Structure of Hadrons from Lattice QCD using Pseudo-distributions

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* GPDs and GFFs

Outline

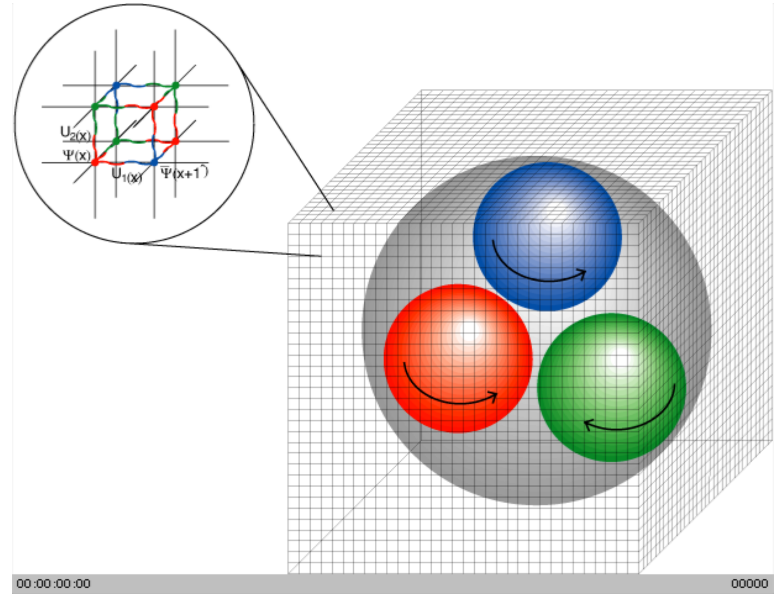
- Lattice QCD
- Hadron Structure on Euclidean Lattice
- Short-distance factorization and pseudo-PDFs
  - Unpolarized nucleon PDF at physical point.
- Understanding systematic effects
  - Distillation + momentum smearing to reach high momenta
- Isoscalar structure of the nucleon - *gluon distribution*
- 3D Structure - GPDs and GFFs
- Summary

Lattice QCD - I

- Continuum Euclidean space time replaced by four-dimensional lattice, or grid, of "spacing" a
- Gauge fields are represented at SU(3) matrices on the links of the lattice - work with the elements rather than algebra

$$U_\mu(n) = e^{iaT^a A^a_\mu(n)}$$

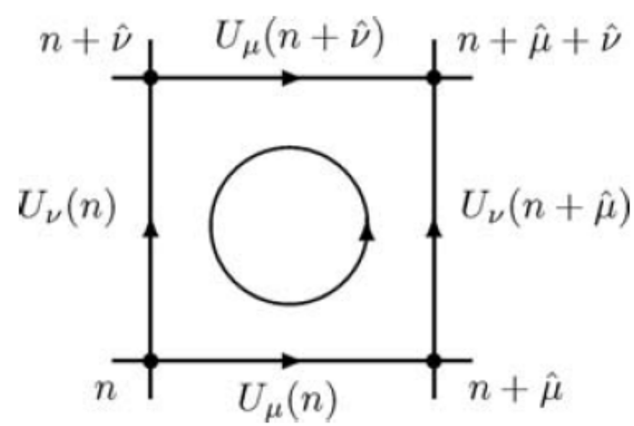
Quarks ψ, ψ are Grassmann Variables, associated with the sites of the lattice



Work in a finite 4D space-time volume

- Volume V sufficiently big to contain, e.g. proton
- Spacing a sufficiently fine to resolve its structure

$$\begin{aligned} V &\simeq (6\ \mathrm{fm})^4 \\ a &\leq 0.1\mathrm{fm} \end{aligned}$$



Lattice QCD - II

Observables in lattice QCD are then expressed in terms of the path integral as

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \prod_{n,\mu} dU_\mu(n) \prod_n d\psi(n) \prod_n d\bar{\psi}(n) \mathcal{O}(U,\psi,\bar{\psi}) e^{-\left(S_G[U]+S_F[U,\psi,\bar{\psi}]\right)}$$

Integrate out the Grassmann variables:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \prod_{n,\mu} dU_\mu(n) \mathcal{O}(U, G[U]) \det M[U] e^{-S_G[U]}$$

Importance Sampling

where $G(U,x,y)^{ij}_{\alpha\beta} \equiv \langle \psi^i_\alpha(x) \bar{\psi}^j_\beta(y) \rangle = M^{-1}(U)$

- Generate an ensemble of gauge configurations

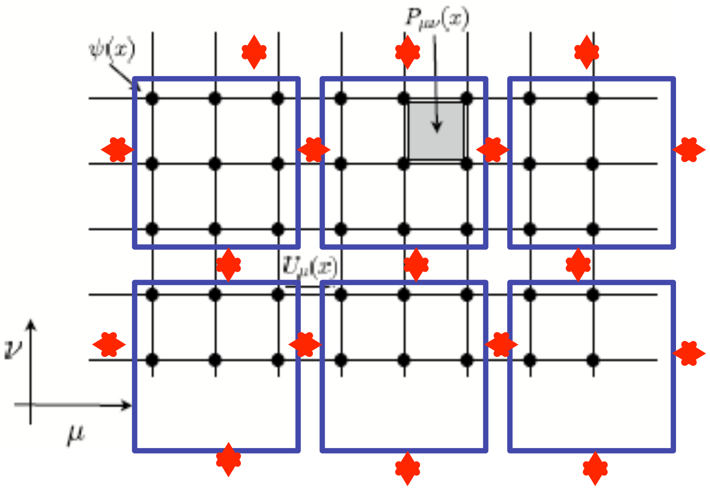
$$P[U] \propto \det M[U] e^{-S_G[U]}$$

This is REAL for Euclidean space QCD - *but see later*

- Calculate observable

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U^n, G[U^n])$$

Gauge Generation



Highly regular problem, with simple boundary conditions – *very efficient use of massively parallel computers using data-parallel programming.*

Change single *$U_\mu(x)$*

$$P[U] \propto \det M[U] e^{-S_G[U]}$$

- **Local change in action**
- **Whole lattice**

Non-local updating procedure, e.g. *Hybrid Monte Carlo/Molecular Dynamics*

For equilibrated, independent configs: $\sigma \simeq 1/\sqrt{N}$ **"gauge noise": noise below which you cannot go!**

$$\mathrm{Cost}_{\mathrm{traj}} = \left[\left(\frac{\mathrm{fm}}{a}\right)^4 \left(\frac{L_s}{\mathrm{fm}}\right)^3 \left(\frac{L_t}{\mathrm{fm}}\right)\right]^{5/4} \cdot \left\{B \cdot \left[\left(\frac{\mathrm{fm}}{a}\right) \cdot \left(\frac{\mathrm{MeV}}{m_\pi}\right)\right]^\gamma + A\right\}$$

Hierarchy of Computations

Capability Computing -
Gauge Generation

e.g. Frontier at ORNL

$$P[U] \propto \det M[U] e^{-S_G[U]}$$

Several *V, a, T, m*$_\pi$

Capacity Computing -
Observable Calculation

e.g. Cluster at JLab
\+ Frontier

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U^n, G[U^n])$$

$$\text{e.g. } C(t) = \sum_{\vec{x}} \langle N(\vec{x}, t) \bar{N}(0) \rangle$$

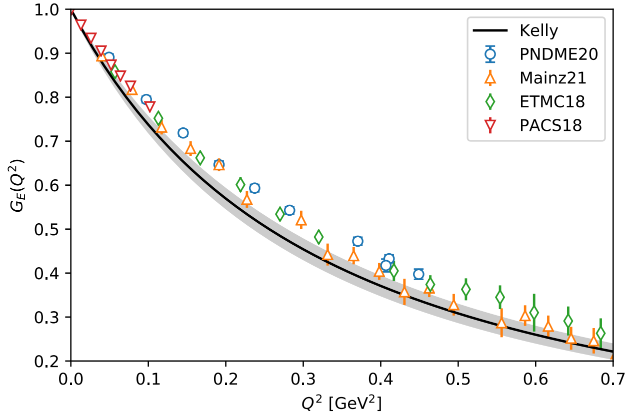
"Desktop" Computing -
Physical Parameters

e.g. Mac at your desk

$$C(t) = \sum_n A_n e^{-E_n t}$$

$$M_N(a, m_\pi, V)$$

Rich Menu of calculations….

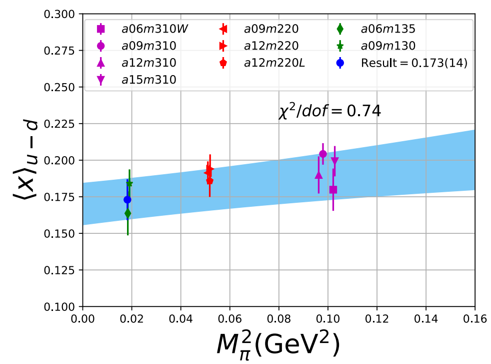


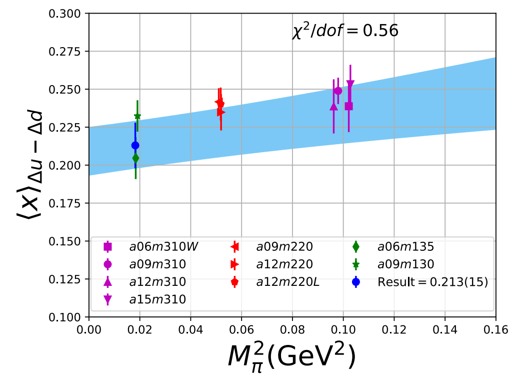
Isovector Sach's Form Factor

D.Djukanovic, Lattice 2022

Momentum and spin fractions of nucleon

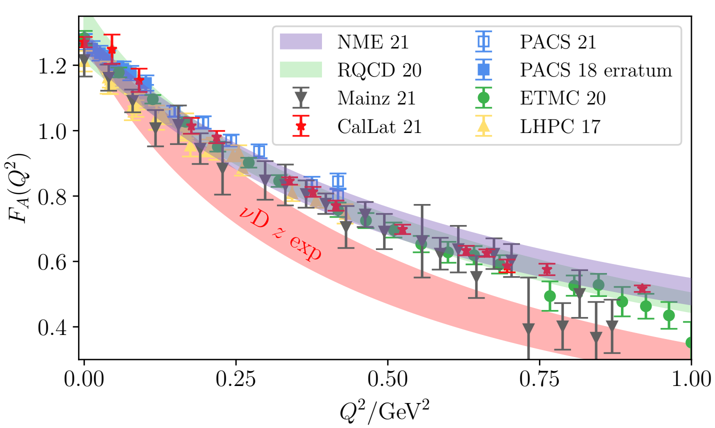
S.Mondal *et al., Phys. Rev. D 102, 054512 (2020)*





Axial-vector form factors - neutrino program

A.S. Meyer, A. Walker-Loud, C.Wilkinson, arXiv:2201.01839



Each characterized by matrix element of local operator → *calculable on Euclidean lattice.*

PDFs, GPDs, TMDs?

