Thermal Effects in Nuclear Structure and Weak Decays

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Hot nuclei and how to describe them

Mean-field theory at finite temperature

Start from grand-canonical potential:

$$\Omega = E - TS - \sum_q \lambda_q N_q$$

$$E = \mathrm{Tr}[\hat{D}\hat{H}]$$

internal energy

$$S = -k_B \mathrm{Tr}[\hat{D}\mathrm{lnD}]$$

entropy

$$N = \mathrm{Tr}[\hat{D}\hat{N}]$$

particle number

We consider a two-body Hamiltonian:

$$\hat{H} = \sum_{ij} t_{ij} c_i^\dagger c_j + \frac{1}{4} \sum_{ijkl} \bar{v}_{ijkl} c_i^\dagger c_j^\dagger c_l c_k$$

And density operator of independent particles:

$$\hat{D}_{\mathrm{MF}} = \prod_i [f_i \hat{n}_i + (1 - f_i)(1 - \hat{n}_i)] \qquad f_i = [1 + \exp[\beta(\varepsilon_i - \lambda_q)]]^{-1}]] \qquad \hat{n}_i = c_i^\dagger c_i$$

Finite-temperature Hartree-Fock equations are obtained by minimizing $\delta_\rho \Omega$

$$\delta_\rho \Omega = \mathrm{Tr}[(h - \varepsilon)\delta\rho] = 0$$

Mean-field Hamiltonian: $h = t + \sum_{kl} \bar{v}_{ikjl}\rho_{lk} - \lambda_q$

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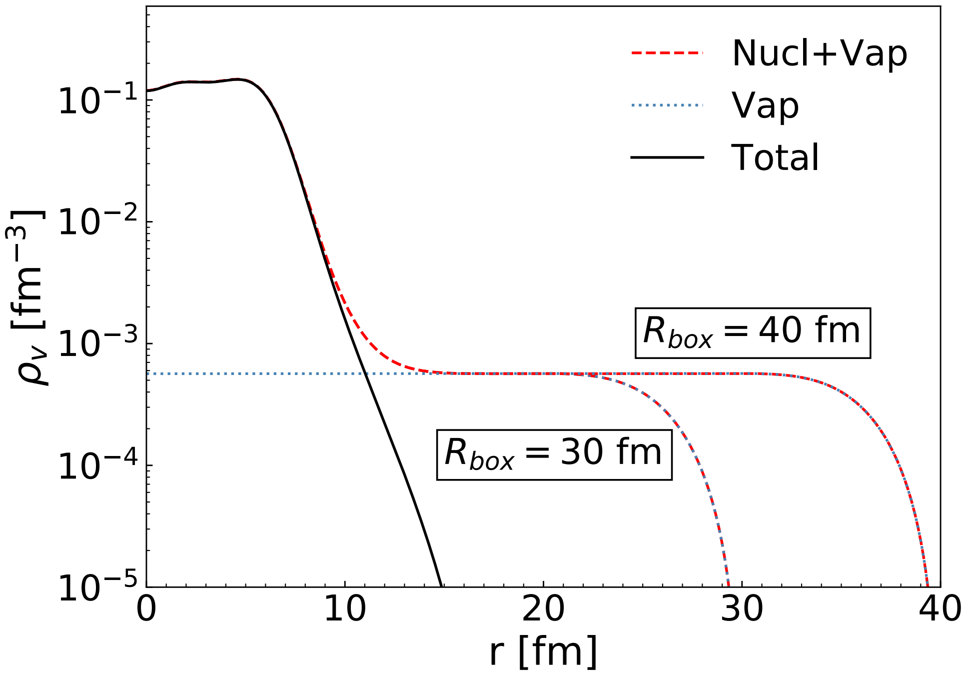
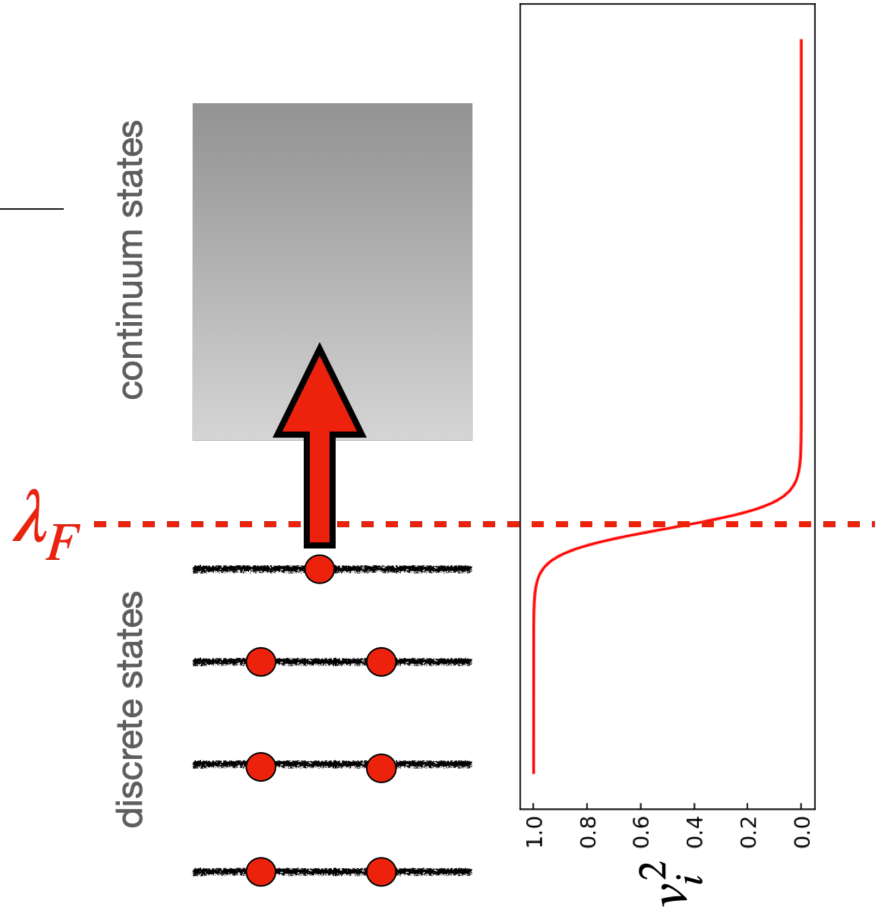


T = 1 MeV for ^{202}Sm



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Mean-field Hamiltonian: $h = t + \sum_{kl} \bar{v}_{ikjl}\rho_{lk} - \lambda_q$

Minimization should be performed with the subtracted grand-canonical potential:

$$\Delta\Omega = \Omega(\rho) - \Omega(\tilde{\rho})$$

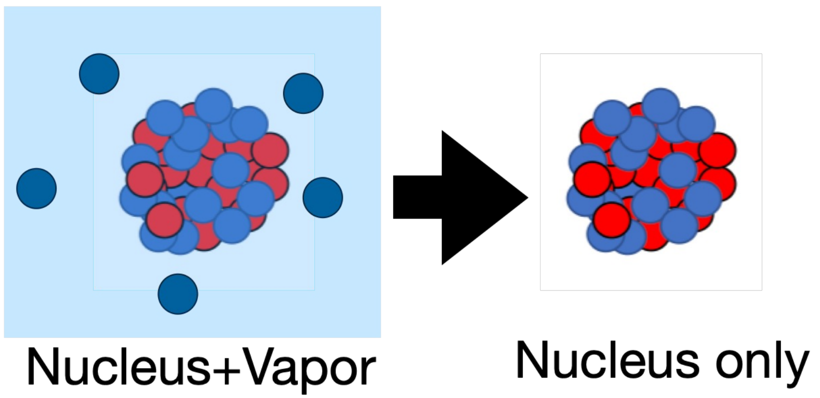
And we obtain equations for two systems

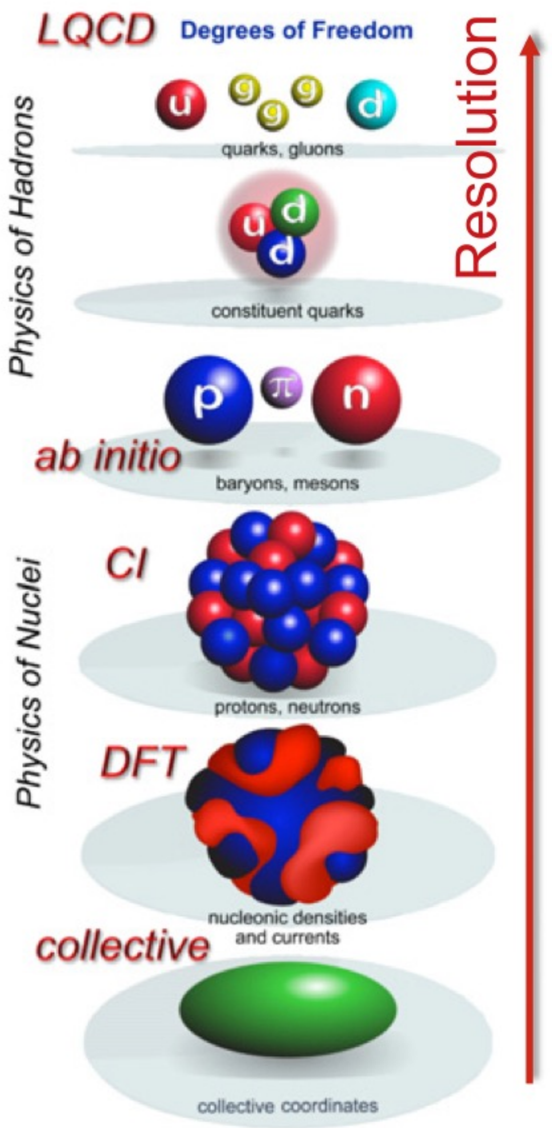
$\delta_\rho \Delta\Omega = 0 \longrightarrow$ Nucleus + Vapor System

$\delta_{\tilde{\rho}} \Delta\Omega = 0 \longrightarrow$ Vapor System

[P. Bonche, S. Levit and D. Vautherin, NPA 427, 278 (1984)]

Treating the continuum:





Hot QCD
Cold QCD

Hadron-Nuclear interface

Nuclear structure
Nuclear reactions

Nuclear astrophysics
New standard model

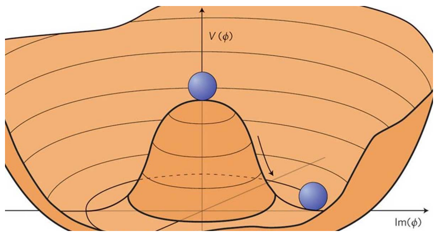
Applications of nuclear science

Nuclear DFT

- two fermi liquids
- self-bound
- superfluid
- breaks translational inv.

