N^2LO contributions to Neutrinoless Double Beta Decay Nuclear Matrix Elements

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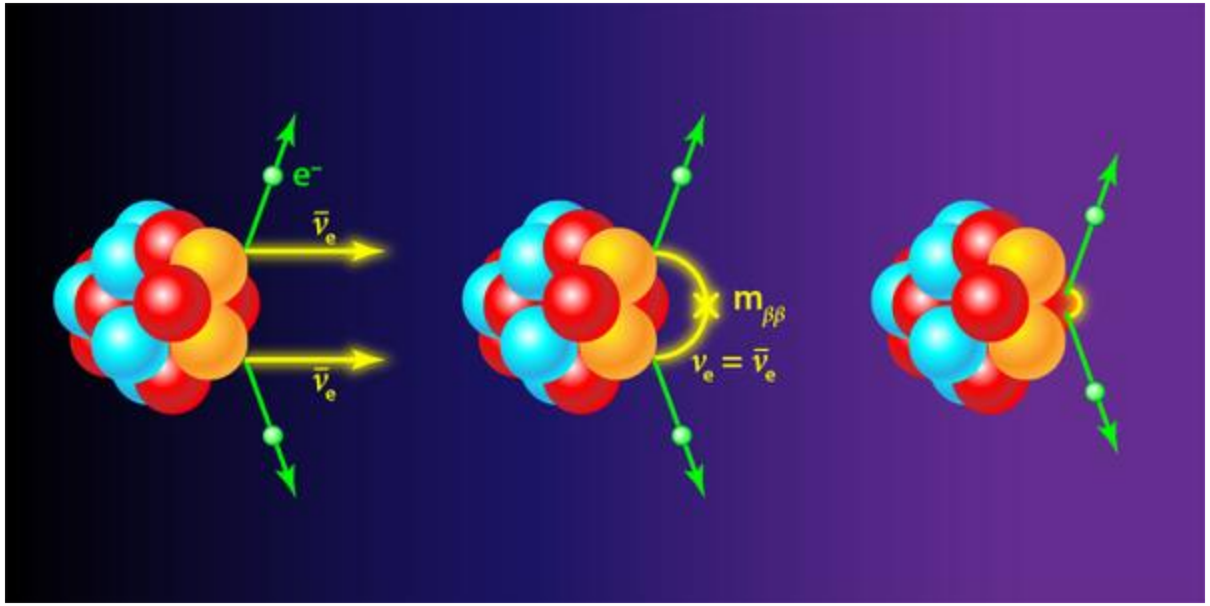
Outline

1. Refresher on $0\nu\beta\beta$ and ab initio nuclear theory
2. VS-IMSRG Methodology
3. Operators in χEFT
4. Results & discussion of N^2LO operator implementation

$0\nu\beta\beta$ refresher

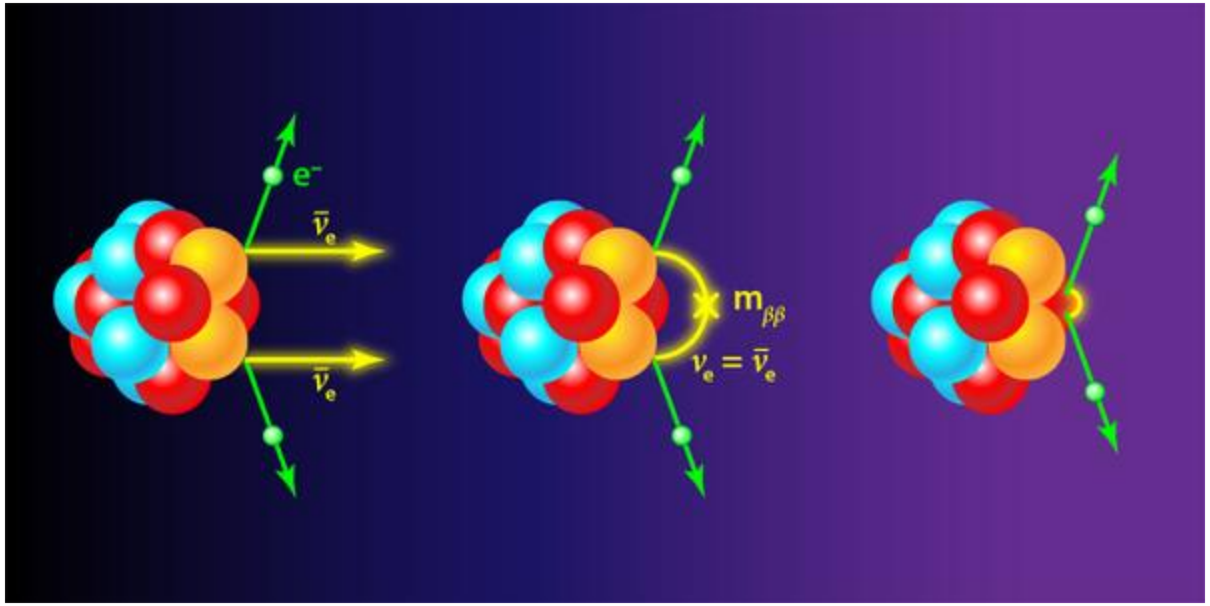
- Two simultaneous beta decays, but no neutrinos emitted
- Neutrinos annihilate each other – Majorana property
- Lepton number violating ($\Delta L=2$)



$0\nu\beta\beta$ refresher

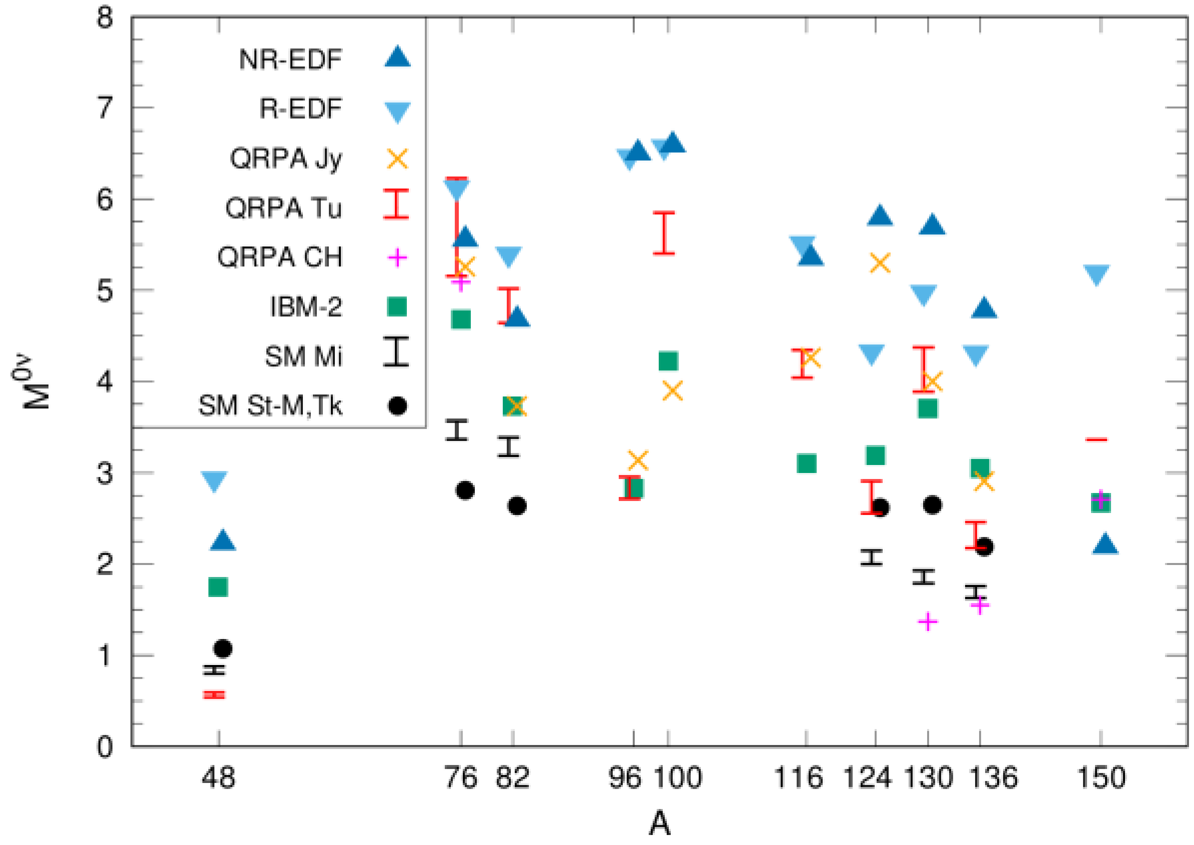
$$\left[T_{1/2}^{0\nu}\right]^{-1} = g_A^4 \frac{\langle m_{\beta\beta}\rangle^2}{m_e^2} G^{0\nu} \boxed{|M^{0\nu}|^2}$$

