Few-Body Current in Effective Field Theory for Neutrinoless Double Beta Decay

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New Physics Searches at the Precision Frontier
Institute for Nuclear Theory
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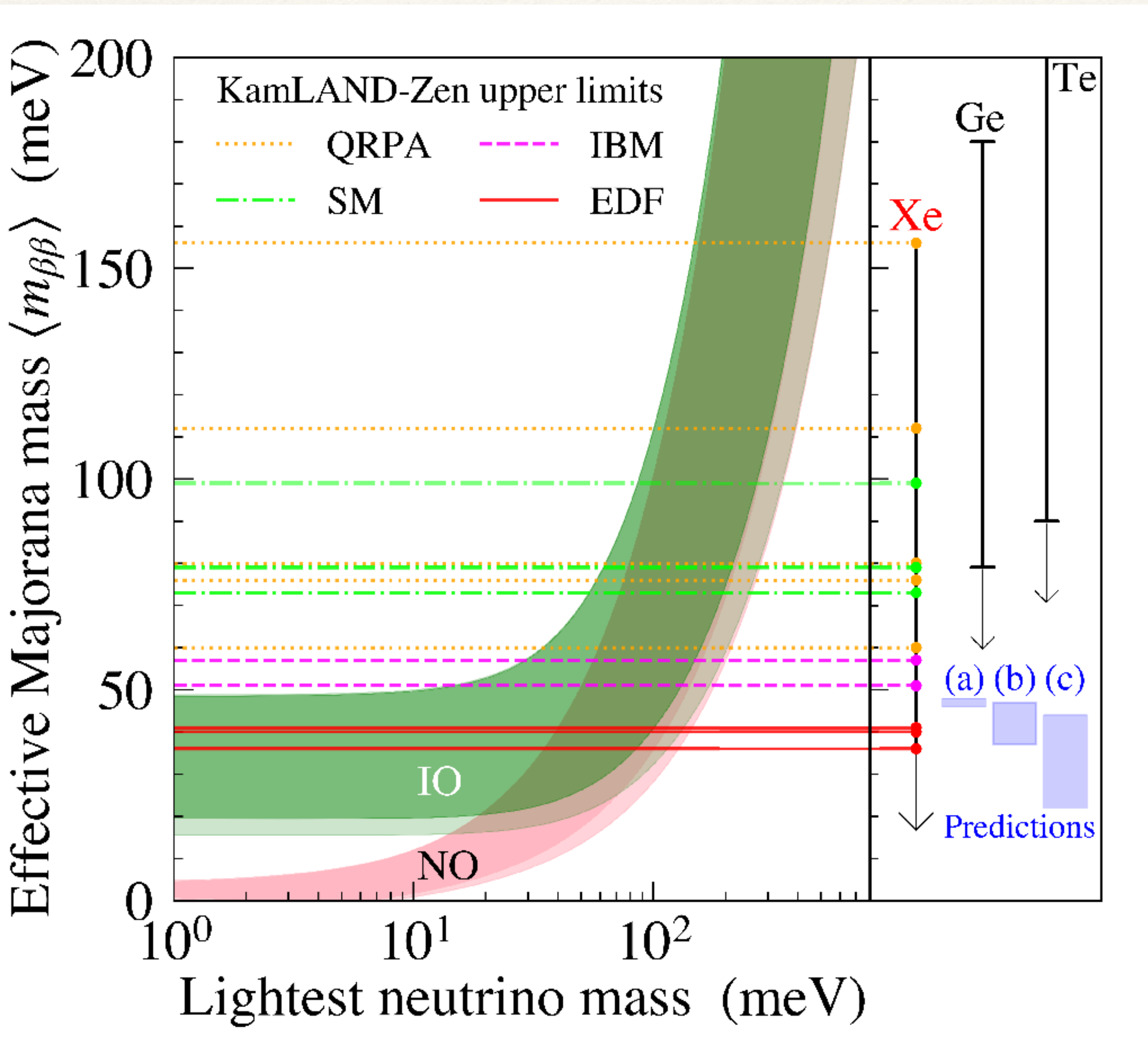
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

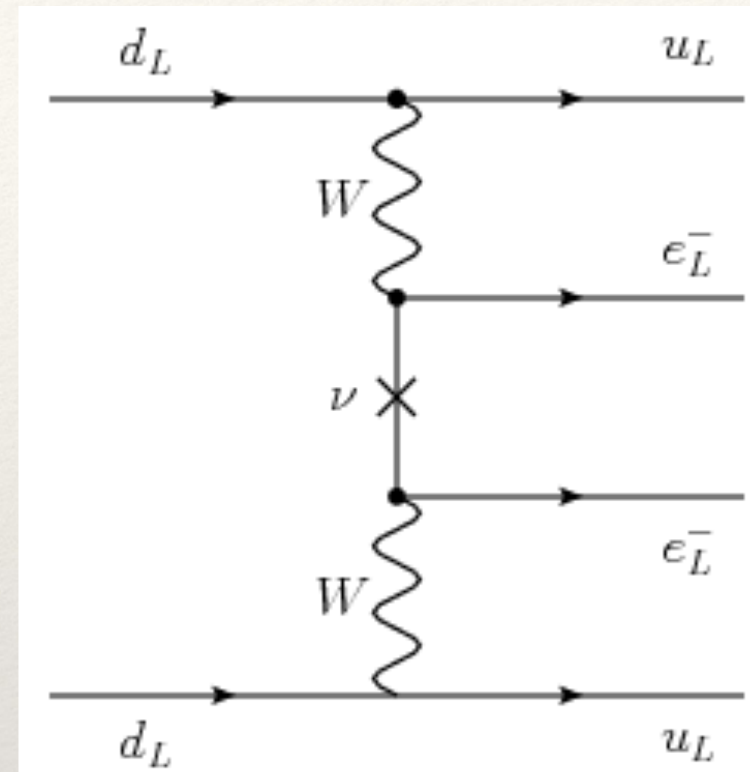
JG|U

Cluster of Excellence PRiSMA+ Precision Physics, Fundamental Interactions and Structure of Matter

Experimental Searches

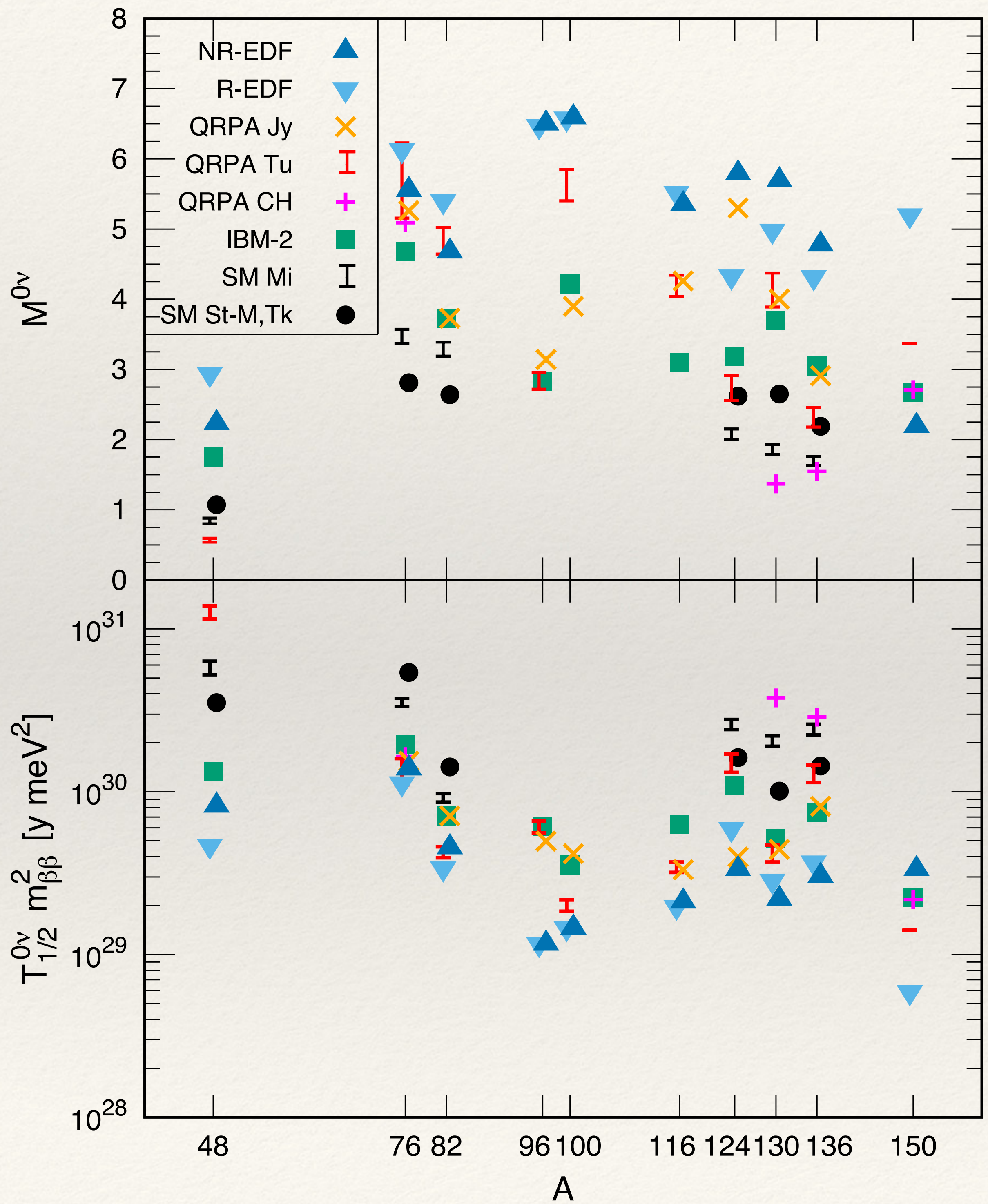


KamLAND-Zen PRL 130



Isotope	$T^{0\nu}_{1/2}$ ($\times 10^{25}$ y)	$\langle m_{\beta\beta} \rangle$ (eV)	Experiment
^{48}Ca	$> 5.8 \times 10^{-3}$	$< 3.5 - 22$	ELEGANT-IV
^{76}Ge	> 8.0	$< 0.12 - 0.26$	GERDA
	> 1.9	$< 0.24 - 0.52$	MAJORANA DEMONSTRATOR
^{82}Se	$> 3.6 \times 10^{-2}$	$< 0.89 - 2.43$	NEMO-3
^{96}Zr	$> 9.2 \times 10^{-4}$	$< 7.2 - 19.5$	NEMO-3
^{100}Mo	$> 1.1 \times 10^{-1}$	$< 0.33 - 0.62$	NEMO-3
^{116}Cd	$> 1.0 \times 10^{-2}$	$< 1.4 - 2.5$	NEMO-3
^{128}Te	$> 1.1 \times 10^{-2}$	—	—
^{130}Te	> 1.5	$< 0.11 - 0.52$	CUORE
^{136}Xe	> 10.7	$< 0.061 - 0.165$	KamLAND-Zen
	> 1.8	$< 0.15 - 0.40$	EXO-200
^{150}Nd	$> 2.0 \times 10^{-3}$	$< 1.6 - 5.3$	NEMO-3