- mean-field $\Rightarrow$ one-body densities
- zero-range $\Rightarrow$ local densities
- finite-range $\Rightarrow$ gradient terms
- particle-hole and pairing channels
- A broken-symmetry generalized product state does surprisingly good job for nuclei.

[Inference Review]



W. Nazarewicz, J. Phys. G: Nucl. Part. Phys. 43, 044002 (2016)

[Slide by W. Nazarewicz]

Practical calculations

Interactions that I employ are covariant:

$$[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+\beta M^{*}+V]\psi_i=\varepsilon_i\psi_i$$

scalar field — vector field

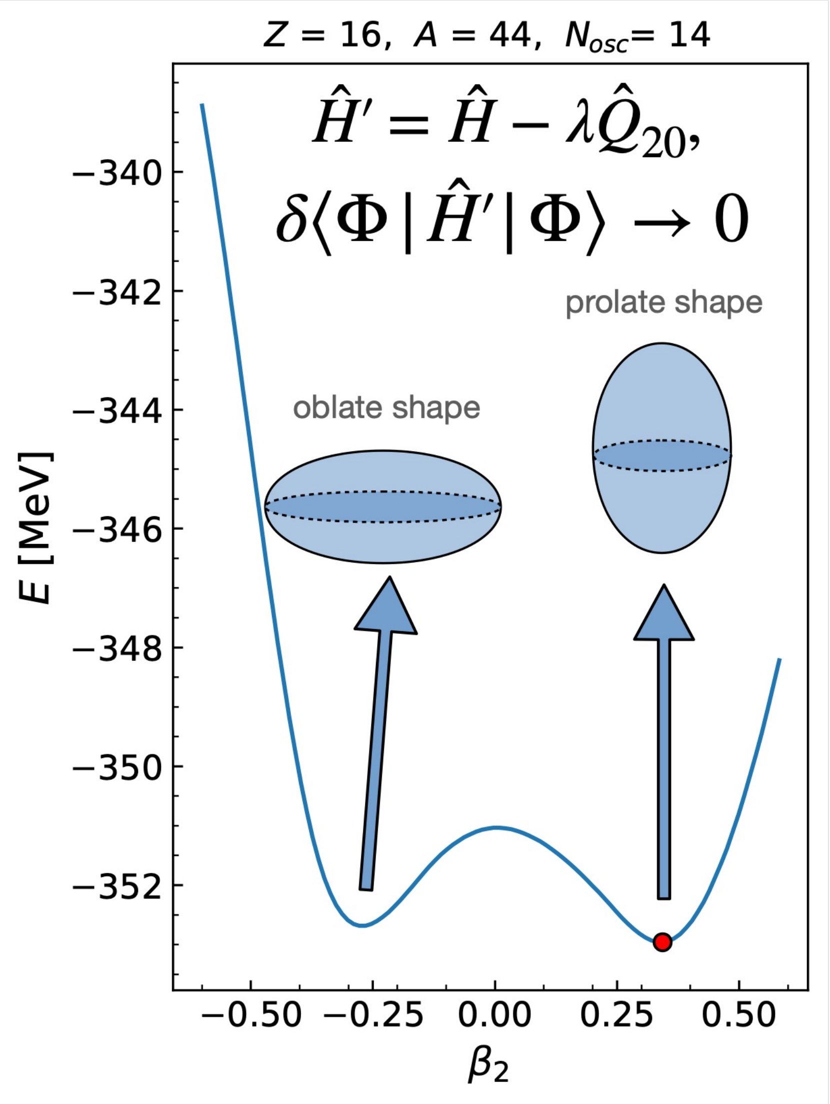
Pairing is treated within the Relativistic-Hartree Bogoliubov theory:

pairing field

$$\begin{pmatrix} h-\lambda & \Delta \\ -\Delta^{*} & -h-\lambda \end{pmatrix}\begin{pmatrix} U \\ V \end{pmatrix}=E\begin{pmatrix} U \\ V \end{pmatrix}$$

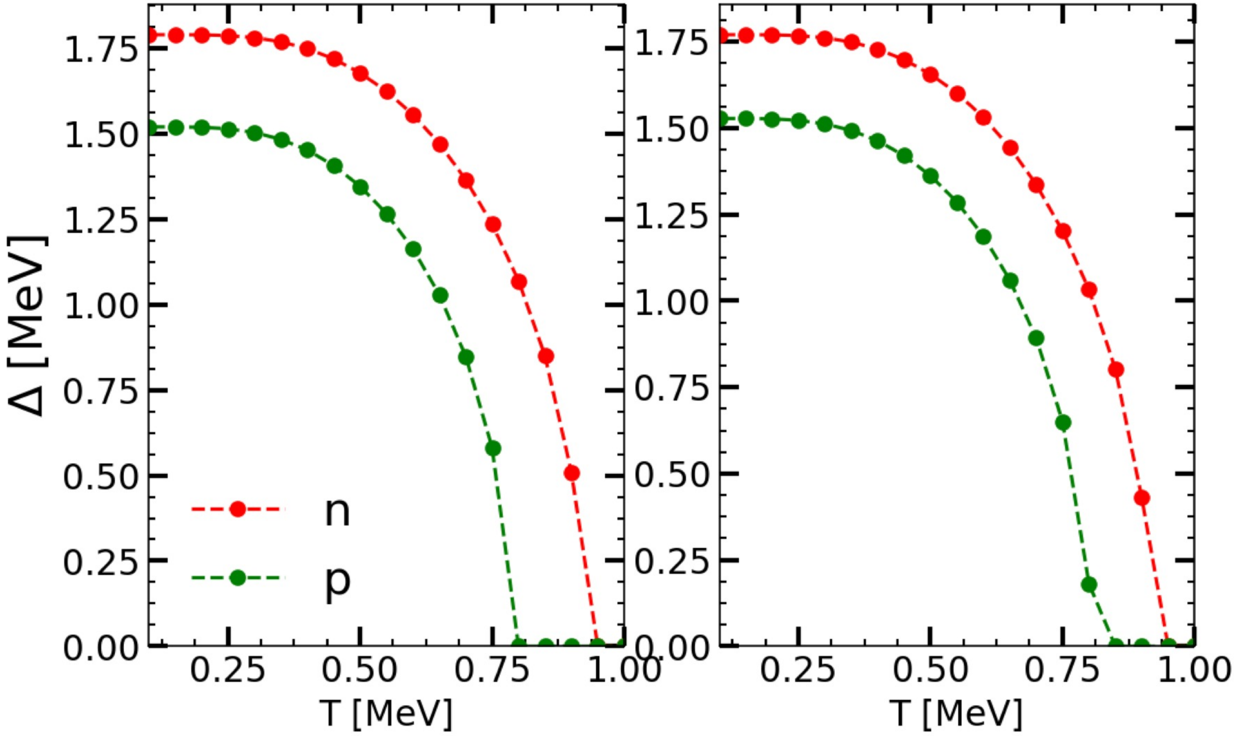
Nuclear shape in the intrinsic frame is determined through constrained Lagrangian calculations

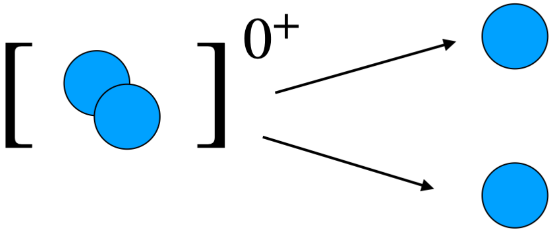
Potential energy surface (PES):



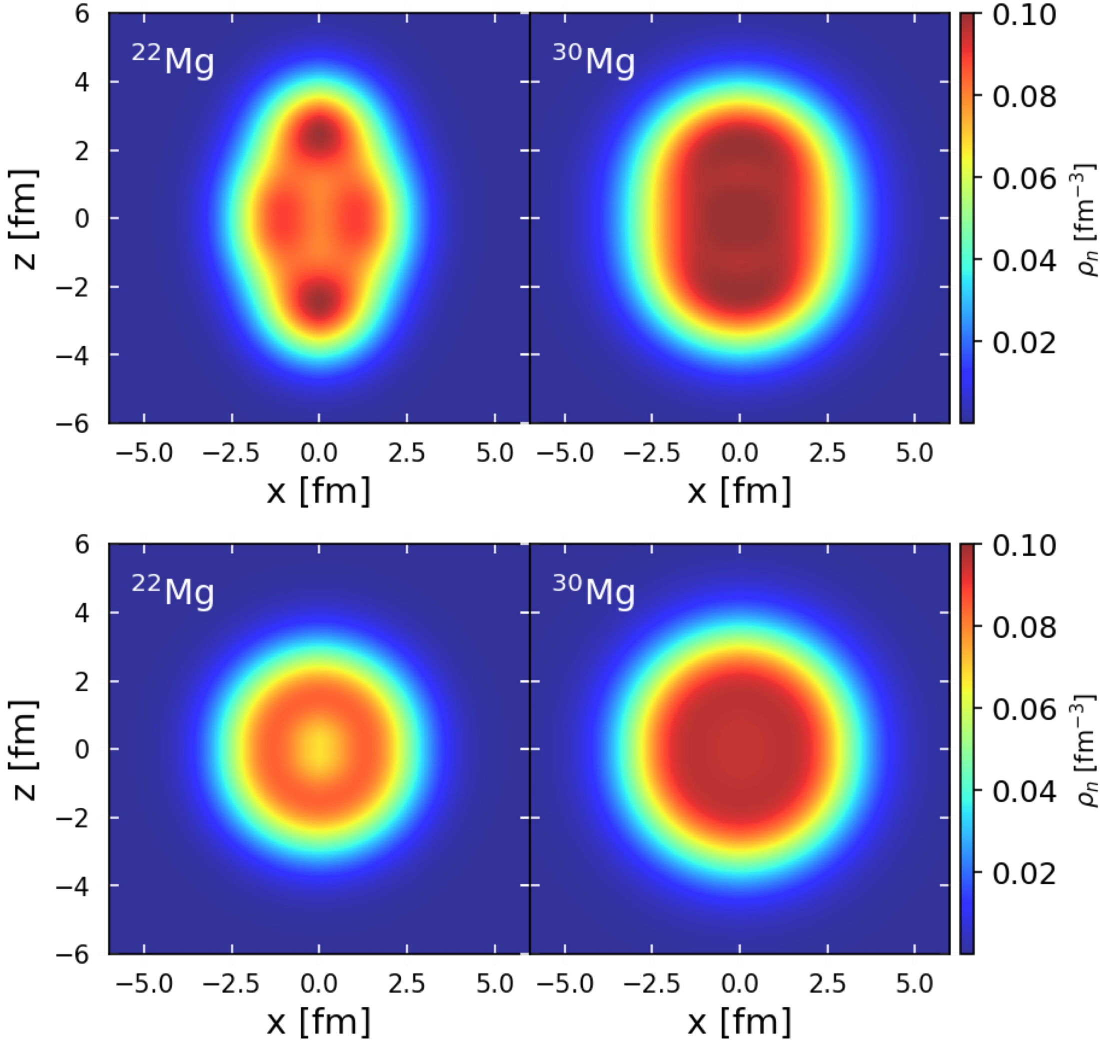
Nucleus at finite temperature

Pairing collapses (superfluid → normal):