(light neutrino exchange model)



Ab Initio Nuclear Theory

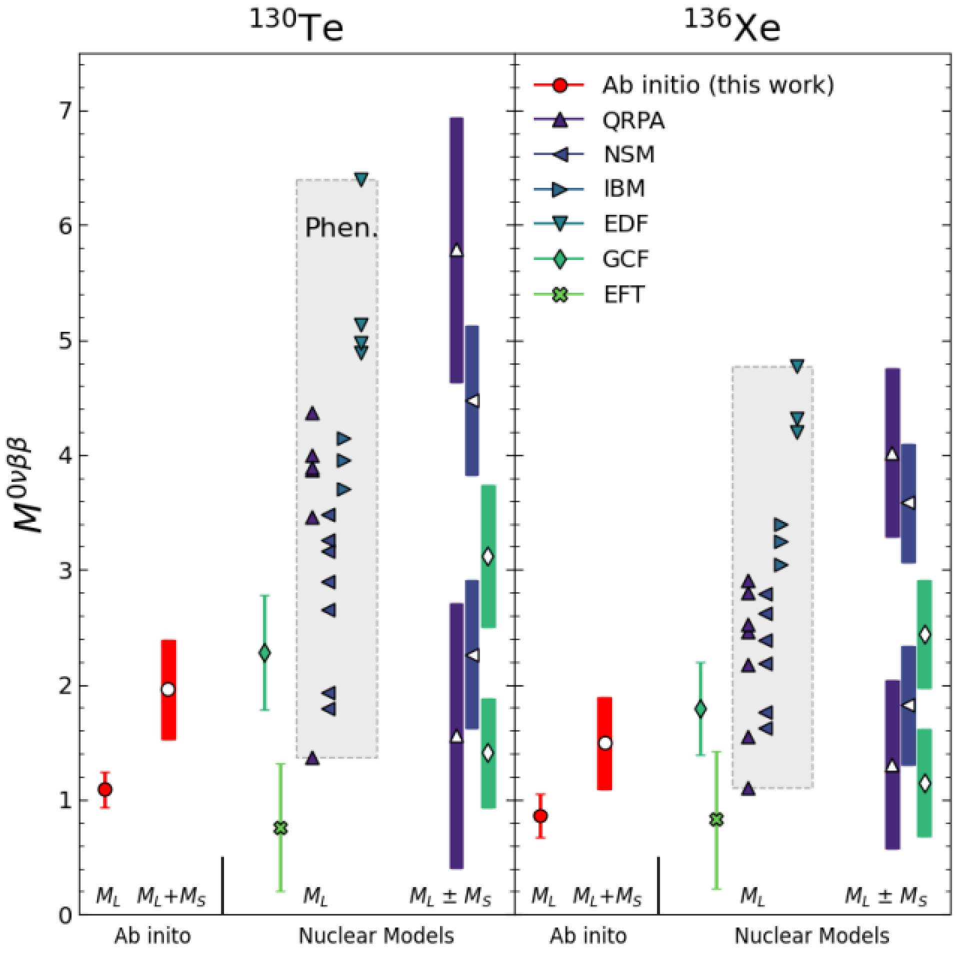
- Two routes for determining matrix element – phenomenological and ab initio
- Phenomenological requires fitting to $2\nu\beta\beta$ data to obtain an effective Hamiltonian to use with shell model



J. Engel & J. Menéndez, 2017 *Rep. Prog. Phys.* **80** 046301

Ab Initio Nuclear Theory

- Ab initio methods based in Chiral Effective Field Theory (χEFT)
- Goal is also to obtain an effective Hamiltonian, but not by fitting to data
- Smaller results spread, systematically improvable & rigorous uncertainty propagation possible



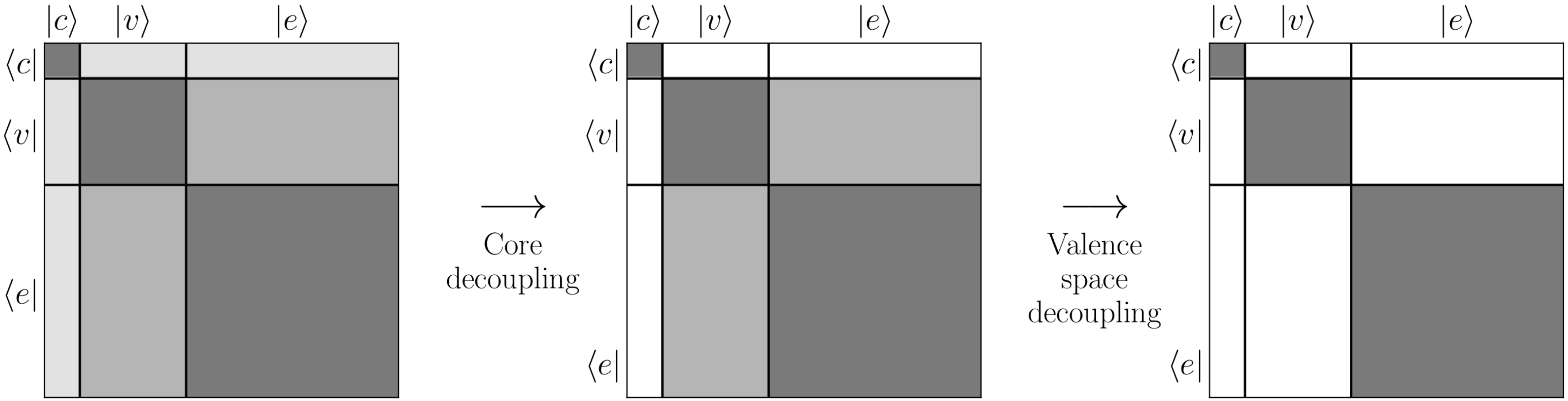
A. Belley et al, 2023 arXiv:2307.15156

VS-IMSRG methodology

$$M^{0\nu} = \langle f | \mathcal{O}^{0\nu} | i \rangle$$

VS-IMSRG methodology

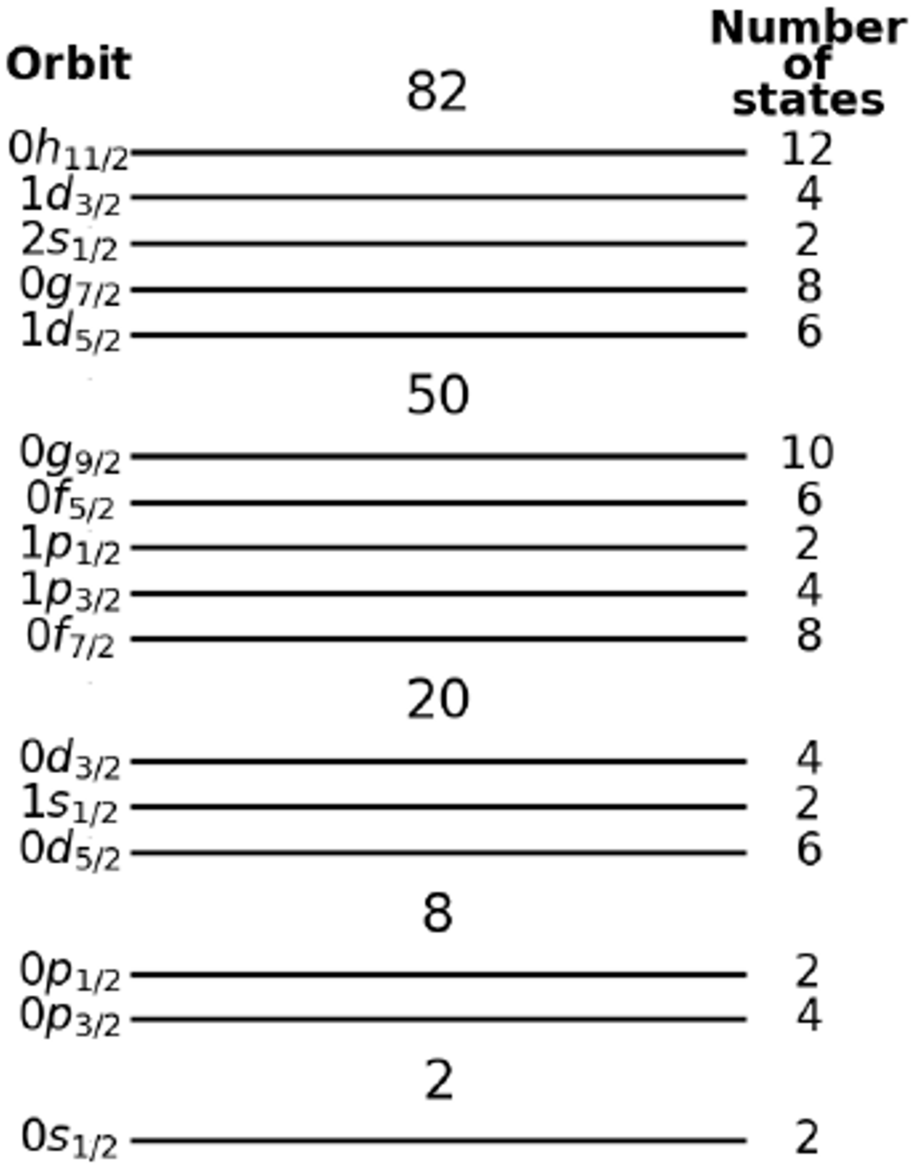
$$\hat{H}(s) = \hat{U}(s)\hat{H}(s-1)\hat{U}^{\dagger}(s)$$
$$\hat{O}(s) = \hat{U}(s)\hat{O}(s-1)\hat{U}^{\dagger}(s)$$



