QED in 2+1 dimensions with qubits and qumodes

Tommaso Rainaldi – Dec 1, 2025

INT – Seattle, WA

Stony Brook University

Based on

Simulating quantum electrodynamics in 2+1 dimensions with qubits and qumodes

Victor Ale, Tommaso Rainaldi, Enrique Rico, Felix Ringer, George Siopsis

We develop a hybrid qubit-qumode framework for simulating quantum electrodynamics in 2+1 dimensions. In this approach, fermionic matter fields are represented by qubits, while U(1) gauge fields are encoded in continuous-variable bosonic modes whose canonical quadratures capture the electric and vector-potential components of the theory. To reconcile the non-compact phase space of the qumodes with the compact U(1) gauge symmetry, we introduce and compare two complementary constraint-enforcement strategies: (i) a squeezing-based projection that confines qumode states to the unit circle through an effective modification of the inner product, and (ii) a method that dynamically enforces compactness via a penalty Hamiltonian term. We construct the corresponding hybrid Hamiltonian, derive its decomposition into experimentally accessible qubit-qumode gates, and analyze its spectrum in the analytically tractable single-plaquette limit. The hybrid formulation reproduces the correct gauge-invariant dynamics and provides a scalable route toward simulating Abelian lattice gauge theories coupled to fermionic matter on near-term hybrid quantum architectures. Ground-state preparation and convergence are demonstrated using a continuous-variable extension of the Quantum Imaginary Time Evolution (QITE) algorithm, establishing a general framework for hybrid discrete-continuous quantum simulations of lattice gauge theories.

And previous work by Ringer, Siopsis et al

Lots of interest towards physics

Quantum simulations of hadron dynamics in the Schwinger model using 112 qubits

Roland C. Farrell[1,*], Marc Illa[1,†], Anthony N. Ciavarella[1,2,‡], and Martin J. Savage[1,§]

Entanglement in a holographic Schwinger pair with confinement

Sebastian Grieninger[1,*], Dmitri E. Kharzeev[1,2,†], and Ismail Zahed[1,‡]

Quantum-classical computation of Schwinger model dynamics using quantum computers

N. Klco[1,*], E. F. Dumitrescu[2], A. J. McCaskey[3], T. D. Morris[4], R. C. Pooser[2], M. Sanz[5], E. Solano[5,6], P. Lougovski[2,†], and M. J. Savage[1,‡]

Efficient representation for simulating U(1) gauge theories on digital quantum computers at all values of the coupling

Christian W. Bauer[1,*] and Dorota M. Grabowska[2,†]

Quantum computation of SU(2) lattice gauge theory with continuous variables

Victor Ale[a], Nora M. Bauer[a], Raghav G. Jha[b], Felix Ringer[b,c,d] and George Siopsis[a]

Simulating quantum field theories on continuous-variable quantum computers

Steven Abel[1,2,*], Michael Spannowsky[1,†], and Simon Williams[1,‡]

Letter | Published: 22 June 2016

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

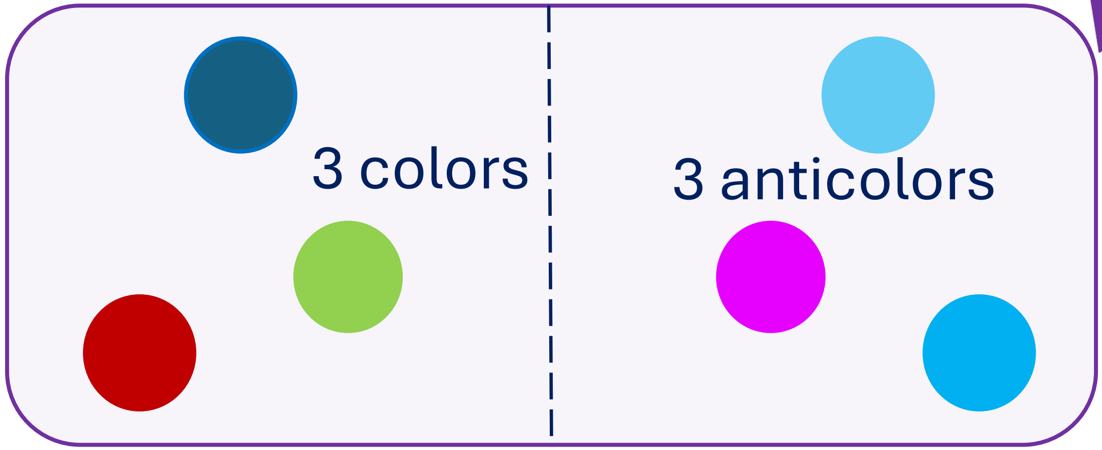
Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard, Markus Heyl, Philipp Hauke, Marcello Dalmonte, Thomas Monz, Peter Zoller & Rainer Blatt

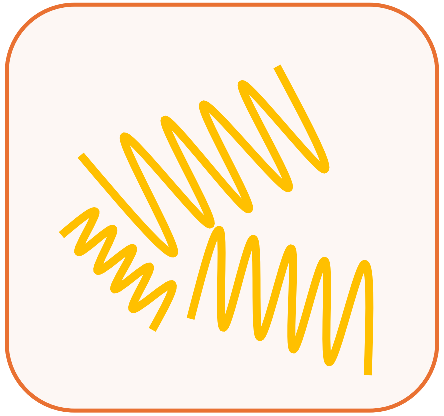
A resource efficient approach for quantum and classical simulations of gauge theories in particle physics

Jan F. Haase[1,2], Luca Dellantonio[1,2], Alessio Celi[3,4], Danny Paulson[1,2], Angus Kan[1,2], Karl Jansen[5], and Christine A. Muschik[1,2,6]

One major goal: Quantum Chromodynamics

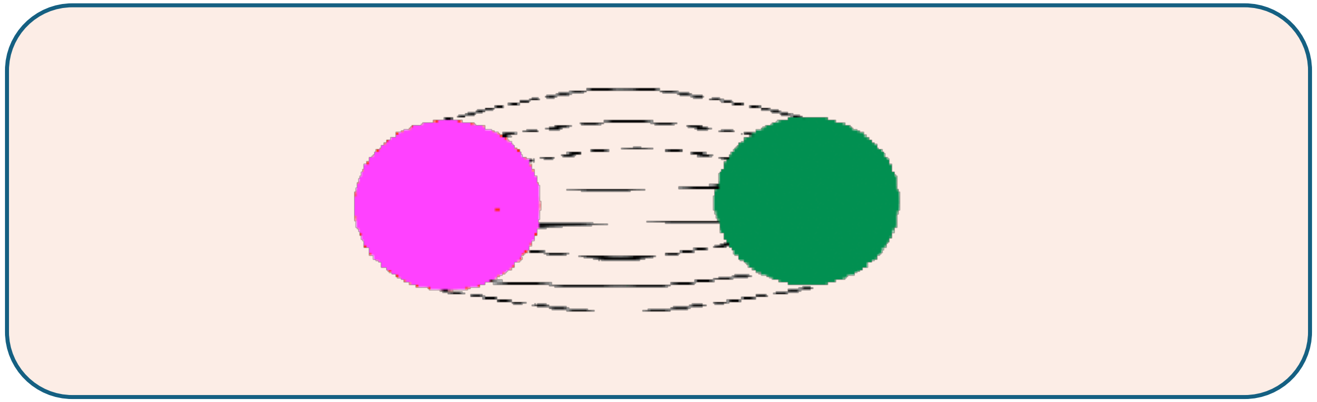
$$\mathcal{L}_{QCD} = \underbrace{\sum_f \bar{\psi}_{i,f}\left(i\gamma^\mu (D_\mu)_{ij} - m_f \delta_{ij}\right)\psi_{j,f}}_{quarks\ \&\ antiquarks} - \underbrace{\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu}}_{gluons}$$





The difficulties with QCD

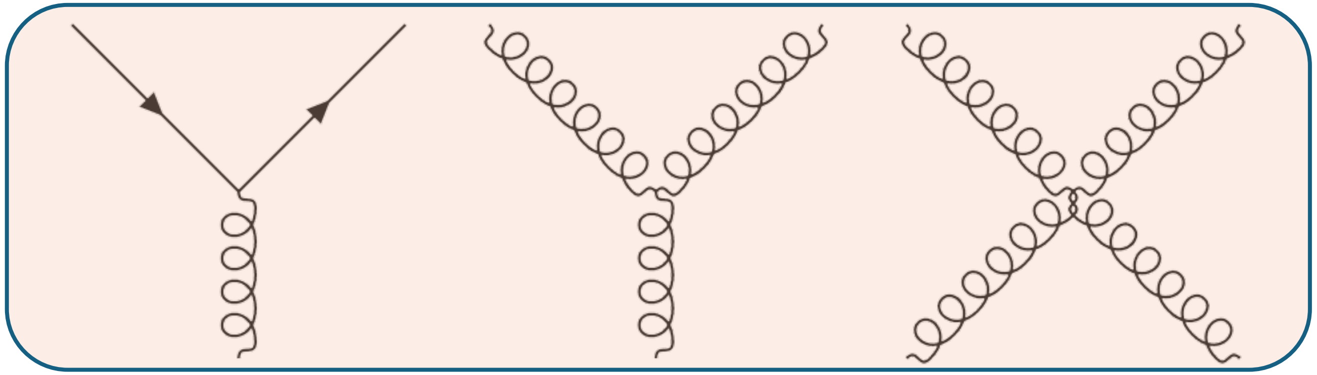
1 Quark and gluons are never observed (**color confinement**)



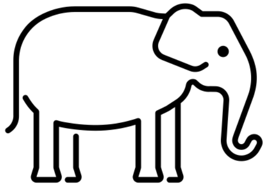
2 The interaction is **strong** (of order 1)

