Renormalization Group Improvement and the Positive Side of Nuclear Effective Field Theories

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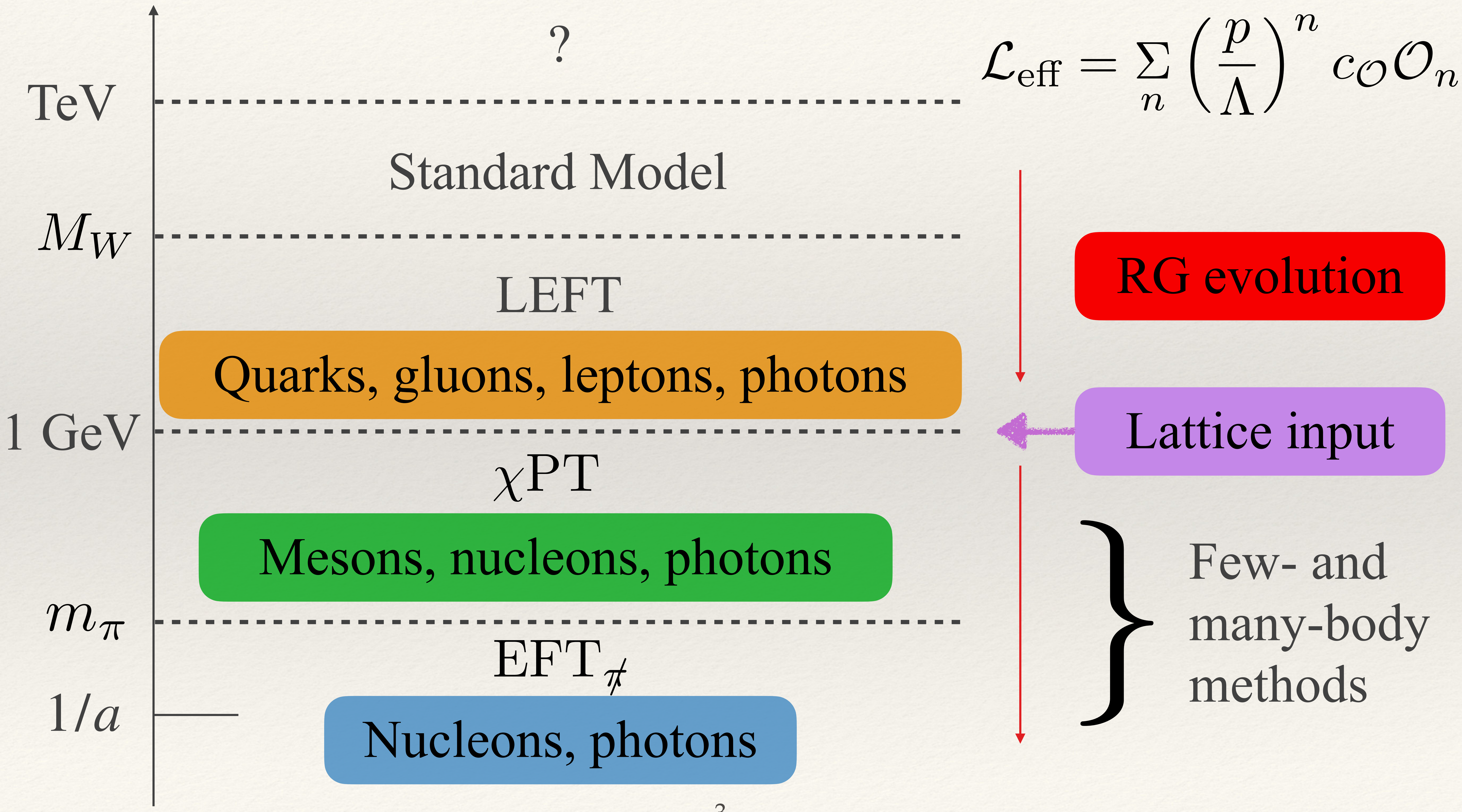
Part I: Renormalization Group Improvement of Nuclear EFT

In collaboration with Immo Reis

Few Body Syst. 65 (2024) 3, 79

Phys. Rev. C 111 (2025) 6, 064001

Effective Field Theory



$$\mathcal{L}_{\text{eff}} = \sum_n \left(\frac{p}{\Lambda}\right)^n c_{\mathcal{O}} \mathcal{O}_n$$

