Quantifying and interpreting gravitational waveform error

INT Workshop 22r-2a "Neutron Rich Matter on Heaven and Earth"

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Physics and Astronomy

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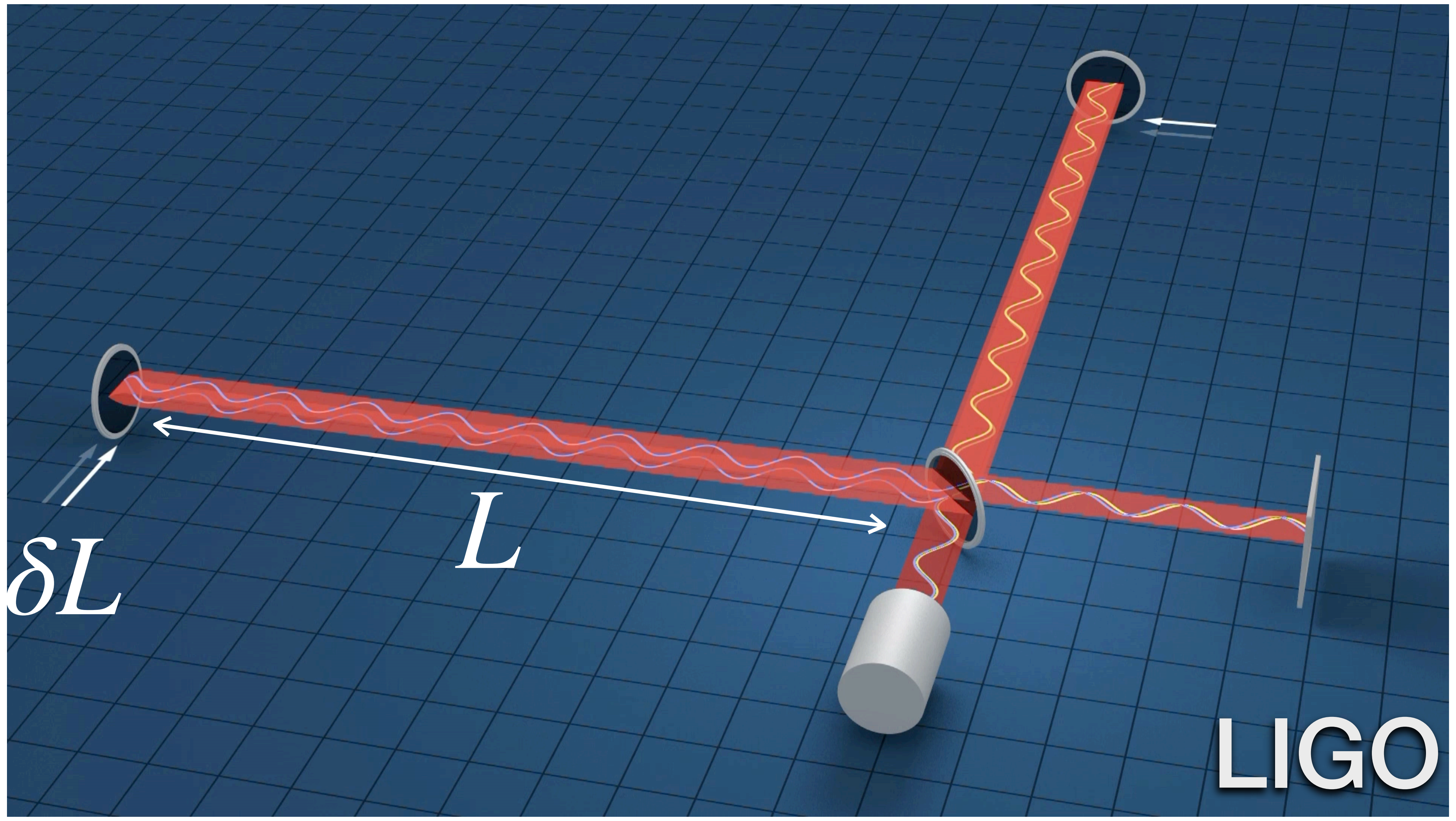
A very short orientation to GW observation

$$h = 2\frac{\delta L}{L} \sim \frac{G}{c^2}\frac{M}{d_L}\frac{v_\perp^2}{c^2}$$



Gravitational waves come from oscillatory sources where mass is changing across your line of sight

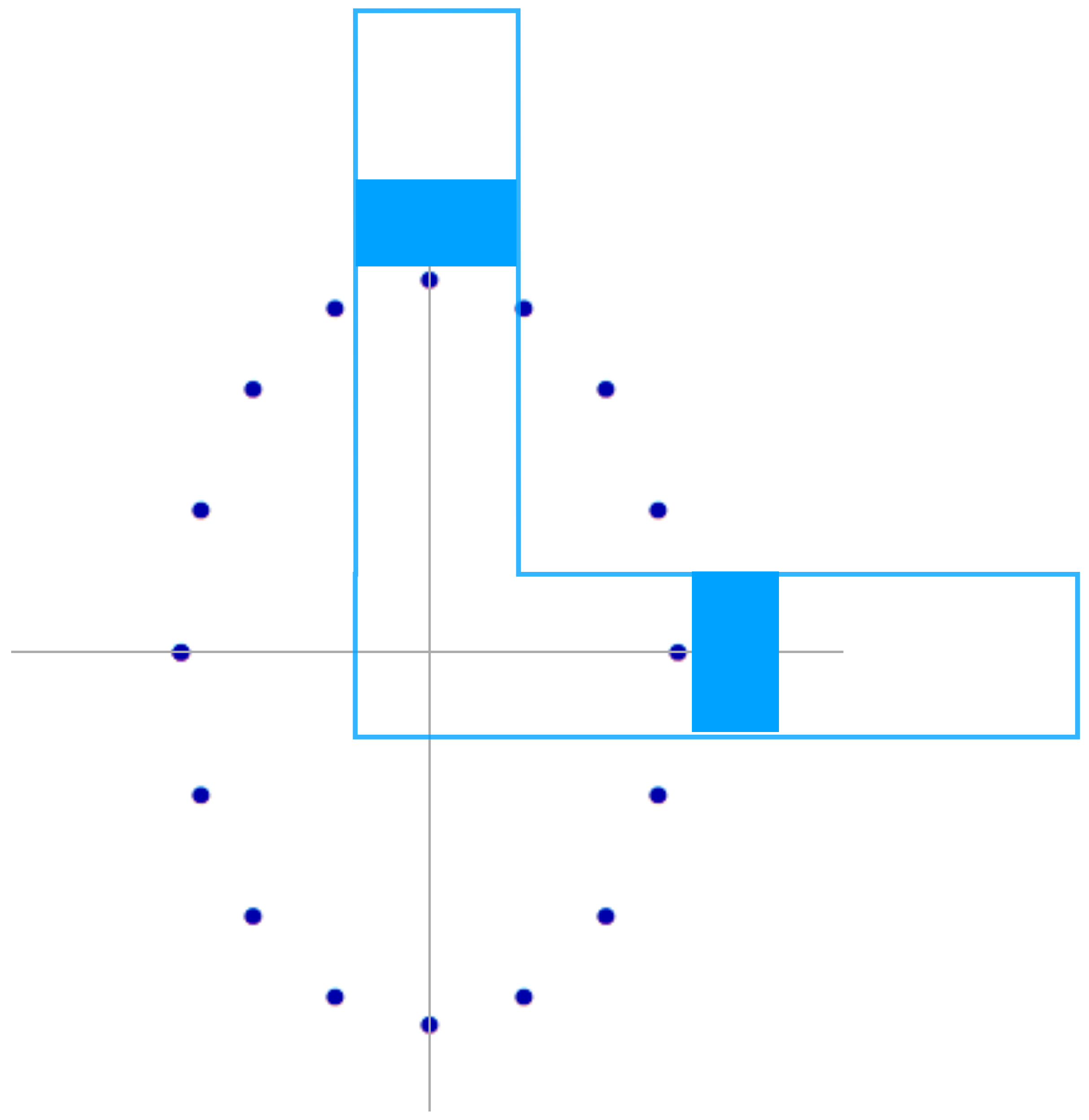
Amplitude falls off with distance from source

Keplerian orbits at separation a,

$$h \sim 10^{-21} \frac{100\ \text{Mpc}}{d_L} \frac{M}{1.4\, M_\odot} \frac{R_S}{a}$$

Size of BH with mass M
$R_S = 2GM/c^2$

$$f_{GW} = 2f_{\mathbf{orb}}$$



Massive objects orbit

Response of test masses

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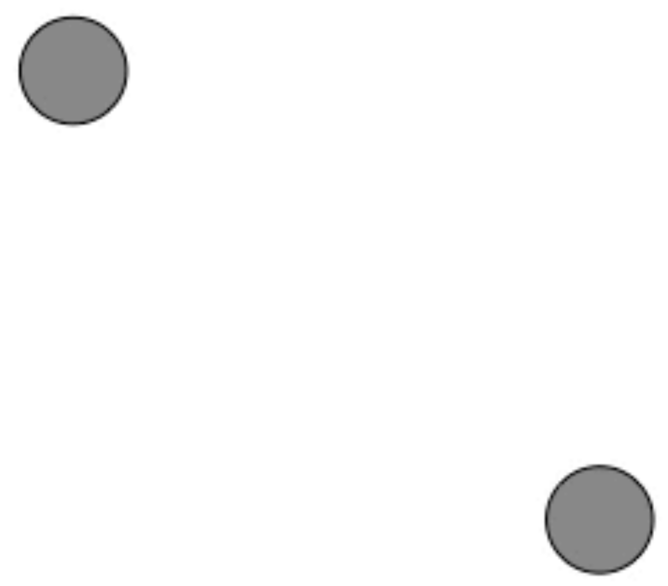

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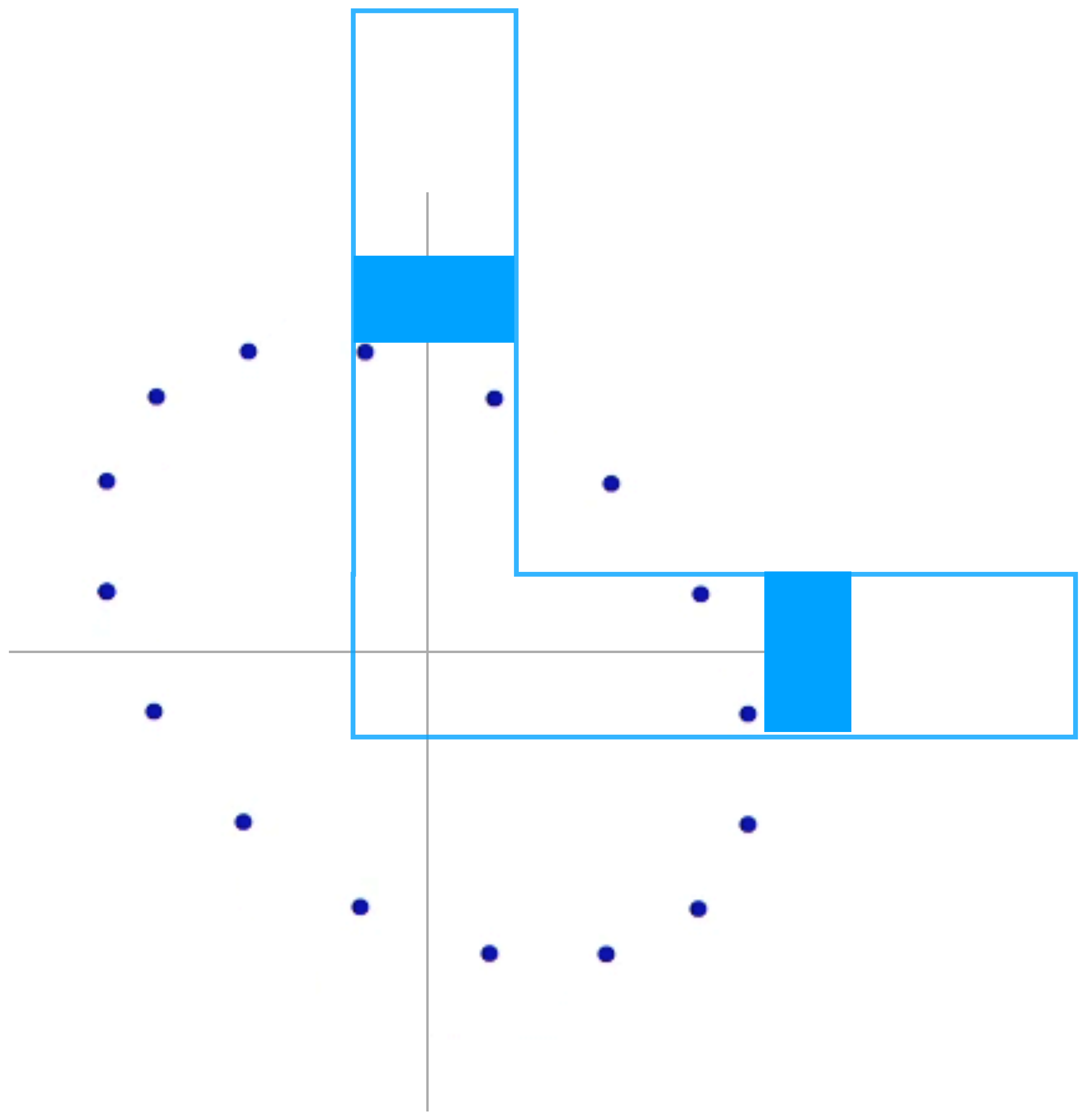

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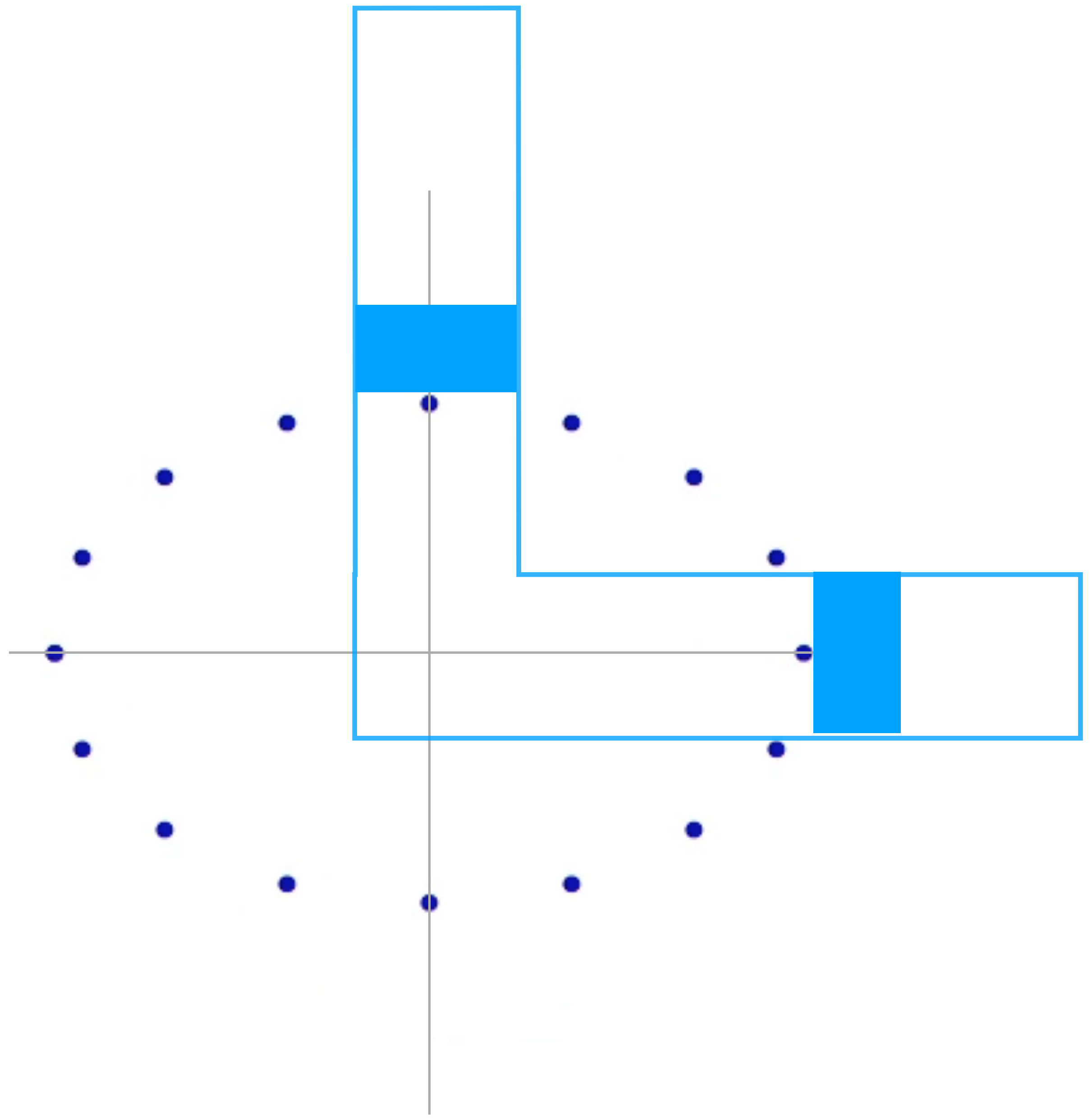

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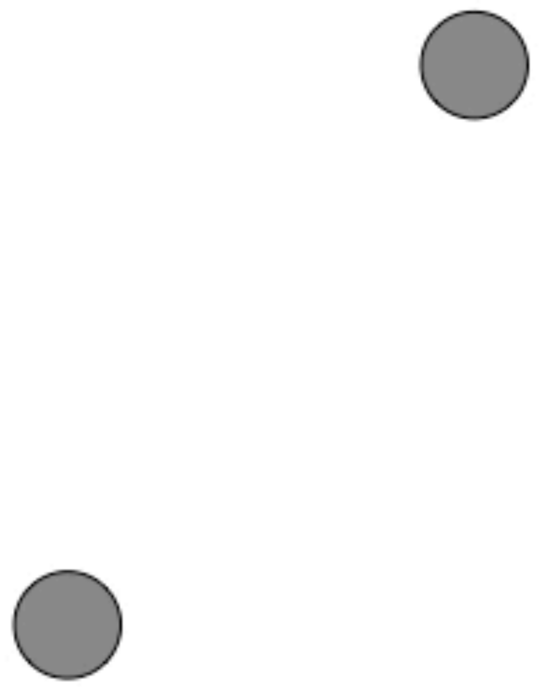