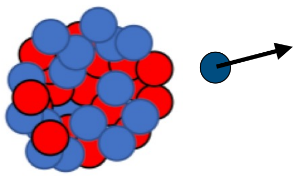




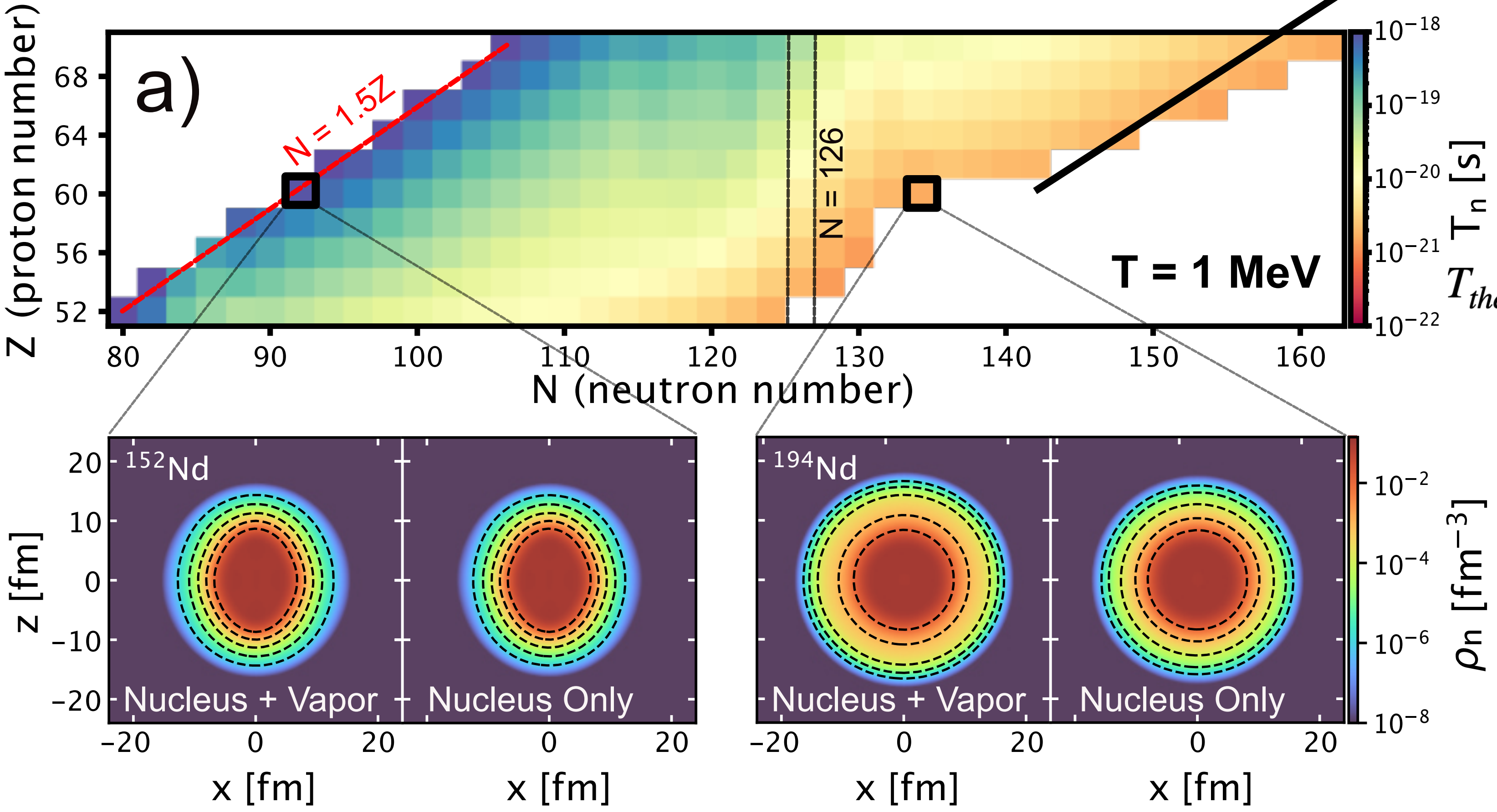
Shape collapses to spherical configurations:



Neutron emission lifetimes:



continuum treatment important for drip-line nuclei



$T_{thermalization} \sim 10^{-22}$ s

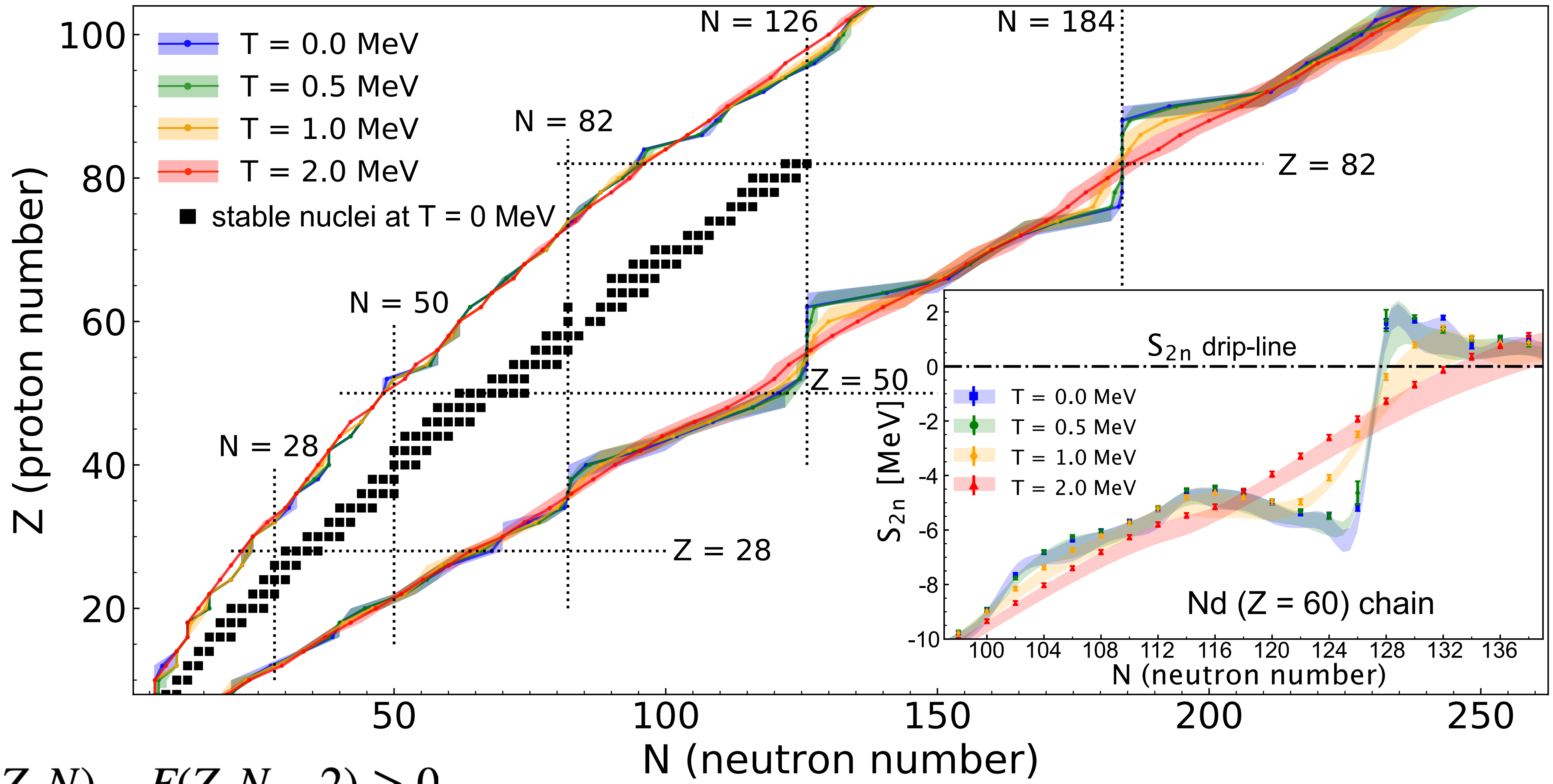
$$\Gamma_n/\hbar = n_{gas}\langle\sigma v\rangle$$

density of neutron "gas"

[AR, E. Yüksel, T. Nikšić and N. Paar, Nat. Commun. 14, 4834 (2023).]

[AR, E. Yüksel, T. Nikšić and N. Paar, Phys. Rev. C 109, 014318 (2024).]

What happens with drip-lines as the temperature is increased?