Dark = higher density of non-zero nuclear matrix elements

Shell Model

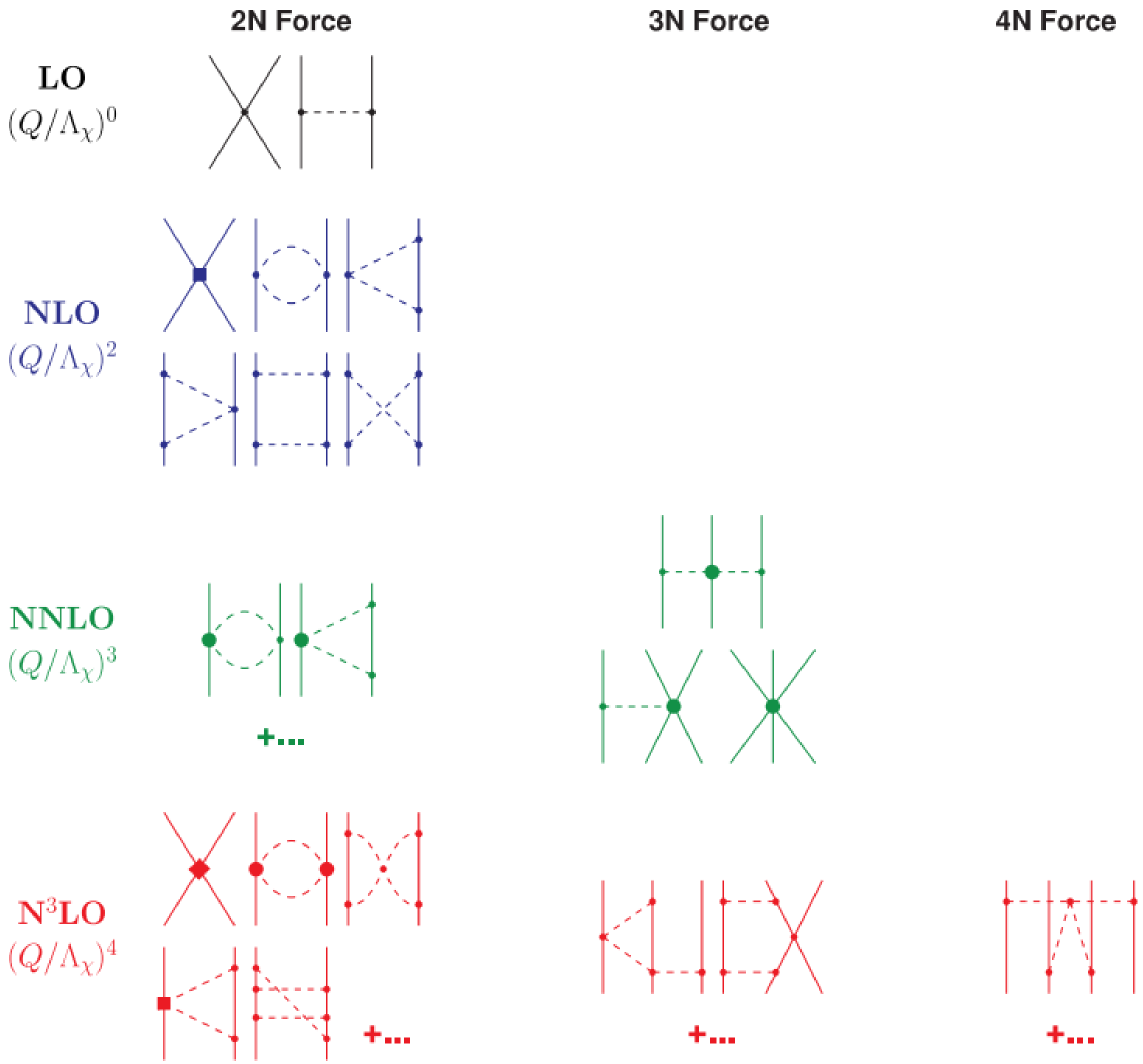
- Then use a shell model code (e.g. KSHELL) to fully diagonalise Hamiltonian and use with the nuclear shell model, yielding wavefunctions
- Compute overlap of wavefunctions with operator to get matrix element



χEFT Force Hierarchy

- Diagrams in χEFT are ranked by size of contribution by counting powers of external momentum/pion mass Q over the chiral symmetry breaking scale Λ_χ

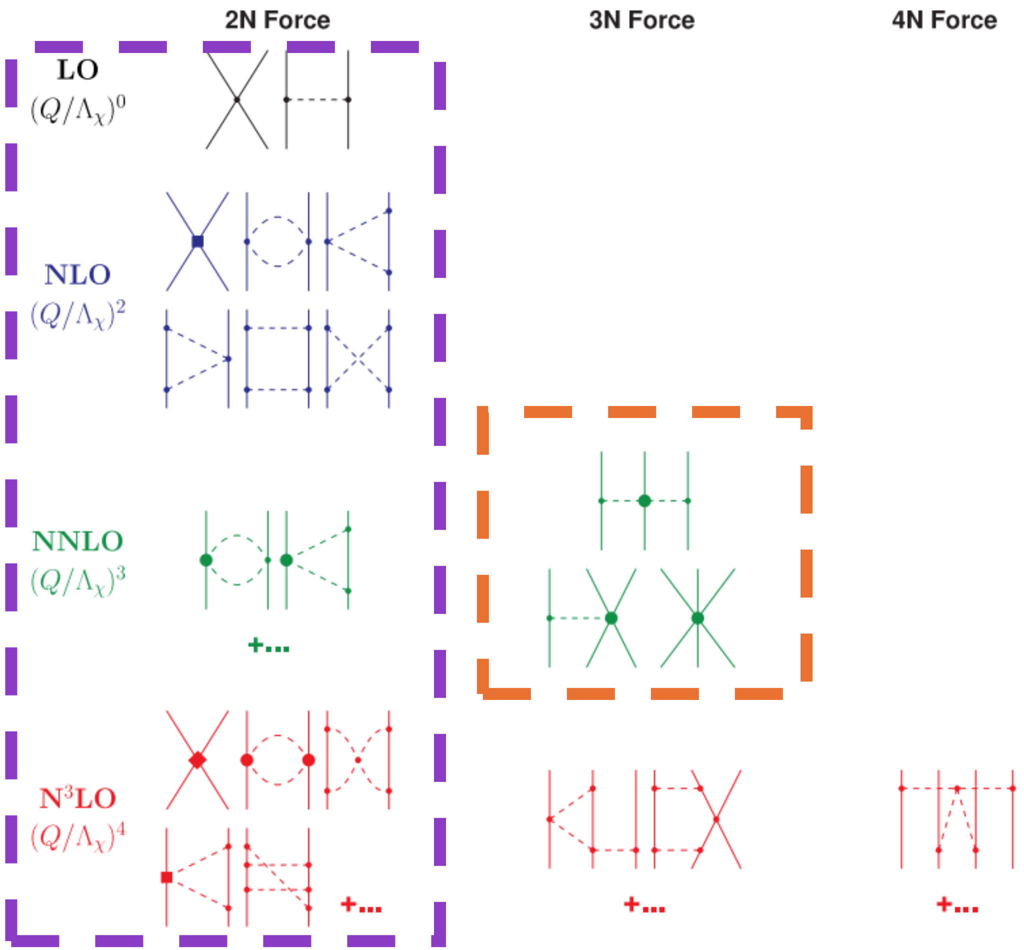
- **This work implements NNLO/N²LO contributions to the 0ν decay operator**