Strong	**1**
Electromagnetic	1/137
Weak	10^{-6}
Gravity	10^{-39}

3 Unlike photons, **gluons interact with themselves**



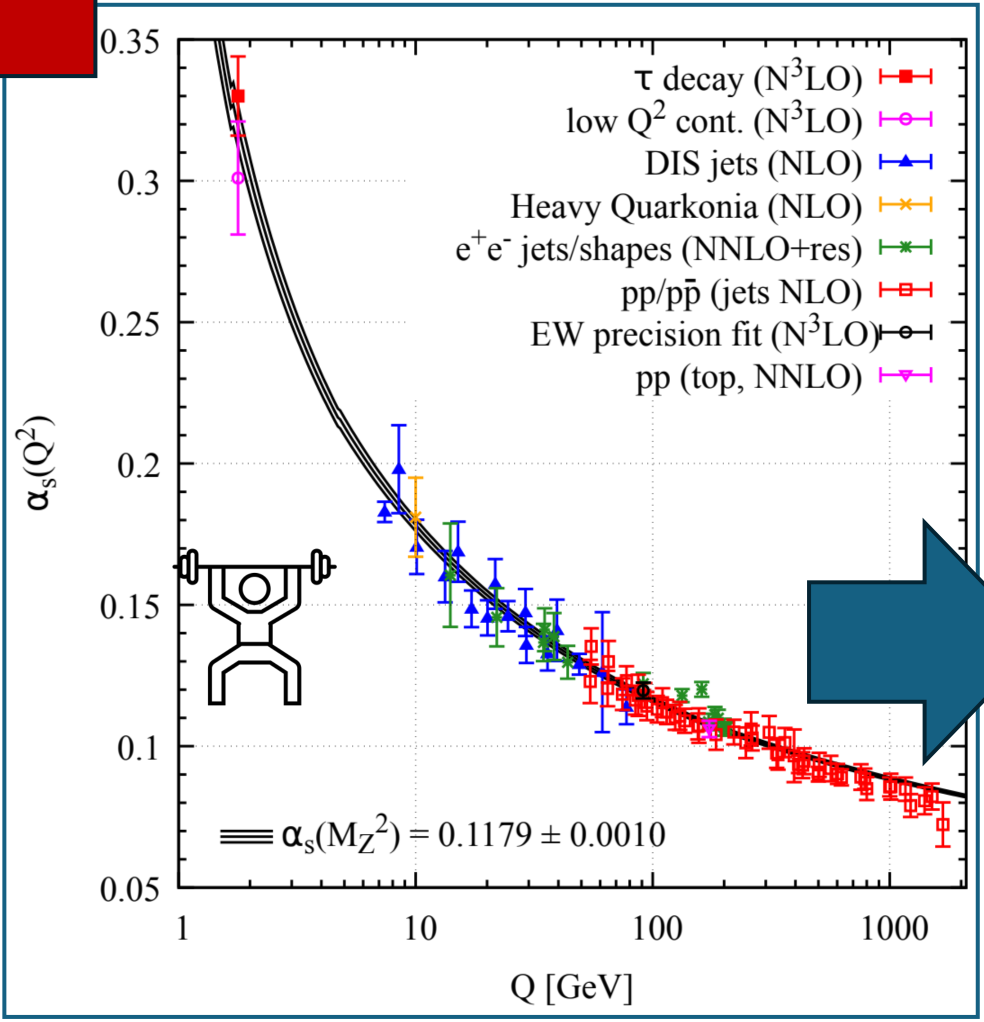
Perturbative vs nonpertubative

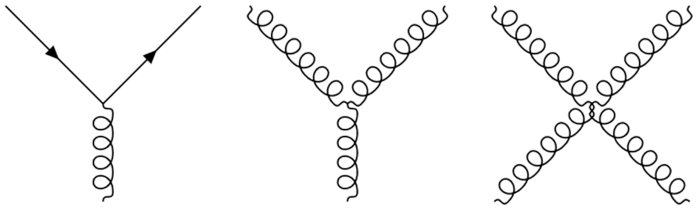


Large distance

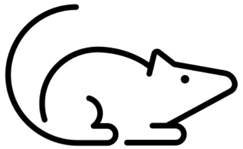
Low energy

Highly nonperturbative





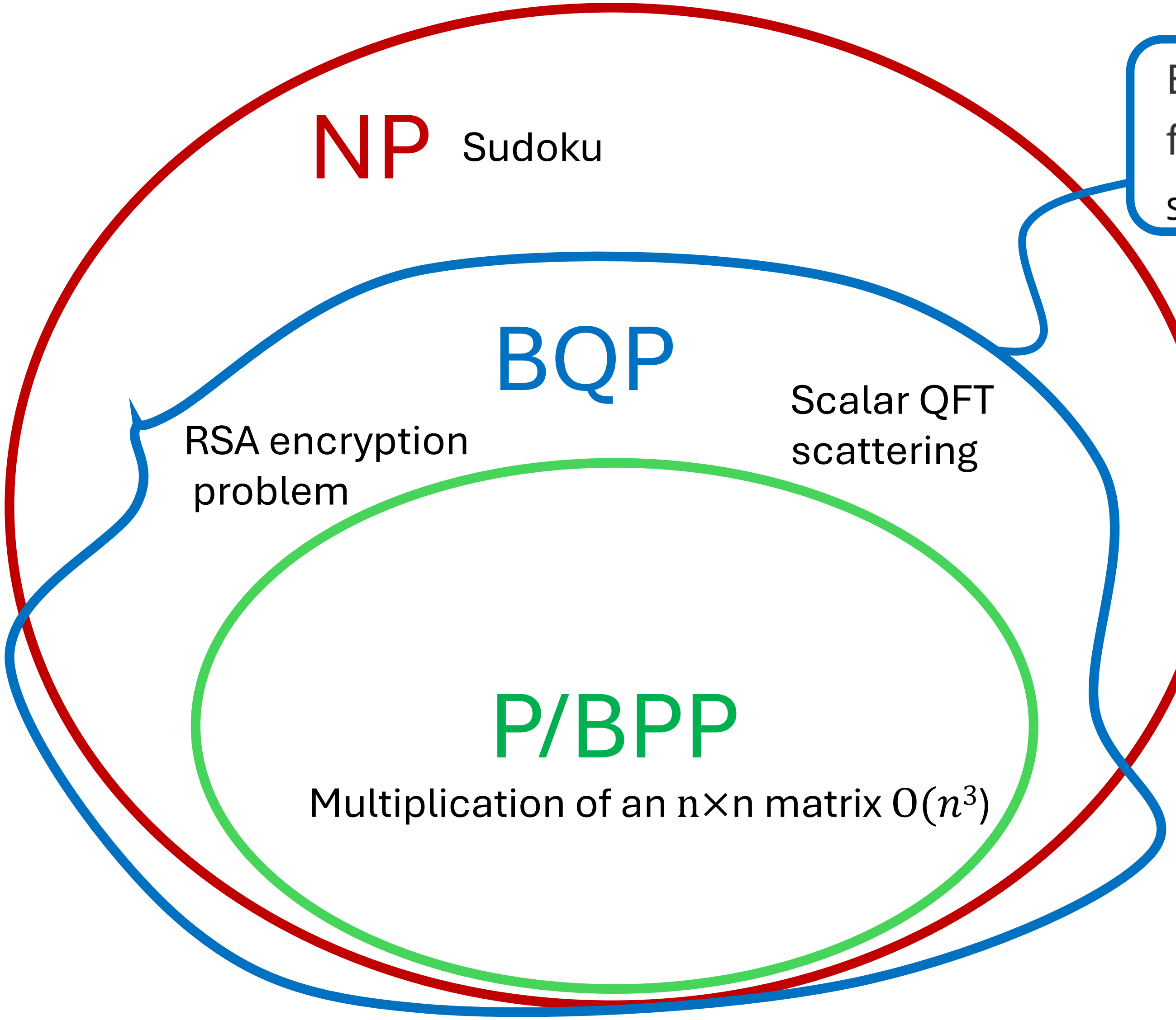
Perturbative



Small distance

High energy

Complexity of QCD?



BQP-completeness of scattering in scalar quantum field theory (with arbitrary operator insertions)

Stephen P. Jordan[1,2], Hari Krovi[3], Keith S. M. Lee[4], and John Preskill[5]

Solve in polynomial time with a quantum computer

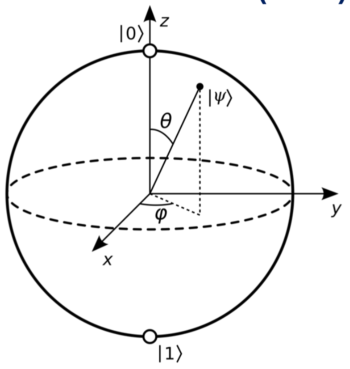
Verifiable in polynomial time

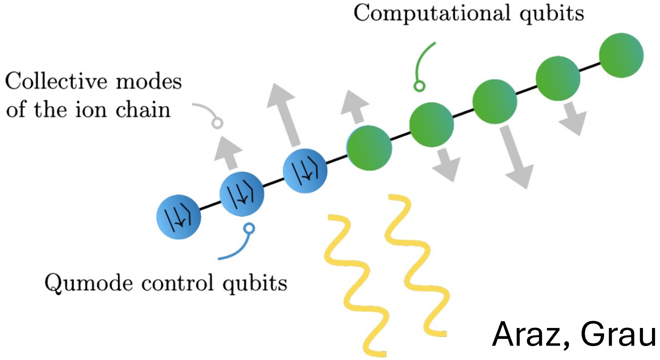
Solve in polynomial time

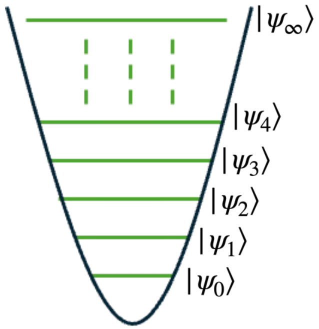
Qubits, (qudits) and qumodes

Elementary units for quantum computation

- **Qubits**
  - Realization with: superconducting circuits, cold atoms, trapped ions, topological qubits, …
  - Two-dimensional Hilbert space per qubit
  - Digital **gate-based** computing with **discrete variables** (DV)
- **Qumodes**
  - Realization with: photonics, trapped ions, superconducting circuits,…
  - Infinite-dimensional Hilbert space per qumode
  - **Gate-based with continuous variables** (CV)

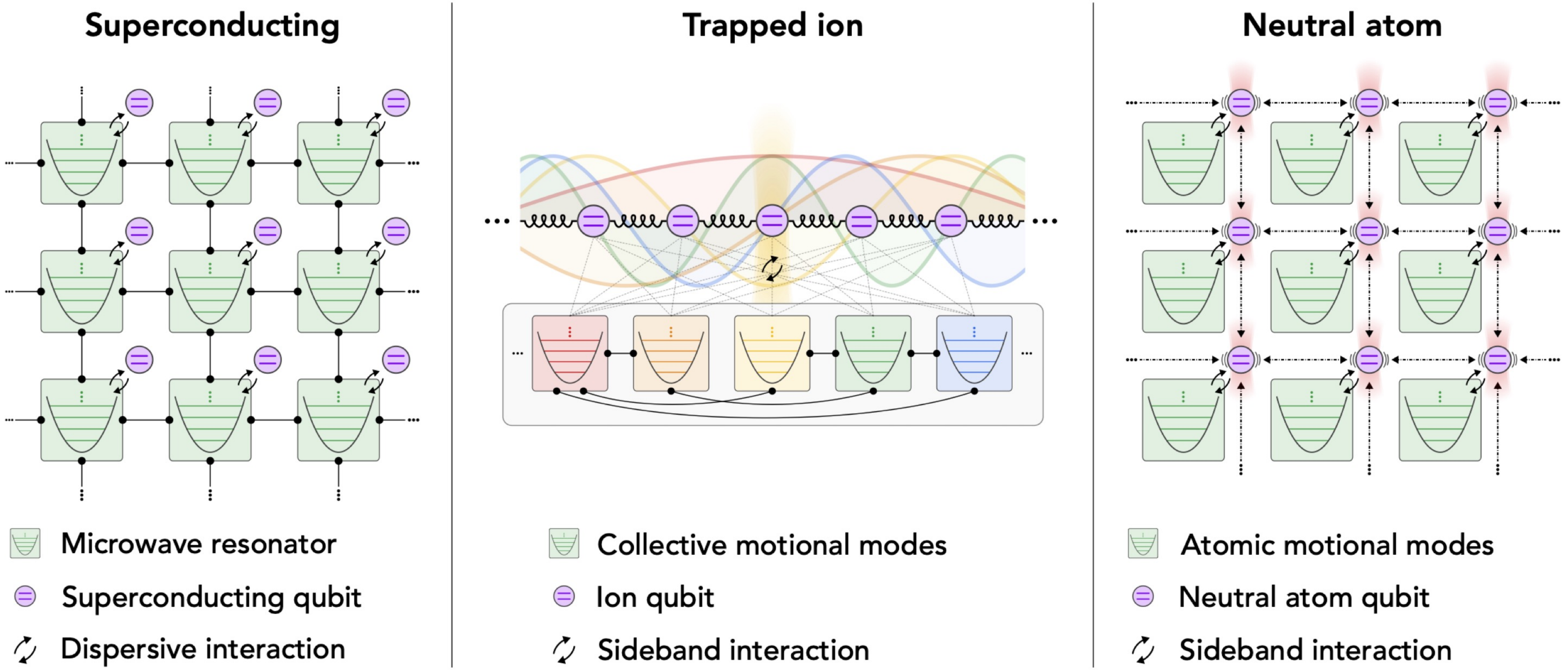






Araz, Grau, Montgomery, Ringer `24

Qubits and qumodes with different platforms



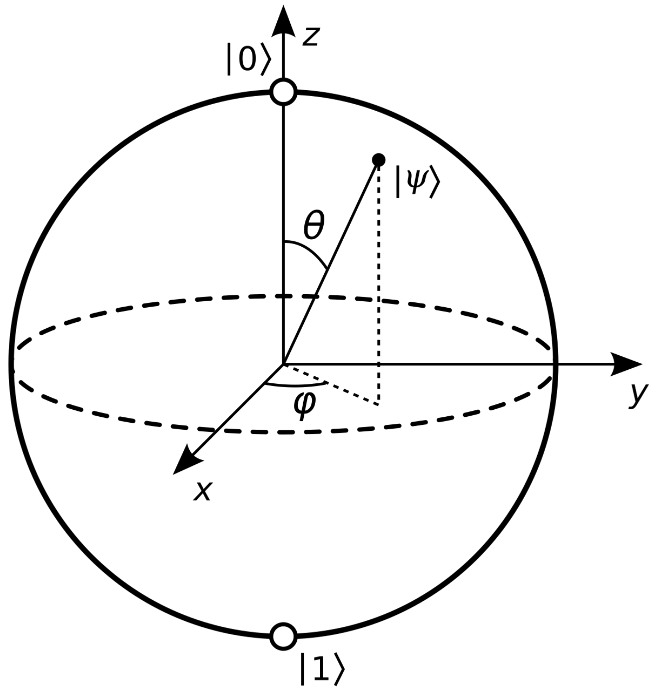
Platform dependence: Native gates, coherence time, connectivity

Chuang, Girvin, Wiebe, et al. `24

Qubit states and operations

Gate type	Operation	Short	Operator
Qubit	Pauli operators		σ^i
	Rotation	$R_i(\theta)$	$e^{i\theta\sigma^i/2}$
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{1}_1-Z_1)(\mathbb{1}_2-X_2)}$
Qumode	Rotation	$R(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$
	Fourier	F	$e^{i\frac{\pi}{2}\hat{a}^\dagger\hat{a}}$
	Displacement	$D(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$
	Single-mode squeezing	$S(z)$	$e^{(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Beam splitter	$BS(z)$	$e^{z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger}$
	Kerr	$K(z)$	$e^{iz(\hat{a}^\dagger\hat{a})^2}$
	Cross-Kerr	$CK(z)$	$e^{iz\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$
	Quadratic phase	$P(\theta)$	$e^{i\frac{\theta}{2}\hat{q}^2}$
	Cubic phase	$V(\theta)$	$e^{i\frac{\theta}{3}\hat{q}^3}$
Hybrid	Red sideband/Jaynes Cummings	$RSB(z)$	$e^{iz\hat{a}X^++iz^*\hat{a}^\dagger X^-}$
	Blue sideband/Anti-Jaynes Cummings	$BSB(z)$	$e^{iz\hat{a}^\dagger X^++iz^*\hat{a}X^-}$
	Controlled rotation	$CR(\theta)$	$e^{i\theta Z\hat{a}^\dagger\hat{a}}$
	Controlled displacement	$CD(z)$	$e^{Z(z\hat{a}^\dagger-z^*\hat{a})}$
	Controlled squeezing	$CS(z)$	$e^{Z(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Controlled beam splitter	$CBS(z)$	$e^{Z(z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger)}$

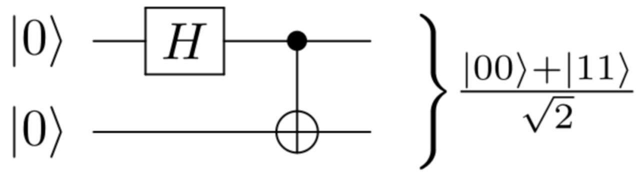
Gate = Unitary operation



Superposition

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

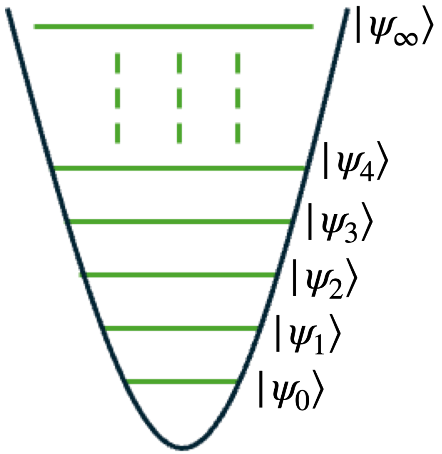
Entanglement



Qumode states and operations

[Lloyd and Braunstein `99]