Parton Distribution Functions (PDFs)

Describe the longitudinal momentum distribution of the partons (quarks and gluons) within the hadron, e.g. nucleon, pion,…

Hadron Structure: No-go Theorem?

- First Challenge:
  - Euclidean lattice precludes calculation of light-cone/time-separated correlation functions

PDFs, GPDs, TMDs

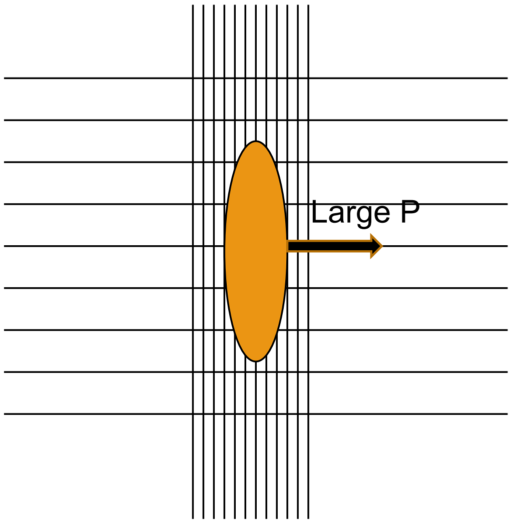
$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)} \psi(0) \mid P \rangle$$

So…. …Use *Operator-Product-Expansion to formulate in terms of Mellin Moments* with respect to Bjorken x.

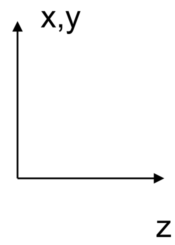
$$\longrightarrow \quad \langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5)D_{\mu_2}\dots D_{\mu_n}\psi \mid P \rangle \to P_{\mu_1}\dots P_{\mu_n} a^{(n)}$$

- Second Challenge:
  - Discretised lattice: power-divergent mixing for higher moments

PDFs from Euclidean Lattice



Large-Momentum Effective Theory (LaMET)



"Equal time" correlator

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')}\psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

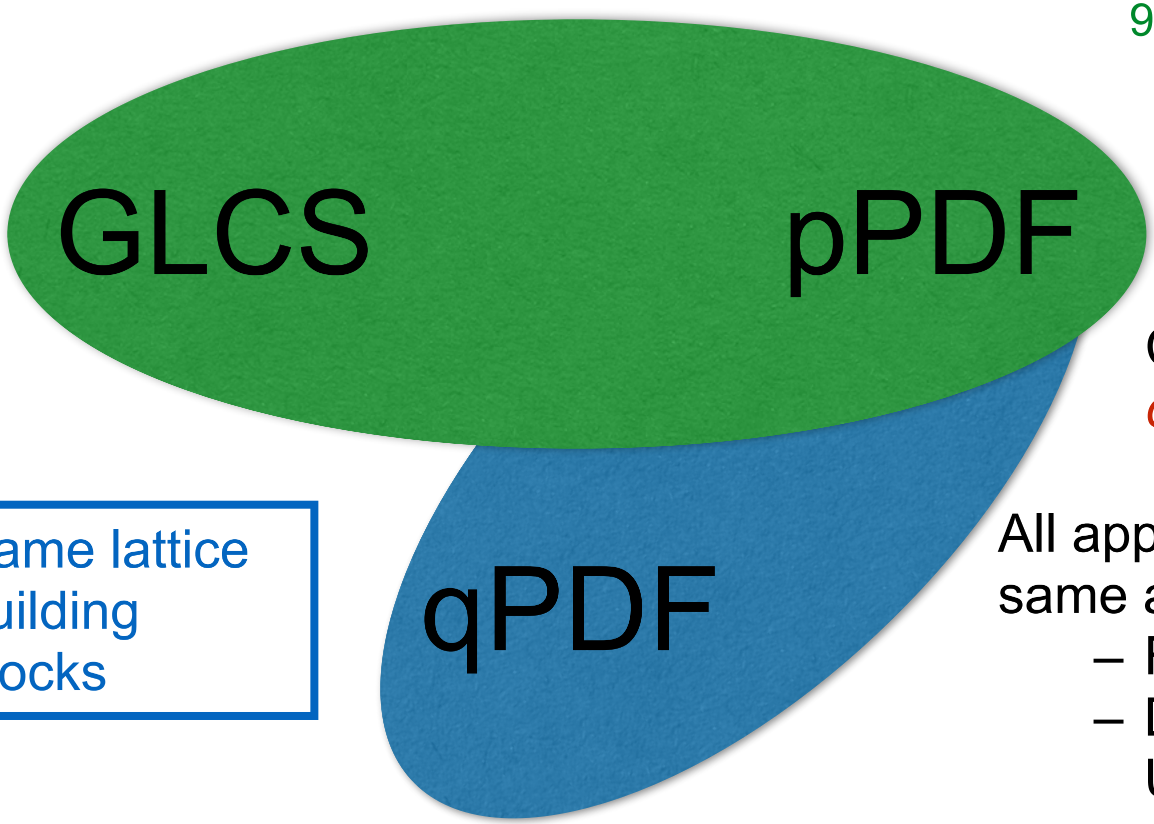
"quasi-PDF Approach"

PDFs, GPDs and TMDs

Ma and Qiu, Phys. Rev. Lett. 120 022003

A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

Light cone reduces to a point



Characterized by *short-distance factorization*

Same lattice building blocks

All approaches should give same after:

- Finite volume
- Discretization Uncertainties
- *Infinite momentum*

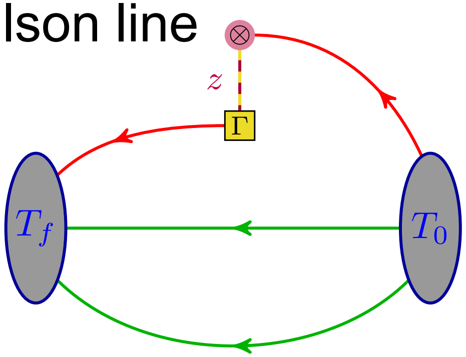
X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

Pseudo-PDFs

Lattice "building blocks" that of quasi-PDF approach.

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

Wilson line



A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

B.Ioffe, PL39B, 123 (1969); V.Braun *et al*, PRD51, 6036 (1995)

$$M^\alpha(p, z) = \langle p \mid \bar{\psi}\gamma^\alpha U(z;0)\psi(0) \mid p\rangle$$

$p = (p^+, m^2/2p^+, 0_T)$ $\qquad$ $z = (0, z_-, 0_T)$

$$\Downarrow$$

$$M^\alpha(z, p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to *space-like z*

Pseudo-PDFs

To deal with UV divergences, introduce reduced distribution

$$\mathfrak{M} = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0,z^2)} \equiv \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu,0)}\right) / \left(\frac{\mathcal{M}(0,z^2)}{\mathcal{M}(0,0)}\right)$$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(u\nu, \mu^2)$$

Computed on lattice

Perturbatively calculable

Ioffe-time Distribution

$$Q(\nu,\mu) = \mathfrak{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) B(u) + L(u)\right] \mathfrak{M}(u\nu, z^2).$$

K. Orginos et al., PRD96 (2017), 094503

Match data at different *z*

$\Downarrow$

$$Q(\nu) = \int_{-1}^{1} dx\, q(x) e^{i\nu x}$$

$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} Q(\nu)$$

Need data for all ν, *or additional physics input*

Inverse problem

ITD ↔ PDF

Ioffe-Time Distribution to PDF

J.Karpie, K.Orginos, A.Radyushkin, S.Zafeiropoulos, Phys.Rev.D 96 (2017)

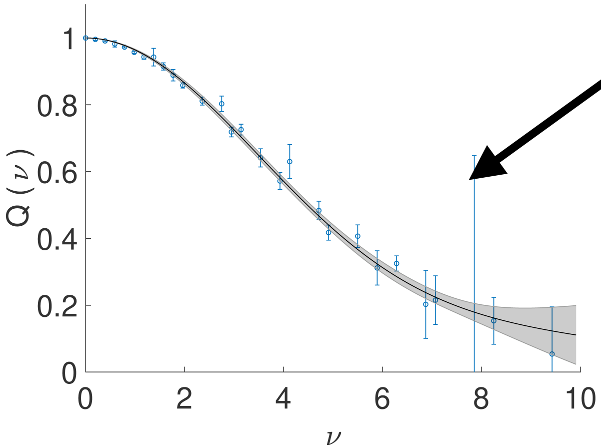
B.Joo *et al.*, HEP 12 (2019) 081, J.Karpie et al., *Phys.Rev.Lett.* 125 (2020) 23, 232003

To extract PDF requires additional information - *use a phenomenologically motivated parametrization*

$$f(x) = x^a(1-x)^b P(x)$$

MSTW, CJ

$$P(x) = \frac{1 + c\sqrt{x} + dx}{B(a+a, b+1) + cB(a+1.5, b+1) + dB(a+2, b+1)}$$



