{(-2)}_{abcd}(\mathbf{p}', \mathbf{p}) = \frac{4\pi\alpha}{(\mathbf{p}' - \mathbf{p})^2} \delta_{ab} \delta_{cd}$$

$$V^{(0)}_{abcd}(\mathbf{p}', \mathbf{p}) = \frac{1}{4} C^{(pp)}_0 \sigma^2_{ac} \sigma^2_{db}$$

"Matching" to AV18

- The LECs need to be matched at $\mu = m_\pi$ at $O(\alpha^0)$

$$C_0^{(s)}(\mu_{\rm match} = m_\pi) = \frac{4\pi a_s}{M_N}$$

$$C_2^{(s)}(\mu_{\rm match} = m_\pi) = \frac{2\pi a_s^2 r_s}{M_N} \qquad C_4^{(s)}(\mu_{\rm match} = m_\pi) = \frac{\pi a_s^3 r_s^2}{M_N}$$

- Use Argonne v18 with $\alpha = 0$ as proxy for lattice QCD—"integrate out the pion"

$$a_0 = -23.084\,\text{fm} \qquad r_0 = 2.703\,\text{fm} \qquad a_{pp} = -17.164\,\text{fm}$$
$$a_1 = 5.402\,\text{fm} \qquad r_1 = 1.752\,\text{fm} \qquad r_{pp} = 2.865\,\text{fm}$$

- Next: Run μ from $m_\pi \to p$ at $O(\alpha)$ and calculate matrix elements

Wirigna et al. PRC 51

Summing Logs in Bound States

1. Obtain anomalous dimensions for the couplings in perturbation theory

2. Integrate the RG equations from matching scale to typical bound state velocity

3. Calculate bound state matrix elements

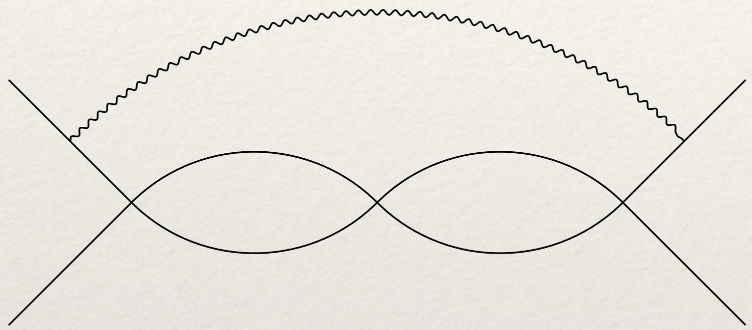
$$L \sim \log(m_L/m_H)$$

LO	1		
NLO	$\alpha\, L$	α	
NNLO	$\alpha^2\, L^2$	$\alpha^2\, L$	α^2
$\ldots$	$\ldots$	$\ldots$	$\ldots$
N^{k}LO	$\alpha^k\, L^k$	$\alpha^k\, L^{k-1}$	$\alpha^k\, L^{k-2}$

Cohen arXiv:1903.03622

Loop Corrections and the Velocity Renormalization Group

- Perturbation theory generates two (possibly) large logarithms



$$p^2 C_0^3 \left[\lambda_S \log\frac{\mu}{p} + \lambda_{US} \log\frac{\mu}{E}\right]$$

- Introduce two correlated scales in dimensional regularization

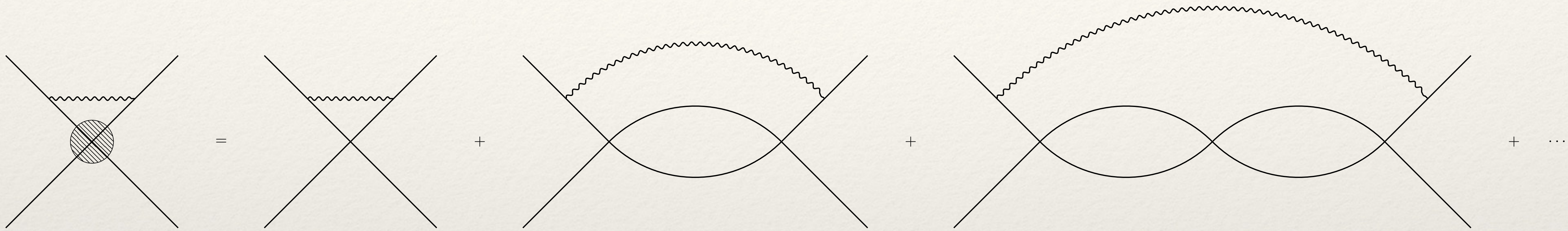
$$\mu_S = M_N \nu \qquad\qquad \mu_U = M_N \nu^2$$

- Run in ν—sum soft and ultrasoft logarithms simultaneously

$$\nu\frac{dV}{d\nu} = \gamma_S + 2\gamma_U \qquad \mu_U \frac{dV}{d\mu_U} = \gamma_U \qquad \mu_S \frac{dV}{d\mu_S} = \gamma_S$$