Gate type	Operation	Short	Operator
Qubit	Pauli operators		σ^i
	Rotation	$\mathrm{R}_i(\theta)$	$e^{i\theta\sigma^i/2}$
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{1}_1-Z_1)(\mathbb{1}_2-X_2)}$
Qumode	Rotation	$\mathrm{R}(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$
	Fourier	F	$e^{i\frac{\pi}{2}\hat{a}^\dagger\hat{a}}$
	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$
	Single-mode squeezing	$\mathrm{S}(z)$	$e^{(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Beam splitter	$\mathrm{BS}(z)$	$e^{z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger}$
	Kerr	$\mathrm{K}(z)$	$e^{iz(\hat{a}^\dagger\hat{a})^2}$
	Cross-Kerr	$\mathrm{CK}(z)$	$e^{iz\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$
	Quadratic phase	$\mathrm{P}(\theta)$	$e^{i\frac{\theta}{2}\hat{q}^2}$
	Cubic phase	$\mathrm{V}(\theta)$	$e^{i\frac{\theta}{3}\hat{q}^3}$
Hybrid	Red sideband/Jaynes Cummings	$\mathrm{RSB}(z)$	$e^{iz\hat{a}X^++iz^*\hat{a}^\dagger X^-}$
	Blue sideband/Anti-Jaynes Cummings	$\mathrm{BSB}(z)$	$e^{iz\hat{a}^\dagger X^++iz^*\hat{a}X^-}$
	Controlled rotation	$\mathrm{CR}(\theta)$	$e^{i\theta Z\hat{a}^\dagger\hat{a}}$
	Controlled displacement	$\mathrm{CD}(z)$	$e^{Z(z\hat{a}^\dagger-z^*\hat{a})}$
	Controlled squeezing	$\mathrm{CS}(z)$	$e^{Z(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Controlled beam splitter	$\mathrm{CBS}(z)$	$e^{Z(z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger)}$



$$a, a^\dagger \Longleftrightarrow \hat{q}, \hat{p}$$

$$[a, a^\dagger] = 1 \Longleftrightarrow [\hat{q}, \hat{p}] = i$$

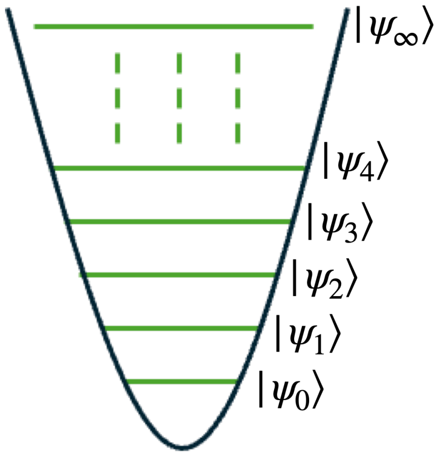
Ladder operators

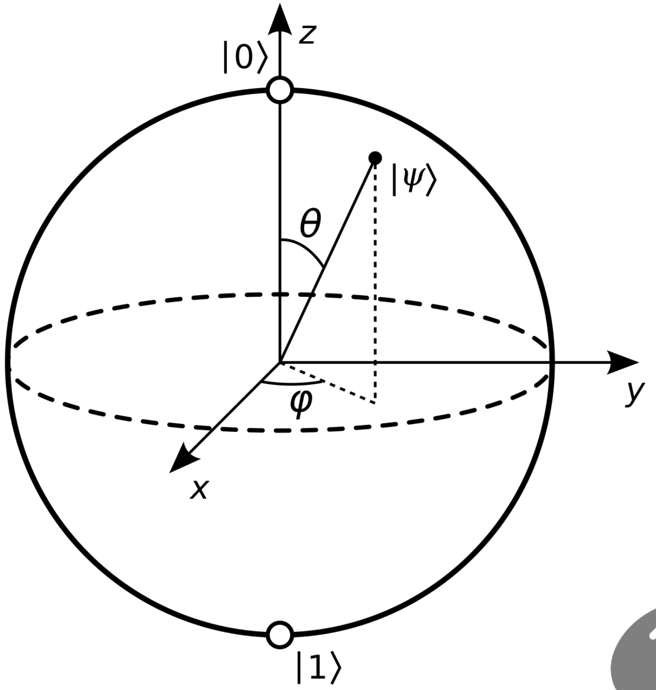
Quadratures

$$|n\rangle \propto (a^\dagger)^n|0\rangle$$

Hybrid states and operations

Gate type	Operation	Short	Operator
Qubit	Pauli operators		σ^i
	Rotation	$\mathrm{R}_i(\theta)$	$e^{i\theta\sigma^i/2}$
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{I}_1 - Z_1)(\mathbb{I}_2 - X_2)}$
Qumode	Rotation	$\mathrm{R}(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$
	Fourier	F	$e^{i\frac{\pi}{2}\hat{a}^\dagger\hat{a}}$
	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger - z^*\hat{a}}$
	Single-mode squeezing	$\mathrm{S}(z)$	$e^{(z^*\hat{a}\hat{a} - z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Beam splitter	$\mathrm{BS}(z)$	$e^{z\hat{a}^\dagger\hat{b} - z^*\hat{a}\hat{b}^\dagger}$
	Kerr	$\mathrm{K}(z)$	$e^{iz(\hat{a}^\dagger\hat{a})^2}$
	Cross-Kerr	$\mathrm{CK}(z)$	$e^{iz\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$
	Quadratic phase	$\mathrm{P}(\theta)$	$e^{i\frac{\theta}{2}\hat{q}^2}$
	Cubic phase	$\mathrm{V}(\theta)$	$e^{i\frac{\theta}{3}\hat{q}^3}$
Hybrid	Red sideband/Jaynes Cummings	$\mathrm{RSB}(z)$	$e^{iz\hat{a}X^+ + iz^*\hat{a}^\dagger X^-}$
	Blue sideband/Anti-Jaynes Cummings	$\mathrm{BSB}(z)$	$e^{iz\hat{a}^\dagger X^+ + iz^*\hat{a}X^-}$
	Controlled rotation	$\mathrm{CR}(\theta)$	$e^{i\theta Z\hat{a}^\dagger\hat{a}}$
	Controlled displacement	$\mathrm{CD}(z)$	$e^{Z(z\hat{a}^\dagger - z^*\hat{a})}$
	Controlled squeezing	$\mathrm{CS}(z)$	$e^{Z(z^*\hat{a}\hat{a} - z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Controlled beam splitter	$\mathrm{CBS}(z)$	$e^{Z(z\hat{a}^\dagger\hat{b} - z^*\hat{a}\hat{b}^\dagger)}$





Gaussian is (mostly) equivalent to Clifford

$$S = Z^{1/2}$$

$$H$$

$$\mathrm{CNOT}$$

Clifford gates

$\Leftrightarrow$

$$\mathrm{BS}(\theta,\phi) = e^{-i\frac{\theta}{2}(e^{i\phi}a^\dagger b + e^{-i\phi}ab^\dagger)}$$

$$S(\zeta) = e^{\frac{1}{2}(\zeta^* a^2 - \zeta a^{\dagger 2})}$$

$$R(\theta) = e^{-i\theta\hat{n}}$$

$$\mathrm{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$

Gaussian gates

Universality

Qubit

$$S = Z^{1/2}$$

$$H$$

$$\mathrm{CNOT}$$

$$T = e^{i\frac{\pi}{8}Z}$$

Qumode

$$\mathrm{BS}(\theta,\phi) = e^{-i\frac{\theta}{2}(e^{i\phi}a^\dagger b + e^{-i\phi}ab^\dagger)}$$

$$S(\zeta) = e^{\frac{1}{2}(\zeta^* a^2 - \zeta a^{\dagger 2})}$$

$$R(\theta) = e^{-i\theta\hat{n}}$$

$$\mathrm{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$

$$V(\gamma) = e^{i\frac{\gamma}{3}\hat{x}^3}$$

Universal gate set

Minimal set of gates to create any other

	Instruction Set Name	Minimum gate set
Linear oscillator control	Gaussian	$\mathcal{G} = \{D(\alpha), S(\zeta), \mathrm{BS}(\theta, \varphi) \text{ or } \mathrm{TMS}(r, \varphi)\}$
Universal oscillator control	Cubic	$\mathcal{G} + U_3(z)$
	Quartic	$\mathcal{G} + U_4(z)$
	SNAP	$\{D(\alpha), \mathrm{SNAP}(\vec{\varphi}), \mathrm{BS}(\theta, \varphi) \text{ or } \mathrm{TMS}(r, \varphi)\}$
Universal hybrid control	Phase-Space	$\{\mathrm{CD}(\beta), R_\varphi(\theta), \mathrm{BS}(\theta, \varphi)\}$
	Fock-Space	$\left\{\mathrm{SQR}(\vec{\theta}, \vec{\varphi}), D(\alpha), \mathrm{BS}(\theta, \varphi)\right\}$
	Sideband	$\{R_\varphi(\theta), \mathrm{JC}(\theta), \mathrm{BS}(\theta, \varphi)\}$

Lie brakets of small control set generate universality

$$[\hat{x}^n X, \hat{p}^m Y] = i[\hat{x}^n \hat{p}^m + \hat{p}^m \hat{x}^n] Z$$

Chuang, Wiebe, Girvin et al `24

Hybrid example:

$$\mathrm{CD}_z(\alpha) \equiv e^{\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right)\otimes Z}$$

$$\mathrm{CD}_y(\alpha) \equiv e^{\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right)\otimes Y}$$

$$\mathrm{CD}_x(\alpha) \equiv e^{\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right)\otimes X}$$

$$\mathrm{CD}_x(\alpha)\mathrm{CD}_y(\alpha)\mathrm{CD}_x(-\alpha)\mathrm{CD}_y(-\alpha)$$

$$e^{2i\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right)^2\otimes Z} + \mathcal{O}(\alpha^4)$$

Repeat for higher order polynomials

Lots of details and additional info

Hybrid Oscillator-Qubit Quantum Processors: Instruction Set Architectures, Abstract Machine Models, and Applications

Yuan Liu,[1,2,3,*] Shraddha Singh,[4,5,*] Kevin C. Smith,[6,4,5,*] Eleanor Crane,[1] John M. Martyn,[1] Alec Eickbusch,[4,5,†] Alexander Schuckert,[7] Richard D. Li,[4,5] Jasmine Sinanan-Singh,[1] Micheline B. Soley,[8] Takahiro Tsunoda,[4,5] Isaac L. Chuang,[1,9] Nathan Wiebe,[10,11,12] and Steven M. Girvin[4,5,‡]

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[6]*Brookhaven National Laboratory, Upton, NY 11973, USA*
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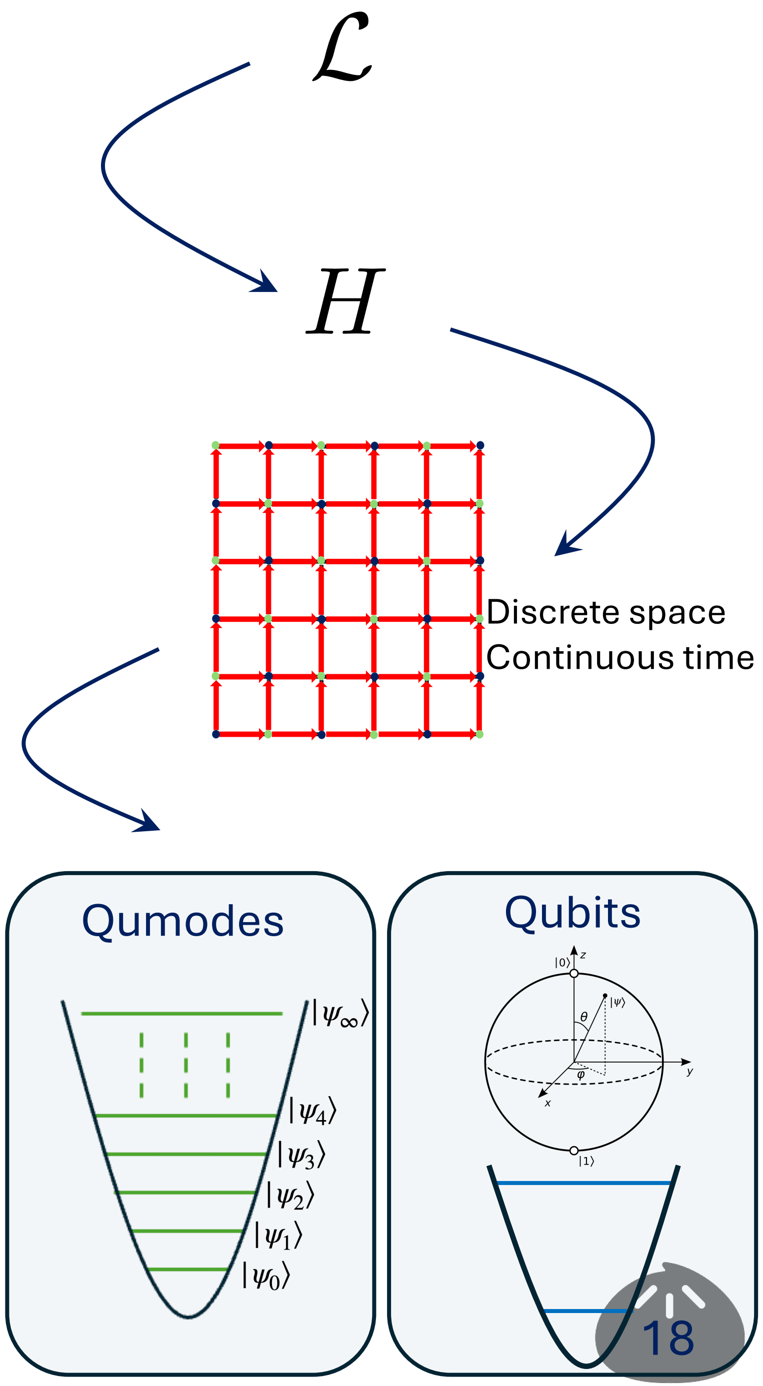
[11]*Canadian Institute for Advanced Research, Toronto ON, Canada*
[12]*Pacific Northwest National Laboratory, Richland WA, USA*

Simulations of physical theories

The steps for the formulation

- The QFT Lagrangian (continuum)
- Switch to Hamiltonian formulation (continuum)
  - Legendre transform
  - Choose gauge condition (gauge fixing)
- Formulate a discrete version of the Hamiltonian
  - Staggered fermion formulation (Kogut-Susskind)
  - Compact gauge dofs at each link
- Map to qubits and qumodes
  - Fermions $\longrightarrow$ Jordan-Wigner transformation (for example)
  - Bosons $\longrightarrow$ QHO spectrum (qumodes)



Some selected challenges to tackle

- Representation of the Hamiltonian
  - **Hybrid qubit-qumode**, qubit-only, qumode-only
- Ground state preparation (or even higher states)
  - **QITE**, VQE (regular, ADAPT, SC-ADAPT,…), **non unitary ansatz**, adiabatic state preparation,…
- Time evolution (Real time dynamics)
  - **Survival probabilities**, correlations, …