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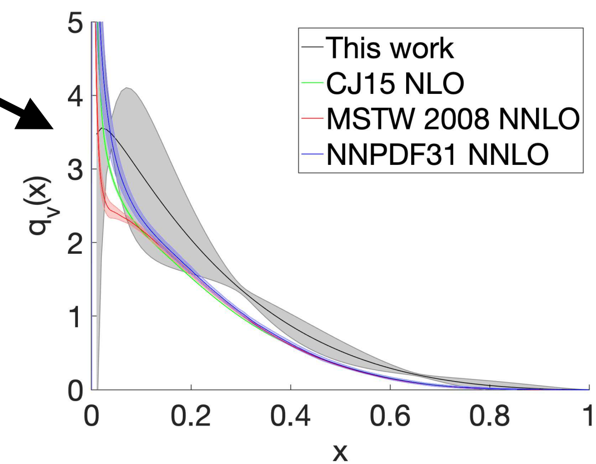
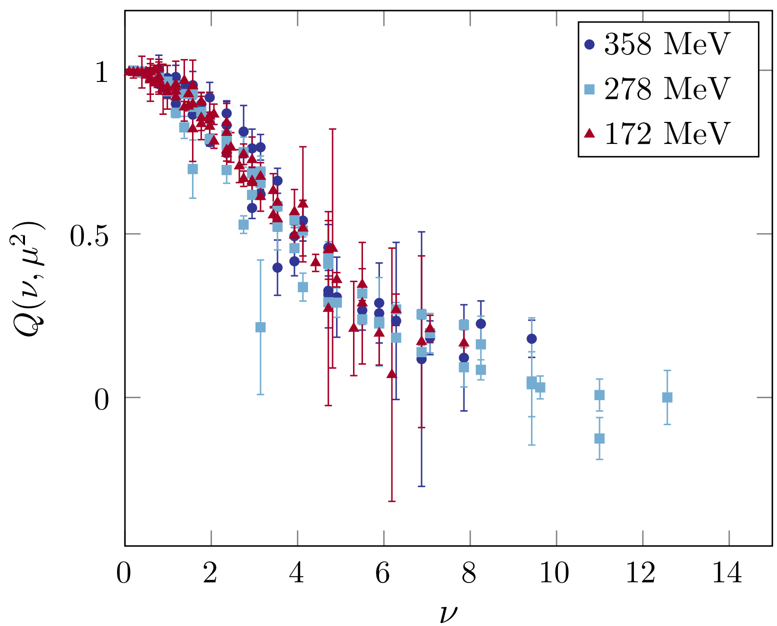
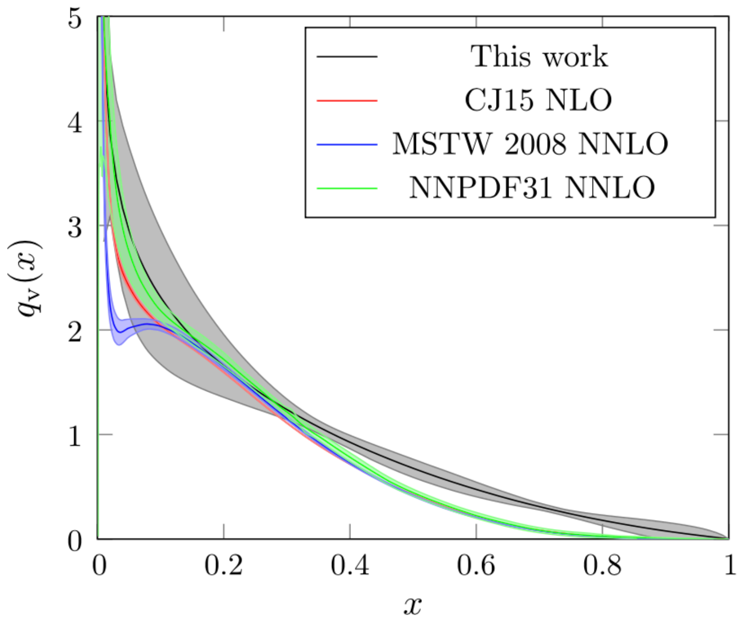


PDFs at Physical Quark Masses

ID	a(fm)	M_π(MeV)	β	c_{SW}	am_l	am_s	$L^3 \times T$	N_{cfg}
$a094m360$	0.094(1)	358(3)	6.3	1.20536588	-0.2350	-0.2050	$32^3 \times 64$	417
$a094m280$	0.094(1)	278(3)	6.3	1.20536588	-0.2390	-0.2050	$32^3 \times 64$	500
$a091m170$	0.091(1)	172(6)	6.3	1.20536588	-0.2416	-0.2050	$64^3 \times 128$	175





B.Joo *et al.*,Phys.Rev.Lett. 125 (2020) 23, 232003

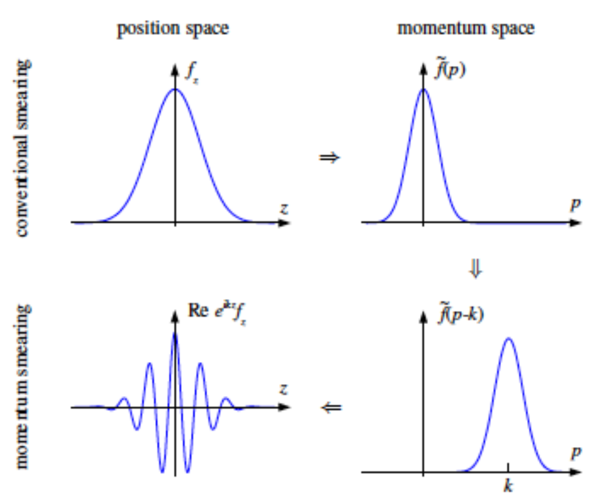
Physical pion

$$q_v(x, \mu^2, m_\pi) = q_v(x, \mu^2, m_0) + a\Delta m_\pi + b\Delta m_\pi^2$$

Challenges of Higher Momenta

Achieving high momenta in a lattice calculation presents several challenges

- Discretization errors
- "Compression" of energy spectrum as spatial momentum increased
- Reduced symmetries for states in motion - parities are mixed, helicity defines the basis
- Poor overlaps of e.g. Jacobi smearing on states in motion - poor signal-to-noise ratio.



Neat solution **Boosted interpolating operators**

Bali *et al.*, Phys. Rev. D 93, 094515 (2016)

Now essentially ubiquitous

Can we combine momentum smearing with distillation to address some of the other issues?

N.B Bali et al does indeed suggest application to distillation.

Look at

- *Nucleon energies and dispersion relation*
- *Nucleon charges*

Distillation

M.Peardon *et al* (Hadspec), Phys.Rev.D 80 (2009) 054506

Low-rank approximation to (typically) Jacobi-smearing kernel

$$-\nabla^2(t)\xi^{(k)}(t) = \lambda^{(k)}(t)\xi^{(k)}(t)$$

Rank →

$$\square(\vec{x},\vec{y};t)_{ab} = \sum_{k=1}^{R_{\mathcal{D}}} \xi_a^{(k)}(\vec{x},t)\,\xi_b^{(k)\dagger}(\vec{y},t)\,,$$

← Spatial Volume!

Components of distillation:

$$\tau_{\alpha\beta}^{(l,k)}(t',t) = \xi^{(l)\dagger}(t')\,M_{\alpha\beta}^{-1}(t',t)\,\xi^{(k)}(t)$$

Perambulators → quark propagation

$$\Phi_{\alpha\beta\gamma}^{(i,j,k)}(t) = \epsilon^{abc}\left(\mathcal{D}_1\xi^{(i)}\right)^a\left(\mathcal{D}_2\xi^{(j)}\right)^b\left(\mathcal{D}_3\xi^{(k)}\right)^c(t)\,S_{\alpha\beta\gamma}$$

Elementals → (baryon) operators

Projection to irrep

↓

$$C_{rs}(t) = \sum_{\vec{x},\vec{y}}\langle 0 \mid \mathcal{O}_r(t,\vec{x})\mathcal{O}_s^\dagger(0,\vec{y}) \mid 0\rangle \equiv \mathrm{Tr}\left[\Phi_r(t)\otimes\tau(t,0)\tau(t,0)\tau(t,0)\otimes\Phi_s(0)\right]$$

Matrix of correlators

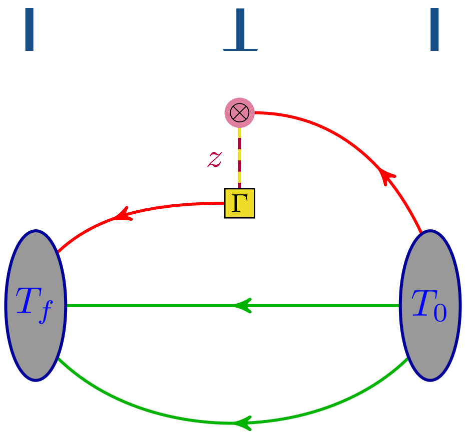
Extension to 3pt functions straightforward

Distillation and Hadron Structure

To control systematic uncertainties, need precise computations over a wide range of momentum.

- Use a low-mode projector to capture states of interest "distillation"
- Enables momentum projection at each temporal point.

Momentum projection

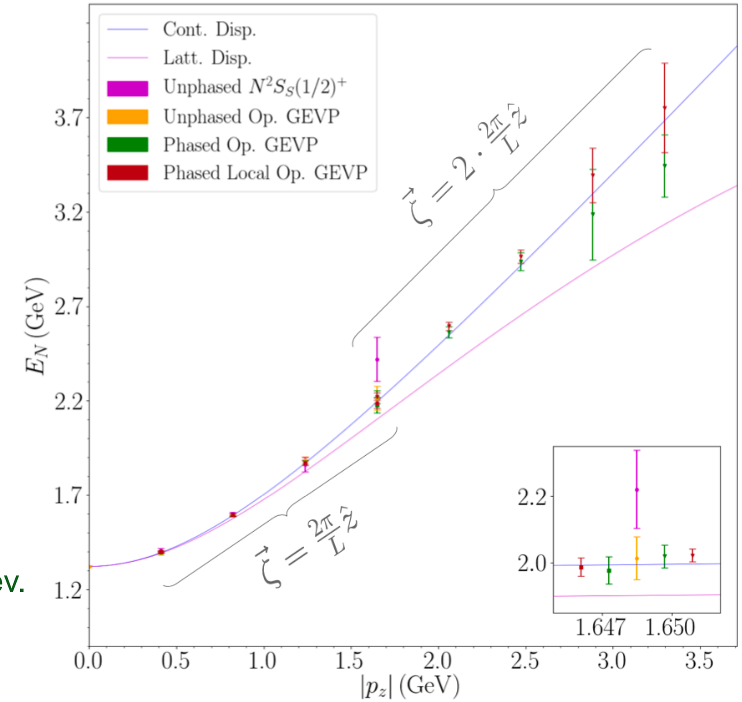


Variational basis

+ momentum smearing

G.Bali *et al*, Phys.Rev.D 93 (2016) 9, 094515

C.Egerer *et al* (Hadstruc), Phys. Rev. D 103, 034502 (2021)



Isovector PDF using Distillation

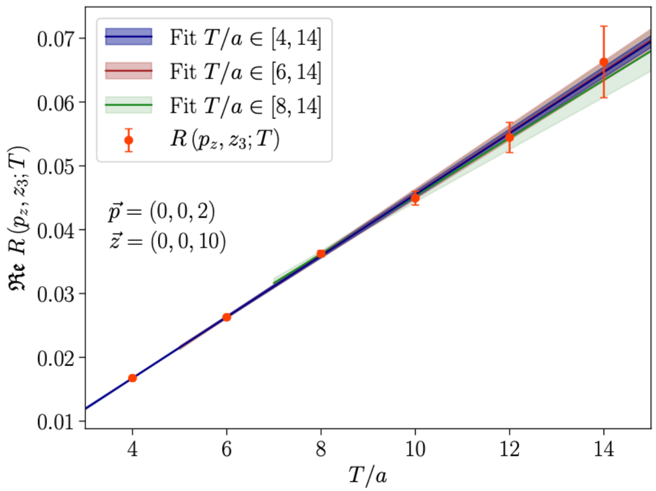
C.Egerer *et al.* (hadstruc), JHEP 11 (2021) 148