Luke et al. PRD 61

Velocity RG Solution

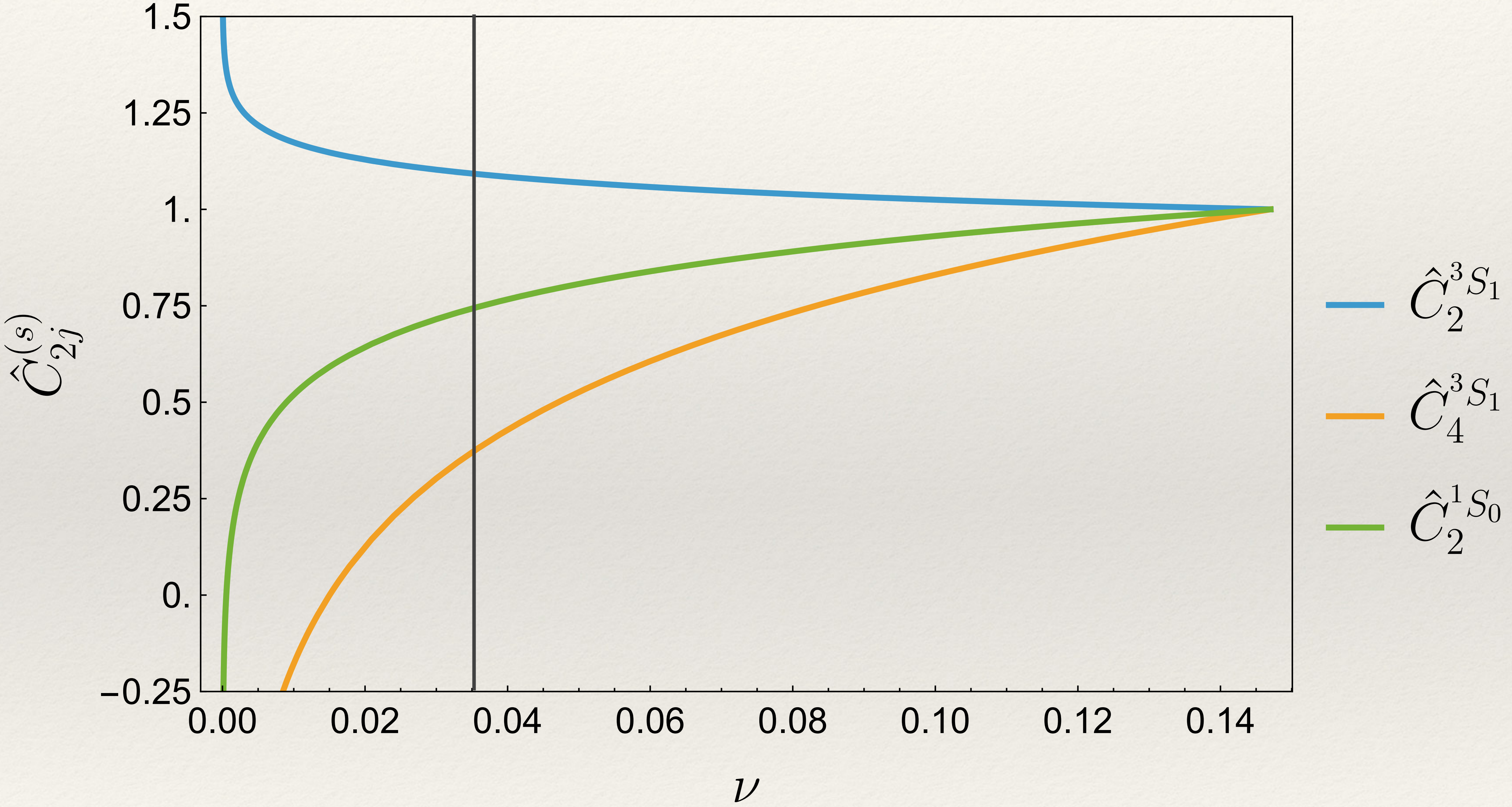


$$C_2(\nu) = C_2\left(\frac{m_\pi}{M_N}\right) - \frac{15}{4}\left(\frac{M_N}{4\pi}\right)^2 C_0^3 \log\left(\frac{\alpha(M_N\nu^2)}{\alpha(m_\pi^2/M_N)}\right)\,,$$

$$C_4(\nu) = C_4\left(\frac{m_\pi}{M_N}\right) + 4\left(\frac{M_N}{4\pi}\right)^4 C_0^5 \log\left(\frac{\alpha(M_N\nu^2)}{\alpha(m_\pi^2/M_N)}\right)$$

