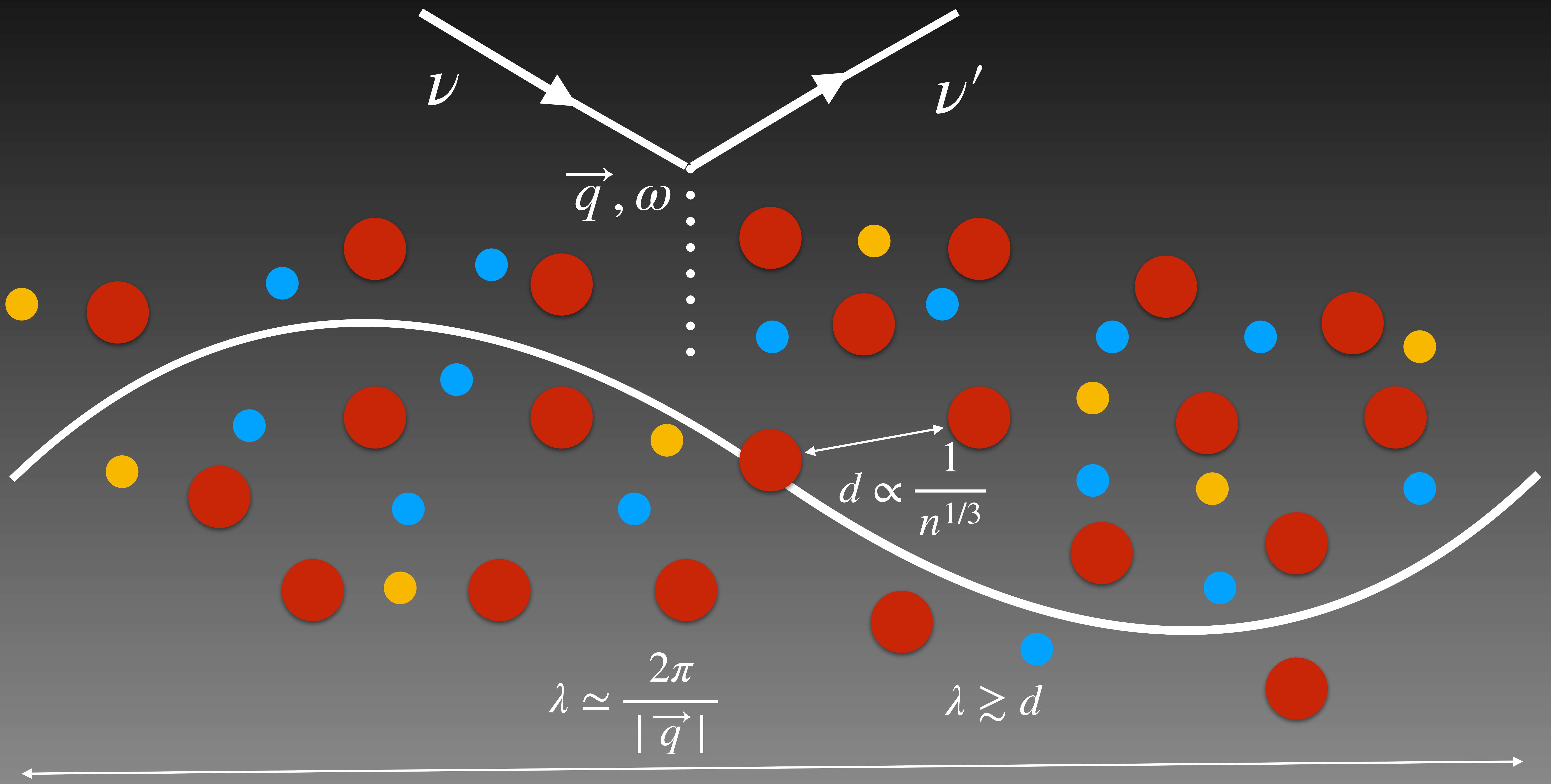
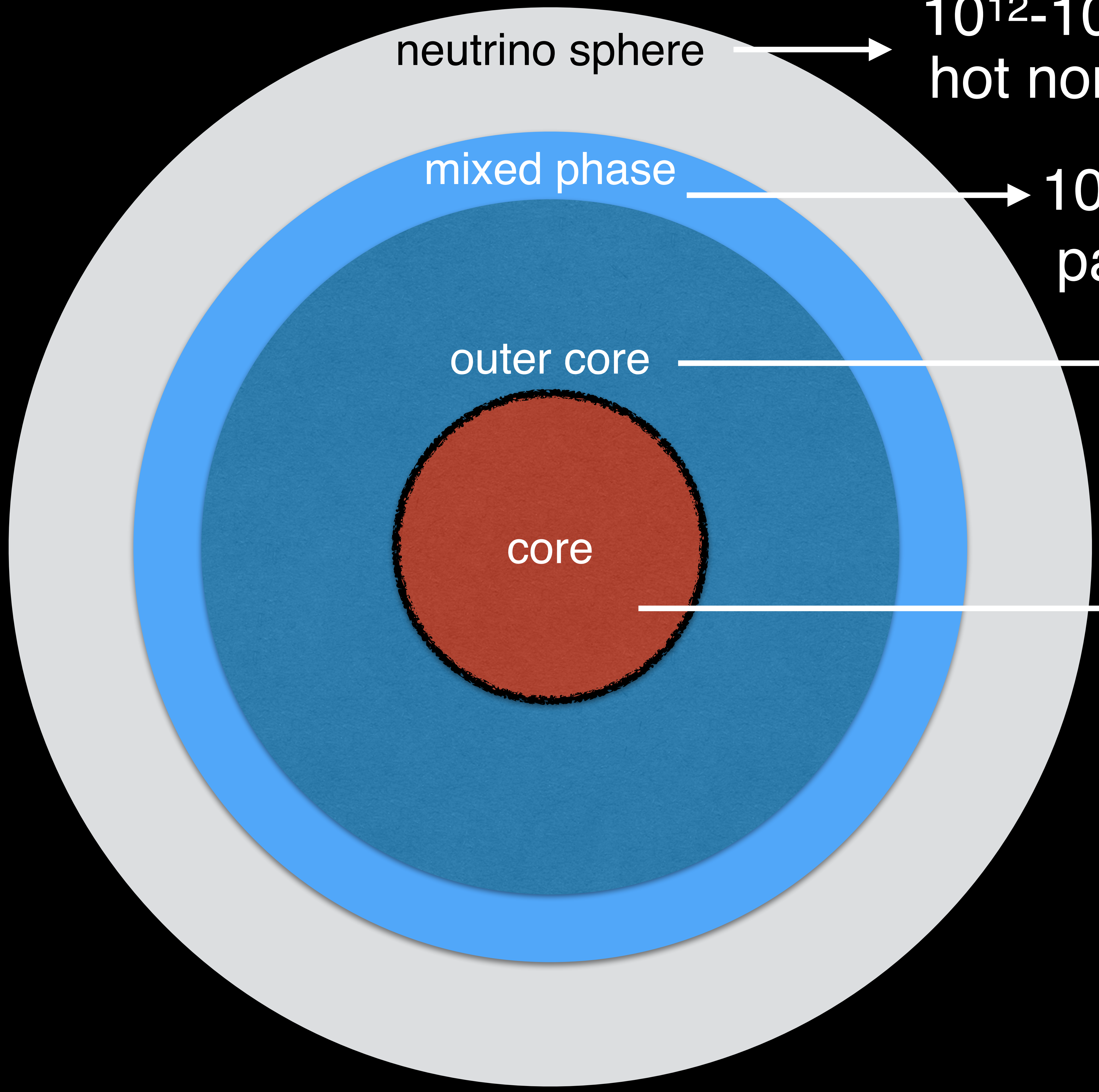
Neutrino interactions in hot and dense matter.



INSTITUTE for
NUCLEAR THEORY

Sanjay Reddy, May 23, 2022. INT Program on The r-process and the nuclear EOS after LIGO-Virgo's third observing run

Neutrino Scattering in Hot Neutron Stars



10^{12}-10^{13} g/cm^3 and T~ 5-10 MeV
hot non-degenerate nucleon gas + few nuclei.

10^{13}-10^{14} g/cm^3 and T~ 5-30 MeV
partially degenerate nucleons + nuclei.

10^{14} - 5x10^{14} g/cm^3 and T~ 5-50 MeV
nuclear matter - Fermi liquid

> 5x10^{14} g/cm^3 Unknown.

Hyperons? Quark Femi-liquid?
Quark Superconductor?
Quarkyonic Matter?

Neutrino Interactions in Dense Matter

Low energy Lagrangian: $\mathcal{L} = \frac{G_F}{\sqrt{2}}\, l_\mu\, j^\mu$ $\qquad l_1 + N_2 \rightarrow l_3 + N_4$

Absorption:	$l_\mu^{cc} = \bar{l}\gamma_\mu(1-\gamma_5)\nu_l$	$j_{cc}^\mu = \bar{\Psi}_p \left(\gamma^\mu(g_V - g_A\gamma_5) + F_2\frac{i\sigma^{\mu\alpha}q_\alpha}{2M}\right)\Psi_n$
Scattering:	$l_\mu^{nc} = \bar{\nu}\gamma_\mu(1-\gamma_5)\nu$	$j_{nc}^\mu = \bar{\Psi}_i\,(\gamma^\mu(C_V^i - C_A^i\gamma_5) + F_2^i\frac{i\sigma^{\mu\alpha}q_\alpha}{2M})\,\Psi_i$

Rate: $\frac{d\Gamma(E_1)}{dE_3 d\mu_{13}} = \frac{G_F^2}{32\pi^2}\frac{p_3}{E_1}(1-f_3(E_3))L_{\mu\nu}\,\mathcal{S}^{\mu\nu}(q_0,q)$

Dynamic structure function: $\mathcal{S}^{\mu\nu}(q_0,q) = \frac{-2\,\mathrm{Im}\,\mathbf{\Pi}^{\mu\nu}(q_0,q)}{1-\exp\left(-(q_0+\Delta\mu)/T\right)}$

Polarization functions: $\mathbf{\Pi}^{\mu\nu}(q_0,q) = -i\int dt\, d^3x\, \theta(t)\, e^{i(q_0 t - \vec{q}\cdot\vec{x})}\langle\, |[j_\mu\,(\vec{x},t), j_\nu(\vec{0},0)]|\,\rangle$

difficult to calculate in general due to the non-perturbative nature of strong interactions.

Sawyer (1970s), Iwamoto & Pethick (1980s),
Burrows & Sawyer, Horowitz & Wehrberger, Raffelt et al., Reddy et al. (1990s),
Benhar, Carlson, Gandolfi, Horowitz, Lavato, Pethick, Reddy, Roberts, Schwenk, Shen, and others (2000s)

Linear Response

Perturbation:

$$\mathcal{H}_{int} = \int d^3x\ \mathcal{O}(x)\ \phi_{ext}(x,t)$$

Response:

$$\delta\rho(\vec{q},\omega) = \Pi^R(\vec{q},\omega)\ \phi_{ext}(\vec{q},\omega)$$

Response function (Polarization function or Generalized Susceptibility)

$$\Pi^R(\vec{q},\omega) = \frac{-i}{\hbar}\int dt\ e^{i\omega t}\ \theta(t)\ \langle[\mathcal{O}(-\vec{q},t),\mathcal{O}(\vec{q},0)]\rangle$$

Response to static and uniform perturbations is related to thermodynamic derivatives.

$$\phi_{ext}(\vec{q}\to 0,\omega=0) = \delta\mu$$

perturbation can be viewed as a change in the chemical potential

Compressibility sum-rule: $\Pi^R(0,0) = \left(\frac{\partial n}{\partial\mu}\right)_T$ where $n = \langle\mathcal{O}(0,0)\rangle$ is the associated density.