$$S_{2n} = F(Z,N) - F(Z,N-2) \geq 0$$

$$S_{2p} = F(Z,N) - F(Z-2,N) \geq 0$$

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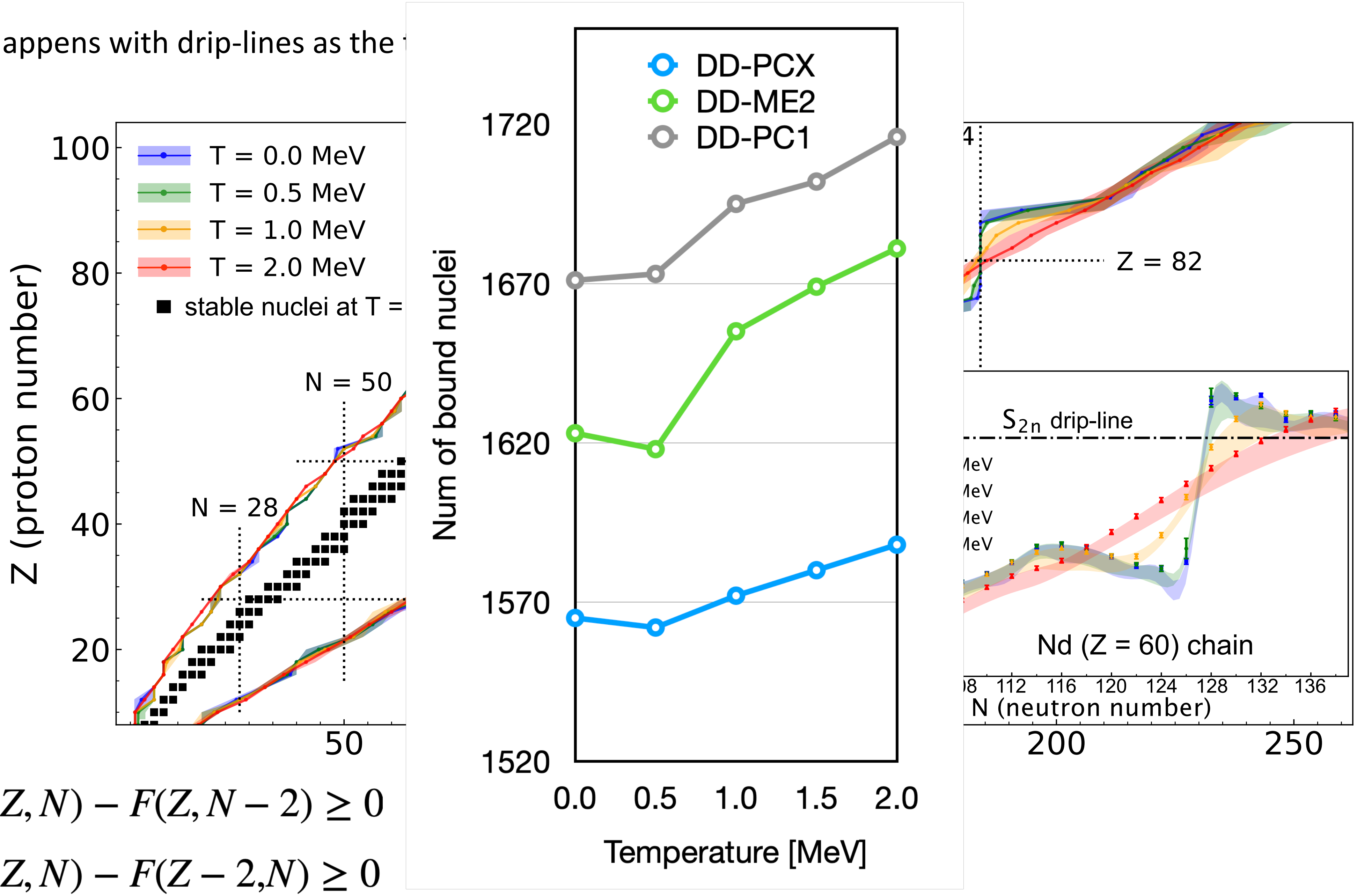

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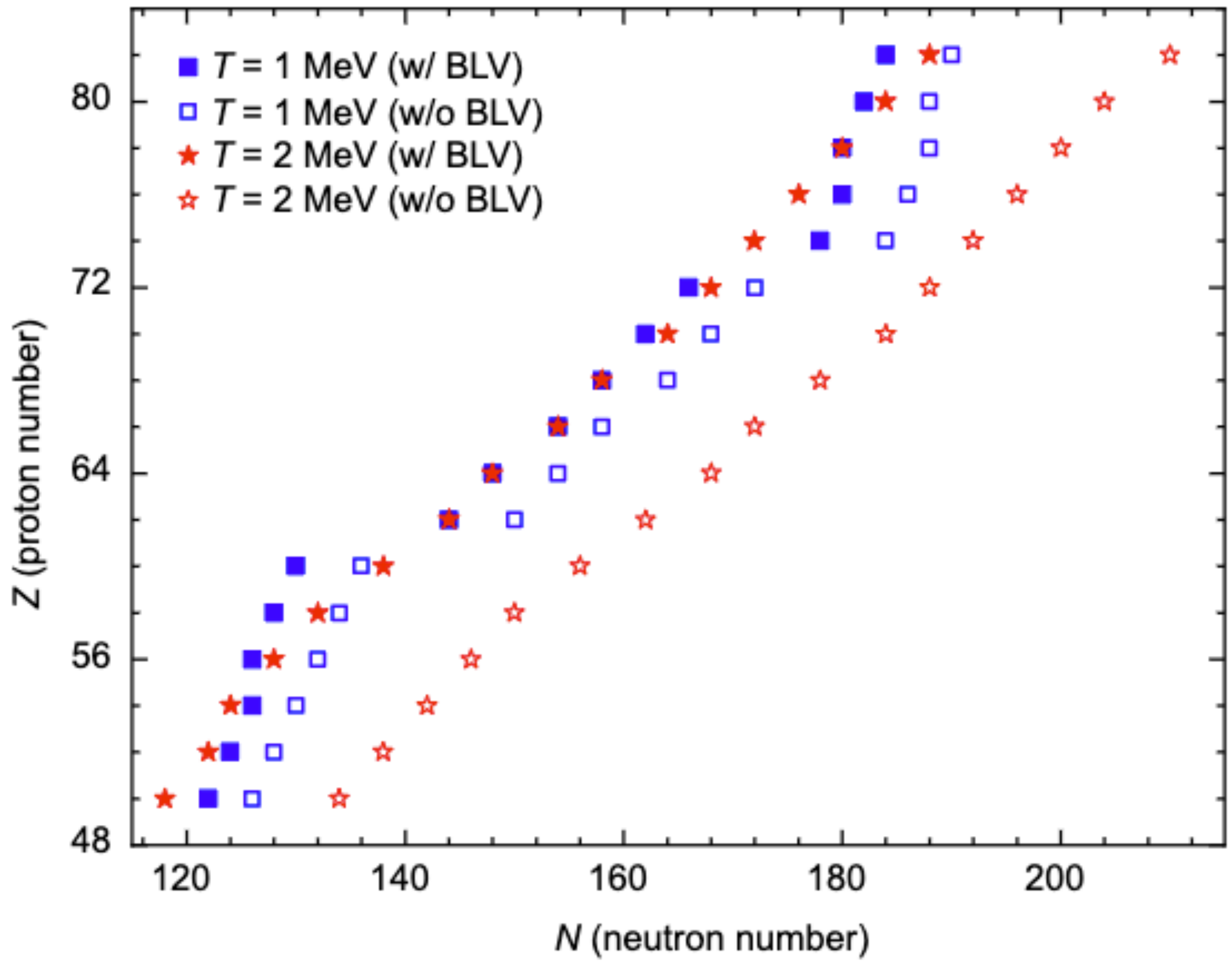
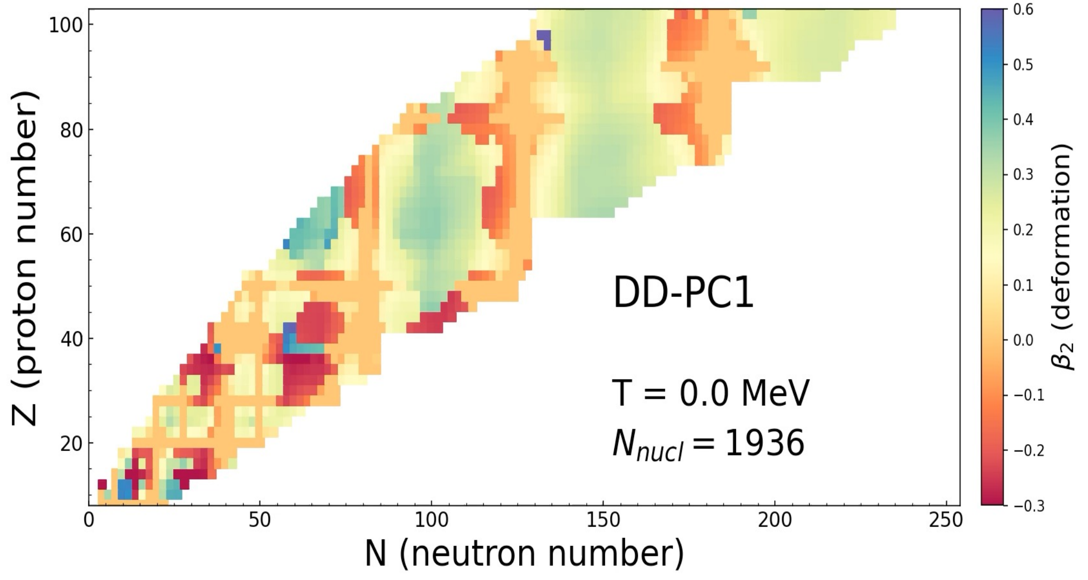
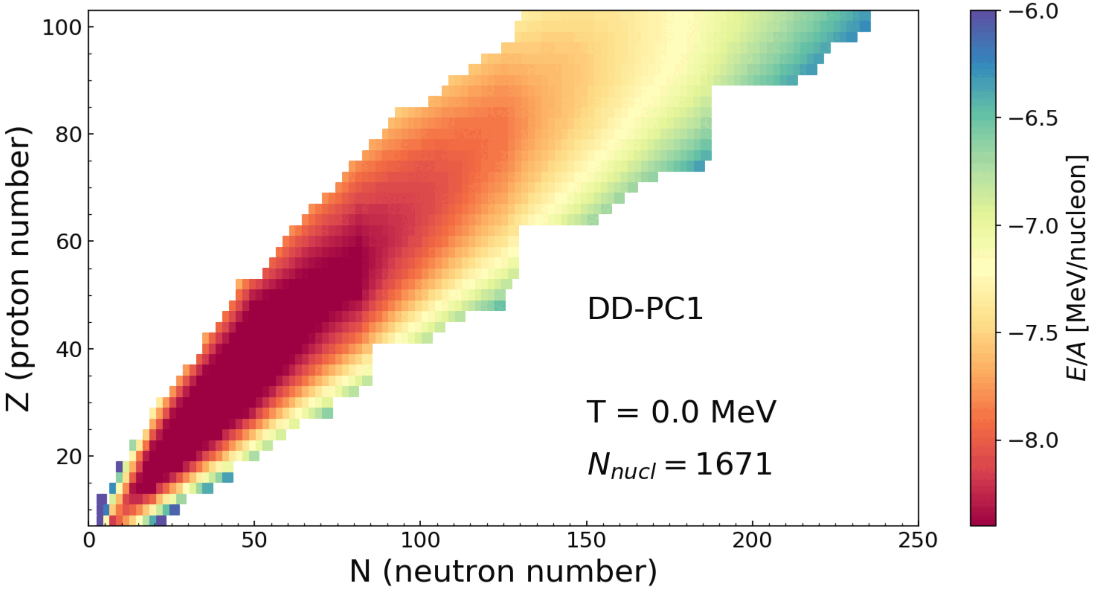


The vapor subtraction procedure is essential for the proper determination of the drip lines. Otherwise, the drip line is significantly extended as temperature increases.