TRR and Reis, FBS 65

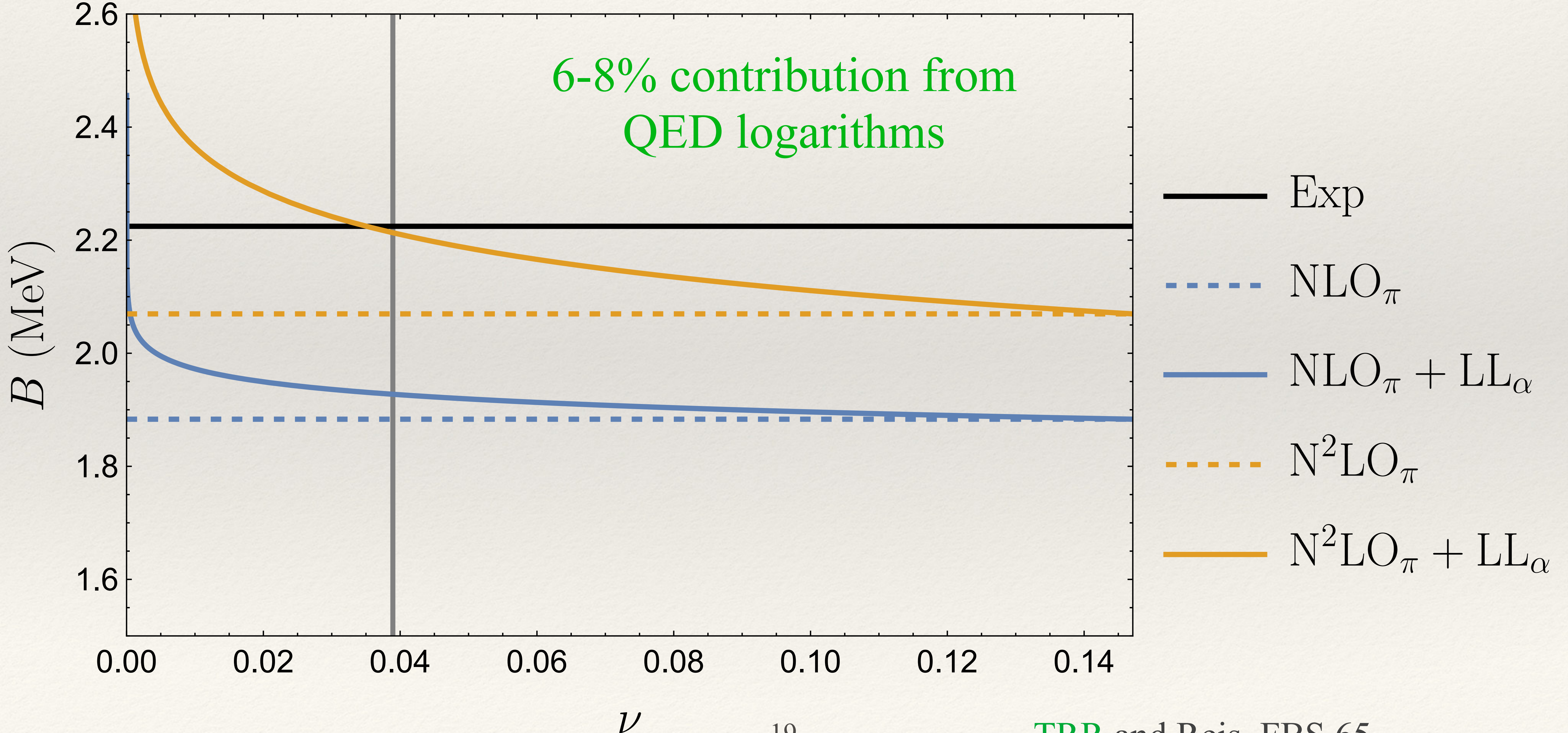
Neutron-proton potential



TRR and Reis, FBS 65

Deuteron Binding Energy

$$B = \frac{1}{M_N}\left(\frac{4\pi}{M_N C_0}\right)^2 + \frac{1}{2\pi}C_2\left(\frac{4\pi}{M_N C_0}\right)^5 + \frac{7}{16\pi^2}M_N C_2^2\left(\frac{4\pi}{M_N C_0}\right)^8 - \frac{1}{2\pi}C_4\left(\frac{4\pi}{M_N C_0}\right)^7$$

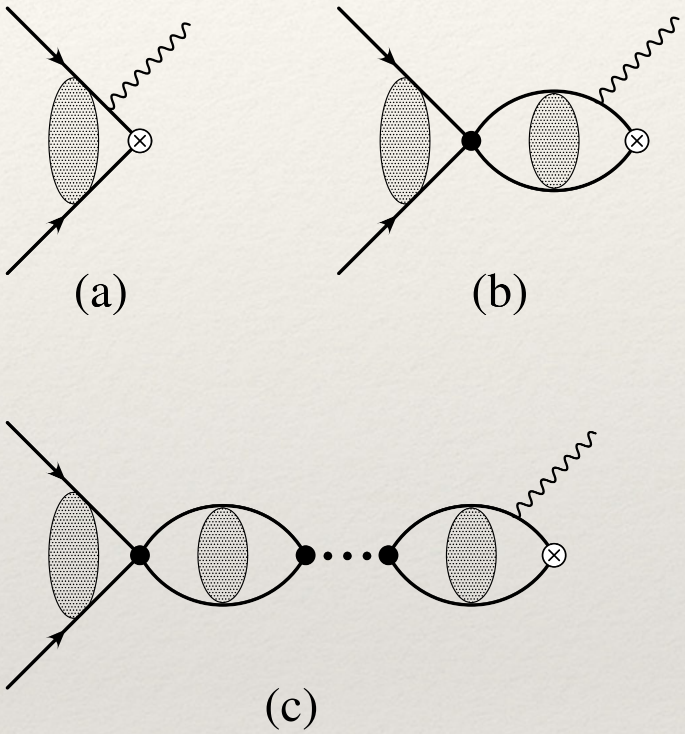


TRR and Reis, FBS 65

Electroweak Matrix Elements: A Case Study in Proton-Proton Fusion

- Typical proton momentum in the sun $p \sim 2.5$ MeV, $E_e \sim 1$ MeV

$$p < a_{pp}^{-1} \ll \sqrt{2M_r B_d} \ll m_\pi$$



- Coulomb interaction is nonperturbative for $v \sim \alpha$

$$A(p) \sim \int_{\vec{k}} \frac{1}{\mathbf{k}^2 + 2M_r B_d} \psi_{\mathbf{p}}(\mathbf{k}) \qquad B(p) \sim \int_{\mathbf{k}_1, \mathbf{k}_2} \frac{\langle \mathbf{k}_1 | G_C | \mathbf{k}_2 \rangle}{\mathbf{k}_1^2 + 2M_r B_d}$$

$$T \sim A + \frac{C_0^{(pp)}}{1 - C_0^{(pp)} J_0(p)} \lambda(p) B(p)$$

$$J_0(p) = i\frac{M_p p}{4\pi} + \frac{M_p^2 \alpha}{4\pi} \log\left(\frac{\mu^2}{-4p^2}\right) - \frac{M_p^2 \alpha}{4\pi}\left[\psi\left(i\frac{M_p \alpha}{2p}\right) + \gamma_E\right]$$

