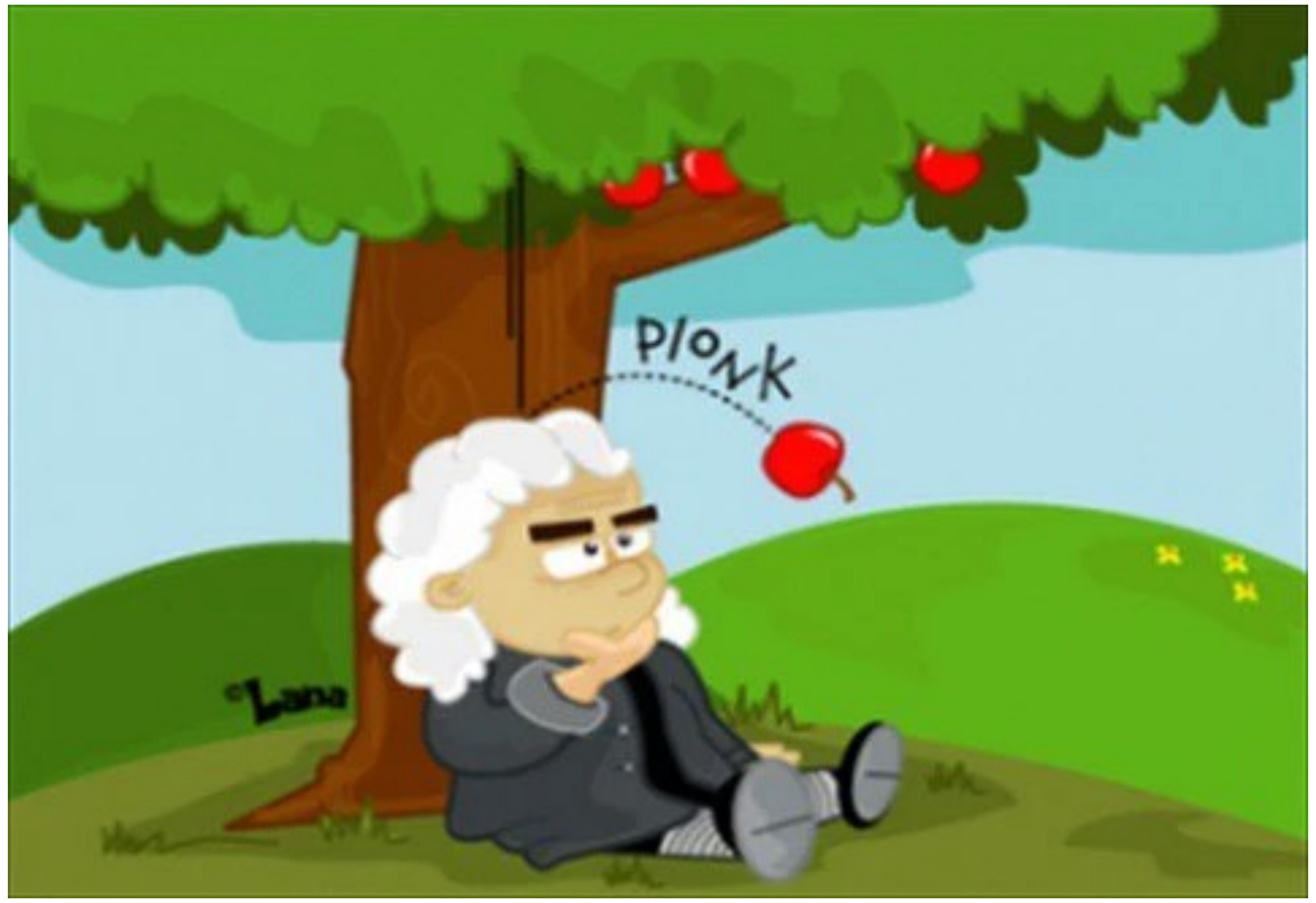
Quantum “Dark” Matter

Surjeet Rajendran

Classical and Quantum Particle Mechanics