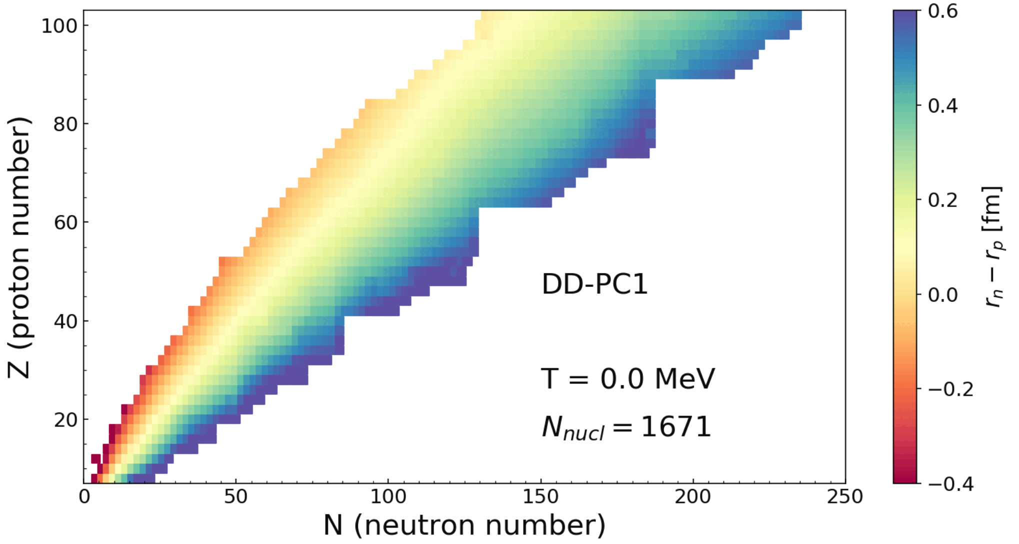
Quadrupole deformation:



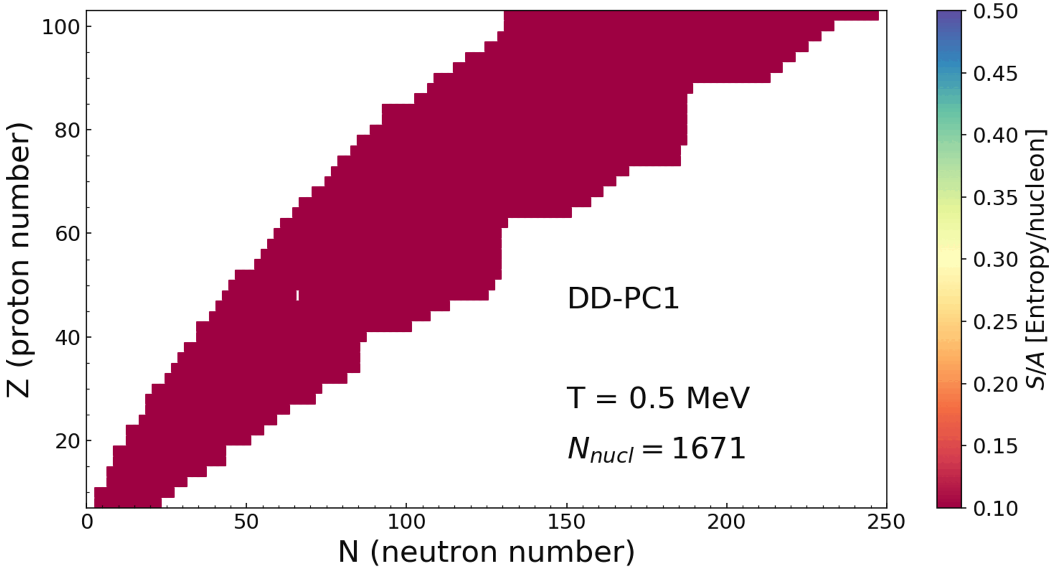
Binding energy per nucleon:



Neutron skin:



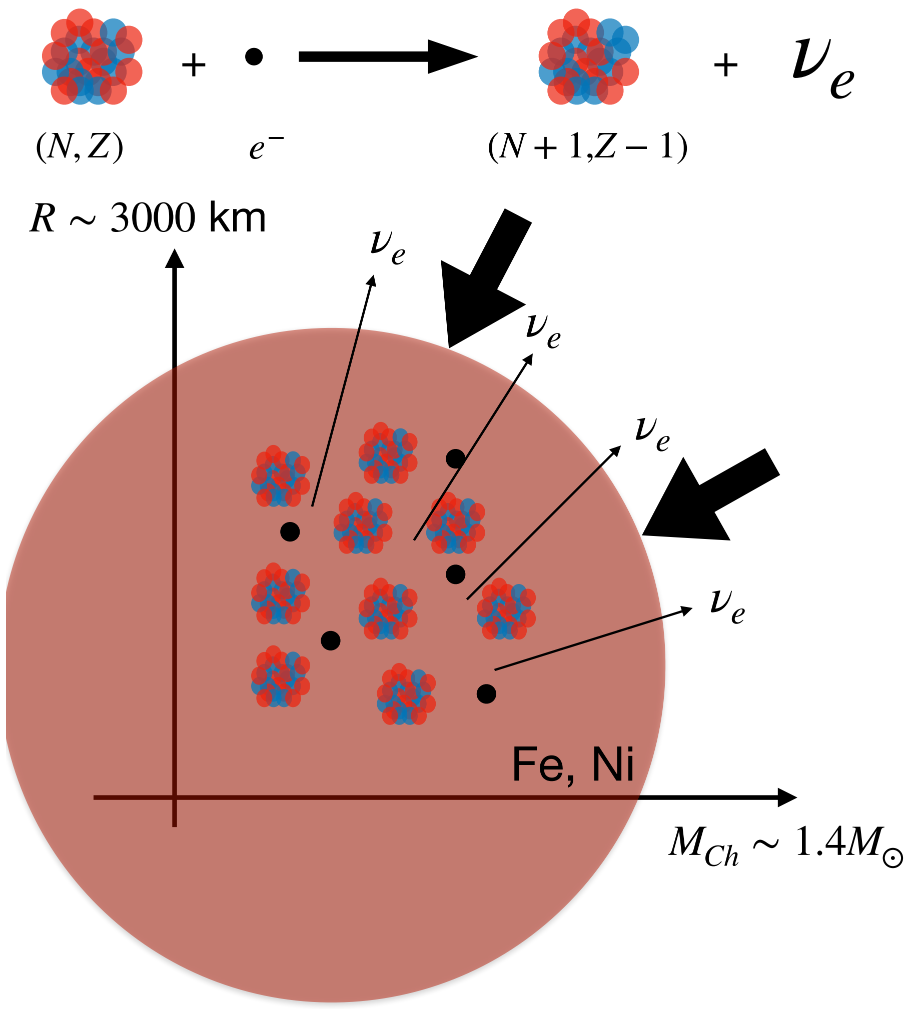
Entropy per nucleon:



Weak decays in stellar conditions

Electron capture (EC) and supernovae environment

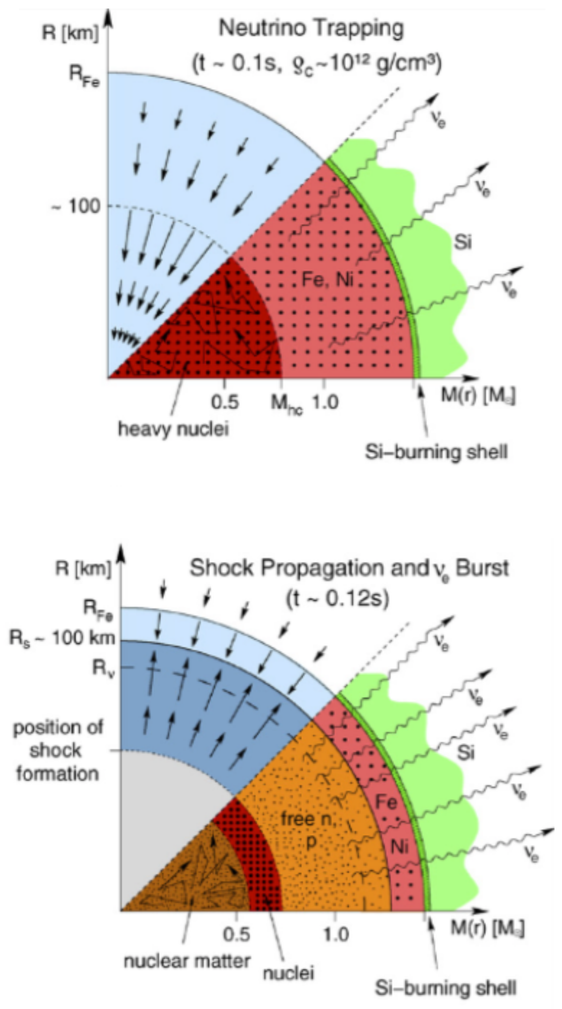
x

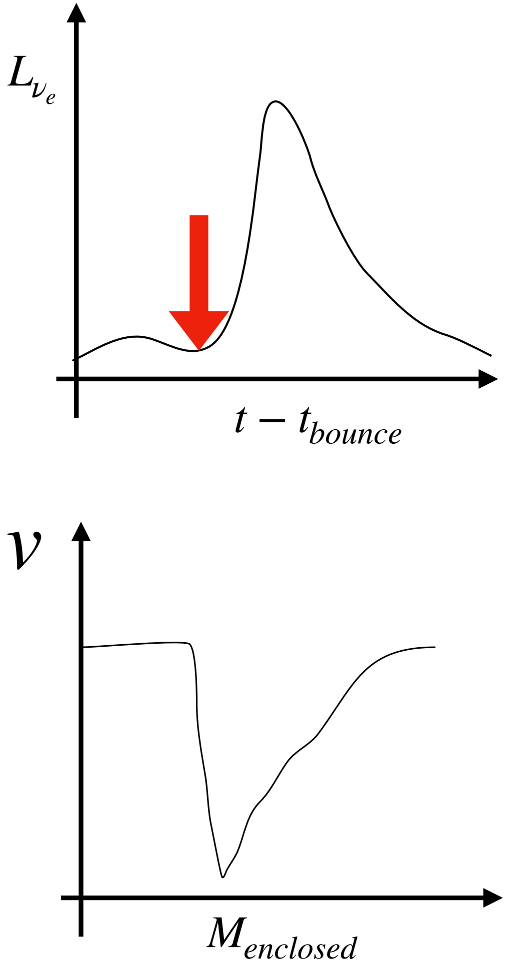


Initial phase of collapse

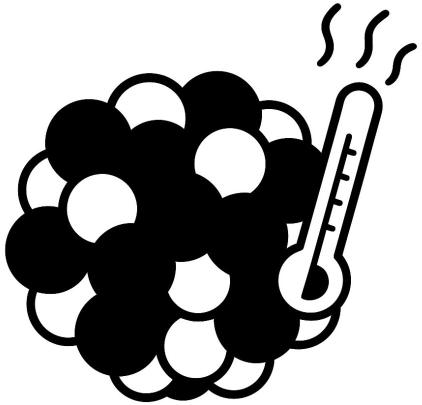
$\rho \sim 10^{10}$ g/cm^3, $T \sim 10$ GK