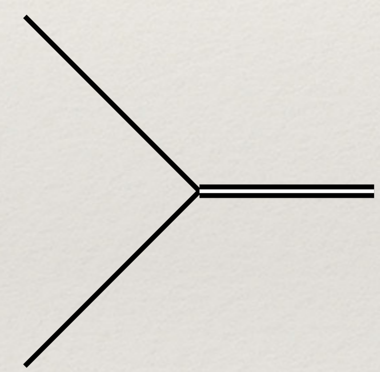
- Want to recover from an EFT perspective—expand in $p/\sqrt{2M_r B_d}$ and resum what has to be resummed

Kong and Ravndal PRC 64, NPA 656

Matching

- Treat deuteron like a fundamental particle
- Wilson coefficient given by hard part ($k_0 \sim \gamma_d$, $\mathbf{k} \sim \gamma_d$) of pionless result— perturbative in α and to all orders in $C_0^{(pp)}$

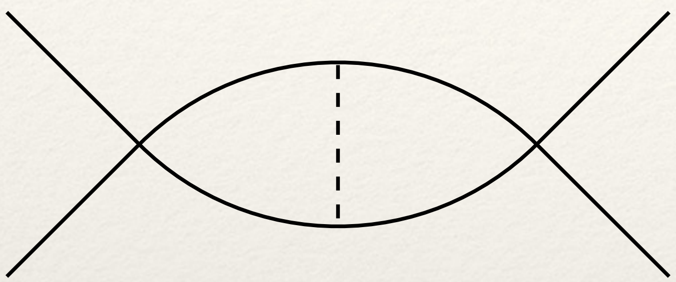
$$\mathcal{L} \supset \frac{1}{2} g_{\text{eff}} L_i \, d^{i\dagger} \left(p\sigma^2 p\right)$$



$$g_{\text{eff}}(\mu = \gamma_d) = \sqrt{\frac{8\pi}{\gamma_d^3}} \left(1 - \frac{M_p C_0^{(pp)}(\gamma_d)}{4\pi}\gamma_d + \frac{M_p \alpha(\gamma_d)}{\gamma_d}\right)$$