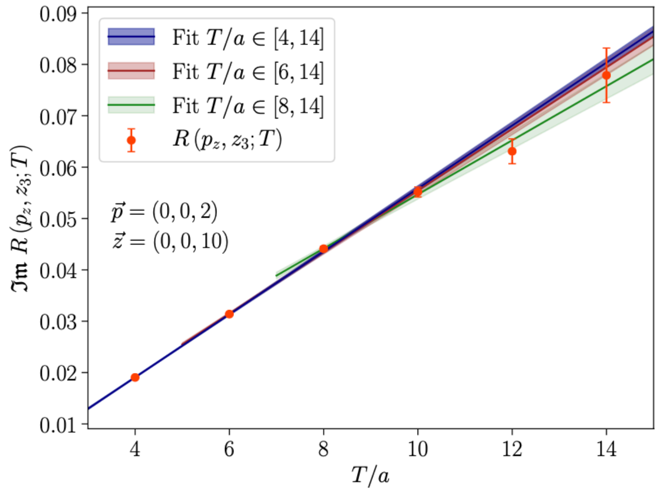
Numerics

ID	a_s (fm)	m_π (MeV)	$L_s^3 \times N_t$	N_{cfg}	N_{srcs}	$R_{\mathcal{D}}$
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	349	4	64

Used throughout rest of this talk

Matrix elements extracted using summation method - *reduced excited-state contributions*





Expand the x-dependence in terms of (shifted) Jacobi Polynomials

$$\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \mathfrak{Re}\int_0^1 dx\, \mathcal{K}_{\rm v}\left(x\nu, z^2\mu^2\right) x^\alpha (1-x)^\beta\, \Omega_n^{(\alpha,\beta)}(x)$$

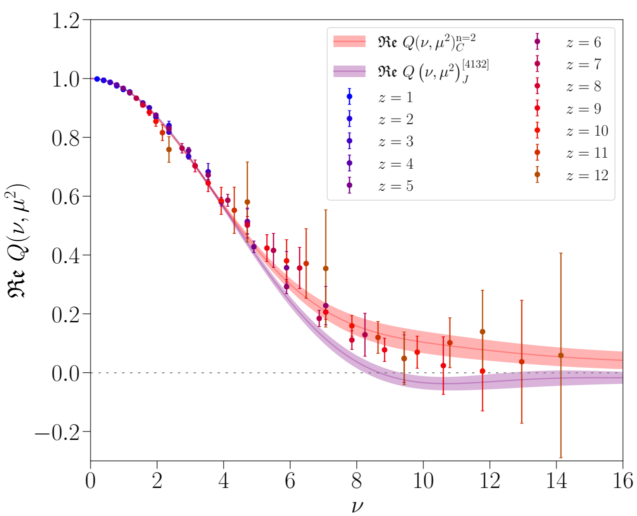
Matching kernel

J.Karpie,K.Orginos,A.Radyushkin,S.Zafeiropoulos, arXiv:2105.13313

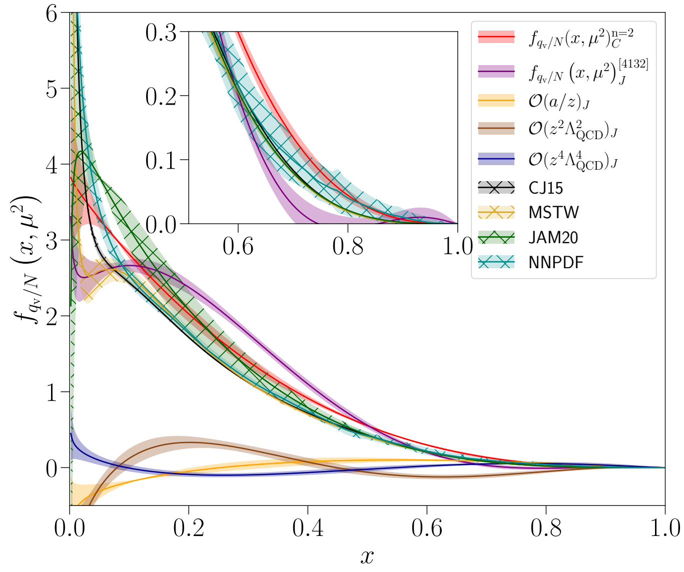
$$\mathfrak{Re}\,\mathfrak{M}_{\rm fit}\left(\nu, z^2\right) = \sum_{n=0}^{\infty}\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{{\rm v},n}^{lt\,(\alpha,\beta)} + \left(\frac{a}{z}\right)\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{az\,(\alpha,\beta)} + z^2\Lambda_{\rm QCD}^2\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{t4\,(\alpha,\beta)}$$

Discretization

Higher twist



$m_\pi \simeq 358$ MeV



Transversity Distribution

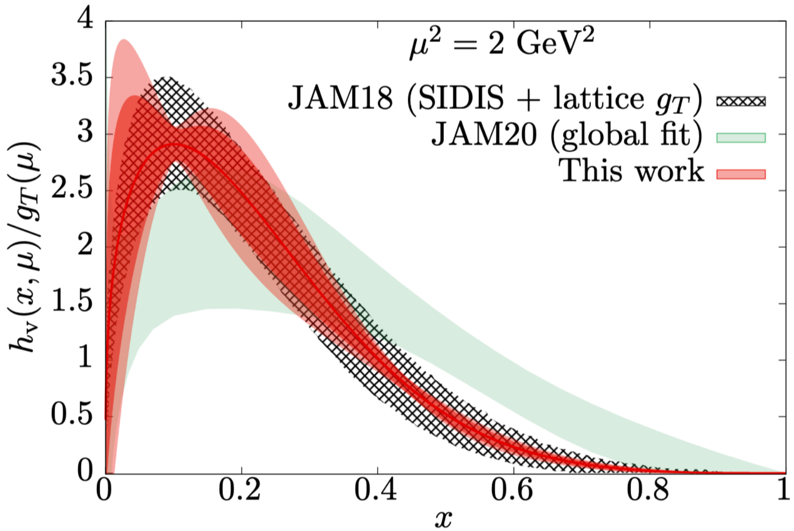
$$2P^+ S^{\rho_\perp} \mathcal{I}(P^+ z^-, \mu) = \langle P, S^{\rho_\perp} | \bar{\psi}(z^-) \gamma^+ \gamma^{\rho_\perp} \gamma_5 W_+(z^-, 0) \psi(0) | P, S^{\rho_\perp} \rangle$$

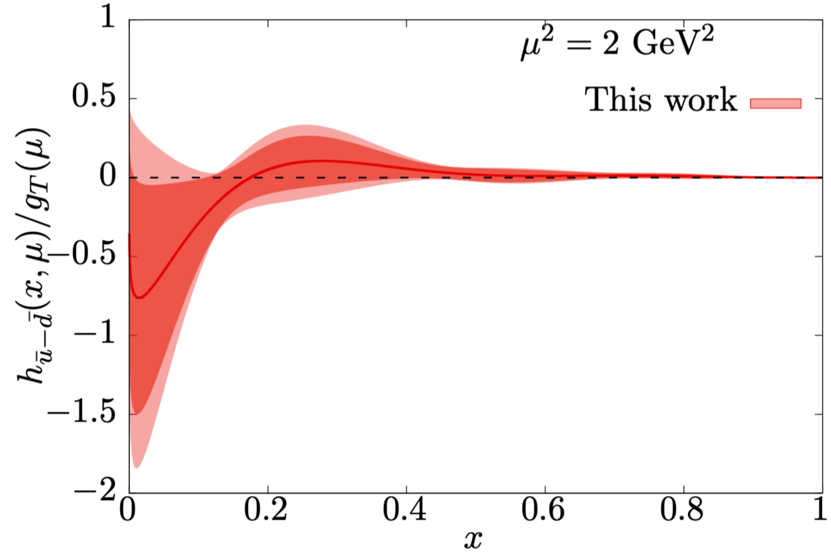
$$h(x, \mu) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} e^{-ix\nu} \mathcal{I}(\nu, \mu)$$

In contrast to unpolarized PDF, there is no conserved current - so express in terms of the (renormalized) tensor charge.

Phys.Rev.D 105 (2022) 3, 034507, Hadstruc Collaboration, (C.Egerer et al).

Isospin symmetric





Helicity Distribution

R.Edwards *et al. (HadStruc)*, JHEP 03 (2023) 086

$$M^{\mu 5}(p,z) = \langle N(p,S)\overline{\psi}(z)\gamma^{\mu}\gamma^{5}W^{(f)}(z,0)\psi(0)\rangle N(p,S)$$

Lorentz invariance

$$M^{\mu 5}(p,z) = -2m_N S^{\mu}\mathcal{M}(\nu,z^2) - 2im_N p^{\mu}(z\cdot S)\mathcal{N}(\nu,z^2) + 2m_N^3 z^{\mu}(z\cdot S)\mathcal{R}(\nu,z^2)$$

Spin polarization

As before, we exploit Lorentz invariance and choose matrix element that can be calculated on a Euclidean lattice

$$M^{35}(p,z_3) = -2m_N S^3[p_z\hat{z}]\{\mathcal{M}(\nu,z_3^2) - ip_z z_3\mathcal{N}(\nu,z_3^2)\} - 2m_N^3 z_3^2 S^3[p_z\hat{z}]\mathcal{R}(\nu,z_3^2)$$

$$M^{35}(p,z_3) = -2m_N S^3[p_z\hat{z}]\{\mathcal{Y}(\nu,z_3^2) + m_N^2 z_3^2\mathcal{R}(\nu,z_3^2)\}$$

$$\widetilde{\mathcal{Y}}(\nu,z_3^2)$$

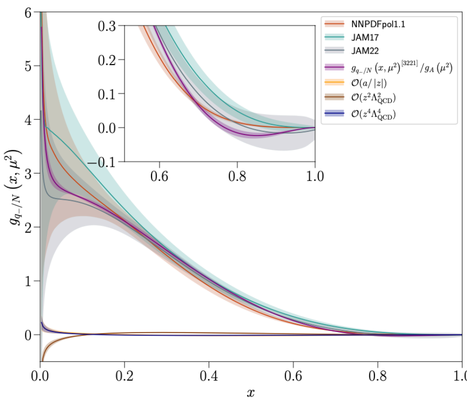
Reduced distribution: $\mathfrak{Y}(\nu,z_3^2) = \left(\frac{\widetilde{\mathcal{Y}}(\nu,z_3^2)}{\widetilde{\mathcal{Y}}(0,z_3^2)|_{p_z=0}}\right) \Big/ \left(\frac{\widetilde{\mathcal{Y}}(\nu,0)|_{z_3=0}}{\widetilde{\mathcal{Y}}(0,0)|_{p_z=0,z_3=0}}\right)$

$$\mathfrak{Y}\left(\nu, z_3^2\right) = \frac{1}{g_A(\mu^2)} \int_0^1 \mathrm{d}u\, \mathcal{C}\left(u, z_3^2\mu^2, \alpha_s\left(\mu^2\right)\right) \mathcal{I}\left(u\nu, \mu^2\right) + \mathcal{O}\left(z_3^2 \Lambda^2_{\mathrm{QCD}}\right)$$