Dynamic Structure Factor

A simpler correlation function $\mathcal{S}(\vec{q},\omega) = \int dt\ e^{i\omega t}\ \langle \mathcal{O}(-\vec{q},t)\mathcal{O}(\vec{q},0)\rangle$

$$= 2\pi\hbar \sum_{m,n} \frac{e^{\beta K_n}}{\mathcal{Z}}\ |\langle n|\mathcal{O}_q|m\rangle|^2\ \delta(K_n - K_m - \hbar\omega)$$

where K_n are eigenvalues of $K = \mathcal{H} - \mu\ N$ (grand canonical Hamiltonian)

Fluctuation-dissipation theorem: $\mathcal{S}(\vec{q},\omega) = \dfrac{-2\hbar\ \mathrm{Im}\ \Pi^R(\vec{q},\omega)}{1 - e^{-\beta\hbar\omega}}$

The dynamic structure factor incorporates all of the many-body effects into the neutrino scattering and absorption rates.

Sum Rules

Static structure factor: $\mathcal{S}_q = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \, \mathcal{S}(q, \omega')$

F-sum rule: $$\int_{-\infty}^{\infty} d\omega' \; \omega' \; \mathrm{Im} \; \Pi^R(q, \omega') = \langle [[\mathcal{H}, \mathcal{O}_q], \mathcal{O}_q] \rangle$$

Compressibility sum-rule: $$-\int_{-\infty}^{\infty} \frac{d\omega'}{\pi} \; \frac{\mathrm{Im} \; \Pi^R(0, \omega')}{\omega'} = \mathrm{Re} \; \Pi^R(0,0) = \left(\frac{\partial n}{\partial \mu} \right)_T$$

At T=0:

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \; \frac{(1 - e^{-\beta \hbar \omega'}) \mathcal{S}(q \to 0, \omega')}{\hbar \omega'} = \int_{0}^{\infty} \frac{d\omega'}{2\pi} \; \frac{\mathcal{S}(q \to 0, \omega')}{\hbar \omega'} = \left(\frac{\partial n}{\partial \mu} \right)_{T=0}$$

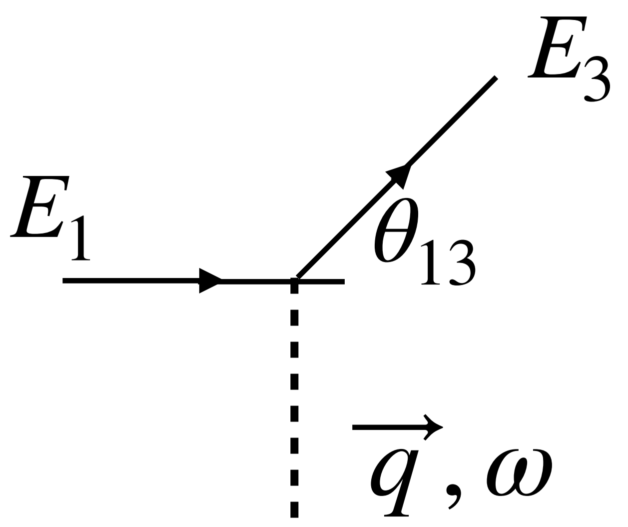
At high temperature: $(1 - e^{-\beta \hbar \omega'}) \simeq \beta \hbar \omega'$

$$\mathcal{S}_{q=0} = \mathrm{lt}_{q \to 0} \int_{-\infty}^{\infty} d\omega' \; \mathcal{S}(q, \omega) = T \; \left(\frac{\partial n}{\partial \mu} \right)_T$$

Neutrino-Nucleon Scattering

Nucleon currents in the non-relativistic limit: $j^\mu_{nc} = \Psi^\dagger \Psi \; \delta^\mu_0 + \Psi^\dagger \sigma_k \Psi \; \delta^\mu_k + \mathcal{O}[\frac{p}{M}]$

density ↑ spin-density ↑



dynamic response functions

$$\frac{d\Gamma(E_1)}{d\Omega dE_3} = \frac{G_F^2}{4\pi^2} \; E_3^2 \; \left[C_V^2(1+\cos\theta_{13}) S_\rho(\omega, q) + C_A^2(3-\cos\theta_{13}) S_\sigma(\omega, q)\right]$$

Integrate over the final neutrino energy:

$$\frac{d\Gamma(E_1)}{dq} = \frac{G_F^2}{\pi} \; q \; \left(C_V^2 I_\rho(q) + C_A^2 I_\sigma(q)\right)$$

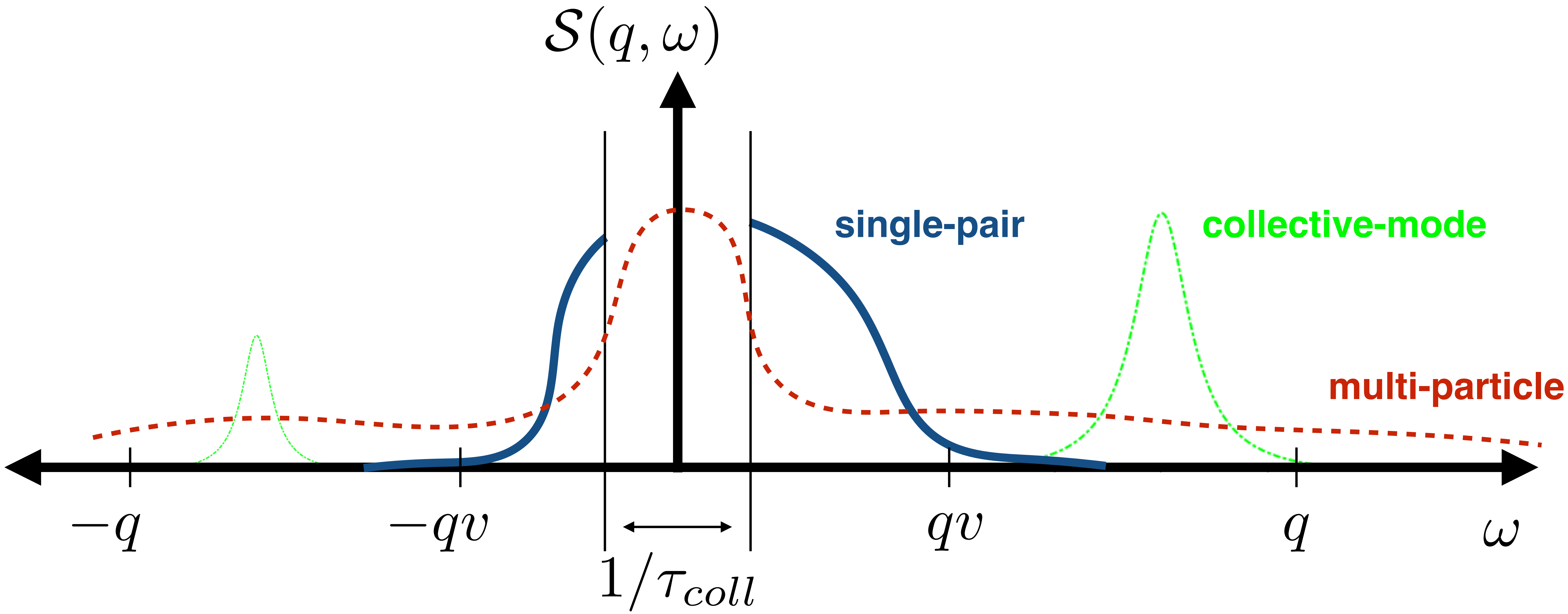
The “static” response functions are :

$$I_\rho(q) = \tilde{S}_\rho(q) \; \left(1 - \frac{q^2}{4E_1^2} - \frac{\langle \omega_\rho(q) \rangle}{E_1} + \frac{\langle \omega_\rho^2(q) \rangle}{4E_1^2}\right)$$

$$I_\sigma(q) = \tilde{S}_\sigma(q) \; \left(1 + \frac{q^2}{4E_1^2} - \frac{\langle \omega_\sigma(q) \rangle}{E_1} - \frac{\langle \omega_\sigma^2(q) \rangle}{4E_1^2}\right)$$

$$\tilde{S}_\alpha(q) = \int_{-q}^{\omega_{max}} d\omega \; S_\alpha(\omega, q) \qquad \langle \omega_\alpha^n \rangle = \frac{\int_{-q}^{\omega_{max}} d\omega \; \omega^n \; S_\alpha(\omega, q)}{\tilde{S}_\alpha(q)}$$

General Structure of the Dynamic Response



In dense nuclear matter single-pair, multi-particle and collective modes contribute to the response.

- At small ω response is governed by hydrodynamics.
- Single-pair response dominates for $|\omega\tau_{coll}| > 1$ and $|\omega| < qv$.
- Multi-particle response dominates for $|\omega| > qv$.
- Collective modes arise due to interactions.

Hydrodynamic Response

In the low energy and long-wavelength limit, the density response function is given by hydrodynamics:

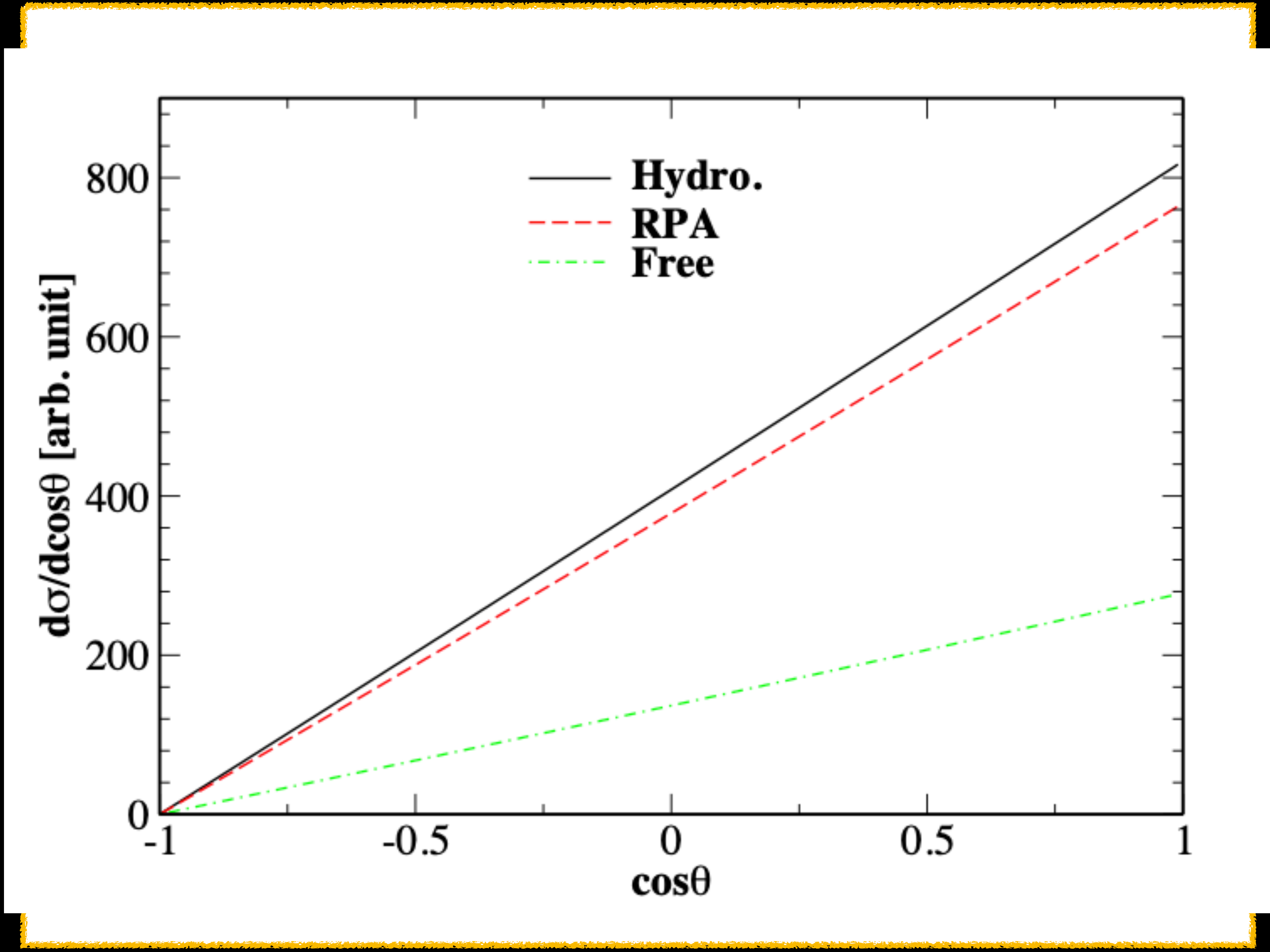
$$\text{Im}\ \Pi^R(q,\omega) = \frac{2F_2\omega}{3m^2c^2}\left(\frac{(\gamma-1)\Gamma_\kappa}{\omega^2+\Gamma_\kappa^2} + \frac{2\Gamma_\eta\, c^2q^2 - \Gamma_\kappa\,(\gamma-1)(\omega^2-c^2q^2)}{(\omega^2-c^2q^2)^2+(2\omega\Gamma_\eta)^2}\right)$$

c is the speed of sound in the fluid,

Γ_η is the damping rate due to shear-viscosity

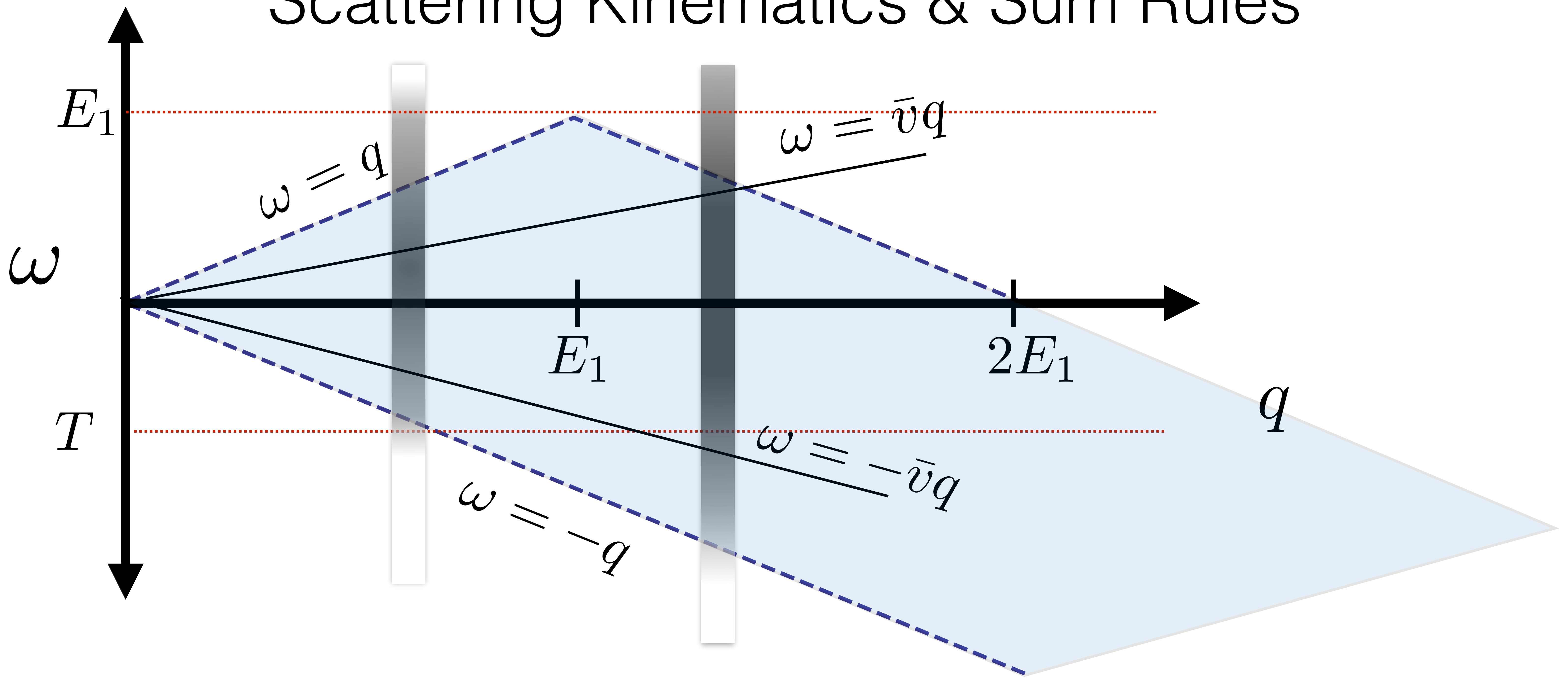
Γ_κ is the damping rate related to the thermal conductivity

- Differential scattering rates can be related to macroscopic properties.
- Difficult to capture the spectral features using approximate diagrammatic calculations.
- A simple resummation of 1p-1h diagrams called random phase approximation (RPA) can capture integrated quantities - angular distributions and total cross-sections quite well.



Shen and Reddy (2013)

Scattering Kinematics & Sum Rules



Neutrinos only probe the space-like region with $|\omega| < q$

In general $\tilde{S}_\alpha(q) = \int_{-q}^{\omega_{max}} d\omega \; S_\alpha(\omega, q) \; < \quad S_\alpha(q) = \int_{-\infty}^{\infty} d\omega \; S_\alpha(\omega, q)$

In practice for conserved currents at long-wavelengths:

$$\tilde{S}_\rho(q \to 0) = S_\rho(q \to 0)$$

$$\tilde{S}_\rho(q) \simeq S_\rho(q)$$

At high temperature recall that $S_\rho(q \to 0) = T \left(\frac{\partial n}{\partial \mu}\right)_T$

Spin is not conserved by strong interactions

F-sum Rule: $F_\alpha(q) = \int d\omega \; \omega S_\alpha(\omega, q) = \frac{1}{2}\langle[\mathcal{O}_\alpha^\dagger, [\mathcal{O}_\alpha, H]]\rangle$

$$F_\rho(q) = n \; \frac{q^2}{2m}$$

1p-1h contribution dominates.
No time-like response.

$$\tilde{S}_\rho(q) = \int_{-q}^{\omega_{max}} d\omega \; S_\rho(\omega, q) \simeq S_\rho(q)$$

$$F_\sigma(q) = C + \tilde{n} \; \frac{q^2}{2m}$$

2p-2h contribution
Finite time-like response.

$$\tilde{S}_\sigma(q) = \int_{-q}^{\omega_{max}} d\omega \; S_\sigma(\omega, q) < S_\sigma(q)$$

- Thermodynamic derivates may not be adequate to accurately describe the long wavelength spin response.
- Some dynamical information is needed to calculate neutrino scattering rates in the medium.

Neutrino Processes in the Neutrino-sphere

Hierarchy of length scales at T=5 MeV and $\rho = 10^{12}$ g/cm^3

Neutrino Wavelength

$$\Lambda_\nu = \frac{2\pi\hbar}{\langle E_\nu \rangle} \approx 100 \text{ fm}$$

Inter-nucleon distance $$a = \left(\frac{3}{4\pi n_n} \right)^{1/3} \approx 10 \text{ fm}$$

De Broglie wavelength $$\Lambda_D = \frac{2\pi\hbar}{\langle p_n \rangle} \approx 4 \text{ fm}$$

Range of the n-n interaction $$r_{nn} \approx 2 \text{ fm}$$

- The matter is dilute, but interactions are strong and non-perturbative.
- nucleon-nucleon scattering length is large ~ 20 fm.
- The small expansion parameter is the fugacity $z=e^{\mu/T}$ - virial expansion.

Long-wavelength Response using the Virial EoS

Assumes that scattering is elastic to include all correlations through the static structure factors.

$$\tilde{S}_\rho(q) = \int_{-q}^{\omega_{max}} d\omega \; S_\rho(\omega, q) \simeq S_\rho(q) \qquad \tilde{S}_\rho(q \to 0) = S_\rho(q \to 0)$$

Calculate the static structure factors using the compressibility or thermodynamic sum rule

$$S_\rho(q \to 0) = T \left(\frac{\partial n}{\partial \mu}\right)_T$$

Sawyer (1975, 1979)
Horowitz and Schwenk (2005), Horowitz et al. (2017)

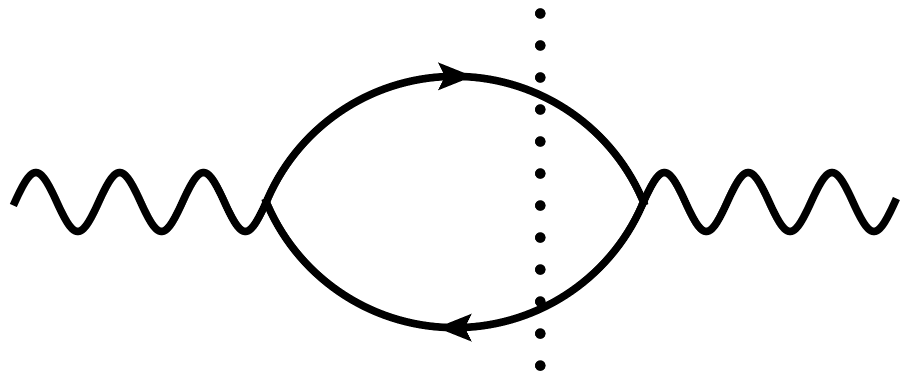
This is a good approximation for the density response relevant to neutral current reactions in the neutrino sphere.

The spin response and charged current reactions require some dynamical input.