Many more …

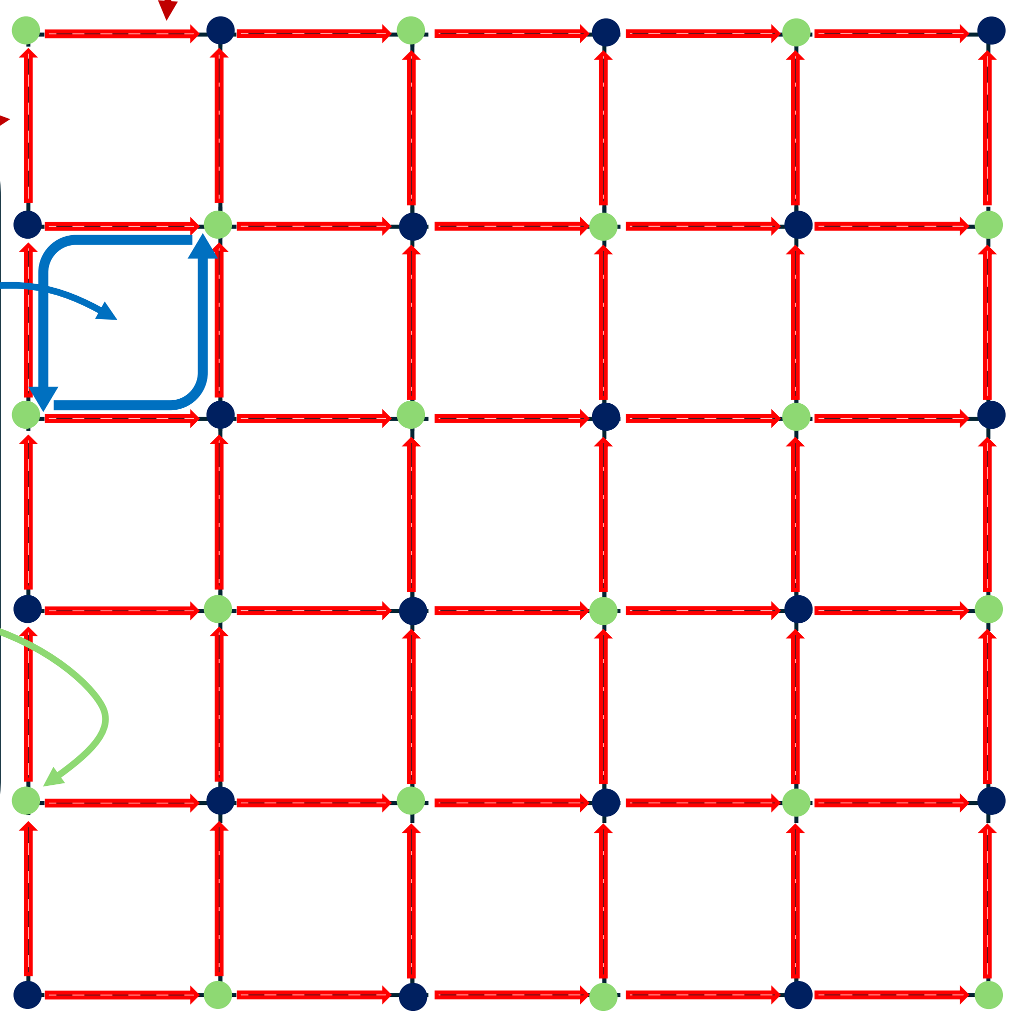
QED in 2+1

$$H_E = \frac{g^2}{2}\sum_{\mathbf{n}}\left(E^2_{\mathbf{n},\mathbf{e}_x} + E^2_{\mathbf{n},\mathbf{e}_y}\right),$$

$$H_B = -\frac{1}{2g^2a^2}\sum_{\mathbf{n}}\left(P_{\mathbf{n}} + P^\dagger_{\mathbf{n}}\right),$$

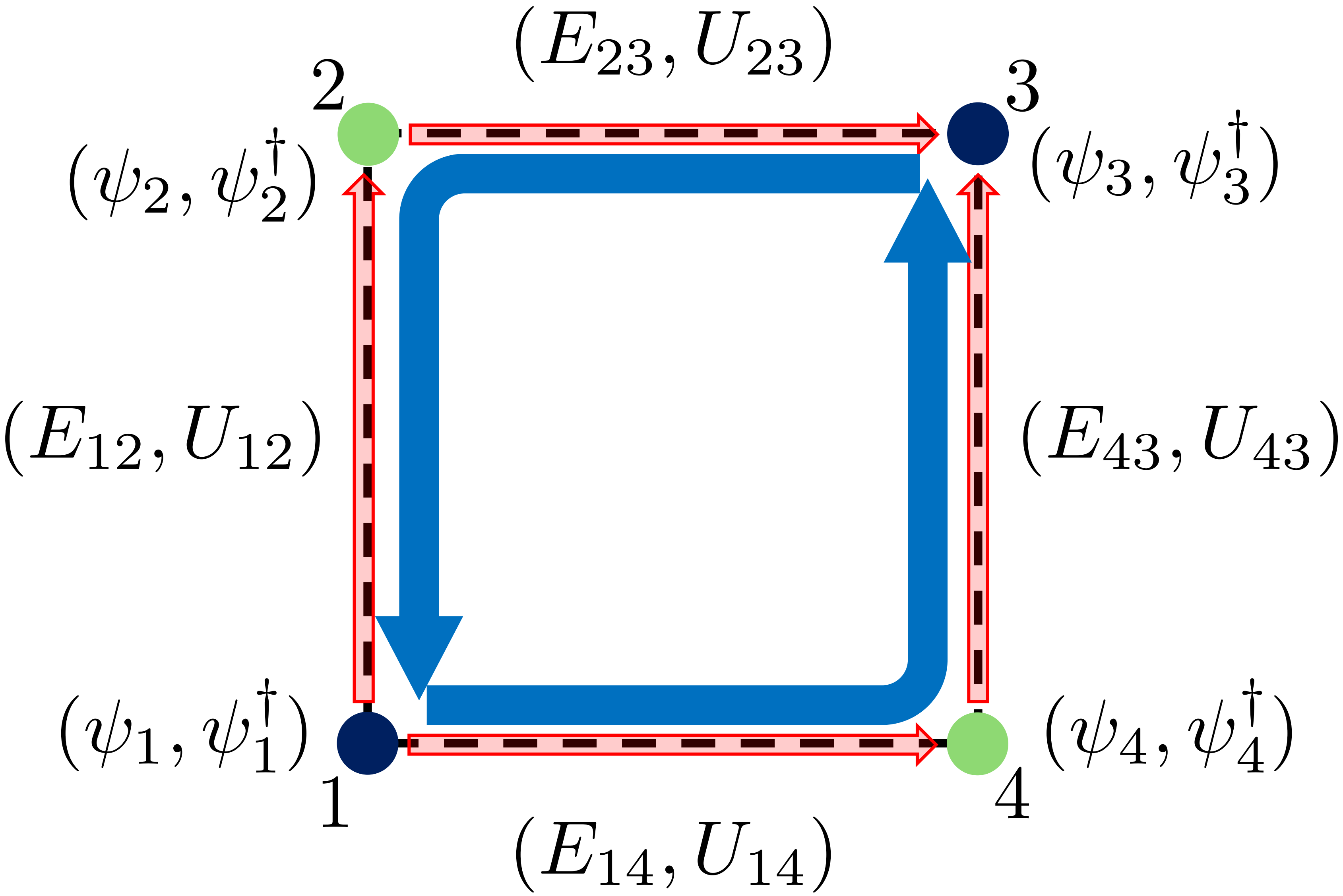
$$H_M = m\sum_{\mathbf{n}}(-1)^{n_x+n_y}\Psi^\dagger_{\mathbf{n}}\Psi_{\mathbf{n}},$$

$$H_K = \kappa\sum_{\mathbf{n}}\sum_{\mu=x,y}\left[\Psi^\dagger_{\mathbf{n}}\left(U^\dagger_{\mathbf{n},e_\mu}\right)^q\Psi_{\mathbf{n}+\mathbf{e}_\mu} + \text{h.c.}\right]$$



$$U_j = e^{i\chi_j} \in U(1)$$

One plaquette



Bosonic CCR

$$[E_j, \chi_k] = -i\delta_{jk}$$

$$\updownarrow$$

$$[E_j, U_k] = U_k \delta_{jk}$$

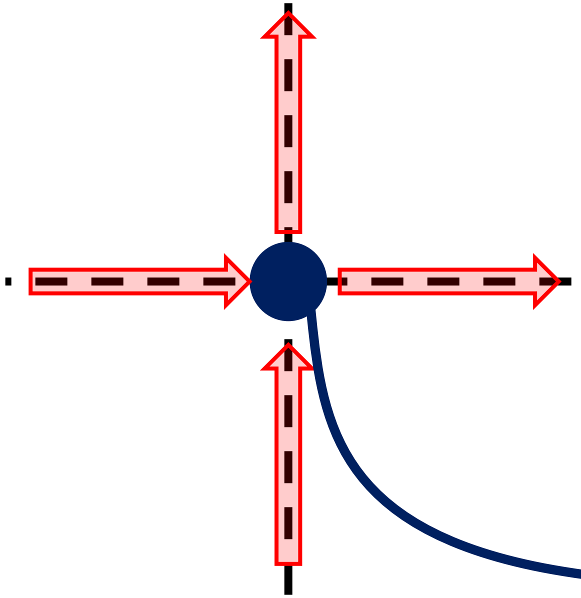
Fermionic CAR

$$\{\psi_j, \psi_k^\dagger\} = \delta_{jk}$$

Gauss Law

$$\vec{\nabla} \cdot \vec{E} = \rho$$

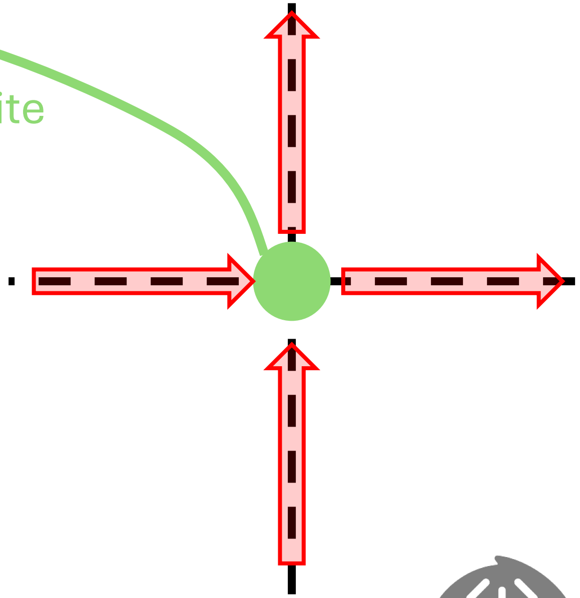
$$\boxed{E_{\mathbf{n},\hat{e}_x} + E_{\mathbf{n},\hat{e}_y} - E_{\mathbf{n},-\hat{e}_x} - E_{\mathbf{n},-\hat{e}_y} = Q_{\mathbf{n}}}$$



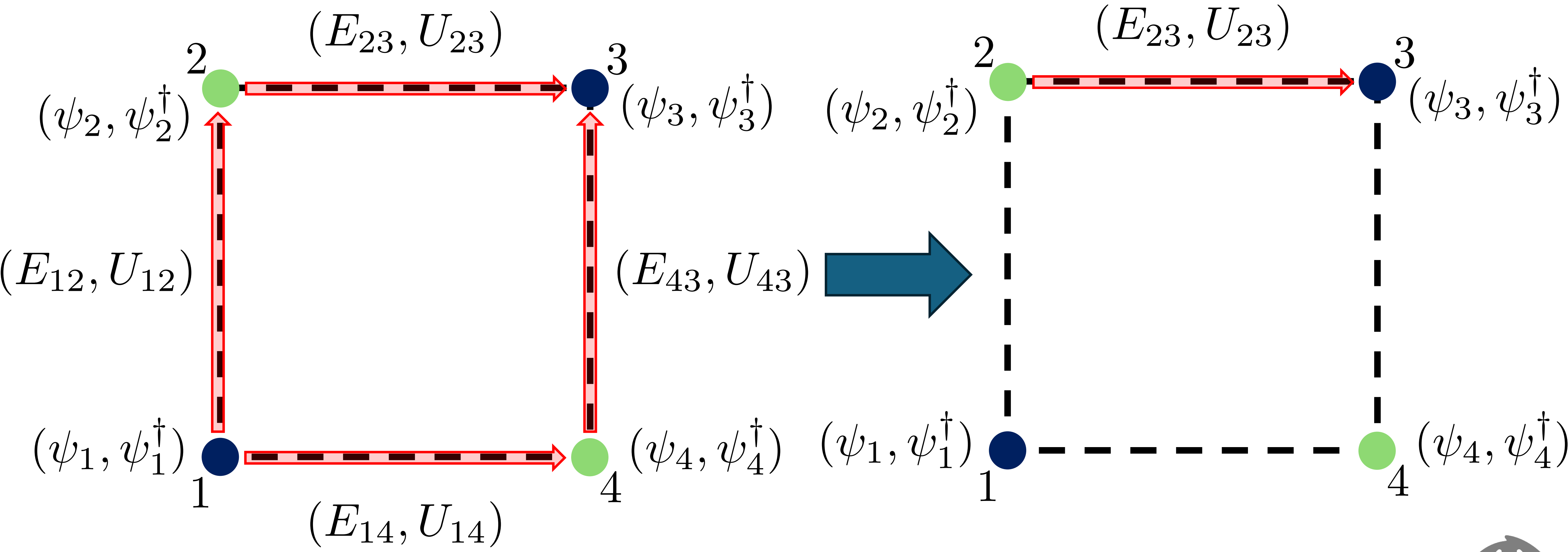
Odd site

$$Q_{\mathbf{n}} = \psi^{\dagger}_{\mathbf{n}}\psi_{\mathbf{n}} - \frac{\mathbb{I}}{2}\left[1 - (-1)^{n_x+n_y}\right]$$

Even site



Gauss Law



(E_{23}, U_{23})
2
3
$(\psi_2, \psi_2^\dagger)$
$(\psi_3, \psi_3^\dagger)$
(E_{12}, U_{12})
(E_{43}, U_{43})
$(\psi_1, \psi_1^\dagger)$
$(\psi_4, \psi_4^\dagger)$
1
4
(E_{14}, U_{14})
(E_{23}, U_{23})
2
3
$(\psi_2, \psi_2^\dagger)$
$(\psi_3, \psi_3^\dagger)$
$(\psi_1, \psi_1^\dagger)$
$(\psi_4, \psi_4^\dagger)$
1
4