R. Machleidt & D.R. Entem, 2011 *Phys. Rep.* **503** (1) 1-75

EM(1.8/2.0) 'Magic' Interaction

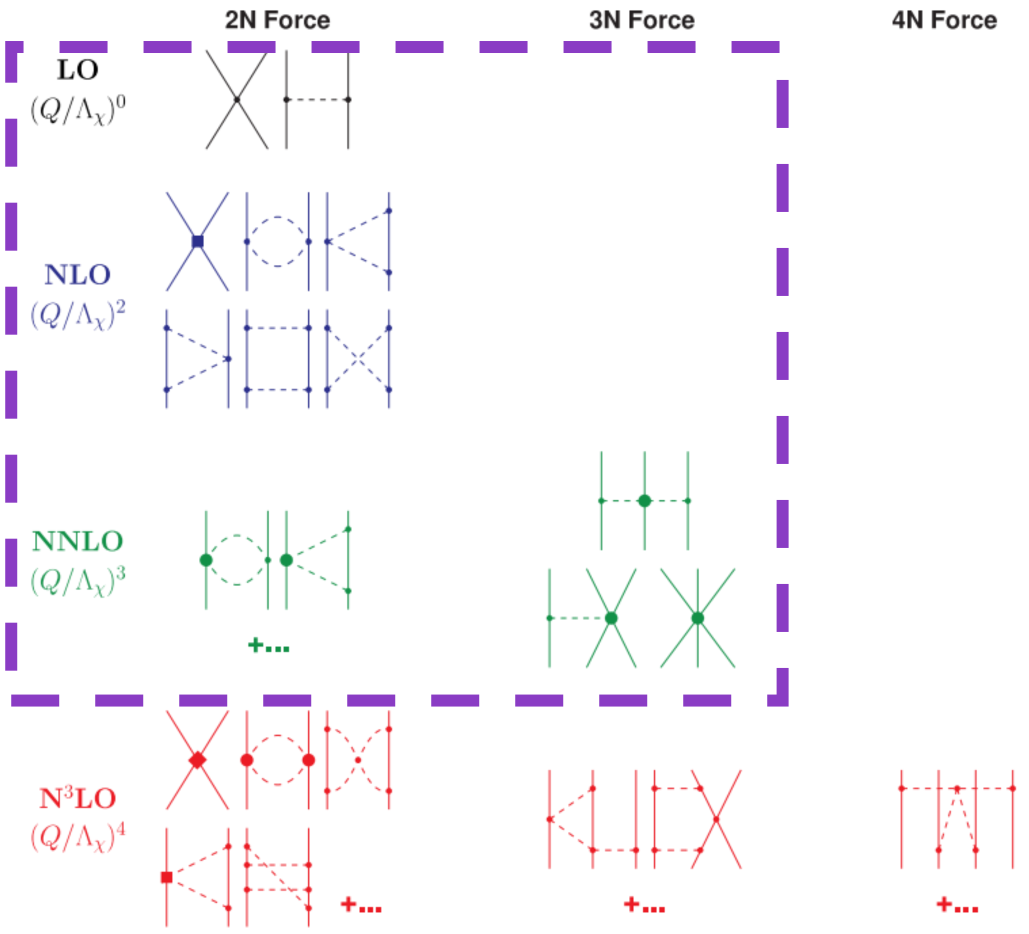
- Hamiltonian made from a 2N force components up to N^{3}LO, SRG evolved at scale 1.8fm^{-1}, and a non-evolved 3N N^{2}LO component with cutoff 2.0fm^{-1}



R. Machleidt & D.R. Entem, 2003 *Phys. Rev. C* **68**, 041001(R)

$\Delta N^2LO_{GO}(394)$ 'DeltaGO' Interaction

- Explicitly accounts for Δ-isobars
- Hamiltonian made from 2N and 3N N^2LO processes at low cutoff (Λ = 394MeV)



W. G. Jiang et al., 2020 *Phys. Rev. C* **102**, 054301

N^2LO Nuclear Potentials

V. Cirigliano et al, 2018 *Phys. Rev. C* **97**, 065501

$$V_{\nu,2}^{(a,b)} = \tau^{(a)+}\tau^{(b)+}\left(\mathcal{V}_{VV}^{(a,b)} + \mathcal{V}_{AA}^{(a,b)} + \tilde{\mathcal{V}}_{AA}^{(a,b)}\,\log\frac{m_\pi^2}{\mu_{\mathrm{us}}^2} + \mathcal{V}_{CT}^{(a,b)}\right)$$

Isospin raising operators

Vector current

Axial current

Counterterm

N²LO Nuclear Potentials

V. Cirigliano et al, 2018 *Phys. Rev. C* **97**, 065501

$$V_{\nu,2}^{(a,b)} = \tau^{(a)+}\tau^{(b)+}\left(\mathcal{V}_{VV}^{(a,b)} + \mathcal{V}_{AA}^{(a,b)} + \tilde{\mathcal{V}}_{AA}^{(a,b)}\log\frac{m_\pi^2}{\mu_{\text{us}}^2} + \mathcal{V}_{CT}^{(a,b)}\right)$$

Isospin raising operators

Vector current

Axial current

Counterterm

N^2LO Nuclear Potentials

V. Cirigliano et al, 2018 *Phys. Rev. C* **97**, 065501

$$\tilde{\mathcal{V}}_{AA}^{(a,b)} = 2\,\frac{g_A^4}{(4\pi F_\pi)^2}\,\frac{\boldsymbol{\sigma}^{(a)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q}\;+\;\mathbf{q}^2\;\mathbf{1}^{(a)}\times\mathbf{1}^{(b)}}{\mathbf{q}^2+m_\pi^2} - \frac{g_A^2}{(4\pi)^2}\,48C_T\,\mathbf{1}^{(a)}\times\mathbf{1}^{(b)}$$

N^{2}LO Nuclear Potentials

V. Cirigliano et al, 2018 *Phys. Rev. C* **97**, 065501

$$\tilde{\mathcal{V}}_{AA}^{(a,b)} = 2\,\frac{g_A^4}{(4\pi F_\pi)^2}\,\frac{\boldsymbol{\sigma}^{(a)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q} \;+\; \mathbf{q}^2\;\mathbf{1}^{(a)}\times\mathbf{1}^{(b)}}{\mathbf{q}^2+m_\pi^2} - \frac{g_A^2}{(4\pi)^2}\,48C_T\,\mathbf{1}^{(a)}\times\mathbf{1}^{(b)}$$

$$\tilde{\mathcal{V}}_{AA}^{(a,b)} = \underbrace{\left[\frac{2}{3}\frac{g_A^4}{(4\pi F_\pi)^2}\frac{\mathbf{q}^2}{\mathbf{q}^2+m_\pi^2}\right]\left(\boldsymbol{\sigma}^{(a)}\cdot\boldsymbol{\sigma}^{(b)}\right)}_{\text{Gamow-Teller Term}} - \underbrace{\left[\frac{2}{3}\frac{g_A^4}{(4\pi F_\pi)^2}\frac{\mathbf{q}^2}{\mathbf{q}^2+m_\pi^2}\right]\left(-3\frac{(\boldsymbol{\sigma}^{(a)}\cdot\mathbf{q})(\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q})}{\mathbf{q}^2}+\boldsymbol{\sigma}^{(a)}\cdot\boldsymbol{\sigma}^{(b)}\right)}_{\text{Tensor Term}}$$

$$+\underbrace{\left[2\frac{g_A^4}{(4\pi F_\pi)^2}\frac{\mathbf{q}^2}{\mathbf{q}^2+m_\pi^2} - \frac{g_A^2}{(4\pi)^2}48C_T\right]\mathbf{1}^{(a)}\times\mathbf{1}^{(b)}}_{\text{Fermi Term}}$$

N^2LO Nuclear Potentials

V. Cirigliano et al, 2018 *Phys. Rev. C* **97**, 065501

$$\tilde{\mathcal{V}}_{AA}^{(a,b)} = \left[\frac{2}{3}\frac{g_A^4}{(4\pi F_\pi)^2}\frac{\mathbf{q}^2}{\mathbf{q}^2+m_\pi^2}\right]\left(\boldsymbol{\sigma}^{(a)}\cdot\boldsymbol{\sigma}^{(b)}\right) - \left[\frac{2}{3}\frac{g_A^4}{(4\pi F_\pi)^2}\frac{\mathbf{q}^2}{\mathbf{q}^2+m_\pi^2}\right]\left(-3\frac{(\boldsymbol{\sigma}^{(a)}\cdot\mathbf{q})(\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q})}{\mathbf{q}^2} + \boldsymbol{\sigma}^{(a)}\cdot\boldsymbol{\sigma}^{(b)}\right)$$

Gamow-Teller Term

Tensor Term

$$+\left[2\frac{g_A^4}{(4\pi F_\pi)^2}\frac{\mathbf{q}^2}{\mathbf{q}^2+m_\pi^2} - \frac{g_A^2}{(4\pi)^2}48C_T\right]\mathbf{1}^{(a)}\times\mathbf{1}^{(b)}$$

Fermi Term

$$f_{\mathrm{local}}^{\mathrm{NN}}(\mathbf{q}) = \exp\left[-\left(\frac{\mathbf{q}}{\Lambda}\right)^{2n}\right]$$