Pseudo-potential for Hot & Dilute Nuclear Matter

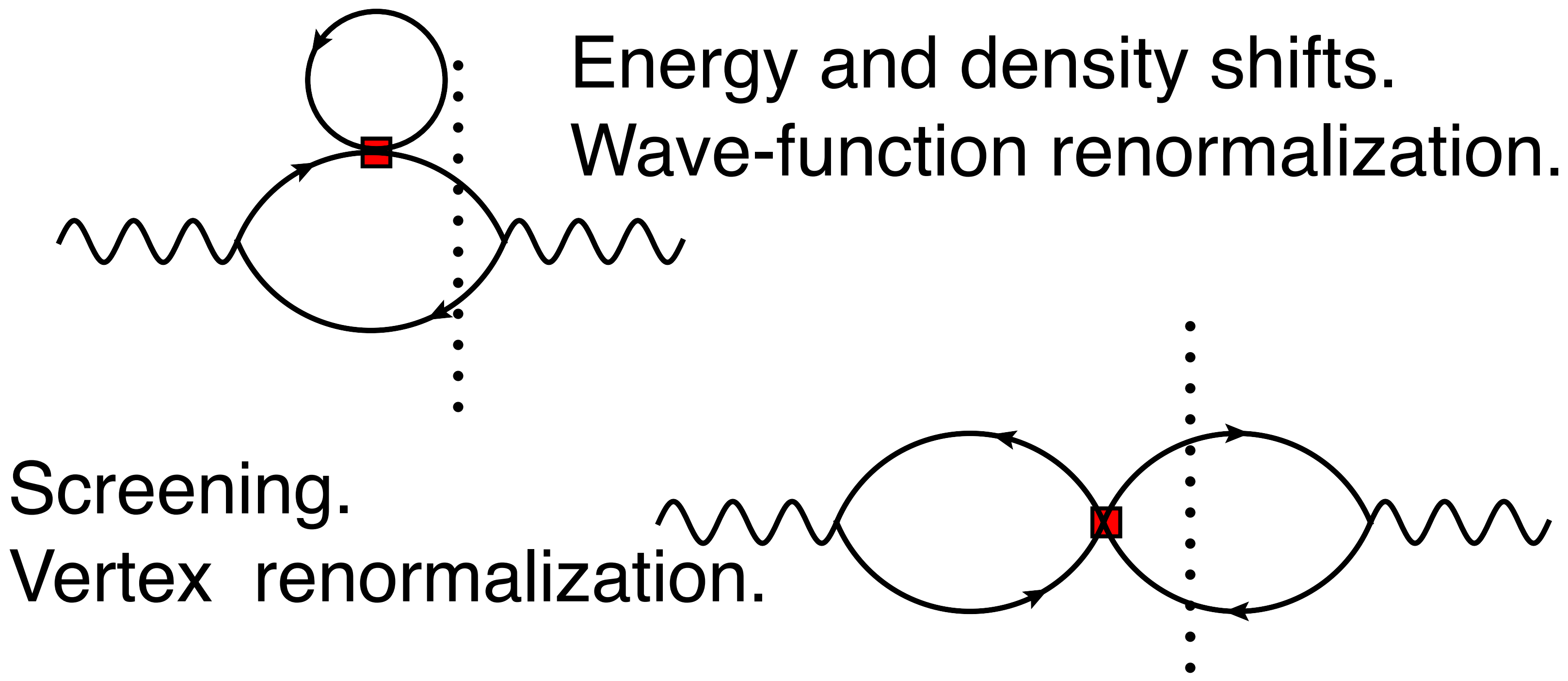
The dynamic structure factor calculable using standard diagrammatic "perturbation" theory - with a twist.
Interactions represented by a pseudo-potential:

$$\mathcal{V}_{ps} \propto \frac{\delta(p_{rel})}{p_{rel}\ M}$$



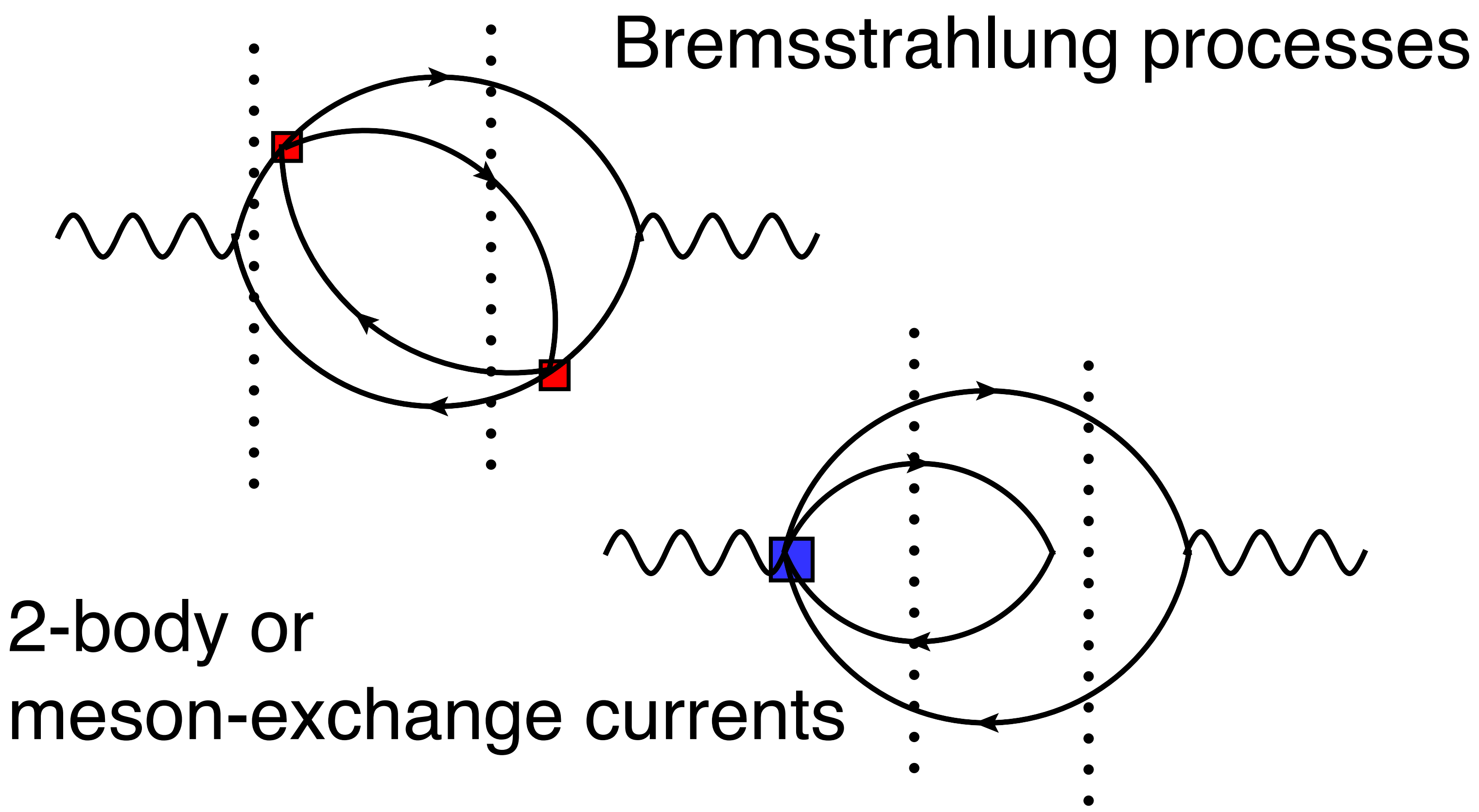
Leading order diagram neglects interactions.

$\mathcal{O}[z]$



Includes interactions at leading order. Consistent with the viral expansion.

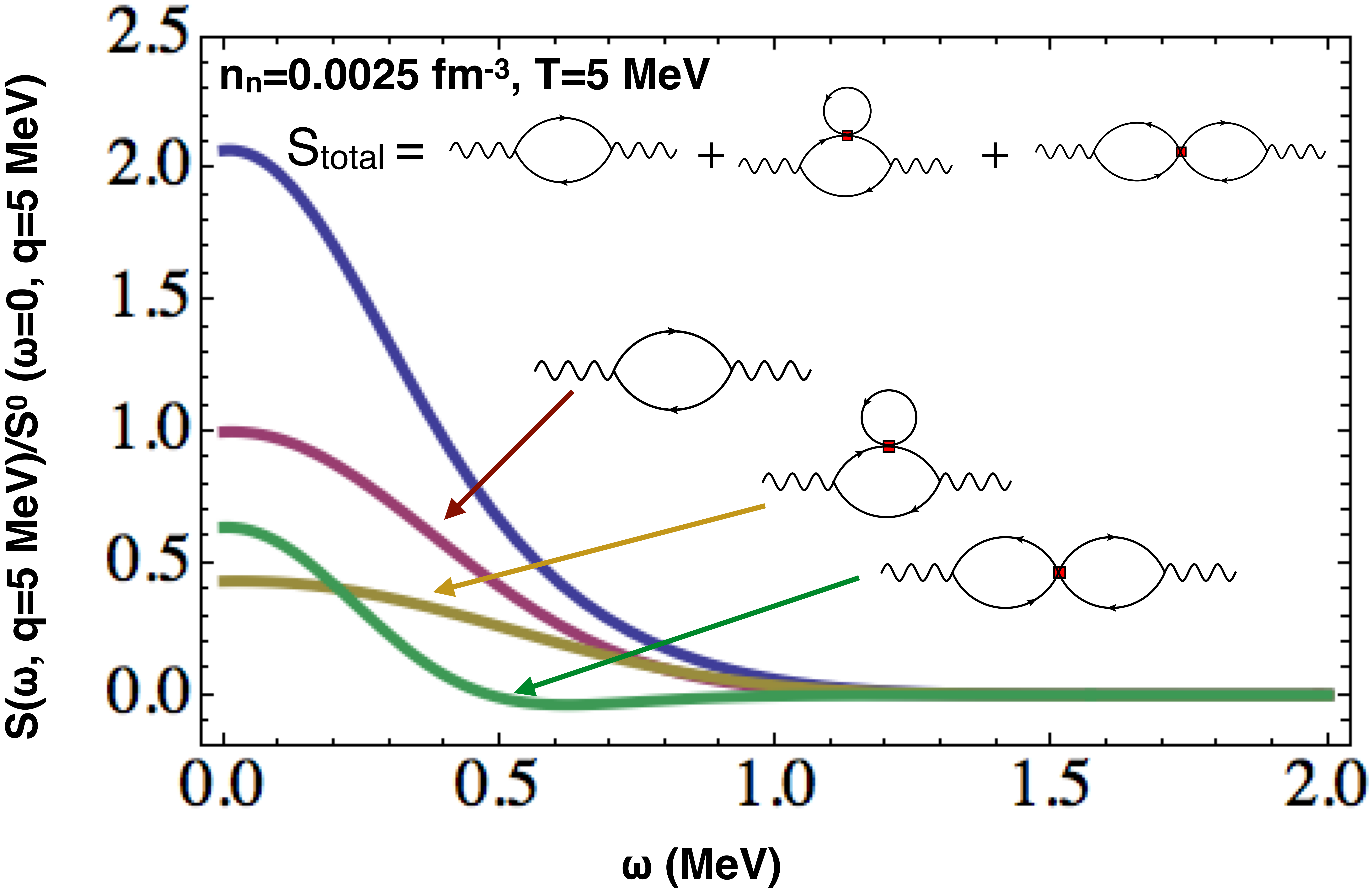
$\mathcal{O}[z^2\ \mathcal{V}_{ps}]$



Includes 2p-2h excitations and 2-body currents. These corrections are beyond the leading order viral expansion.