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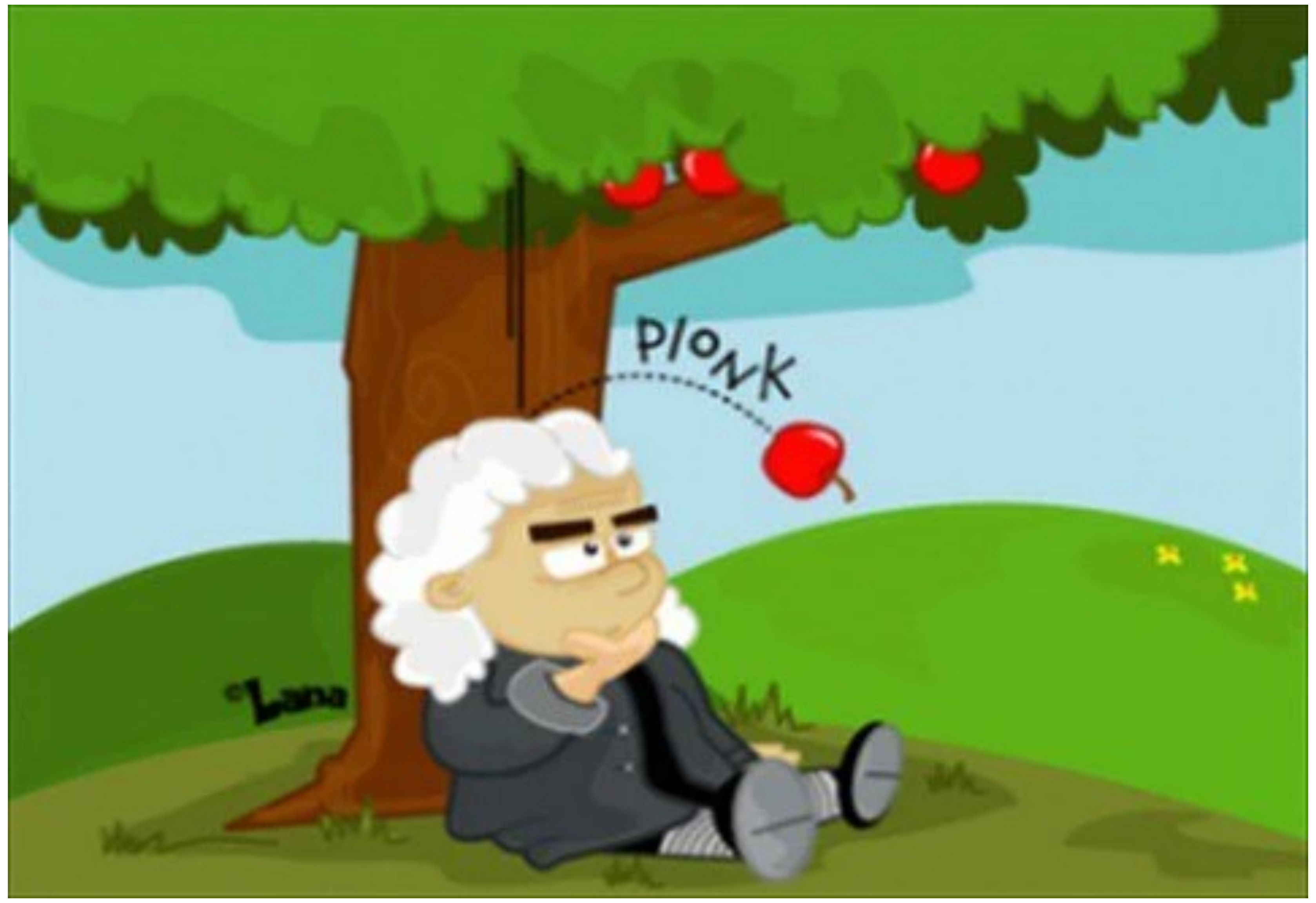
Classical and Quantum Particle Mechanics