$\Lambda = 500$ MeV $\quad n = 4$

$\Lambda = 394$ MeV $\quad n = 4$

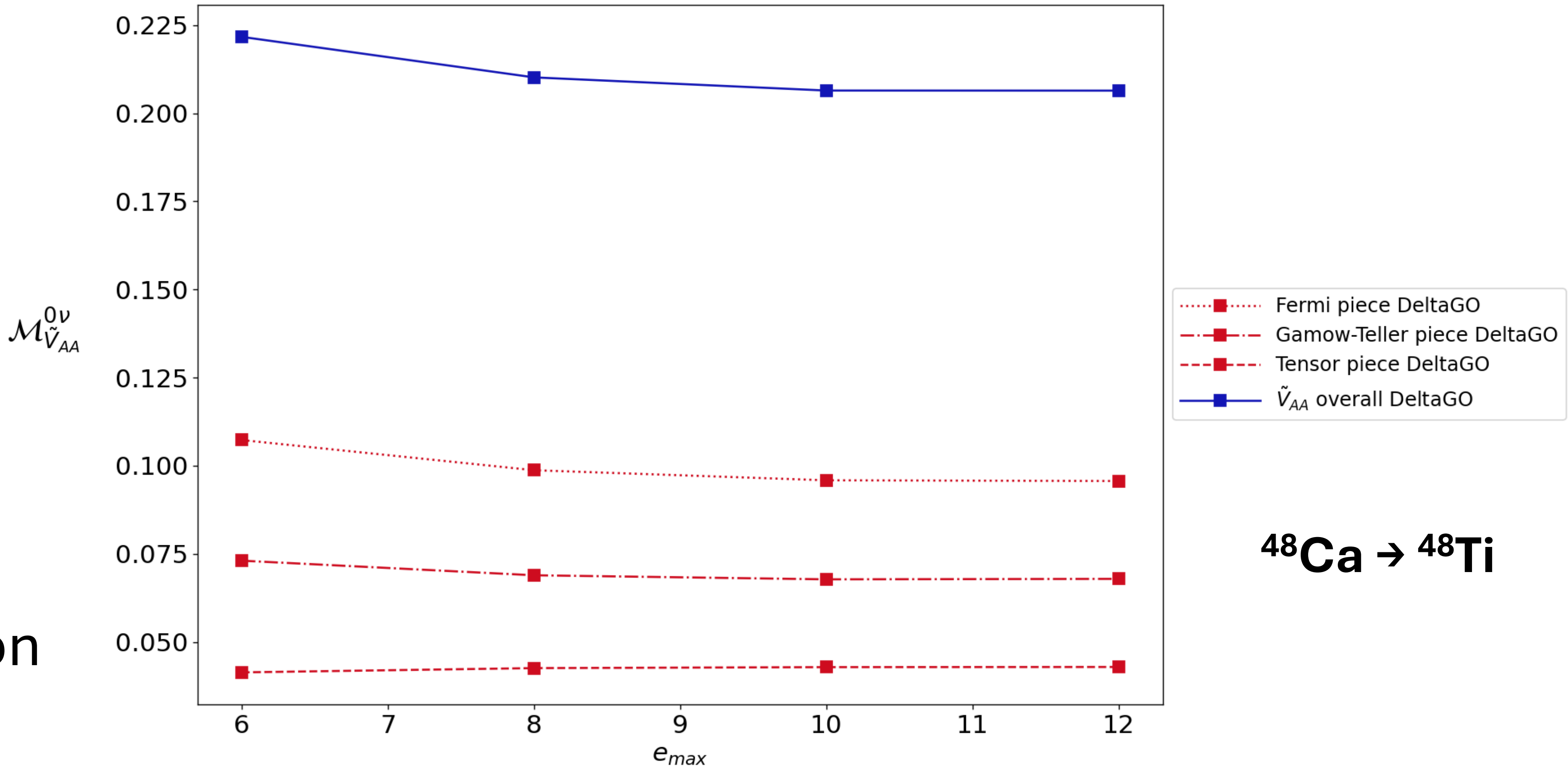
Chosen to be consistent with Magic Interaction

“ ”

DeltaGO Interaction

$\tilde{V}_{AA}$ term of N^2LO NME

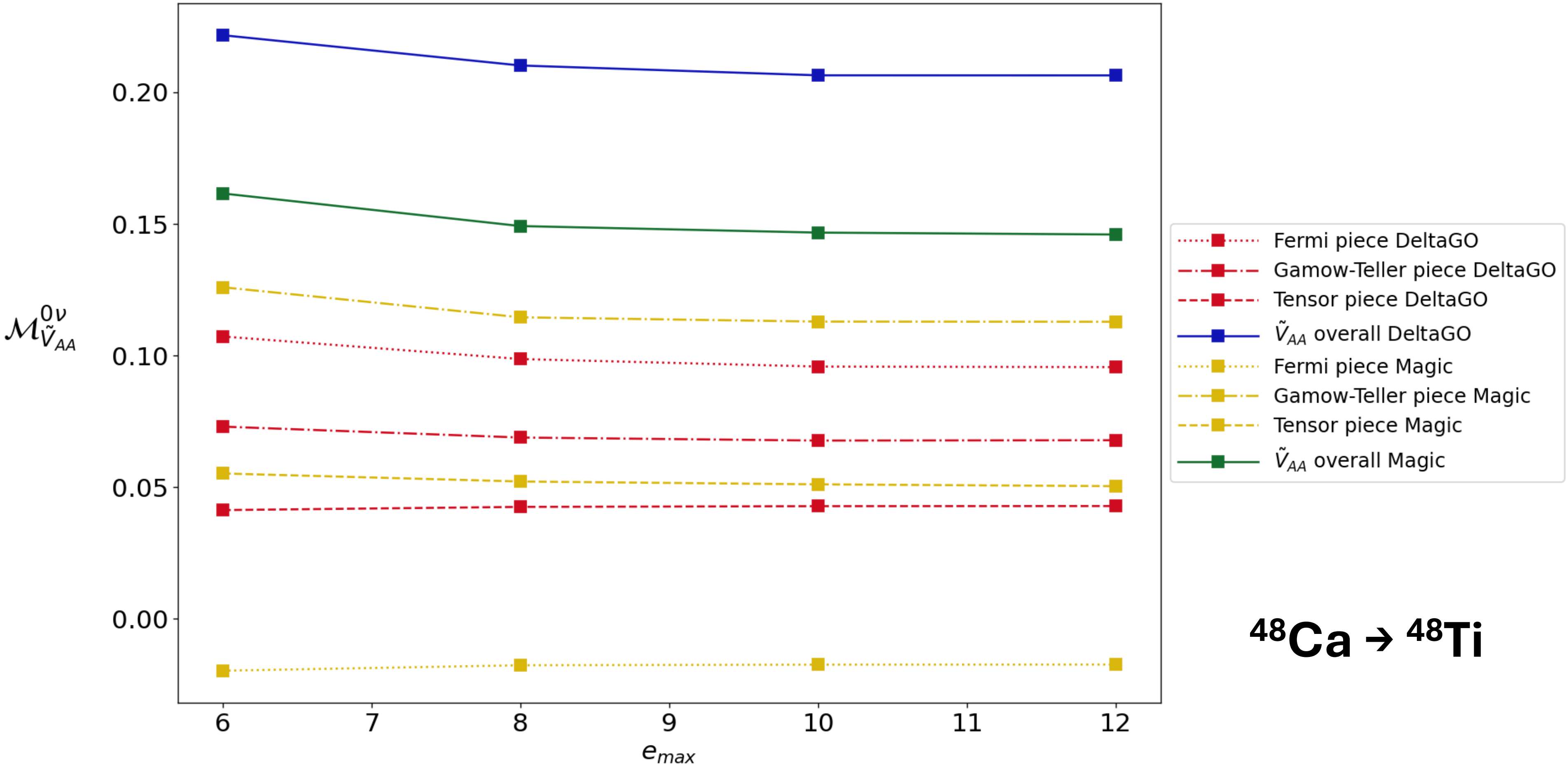
- DeltaGO interaction
- Ultrasoft renormalisation scale = $2m_\pi$