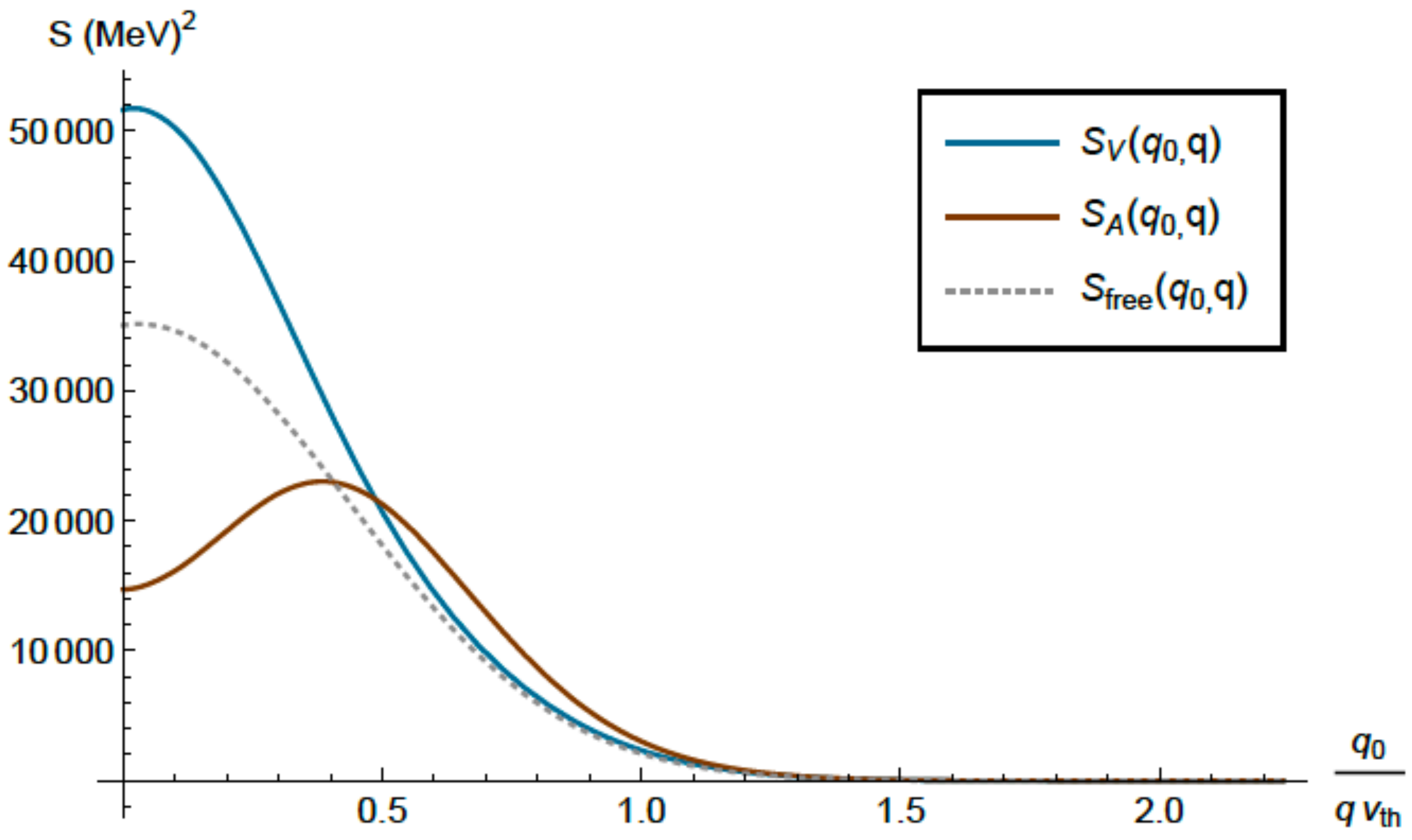
$\mathcal{O}[z^2\ \mathcal{V}_{ps}^2]$

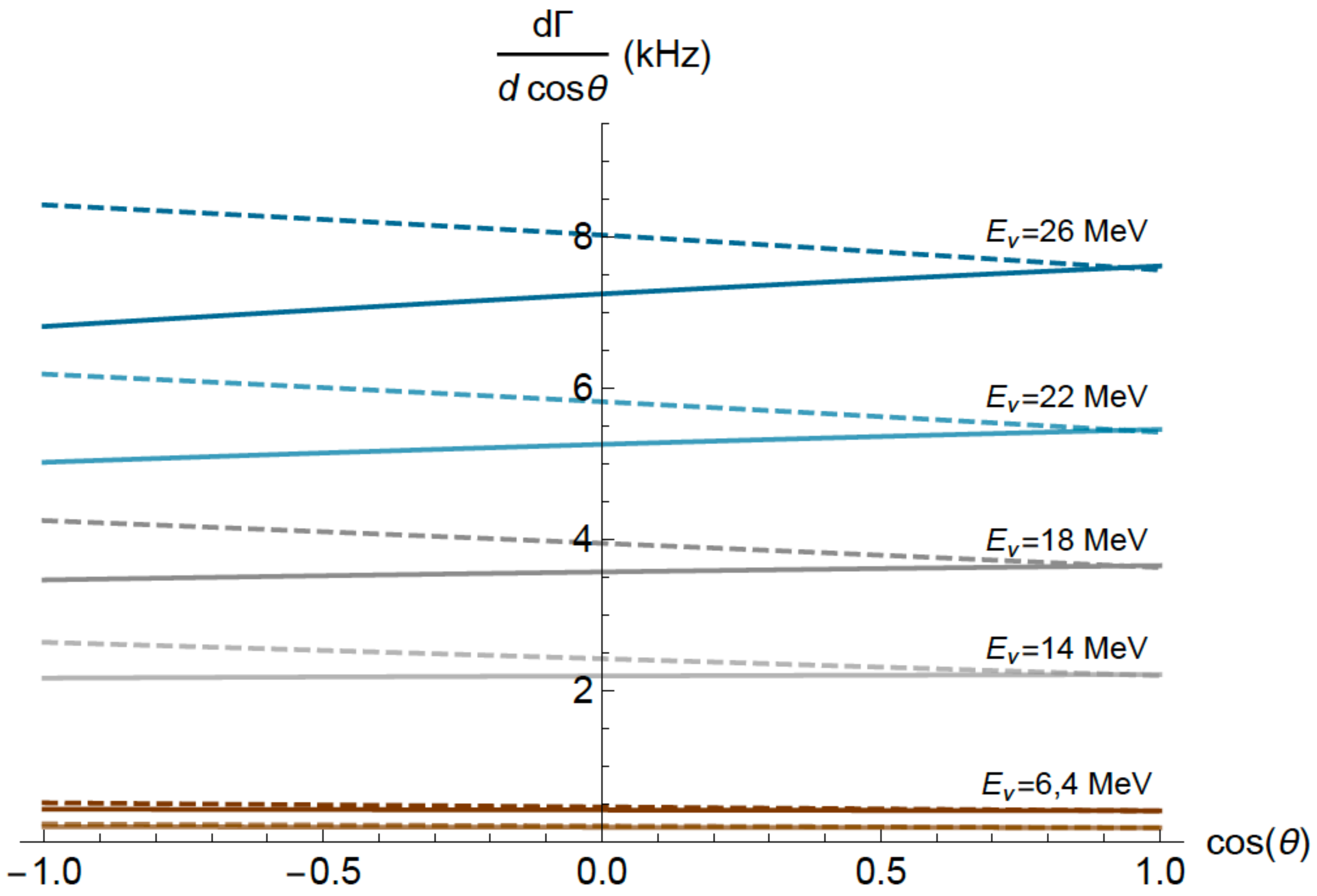
Dynamic Structure Factor with Pseudo-potential



Bedaque, Reddy, Sen & Warrington (2018)

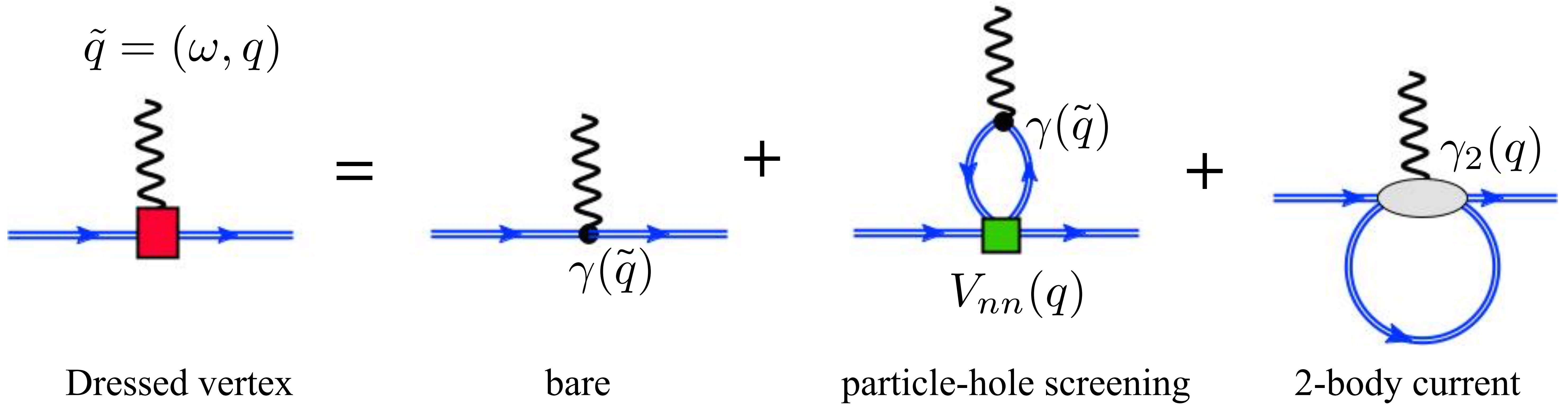
Neutrino Scattering in the Neutrino-sphere





Bedaque, Reddy, Sen & Warrington (2018)