EFT Requirements

1. All particles that are near the mass-shell are included

2. Homogeneous power counting governed by a single ratio of scales

3. Renormalization group invariance up to the order we are working

4. Symmetries (gauge, discrete, internal) are preserved

Nonrelativistic EFT with Virtual Photons

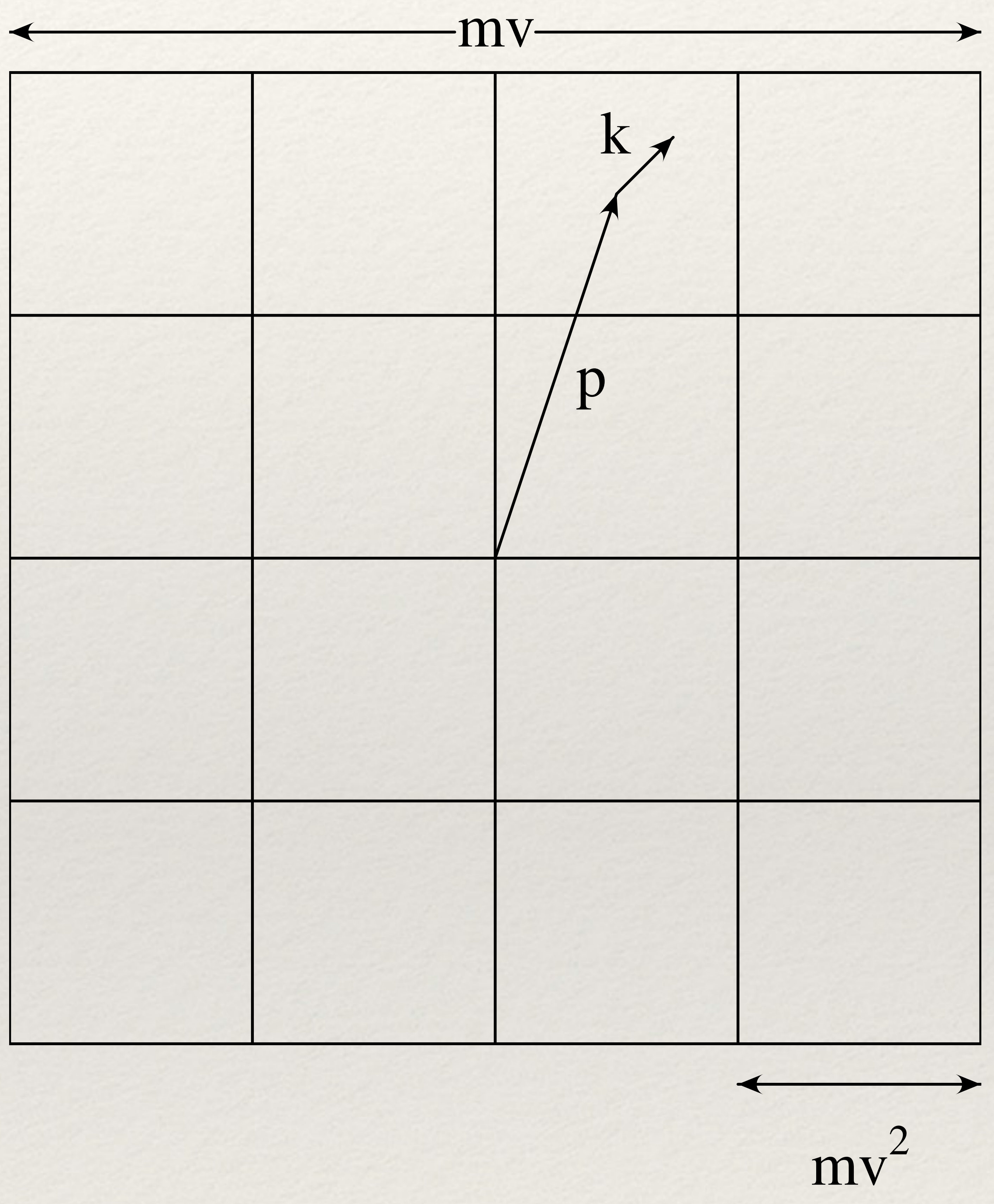
- Bound state properties are easier with nonrelativistic theories

- Energy and momenta are different but correlated scales

$$v \sim p/M = \sqrt{E/M}$$

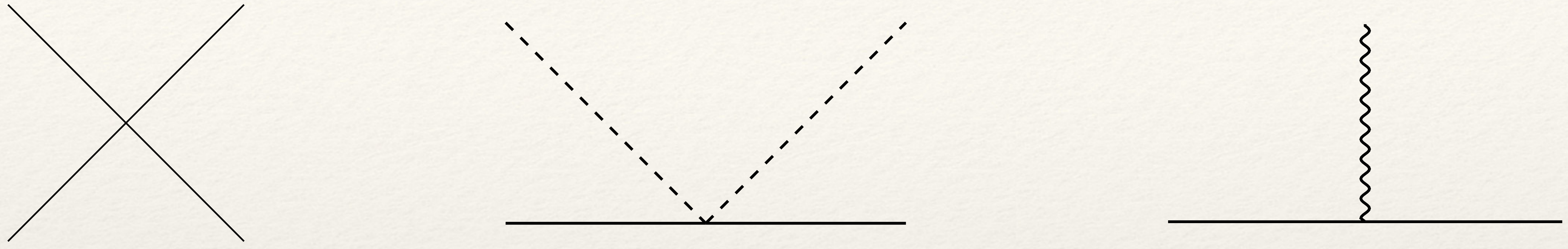
- Homogeneous power counting requires mode separation

$$\begin{aligned} \text{potential} &\sim (Mv^2, Mv) : N_{\mathbf{p}} \\ \text{soft} &\sim (Mv, Mv) : A_p \\ \text{ultrasoft} &\sim (Mv^2, Mv^2) : A \end{aligned}$$



Caswell, Lepage, Labelle, Luke, Manohar, Rothstein, Savage, Grießhammer, Grinstein, Stewart, Hoang, Pineda, Soto, Brambilla, Vairo

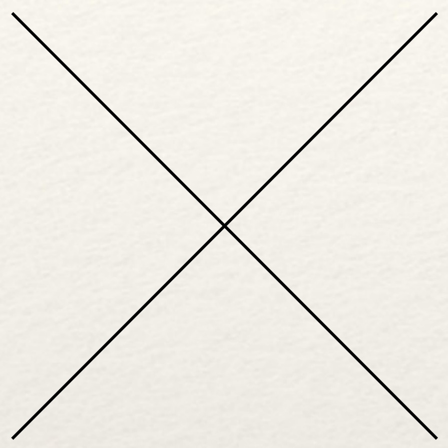
Velocity EFT Lagrangian



$$\mathcal{L} = \sum_p N_p^\dagger \left(iD_0 - \frac{(p - iD)^2}{2M_N} \right) N_p - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_p \left| p^\mu A_p^\nu - p^\nu A_p^\mu \right|^2$$
$$- \sum_{p',p} V(p',p) - \frac{4\pi\alpha}{2M_N} \sum_{q,q',p,p'} A_{q'} \cdot A_q N_{p'}^\dagger Q N_p$$
$$+ \frac{e}{2M_N} \epsilon^{ijk} \left(\nabla^j A^k \right) \sum_p N_p^\dagger \sigma^i \left[\kappa_0 + \kappa_1 \tau^3 \right] N_p \,,$$