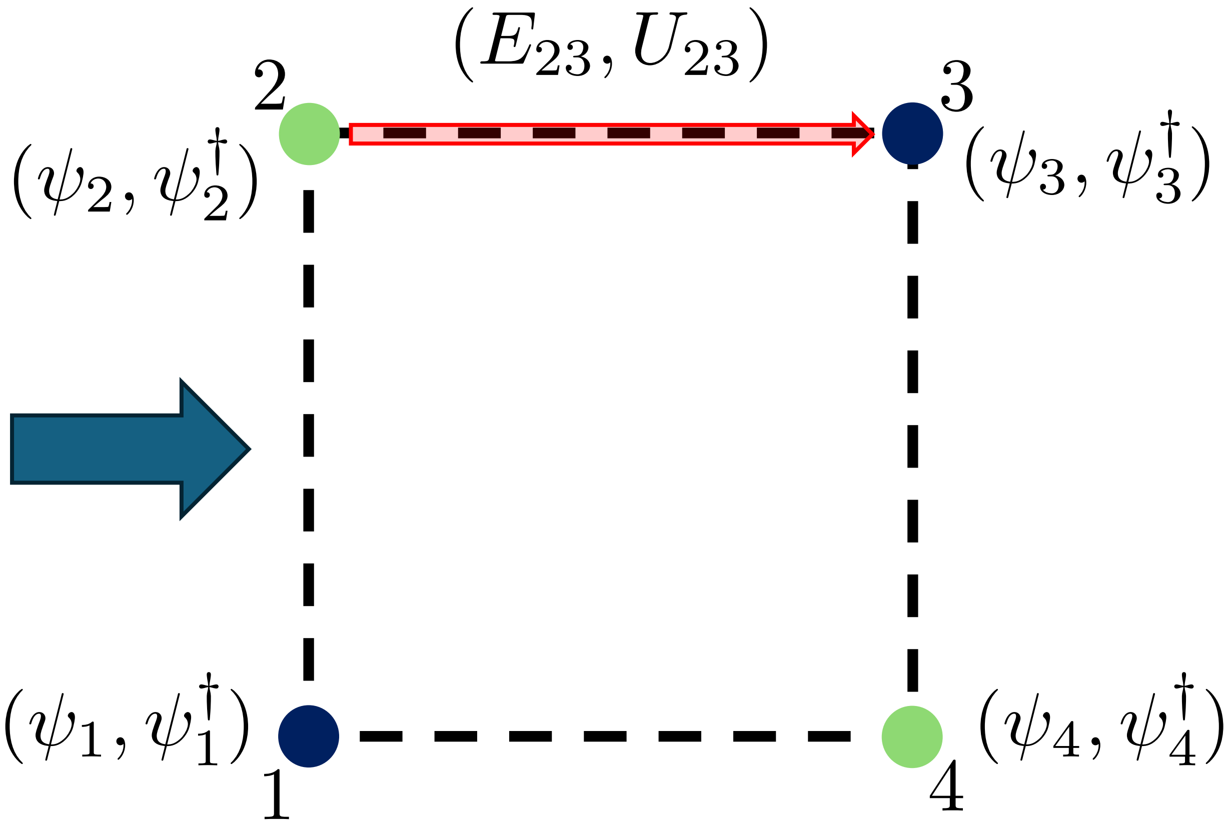
Gauss Law

- Only dynamical dofs survive
- Resource reduction

Only **one** dynamical gauge link **per plaquette**

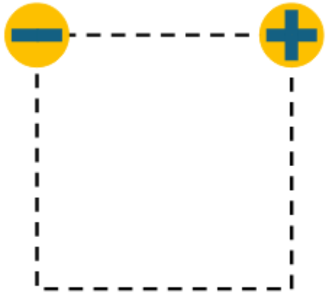
Hamiltonian is block diagonal in total charge sectors

$$\mathcal{H}^{(\text{fermions})}_{\text{physical}} = \mathcal{H}^{(\text{fermions})}\Big|_{Q_{\text{Tot}}=0}$$

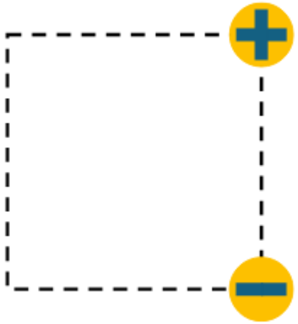


Zero-charge fermion states

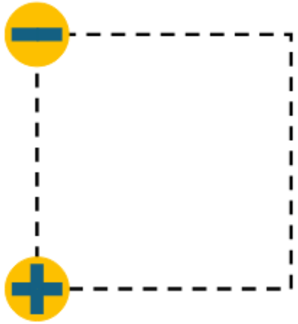
$$|v_1\rangle \equiv |\Omega_f\rangle = |vvvv\rangle$$



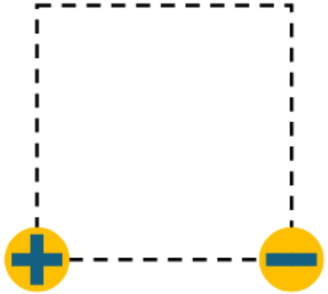
$$, |v_2\rangle \equiv \Psi_2 \Psi_3^\dagger |\Omega_f\rangle = |ve^-e^+v\rangle ,$$



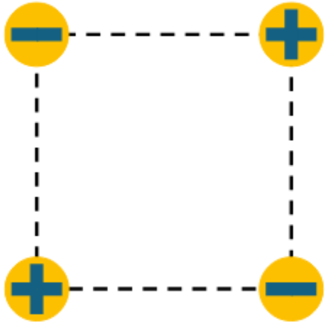
$$|v_3\rangle \equiv \Psi_3^\dagger \Psi_4 |\Omega_f\rangle = |vve^+e^-\rangle$$



$$|v_4\rangle \equiv \Psi_1^\dagger \Psi_2 |\Omega_f\rangle = |e^+e^-vv\rangle$$



$$, |v_5\rangle \equiv \Psi_1^\dagger \Psi_4 |\Omega_f\rangle = |e^+vve^-\rangle$$



$$, |v_6\rangle \equiv \Psi_1^\dagger \Psi_2 \Psi_3^\dagger \Psi_4 |\Omega_f\rangle = |e^+e^-e^+e^-\rangle$$

Only 6 out of $2^4 = 16$

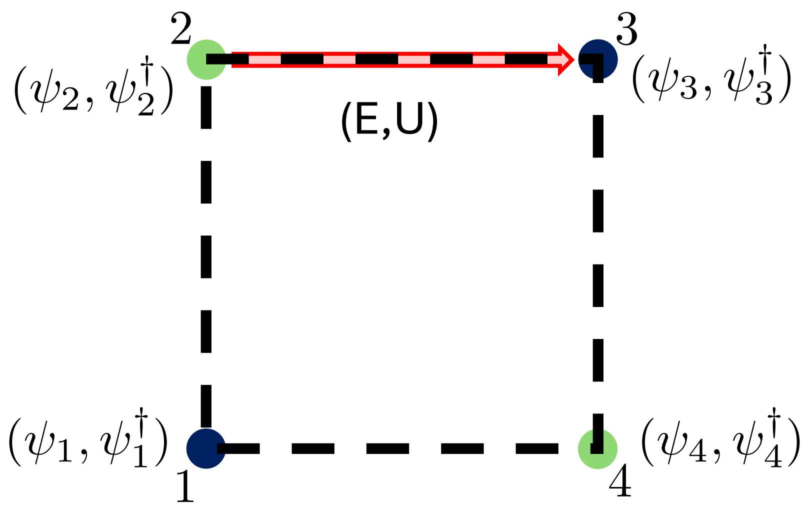
One-plaquette Hamiltonian after Gauss Law

$$H_E = \frac{g^2}{2}\left[E^2 + (E - Q_2)^2 + (E + Q_3)^2 + (E - Q_1 - Q_2)^2\right]$$

$$H_B = -\frac{1}{2g^2}\left(U + U^\dagger - 2\right)$$

$$H_M = m_0 \sum_{i=1}^{4} (-1)^{1+i} \Psi_i^\dagger \Psi_i,$$

$$H_K = \frac{1}{2}\left(\Psi_1^\dagger \Psi_2 + \Psi_1^\dagger \Psi_4 + \Psi_2 U^\dagger \Psi_3^\dagger + \Psi_4 \Psi_3^\dagger\right) + \text{h.c.}$$



Note:

Bosonic CCR

$$[E_j, \chi_k] = -i\delta_{jk}$$

$$\Updownarrow$$

$$[E_j, U_k] = U_k\delta_{jk}$$

There is no finite dimensional faithful representation of the Weyl/CCR algebra

Fermionic CAR

$$\{\psi_j, \psi_k^\dagger\} = \delta_{jk}$$

There exists finite dimensional faithful representations of the CAR algebra

Mapping fermions to qubits

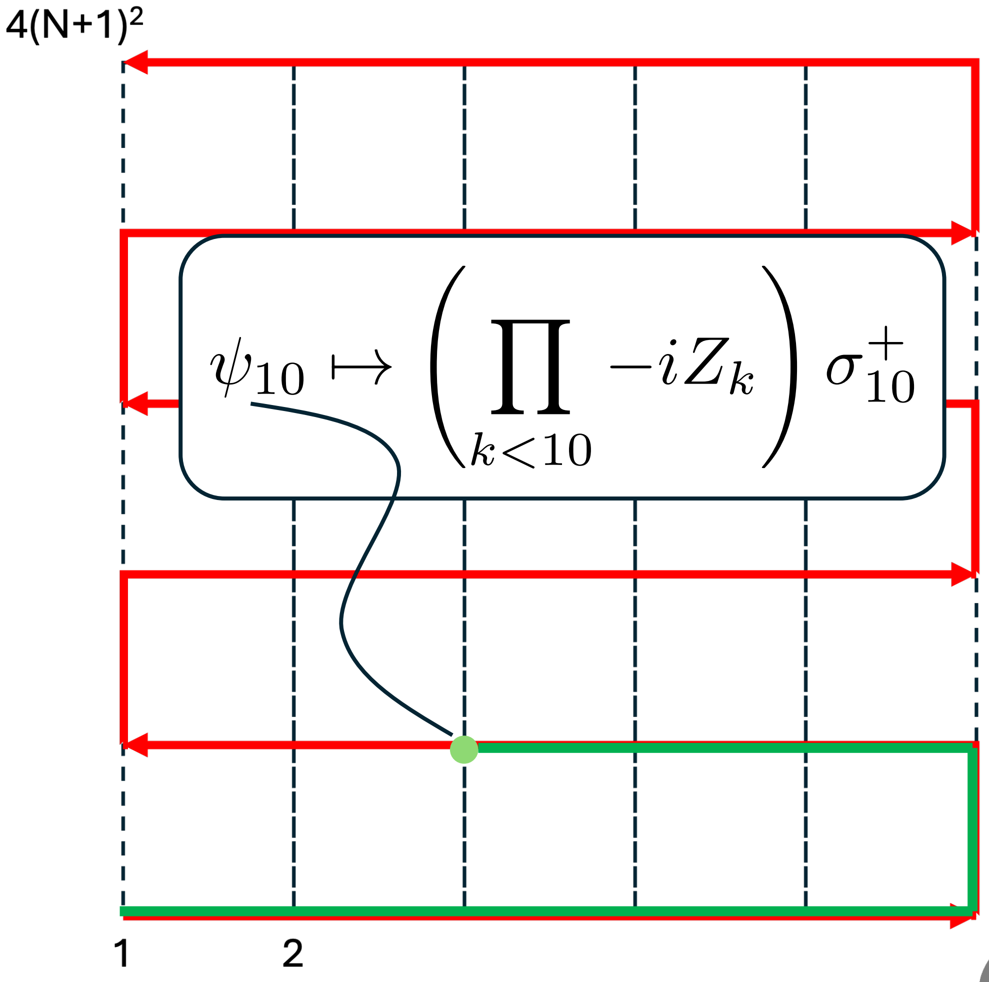
Task: Find a representation of the fermionic CAR in terms of Pauli matrices

$$\{\psi_j, \psi_k^\dagger\} = \delta_{jk}$$

Jordan-Wigner transformation (not unique)

$$\psi_j \mapsto \left(\prod_{k<j} -iZ_k \right) \sigma_j^-$$

$$\psi_j^\dagger \mapsto \left(\prod_{k<j} +iZ_k \right) \sigma_j^+$$



Mapping bosons to qumodes

Task: Find a representation of the bosonic CCR in terms of qumodes

$$[E_j, \chi_k] = -i\delta_{jk} \quad \text{or} \quad [E_j, U_k] = U_k\delta_{jk}$$

Qumodes are inherently non-compact but U(1) is a compact group !!

One possible solution:

Use two qumodes per link variable and then constrain them to the unit circle

$$U_j \mapsto \hat{q}_j^0 + i\hat{q}_j^1$$

and

$$E_j \mapsto J_j \equiv \hat{q}_j^0\hat{q}_j^1 - \hat{q}_j^1\hat{q}_j^0$$

$$(\hat{q}_j^0)^2 + (\hat{q}_j^1)^2 = \mathbb{I}$$

The final Hamiltonian to simulate

$$H_E^{(Q)} = \frac{g^2}{2}\left[J^2 + \boxed{(J-Q_2)^2 + (J+Q_3)^2 + (J-Q_1-Q_2)^2}\right],$$

$$H_B^{(Q)} = \frac{1}{2g^2}[(q^0-1)^2 + (q^1)^2],$$

Qubits and qumodes

$$H_M^{(Q)} = m_0 \sum_{i=1}^{4} (-1)^{1+i} Q_i,$$

$$H_K^{(Q)} = \frac{1}{2}\left(X_1^+ X_2 + \boxed{(q^0 - iq^1) X_3^+ X_2^-} + X_3^+ X_4^- + X_1^+ Z_2 Z_3 X_4^-\right) + \text{h.c.}$$

Hybrid Hamiltonian for a general lattice

Qumodes

$$H_E^{(Q)} = g^2 \sum_{\boldsymbol{n},\boldsymbol{n}'} \left(\boxed{\mathcal{H}^{(2)}_{\boldsymbol{n}\boldsymbol{n}'} J_{\boldsymbol{n}} J_{\boldsymbol{n}'}} + \boxed{\mathcal{H}^{(1)}_{\boldsymbol{n}\boldsymbol{n}'} Q_{\boldsymbol{n}} J_{\boldsymbol{n}'}} + \boxed{\mathcal{H}^{(0)}_{\boldsymbol{n}\boldsymbol{n}'} Q_{\boldsymbol{n}} Q_{\boldsymbol{n}'}} \right) ,$$

$$H_B^{(Q)} = \frac{1}{2g^2} \sum_{\boldsymbol{n}} \boxed{(\boldsymbol{q}_{\boldsymbol{n}+e_y} - \boldsymbol{q}_{\boldsymbol{n}})^2} ,$$

Qubits

Qubit and qumodes

$$H_M^{(Q)} = m_0 \sum_{\boldsymbol{n}} \boxed{(-)^{n_x+n_y} Q_{\boldsymbol{n}}} ,$$