10^{14} g/cm^3 uniform neutron-rich matter

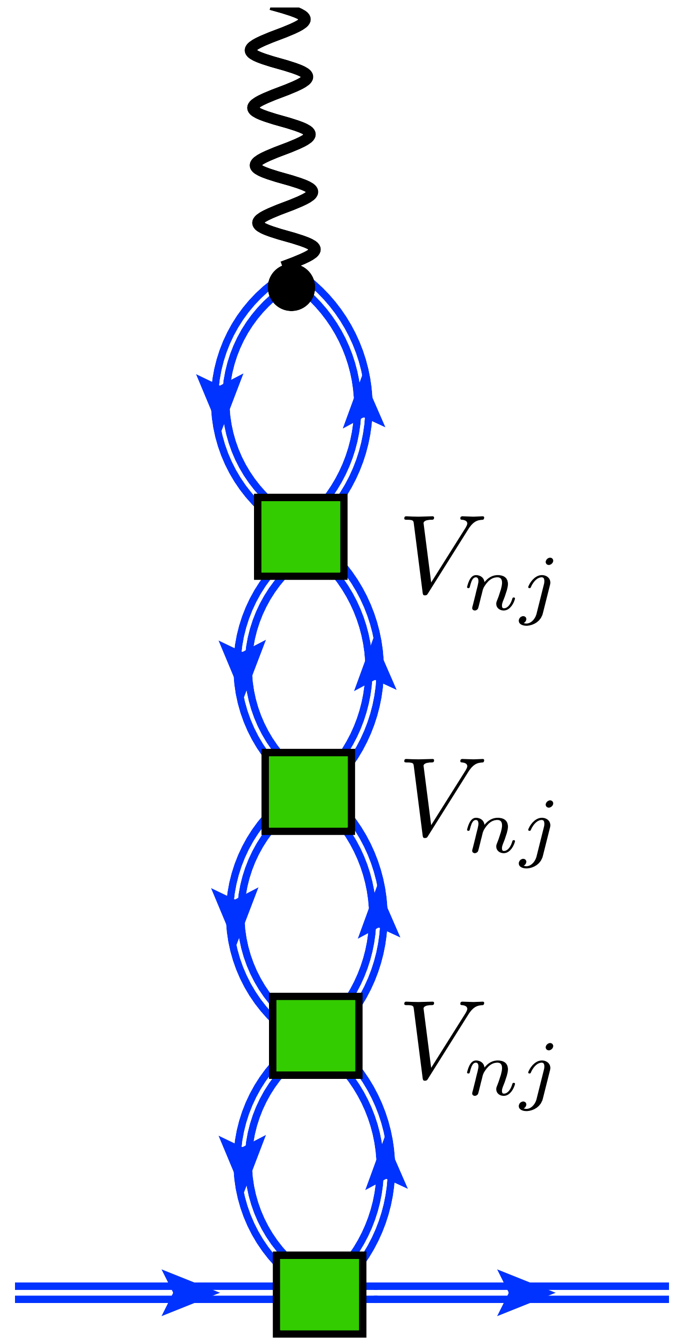


- Corrections due to screening, 2-body currents and 2p-2h excitations are all large. No expansion parameter - results rely on (uncontrolled) many-body approximations.

- Need re-summations - Random Phase Approximation or RPA.

- Earlier work using simple models suggests that both the density and spin response are altered by interactions by factors of 2-4.

- More systematic work with EFT-based interactions is needed.



Spin-Response of Neutron Matter: Guidance from Quantum Monte Carlo

Going beyond RPA: Sum-rules can be calculated with QMC.

$$S_\sigma^n = \int_{-\infty}^{\infty} d\omega\ \omega^n\ S(\omega, q \simeq 0) \qquad \bar{\omega}_0 = \frac{S_\sigma^0}{S_\sigma^{-1}} \qquad \bar{\omega}_1 = \frac{S_\sigma^1}{S_\sigma^0}$$

Density (fm^{-3})	S_σ^{-1} (MeV^{-1})	S_σ^0	S_σ^{+1} (MeV)	$\bar{\omega}_0$ (MeV)	$\bar{\omega}_1$ (MeV)
$n = 0.12$	0.0057(9)	0.20(1)	8(1)	35(9)	40(8)
$n = 0.16$	0.0044(7)	0.20(1)	11(1)	46(11)	55(8)
$n = 0.20$	0.0038(6)	0.18(1)	14(1)	47(12)	78(10)

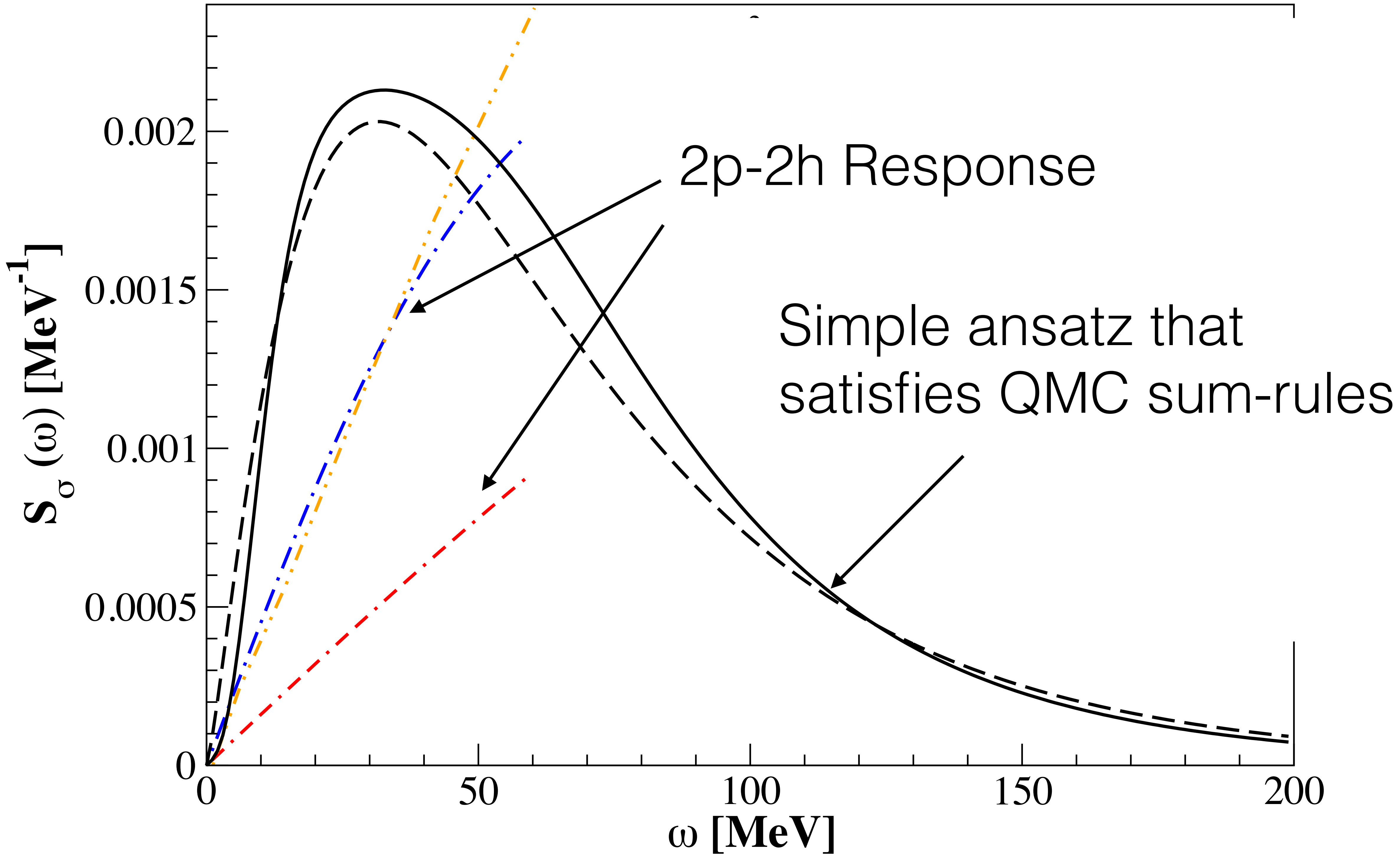
Shen, Gandolfi , Carlson, Reddy (2012)

In the vicinity of nuclear density QMC sum-rules indicate significant strength at

$$\omega \simeq 30 - 50 \text{ MeV}$$

Energy scale is large compared to

$$\frac{q^2}{2m} \quad \text{or} \quad q \times v_F$$



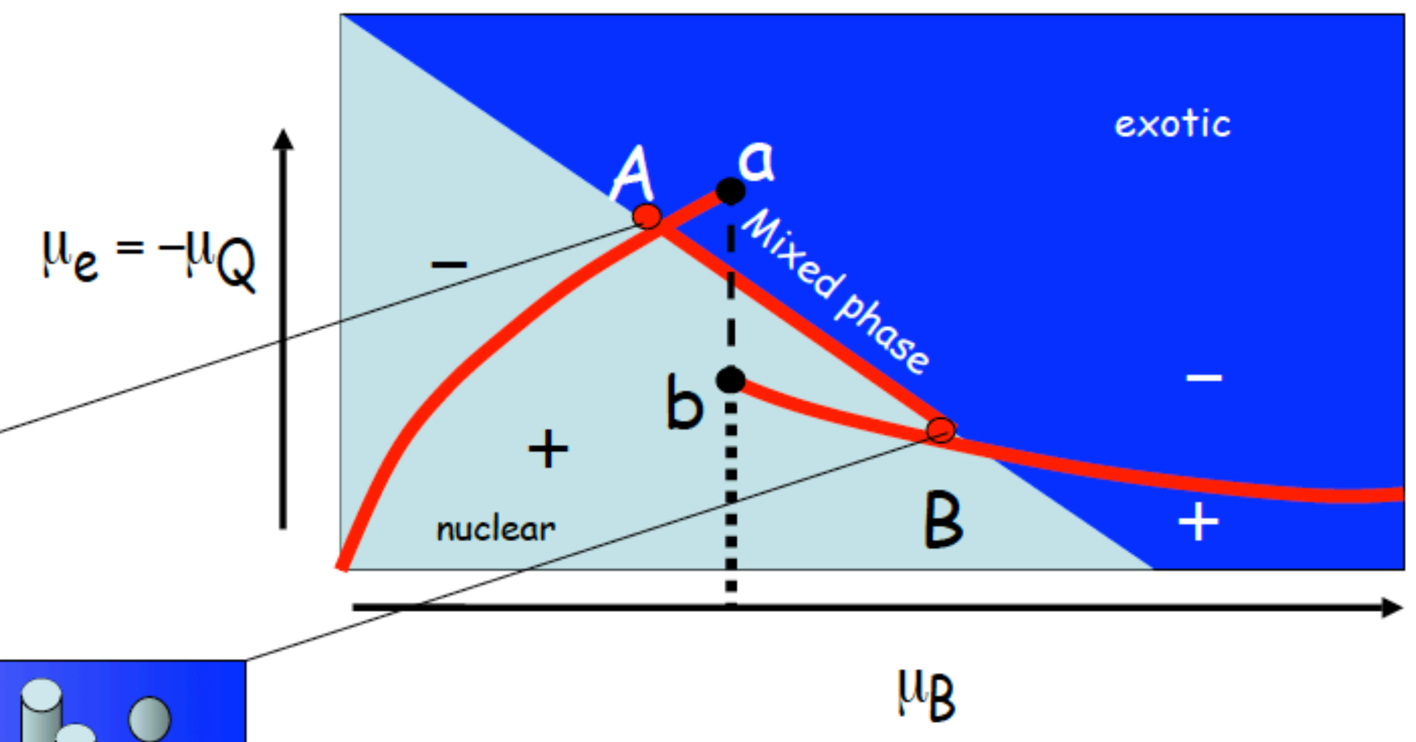
Reddy, Bertsch, and Prakash (2000)