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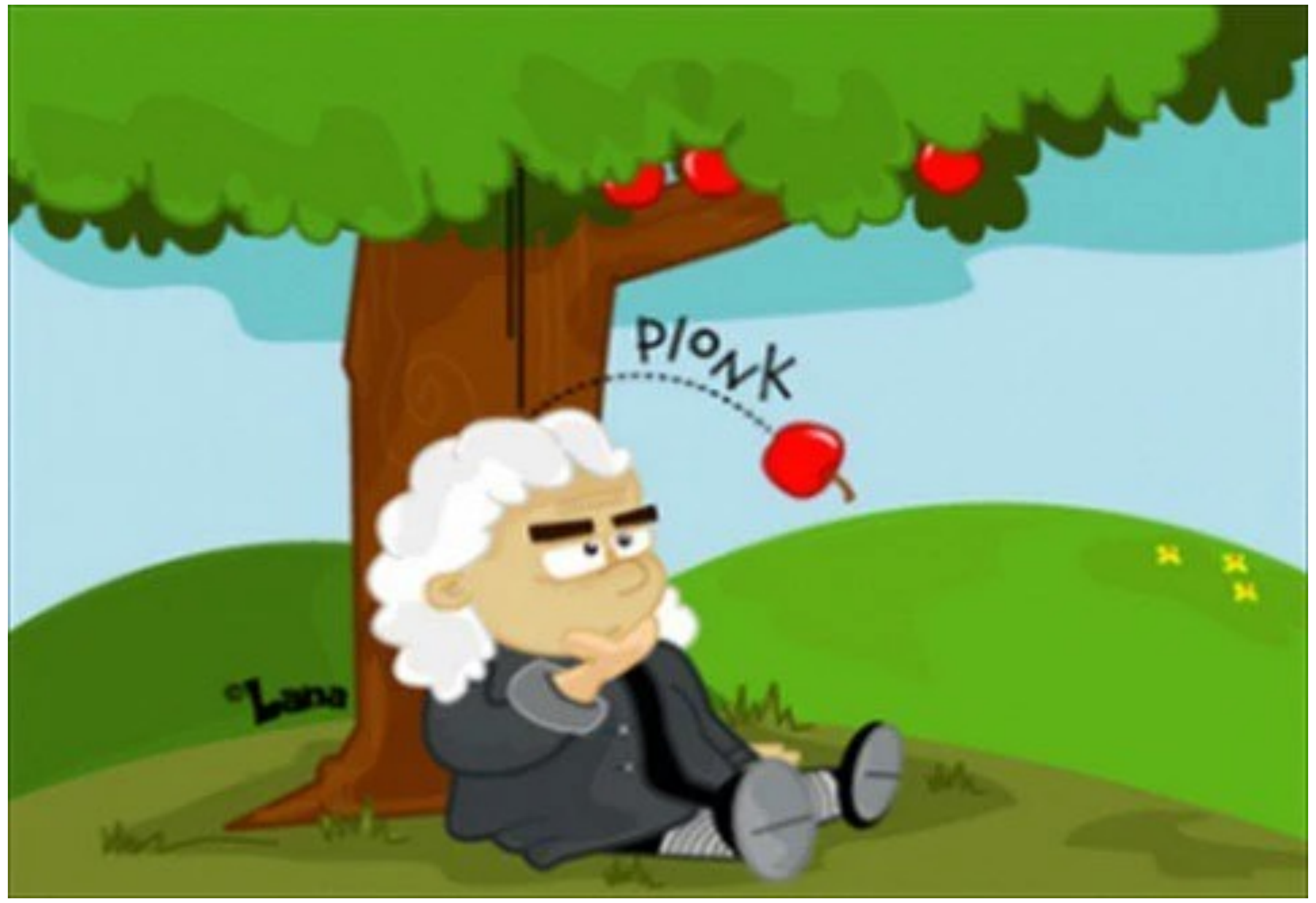

$$\vec{F} = m\vec{a}$$

$$\frac{d\langle\Psi|\hat{p}|\Psi\rangle}{dt} = -\langle\Psi|\frac{dV}{dx}|\Psi\rangle$$