Landscape of Nuclear Matrix Elements

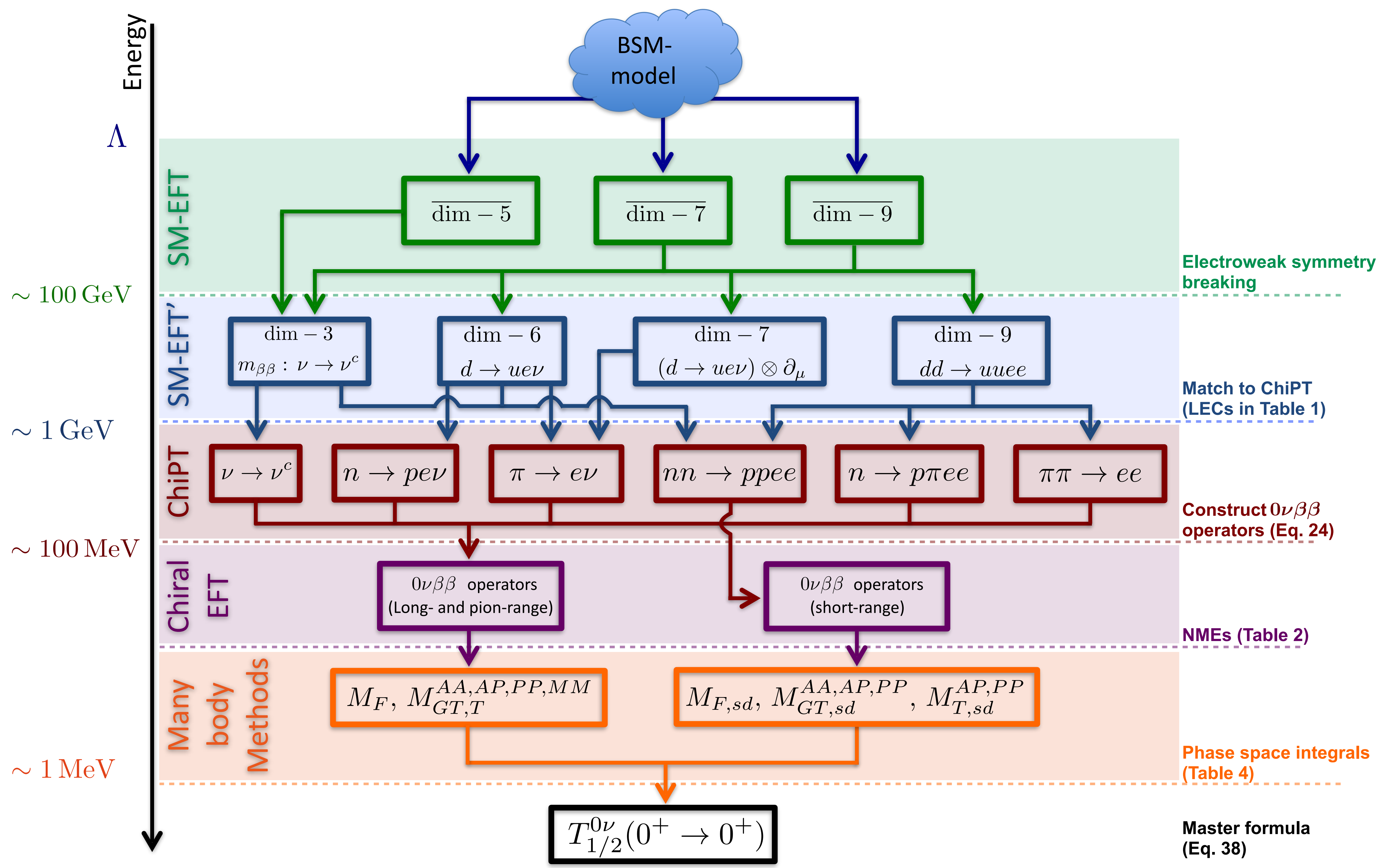


$$\left[T^0_{1/2}\right]^{-1} = G_{0\nu}\,\boxed{\left|M_{0\nu}\right|^2}\,m^2_{\beta\beta}$$

- Sensitive to nuclear matrix elements—**need to be under control theoretically**

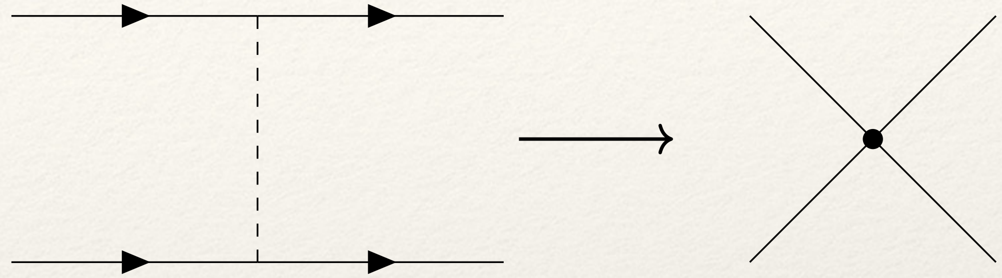
Engel and Menendez, Rept. Prog. Phys. 80

Effective Field Theory From the Top Down



 Cirigliano et al., JHEP 12

Pionless Effective Field Theory



- Nucleon degrees of freedom
- Valid for $p \ll m_\pi$

$$\mathcal{L}_{NN} = N^\dagger \left(iD_0 + \frac{1}{2m_N} D^2 \right) N - C_0 \left(N^T P N \right)^\dagger \left(N^T P N \right) + \cdots$$

- Tower of contact terms with desired symmetries, i.e. Galilean, gauge, isospin, parity, time-reversal etc.

Kaplan, Savage, and Wise, 1998; van Kolck, 1999

Chiral Effective Field Theory

- $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_V$ realised nonlinearly

$$\mathcal{L}_{\mathrm{ChEFT}} = \mathcal{L}_\pi + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \cdots$$

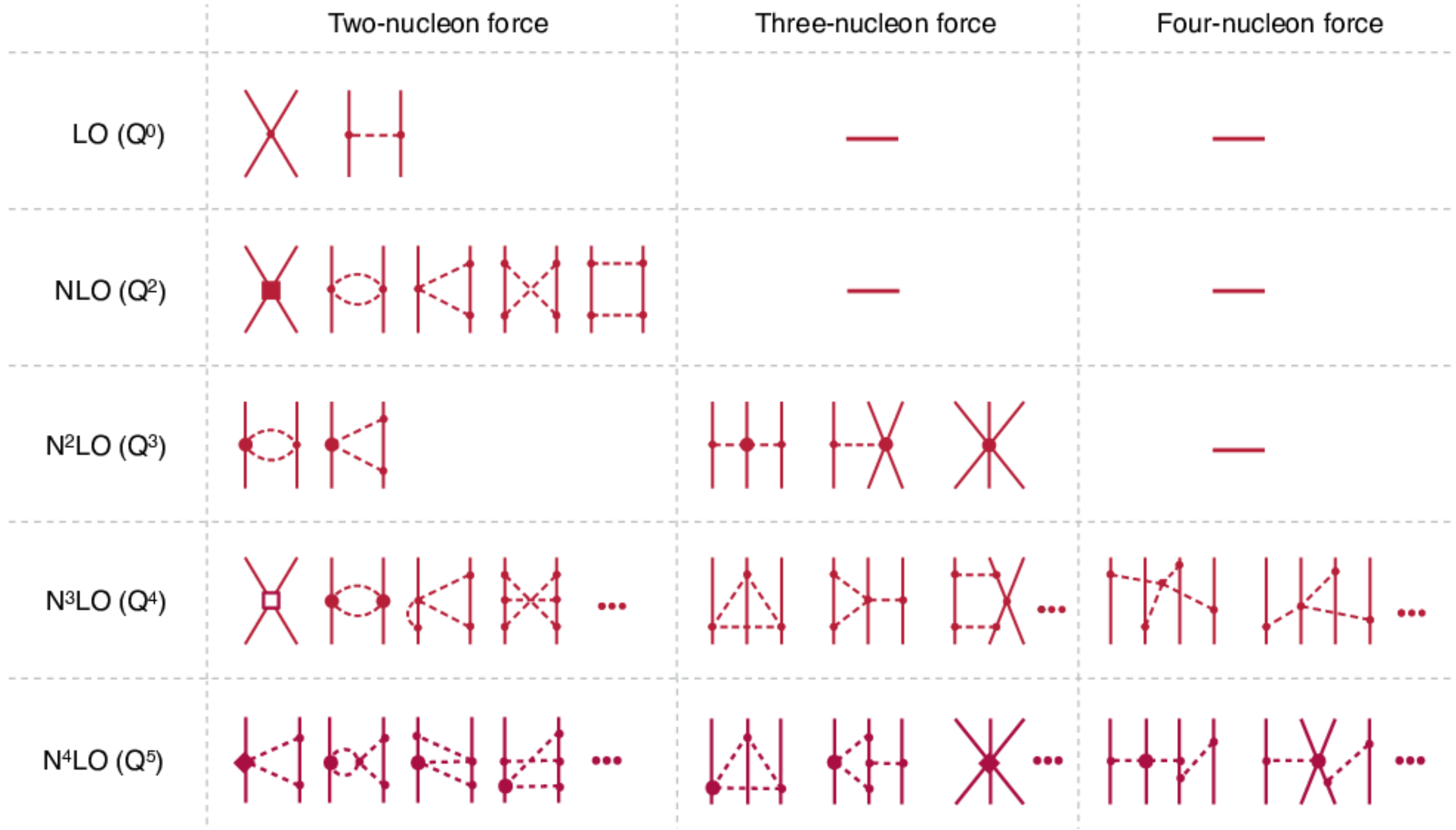
$$\mathcal{L}_\pi = \frac{F^2}{4}\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right) + \frac{F^2}{4}\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right) + \cdots$$

$$\mathcal{L}_{\pi N} = \bar{N}_v \left(iv \cdot D + g_A S_v \cdot u\right) N_v + O(1/m_N) + \cdots$$

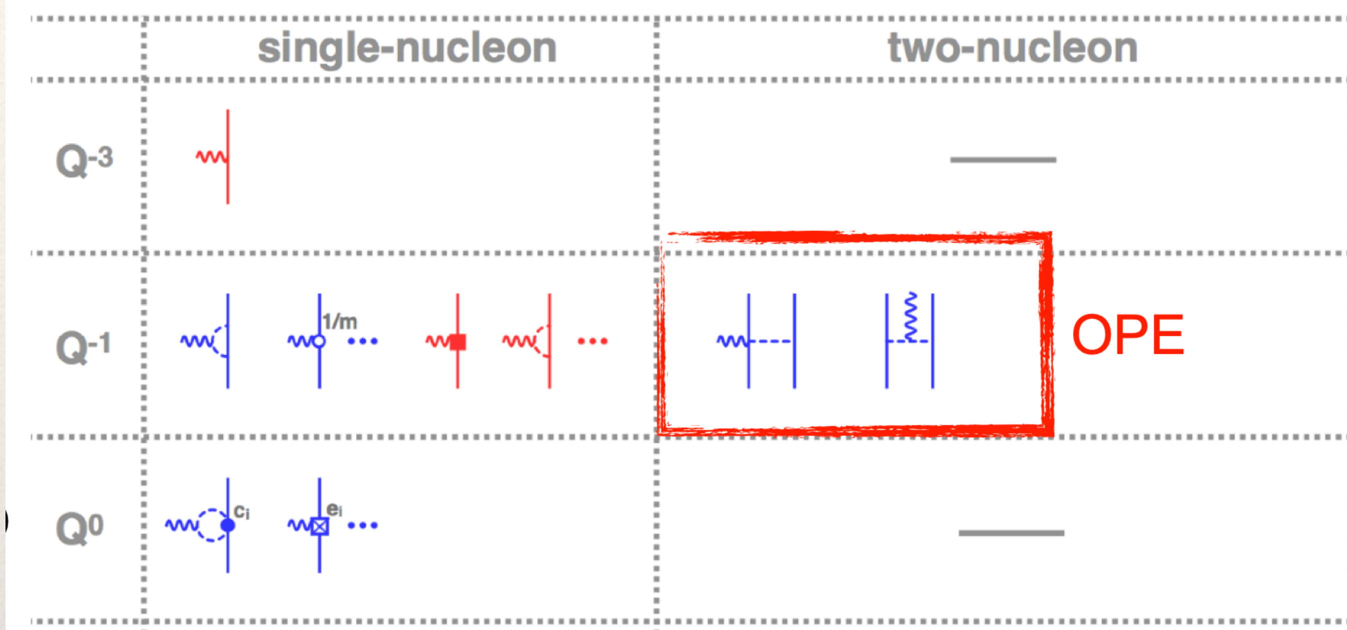
- *Ab initio* program: Construct chiral potentials and solve many-body Schrödinger equation