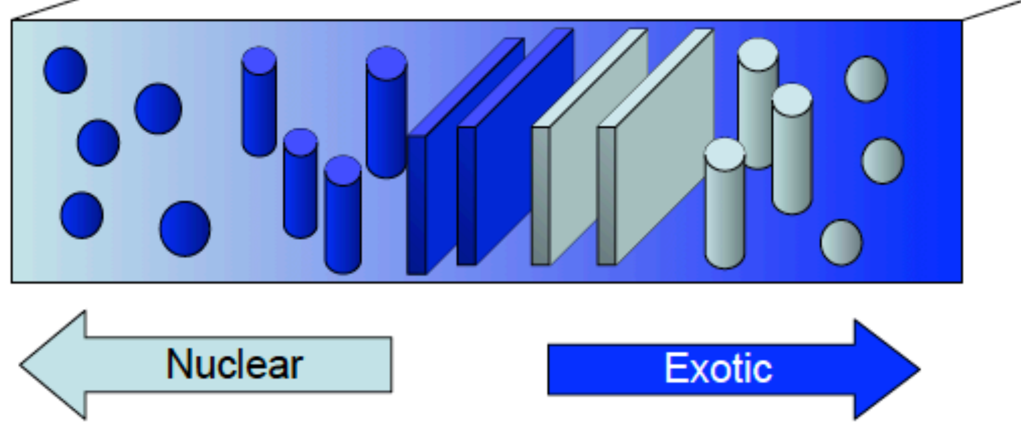
Bryce Fore and S. Reddy (2020)

Reddy (1998), Pons, Reddy, Ellis, Prakash, Lattimer (2000)

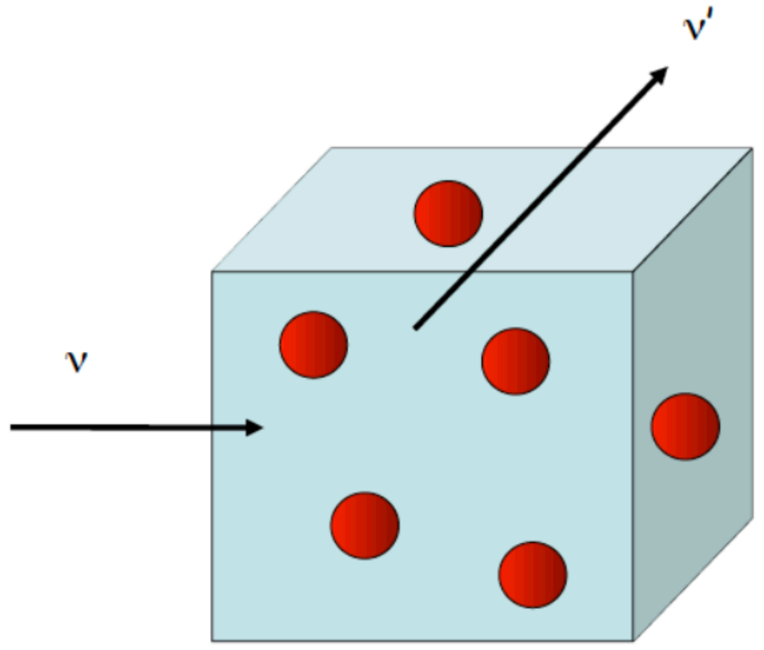
Reddy, Prakash, Lattimer (1998)

Carter and Reddy (2000), Kundu and Reddy (2004)

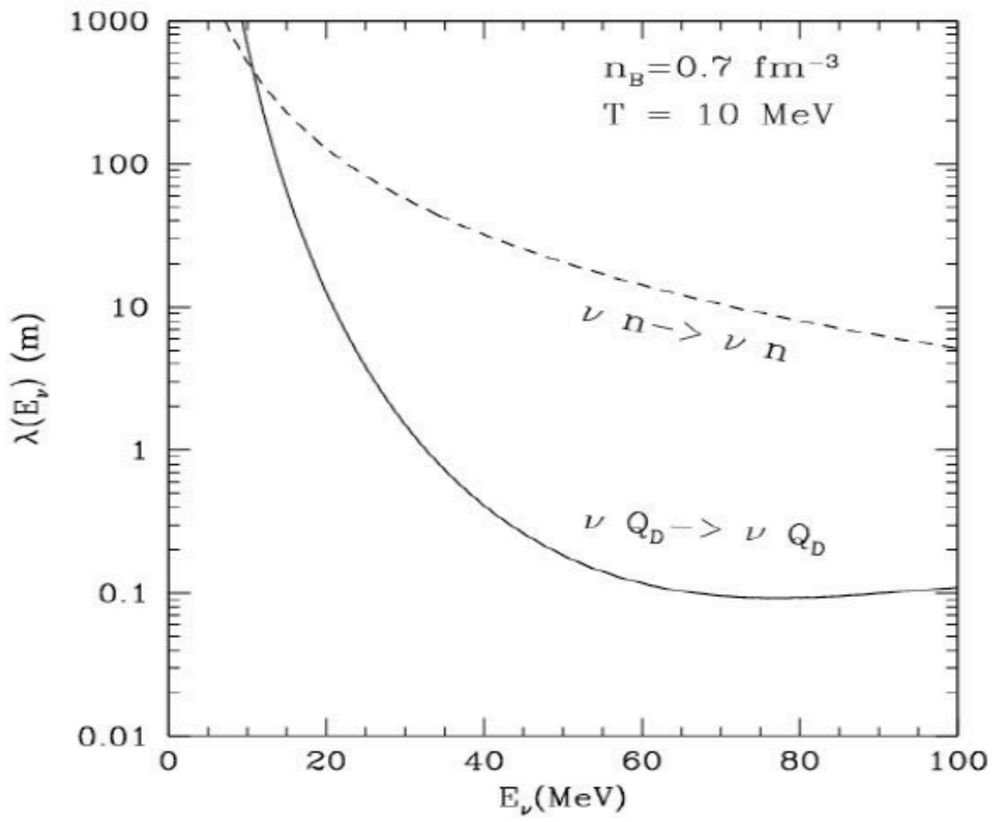




Mixed Phase:
Heterogeneous phase with structures of size 5-10 fm.



quark droplets in nuclear matter.



$$\frac{d\sigma}{d\cos(\theta)} = N_D \frac{G_F^2}{16\pi} S_q\, Q_W^2 E_\nu^2 (1+\cos(\theta))$$

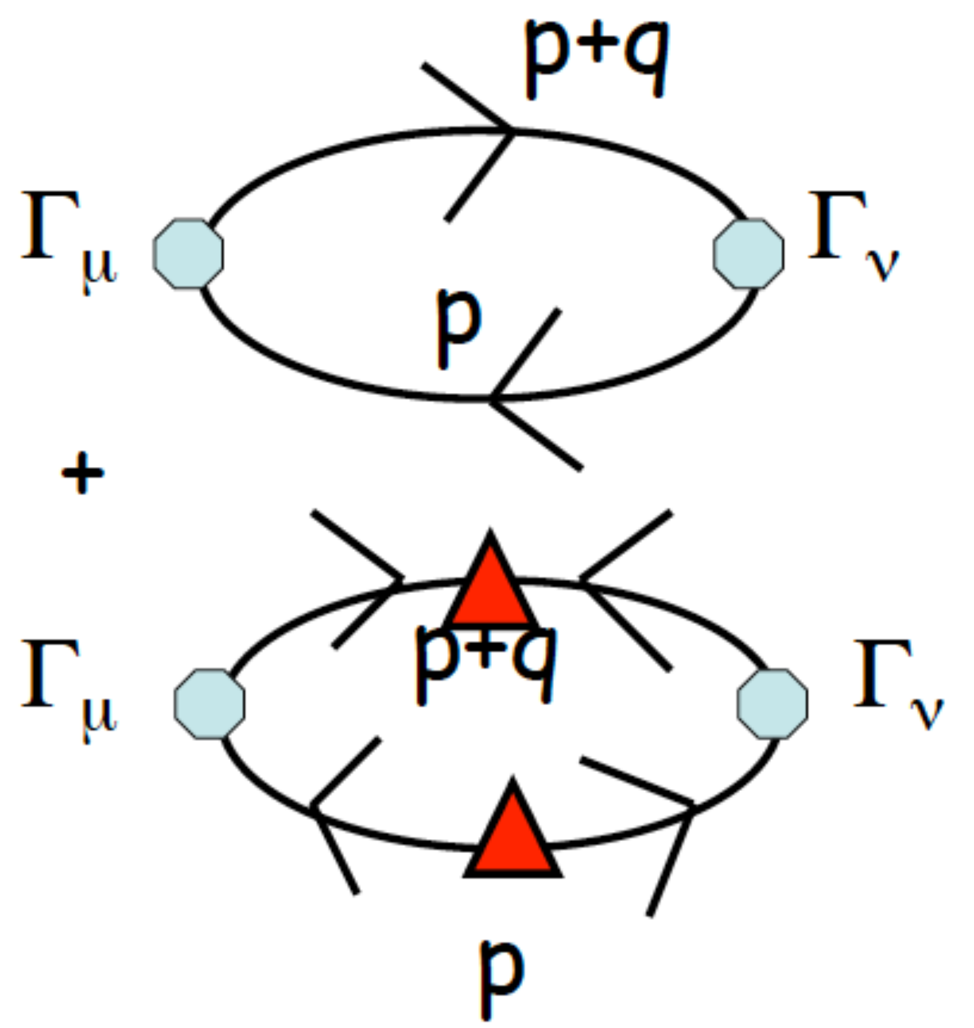
number density of droplets

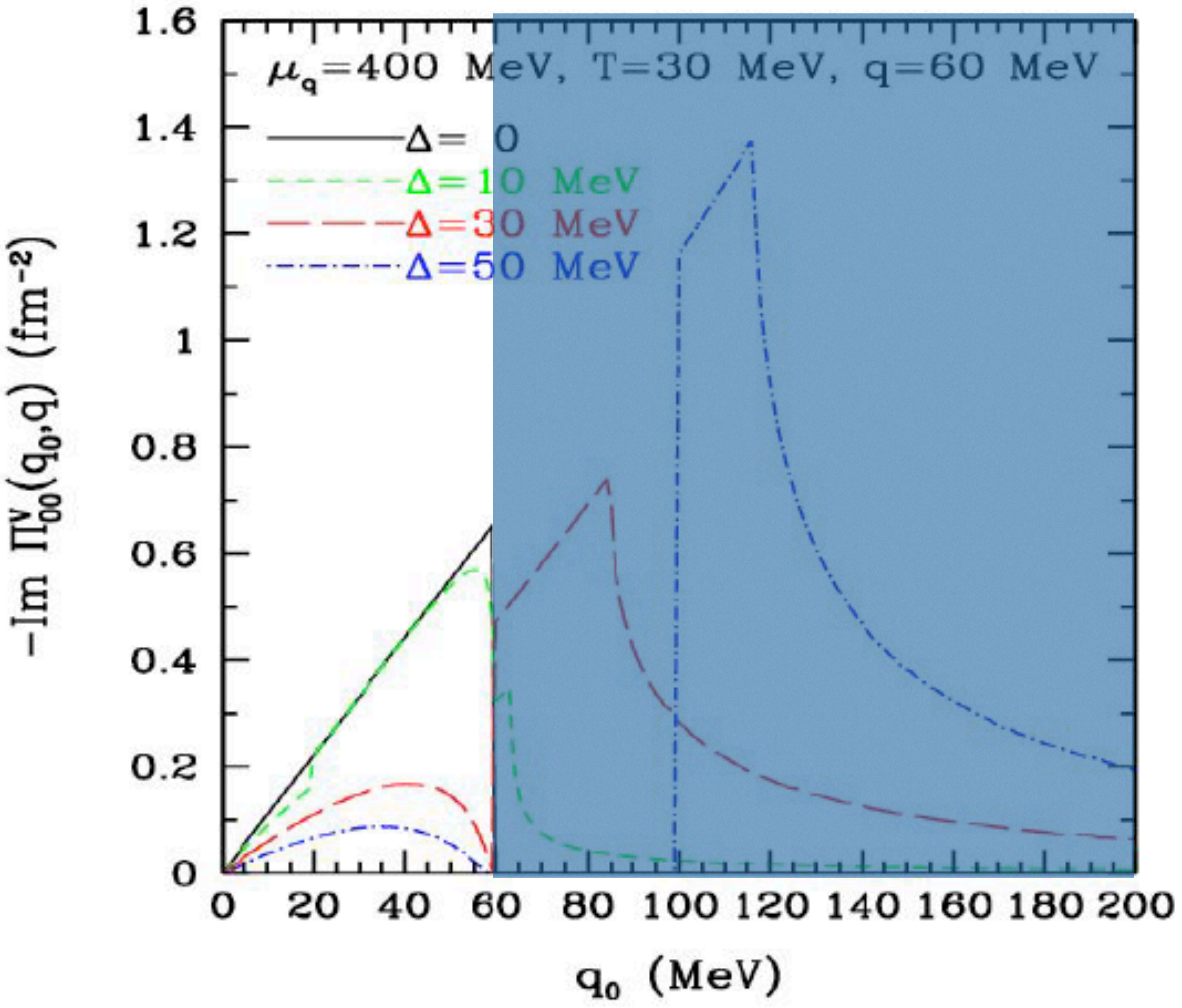
weak charge of the droplet.