$^{48}Ca \rightarrow {}^{48}Ti$

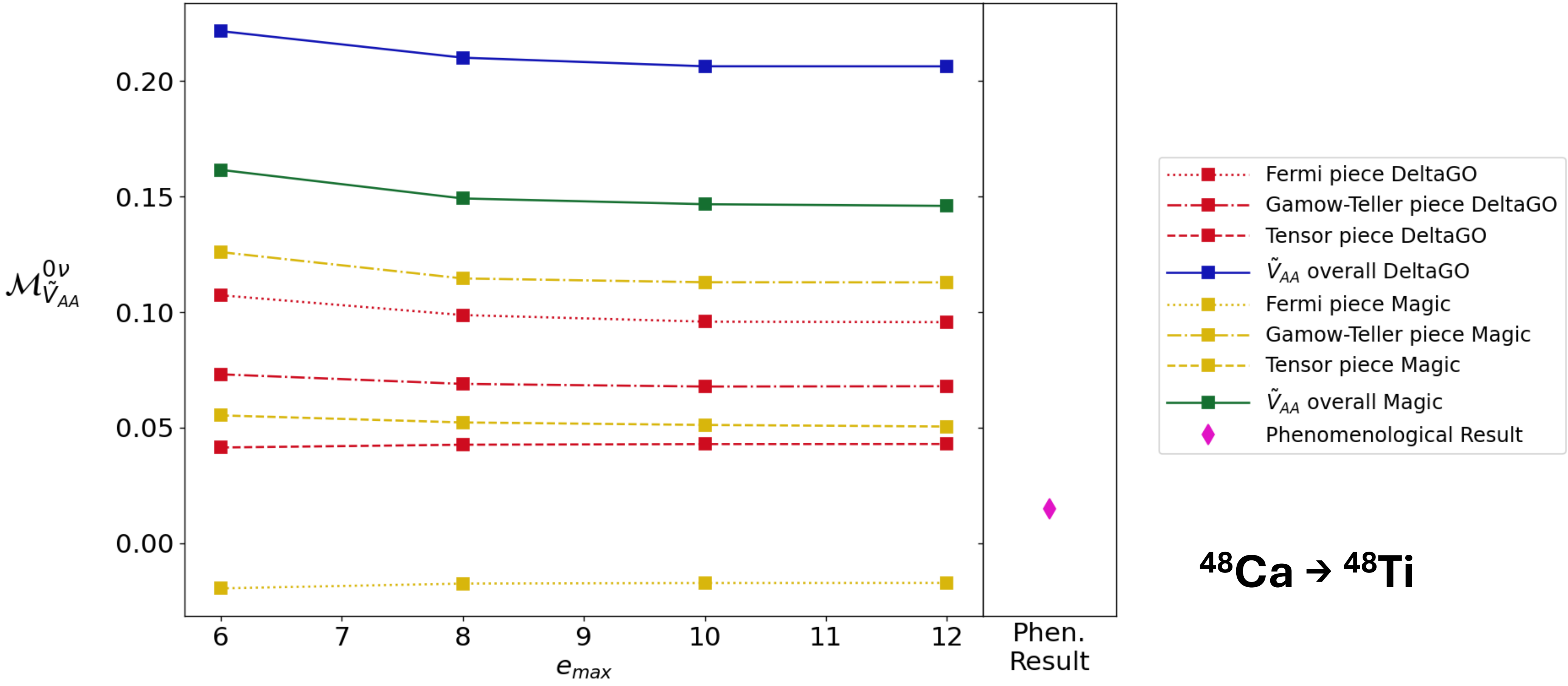
Magic vs DeltaGO for $\tilde{V}_{AA}$ term

- Ultrasoft renormalisation scale = $2m_\pi$
- Results similar between interactions as anticipated



$^{48}Ca \rightarrow {}^{48}Ti$

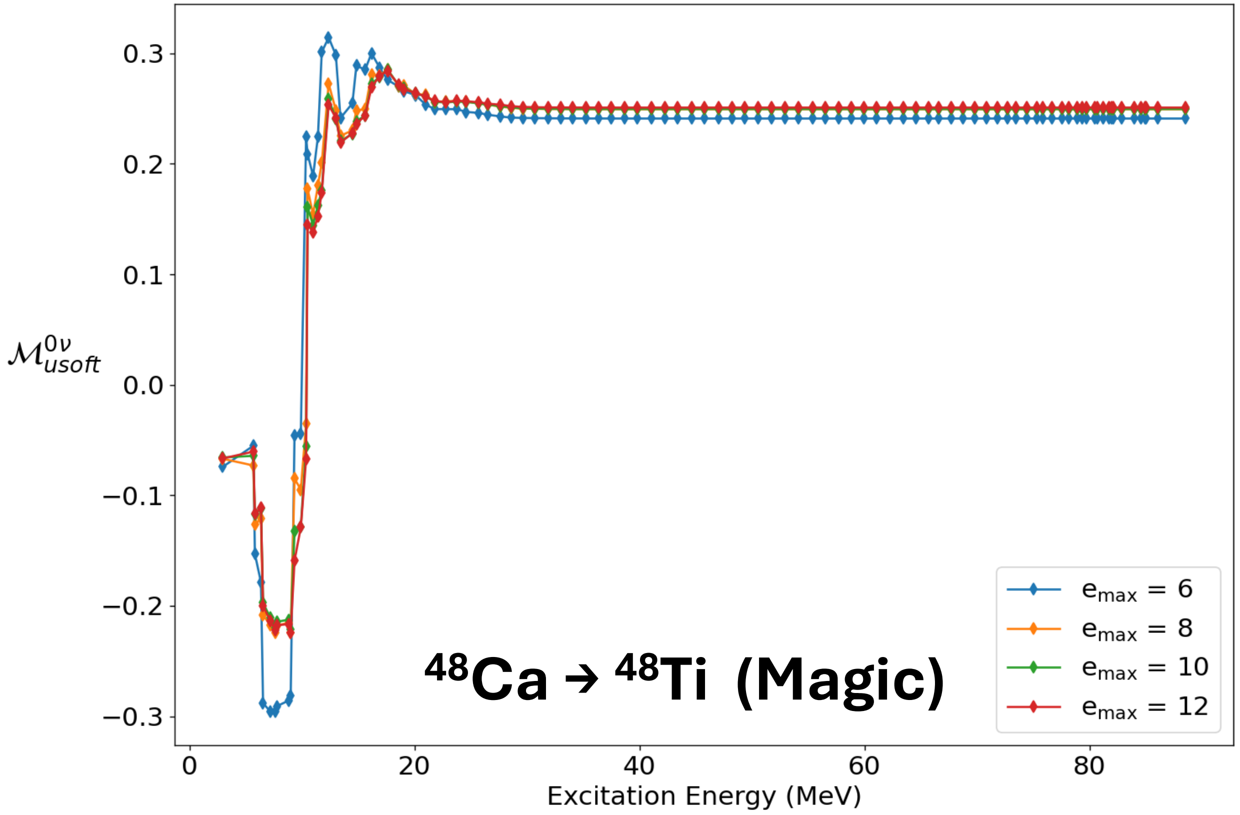
Ab initio vs Phenomenological result

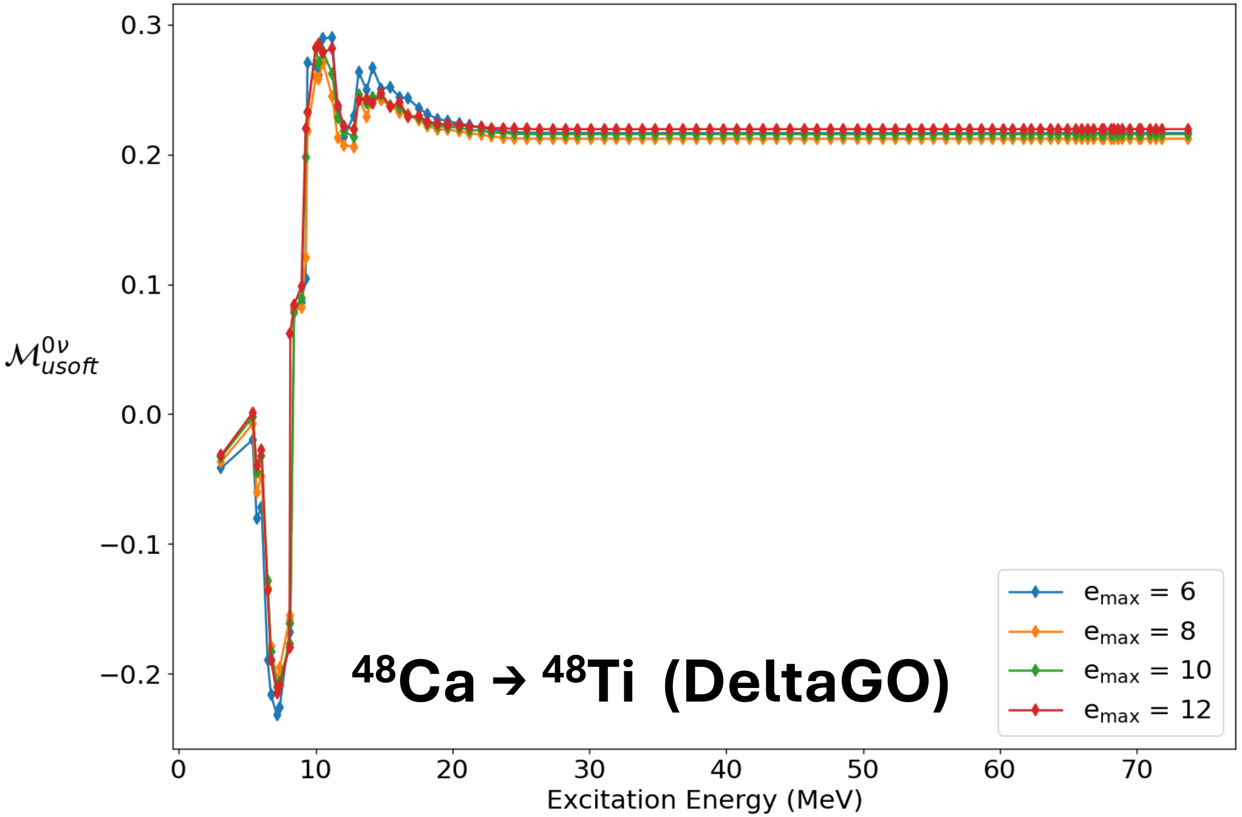


$^{48}Ca \rightarrow {}^{48}Ti$

Credit: Daniel Castillo Garcia (unpublished)