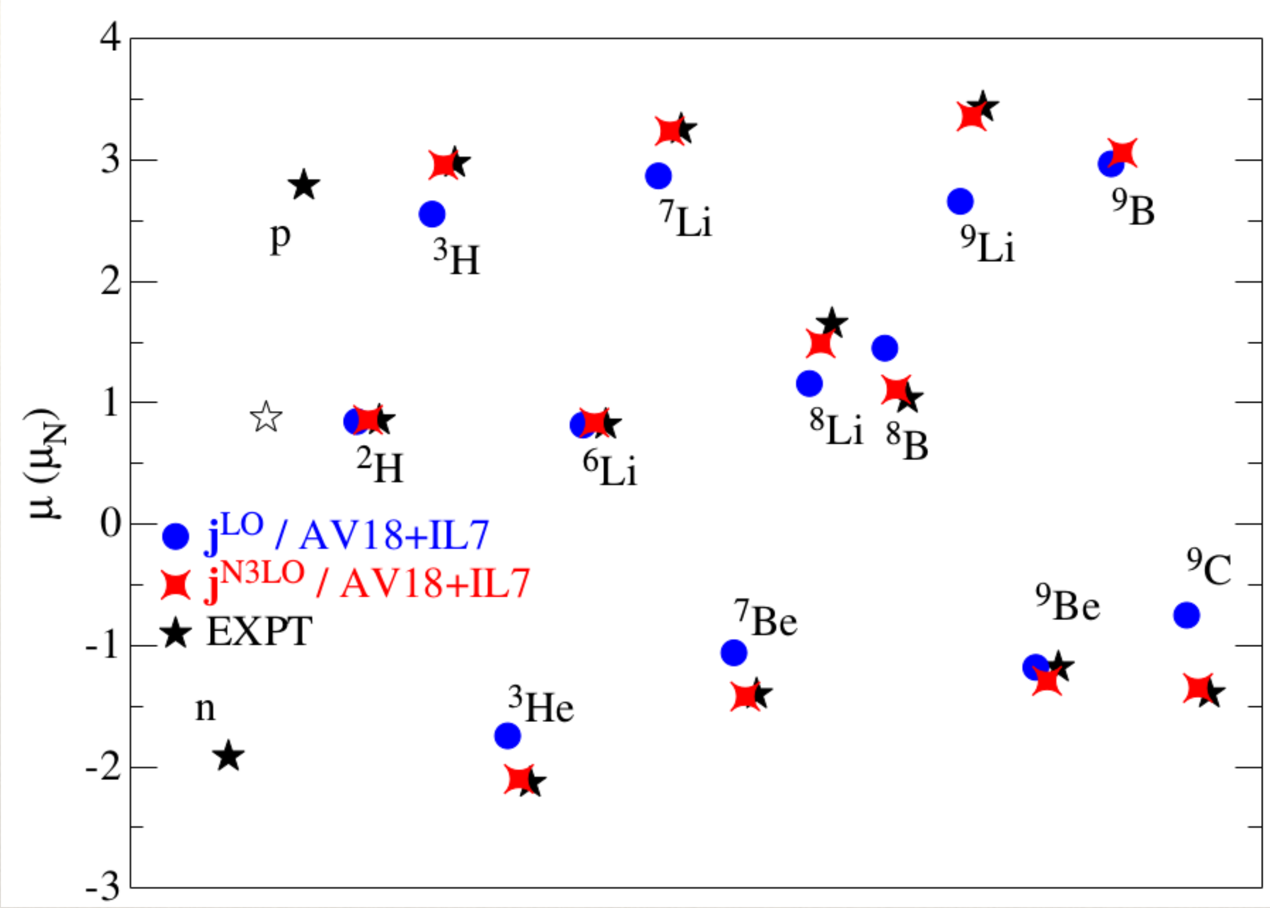
Weinberg, 1968, 1990, 1991; Gasser and Leutwyler, 1983, 1984; Gasser et al. 1987; Jenkins and Manohar, 1991

Chiral forces

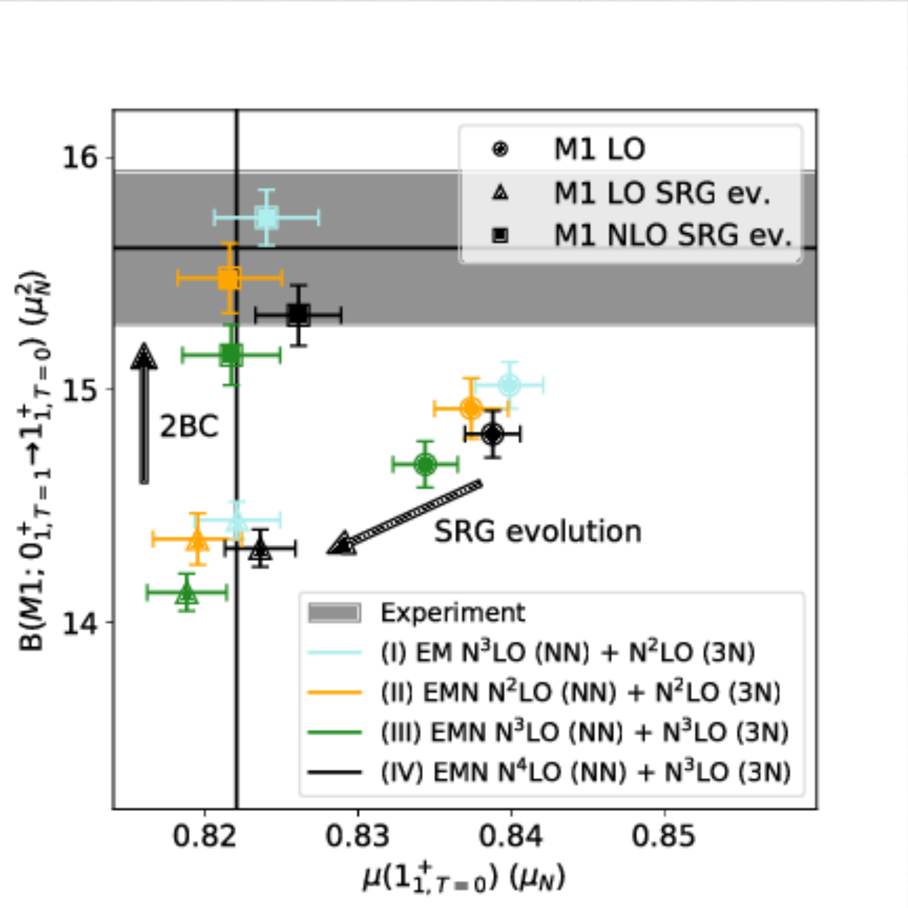


Electromagnetic Few-body currents





Pastore et al. PRC 90



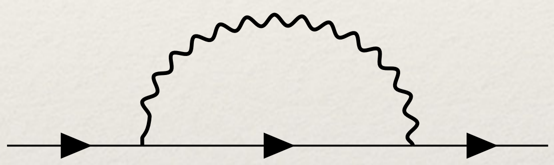
Bacca et al. PRL 126

2-body currents required to satisfy current conservation

Spurions

- Virtual photons/leptons require new counterterms

$$\mathcal{L} = i\bar{q}_L\gamma^\mu\partial_\mu q_L + i\bar{q}_R\gamma^\mu\partial_\mu q_R - \bar{q}_L M^\dagger q_R - \bar{q}_R M q_L$$
$$+ ieA_\mu \left[\bar{q}_L\gamma^\mu Q_L q_L + \bar{q}_R\gamma^\mu Q_R q_R\right]$$



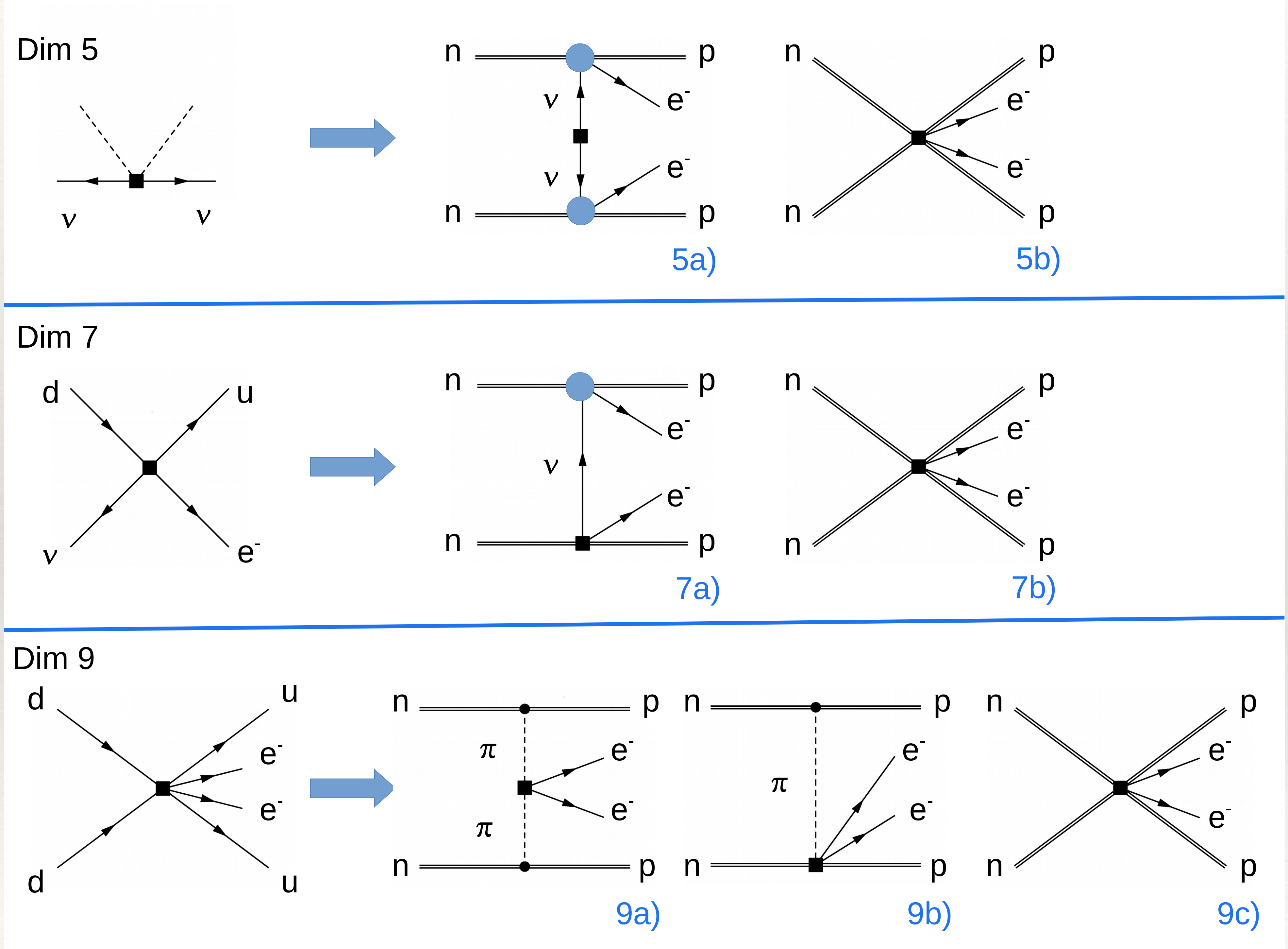
$$M \mapsto RML^\dagger \quad \Rightarrow \quad \begin{array}{l} Q_R \mapsto RQ_R R^\dagger \\ Q_L \mapsto LQ_R L^\dagger \end{array}$$