Coherent scattering from the droplets is large. Greatly reduces the neutrino mean free paths.

Neutrino Scattering in Superconducting Quark Matter.

$$\Pi_{\mu\nu}(q_0,q) = -i\int d^4p\ \mathrm{Tr}[G(p+q)\Gamma_\mu G(p)\Gamma_\nu]$$





Pairing modifies particle propagation. Particles can be absorbed or emitted from the condensate.

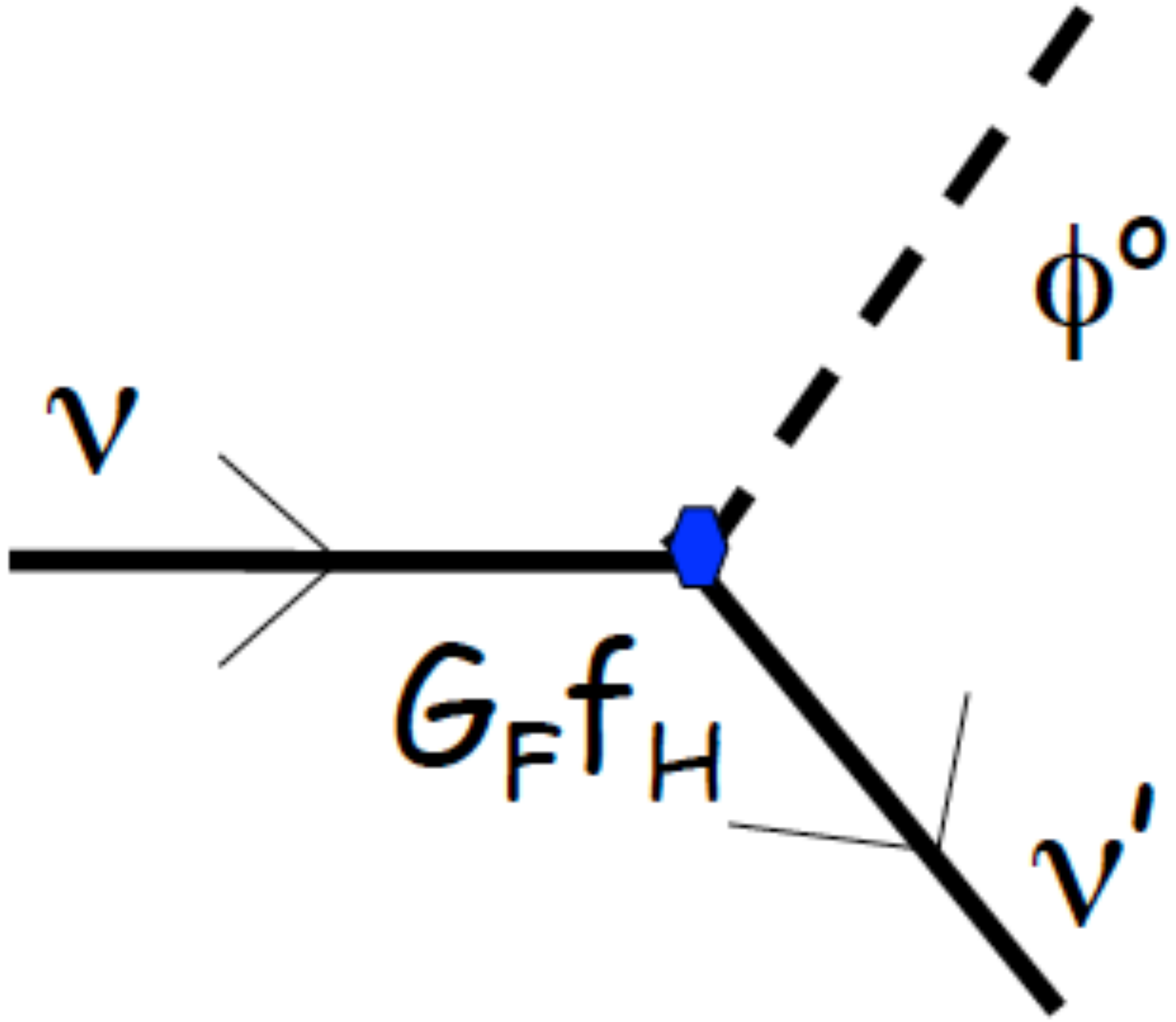
Energy gap modifies the energy spectrum.

Response moves to high energy (time-like).
Neutrino scattering is exponentially suppressed.

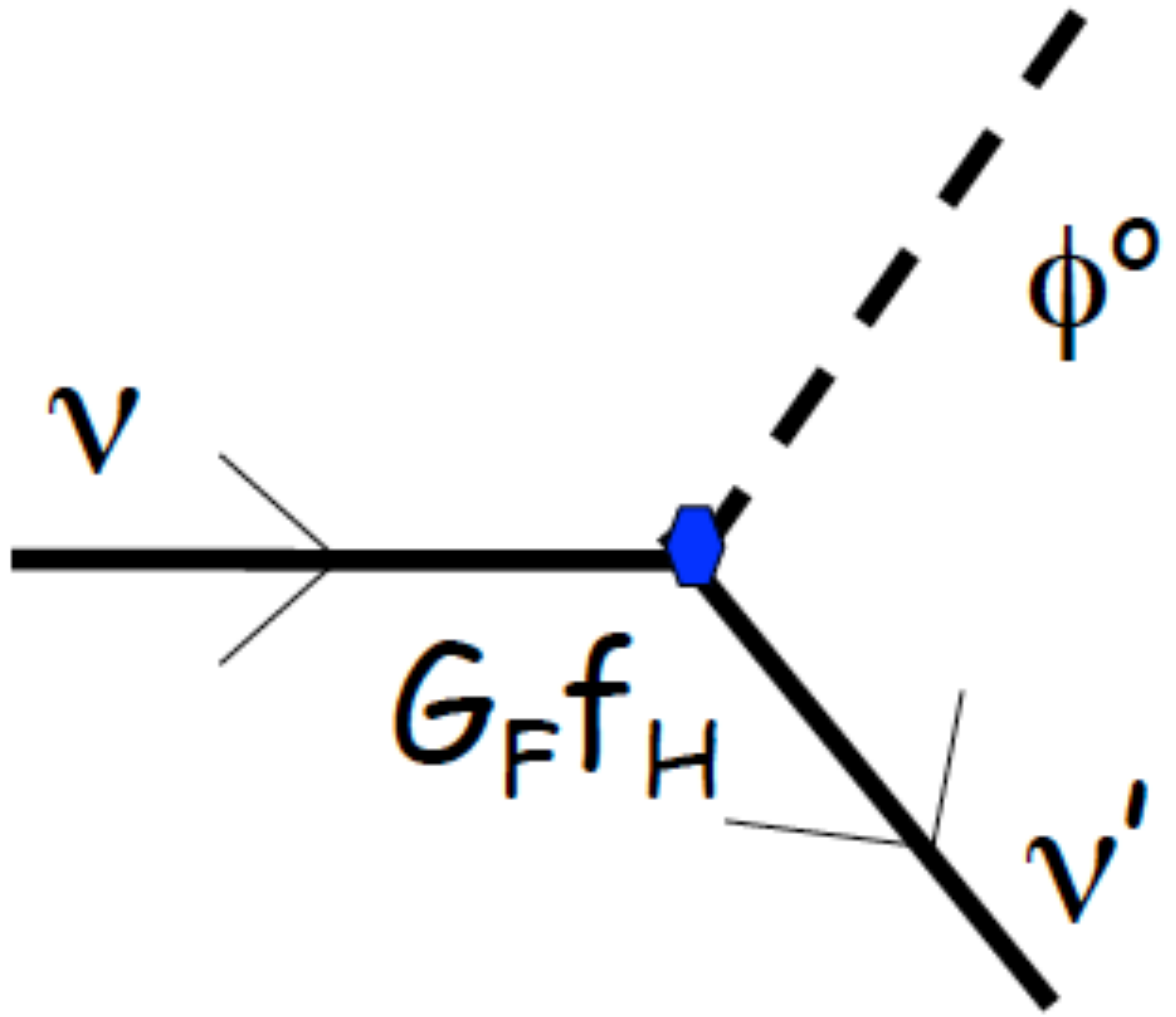
Carter & Reddy (2000)

Neutrino Scattering in Superfluid Quark Matter.

- Superfluid state has a Goldstone boson.
- Neutrinos couple to these modes.
- Arises naturally in RPA.
- At $T \ll T_c$ this is the only relevant mode for neutrino scattering.



Ultra Dense Matter is Opaque to Neutrinos but Transparent to Photons!



$$\frac{1}{\lambda_{\nu\to H\nu}(E_\nu)} = \frac{256}{45\pi}\left[\frac{v(1-v)^2(1+\frac{v}{4})}{(1+v)^2}\right] G_F^2\, f_H^2\ \ E_\nu^3$$

$$SU(3)_{color} \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)_B$$
$$\Downarrow$$
$$SU(3)_{color+L+R} \otimes Z_2$$

More Opaque than the Normal Phase !

phase	process	λ(T=5 MeV)	λ(T=30 MeV)
Nuclear	$\nu n \to \nu n$	200 m	1 cm
Matter	$\nu_e n \to e^- p$	2 m	4 cm
Unpaired	$\nu q \to \nu q$	350 m	1.6 m
Quarks	$\nu d \to e^- u$	120 m	4 m
CFL	λ_{3B}	100 m	70 cm
	$\nu\phi \to \nu\phi$	>10 km	4 m

Conclusions

- Effects due to nuclear interactions on the density, spin, and isospin susceptibility impact neutrino transport and spectra in supernovae and mergers.
- There is a systematic approach to calculating the dynamic structure factors at densities and temperatures of interest to the neutrino-sphere.
- Sum rules from ab initio theory can be useful to construct reliable models for the dynamic response.
- Phase transitions can have a strong influence on neutrino interactions.