Ultrasoft Nuclear Matrix Element

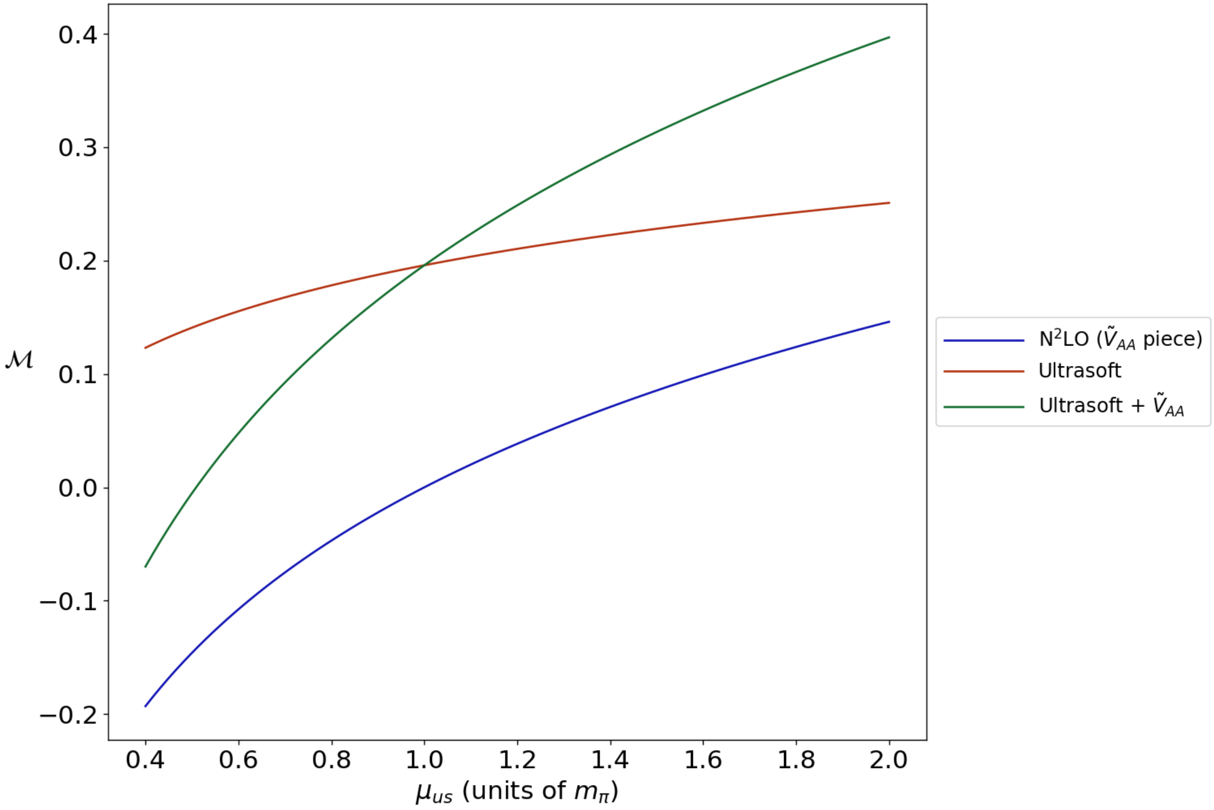




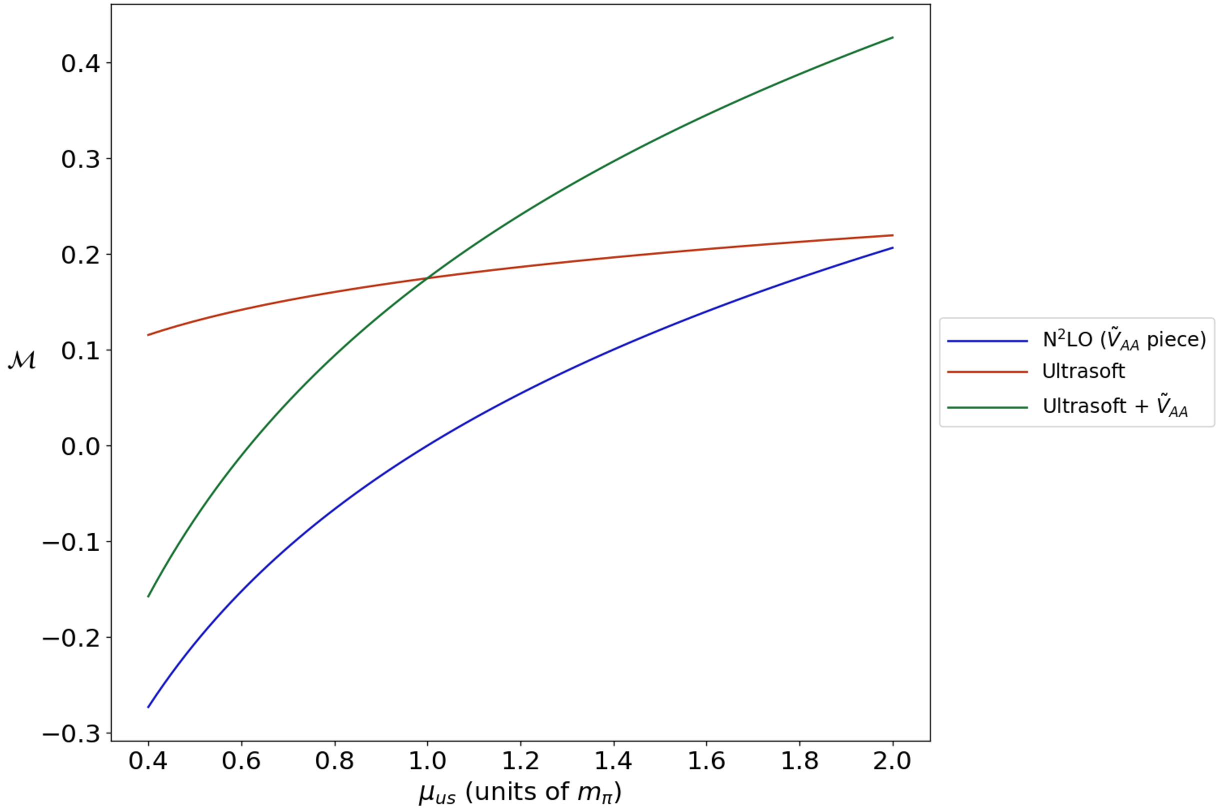
$\mu_{us} = 2m_\pi$

Renormalisation Scale Dependence

Preliminary!



^{48}Ca → ^{48}Ti (Magic)



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Renormalisation Scale Dependence

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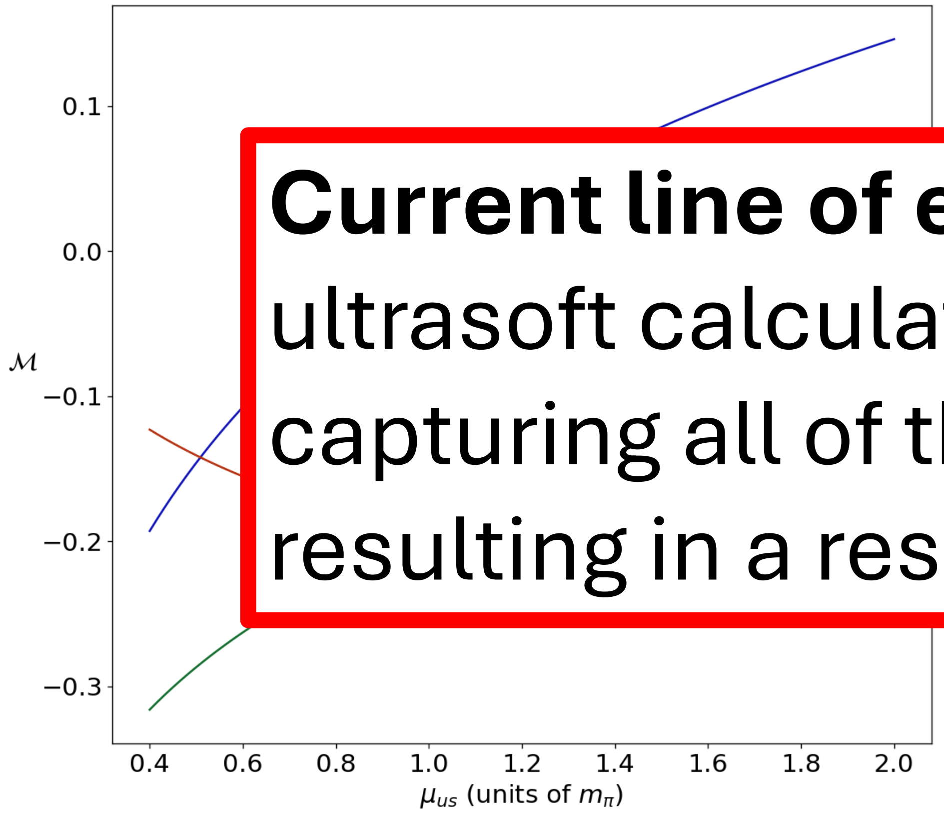