TRR and Reis, FBS 65

Neutron-Proton Potential



$$V_{pn} = \sum_{v=-1} \sum_{p',p} V^{(v)}_{abcd}(p',p) p^{\dagger}_{p',a} p_{p,b} n^{\dagger}_{-p',c} n_{-p,d}$$

$$V^{(-1)}_{abcd} = C^{(S=1)}_{0,pn} P^{(1)}_{ab,cd} + C^{(S=0)}_{0,pn} P^{(0)}_{ab,cd}$$

$$V^{(0)}_{abcd} = \frac{1}{2}\left(p'^2 + p^2\right)\left[C^{(S=1)}_{2,pn} P^{(1)}_{ab,cd} + C^{(S=0)}_{2,pn} P^{(0)}_{ab,cd}\right]$$

$$V^{(1)}_{abcd} = \frac{1}{4}\left(p'^2 + p^2\right)^2\left[C^{(S=1)}_{4,pn} P^{(1)}_{ab,cd} + C^{(S=0)}_{4,pn} P^{(0)}_{ab,cd}\right]$$

Kaplan, Savage, Wise NPB 478, PLB 424, NPB 534
van Kolck NPA 645

“Matching” to AV18

- The LECs need to be matched at $\mu = m_\pi$ at $O(\alpha^0)$

$$C_0^{(s)}(\mu_{\mathrm{match}} = m_\pi) = \frac{4\pi a_s}{M_N}$$

$$C_2^{(s)}(\mu_{\mathrm{match}} = m_\pi) = \frac{2\pi a_s^2 r_s}{M_N} \qquad C_4^{(s)}(\mu_{\mathrm{match}} = m_\pi) = \frac{\pi a_s^3 r_s^2}{M_N}$$

- Use Argonne v18 as proxy for lattice QCD—“integrate out the pion”

$$a_0 = -23.084\,\mathrm{fm} \qquad r_0 = 2.703\,\mathrm{fm}$$

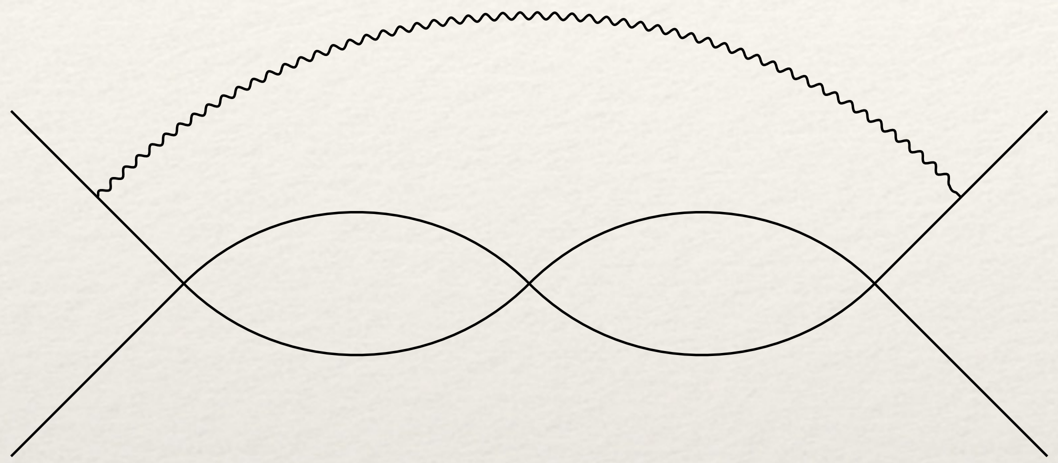
$$a_1 = 5.402\,\mathrm{fm} \qquad r_1 = 1.752\,\mathrm{fm}$$

- Next: Run μ from $m_\pi \to p$ at $O(\alpha)$ and calculate matrix elements

Wirigna et al. PRC 51

Loop Corrections and the Velocity Renormalization Group

- Perturbation theory generates two (possibly) large logarithms



$$p^2 C_0^3 \left[\lambda_S \log \frac{\mu}{p} + \lambda_{US} \log \frac{\mu}{E} \right]$$

- Introduce two correlated scales in dimensional regularization

$$\mu_S = M_N \nu \qquad \mu_U = M_N \nu^2$$

- Run in ν—sum soft and ultrasoft logarithms simultaneously

$$\nu \frac{dV}{d\nu} = \gamma_S + 2\gamma_U \qquad \mu_U \frac{dV}{d\mu_U} = \gamma_U \qquad \mu_S \frac{dV}{d\mu_S} = \gamma_S$$