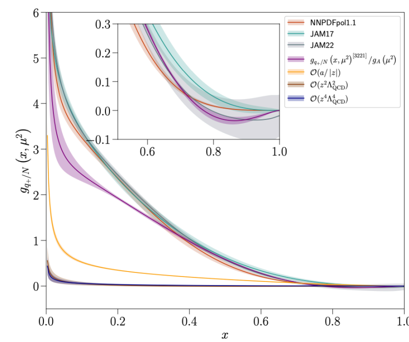
Not conserved current - normalize to g_A

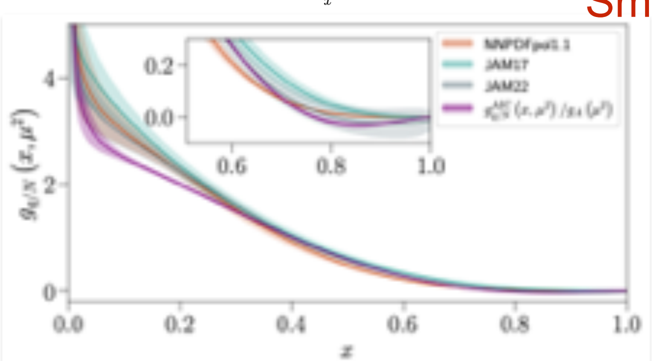
where $\mathcal{I}\left(\nu, \mu^2\right) = \int_{-1}^{1} \mathrm{d}x\; e^{i\nu x} g_{q/N}\left(x, \mu^2\right)$

Valence quark helicity distribution, together with contamination terms

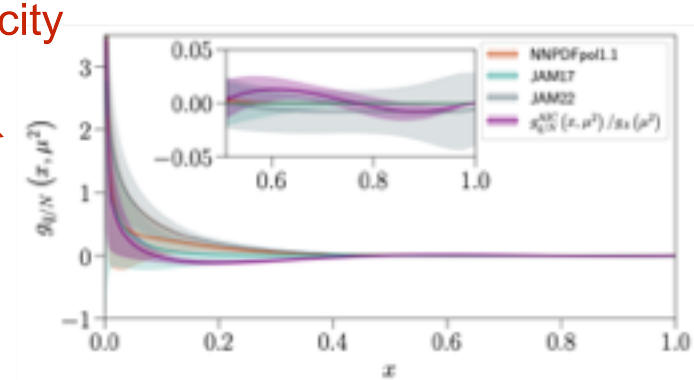


CP-odd helicity distribution, together with contamination terms





Small NS anti-quark helicity



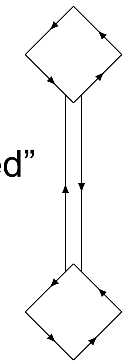
Gluon PDF

Gluon Contribution to unpolarized PDF

c.f. Z.Fan, H-W-Lin, arXiv:2104.06372, arXiv:2007.16113

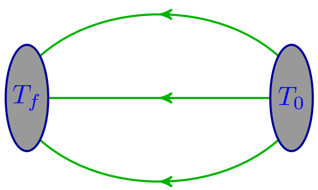
T.Khan *et al.* (Hadstruc), *Phys.Rev.D* 104 (2021) 9, 094516

“disconnected”



$$M_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p|\, G_{\mu\alpha}(z)\, W[z,0]\, G_{\lambda\beta}(0)\, |p\rangle$$

$$O_g(z) = G_{ji}(z)\, U(z,0)\, G_{ij}(0)\, U(0,z) - G_{ti}(z)\, U(z,0)\, G_{it}(0)\, U(0,z).$$



Two-point functions as in isovector case

Reduced matrix element: $\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)|_{z=0}}\right) / \left(\frac{\mathcal{M}(0, z^2)|_{p=0}}{\mathcal{M}(0,0)|_{p=0,z=0}}\right)$

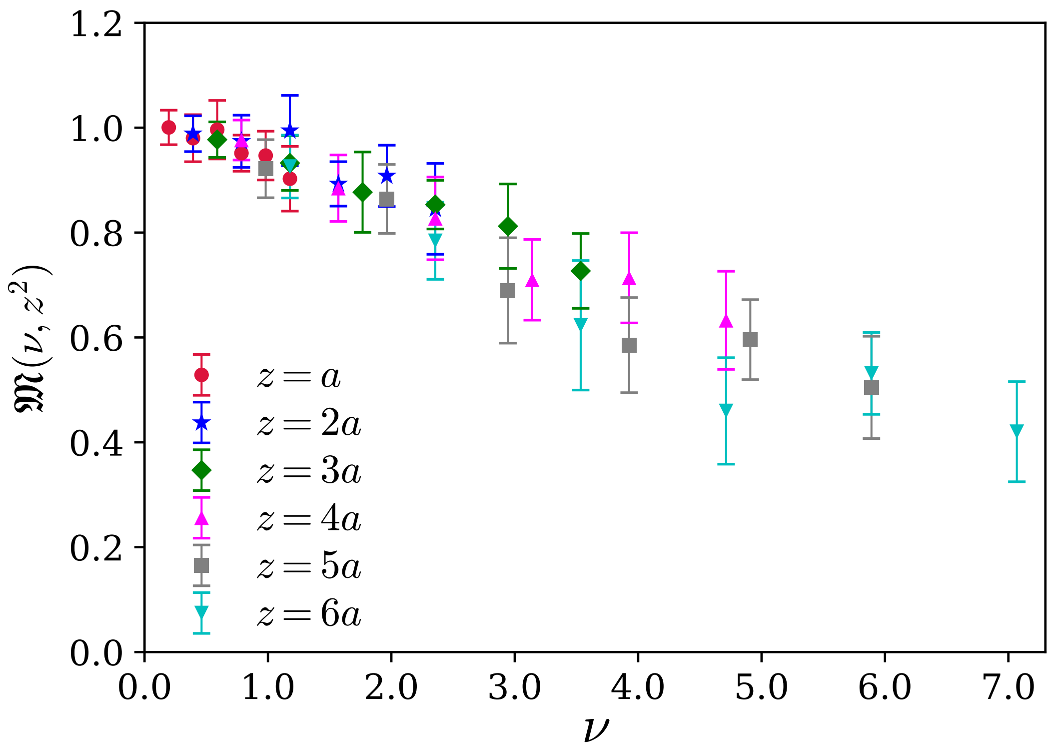
Flavor-singlet quantities are subject to severe signal-to-noise problems compared with isovector measures:

- Use distillation and many more measurements per configuration - *sampling of lattice*
- Use of summed Generalized Eigenvalue Problem (sGEVP) - *better control over excited state contributions*
- Use of *Gradient Flow - smoothing of short-distance fluctuations*

Ioffe-time distributions

Use Gradient flow - to further reduce UV fluctuations

Insert flowed link variable $\dot{V}_\mu(\tau, x) - -g_0^2\{\partial_{x,\mu} S(V_\mu(\tau, x))V_\mu(\tau, x)\}V_\mu(\tau, x)$



$\tau \longrightarrow 0$

ITD to PDF

Matching: I.Balitsky,W.Morris,A.Radyushkin,Phys.Lett.B 808 (2020) 135621

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{I}_g(\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} - \frac{\alpha_s N_c}{2\pi}\int_0^1 du\, \frac{\mathcal{I}_g(u\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} \left\{ \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) B_{gg}(u) + 4\left[\frac{u+\ln(\bar{u})}{\bar{u}}\right]_+ + \frac{2}{3}\left[1-u^3\right]_+ \right\}$$

N.B neglecting quark-gluon mixing

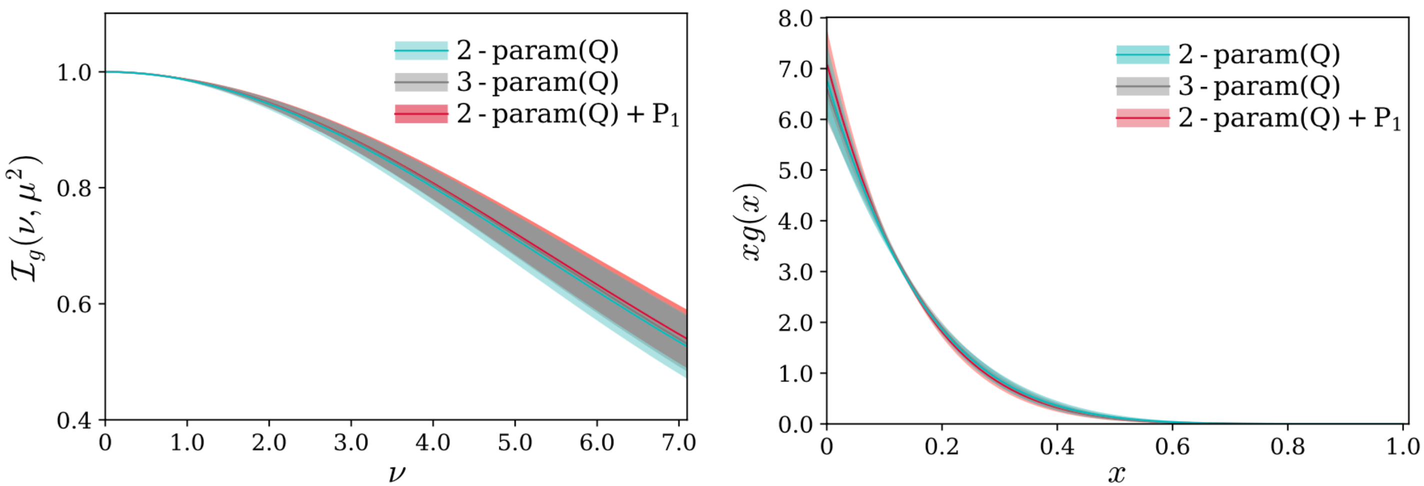
Implementation for obtaining the PDFs follows that of the isovector distribution