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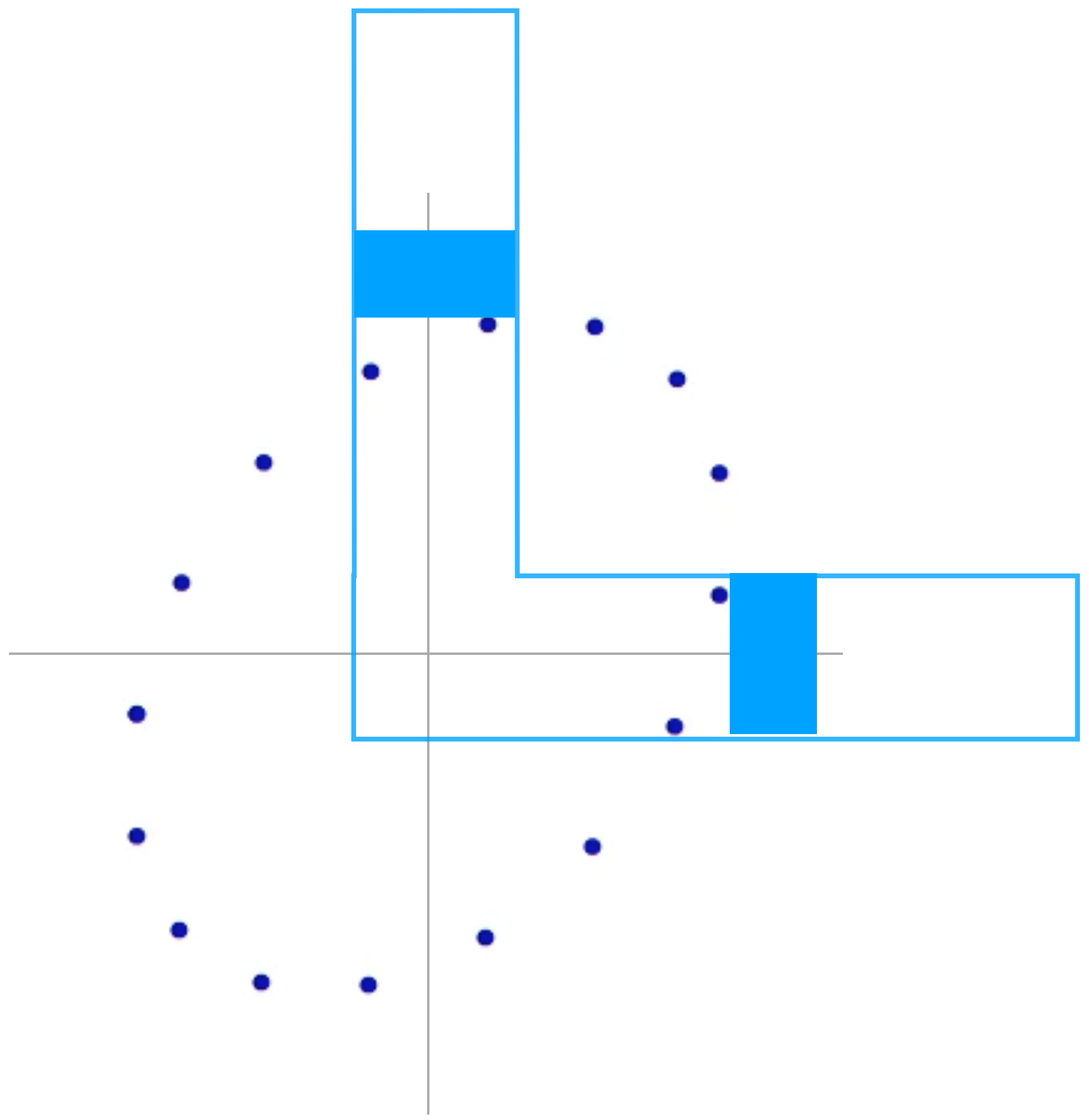

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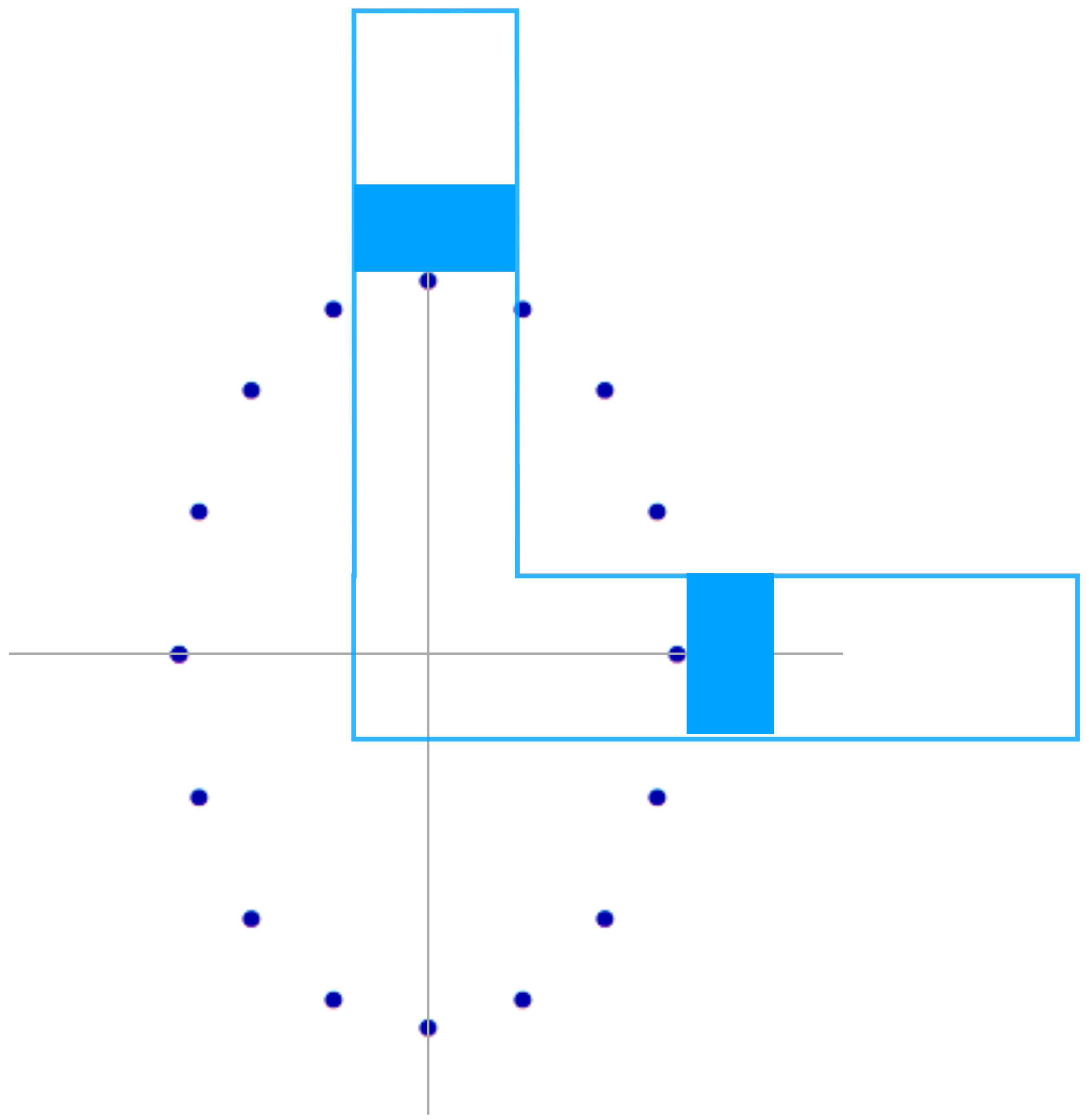

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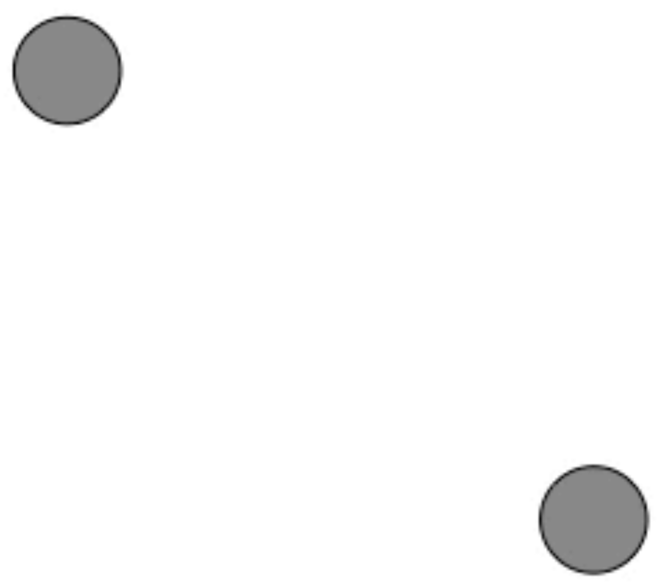

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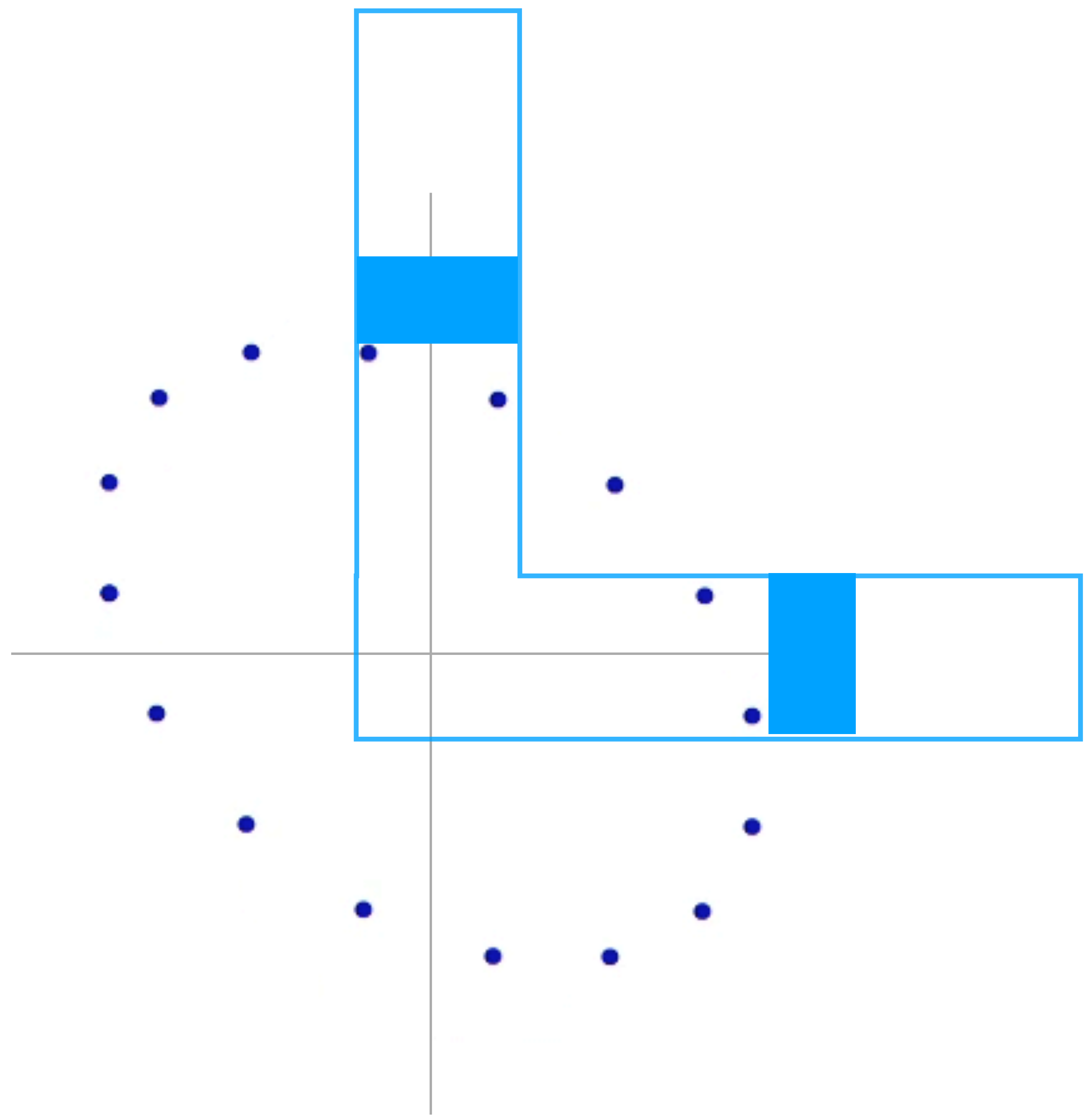

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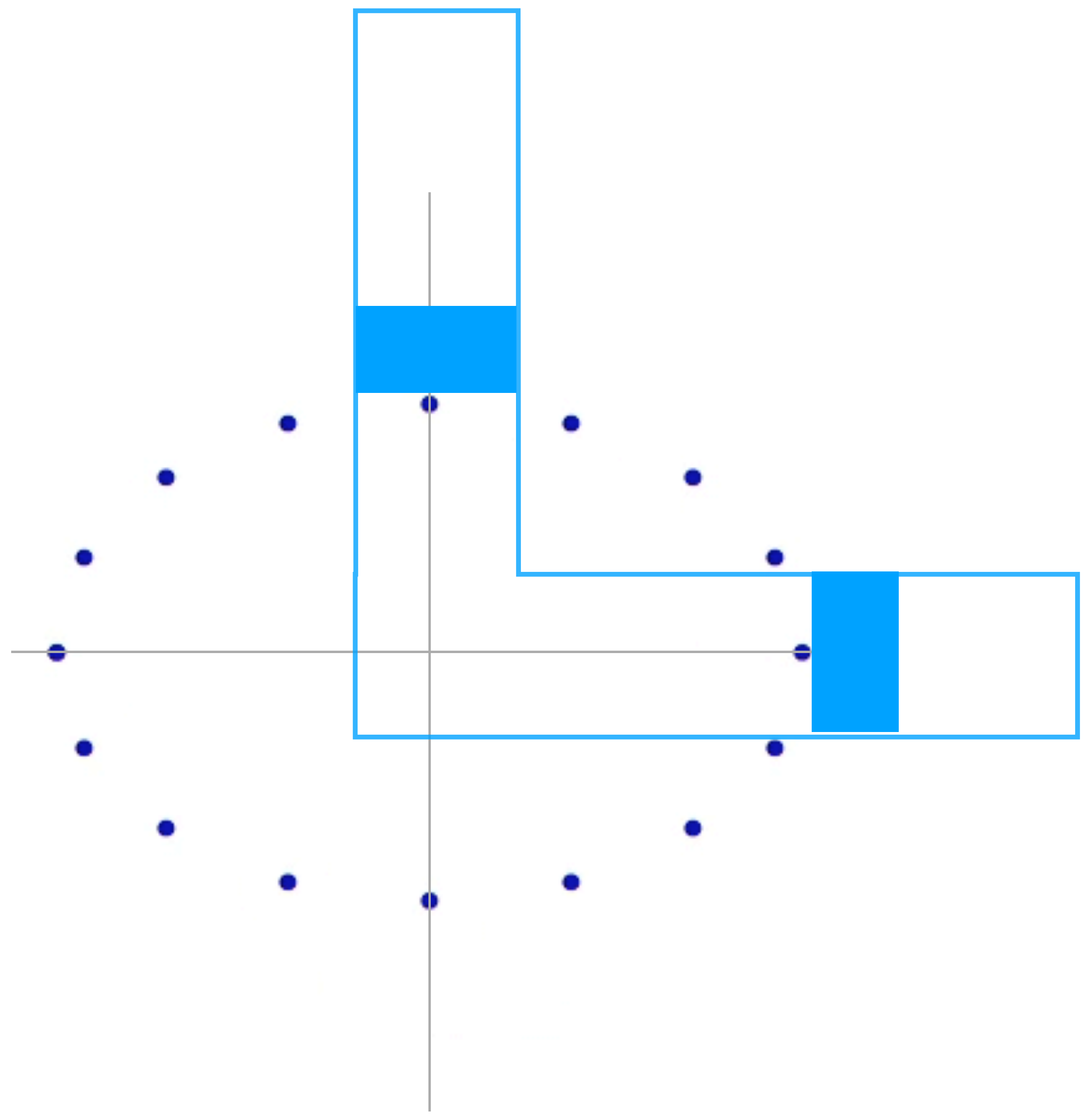

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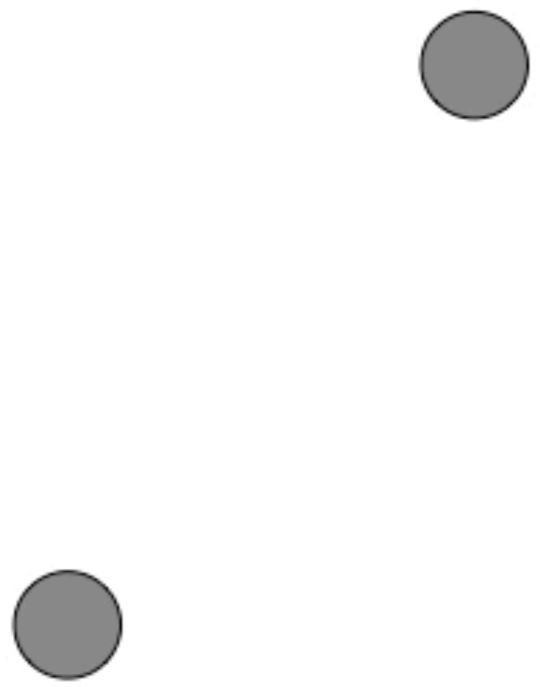