[H. T. Janka et al., Physics Reports, 442, 38-74 (2007)]





Expression for electron capture rate in stellar environment:



$$\lambda_{ec} = \frac{(m_e c^2)^3}{\pi^2 \hbar^3} \frac{1}{Z} \sum_{if} e^{\beta E_i} \int_{W_0^{th(i,f)}}^{\infty} pW \sigma_{ec}^{(i,f)}(W) f_e(W, \mu_e) dW$$

$\sigma_{ec}^{(i,f)}$ Requires knowledge on nuclear matrix elements at finite-temperature $k_B T > 0$

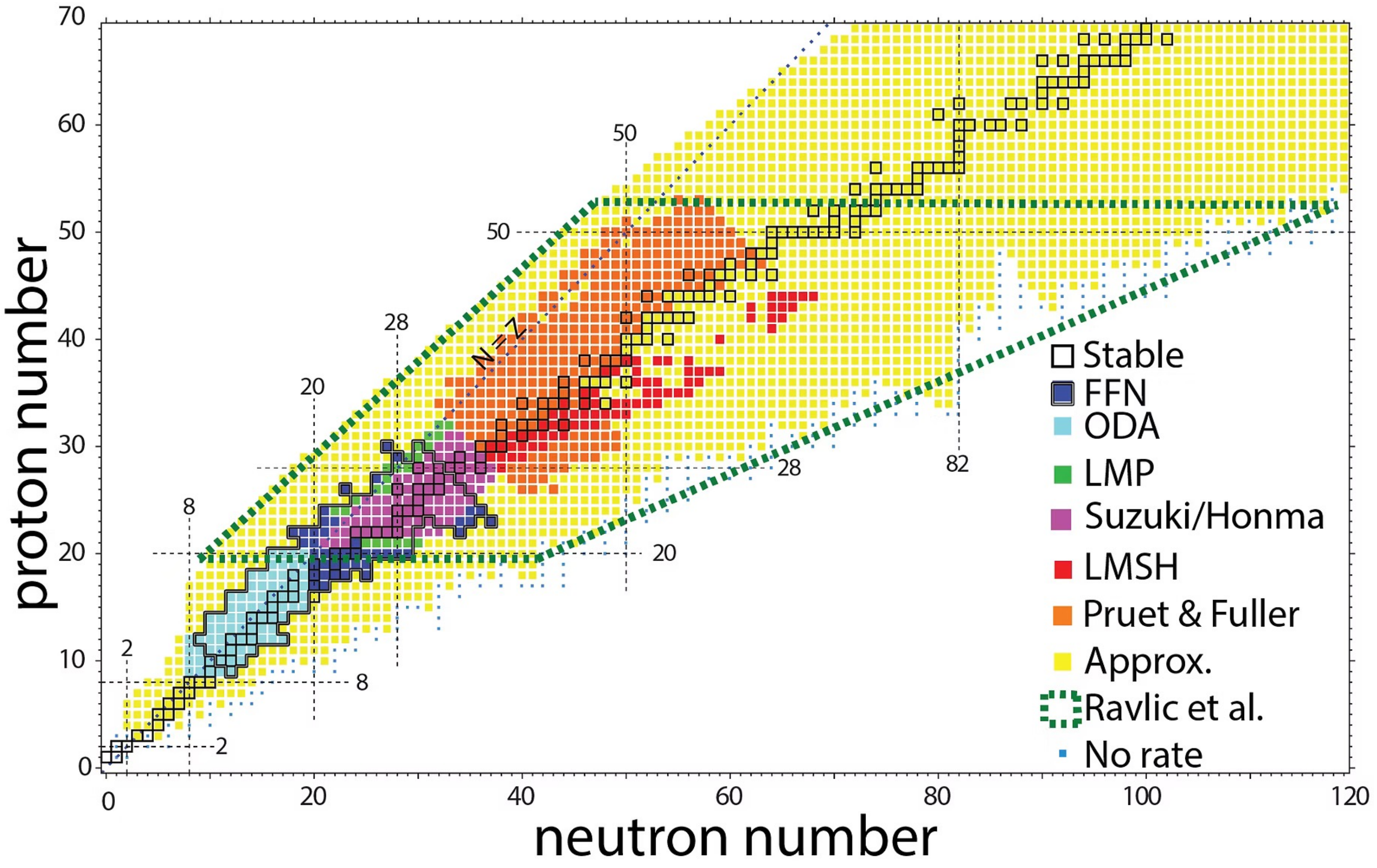
→ Calculated using finite-temperature QRPA (FT-QRPA or FT-PNRQRPA)

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} \Rightarrow \begin{pmatrix} C & a & b & D \\ a^\dagger & A & B & b^T \\ b^\dagger & B^* & A^* & a^T \\ D^* & b^* & a^* & C^* \end{pmatrix} \begin{pmatrix} P \\ X \\ Y \\ Q \end{pmatrix} = E \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} P \\ X \\ Y \\ Q \end{pmatrix}$$

$f_e(W, \mu_e)$ Fermi-Dirac factor – takes into account density dependence of rates

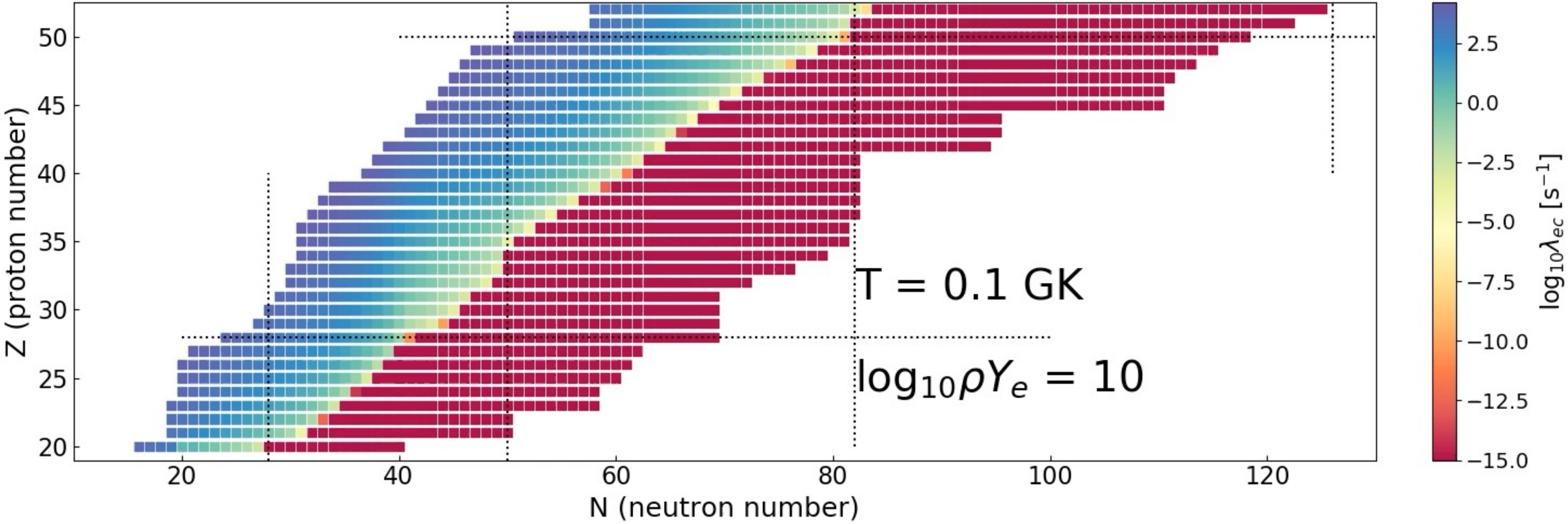
[AR, E. M. Ney, J. Engel, and N. Paar, Eur. Phys. J A 61(2) 37 (2025).]
[AR, E. Yüksel et al. PRC 102, 065804 (2020).]

Large scale calculations of EC rates for core-collapse supernovae:



Data available at: www.remcozegers.com/weak-rate-library

- Dataset including around 1700 nuclei (from Z = 20 to 52)
- Includes allowed ($\Delta J^{\pi} = 0^{+}, 1^{+}$) and first-forbidden transitions ($\Delta J^{\pi} = 0^{-}, 1^{-}, 2^{\wedge} -$)
- FT-QRPA with relativistic mom-dep D3C* interaction