$\Uparrow$

Ehrenfest's Theorem

$$i\frac{\partial|\Psi\rangle}{\partial t} = H|\Psi\rangle$$

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Quantum Supremacy

Classical and Quantum Field Theory

Classical Field Theory
(e.g. Klein Gordon)

$$\frac{\partial S}{\partial \phi} = 0$$

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Schrodinger Picture of Quantum Field Theory

$|\chi(t)\rangle$ **Quantum State of Fields (e.g. in Fock states)**

$\phi(x)$ **Time Independent Operators**

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$$H = \int d^3x\, \mathcal{H}\left(\phi(x), \pi(x)\right)$$

Time Evolution

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Analog of Ehrenfest Theorem?

$$i \frac{\partial |\chi(t)\rangle}{\partial t} = H |\chi(t)\rangle$$

Schwinger Dyson Equations

Quantum Supremacy

Schwinger Dyson

$$Z = \int D\phi e^{iS[\phi]}$$

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Change Variables: $\phi \to \phi + \delta\phi \implies \delta Z = 0$

$$\delta Z = \int D\phi \left(\frac{\partial S}{\partial \phi}\right) e^{iS[\phi]} \delta\phi$$

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Gauge theories?

Classical and Quantum Gauge Theories

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Need to Gauge Fix to define Path Integral

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Not the Same ????????

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Outline

1. Classical Mini Superspace Cosmology

2. Path Integral

3. Hamiltonian

4. Wheeler deWitt

5. Conclusions

Mini Superspace Cosmology

Homogeneous Universe

Study homogeneous space-times in General Relativity

$$g_{\mu\nu} = -N\left(t\right)^2 dt^2 + a\left(t\right)^2 \left(dx^2 + dy^2 + dz^2\right)$$

Couple this to homogenous sources of matter, for e.g. rolling scalar field ϕ

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

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Classical Equations?

2 FRW + 1 Scalar Field

Classical Equations

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$$\frac{\partial S}{\partial a} = 0 \implies \partial_t\left(\frac{\partial \mathcal{L}}{\partial \dot{a}}\right) - \frac{\partial \mathcal{L}}{\partial a} = 0 \text{ i.e. } \left(\frac{\ddot{a}}{a} + \cdots = 0\right)$$

2nd FRW Equation

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Klein Gordon

$$\frac{\partial S}{\partial N} = 0 \implies \frac{\partial \mathcal{L}}{\partial N} = 0 \text{ i.e. } \left(\left(\frac{\dot{a}}{a}\right)^2 + \cdots = 0\right)$$

1st FRW Equation (Hubble)

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1st FRW Equation (Hubble)

Classical Solutions are over-constrained (3 equations for 2 variables)

Solve: Klein Gordon + 2nd FRW with boundary condition from 1st FRW

Path Integral

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$\dot{N} = 0$, enforce via Lagrange multiplier

$$\langle \phi_f, a_f | T\left(t_2; t_1\right) | \phi_i, a_i \rangle = \int_{\phi(t_1)=\phi_i, a(t_1)=a_i, N(t_1)=N_1}^{\phi(t_2)=\phi_f, a(t_2)=a_f, N(t_2)=N_2} D\phi Da DN D\lambda e^{i\int_{t=t_1}^{t=t_2}\left(\mathcal{L}-\lambda \dot{N}\right)}$$

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$$\dot{N}=0\implies N\left(t_2\right)=N_0,N\left(t_1\right)=N_0$$

Path Integral

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Can show that path integral over a(t), ϕ(t) are finite

$$\langle \phi_f, a_f | T(t_2; t_1) | \phi_i, a_i \rangle = e^{iP(\phi_f, a_f, t_2; \phi_i, a_i, t_1; N_0)}$$

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Path Integral explicitly depends upon random gauge choice N_0

Path integral obviously not gauge invariant

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Is this a problem?

Gauge Invariant Physics

$$\langle \phi_f , a_f | T \left(t_2 ; t_1 \right) | \phi_i , a_i \rangle$$

Path integral defines the time evolution operator

$$g_{\mu\nu} = -N \left(t \right)^2 dt^2 + a \left(t \right)^2 \left(dx^2 + dy^2 + dz^2 \right)$$

Time coordinate is a gauge choice in General Relativity

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Perfectly Reasonable for the time evolution operator to be gauge dependent

We need gauge invariant physics. What does this mean?