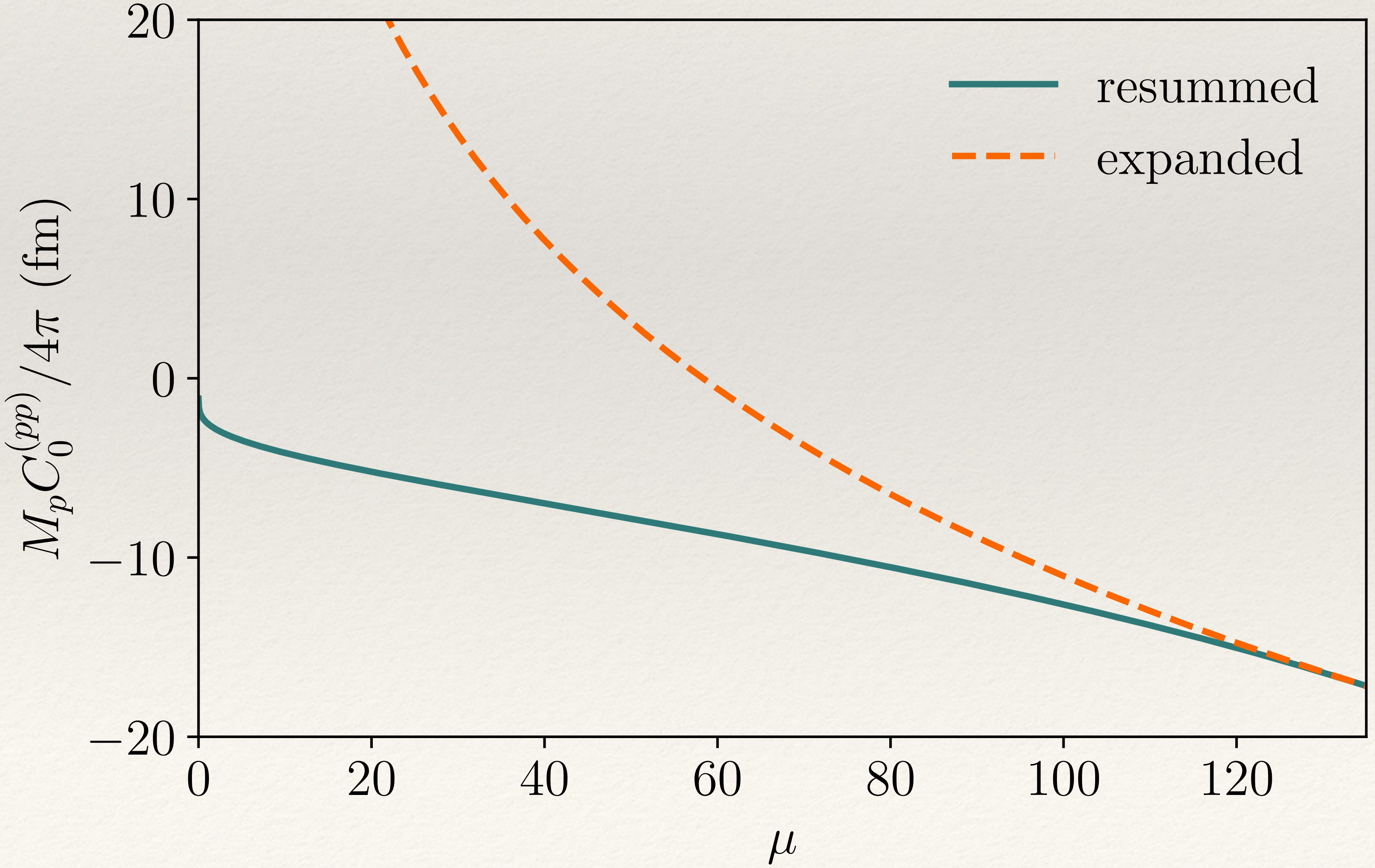
$$\gamma_d = \sqrt{2M_r B_d}$$

$C_0^{(pp)}$ Anomalous Dimension

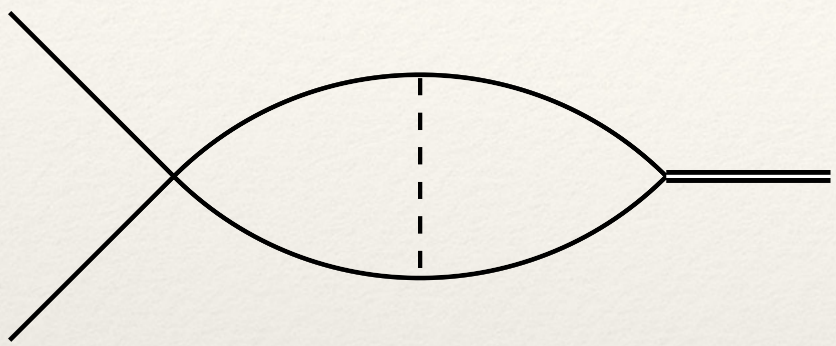


$$\beta(C_0^{(pp)}) = -\frac{M_p^2 \alpha}{2\pi} C_0^{(pp)2}$$

$$C_0^{(pp)} = \frac{C_0^{(pp)}(m_\pi)}{1 + \frac{3M_p^2}{4} C_0^{(pp)}(m_\pi) \log\left(\frac{\alpha(\mu)}{\alpha(m_\pi)}\right)}$$