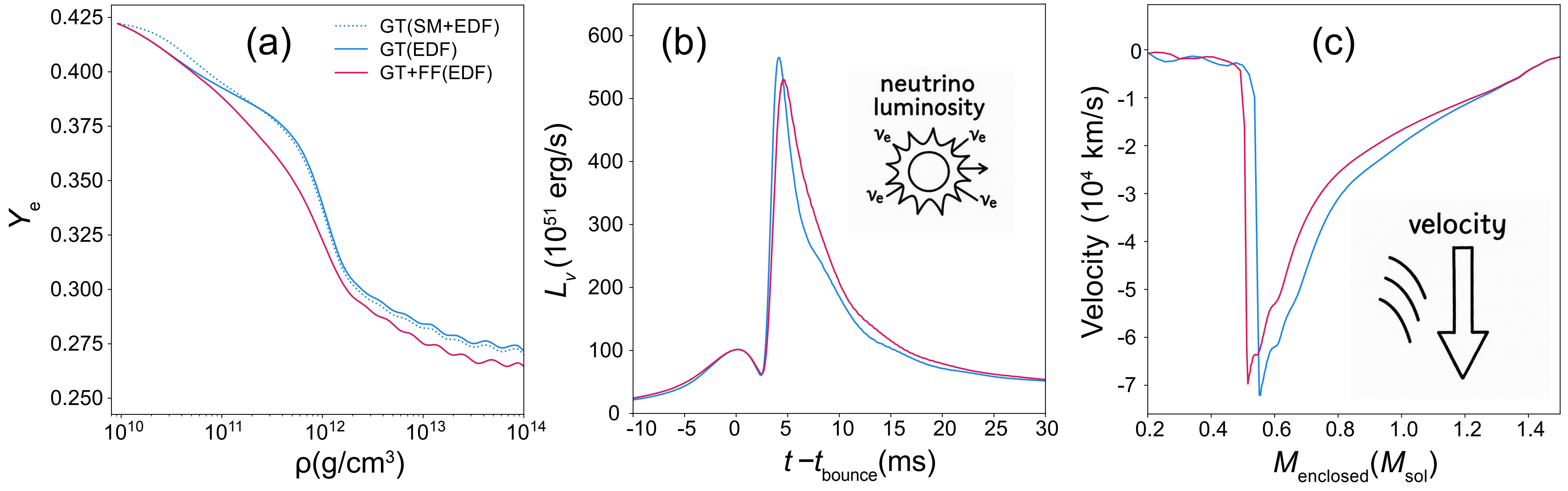


GT(EDF) – FT-QRPA with Gamow-Teller only

GT+FF(EDF) – FT-QRPA with Gamow-Teller + first-forbidden

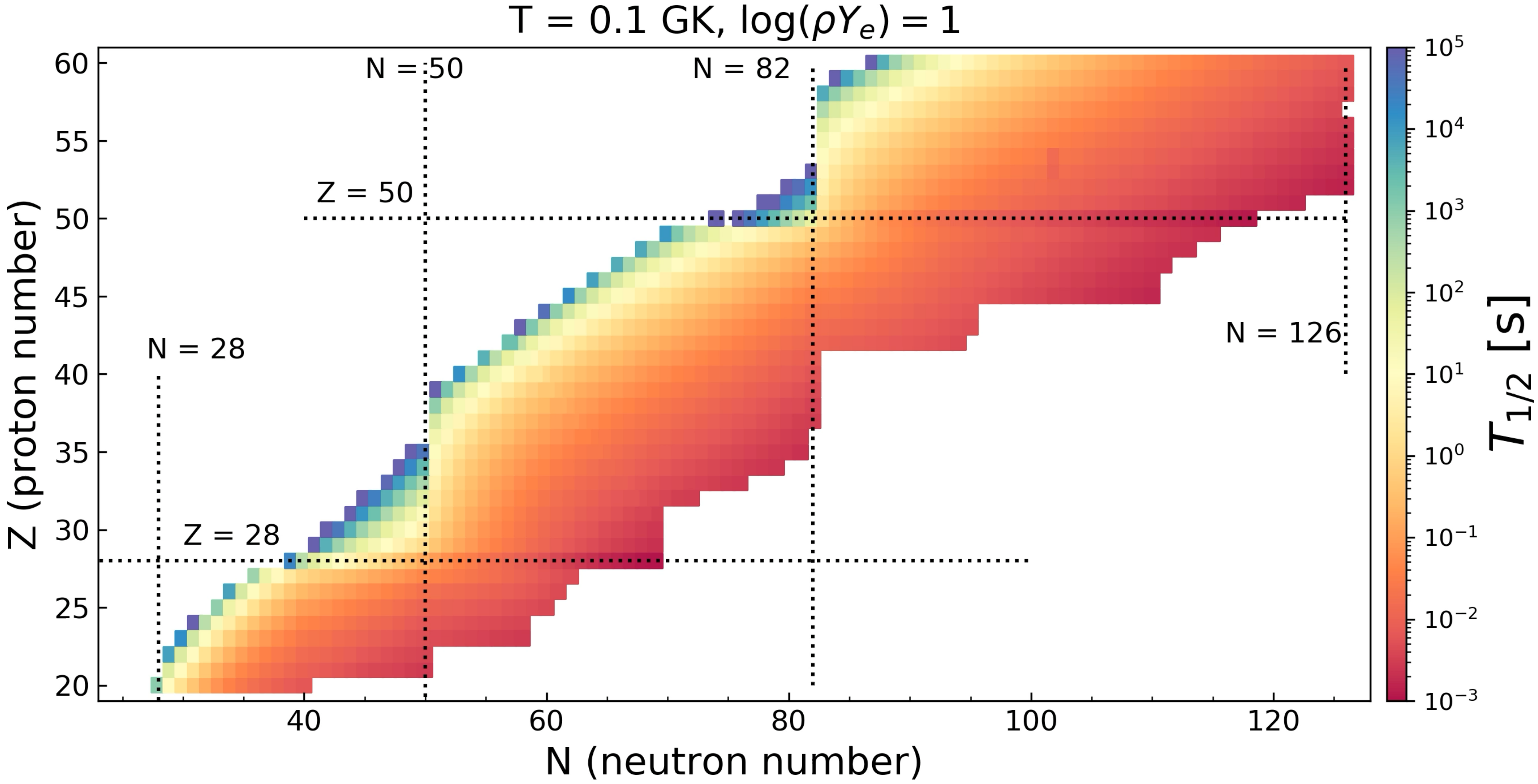
GT(SM+EDF) – FT-QRPA with Gamow-Teller + shell-model where available

$$Y_e = \frac{\#\text{of electrons}}{\#\text{of barions}}$$



[AR, S. Giraud, R. Zegers and N. Paar, PRC 112, L032801 (2025).]

What about stellar β-decay rates ?



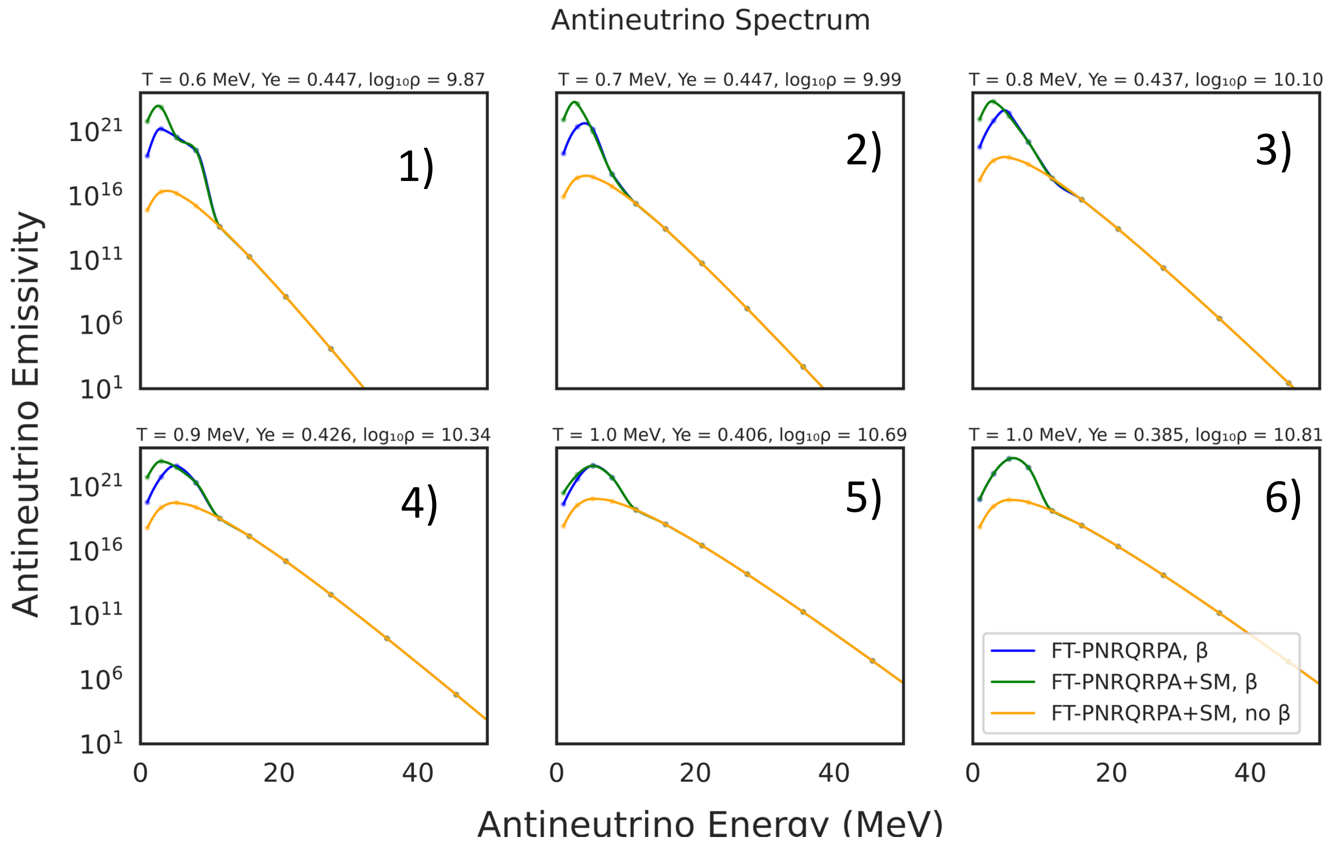
Data available at: https://github.com/ascsn/rpx

β-decay rates and supernovae environment

Work by Troy Dasher @ MSU

NuLib + GR1D framework

[*CQG 27*(11), 114103 (2010), Astrophys. J. Suppl. Ser. 219, 24 (2015).]



Antineutrino emissivity:

$$\frac{d\epsilon_{\bar{\nu}_e}}{d\Omega dE_{\bar{\nu}_e}} = \sum_i \frac{E_{\bar{\nu}_e}}{4\pi} Y_i n_i(E_{\bar{\nu}_e})$$

abundance

spectral function
(depends on rate)

[T. Dasher, AR, S. Lalit, E. O'Connor, K. Godbey, Phys. Rev. D 113, 123041 (2026).]