Gauge Invariant Physics

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What happens if $N_0 \to \tilde{N}_0$?

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Can Explicitly Show:

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$$\text{where } \tilde{t} = \frac{N_0}{\tilde{N}_0} t$$

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$$|\phi_i, a_i\rangle \to \sum_{\phi_f, a_f} c_{\phi_f, a_f}(t_2, t_1) |\phi_f, a_f\rangle = \sum_{\phi_f, a_f} c_{\phi_f, a_f}\left(\tilde{t}_2, \tilde{t}_1\right) |\phi_f, a_f\rangle$$

$$|\Psi(t_1)\rangle = |\tilde{\Psi}\left(\tilde{t}_1\right)\rangle = |\Sigma\rangle \to |\Psi(t_2)\rangle = |\tilde{\Psi}\left(\tilde{t}_2\right)\rangle = |\Omega\rangle$$

Gauge Invariant Physics

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Fix manifold (R^1).

Pick any choice of time co-ordinate on the manifold

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$$\text{General Gauge: } N(t) = f(t)$$

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Moral: Gauge invariant physics from gauge dependent path integral
Reasonable since time is gauge choice!

Schwinger Dyson Equations

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$$\phi \to \phi + \delta\phi, a \to a + \delta a, N \to N + \delta N$$

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2nd FRW Equation

Heisenberg Picture

$$\langle \Psi | \partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right) - \frac{\partial \mathcal{L}}{\partial \phi} | \Psi \rangle = 0$$

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$$\langle \Psi | \frac{\partial \lambda}{\partial t} - \frac{\partial \mathcal{L}}{\partial N} | \Psi \rangle = 0$$

Tells you how λ evolves in the path integral - not 1st FRW Equation?????

Hamiltonian

Hamiltonian Construction

$$g_{\mu\nu} = -N\left(t\right)^2 dt^2 + a\left(t\right)^2 \left(dx^2 + dy^2 + dz^2\right)$$

Physical Degrees of freedom: a(t), ϕ(t)
Gauge Freedom: N(t)

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Quantize These

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Quantize These

$$\Pi_N = \frac{\partial \mathcal{L}}{\partial \dot{N}} = 0$$

What to do with N?

Hamiltonian Construction

$$\Pi_a = \frac{\partial \mathcal{L}}{\partial \dot{a}}, \Pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

Quantize These

$$[a, \Pi_a] = [\phi, \Pi_\phi] = i$$

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Hamiltonian Construction

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Fully non-linear General Relativity - no "free" theory with "free" kinetic term

But, can still construct Hilbert space with Fock states of A, B - these are operator level statements independent of kinetic terms of the theory

$$A|0\rangle = 0, A^\dagger|0\rangle = |1\rangle \text{ etc.}$$

Hamiltonian Construction

$$S = \int dt \mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

Construct Canonical Hamiltonian from this Lagrangian

$$H_N = N\, H_0\left(a, \Pi_a, \phi, \Pi_\phi\right)$$

Hamiltonian Construction

$$S = \int dt \mathcal{L} \left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right) \right)$$

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What is N? Different values of N yield different Hamiltonians
Different physics? Gauge Invariance?

If N is a non-trivial operator on the Fock space, no way to make physics gauge invariant
Possible choice: N is a c-number
But still, different choices of N yield different Hamiltonians!

Schrodinger Equation

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = N H_0 |\chi(t)\rangle$$

N is a c-number, different choices of N yield different Hamiltonians

Schrodinger Equation

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

N is a c-number, different choices of N yield different Hamiltonians

$$i\frac{1}{N}\frac{\partial|\chi(t)\rangle}{\partial t} = H_0|\chi(t)\rangle$$

$$d\tilde{t} = N dt$$

Schrodinger Equation

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Different choices of N correspond to different choices of time co-ordinate.
Gauge invariant physics!