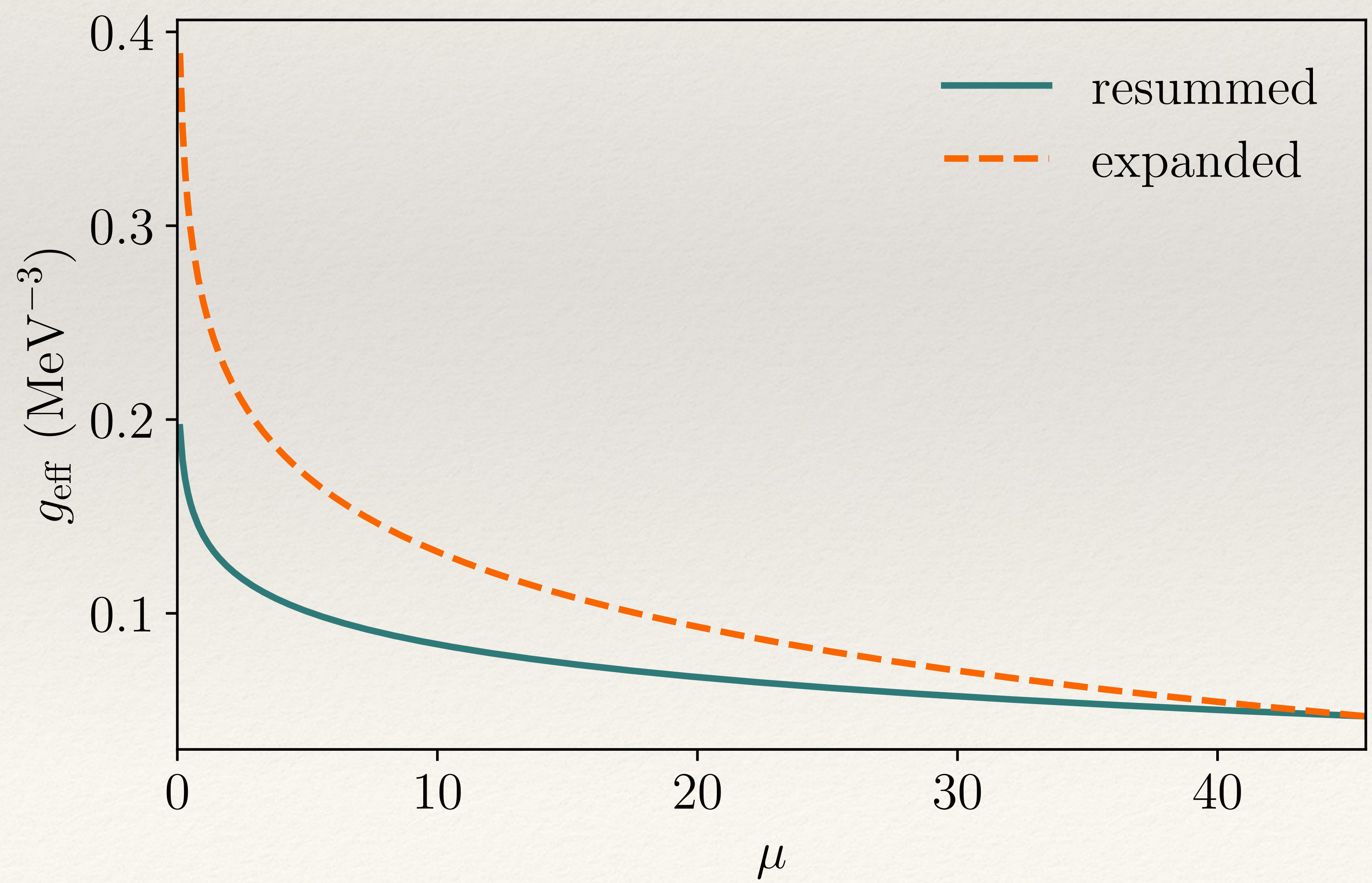


g_{eff} Anomalous Dimension

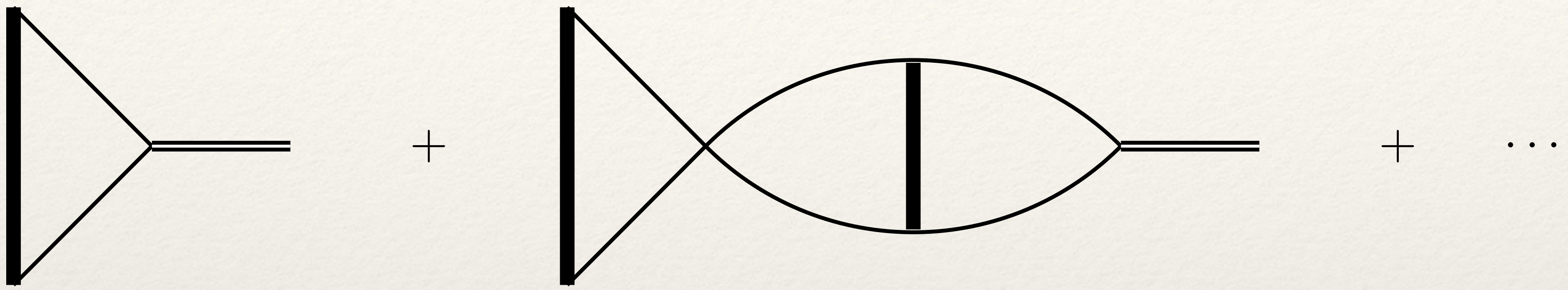


$$\beta(g_{\text{eff}}) = \frac{M_p^2 \alpha}{2\pi} C_0^{(pp)} g_{\text{eff}}$$

$$g_{\text{eff}}(\mu) = g(\gamma_d) \left[1 + \frac{3M_p^2}{4} C_0^{(pp)} \log \frac{\alpha(\mu)}{\alpha(\gamma_d)} \right]$$