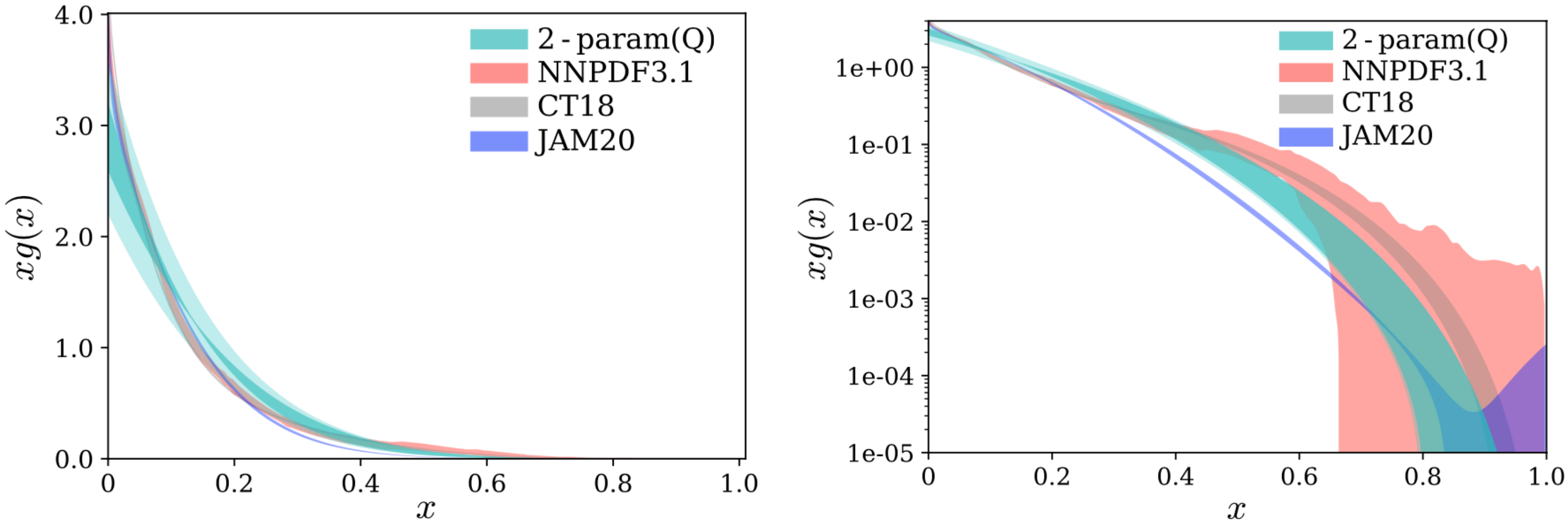
– *Expand in Jacobi Polynomials*

$x^\alpha(1-x)^\beta$

$+J_1^{\alpha,\beta}$

$+a/|z|$





Require normalization of $xg(x)$ $\langle x \rangle_g^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV}) = 0.427(92)$

C.Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017)

Gluon Helicity PDF

Matrix elements of spatially separated gluon fields

$$\tilde{m}_{\mu\alpha;\lambda\beta} = \langle p, s \,|\, G_{\mu\alpha}(x) W[z,0] \tilde{G}_{\alpha\beta}(0) \,|\, p, s \rangle$$

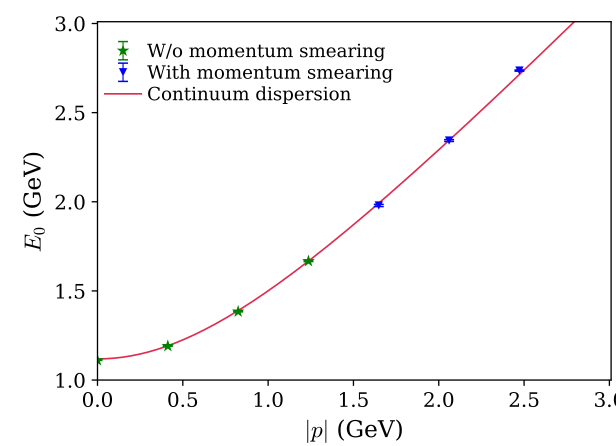
Combination corresponding to polarized gluon distribution

$$\tilde{M}_{\mu\alpha;\lambda\beta}(z,p,s) = \tilde{m}_{\mu\alpha;\lambda\beta}(z,p,s) - \tilde{m}_{\mu\alpha;\lambda\beta}(-z,p,s)$$

Ioffe-time distribution is related to gluon distribution through inverse problem

$$\tilde{\mathcal{I}}(\nu) = \frac{i}{2}\int_{-1}^{1} e^{-ix\nu} x \Delta g(x)$$

ID	a (fm)	m_π (MeV)	$L^3 \times N_t$	N_{cfg}	N_{srcs}
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	1901	64



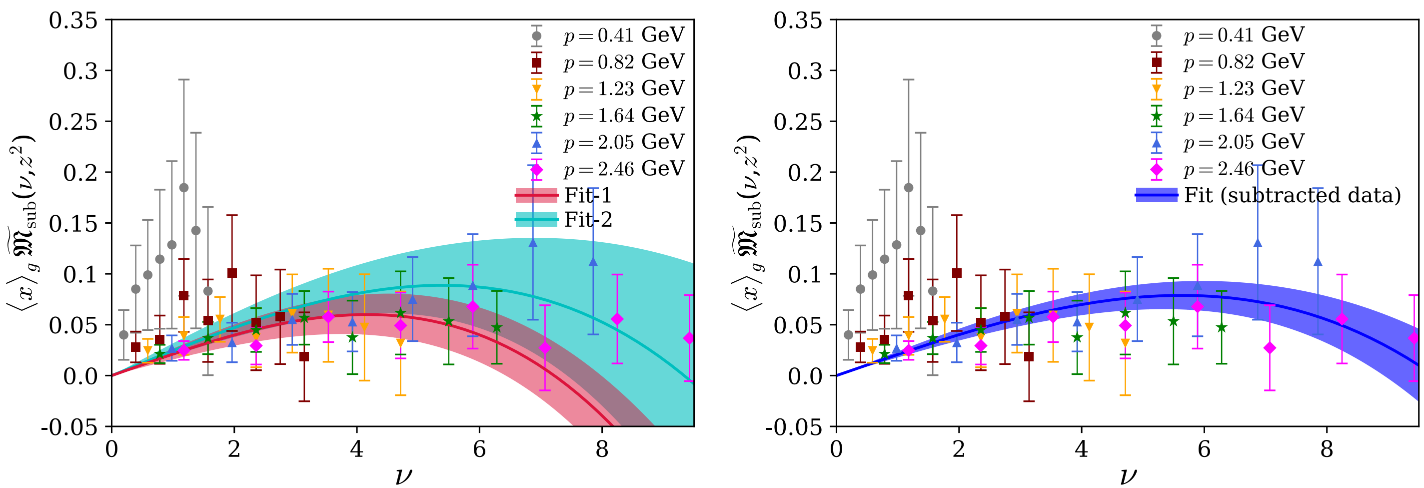
$$\widetilde{\mathfrak{M}}(\nu, z^2) = \left[\widetilde{\mathcal{M}}_{sp}^{(+)}(\nu, z^2) - \nu \widetilde{\mathcal{M}}_{pp}(\nu, z^2)\right] + \frac{m_p^2}{p_z^2} \nu \widetilde{\mathcal{M}}_{pp}(\nu, z^2)\,,$$

"Nuisance term"

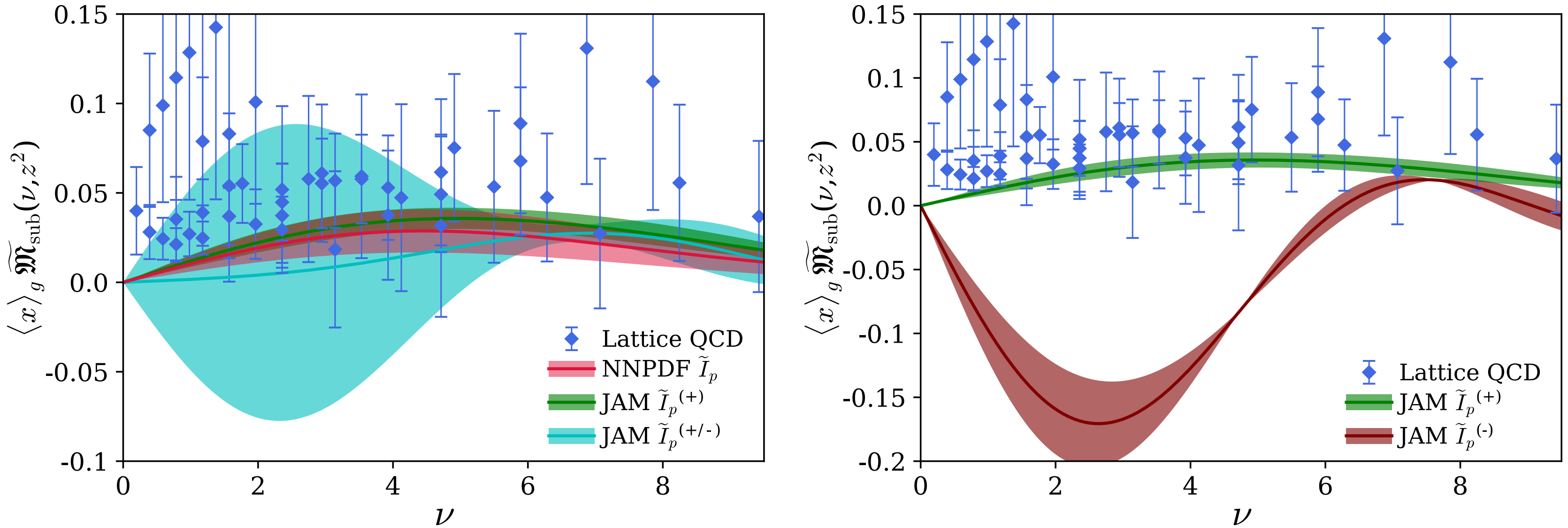
Rather than fitting to $\tilde{\mathscr{M}}$ directly define subtracted matrix element

$$\widetilde{\mathcal{M}}_{\rm sub}(z,p_z) = \widetilde{\mathcal{M}}_{sp}^{(+)}(\nu,z^2) - \nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2) - \nu\frac{m_p^2}{p_z^2}\left[\widetilde{\mathcal{M}}_{pp}(\nu,z^2) - \widetilde{\mathcal{M}}_{pp}(\nu=0,z^2)\right]$$