- Pion mass splitting

$$\mathcal{L}_\pi^{\mathrm{EM}} = e^2 C \mathrm{Tr}\left(QUQU^\dagger\right)$$

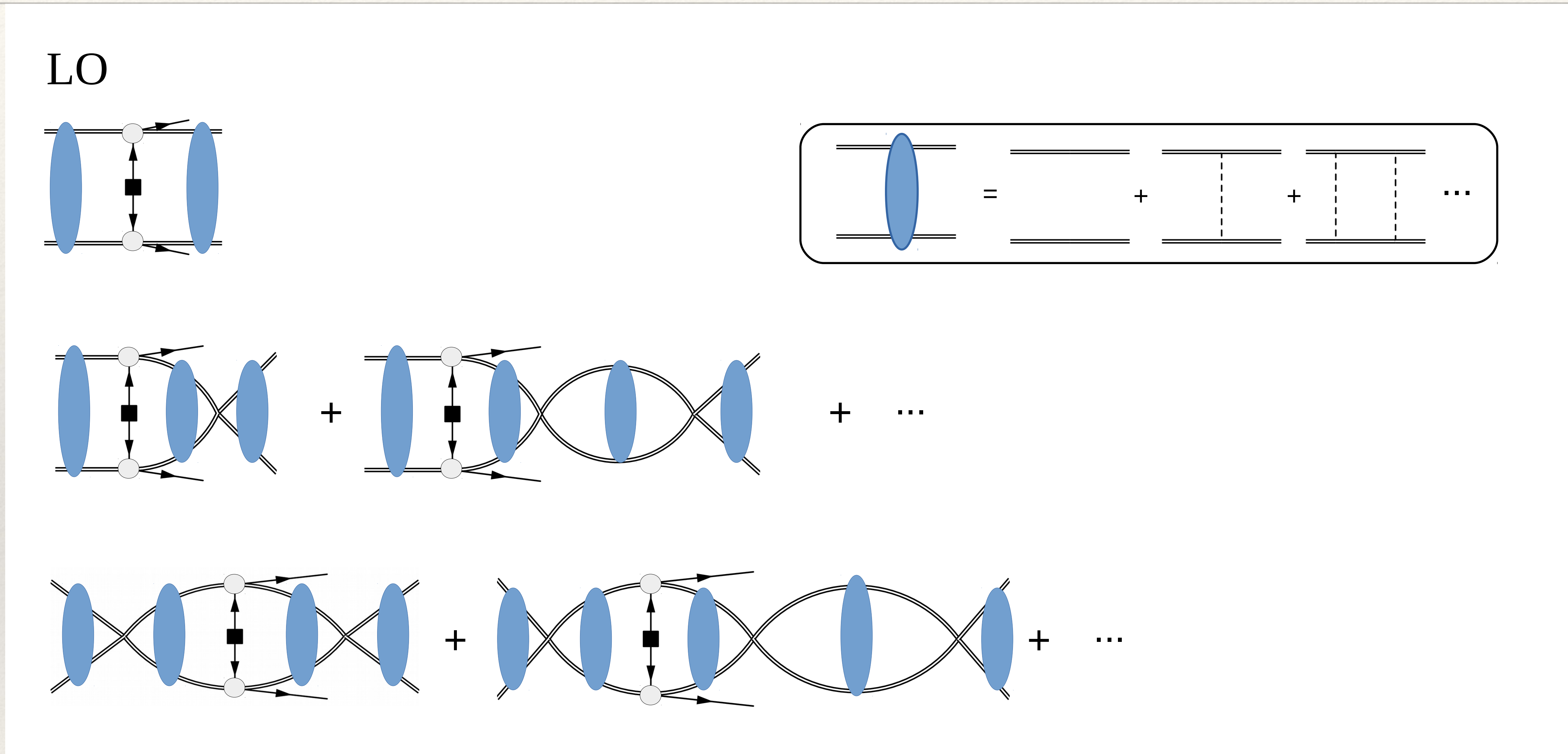
Urech, 1995; Knecht and Urech, 1998;
Neufeld and Rupertsberger, 1995, 1996;
Ecker et al., 1989; Knecht et al. 2000; Meißner et al. , 1997, 1998, 1999

Few-body currents for $0\nu\beta\beta$



Cirigliano et al. JHEP 12

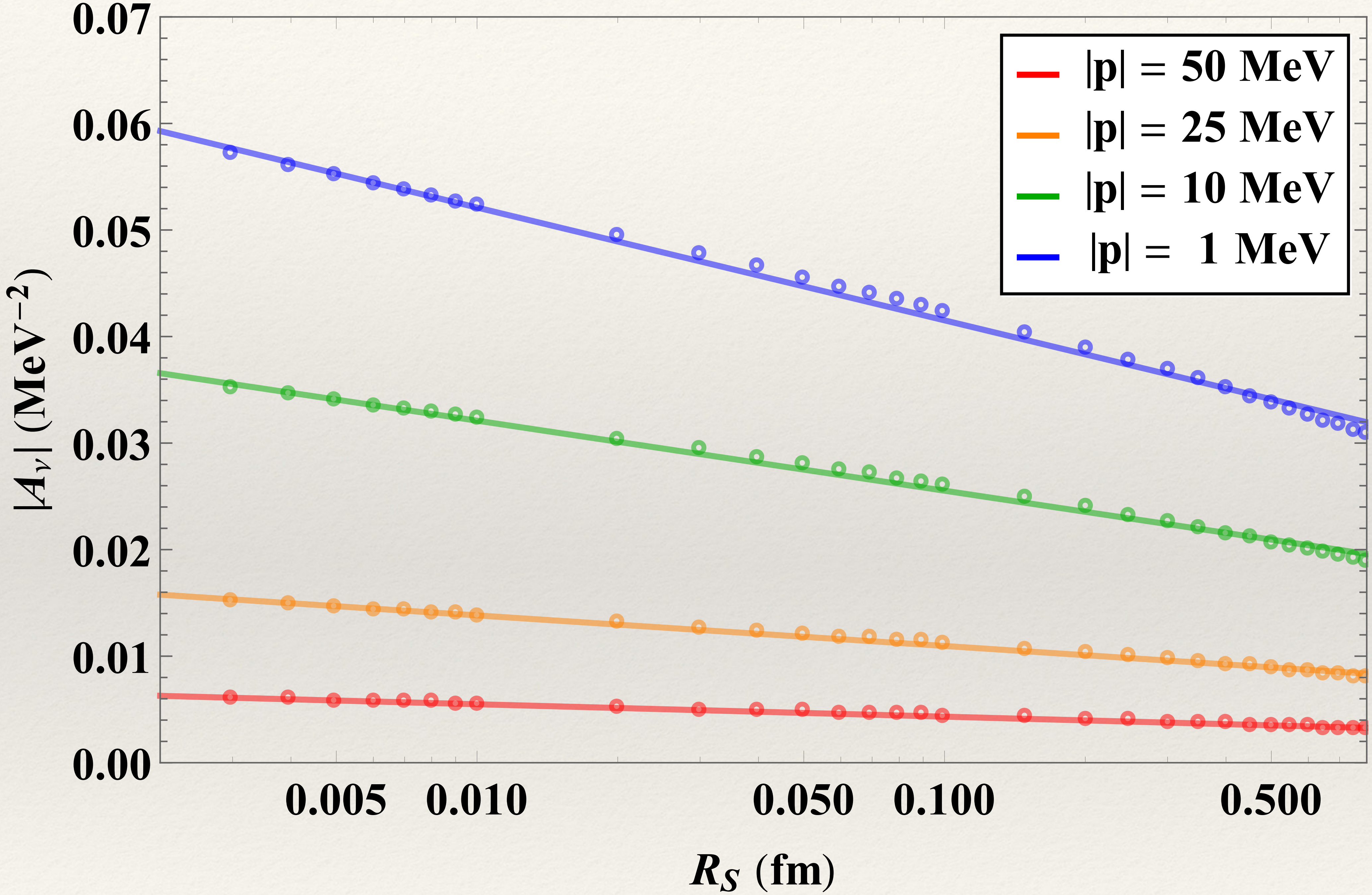
Light Majorana Exchange



$$V_{\nu L}^{^1S_0}(q) = \frac{\tau^{(1)+}\tau^{(2)+}}{q^2}\left[1 + 2g_A^2 + \frac{g_A^2 m_\pi^4}{(q^2+m_\pi^2)^2}\right]$$

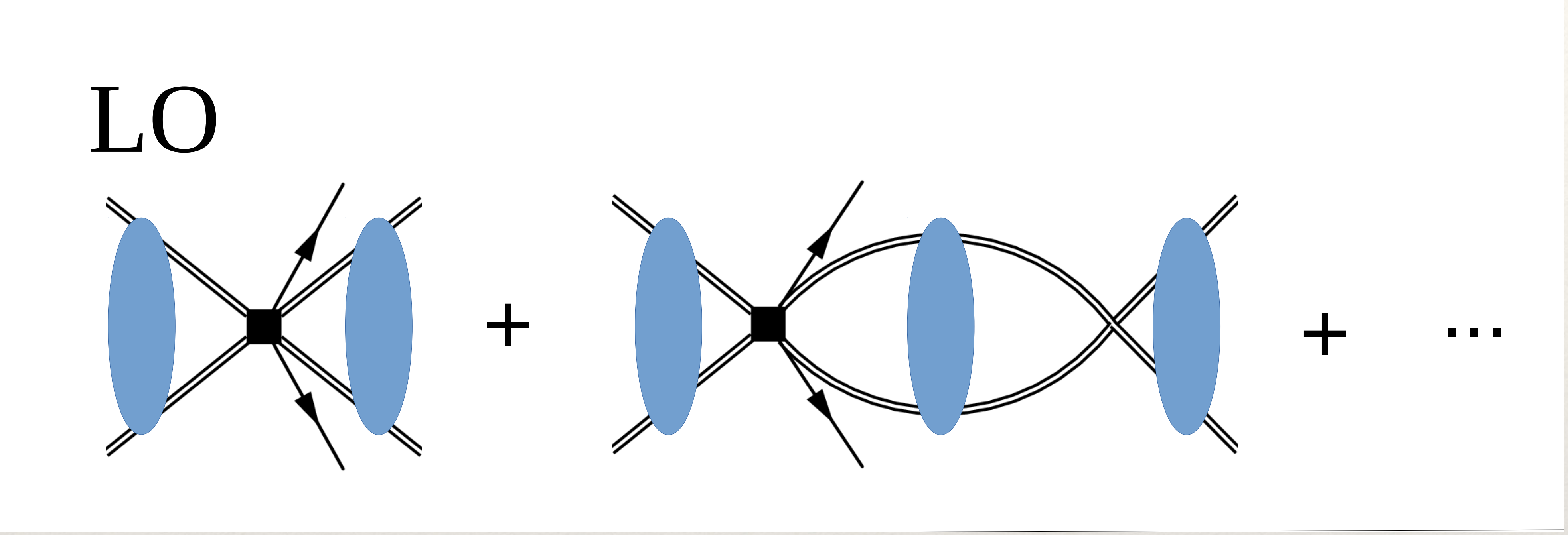
Cirigliano et al. PRC 97, PRL 120, PRC 100

Cutoff Dependence



Cirigliano et al. PRC 100

Renormalization at LO



$$\mathcal{L}^{NN}_{\Delta L=2} = \left(2\sqrt{2}G_F V_{ud}\right)^2 m_{\beta\beta}\bar{e}_L C\bar{e}_L^T \frac{g_\nu^{NN}}{4}\left[\left(N^\dagger u\tau^+ u^\dagger N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tau^+\tau^+\right)\left(N^\dagger\tau^a N\right)^2\right] + \mathrm{H.c}$$