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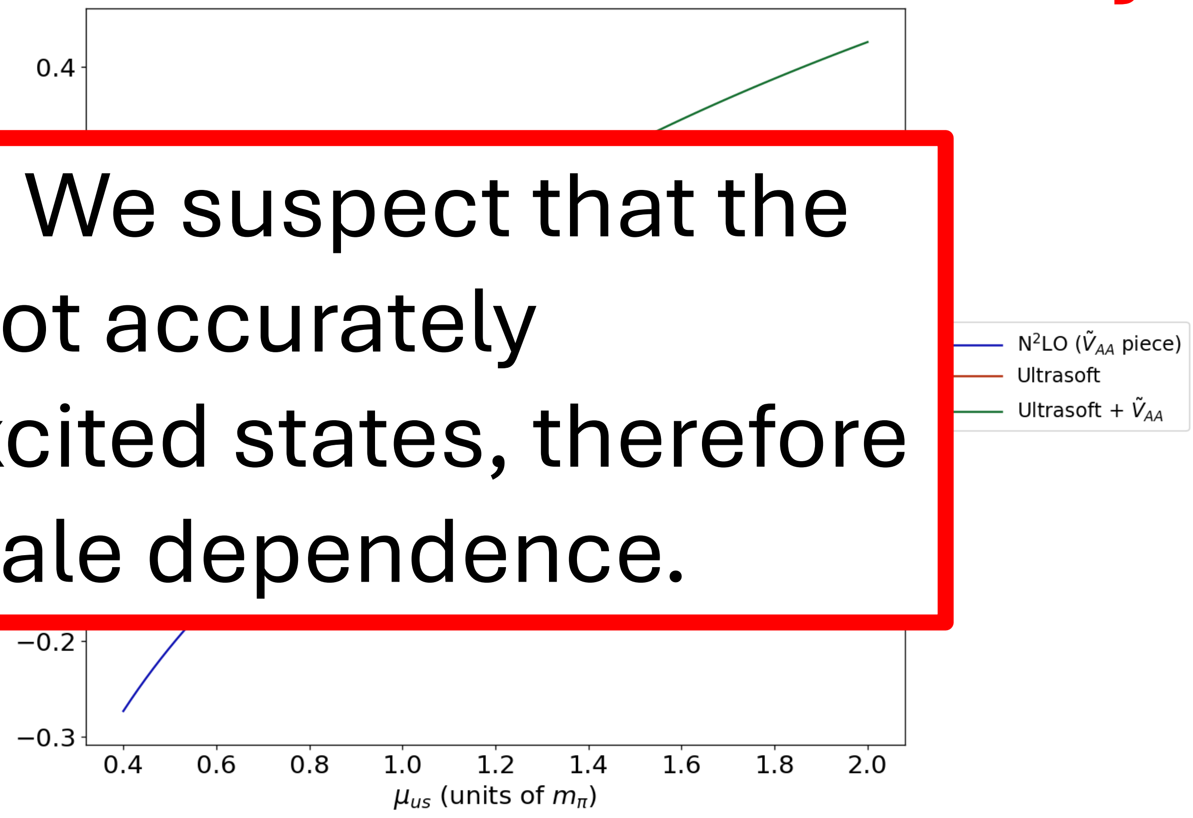

^{48}Ca → ^{48}Ti (DeltaGO)

Current line of enquiry: We suspect that the ultrasoft calculation is not accurately capturing all of the 1+ excited states, therefore resulting in a residual scale dependence.

Summary

- Initial implementation of $\tilde{V}_{AA}$ N^2LO nuclear potential achieved
- Examined how NME varies depending on chosen interaction
- Ultrasoft scale dependence not sufficiently neutralised – investigating accuracy of US calculation as a possible cause

Acknowledgements: Taiki Shickele, Alex Todd, Daniel Castillo Garcia, Antoine Belley, Jason Holt

$$\mathcal{V}_{AA}^{(a,b)} = \frac{g_A^2}{(4\pi F_\pi)^2} \frac{\boldsymbol{\sigma}^{(a)} \cdot \mathbf{q}\, \boldsymbol{\sigma}^{(b)} \cdot \mathbf{q}}{m_\pi^2} \left\{ \frac{g_A^2}{1+\hat{q}} (L_\pi - 4) + \frac{1}{(1+\hat{q})^2} \right\}$$

$$+ \frac{\mathbf{1}^{(a)} \times \mathbf{1}^{(b)}}{(4\pi F_\pi)^2} \left\{ -\frac{3}{4}(1-g_A^2)^2 L_\pi + g_A^4 f_4(\hat{q}) + g_A^2 f_2(\hat{q}) + f_0(\hat{q}) + 24 g_A^2 F_\pi^2 C_T \left\{ L_\pi + 1 \right\} \right\}$$

$$\mathcal{V}_{VV}^{(a,b)} = -\frac{g_A^2}{(4\pi F_\pi)^2} \frac{\boldsymbol{\sigma}^{(a)} \cdot \mathbf{q}\, \boldsymbol{\sigma}^{(b)} \cdot \mathbf{q}}{m_\pi^2} \times \left\{ \frac{2(1-\hat{q})^2}{\hat{q}^2(1+\hat{q})} \log(1+\hat{q}) - \frac{2}{\hat{q}} + \frac{7-3\hat{q}L_\pi}{(1+\hat{q})^2} + \frac{L_\pi}{1+\hat{q}} \right\},$$

$$\mathcal{V}_{CT}^{(a,b)} = \frac{g_A^2}{(4\pi F_\pi)^2} \frac{\boldsymbol{\sigma}^{(a)} \cdot \mathbf{q}\, \boldsymbol{\sigma}^{(b)} \cdot \mathbf{q}}{m_\pi^2} \left[\frac{5}{6} g_\nu^{\pi\pi} \frac{\hat{q}}{(1+\hat{q})^2} - g_\nu^{\pi N} \frac{1}{1+\hat{q}} \right] - \frac{2 g_\nu^{NN}}{(4\pi F_\pi)^2} \mathbf{1}^{(a)} \times \mathbf{1}^{(b)}$$

$$f_0(\hat{q}) = -\frac{1+8\hat{q}}{6\hat{q}} + \frac{(1+\hat{q})(1+8\hat{q}+\hat{q}^2)}{6\hat{q}^2} \log(1+\hat{q}) - \frac{1}{24}(4+\hat{q})(5+2\hat{q}) g(\hat{q})$$

$$f_2(\hat{q}) = \frac{1+8\hat{q}}{3\hat{q}} + \frac{(1+\hat{q})^2(-1+5\hat{q})}{3\hat{q}^2} \log(1+\hat{q}) - \frac{1}{12}(40+47\hat{q}+10\hat{q}^2) g(\hat{q})$$

$$f_4(\hat{q}) = -\frac{1}{6}\left(20 + \frac{1}{\hat{q}} - \frac{12}{4+\hat{q}}\right) - \frac{-1+14\hat{q}+78\hat{q}^2+62\hat{q}^3+23\hat{q}^4}{6\hat{q}^2(1+\hat{q})} \log(1+\hat{q})$$

$$+ \frac{1}{24(4+\hat{q})}(640+912\hat{q}+375\hat{q}^2+46\hat{q}^3) g(\hat{q}),$$

$$g(\hat{q}) = \frac{4}{\sqrt{\hat{q}(4+\hat{q})}} \operatorname{arctanh}\left(\sqrt{\frac{\hat{q}}{4+\hat{q}}}\right)$$

$$L_\pi = \log \frac{\mu^2}{m_\pi^2} \qquad \hat{q} = -q^2/m_\pi^2$$

V. Cirigliano et al, 2018 *Phys. Rev. C* **97**, 065501