Luke et al. PRD 61

Summing Logarithms with the RG

1. Obtain anomalous dimensions for the couplings in perturbation theory

2. Integrate the RG equations from matching scale to typical bound state velocity

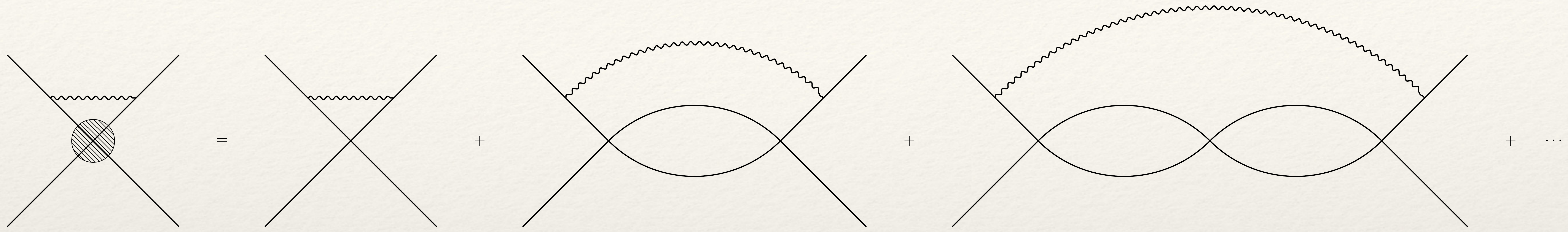
3. Calculate bound state matrix elements

$$L \sim \log(m_L/m_H)$$

LO	1		
NLO	$\alpha\, L$	α	
NNLO	$\alpha^2\, L^2$	$\alpha^2\, L$	α^2
$\ldots$	$\ldots$	$\ldots$	$\ldots$
N^{k}LO	$\alpha^k\, L^k$	$\alpha^k\, L^{k-1}$	$\alpha^k\, L^{k-2}$

Cohen arXiv:1903.03622

Velocity RG Solution



$$C_2(\nu) = C_2\left(\frac{m_\pi}{M_N}\right) - \frac{27}{8}\left(\frac{M_N}{4\pi}\right)^2 C_0^3 \log\left(\frac{\alpha(M_N\nu^2)}{\alpha(m_\pi^2/M_N)}\right) ,$$

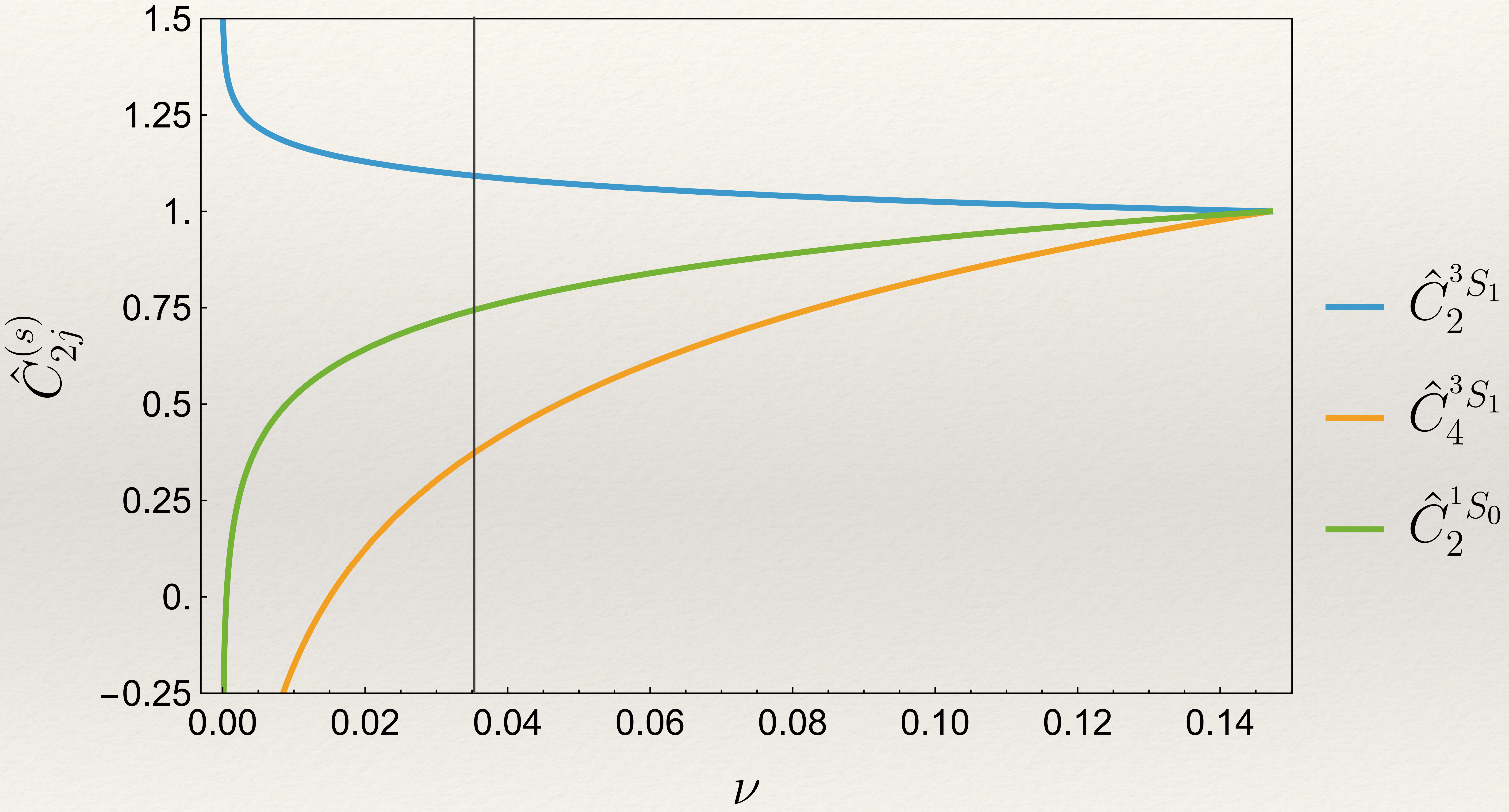
$$C_4(\nu) = C_4\left(\frac{m_\pi}{M_N}\right) + \frac{15}{4}\left(\frac{M_N}{4\pi}\right)^4 C_0^5 \log\left(\frac{\alpha(M_N\nu^2)}{\alpha(m_\pi^2/M_N)}\right)$$