Relation to Charge Independence Breaking

- CIB isotensor Lagrangian

$$\mathcal{L}^{NN}_{CIB} = \frac{e^2}{2}\left\{(\mathcal{C}_1+\mathcal{C}_2)\left[\left(N^\dagger\tilde{Q}_+N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger\tau^a N\right)^2\right]\right.$$
$$\left.(\mathcal{C}_1-\mathcal{C}_2)\left[\left(N^\dagger\tilde{Q}_-N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_-^2\right)\left(N^\dagger\tau^a N\right)^2\right]\right\}$$

Epelbaum and Meißner 1999, Walzl et al. 2001

- Chiral symmetry dictates

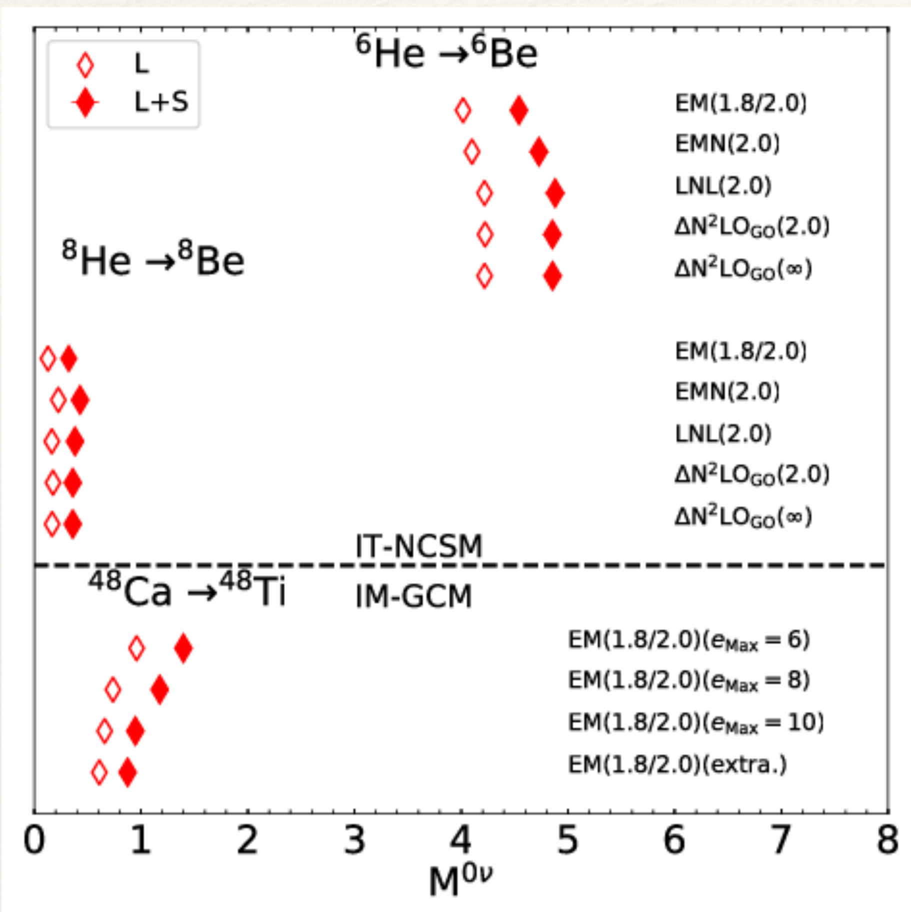
$$g_\nu^{NN} = \mathcal{C}_1$$

- Approximate

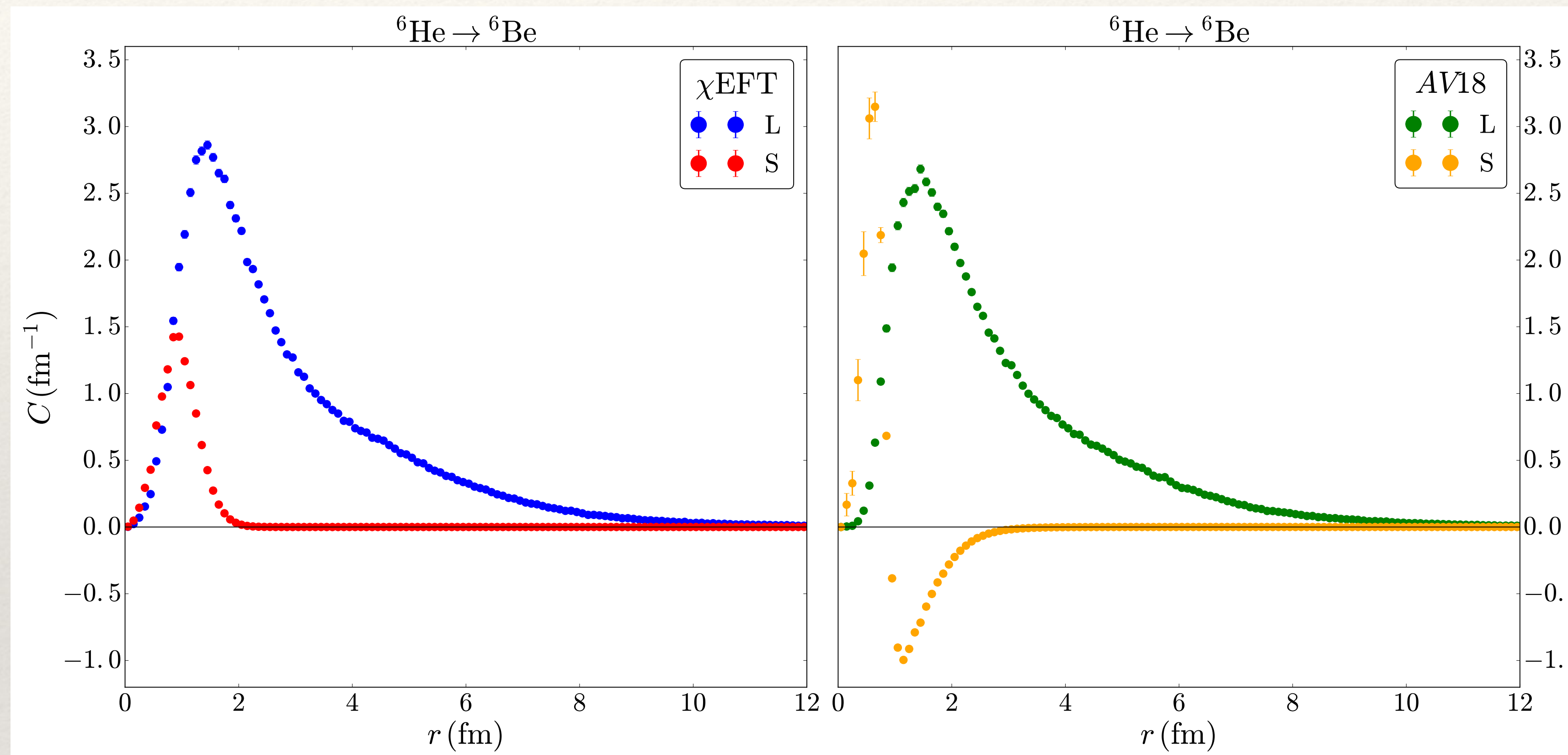
$$g_\nu^{NN} = \frac{1}{2}\left(\mathcal{C}_1+\mathcal{C}_2\right)$$

Cirigliano et al. 2018a, 2018b, 2019, 2021

Impact on Nuclear Matrix Elements



Wirth et al. PRL 127



Cirigliano et al. PRC 100

Low Energy Coefficients

$$\mathcal{L}_{\rm eff} = \sum_n \left(\frac{p}{\Lambda}\right)^n c_{\mathcal{O}} \mathcal{O}_n$$

- LECs must be obtained from:
  - fit to data
    - lacking for many low-energy processes
  - Matching calculations
    - lattice QCD
- Theoretical constraints from large-N_c QCD

Large-N$_c$ QCD

- One-loop beta function

$$\mu \frac{dg}{d\mu} = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3}N_c - \frac{2}{3}N_f\right]$$

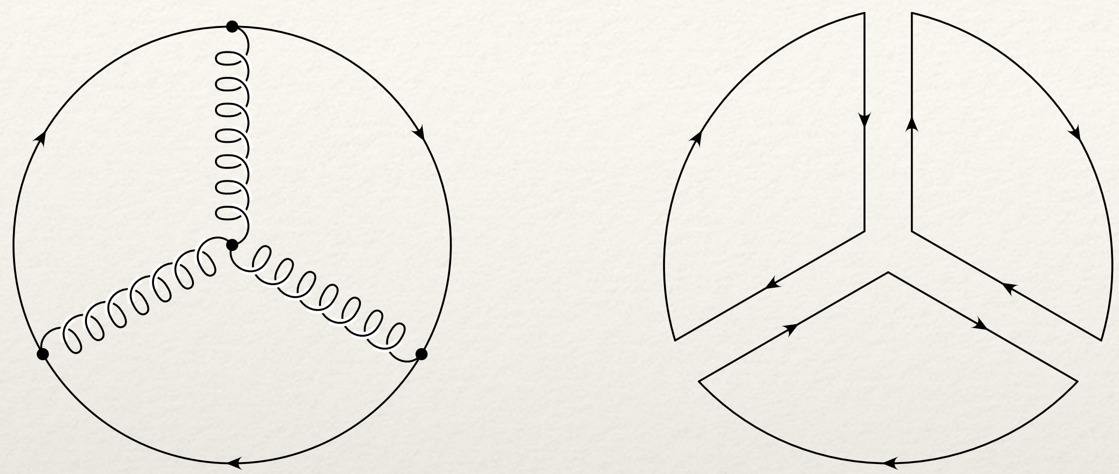
- Rescale coupling constant

$$g \to \frac{g}{\sqrt{N_c}} \quad \Longrightarrow \quad \mu \frac{dg}{d\mu} = -\left(\frac{11}{3} - \frac{2N_f}{3N_c}\right)\frac{g^3}{(4\pi)^2}$$

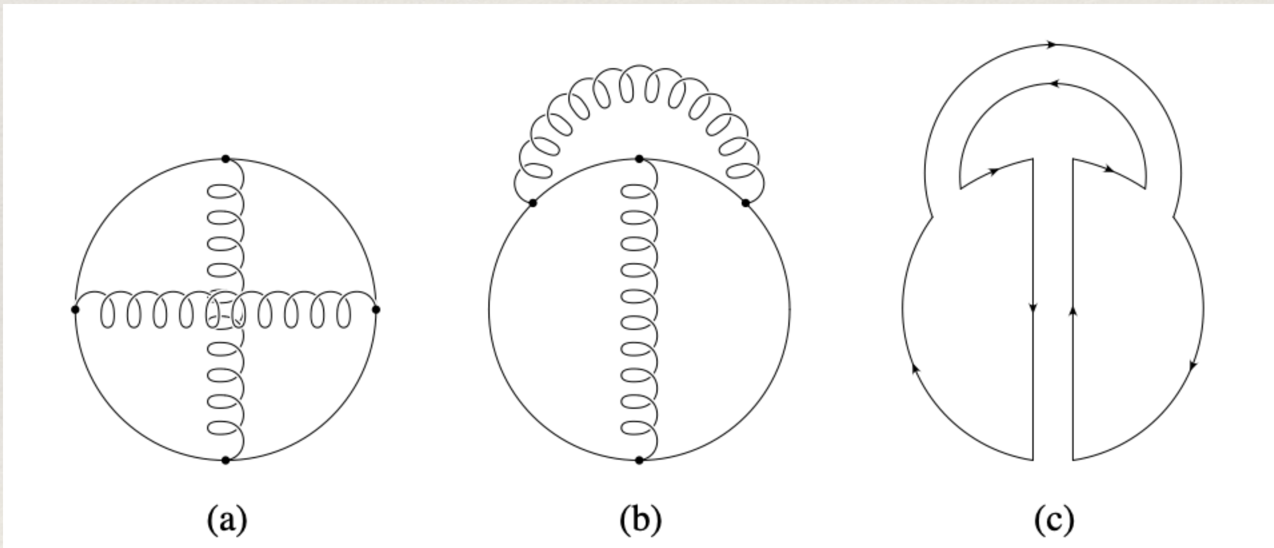
- QCD becomes expansion of planar diagrams
  - Planar gluons $\lesssim O(N_c^2)$
  - Single quark along edge with planar gluons $\lesssim O(N_c)$

't Hooft NPB 72

Double Line Feynman Diagrams



$\sim N_c$



$\sim 1/N_c$

Manohar, 1998

Large-N Constraints in Nuclear EFTs

- Undetermined coefficients must be determined for the EFT to be predictive
- Chiral EFT and pionless EFT possess symmetries of QCD
  - Map scalings to operators with same spin-flavor structure
- Caveats:
  - Δ degenerate with nucleon
  - chiral limit vs. large-N limit $\dfrac{m_\pi}{m_\Delta - m_N}$
- **Fierz transformations can obscure large-N_c scaling**

Mesons

- Stable, fixed mass

Witten NPB 160

- Infinite number of meson states
- Weakly interacting

$$F_0 \sim \sqrt{N_c}$$

  - 3-meson vertex $\sim 1/\sqrt{N_c}$
  - 4-meson vertex $\sim 1/N_c$
- Exotics suppressed?