Still contains nuisance term - but small



Recall ITD ↔ PDF



C.Egerer *et al. (HadStruc)*, Phys.Rev.D 106 (2022) 9, 094511

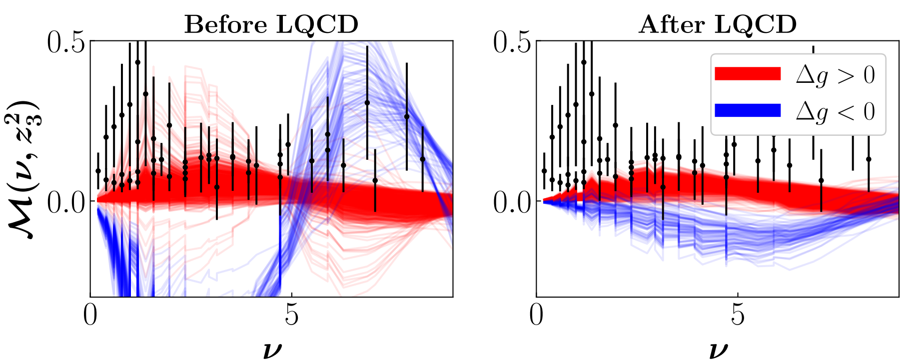
LQCD Calculation of gluon helicity distribution compared with global analyses

Caveat! Mixing with sea quarks not yet included

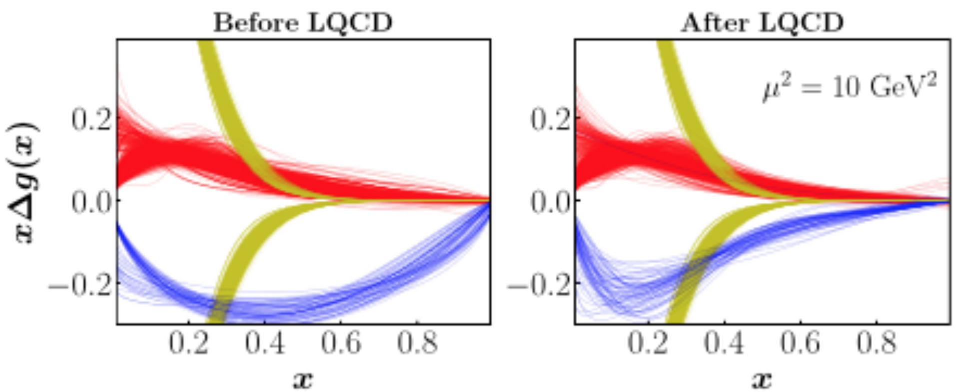
Lattice + Expt

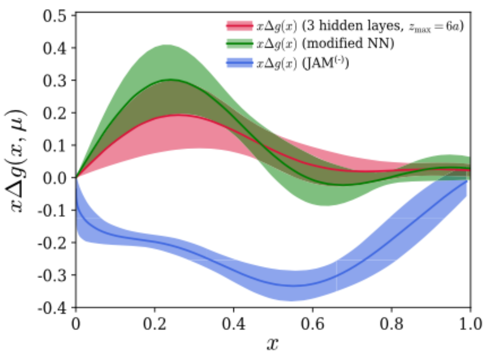
The culmination of QGT is a framework where LQCD + Expt can provide a more faithful description of hadron structure than either alone.

Does QCD admit negative solutions $\Delta g(x) < 0$



J.Karpie *et al.*, Phys.Rev.D 109 (2024) 3, 036031





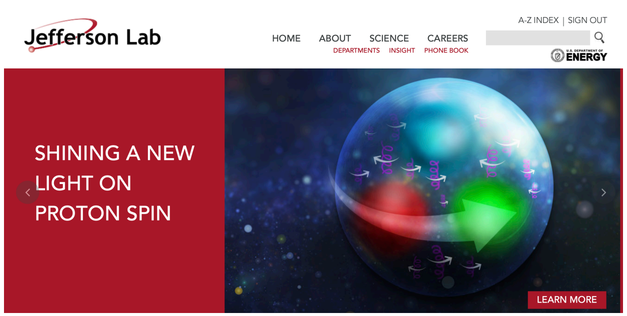
Neural network analysis of **lattice calculation**

T.Khan,T.Liu and R.Sufian, Phys. Rev. D 108, 074502

Significant change in parametrization but insufficient to exclude negative solute in global analysis

Impact of LQCD on global analysis

RHIC polarized jet, large-x JLab + LQCD eliminate negative solutions: N.Hunt-Smith et al., arXiv:2403.08117



See S. Kumano, Monday

3D Hadron Structure

Generalized Parton Distributions (GPDs) provide 3D description in terms of longitudinal momentum fraction and (2D) transverse displacement

- Orbital Angular Moment
- Integrated Generalized Form Factors: distribution of mass, charge, pressure

(Pseudo)-GPDs from Lattice QCD

H.Dutrieux et al., (HadStruc), arXiv:2405.10304 See also S.Bhattacharya *et al.,* Phys.Rev.D 108 (2023) 1 014507

GPDs described by *off-forward* matrix elements of operators *along light cone*

$$F^q(x, p_f, p_i) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \tag{2.1}$$

$$\times \langle N(p_f,\lambda_f)|\, \bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+ \hat{W}\left(-\frac{z}{2},\frac{z}{2};A\right)\psi^q\left(\frac{z}{2}\right)|N(p_i,\lambda_i)\rangle\,|_{z^+=0,\mathbf{z}_\perp=\mathbf{0}_\perp},$$

$$= \frac{1}{2P^+}\overline{u}(p_f,\lambda_f)\left[\gamma^+ H^q(x,\xi,t) + \frac{i\sigma^{+\nu}q_\nu}{2m}E^q(x,\xi,t)\right]u(p_i,\lambda_i)\,, \tag{2.2}$$

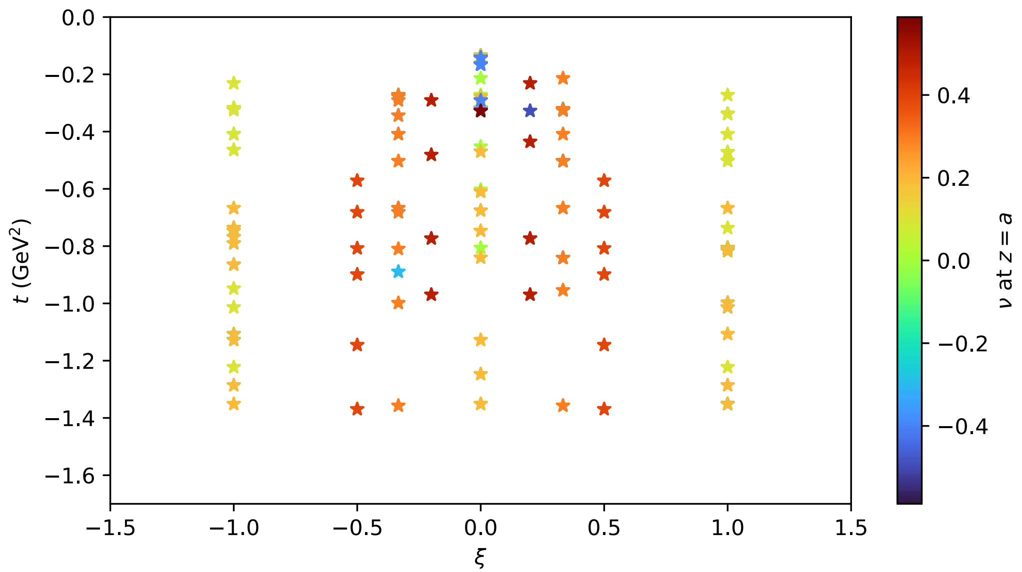
Kinematic variables $P \equiv \frac{1}{2}(p+p')\,,\quad q \equiv p'-p\,,\quad t \equiv q^2\,,\quad \xi \equiv -\frac{q^+}{2P^+}\,.$

Skewness

As for the case of the PDFs, we calculate matrix elements at *space-like separations z.*
We can then express skewness as

$$\xi = -\frac{q \cdot z}{2P \cdot z} = -\frac{q \cdot z}{2\nu}\,.$$

- In contrast to DVCS and DVMP, lattice QCD admits the calculation of GPDs at discrete points in 3D

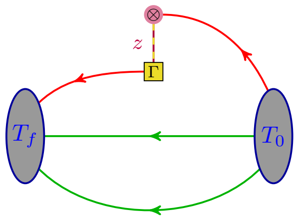


IGPDs and GPDs related through transform

$$\begin{pmatrix} H^q \\ E^q \end{pmatrix}(x,\xi,t) = \int \frac{d\nu}{2\pi} e^{ix\nu} \begin{pmatrix} H^q \\ E^q \end{pmatrix}(\nu,\xi,t)\,,$$

As before, this involves tackling *inverse problem*

Very Computationally and Data Demanding



$$\Xi^{\Gamma(i,j)}_{\alpha\beta;ab}(T_f,T_i;\tau,z) = \sum_{\vec{y}} \xi^{(i)\dagger}_a(T_f)\, D^{-1}_{\alpha\sigma;ac}(T_f;\tau,\vec{y})\, \Gamma(\tau)\, D^{-1}_{\rho\beta;db}(\tau,\vec{x};T_i)\, \xi^{(j)}_b(T_i)$$

Do calculations "on the fly"…

Moments of GPDs

For this first study, we will focus on calculations of the moments of GPDs

$$\int_{-1}^{1} \mathrm{d}x\, x^{n-1} \begin{pmatrix} H^{u-d} \\ E^{u-d} \end{pmatrix} (x, \xi, t) = \sum_{k=0 \text{ even}}^{n-1} \begin{pmatrix} A_{n,k}(t) \\ B_{n,k}(t) \end{pmatrix} \xi^k \,.$$

Moments can be obtained by ν expansion of the Ioffe-time distribution

$$F(\nu, \xi, t, z^2) = \sum_{n=0}^{\infty} \frac{(-i\nu)^n}{n!} F_{n+1}(\xi, t, z^2)$$ where $$F_n(\xi, t, z^2) \equiv \int_{-1}^{1} dx\, x^{n-1} F(x, \xi, t, z^2)$$

We fit the resulting GFFs to a *dipole* $A_{n,k}(t) = A_{n,k}(t=0)\left(1 - \frac{t}{\Lambda_{n,k}^2}\right)^{-2}$

More rigorously, use the so-called *z-expansion.*

Pion mass = 0.36 GeV - Proton mass = 1.12 GeV

No continuum limit - signs of discretization errors / light-cone uncertainty

Matching at 2 GeV with leading logarithmic accuracy

Value at t = 0

GPD H^{u-d}

$A_{1,0}$: 0.97(2)	
$A_{2,0}$: 0.204(4)	
$A_{3,0}$: 0.062(4)	$A_{3,2}$: 0.42(7)
$A_{4,0}$: 0.06(1)	$A_{4,2}$: 0.5(2)