EFT Amplitude

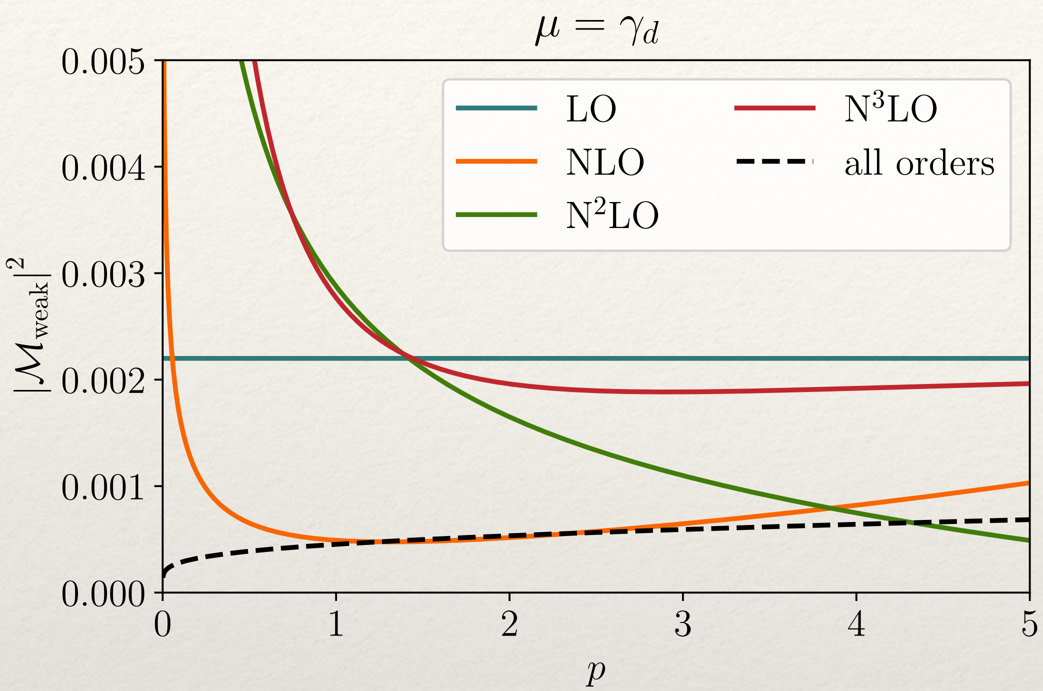


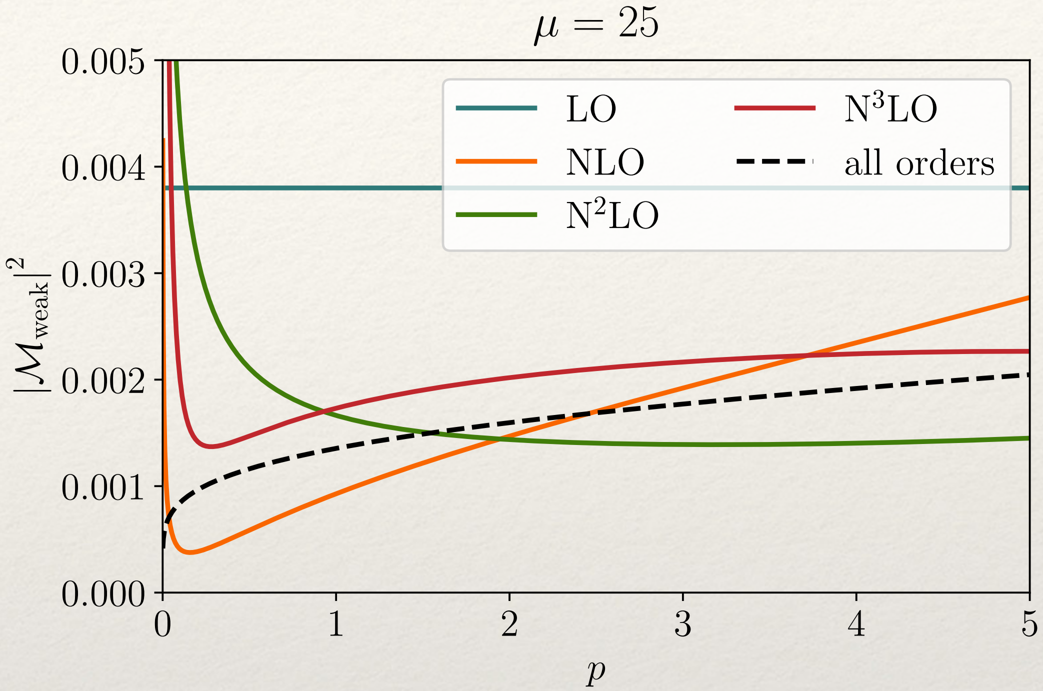
$$T = g_{\text{eff}}\lambda(p)\left[1 + C_0 J_0 + (C_0 J_0)^2 + \cdots\right]$$
$$= g_{\text{eff}}\lambda(p)\frac{1}{1 - C_0 J_0}$$

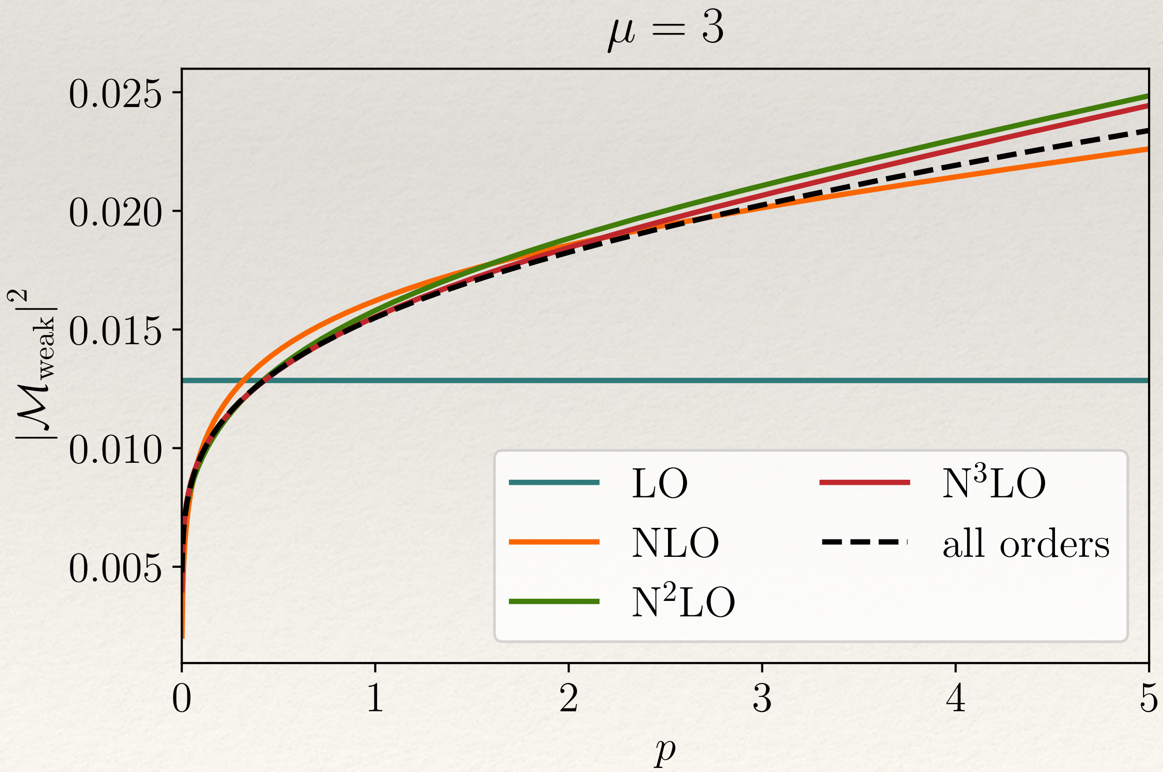
- Expected from EFT principles

$$\langle \mathcal{O}_{\text{full}} \rangle = C \langle \mathcal{O}_{\text{EFT}} \rangle$$