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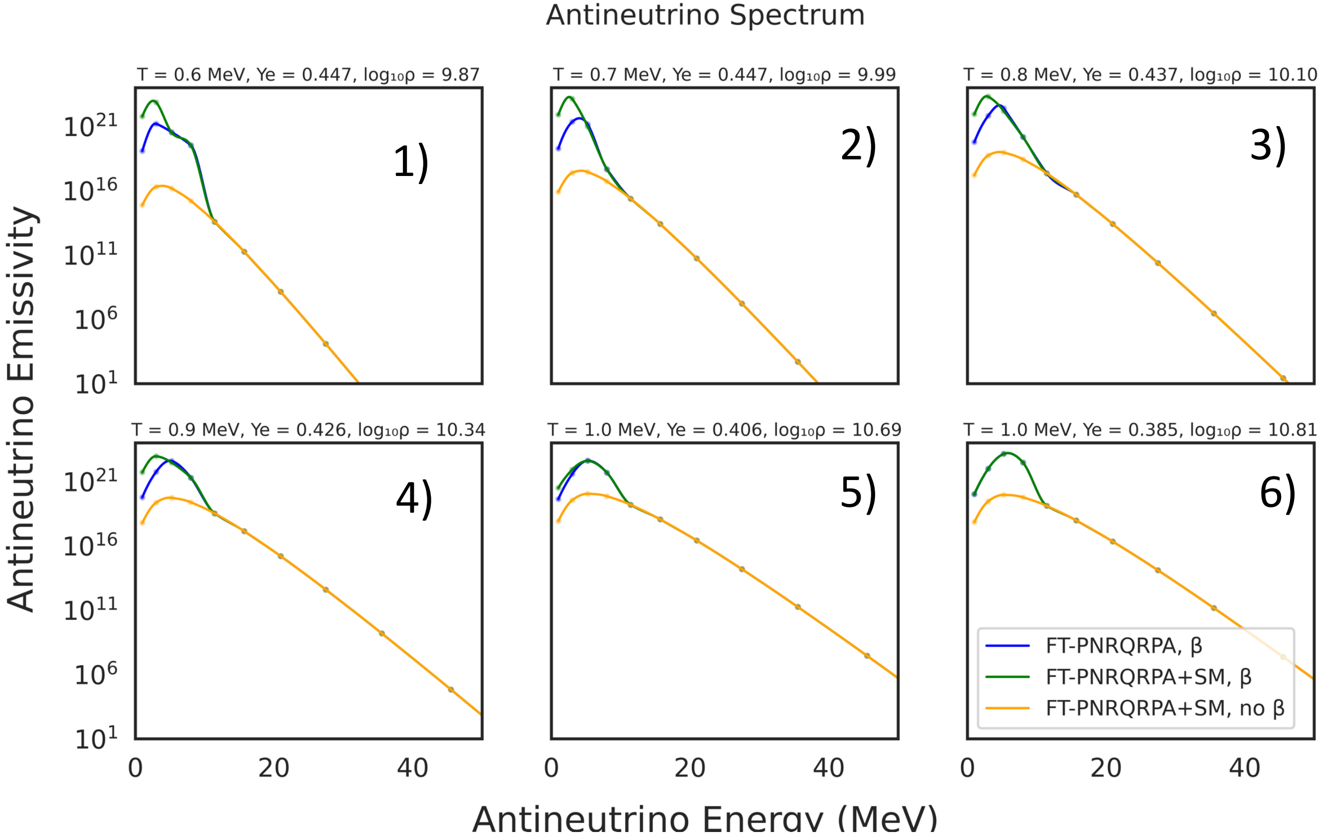
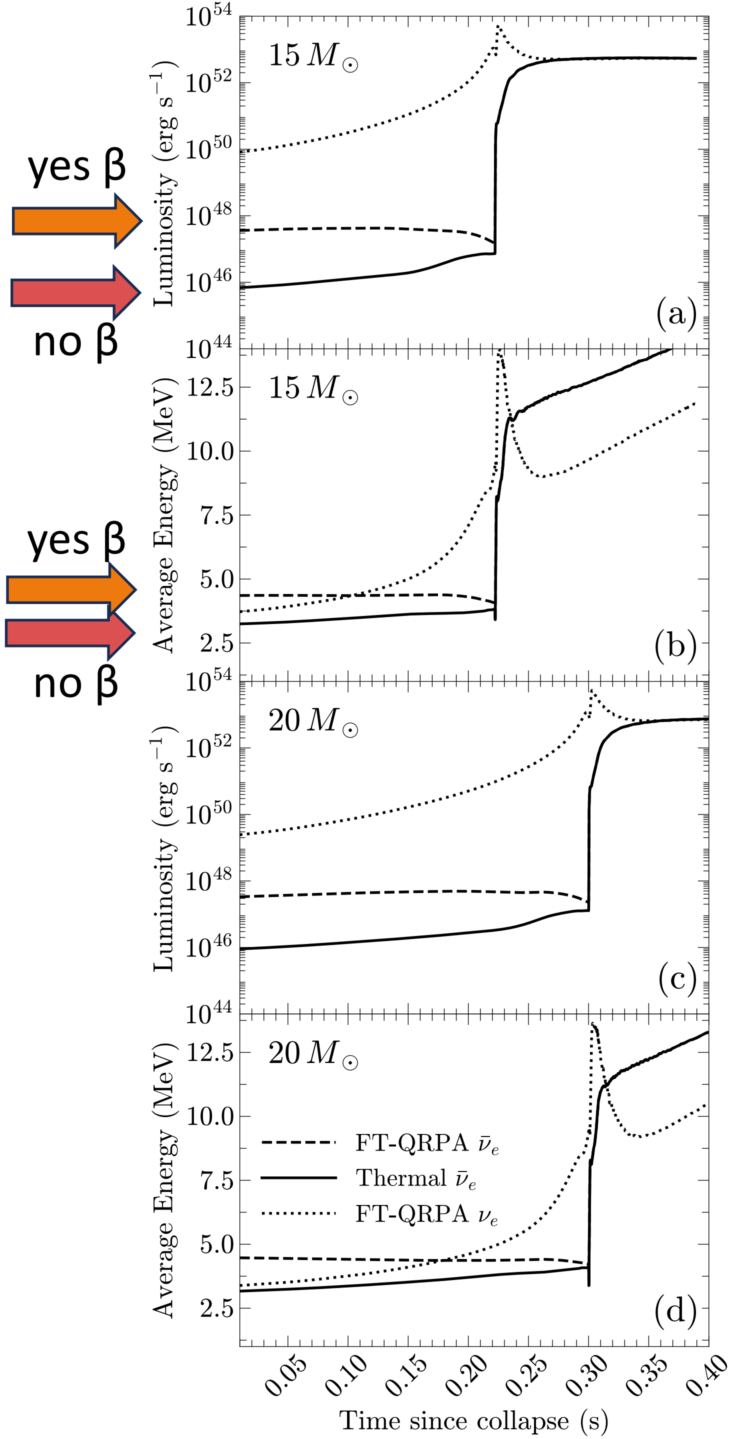
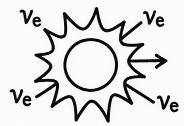


[T. Dasher, AR, S. Lalit, E. O'Connor, K. Godbey, Phys. Rev. D 113, 123041 (2026).]





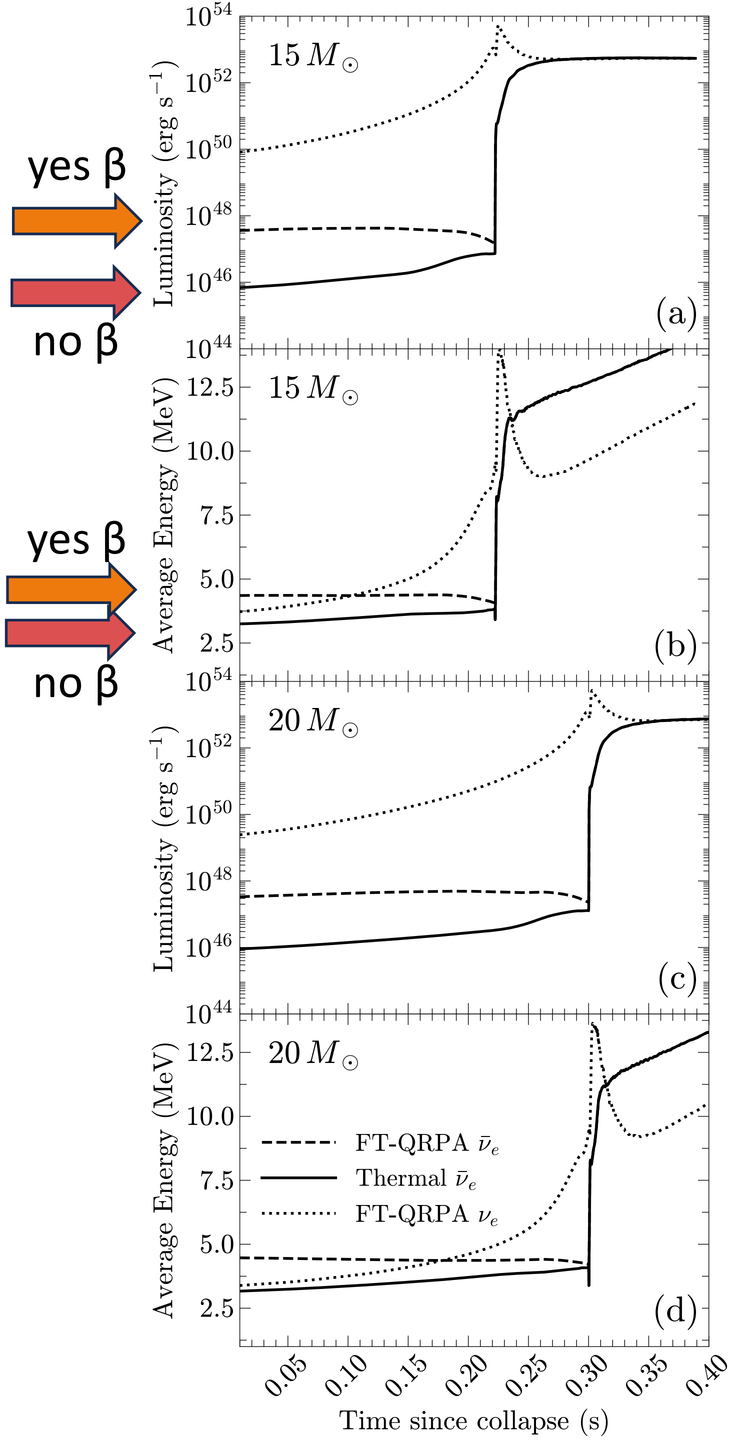
β-decay rates and supernovae environment

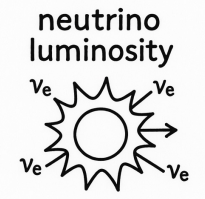
We can study the impact of the enhanced luminosity on
antineutrino detection events

TABLE I. Predicted prebounce IBD event counts at $d = 10$ kpc, assuming no neutrino oscillations. Hyper-K-like rates are scaled from a 100 kt water-Cherenkov detector (30% PMT coverage; `wc100kt30prct`) to the 188 kt Hyper-K fiducial mass. JUNO-lilke rates use the `scint20kt` SNOw-GLoBES configuration. Employed models are $15M_\odot$ progenitor with (`s15_beta`) and without (`s15_nobeta`) β-decay rates included in GR1D simulation, and likewise for $20M_\odot$ progenitor.

Model	Progenitor	Hyper-K-like	JUNO-like
`s15_nobeta`	$15M_\odot$	2.28×10^{-4}	1.01×10^{-4}
`s15_beta`	$15M_\odot$	1.50×10^{-3}	1.61×10^{-3}
`s20_nobeta`	$20M_\odot$	4.30×10^{-4}	2.01×10^{-4}
`s20_beta`	$20M_\odot$	2.40×10^{-3}	2.54×10^{-3}

[T. Dasher, A. Ravlic, S. Lalit, E. O'Connor, K. Godbey, Phys. Rev. D 113, 123041 (2026).]





Thank you for your attention!

Collaborators: Witek Nazarewicz (FRIB/MSU)
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Sudhanva Lalit (FRIB/MSU)
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Lauren Jin, Troy Dasher (FRIB/MSU)
Nils Paar (Zagreb)

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