Expanding logs gives complete sum $\sum_n \alpha^n \log^n(\nu)$

Varying ν probes higher-order logarithmic series

TRR and Reis, FBS 65

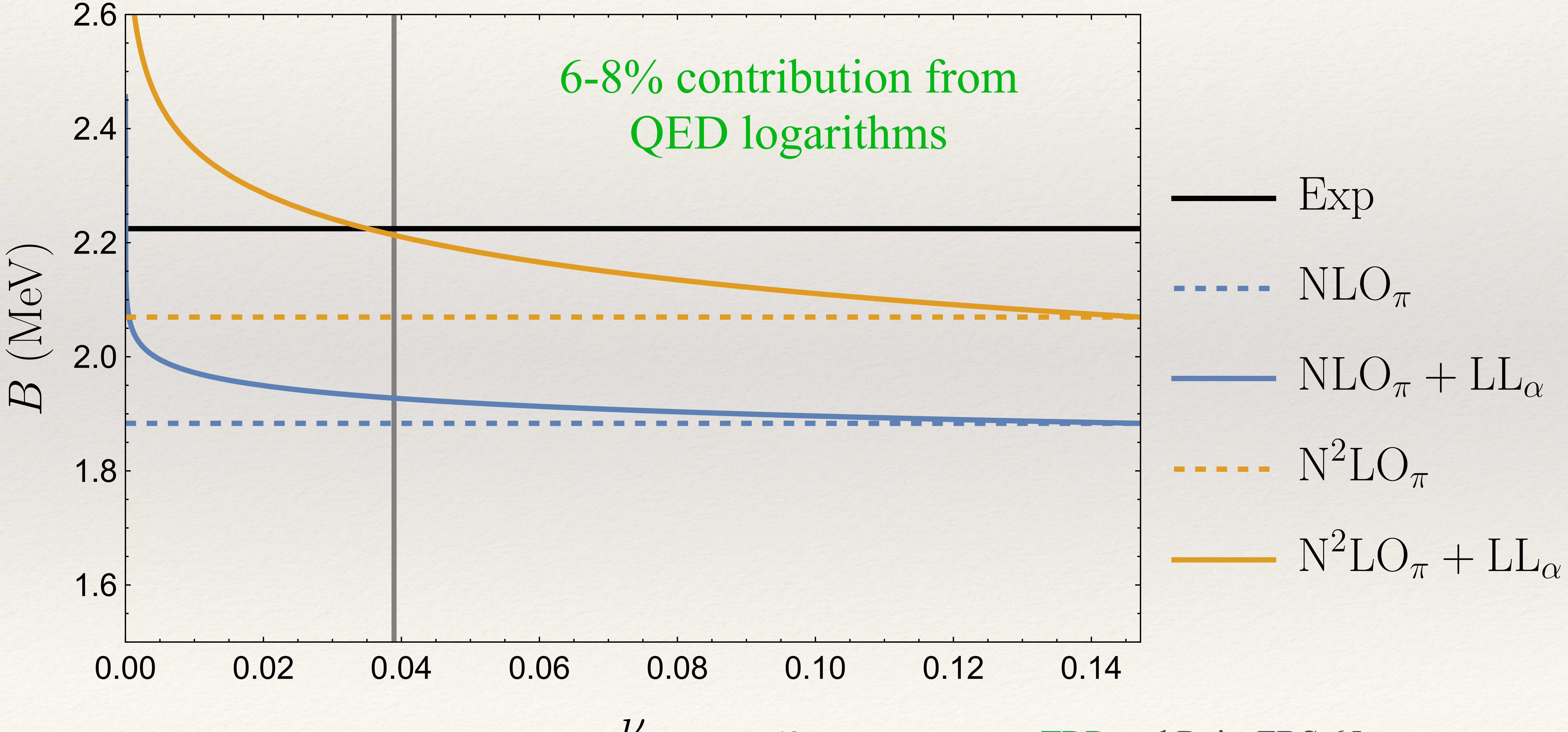
Running potentials



TRR and Reis, FBS 65

Deuteron Binding Energy

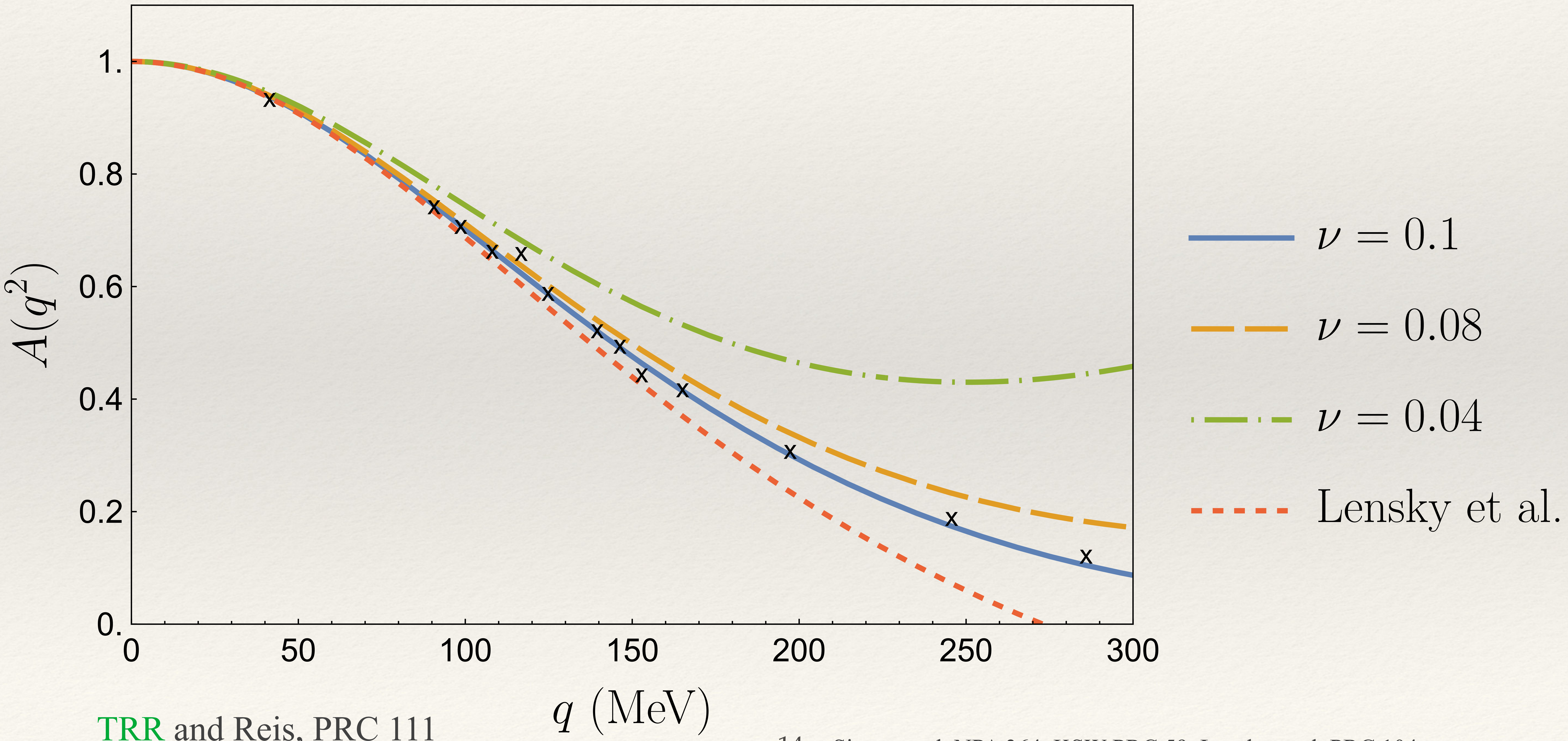
$$B = \frac{1}{M_N}\left(\frac{4\pi}{M_N C_0}\right)^2 + \frac{1}{2\pi}C_2\left(\frac{4\pi}{M_N C_0}\right)^5 + \frac{7}{16\pi^2}M_N C_2^2\left(\frac{4\pi}{M_N C_0}\right)^8 - \frac{1}{2\pi}C_4\left(\frac{4\pi}{M_N C_0}\right)^7$$



TRR and Reis, FBS 65

Electron Scattering Structure Function

$$A(q^2) = F_C^2(q^2) + \frac{2}{3}\eta F_M^2(q^2) + \frac{8}{9}\eta^2 F_Q^2(q^2)$$



TRR and Reis, PRC 111

Simon et al. NPA 364, KSW PRC 59, Lensky et al. PRC 104

Charge Radius

- Conventional definition of charge radius is not scale-independent at $O(\alpha)$ and higher

$$\langle r_d^2 \rangle_C = -6 \frac{dF_C(q^2)}{dq^2}\bigg|_{q^2=0}$$

$$r_{\mathrm{CREMA}} = 2.12562(13)_{\mathrm{exp}}(77)_{\mathrm{th}}$$

CREMA Science 353

		$\nu = \frac{m_\pi}{M_N}$	$\nu = 0.1$	$\nu = 0.06$
r_d (fm)	no truncation error	2.154(6)	2.111(6)	2.049(7)
	with truncation error	2.15(6)	2.11(6)	2.05(6)