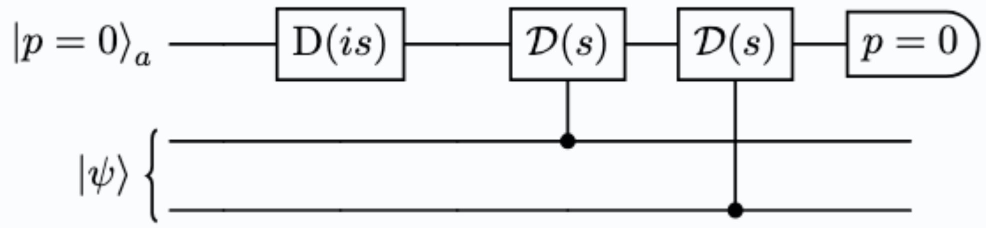
$$H_K^{(Q)} = \frac{1}{2} \sum_{\boldsymbol{n}} \boxed{(q^0_{\boldsymbol{n}} - i q^1_{\boldsymbol{n}}) X^+_{\boldsymbol{n}} X^-_{\boldsymbol{n}+e_x}} + \frac{1}{2} \sum_{\boldsymbol{n}} \boxed{P_{L(\boldsymbol{n},\boldsymbol{n}+e_y)} X^+_{\boldsymbol{n}} X^-_{\boldsymbol{n}+e_y}} + \text{h.c.} ,$$

Enforcing the constraint $(\hat{q}_j^0)^2 + (\hat{q}_j^1)^2 = \mathbb{I}$

IMPORTANTLY $[(\hat{q}_{\mathbf{n}}^0)^2 + (\hat{q}_{\mathbf{n}}^1)^2, H] = 0, \forall \mathbf{n}$

Inner product modification

$$|\psi^{(Q)}_{\text{approx}}(\Lambda)\rangle \propto \int d^2q\, e^{-\frac{\Lambda^2}{g^2}(\boldsymbol{q}^2-1)^2}\psi(\boldsymbol{q})\,|\boldsymbol{q}\rangle\,, \;\; \Lambda = \frac{g}{2}se^r$$



Hamiltonian penalty term

$$H^{(Q)} \mapsto H^{(Q)} + H^{(Q)}_\mu$$

$$H_\mu = \frac{\mu}{2}\sum_{\boldsymbol{n},i}(\boldsymbol{q}^2_{\boldsymbol{n},i} - 1)^2$$

Disfavors unphysical configurations

Time evolution

$$e^{-i\Delta t H^{(Q)}} \approx e^{-i\Delta t H_E^{(Q)}} e^{-i\Delta t H_B^{(Q)}} e^{-i\Delta t H_M^{(Q)}} e^{-i\Delta t H_K^{(Q)}}$$

Trotter-Suzuki formula

$$e^{-itH^{(Q)}} = \left(e^{-i\Delta t H_E^{(Q)}} e^{-i\Delta t H_B^{(Q)}} e^{-i\Delta t H_M^{(Q)}} e^{-i\Delta t H_K^{(Q)}} \right)^{t/\Delta t}$$

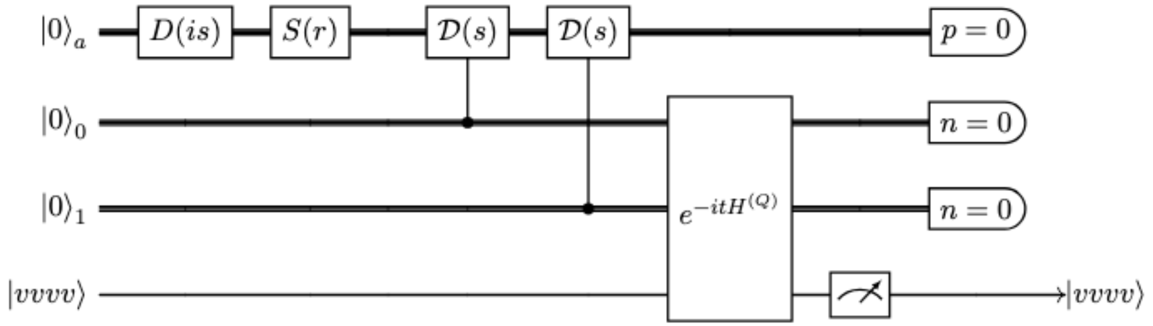
Chain of gates (unitaries) for small time steps

Time evolution

$$e^{-itH^{(Q)}} = \left(e^{-i\Delta t H_E^{(Q)}} e^{-i\Delta t H_B^{(Q)}} e^{-i\Delta t H_M^{(Q)}} e^{-i\Delta t H_K^{(Q)}}\right)^{t/\Delta t}$$

Vacuum of the free theory (not ground state)

$$|V\rangle \equiv |vvvv\rangle \otimes e^{-\frac{\Lambda^2}{g^2}(\hat{\mathbf{q}}^2-1)^2}|0\rangle_0|0\rangle_1$$



Survival probability

$$|\langle V|e^{-itH^{(Q)}}|V\rangle|^2$$

Time evolution

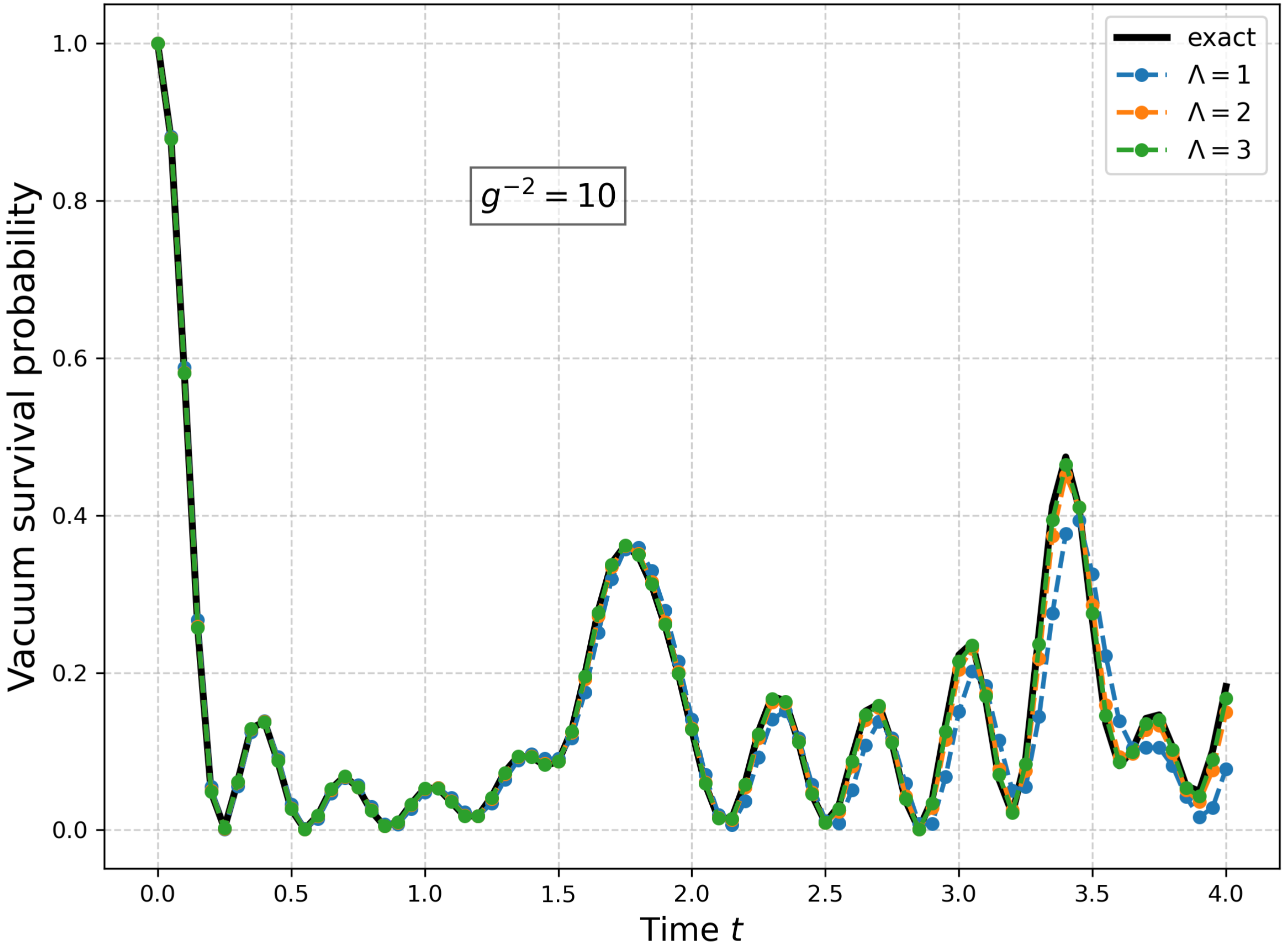
$$e^{-itH^{(Q)}} = \left(e^{-i\Delta t H_E^{(Q)}} e^{-i\Delta t H_B^{(Q)}} e^{-i\Delta t H_M^{(Q)}} e^{-i\Delta t H_K^{(Q)}}\right)^{t/\Delta t}$$

Vacuum of the free theory (not ground state)

$$|V\rangle \equiv |vvvv\rangle \otimes e^{-\frac{\Lambda^2}{g^2}(\hat{\mathbf{q}}^2-1)^2}|0\rangle_0|0\rangle_1$$

Survival probability

$$|\langle V|e^{-itH^{(Q)}}|V\rangle|^2$$



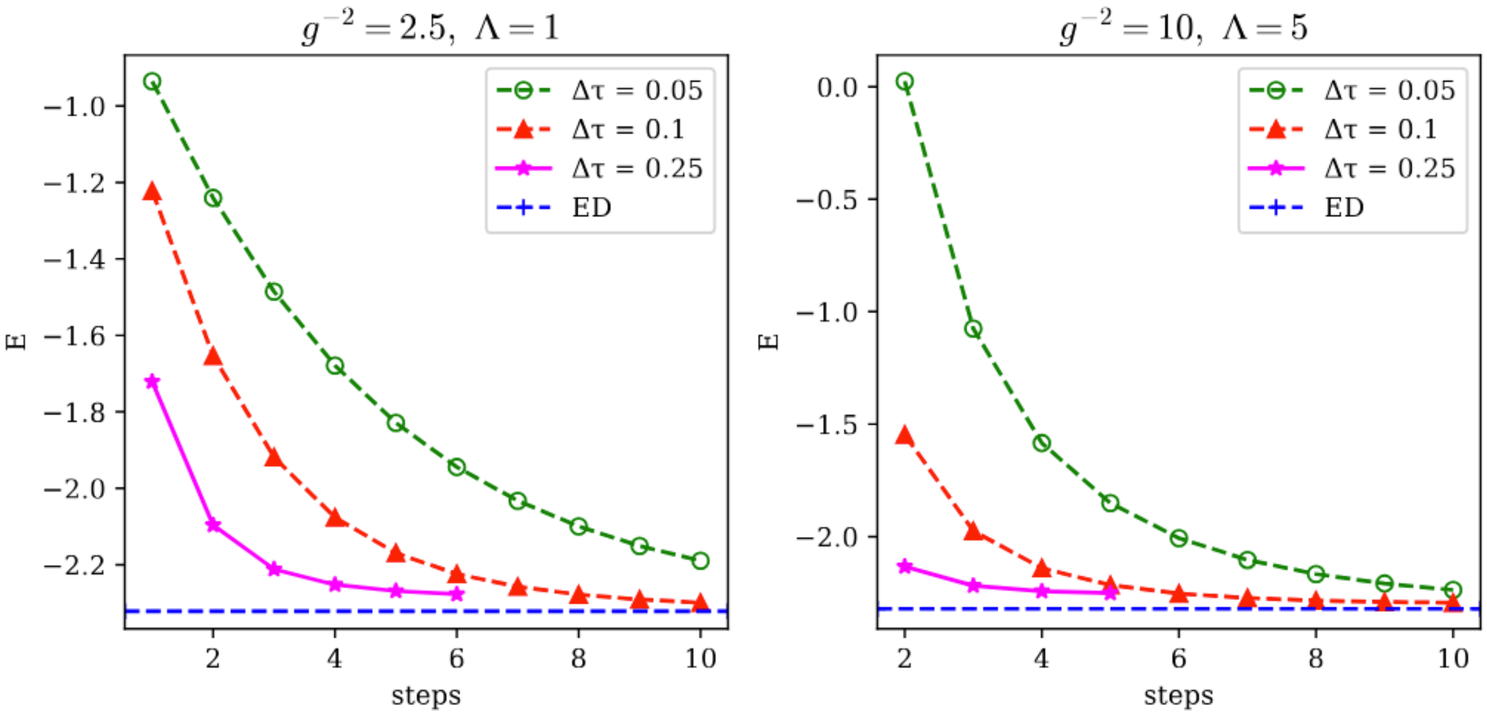
Ground state preparation (Method 1)