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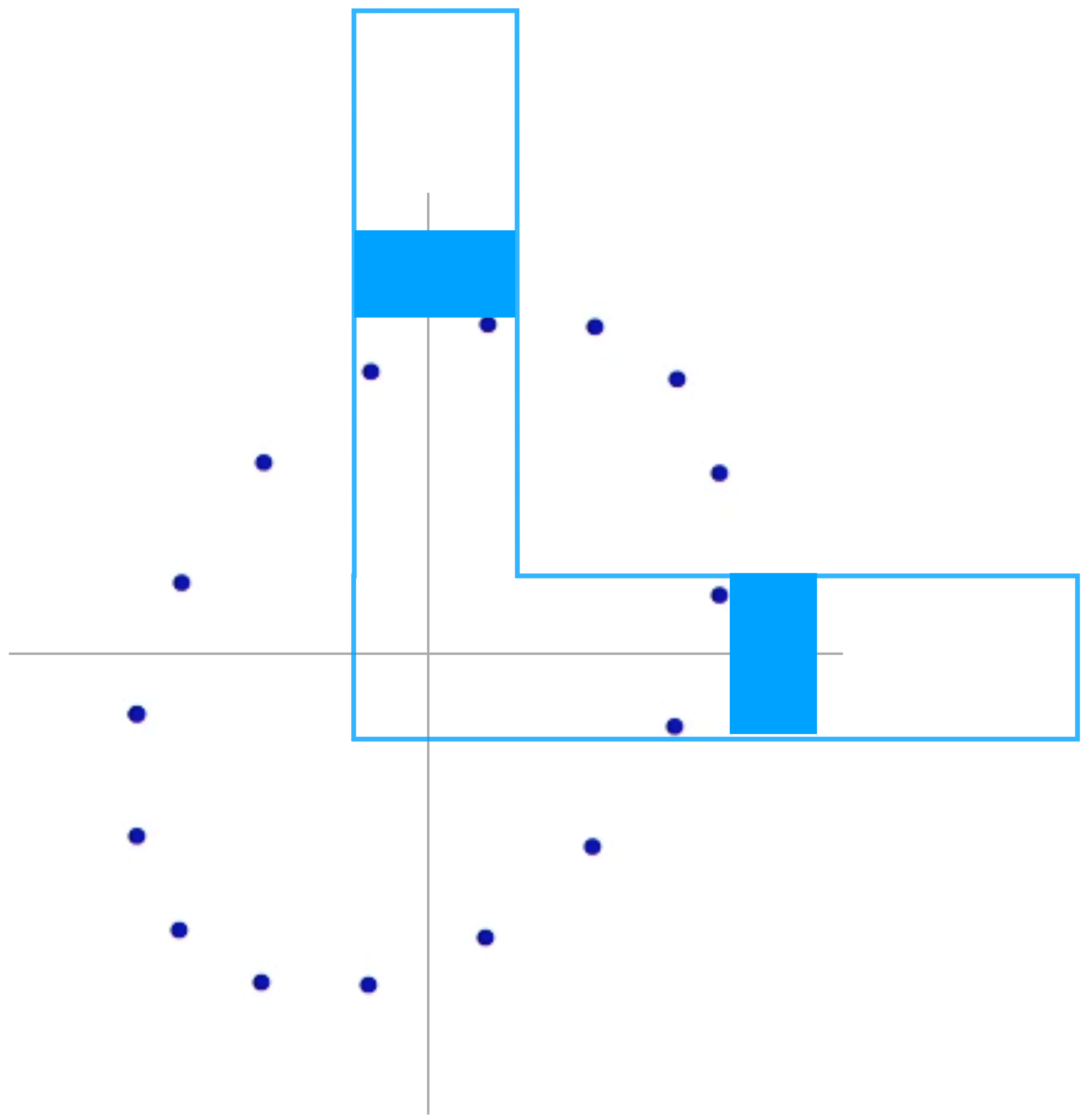

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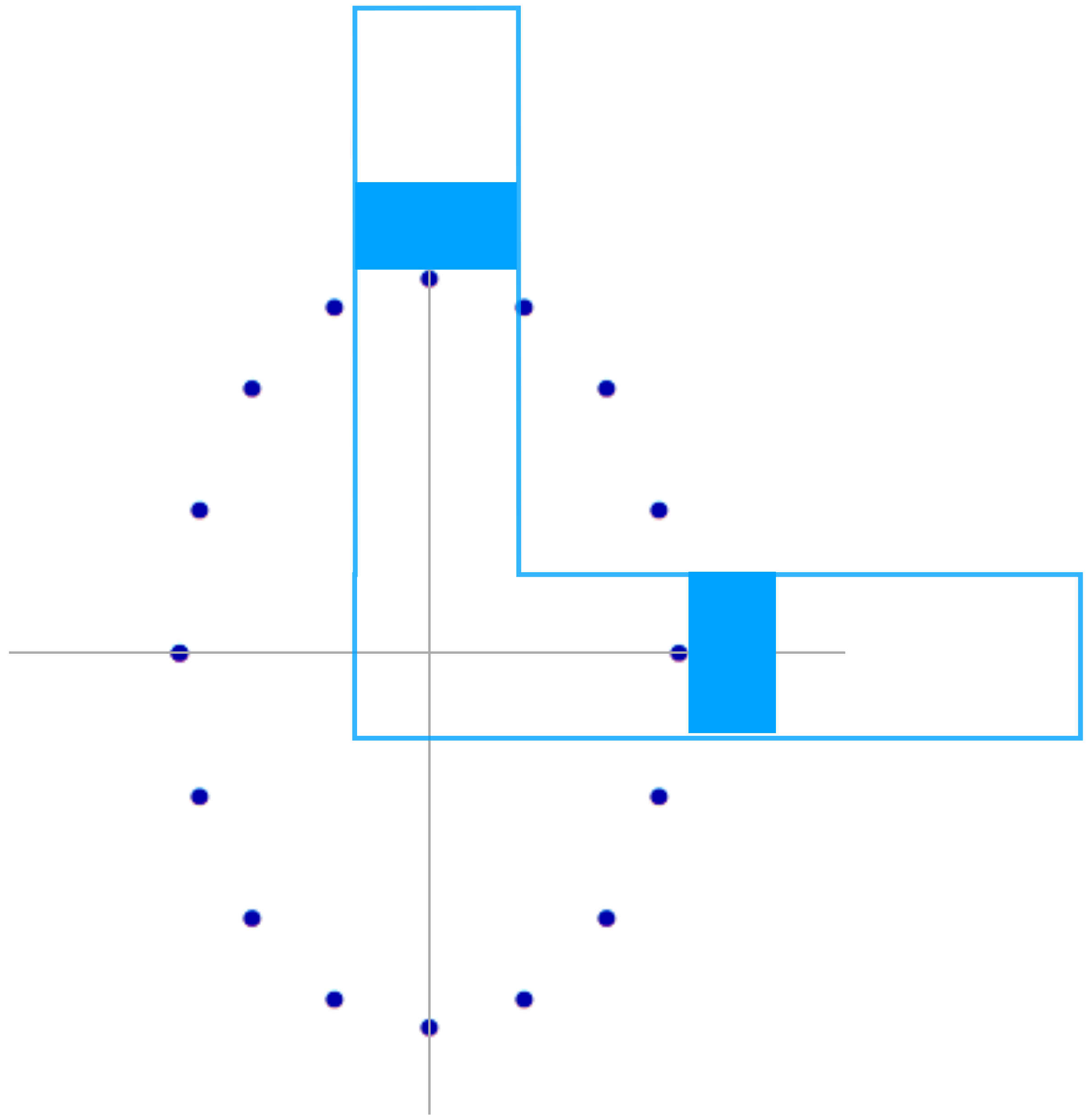
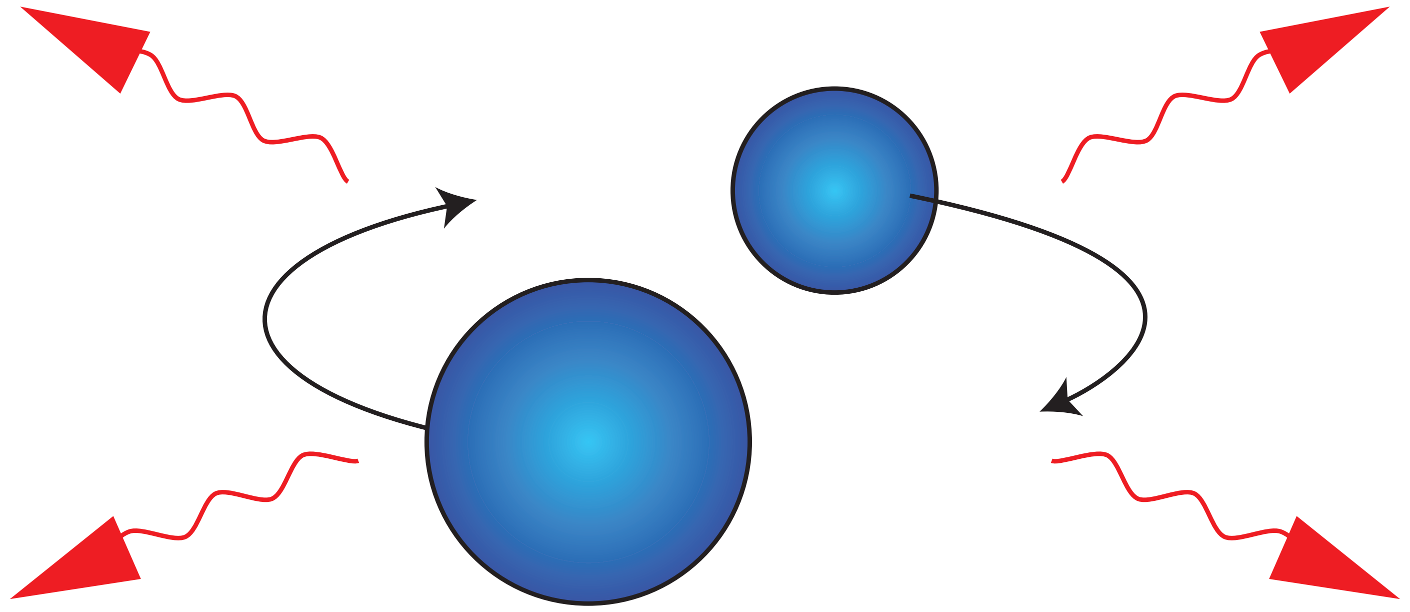


Massive objects orbit

Response of test masses

Binary luminosity

*Binary schematic:
Dave Tsang*



$$\frac{dE}{dt} = \mathscr{L}_{GW} \sim 10^{59}\ \text{erg s}^{-1} \frac{R_S}{a}$$

Binary luminosity

Binary schematic:
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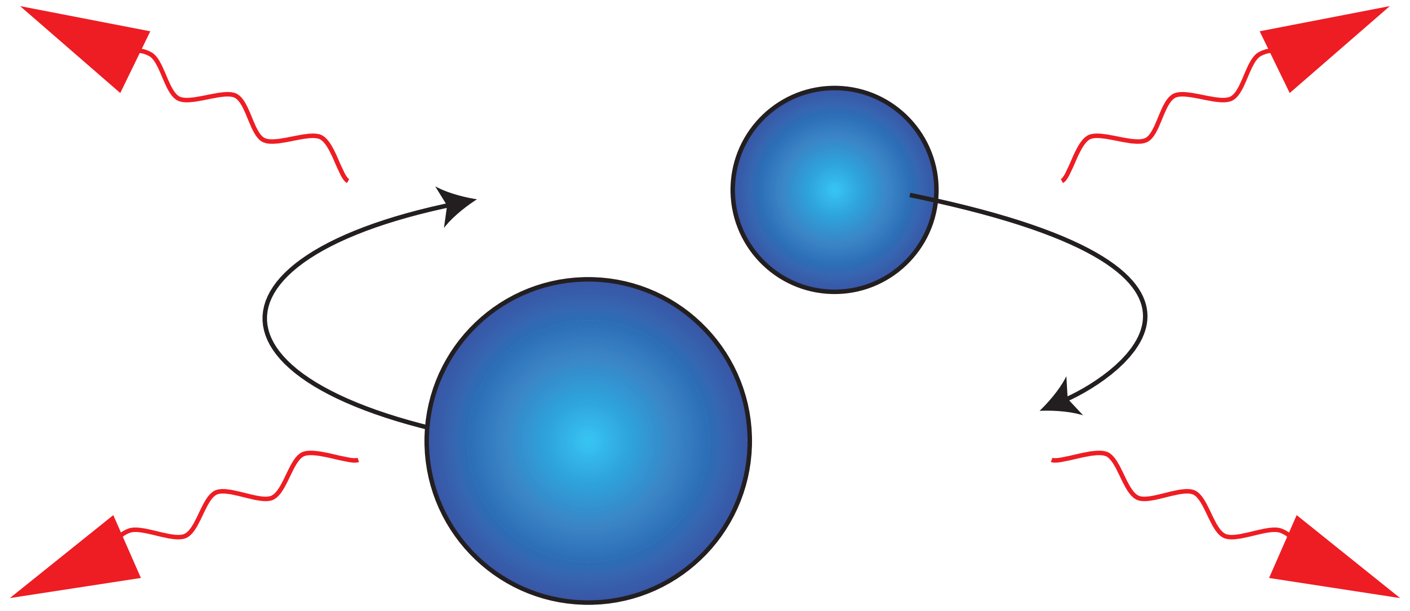
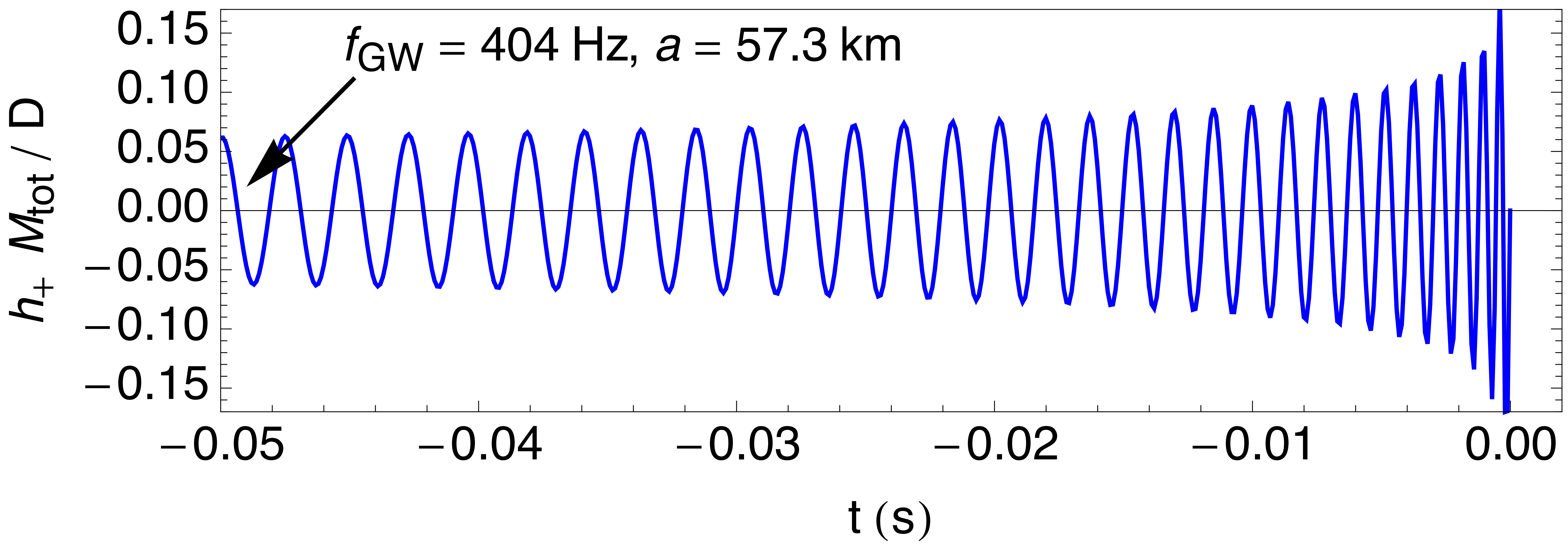


$$\frac{dE}{dt} = \mathscr{L}_{GW} \sim 10^{59}\ \mathrm{erg\ s^{-1}} \frac{R_S}{a}$$

Energy balance: the "chirp"

$$\frac{da}{dt} = \frac{-\mathscr{L}_{GW}}{dE_{orb}(a)/da}$$



Imprint of matter

Surface contours: NS simulation, David Radice

- Orbital energy lost to the deformation of the stars
- Tidal bulges add a little quadrupole luminosity
- Both $\propto \Lambda$ of stars: $\Lambda_i = \frac{\lambda_i}{m_i^5} = \frac{2}{3} k_2 \left(\frac{R_i}{m_i} \right)^5$