2% shift

Radiative Neutron Capture

- First step in Big Bang Nucleosynthesis network

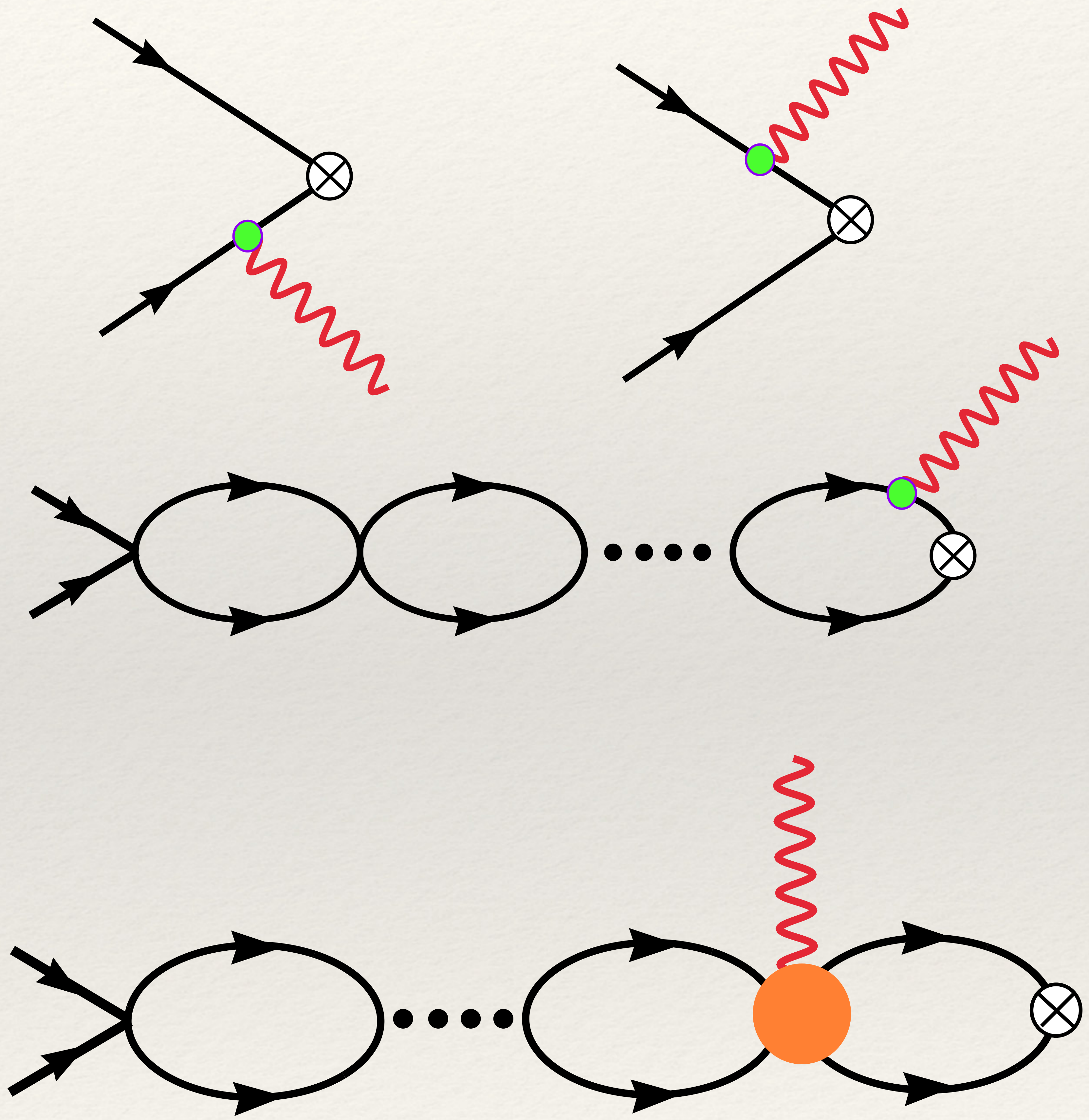
Wagoner, Fowler, Hoyle, Steigman,
Iocco, Mangano, Miele, Pisanti,
Serpico, Cyburt, Fields, Olive, Yeh

- Uncertainty in cross section sets scale for uncertainty in light element abundances

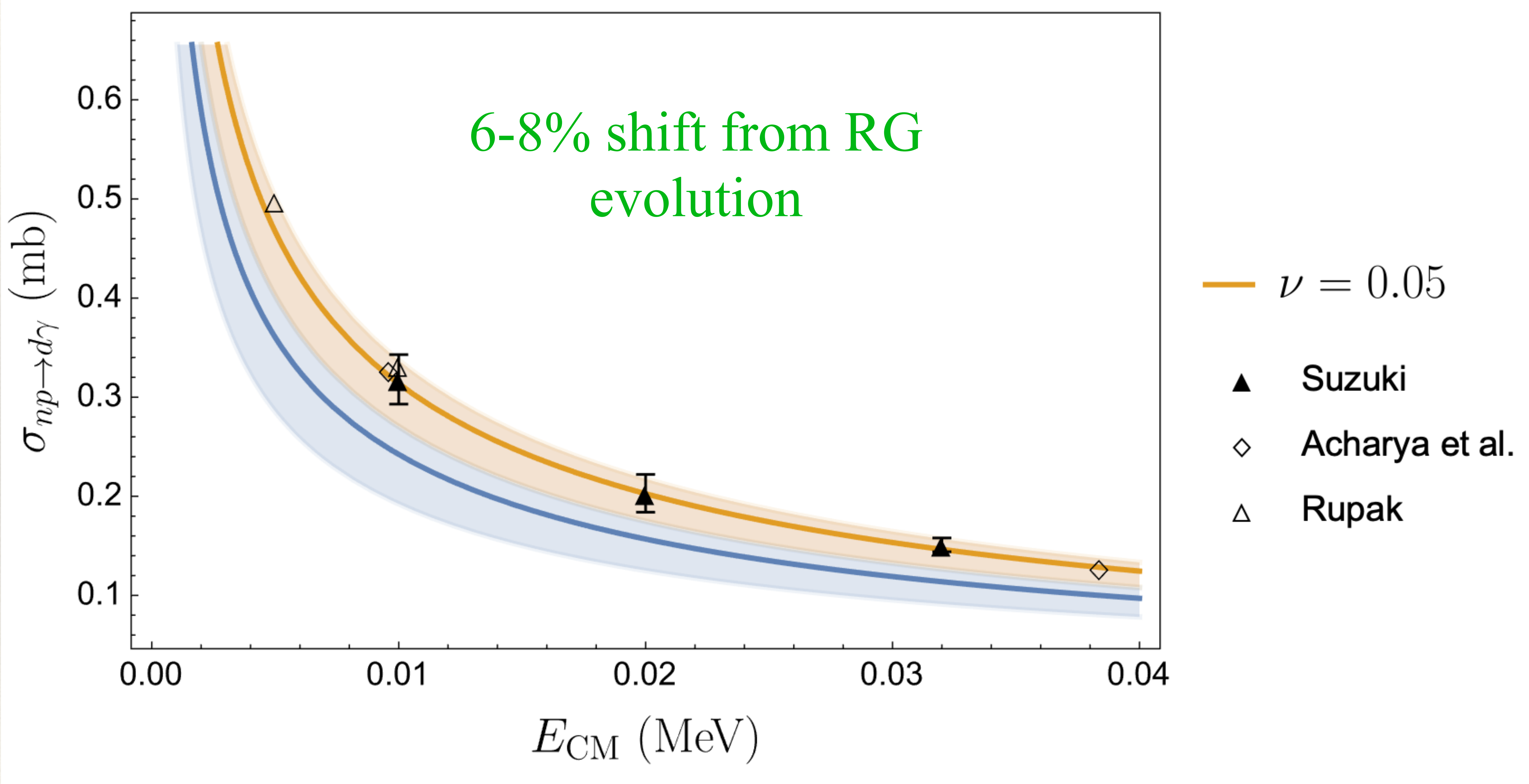
- Could be a probe of new physics with precise Standard Model predictions