GPD E^{u-d}

$B_{1,0}$: 3.44(4)	
$B_{2,0}$: 0.36(2)	
$B_{3,0}$: 0.07(2)	$B_{3,2}$: 0.9(6)
$B_{4,0}$: 0.06(4)	$B_{4,2}$: 1.2(9)

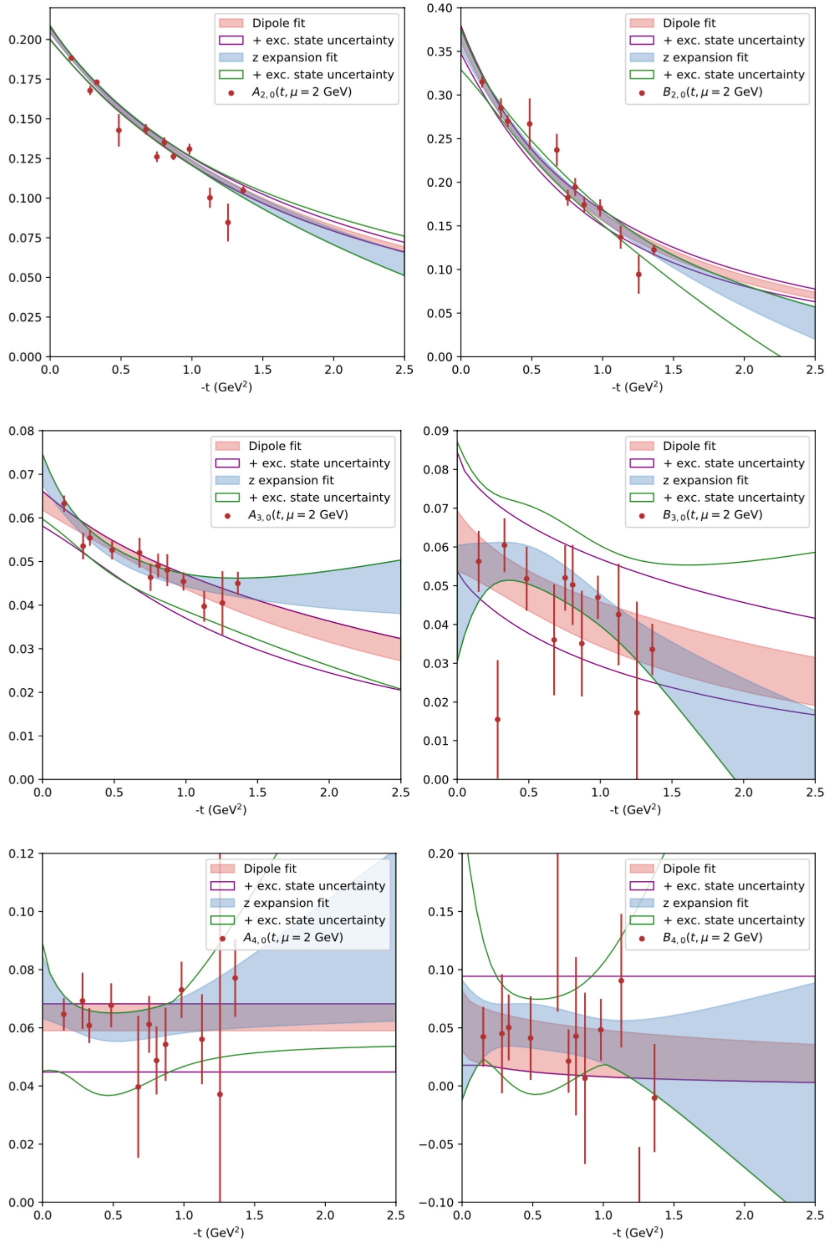
Dipole mass (GeV)

GPD H^{u-d}

$A_{1,0}$: 1.25(2)	
$A_{2,0}$: 1.86(6)	
$A_{3,0}$: 2.2(4)	$A_{3,2}$: 1.07(9)
$A_{4,0}$: Unreliable	$A_{4,2}$: 1.2(2)

GPD E^{u-d}

$B_{1,0}$: 0.982(6)	
$B_{2,0}$: 1.41(8)	
$B_{3,0}$: 2.4(9)	$B_{3,2}$: 1.0(3)
$B_{4,0}$: Unreliable	$B_{4,2}$: 1.1(2)



ξ^0

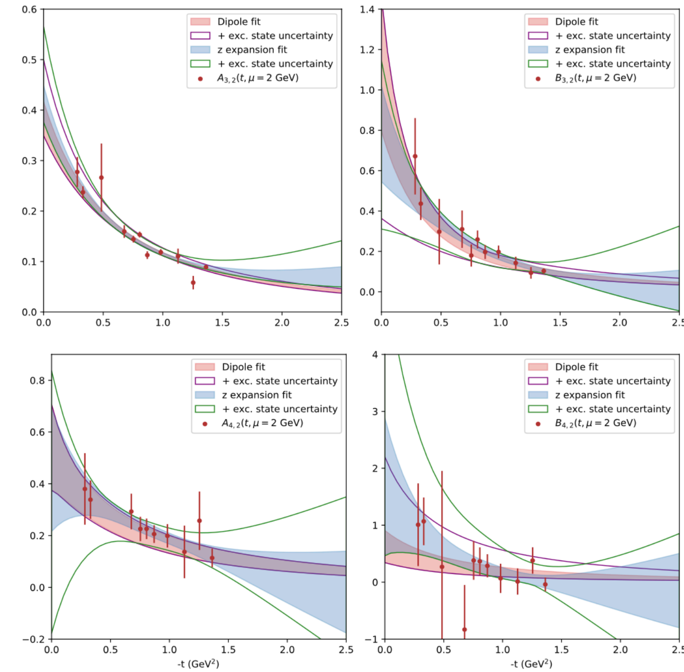
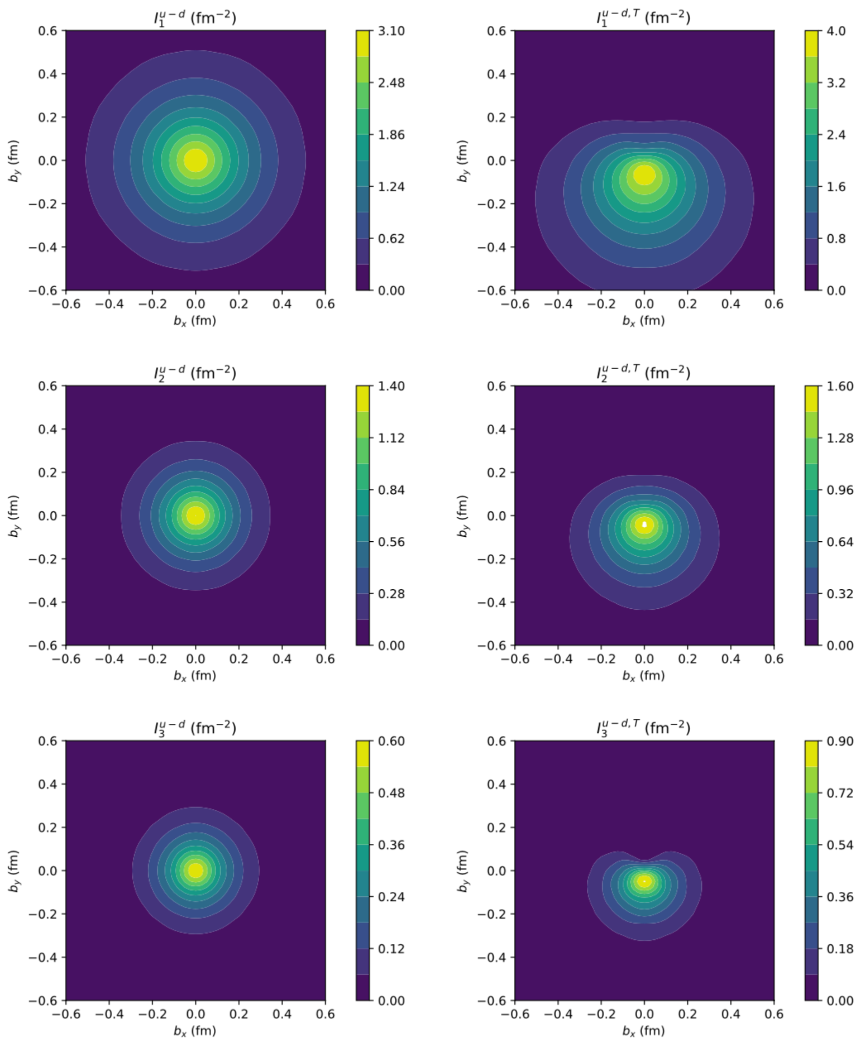


Figure 18. The skewness-dependent generalized form factors. Same caption as Fig. 17.

ξ^2

Translate to Impact-Parameter Space

Transform to impact-parameter space:
narrowing of distribution with increasing moment



Summary

- Realistic calculation of light-cone distributions from LQCD now available
- Focus on understanding systematic contributions in pseudo-PDF framework
- Distillation + boosting enables both far increased reach in momentum, and improved sampling of lattice
  - *Essential in calculations of gluon contributions*
- Are able to isolate leading twist from higher-twist and discretization contamination
- Calculation of isosinglet contributions incomplete - inclusion *of sea-quark distributions*.
- 3D Hadron Structure through GPDs
  - Moment calculation allows higher moments than from local operators
  - Direct calculation of x dependence in progress
  - Next frontier - flavor singlet. Provides access to so-called D-term

FIRST INTERNATIONAL SCHOOL OF HADRON FEMTOGRAPHY

Jefferson Lab | September 16 - 25, 2024

The Center for Nuclear Femtography (CNF) and the Quark and Gluon Tomography (QGT) collaboration have joined forces to launch the First International School of Hadron Femtography. The school will take place at Jefferson Lab September 16-25, 2024. The program is designed to offer comprehensive lectures aimed at early-career experimental and theoretical scientists, including graduate students and post-doctoral researchers.

Acceptance to the program is through competitive application. Support will be provided for accepted participants, funded by CNF, supported by the Commonwealth of Virginia, and QGT, supported by the US Department of Energy. Participants will be housed on site at Jefferson Lab with ample opportunity for interactions with lecturers, and with other participants. Applications are now open, and for full consideration applications must be received by June 24, 2024.

Topics:
QCD Analysis - Theory & Experiment
Processes, DVCS, DVMP and multiparticle final states
Lattice QCD
Imaging Structure & Dynamics
GPD analysis as an Inverse problem
Experimental methodologies
AI for nuclear femtography

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Further details can be found at:
https://www.jlab.org/conference/HadronFemtographySchool
Email: femtoschool@jlab.org