Quantum Imaginary Time Evolution (QITE) algorithm

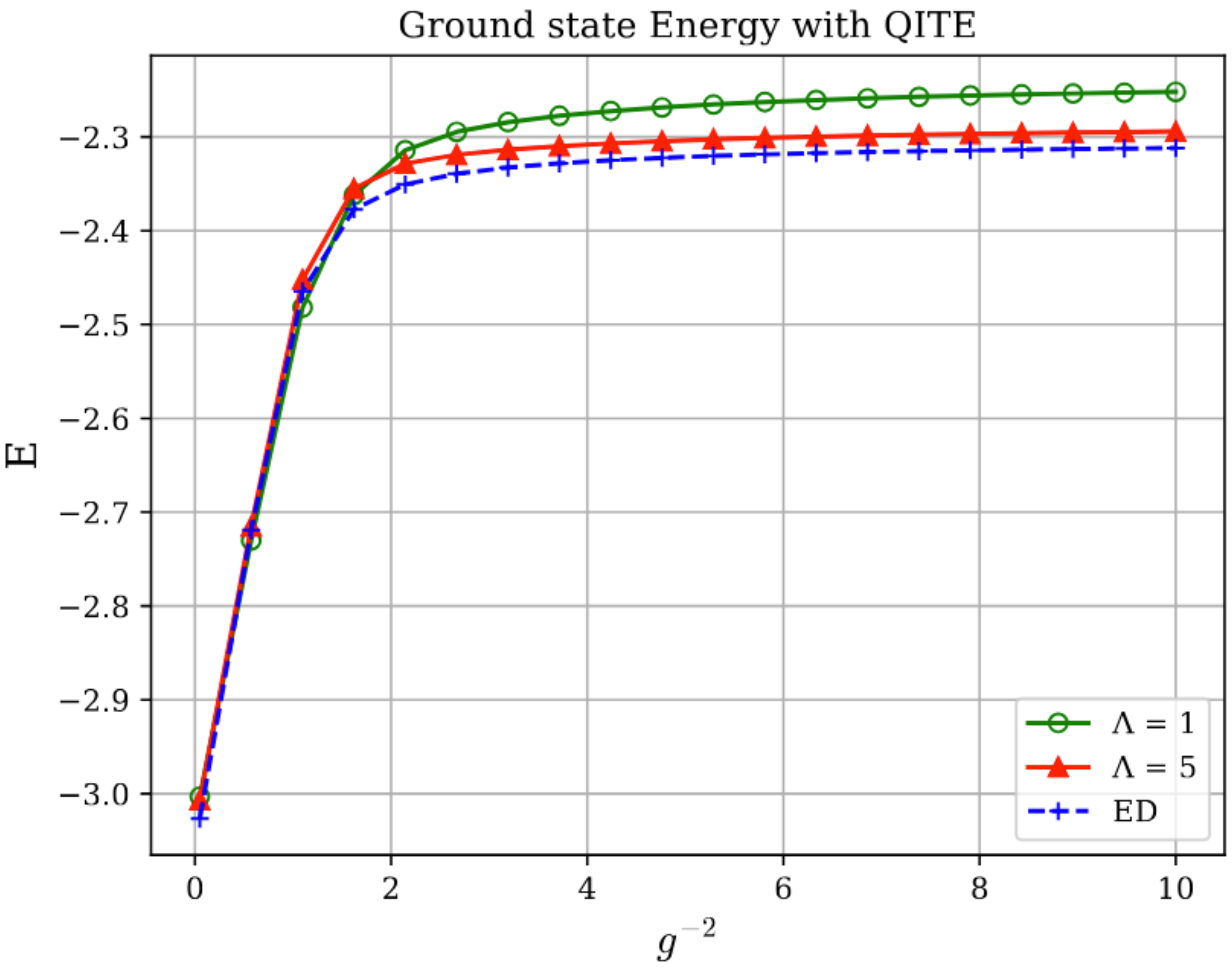
$$\frac{\langle\psi|\, e^{-2\tau H} H\, |\psi\rangle}{\langle\psi|\, e^{-2\tau H}\, |\psi\rangle} = E_0 + \mathcal{O}(e^{-2\tau(E_1 - E_0)})$$

We show the necessary gates and measurements to go to imaginary time

2 METHODS*



Ground state preparation (Method 1)

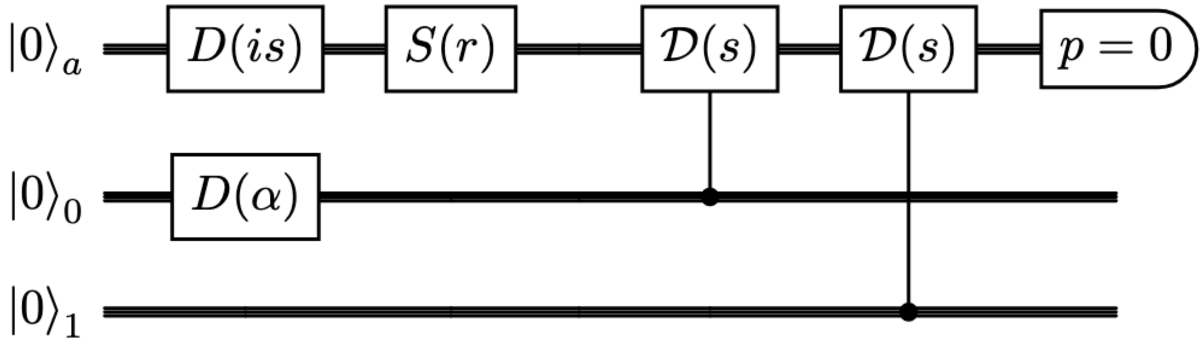


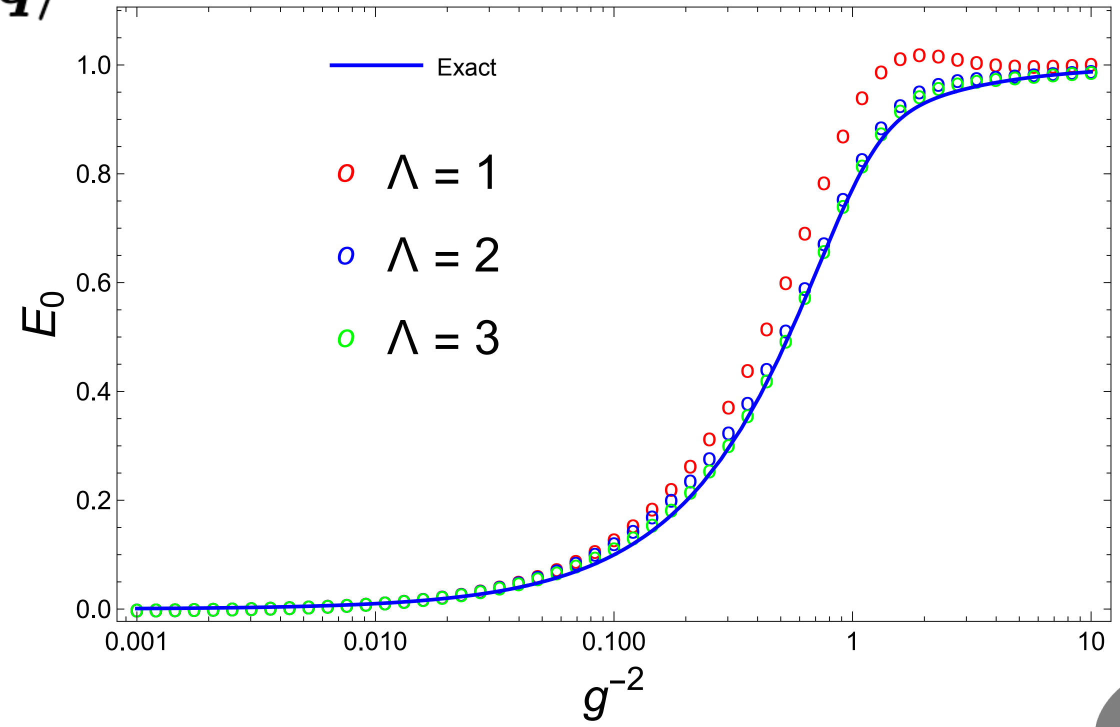
Ground state preparation (Method 2)

Non-unitary ansatz (showing for pure U(1) gauge theory)

$$|\psi_0(\alpha,\Lambda)\rangle \propto \int d^2q\, e^{-\frac{\Lambda^2}{g^2}\left(\boldsymbol{q}^2-1\right)^2}\, \boxed{e^{\alpha q^0}}\, e^{-\frac{1}{2}\boldsymbol{q}^2}\, |\boldsymbol{q}\rangle$$

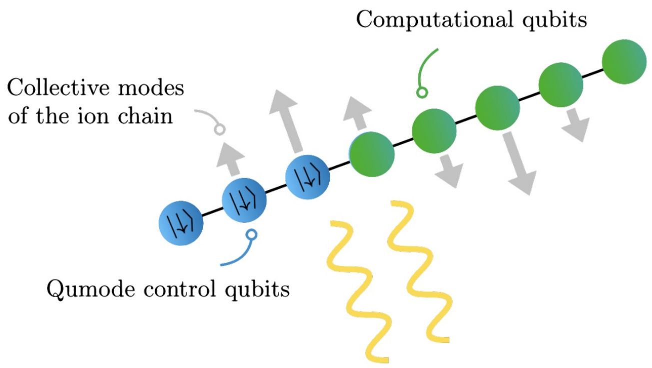
Variational parameter



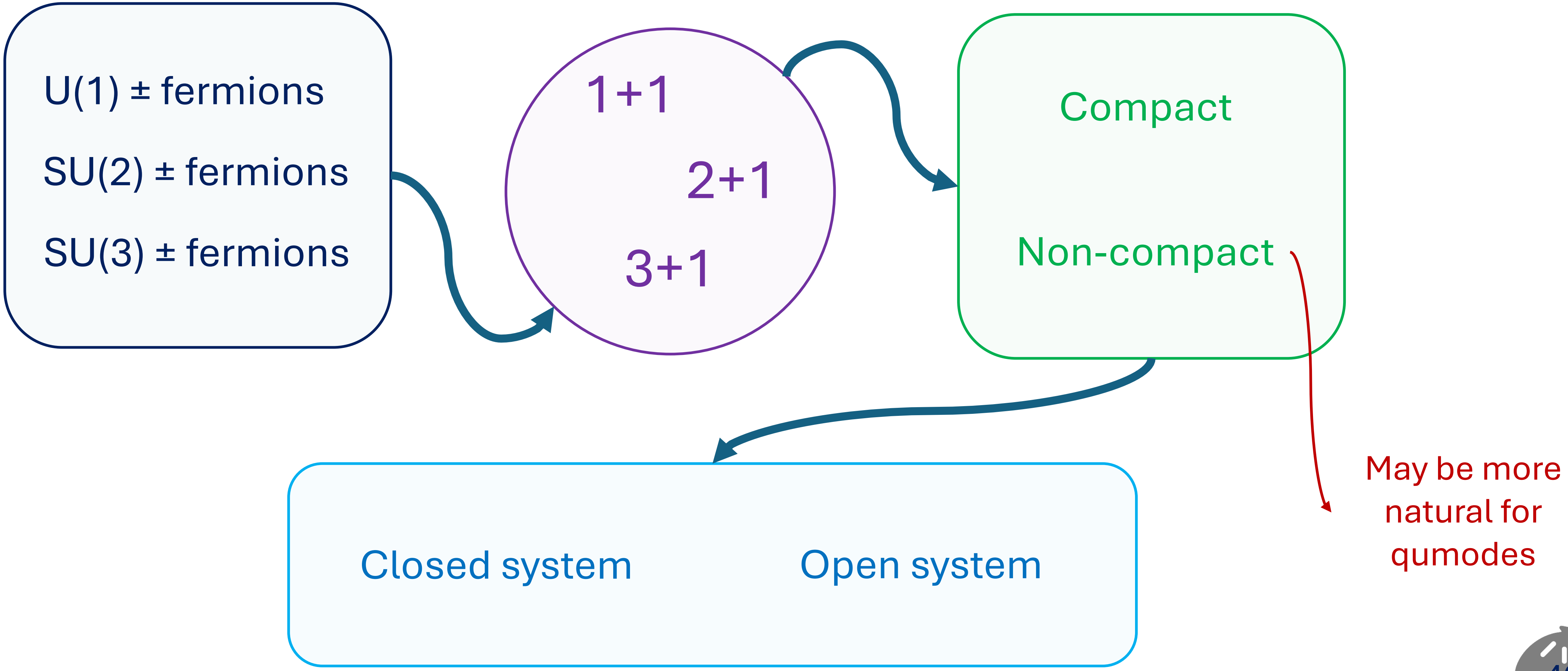


Summary

- Hybrid qubit/qumode map for gauge theories with fermions
- Gauge invariance is preserved
- Feasibility with different hybrid platforms
- Determination of Time evolution and QITE algorithms
- Test some ground state preparation algorithms

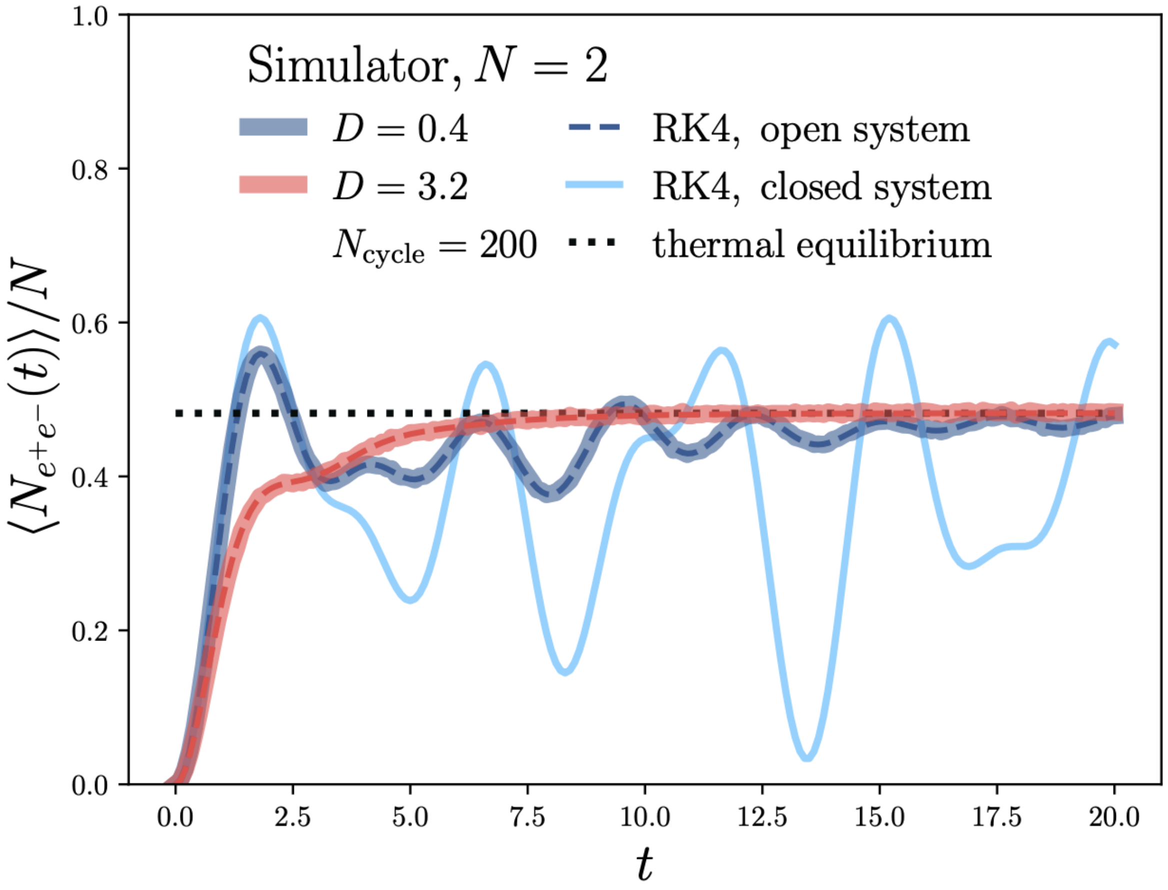


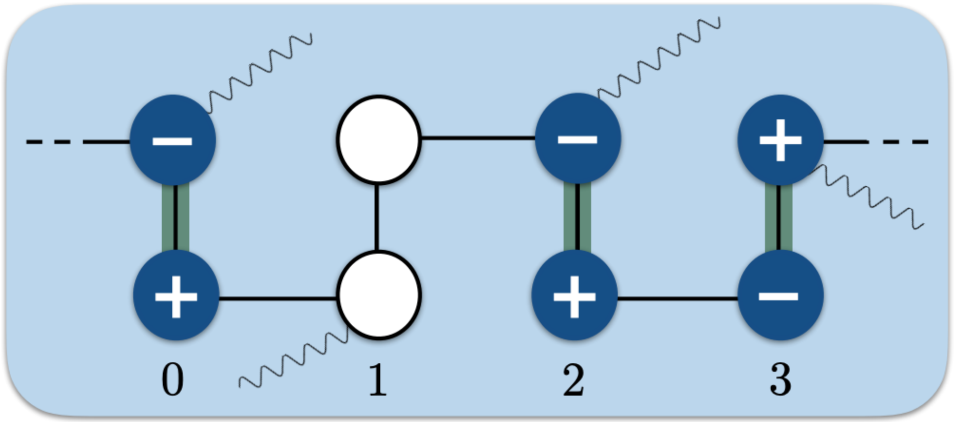
Future: combination of



Non-equilibrium dynamics and thermalization

$$\frac{\mathrm{d}\rho_S(t)}{\mathrm{d}t} = -i\left[H_S, \rho_S(t)\right] + L\rho_S(t)L^\dagger - \frac{1}{2}\left\{L^\dagger L, \rho_S(t)\right\}$$