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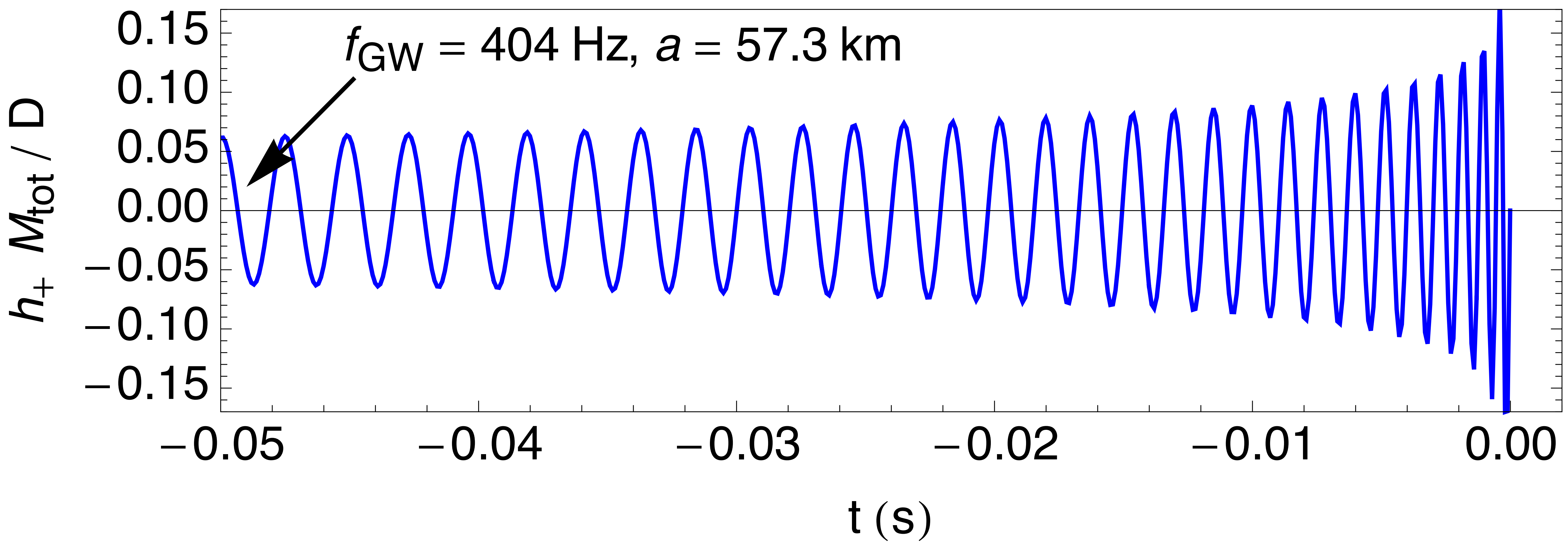

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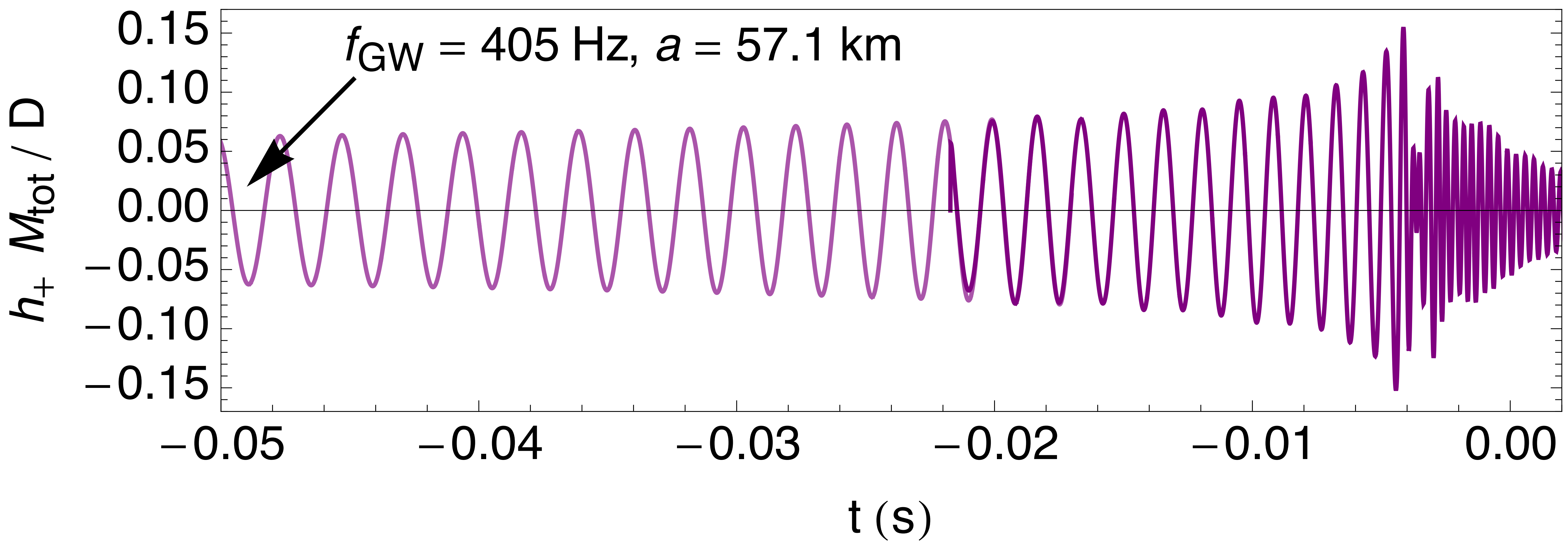
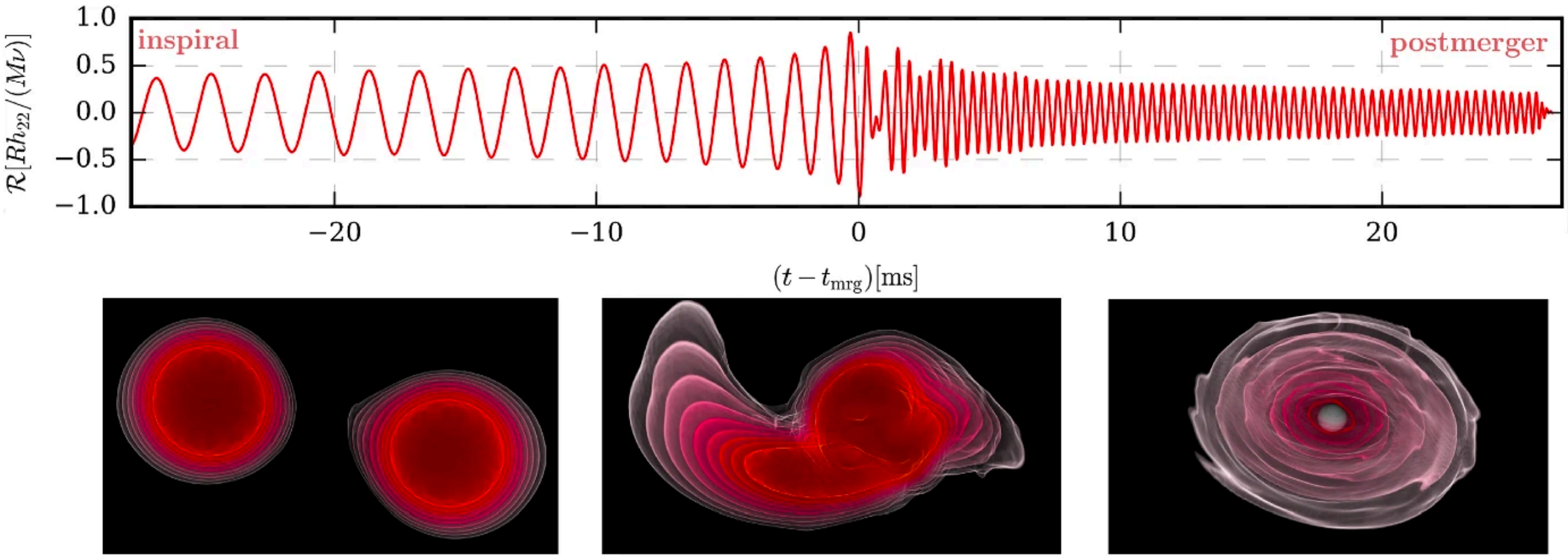
$$\frac{da}{dt} = \frac{-\mathscr{L}_{GW} - \mathscr{L}_{GW,def}}{dE_{orb}(a)/da + dE_{def}(a)/dr}$$





Dietrich, T., Hinderer, T. & Samajdar, A. Gen Relativ Gravit 53, 27 (2021)

- Stars deform in complicated, close interactions:
  - stars are not isolated, deformations are not linear, deformations are not pure quadrupole, star response is dynamic …
- GW analysis currently uses Λ_1, Λ_2 as *effective* matter descriptors in gravitational-wave models based on numerical simulation and high-order analytical expressions

Inference from GW observations

- $\boldsymbol{d}$ data $= \boldsymbol{h}$ signal $+ \boldsymbol{n}$ noise
- Power spectral density of noise $S_n(f)$:

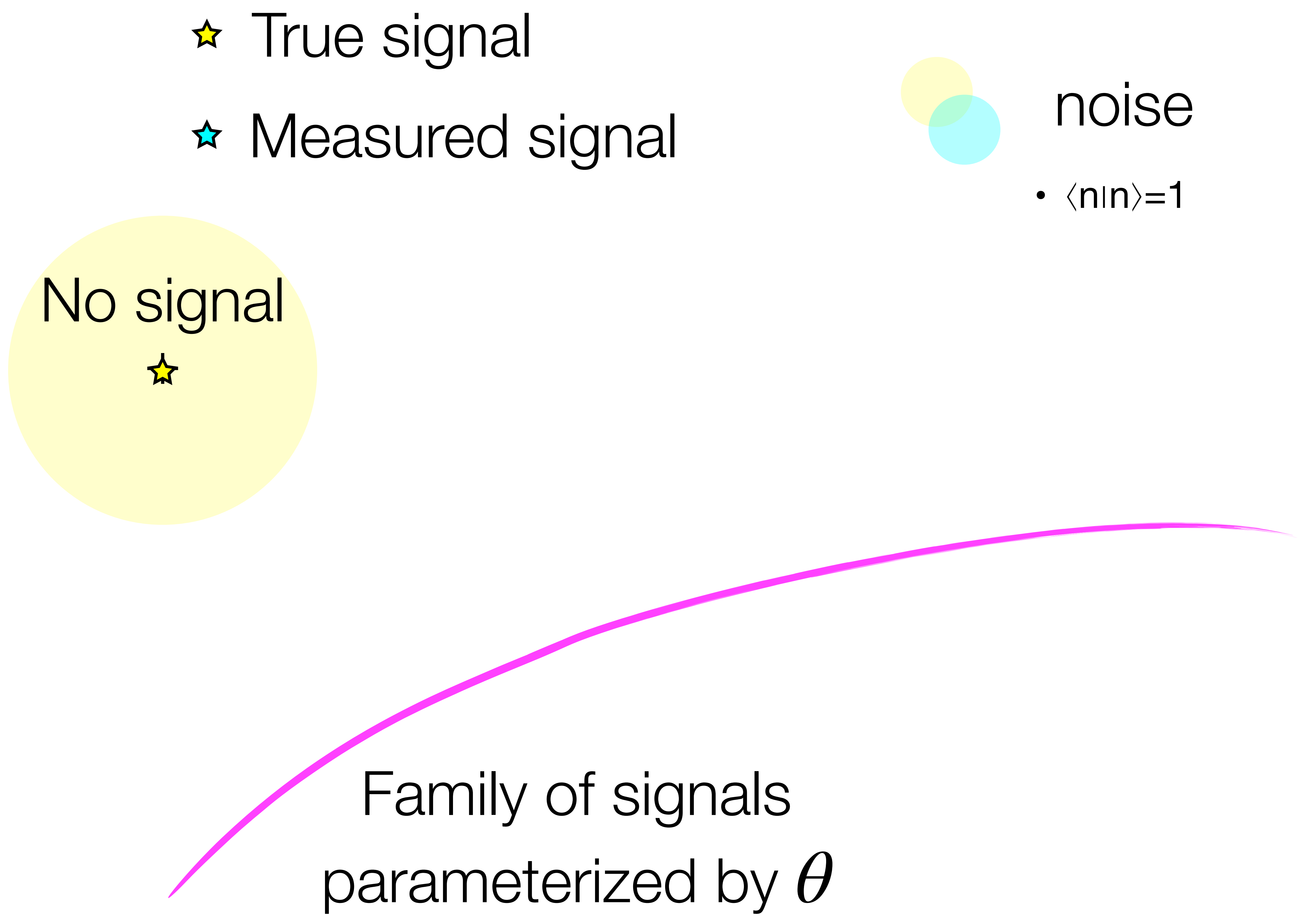
$$\mathscr{L}(\boldsymbol{n}) \propto \exp\left(-\sum_i 2\Delta f \frac{|n_i|^2}{S_n(f_i)}\right)$$

- Likelihood of data given a candidate signal :

$$\mathscr{L}(\boldsymbol{d}|\boldsymbol{h}) \propto \exp\left(-\sum_i 2\Delta f \frac{|d_i - h_i|^2}{S_n(f_i)}\right)$$

$$= \exp\left(-\langle \boldsymbol{d} - \boldsymbol{h}, \boldsymbol{d} - \boldsymbol{h}\rangle\right)$$

- Inner product in Fourier space; $\langle \boldsymbol{n}, \boldsymbol{n}\rangle = 1$



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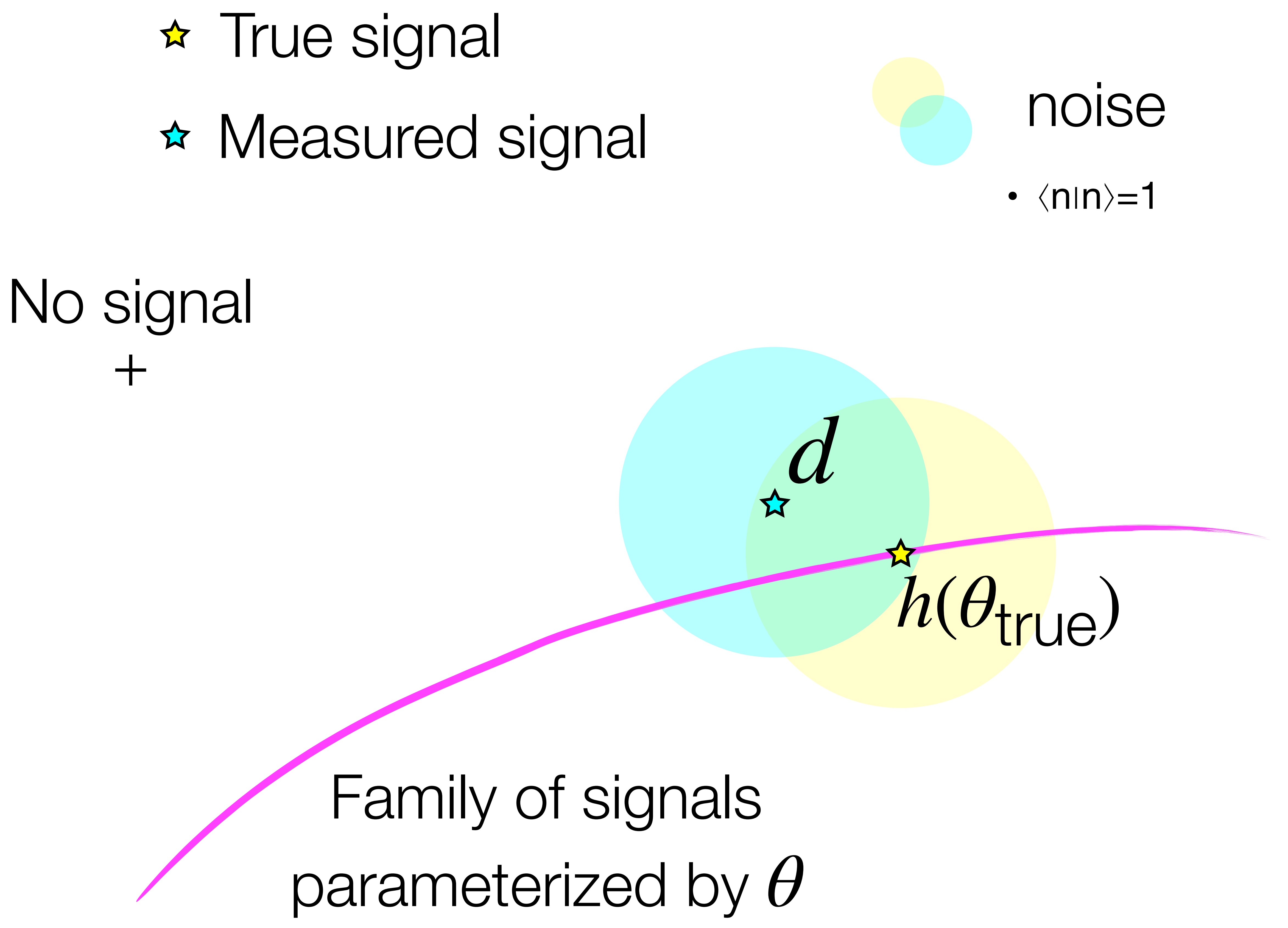