Weinberg PRL 110, Knecht and Peris PRD 88

Large-N_c Baryons

- Baryon must be made of N_c quarks
- Baryon mass $m_B \sim O(N_c)$
- Meson-baryon amplitude $O(N_c^0)$
- Two-baryon interaction $O(N_c)$

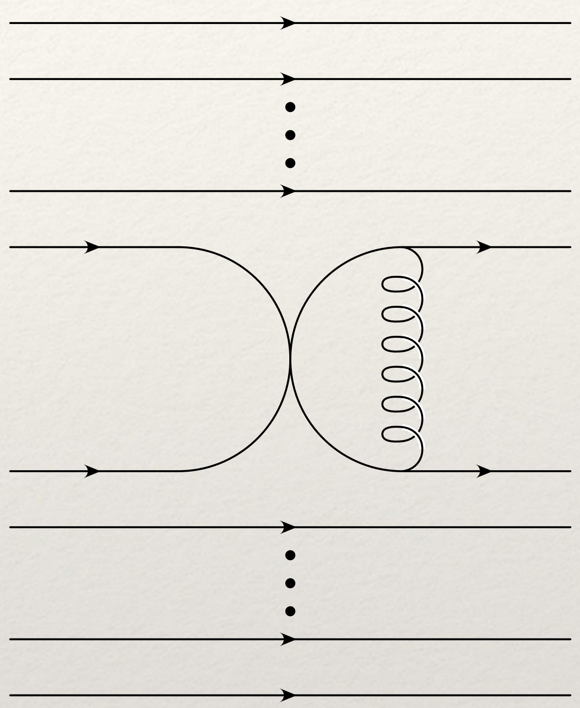


Figure: Baryon-baryon scattering
Manohar, 1998

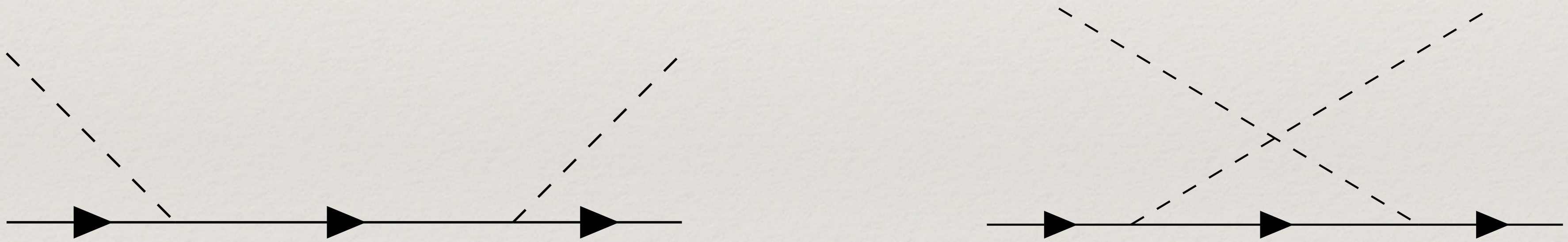
Witten NPB 160

Consistency Conditions

- Axial current matrix elements

$$\langle B'|\, \bar{q}\gamma^i\gamma^5\tau^a q\,|B\rangle = \hat{g}_A N_c \langle B'|\, X^{ia}\,|B\rangle$$

- Baryon-meson scattering amplitude *should be* $O(1)$



$$\frac{\hat{g}_A^2}{F^2} N_c^2 \left[X^{ia}, X^{jb}\right] \Rightarrow \left[X^{ia}, X^{jb}\right] \lesssim O(1/N_c^2)$$

- Baryons transform under contracted $\mathrm{SU}(2N_F)$

Gervais and Sakita PRL 52, PRD 30; Dashen and Manohar PLB 315

Spin-Flavor Symmetry

- Large-N_c baryons transform under SU(4)

$$S^i = q^\dagger \frac{\sigma^i}{2} q \qquad I^a = q^\dagger \frac{\tau^a}{2} q \qquad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q$$

$$\langle B' | \frac{\mathcal{O}^{(n)}}{N_c^n} | B \rangle \sim N_c^{-|I-S|} \qquad \langle B' | \hat{1} | B \rangle \sim \mathcal{O}(N_c)$$

- Expand QCD operators in basis of SU(4) generators

$$\mathcal{O}^{(m)}_{\text{QCD}} = N_c^m \sum_{n,s,t} c_n \left(\frac{S^i}{N_c}\right)^s \left(\frac{I^a}{N_c}\right)^t \left(\frac{G^{ia}}{N_c}\right)^{n-s-t}$$

Dashen PRD 49, 51; Carone PLB 322;
Luty and March-Russell NPB 426

Example: Mass Operator

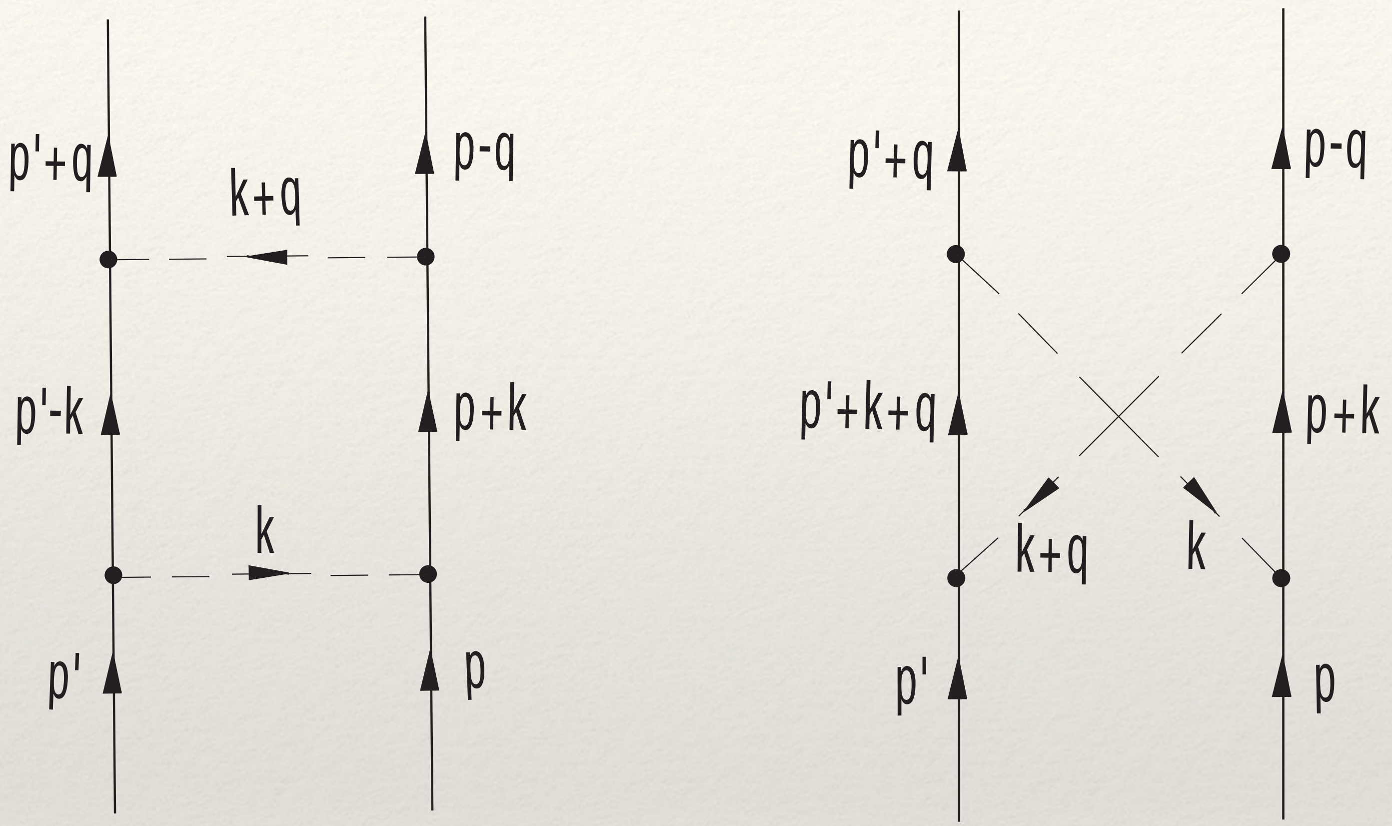
- Mass operator is a spin-isospin singlet

$$M = N_c m_0 + m_2 \frac{J^2}{N_c}$$

$$\begin{aligned} m_N =& N_c m_0 + \frac{3}{4N_c} m_2 \\ m_\Delta =& N_c m_0 + \frac{15}{4N_c} m_2 \end{aligned}$$

- Mass splitting is O(10%)

Large-N Forces and Δ



- Explicit Δ leads to correct scaling

Banerjee, Cohen, Gelman PRC 65

Large-N Lagrangian

TRR et al. PRC 103

- LNV contact term

$$\left(N^\dagger\sigma^i\tau^+N\right)\left(N^\dagger\sigma^i\tau^+N\right) = -3\left(N^\dagger\tau^+N\right)\left(N^\dagger\tau^+N\right)$$

$$g_\nu^{NN} \sim O(N_c)$$

- CIB contact terms

$$\mathcal{L}^{\Delta I=2}_{\text{LO-in-}N_c} = \bar{\mathcal{C}}_3\left[\left(N^\dagger\sigma^i\tilde{Q}_+N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger\sigma^i\tau^aN\right)^2\right]$$
$$\mathcal{L}^{\Delta I=2}_{\text{NLO-in-}N_c} = \bar{\mathcal{C}}_6\left[\left(N^\dagger\sigma^i\tilde{Q}_-N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_-^2\right)\left(N^\dagger\sigma^i\tau^aN\right)^2\right]$$

$$\bar{\mathcal{C}}_3 \sim O(N_c) \qquad \bar{\mathcal{C}}_6 \sim O(1)$$

Large-N$_c$ Consistency

- CIB LECs same size and sign

$$\mathcal{C}_1 = -3\bar{\mathcal{C}}_3 - 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3\left[1 + O(1/N_c)\right]$$
$$\mathcal{C}_2 = -3\bar{\mathcal{C}}_3 + 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3\left[1 + O(1/N_c)\right]$$

- LNV and CIB scale the same way

$$g_\nu^{NN} \sim O(N_c) \qquad\qquad \mathcal{C}_1 + \mathcal{C}_2 \sim O(N_c)$$

$$\Longrightarrow \quad g_\nu^{NN} = \frac{1}{2}\left(\mathcal{C}_1 + \mathcal{C}_2\right)$$