- EFT analyses claim O(1%) uncertainty

Chen et al. NPA, 653, Rupak NPA 678, Acharya and Bacca PLB 827



Radiative Neutron Capture



Cox et al. NP 74, Suzuki et al. 1995, Nagai et al. PRC 56, Tudoric-Ghemo, NPA 92, Bosman et al. PLB 82, Steiler et al. 1986, Michel et al. JPG 1989

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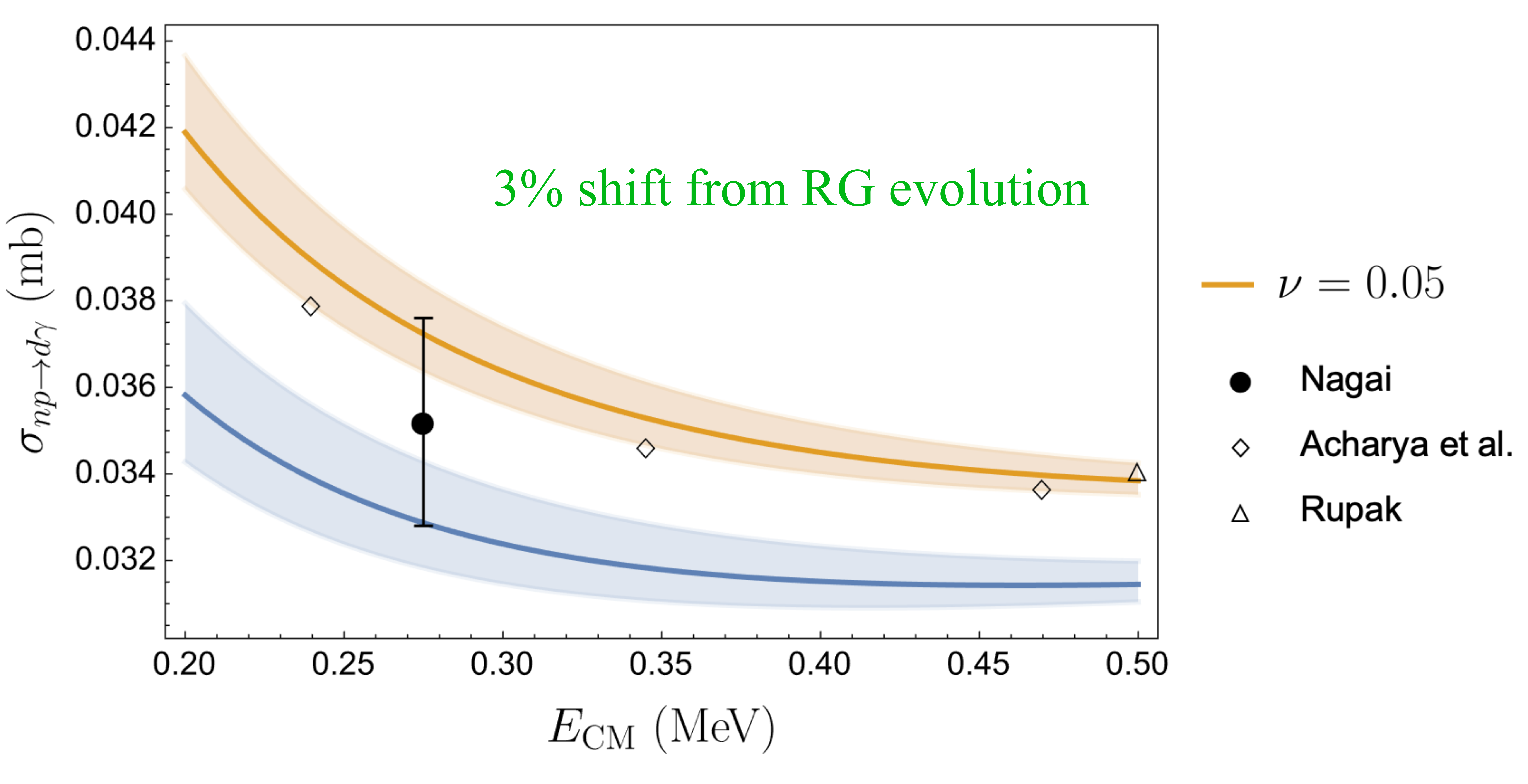

Cox et al. NP 74, Suzuki et al. 1995, Nagai et al. PRC 56, Tudoric-Ghemo, NPA 92, Bosman et al. PLB 82, Steiler et al. 1986, Michel et al. JPG 1989

Part I Summary

- Many lattice QCD results (mesons, baryons) are at the precision where QED needs to be considered
  - RG improvement seems to suggest a similar scenario in EFT
- What precision should lattice community aim for in two-baryon sector?
  - Maybe a few percent is ok?
- How can these effects be incorporated into many-body calculations?
  - Translate to cutoff regulator—(an aside) what about bump function (smooth, compact) regulators?