RG Improving the Amplitude







Uncertainty Estimate

- Matching $< 0.5\,\%$

  $\delta_{\rm NS}$ goes in the matching

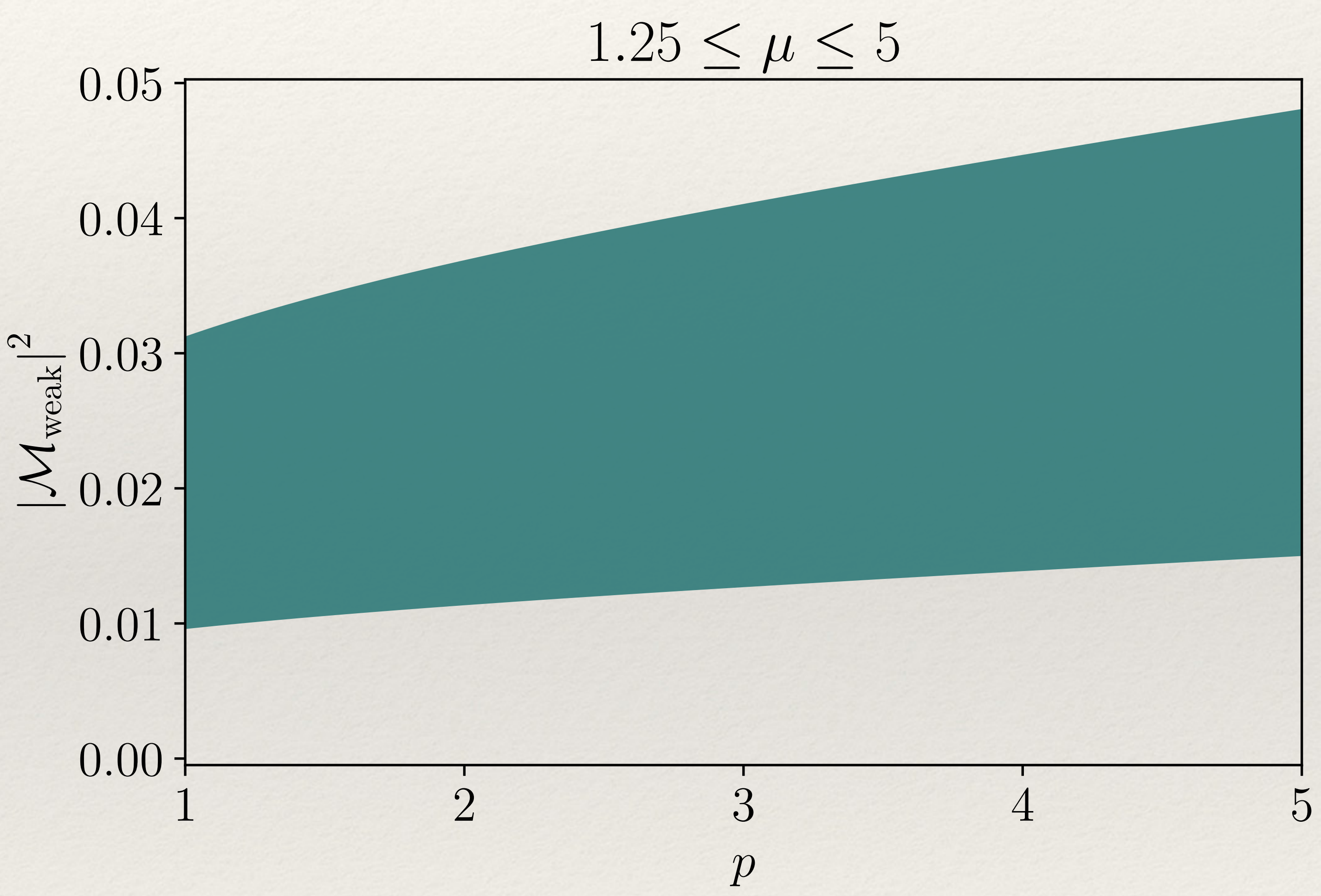
  Combes et al. PRC 110

- Ultrasoft $< 0.1\,\%$

- Soft $\Longrightarrow$ Fermi function and other 0.5 %

  Hill and Plestid PRL 133, PRD 109

- Scale variation $\sim 30\,\%$



Summary

- Nucleons + photons ≈ heavy quarkonium + gluons
- LL_α improved two nucleon interaction—few percent effect from QED logs in deuteron binding
- 30% uncertainty in pp fusion—improve with NLL_α? Power suppressed potentials?
- Embed correlated running in the superallowed framework—are intranuclear corrections understood?

  Cirigliano et al. PRL 133
- Revisit corrections in tritium—KATRIN, Project 8