Comparison to Cottingham

Cirigliano et al., PRL 126, JHEP 05

- Central Cottingham values fall within large-N_c estimate

$$\tilde{\mathcal{C}}_1\,(\mu = m_\pi) = 1.3(6)$$

$$\tilde{\mathcal{C}}_1 + \tilde{\mathcal{C}}_2\,(\mu = m_\pi) = 2.9(1.2)$$

$$\Rightarrow$$

$$\frac{\tilde{\mathcal{C}}_1}{\tilde{\mathcal{C}}_2} \approx 0.81$$

$$\frac{\tilde{\mathcal{C}}_1 - \tilde{\mathcal{C}}_2}{\tilde{\mathcal{C}}_1 + \tilde{\mathcal{C}}_2} \approx 0.1$$

Comparison to CIB Scattering

- Large-N_c with experiment

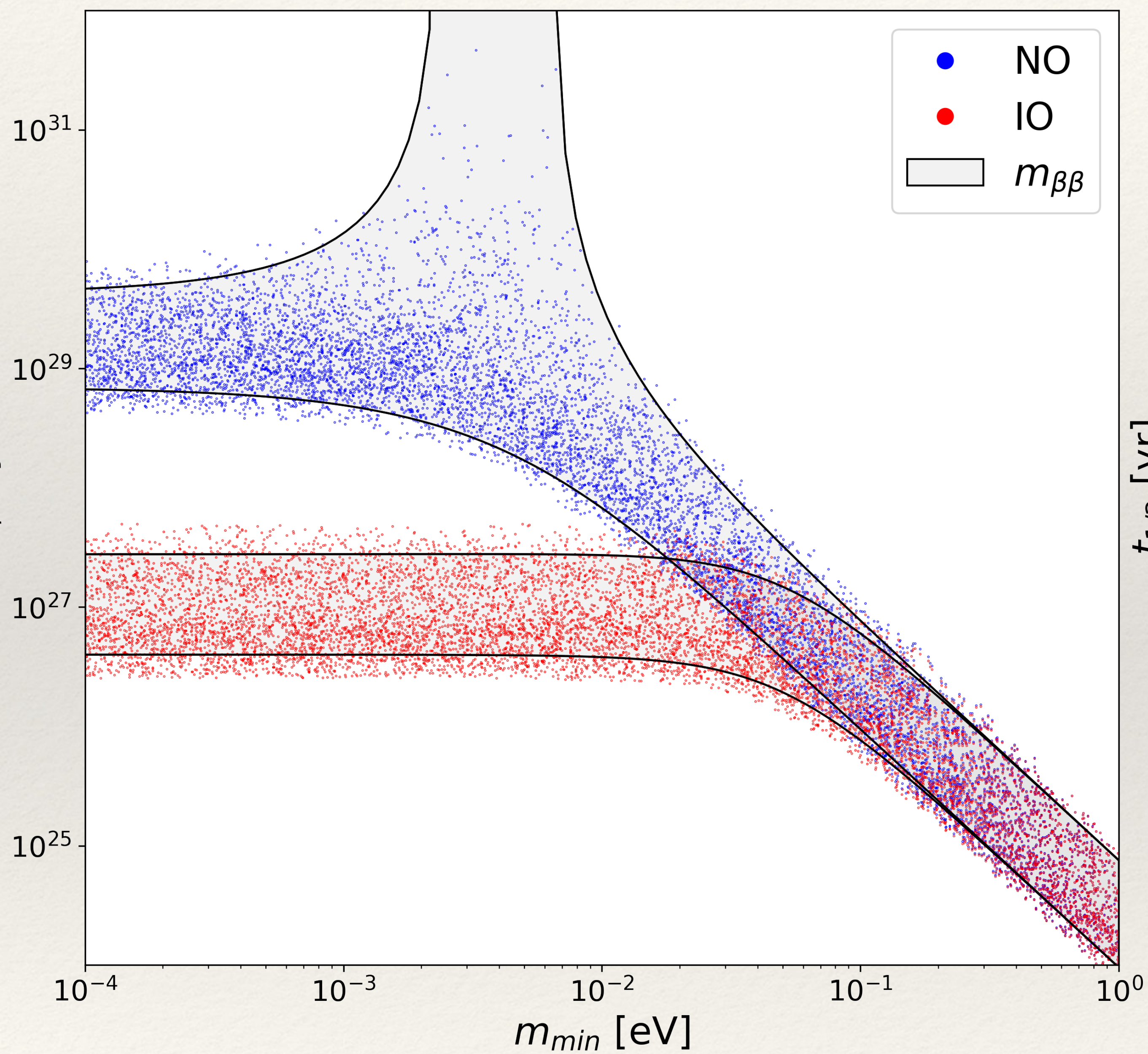
Cirigliano et al., PRL 126, JHEP 05

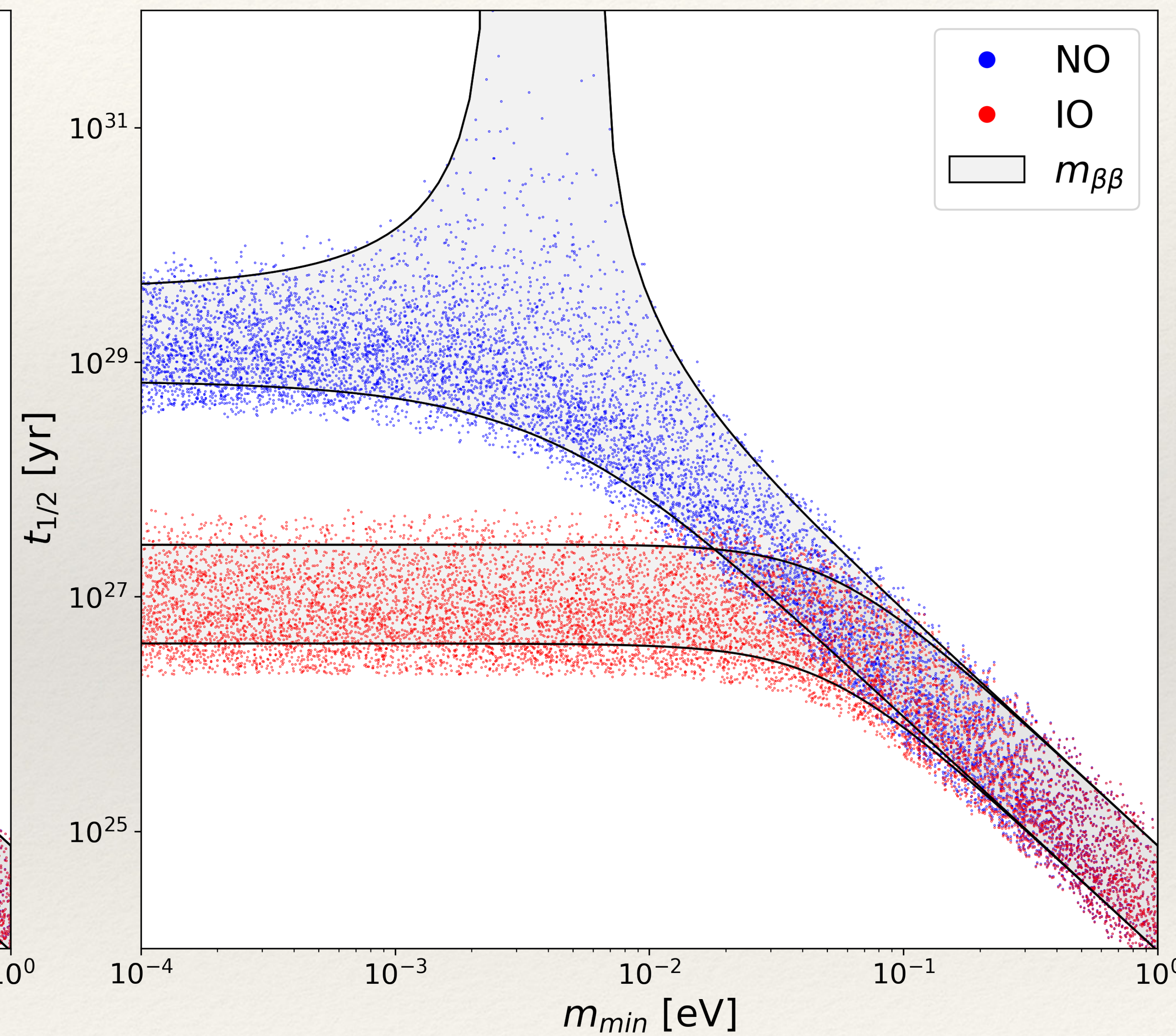
$$\tilde{\mathcal{C}}_1 + \tilde{\mathcal{C}}_2\,(\mu = m_\pi) = 5.1 \qquad \tilde{\mathcal{C}}_1\Big|_{\text{Large-}N_c} \approx 2.5$$

- With room for 30% corrections

$$1.7 \lesssim \tilde{\mathcal{C}}_1 \lesssim 3.3 \qquad 1.8 \lesssim \tilde{\mathcal{C}}_2 \lesssim 3.4$$