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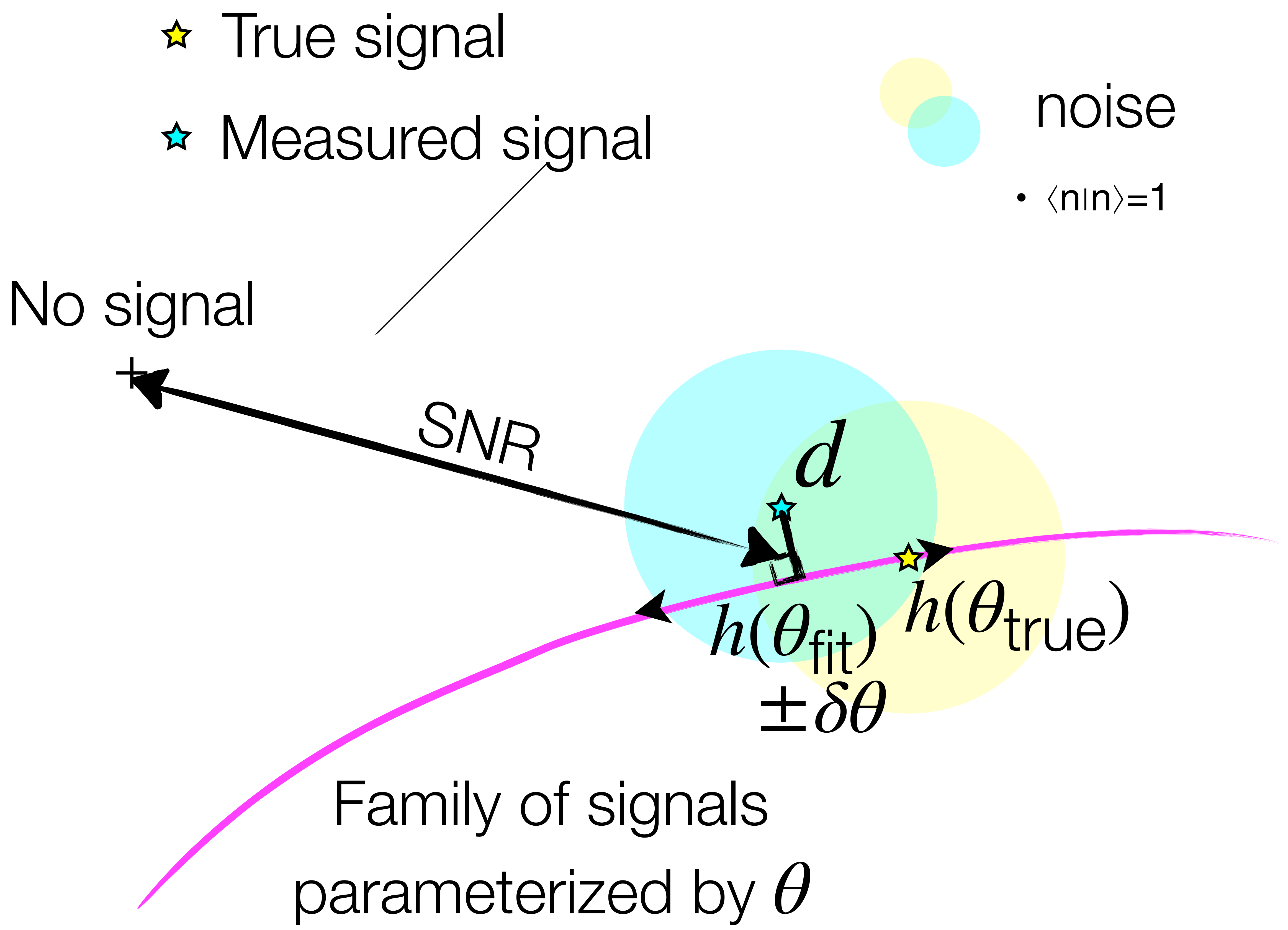


Sources of systematics

- e.g. Vitale et al 2012 Phys. Rev. D **85**, 064034, Ling Sun et al 2020 Class. Quantum Grav. 37 225008, Essick Phys. Rev. D 105, 082002

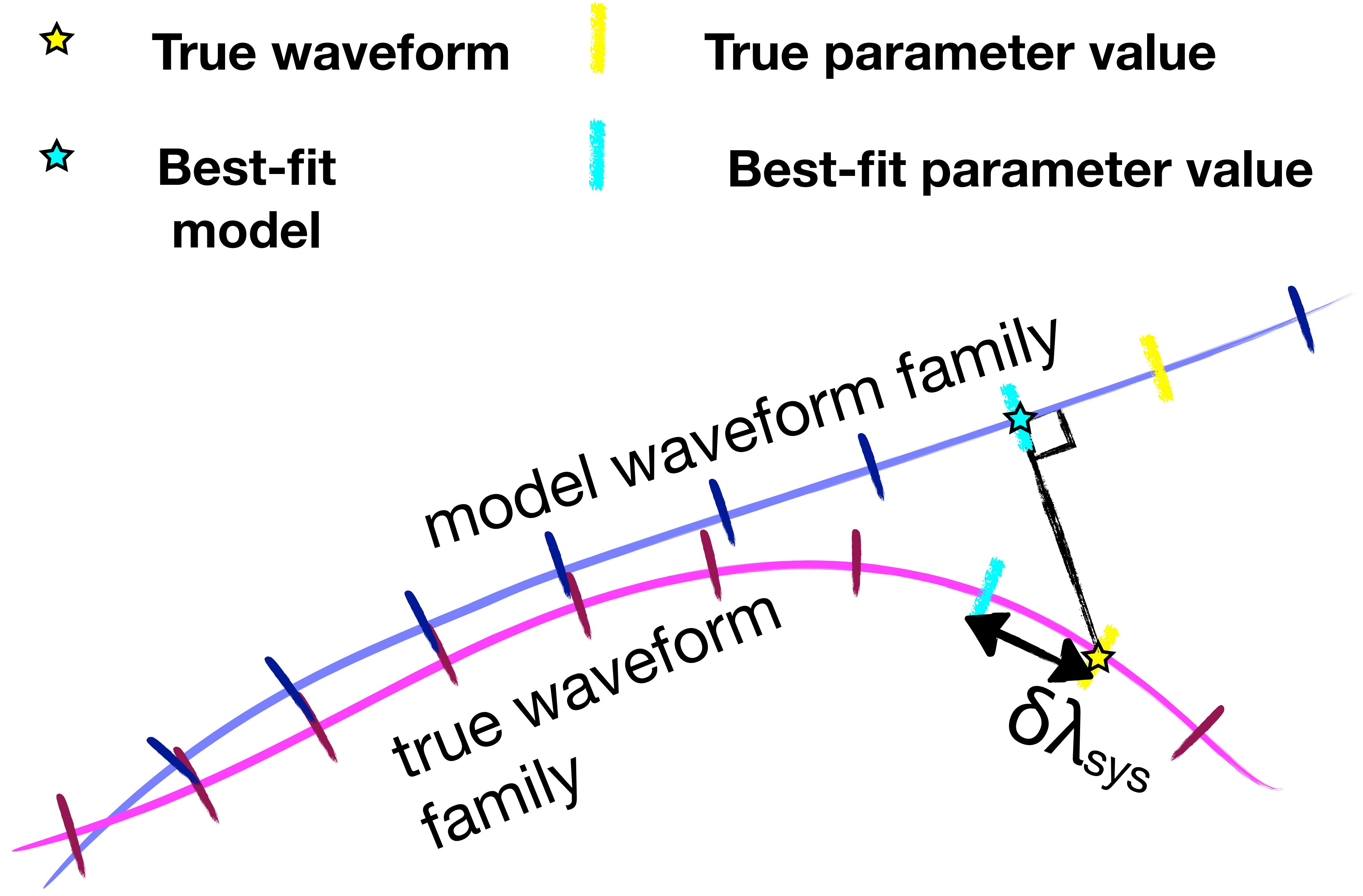
Detector Calibration

Source models

$$\mathscr{L}(d|\theta) \propto \exp\left(-\sum_k \frac{2\left|d_k - h_k(\theta)\right|}{S_k}\right)$$

Non-Gaussian noise distribution

Noise amplitude estimation

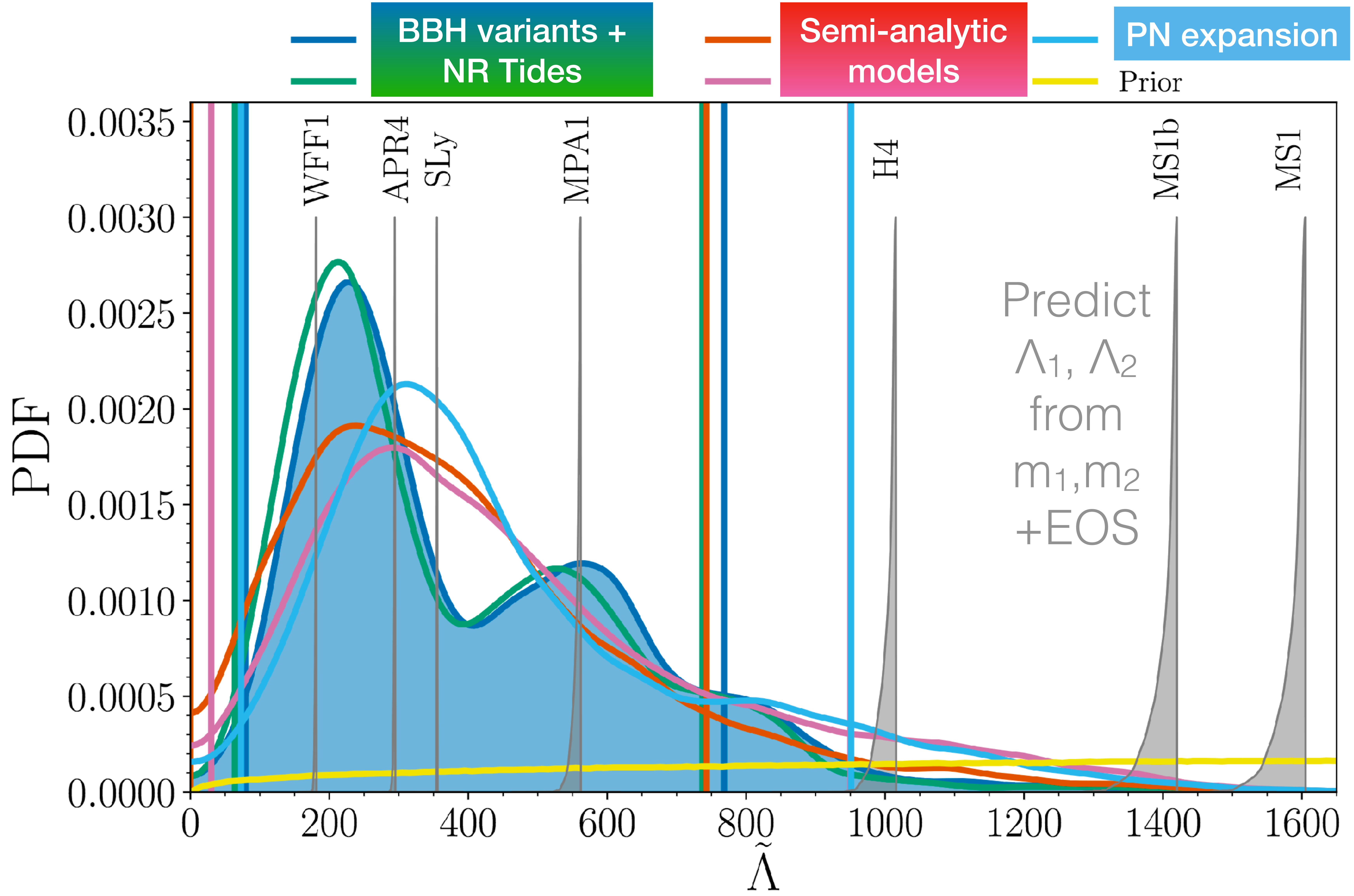


e.g. "glitches"
Sophie Hourihane et al Phys. Rev. D **106**, 042006, Chris Panków et al, Phys. Rev. D 98, 084016 (2018)

e.g. Sylvia Biscoveanu et al Phys. Rev. D 102, 023008 – Published 6 July 2020, Talbot and Thrane Phys. Rev. Research **2**, 043298

Waveform systematics:

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$



Assume low spin ($\chi < 0.05$)

Fiducial WF:
$\tilde{\Lambda} = 330^{+438}_{-251}$

Bars denote 90% highest probability density credible interval

LSC/Virgo GWTC-1 Phys. Rev. X 9, 031040 (2019)

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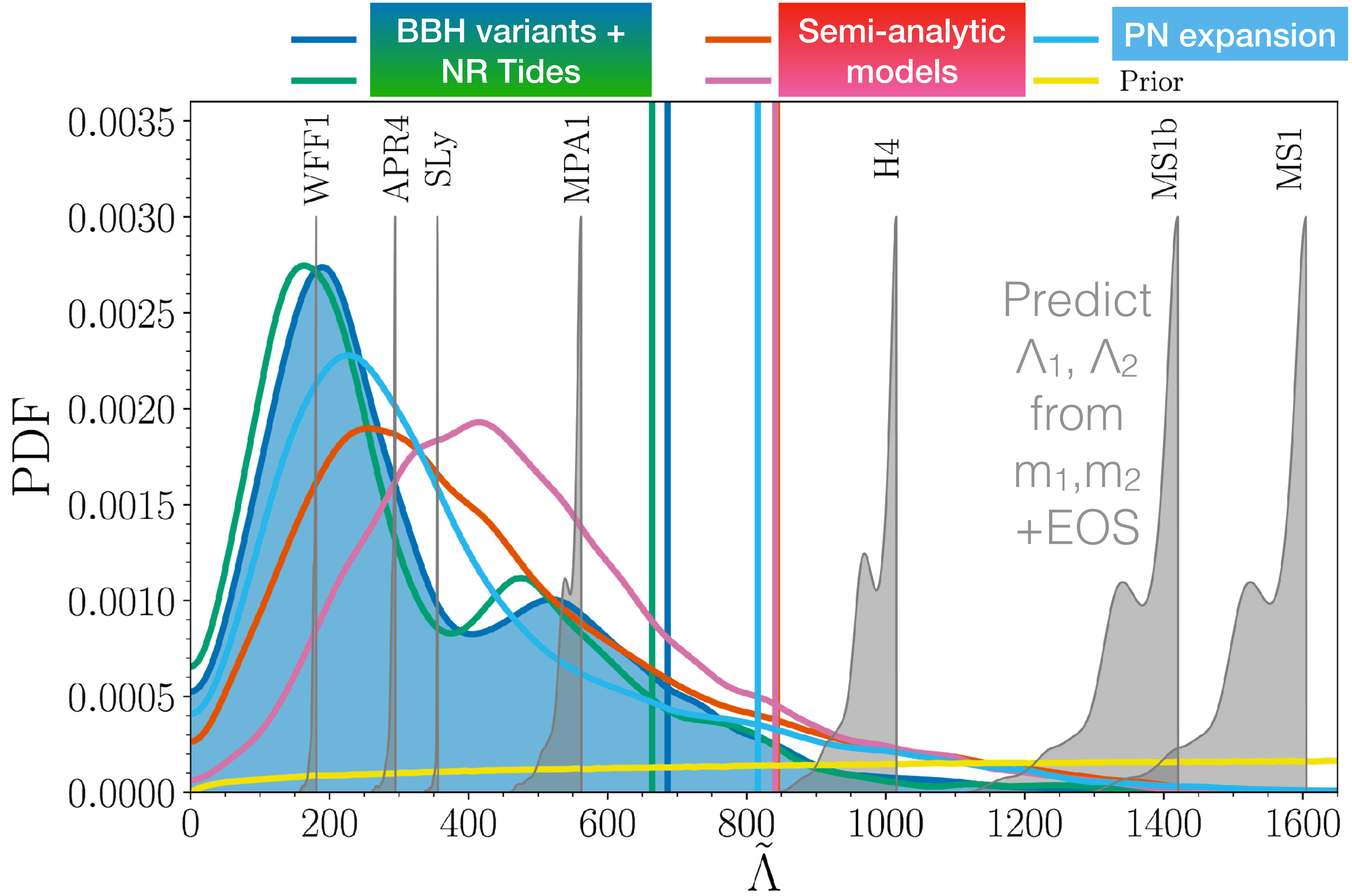

Fiducial WF:
$\tilde{\Lambda} < 686$

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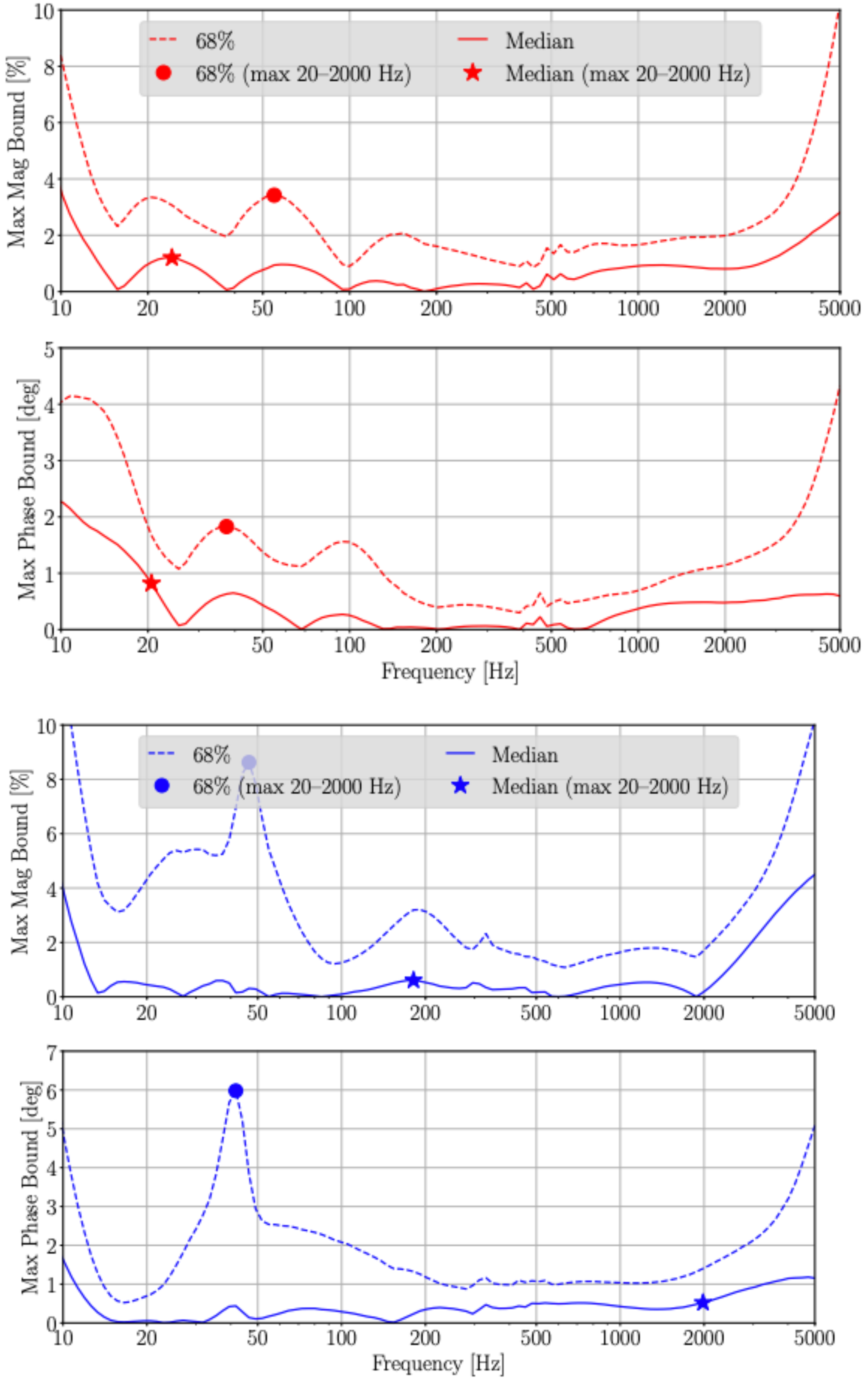
LSC/Virgo GWTC-1 Phys. Rev. X 9, 031040 (2019)