Quantum simulation of non-equilibrium dynamics and thermalization in the Schwinger model

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Thank you

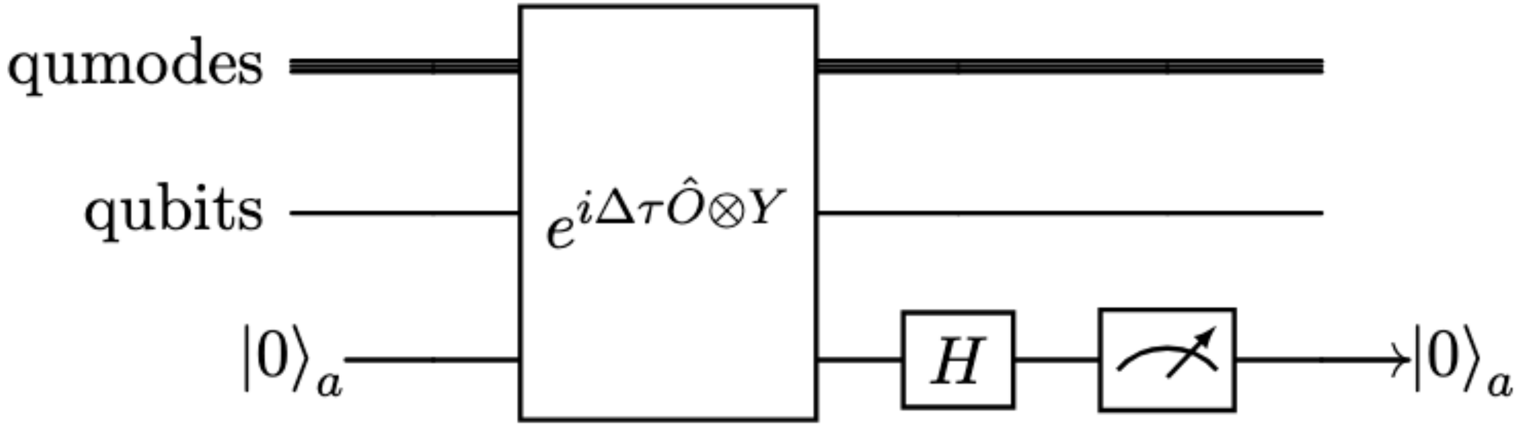
Extra

QITE Method A

Zero qubit state

$${}_a\langle 0|\, H_a e^{i\Delta\tau \hat{O}\otimes Y_a}\, |0\rangle_a = \frac{1}{\sqrt{2}} e^{-\Delta\tau \hat{O}} + \mathcal{O}((\Delta\tau)^2)$$

Need one **ancilla** (extra) **qubit** + postselection (measurement)



QITE Method B

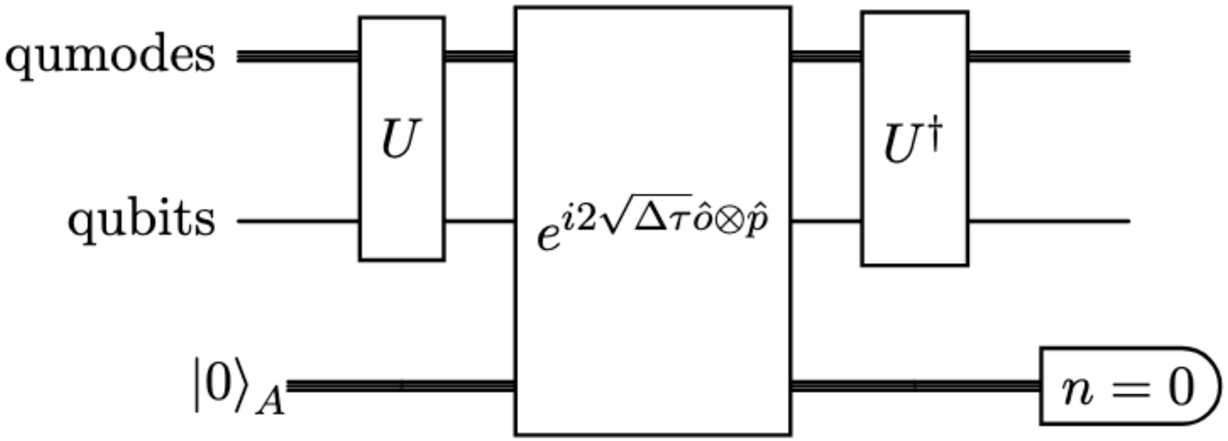
$$_A\langle 0|\, e^{i2\sqrt{\Delta\tau}\hat{o}\otimes\hat{p}_A}\, |0\rangle_A \propto e^{-\Delta\tau\hat{o}^2}$$

Zero Fock state

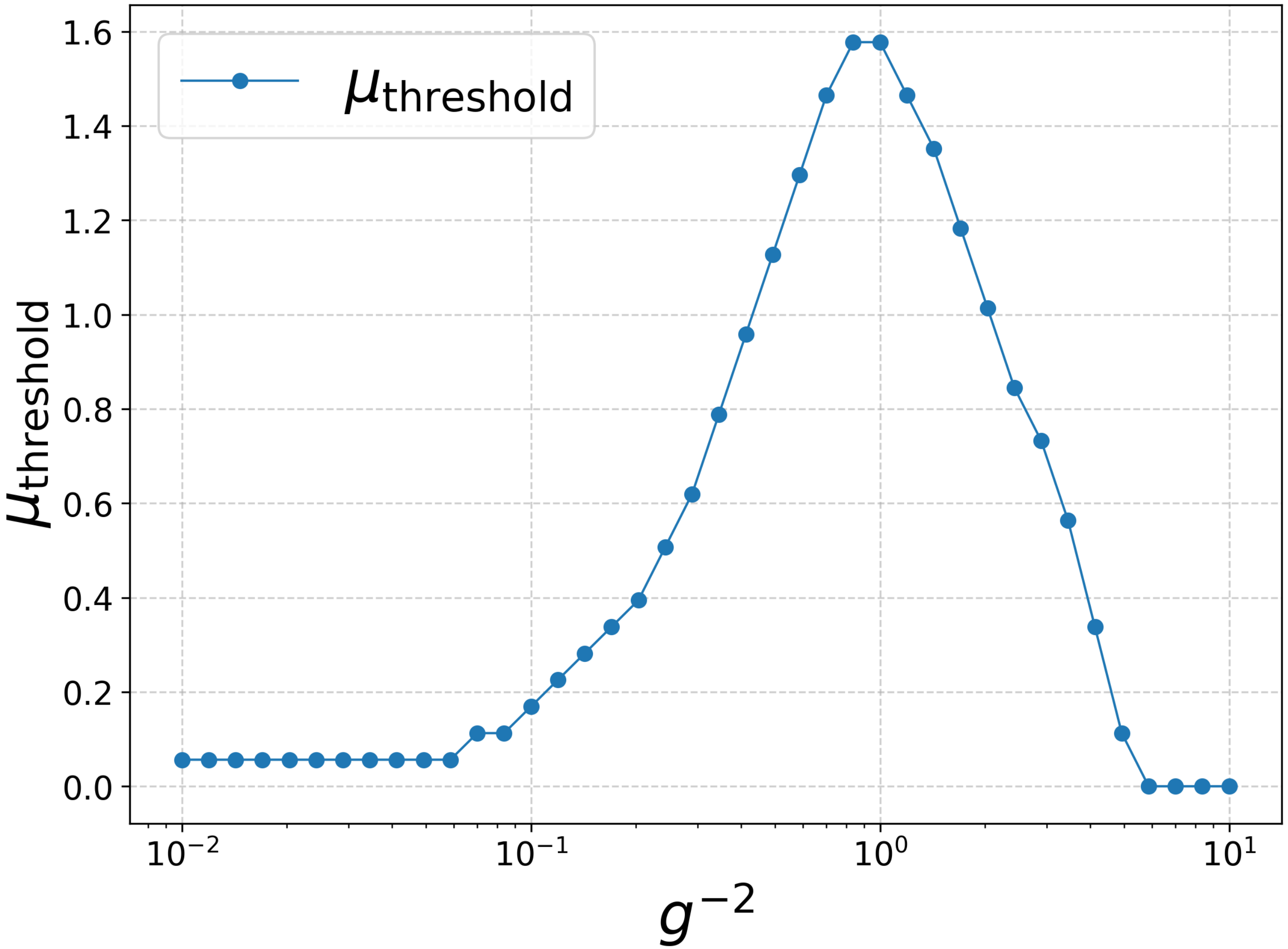
Only useful for operators such that $U^\dagger e^{-\Delta\tau\hat{o}^2} U = e^{-\Delta\tau\hat{O}}$

Target operator

Need one **ancilla** (extra) **qumode** + postselection (measurement)



Penalty term estimate



$$H^{(Q)} \mapsto H^{(Q)} + H_{\mu}^{(Q)}$$

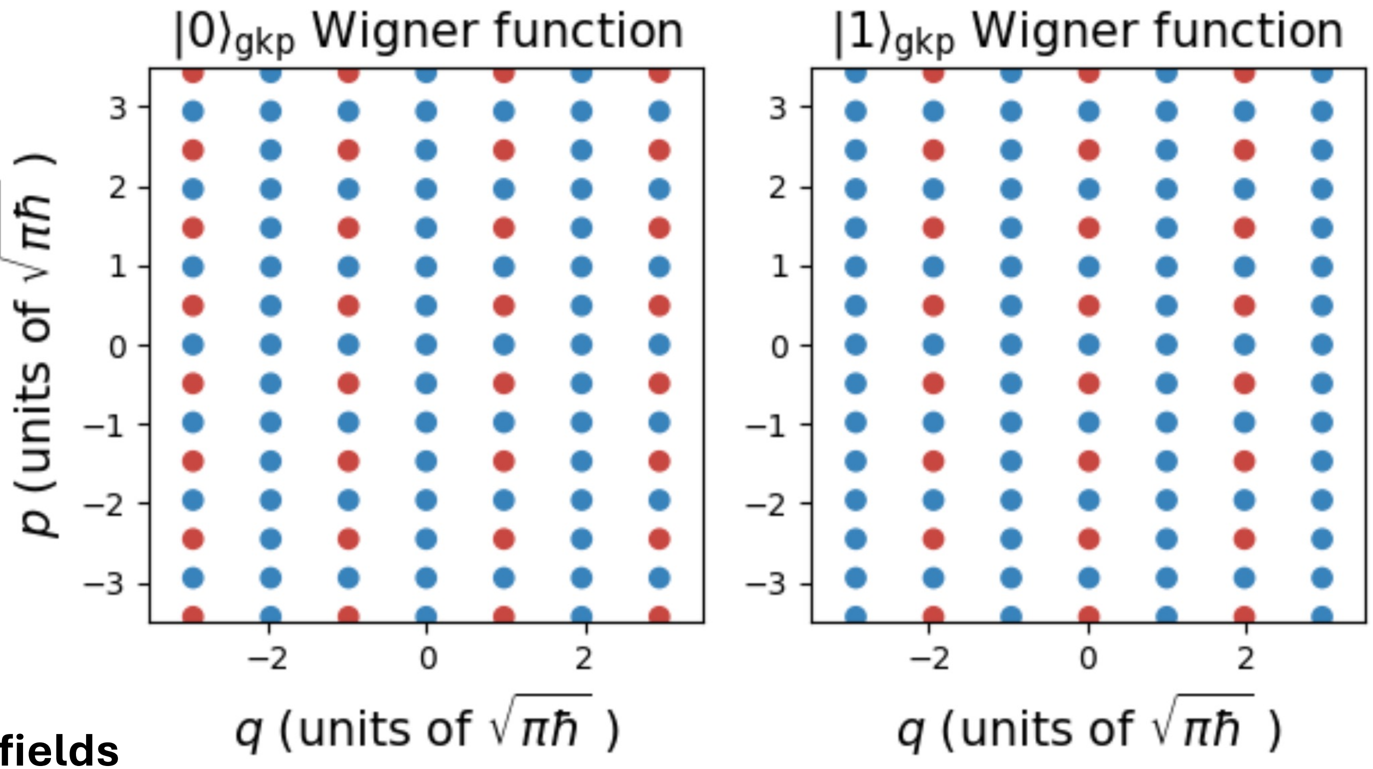
$$H_{\mu} = \frac{\mu}{2} \sum_{\boldsymbol{n},i} (\boldsymbol{q}_{\boldsymbol{n},i}^{2} - 1)^{2}$$

Threshold for 1% error in the ground state energy with

$$\mathrm{R} = \sqrt{(q^0)^2 + (q^1)^2} = 1 \pm 0.05$$

Logical qubits with GKP states (qumodes)

$$|0_L\rangle \equiv \sum_{s\in\mathbb{Z}} |2s\sqrt{\pi}\rangle_x \qquad |1_L\rangle \equiv \sum_{s\in\mathbb{Z}} |(2s+1)\sqrt{\pi}\rangle_x$$



$$X_L = e^{-i\sqrt{\pi}\hat{p}}$$

$$Z_L = e^{i\sqrt{\pi}\hat{q}}$$

$$Y_L = iX_LZ_L$$

Credits: Strawberryfields