$$\langle\phi_f, a_f|T(t_2;t_1)|\phi_i, a_i\rangle = \int_{\phi(t_1)=\phi_i, a(t_1)=a_i}^{\phi(t_2)=\phi_f, a(t_2)=a_f} D\phi D a e^{i\int_{t=t_1}^{t=t_2}(\tilde{\mathcal{L}}(N_0))}$$

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Consistent with Path Integral - you just pick N

Consequence of Schrodinger Equation

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle \implies \frac{d\langle\chi(t)|H_0|\chi(t)\rangle}{dt} = 0$$

Identity, similar to Ehrenfest and Schwinger-Dyson

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Can Show: $\langle\chi(t)|H_0|\chi(t)\rangle = \langle\chi(t)|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi(t)\rangle$

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$$\text{Thus: } \frac{d\langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle}{dt} = 0$$

This is almost the 1st FRW equation - but not quite.
1st FRW equation only satisfied up to overall constant

Initial State

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Create quantum states of a, ϕ

$$|\chi\rangle = f\left(A, A^\dagger, B, B^\dagger\right)|0>$$

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In classical physics, we demand 2 FRW + 1 KG equation to hold - so we restrict initial conditions to obey 1st FRW

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$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

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Quantum Dynamics

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

First order ODE - no issue with time evolving

$$\langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle \neq 0$$

Violating 1st FRW

$$\langle \chi | a^3 \frac{\partial \mathcal{L}}{\partial N} | \chi \rangle = c$$

Quantum Dynamics: $$i \frac{\partial | \chi(t) \rangle}{\partial t} = N H_0 | \chi(t) \rangle$$

Violating 1st FRW

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Quantum Dynamics: $$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

Implied Classical Dynamics

$$\langle\chi|\partial_t\left(\frac{\partial\mathcal{L}}{\partial\dot{\phi}}\right) - \frac{\partial\mathcal{L}}{\partial\phi}|\chi\rangle = 0$$ **2nd FRW Equation**

$$\langle\chi|\partial_t\left(\frac{\partial\mathcal{L}}{\partial\dot{a}}\right) - \frac{\partial\mathcal{L}}{\partial a}|\chi\rangle = 0$$ **Klein Gordon**

$$\langle\chi|\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = \langle\chi|\frac{c}{a^3}|\chi\rangle$$ **1st FRW but with "Dark" Matter**

Quantum "Dark" Matter

$$\langle \chi | a^3 \frac{\partial \mathcal{L}}{\partial N} | \chi \rangle = c$$

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Create quantum states of a, ϕ
Quantum Dynamics: Just 1 first order ODE (Schrodinger)
No reason to constrain initial state!

Failure manifests classically as "dark" matter - though no real particle excitation there. Conservation implies super-selection sector.

Can be positive or negative!

Wheeler deWitt

Can we get all of Einstein's Equations?

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

$$\text{Want } \langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = 0$$

Hamiltonian changes with N

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Universe is in an energy eigenstate

But... Time???

Path Integral Version

$$\langle \phi_f, a_f | T\,(t_2; t_1)\,|\phi_i, a_i\rangle = e^{iP(\phi_f, a_f, t_2; \phi_i, a_i, t_1; N_0)}$$

Gauge fixed path integral implies we lose 1st FRW equation

Path Integral explicitly depends upon random gauge choice N_0

Path integral obviously not gauge invariant

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Path Integral explicitly depends upon random gauge choice N_0

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Integrate over all N_0

Unsurprisingly, this yields infinity

Demanding gauge invariant path integral implies only possible states are static (Wheeler deWitt)

Conclusions

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Quantum Supremacy

Wheeler deWitt

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Quantum Supremacy

Wheeler deWitt

Gauge Dependent Time Evolution With Gauge Invariant Physics

1st FRW Equation only true up to constant

Quantum "Dark" Matter

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Quantum Supremacy

Gauge Dependent Time Evolution With Gauge Invariant Physics

1st FRW Equation only true up to constant

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Wheeler deWitt

Gauge Independent Time Evolution, Infinite path integral

All Equations Hold...but....

Only static states...no time!

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Gauge Independent Time Evolution, Infinite path integral

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Only static states...no time!

Full General Relativity , similar issues in Electromagnetism