Model uncertainty: Calibration and marginalization



Sun et al 2020

- $h_{\text{obs}} = h(f)\big(1 + \delta A(f)\big) \exp\big(i\delta\phi(f)\big)$

- Calibration parameters: e.g. spline functions for $\delta A(f)$ and $\delta\phi(f)$ with priors calibrated to detector model e.g. Sun et al 2020 Class. Quantum Grav. **37** 225008

- Marginalize over calibration uncertainty during GW inference e.g. Vitale et al 2012 Phys. Rev. D **85**, 064034

Waveform error: source physics and the observed signal

From Source to Strain

- Source emission model: Multipole expansion

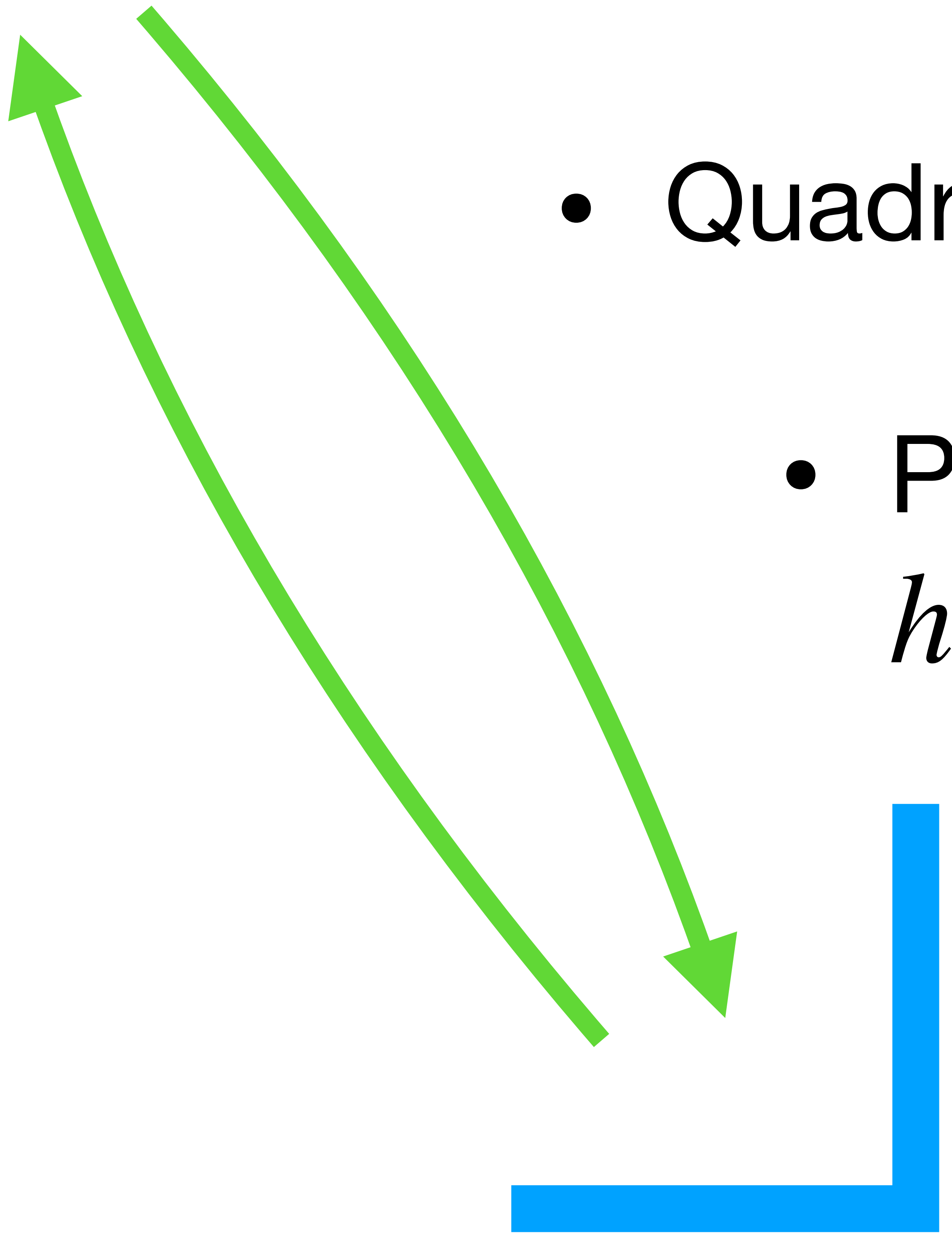
$$h_+(t) - ih_\times(t) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} h_{\ell m}(t) Y_{-2}^{\ell m}(\iota, \varphi)$$

- Quadrupole-dominant: $h_{22}(t) = \mathscr{A}(t)e^{i\psi(t)}$

Need **higher multipoles?**
SPA framework in Mezzasoma and Yunes 2022,
Hughes et al 2021

- Projected onto detectors:
$$h(t) = F_+(\alpha, \delta, \psi_p)h_+(t) + F_\times(\alpha, \delta, \psi_p)h_\times(t)$$

Sky location, orientation, inclination



- Resulting amplitudes: $h(t) = \dfrac{Q(\alpha, \delta, \iota, \psi)}{d_L}\mathscr{A}(t)e^{i\psi(t)}$

"Intrinsic properties"

Intrinsic physics of a GW signal

- Source model $h_{22}(t) = \mathscr{A}(t)e^{i\psi(t)}$ has **instantaneous frequency**: $2\pi F(t) = \dot{\psi}(t)$
- Source masses, spins, tides: **encoded in characteristic functions** of F:

$$\mathscr{A}(F) \equiv \mathscr{A}(T(F)) \quad \text{and} \quad \dot{F}(F) \equiv dF/dT$$

- Signal $\tilde{h}(f) = A(f)e^{i\phi(f)}$ from $\mathscr{A}$, ϕ_c, t_c and integration of dT/dF

$$A(f) = Q(\boldsymbol{\theta}_{\text{ext}})\sqrt{T'(f)}\mathscr{A}(f)$$

$$\phi(f) = \frac{\pi}{4} + \psi(f) - 2\pi f T(f) = \phi_c - 2\pi f t_c + 2\pi \int_f^{f_c} d\tilde{f} \int_{\tilde{f}}^{f_c} \frac{dF}{\dot{F}(F)} \text{s}$$

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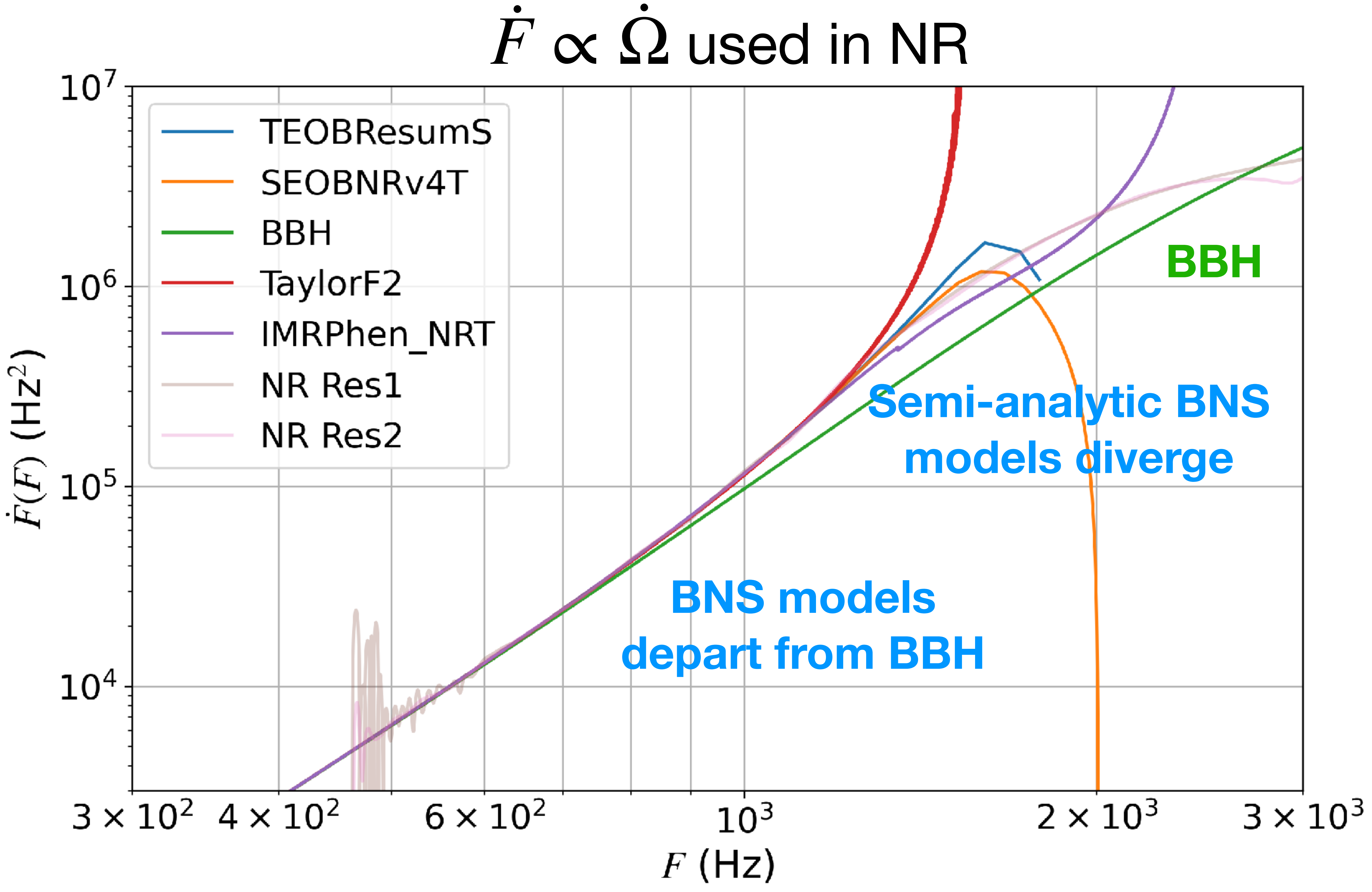
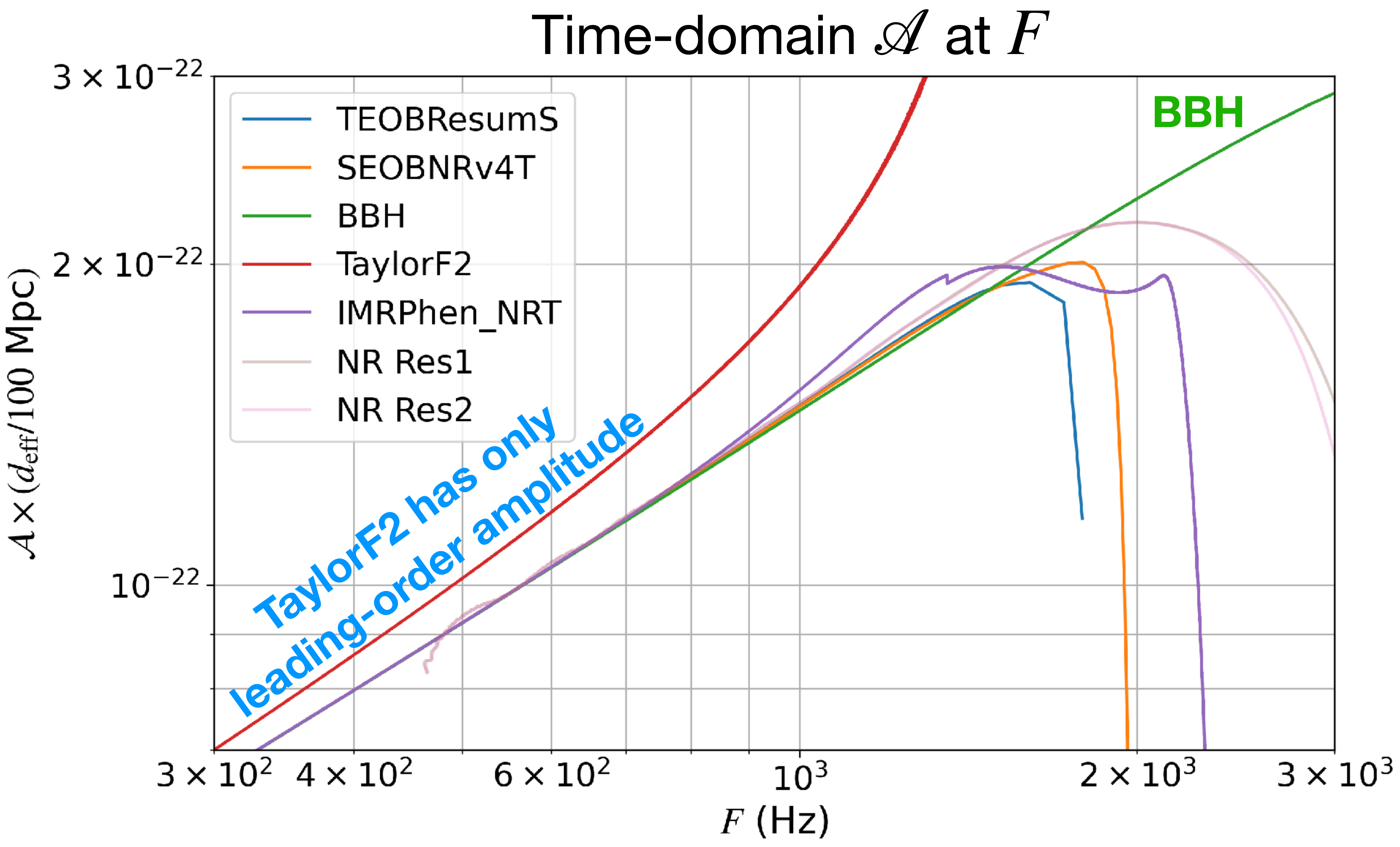
Where do we model the physics well?





NR - high-res CoRe sim 'BAM:0095' with SLy EOS
Spline smoothing for F before taking derivative
m_1 & m_2: 1.349998 for all waveforms shown

From Sly: Λ_1 & Λ_2 = 390.1104
used for TEOBResumS, SEOBNRv4T, TaylorF2, and IMRPhenomPv2_NRT

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Energetics Interpretation

Luminosity and $\mathscr{A}$:

$$\mathscr{A}(F)^2 = \frac{4}{\pi}\frac{1}{d^2}\frac{1}{F^2}\mathscr{L}_{\text{GW}}(F)$$ from integration of $\left|\dot{h}_{\ell m} d\right|^2$

Energy balance and $\dot{F}$:

$$\dot{F}(F) = -\frac{\mathscr{L}(F)}{E'(F)}$$ from system energy as function of emission frequency

Sources of modification: Variation of $E(F)$ or $\mathscr{L}(F)$ from GR source properties, plus

- Additional luminosity $\mathscr{L}(F)$: non-GW energy loss $\mathscr{L}_{MM}$ or $\mathscr{L}_{NR}$
- Internal energy transfers $\delta E_A, \delta E_D$ that modify how E changes with F:

  $$\delta E' = \delta E_A + \frac{t_A}{t_D}\delta E_D$$ (A adiabatic, D dynamic, t timescales)

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Differences between BBH and BNS

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Energy transfers and the Fourier signal

- If there are small, linearizable corrections to the model used for PE:

$$\delta A(f) = \frac{1}{2}\left(\delta E' + \delta \mathscr{L}_{\mathrm{GW}} - \delta \mathscr{L}_{\mathrm{MM}}\right)$$

$$\delta\phi(f) = 2\pi \int_{f}^{f_c} d\tilde{f} \int_{\tilde{f}}^{f_c} dF\, T'(F)\left(\delta E' - \delta \mathscr{L}_{GW} - \delta \mathscr{L}_{MM}\right)$$

- **Applications:**
  - Generically limit unmodeled energy transfers *(not in PE waveform)* in observed systems through constraints on $\delta A, \delta\phi$.
  - Given a model of astrophysical energy transfer (like a resonant mode), can imprint on *any* underlying waveform model

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Example: Coherent waveform recovery

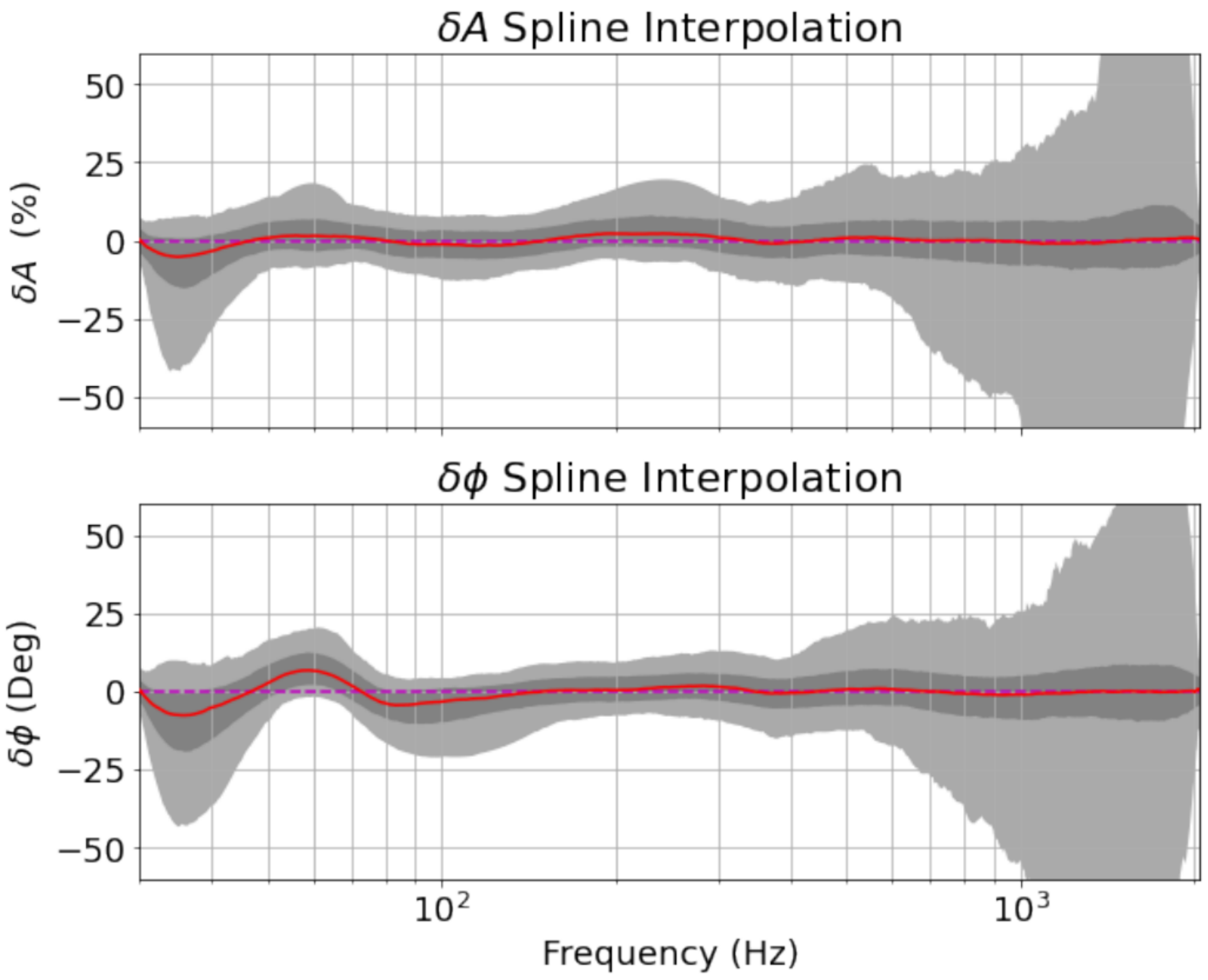


FIG. 12. Spline interpolation of GW170817 with 1 and 2 σ credible intervals (grey) and the median spline interpolant (red) shown.