Impact on the half-life: νDoBe



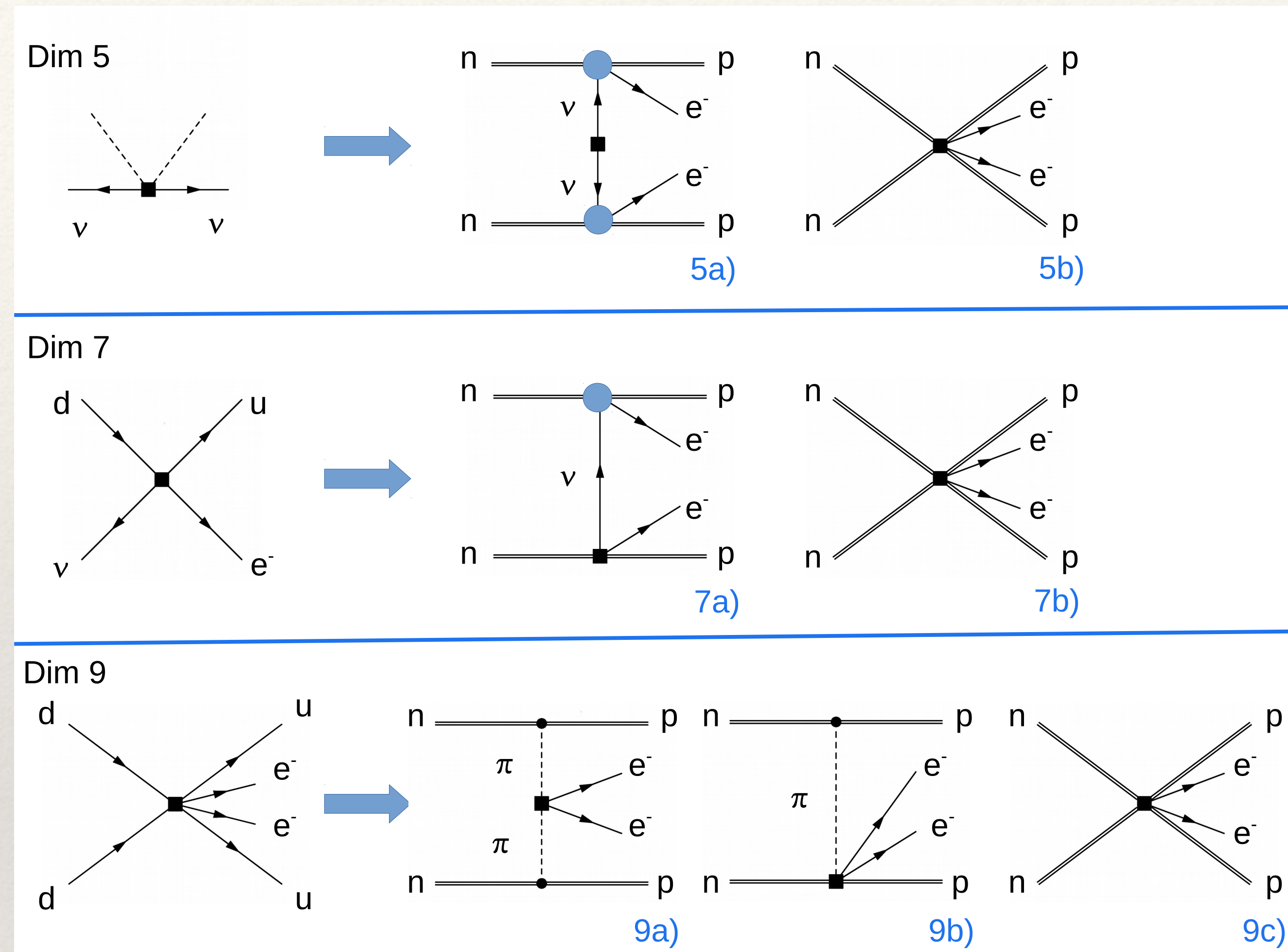


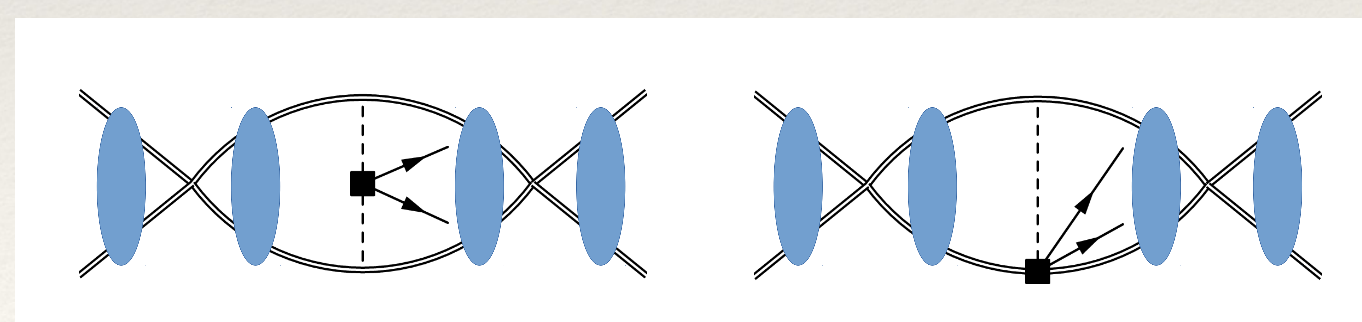
Scholer, de Vries, Graf arXiv:2304.05415 [hep-ph]

Higher dimension operators

- Large-N can be used to estimate sizes of other matrix elements

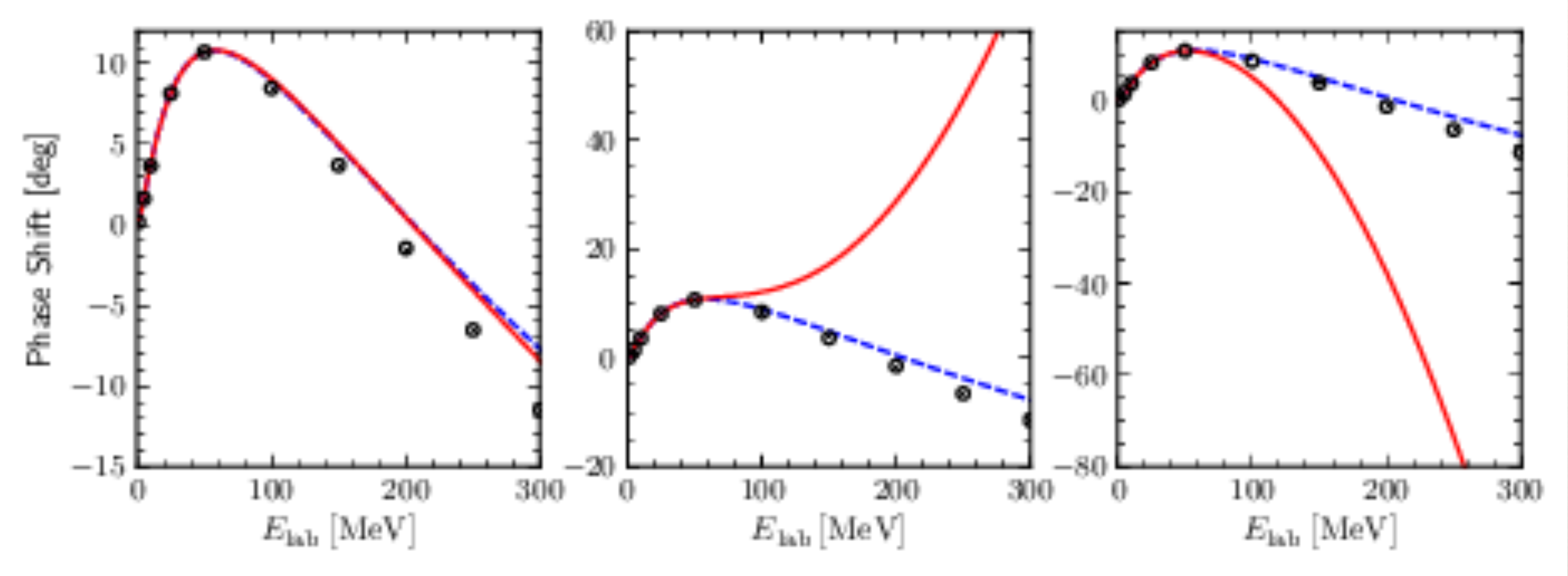
- Higher dimensional operators come with suppressions



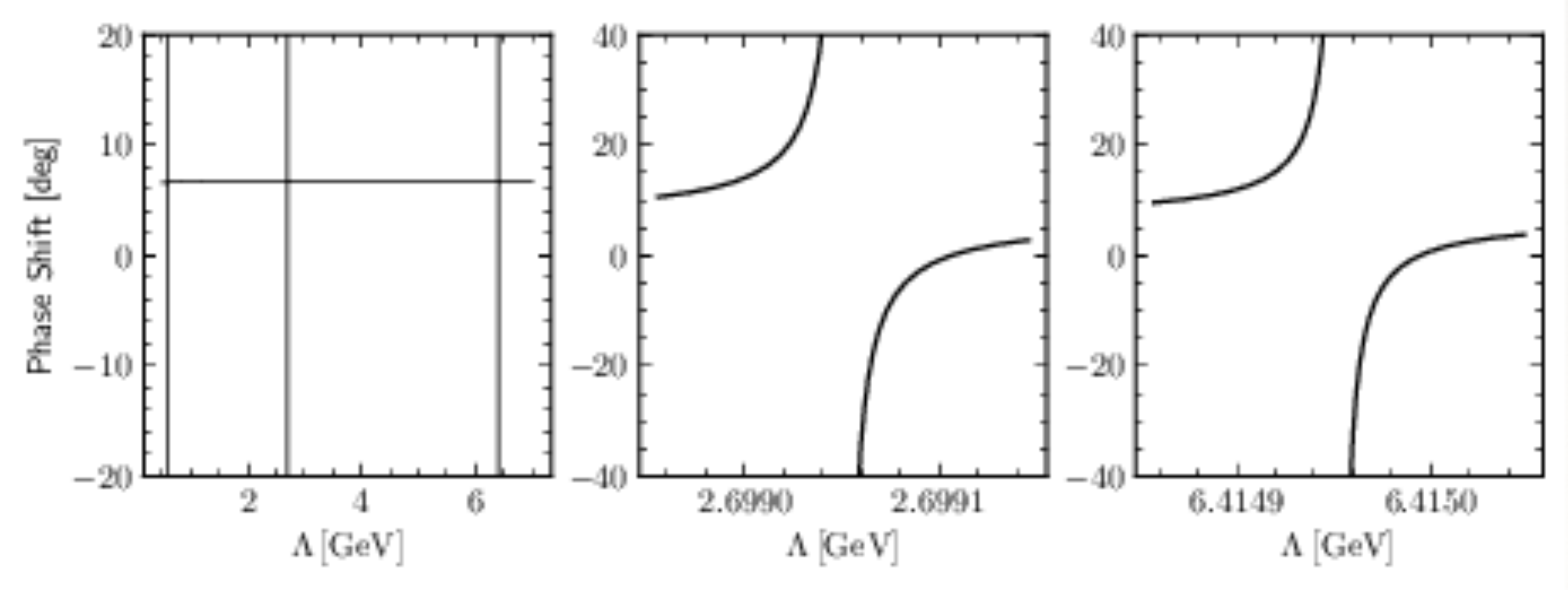


Cirigliano et al. JHEP 12

Renormalization Past LO?



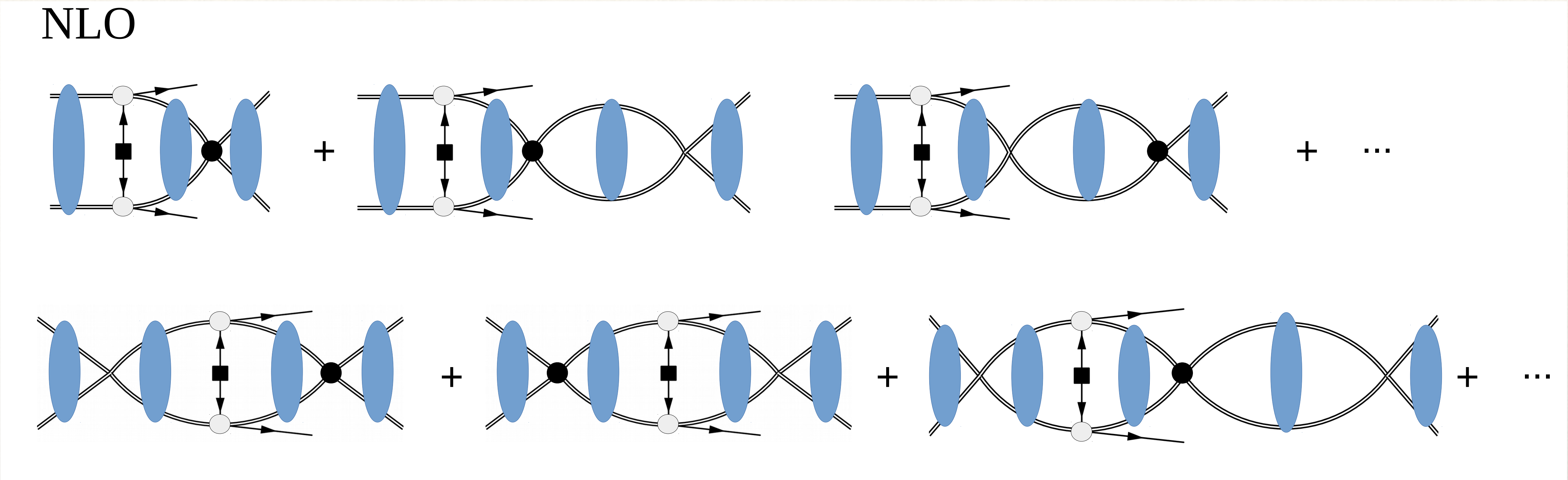
$\Lambda = 12500,\ 12249.69,\ 12249.73$ MeV



$T_{\rm lab} = 130$ MeV

Epelbaum and Gasparyan PRC 107

Light Majorana Exchange: NLO



- Are there "exceptional points" for this correction?

Summary

- Few-body currents have a significant impact in nuclear matrix elements

- Short range part must be determined from lattice, data, or something else…

- Large-N QCD indicates patterns/trends based on *symmetry*
  - ➡ Estimate size of contact for $0\nu\beta\beta$ and CIB