- Edelman et al Phys. Rev. D 103, 042004 (2021): Constraint on coherent departures from waveform model

- Generic signal modification described with splines for $\delta A, \delta\phi$, constraint for GWTC-1

- **Interpretation with waveform energetics: GW170817** phase shift $\delta\phi \sim$ 5 deg at 60 Hz is compatible with a **resonant energy transfer** of $\delta E = \Delta E/E \sim 0.001$ relative to the orbital energy $E(60 \text{ Hz}) \simeq -0.006 M_\odot c^2$ (Scale of $\Delta E \sim 10^{49}$ erg)

What accuracy do current models give? and do future detectors need?

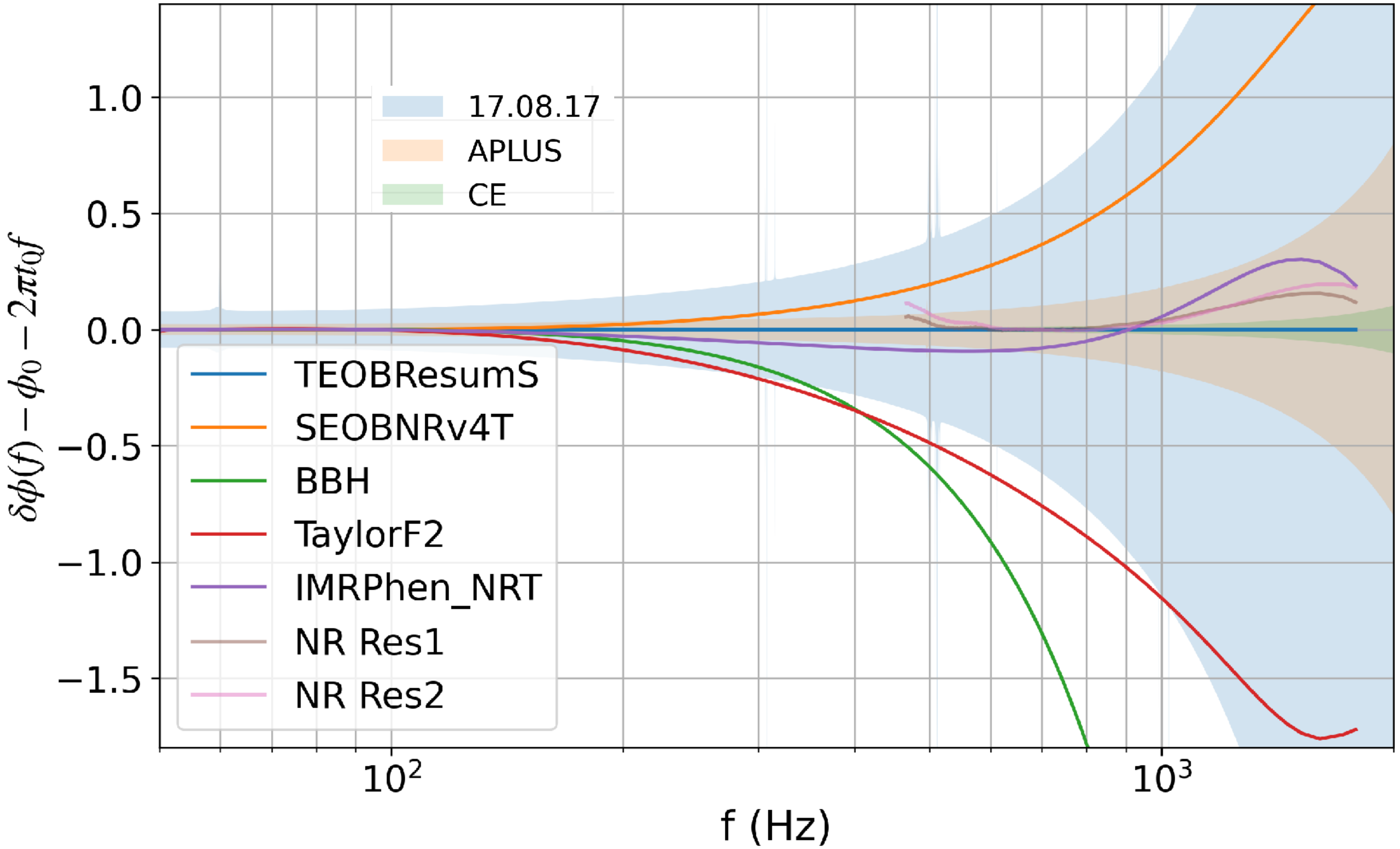
Indistinguishability

- Two waveform models have waveform difference $\delta \boldsymbol{h} = \boldsymbol{h}_1 - \boldsymbol{h}_2$
- "Indistinguishable" if $\langle \delta \boldsymbol{h} \,|\, \delta \boldsymbol{h} \rangle < 1$ "less than noise"
- $h_{\text{model}}(f) = h_{\text{true}}(f)\Big(1 + \delta A(f)\Big) \exp\Big(i\delta\phi(f)\Big)$
  - $\delta \boldsymbol{h}$ from δA is $A(f)\delta A(f)$
  - $\delta \boldsymbol{h}$ from $\delta \phi$ via $A(f)\bigg(1 - \exp\Big(i\delta\phi(f)\Big)\bigg)$
- In waveforms, distinguishability is historically assessed via *mismatch*

Waveform phase impact

- Assessing mismatch *only* - an integrated quantity - obscures the frequency dependence of waveform difference & comparison to $S_n(f)$
- Phase differences of $\phi_0 + 2\pi f t_0$ absorbed by marginalization over time and phase ϕ_c, t_c
- Here: compare **residual phase** $\delta\phi_{\rm res}$ without max likelihood fit $\phi_0 + 2\pi f t_0$ (weight by variance $S_n(f)/A(f)^2$)

$\delta\phi_{\rm res}$ relative to TEOBResumS



NR waveforms: relative to 700 Hz for reference (not long enough to fit ϕ_0, t_0)

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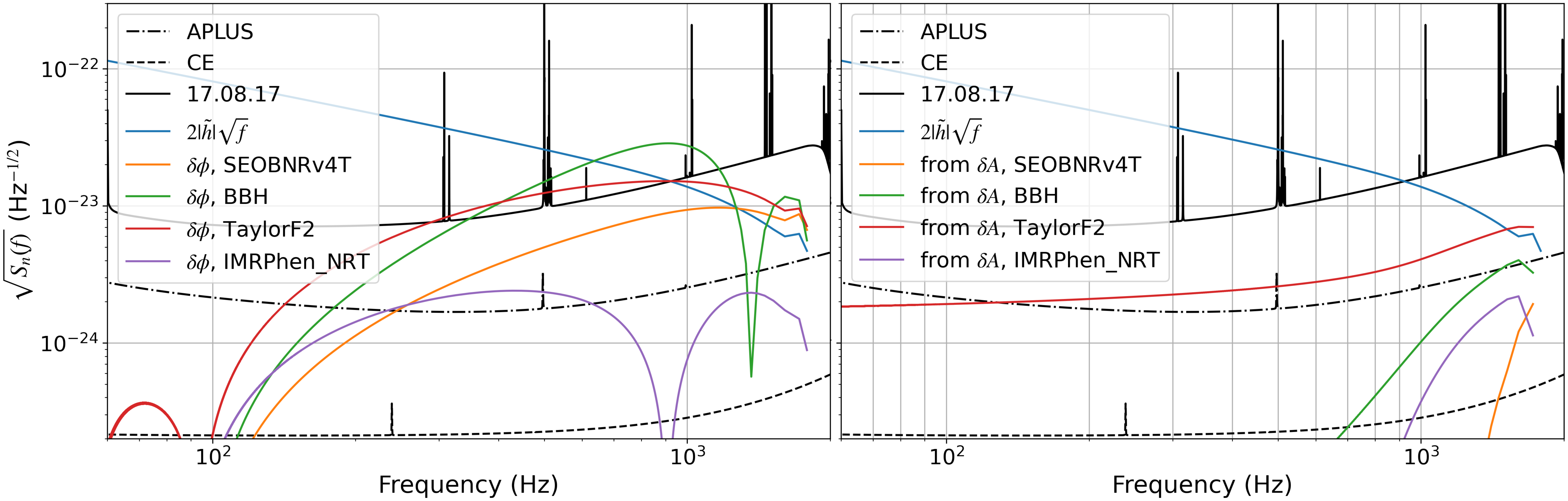
Impact of model differences

Characteristic strain $4f\tilde{h}(f)$ from $\delta\phi$ and δA can be compared to $S_n(f)$

Below: $\delta \boldsymbol{h}$ of waveform models for a 1.35-1.35 $M_\odot$ BNS relative to TEOBResumS for source at 100 Mpc



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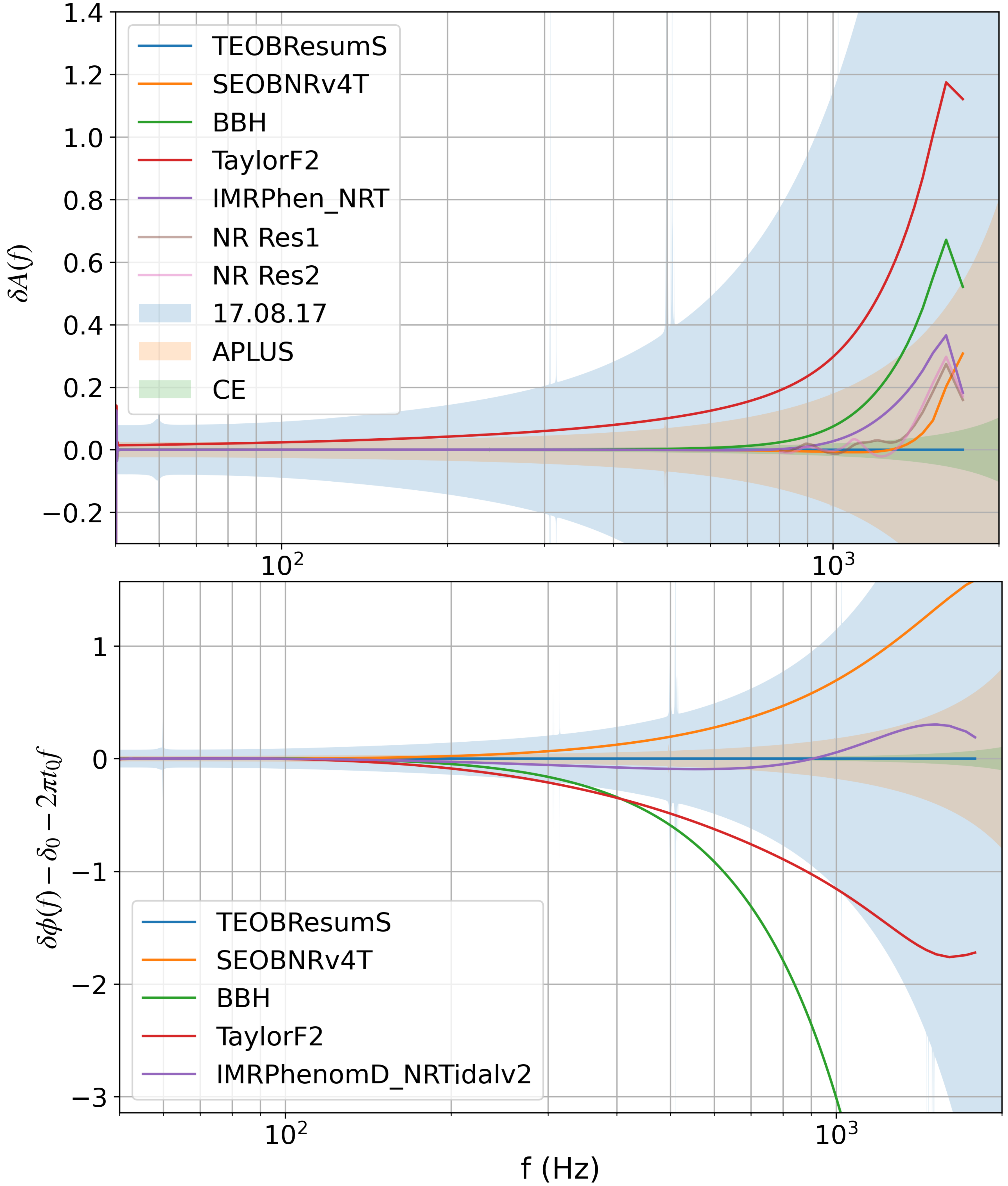
Small-angle approximation

- Indistinguishability condition from characteristic strain: $2\sqrt{f}\,|\,\delta\tilde{h}(f)\,| < \sqrt{S_n(f)}$ sets shaded regions for reference detectors, signal $d_{\text{eff}} = 100$ Mpc

- Waveform differences that remain in the shaded regions should be indistinguishable in that detector



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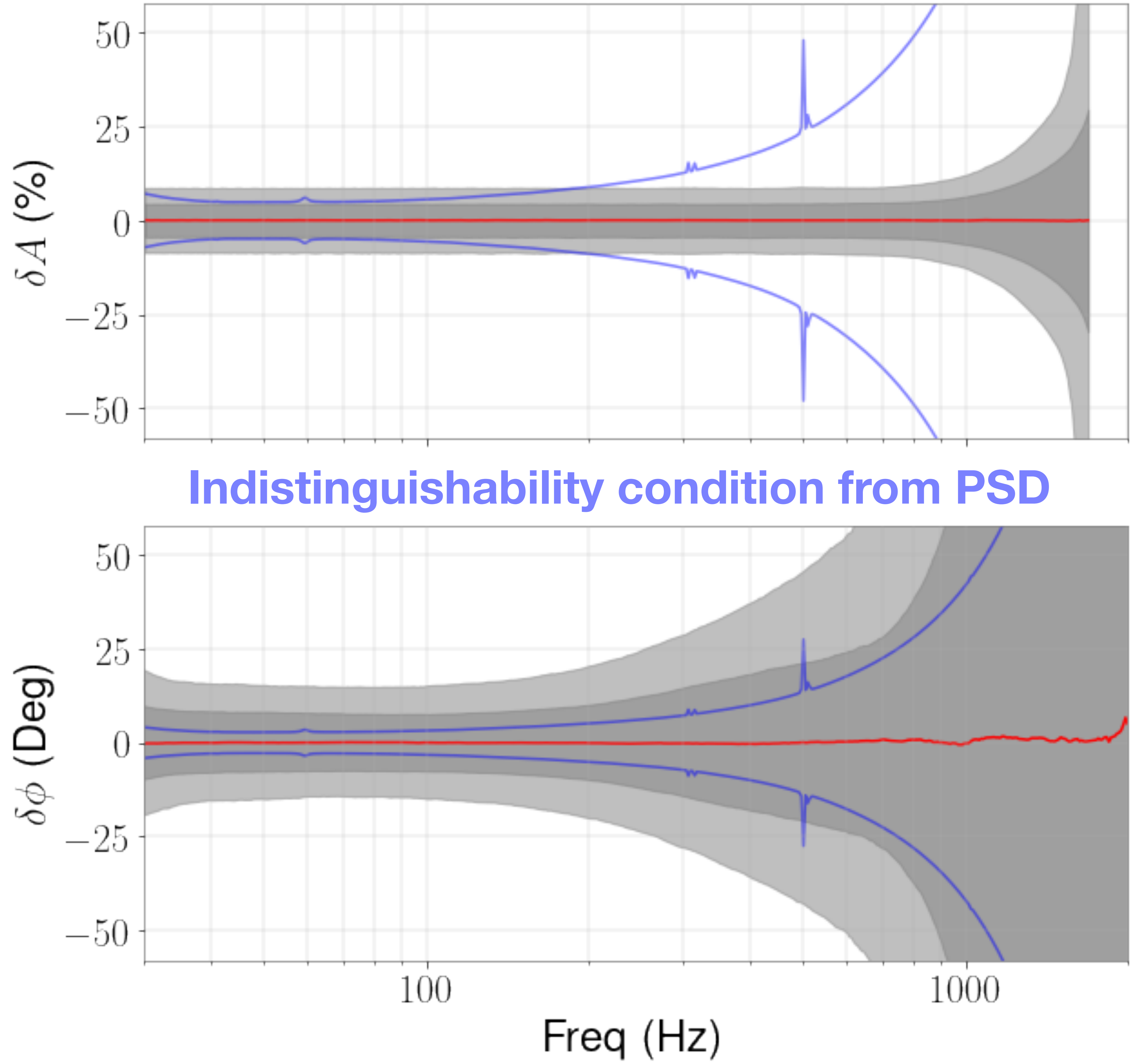
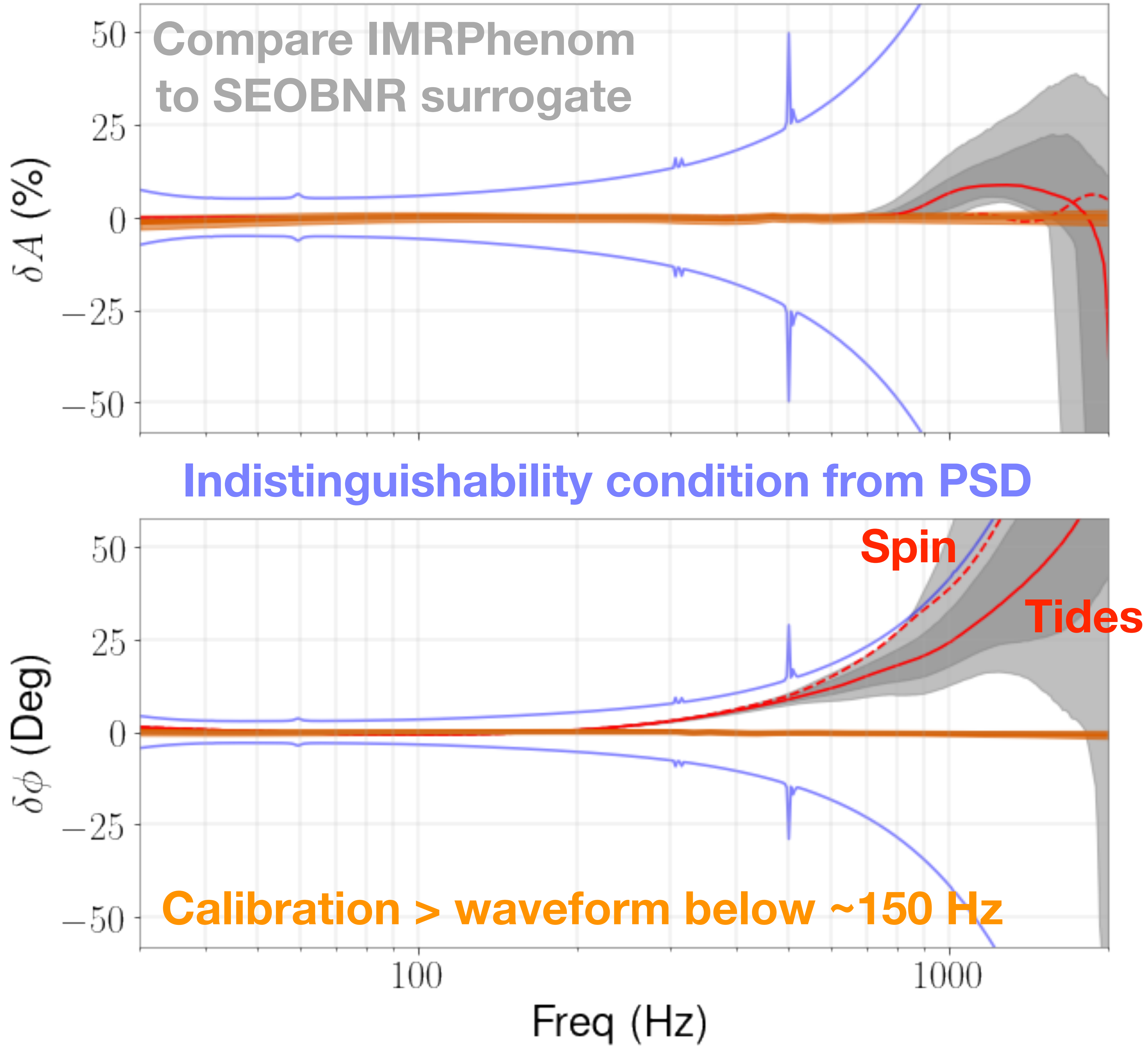
Statistical/systematic: GW170817

Beyond the characteristic strain estimate

Random draws from posterior distributions



Calibration envelopes and waveform systematics



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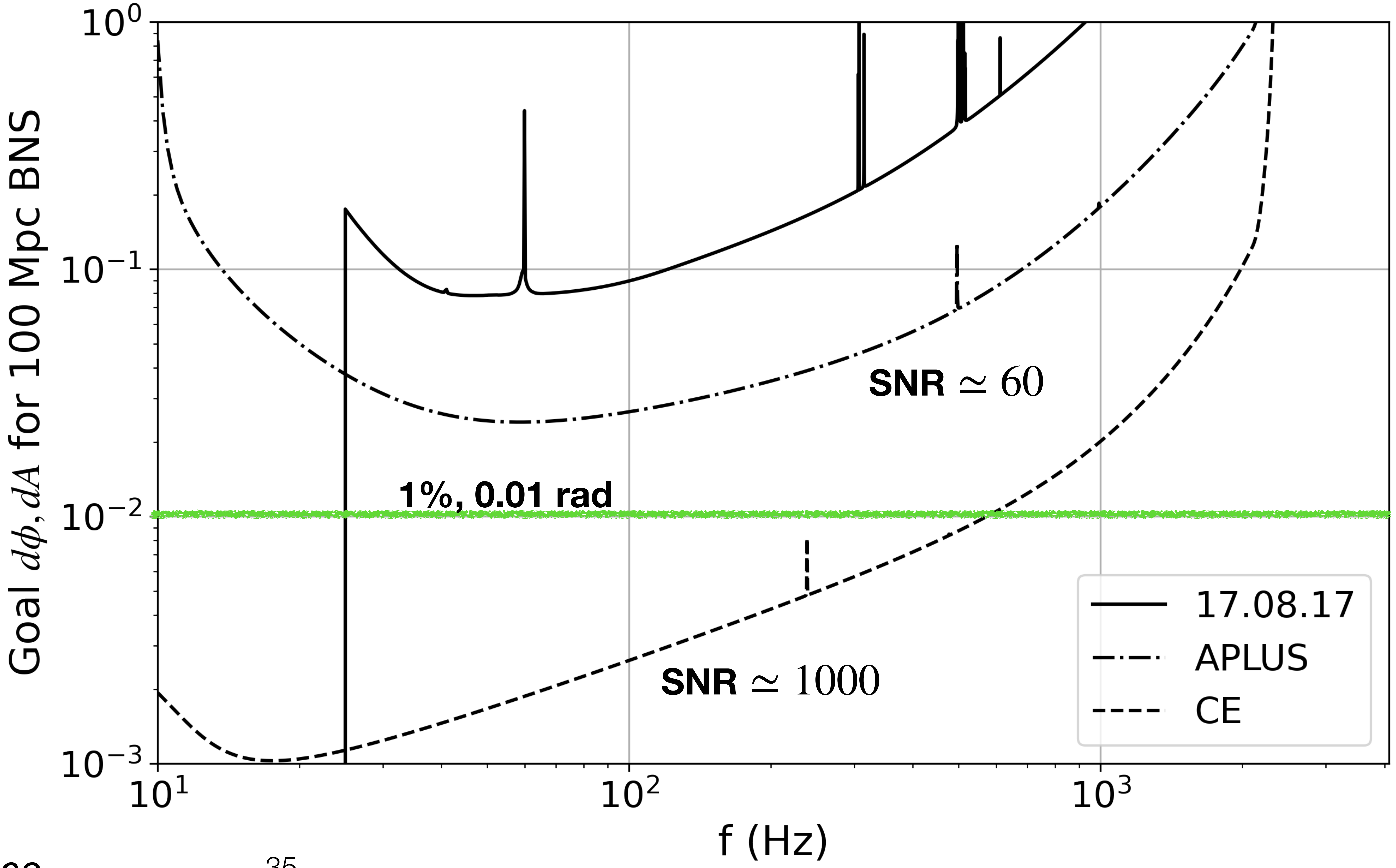
Goals for calibration & waveforms

- Frame: Model error has potential to impact source analysis if $\delta h = h_{\text{true}} - h_{\text{model}}$ generates **characteristic strain larger than detector noise** at a given frequency
- Goal δA (fractional) and $\delta\phi$ (radians) shown
- 'Model' of detector (calibration) or source (waveform)
- ***PE Goal: Use the data to inform the analysis***

$$h_{\text{model}}(f) = h_{\text{true}}(f)\left(1 + \delta A(f)\right) \exp\left(i\delta\phi(f)\right)$$



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Thank you!

Model consistency: amplitude

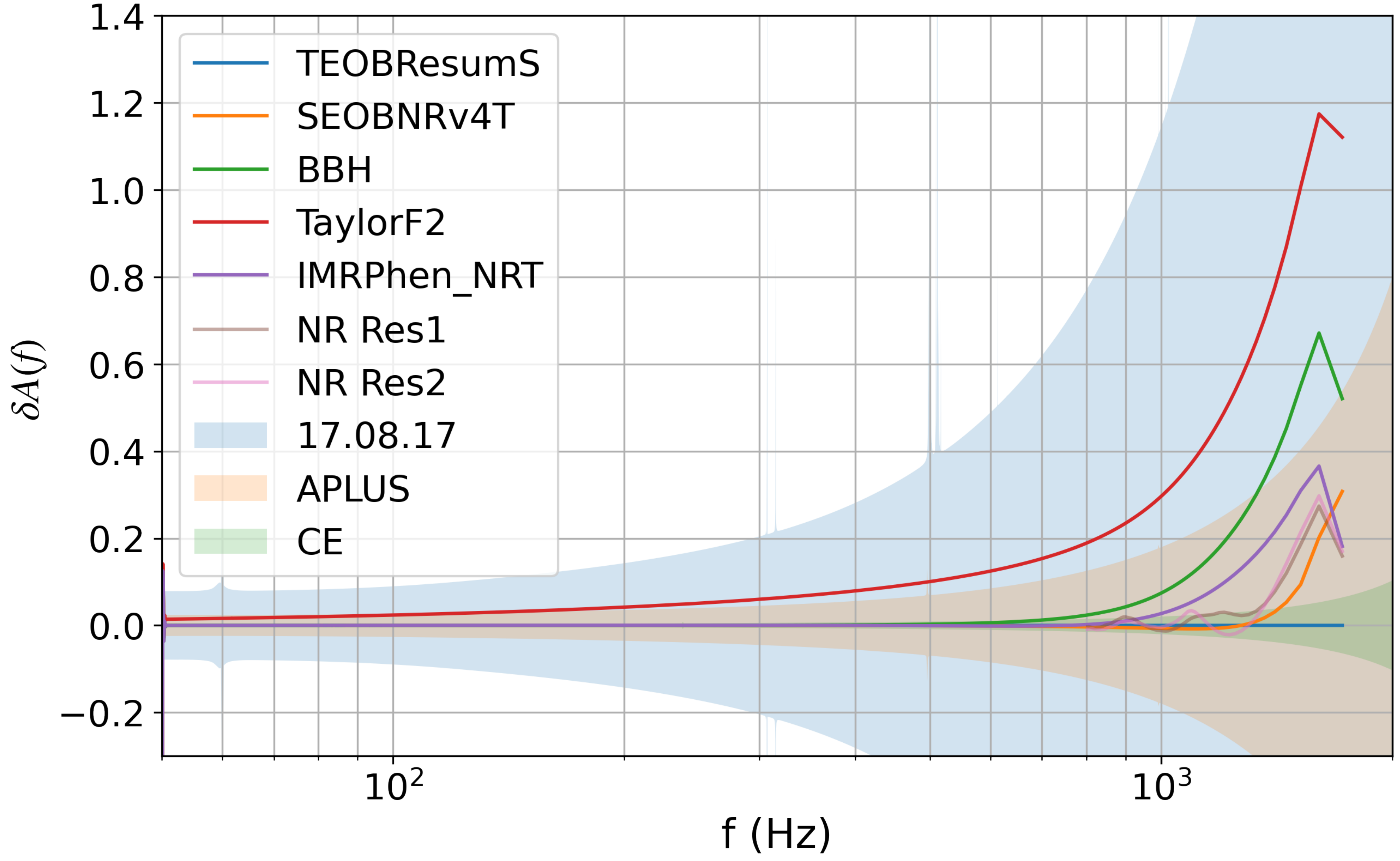
- $\tilde{h}(f) = A(f)e^{i\phi(f)}$
- in SPA:

$$A(f) = Q(\boldsymbol{\theta}_{\mathrm{ext}})\left(T'(f)\right)^{1/2}\mathscr{A}(f)$$

$$\delta A(f) = \frac{A(f) - A_{\mathrm{ref}}(f)}{A_{\mathrm{ref}}(f)}$$

- PSD reference: condition for indistinguishability at 100 Mpc

δA relative to TEOBResumS



- TaylorF2 only has leading order amplitude
- Other differences in amplitude are small

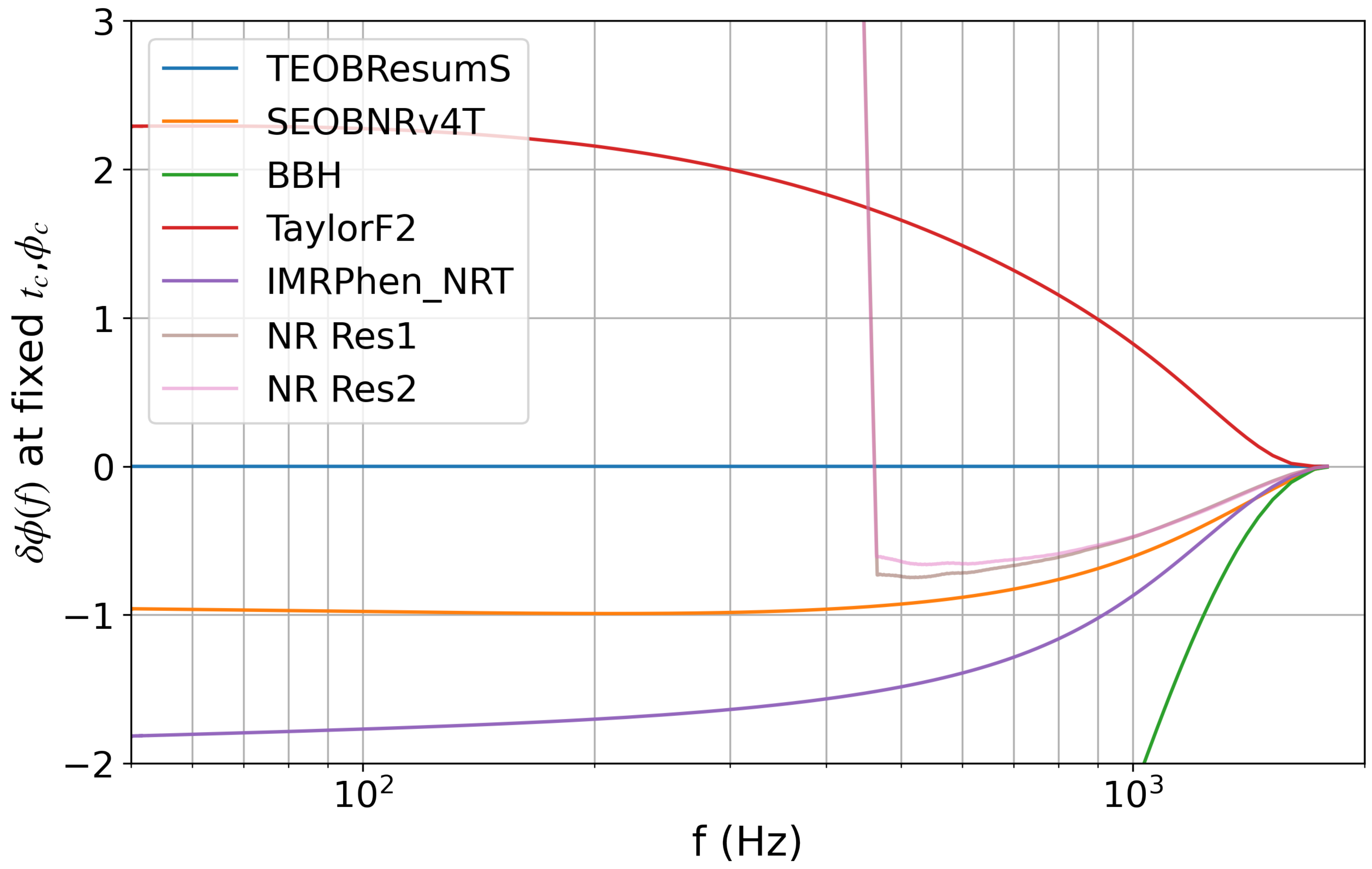
Model consistency: phase

- Relative to **fixed** f_c of coalescence (same t_c and ϕ_c for all signals)

- in SPA:
$$\phi(f) = \frac{\pi}{4} + \psi(f) - 2\pi f T(f)$$
$$= \phi_c - 2\pi f t_c + 2\pi \int_f^{f_c} d\tilde{f} \int_{\tilde{f}}^{f_c} dF\, T'(F)$$

- $\delta\phi(f) = \phi(f) - \phi_{\text{ref}}(f)$

- Can compute for NR from time and phase accumulation f to f_c



- Intrinsic model error compared to other models or numerical simulation relative to TEOB f_c
- other WF models' f_c would give different alignment and potentially better NR/model agreement

SPA validity conditions