

General purpose Equations of State

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Overview

- Update on general purpose EOS
- Thermal properties of nucleonic models
- Thermal properties of models with exotic d.o.f.

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Update on General purpose EOS

- from 1991 to 2010 only two EOS:
 - (1) Lattimer & Swesty (LS) [NPA 535, 331 (1991)];
 - (2) Shen, Toki, Oyamatsu & Sumiyoshi (STOS) [NPA 637, 435 (1998)]
- "finite-T" EOS have been "built" by Γ -law [Bauswein+ (2010)];
- starting with 2010 the situation improved a lot, e.g., on CompOSE*
 - more than 100 EOS are now available,
 - more than 40 EOS account for Λ , Y , Δ , π , K ; different blends; hadron to quark phase trans.
 - effective interactions: Skyrme, Covariant Density Functionals, ab-initio;
 - various NM properties, i.e. n_{sat} , E_{sat} , K_{sat} , J_{sym} , L_{sym} , K_{sym} , m_{eff}
- simulations of CCSN, BNS, stellar BH formation; effect of heavy baryons and a hadron to quark phase trans.; for non-rel. EOS $m_{rmeff};L$ impacts the evolution [Yasin+, PRL (2020)] [Schneider+, ApJ (2020)] [Schneider+, ApJ (2020)]
- **Our purpose:** Systematic investigation of thermal effects

* <https://compose.obspm.fr>

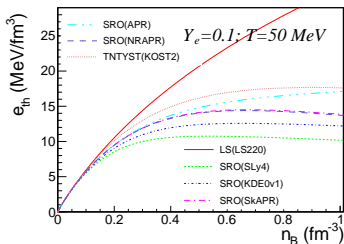
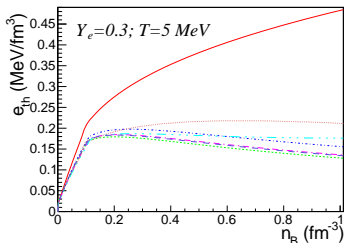
Nucleonic models

- 13 models
- 7 non-relativistic: SRO(APR), TNTYST(KOST2), LS(220), SRO(SLy4), SRO(KDE0v1), SRO(SkAPR), SRO(NRAPR),
red: variational; blue: Skyrme-like
- 6 CDFT: HS(DD2), HS(IUF), SFH(SFH_o), SFH(SFH_x), SNSH(TM1e), SHO(FSU2)
- $L_{sym} = 58.7 \pm 28.1$ MeV; $K_{sat} = 230 \pm 40$ MeV,
- $1.94 \leq M_{max}/M_{\odot} \leq 2.21$
- models are available on CompOSE*

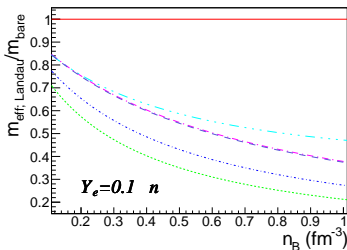
* <https://compose.obspm.fr/>
format suitable for input in numerical simulations

Non-relativistic models (I): thermal energy

$$e_{\text{th}} = e(n_B, Y_e, T) - e(n_B, Y_e, 0)$$

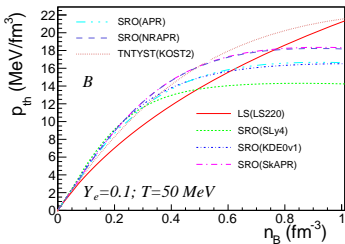
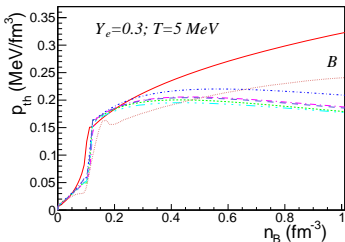


- low n_B , high T : ideal gas
 $e_{\text{th}} = \alpha T$, $\lim_{n_B \rightarrow 0} e_{\text{th}} \rightarrow 0$; no EOS-dep.;
- low n_B , low T : matter in clustered; dependence on eff. int. and NSE
- high n_B : strong n_B -, T , EOS-dep.
- models with large $m_{\text{eff};L} \rightarrow$ large e_{th}
 [Constantinou+, PRC (2014); PRC (2015)]

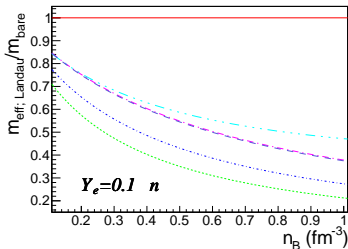


Non-relativistic models (I): thermal pressure

$$p_{\text{th}} = p(n_B, Y_e, T) - p(n_B, Y_e, 0)$$



- low n_B , high T : ideal gas
 $p_{\text{th}} = \alpha T$, $\lim_{n_B \rightarrow 0} p_{\text{th}} \rightarrow 0$; no EOS-dep.;
- low n_B , low T : matter in clustered;
 dependence on eff. int. and NSE
- high n_B : strong n_B -, T , EOS-dep.
- p_{th} depends on $m_{\text{eff};L}$, $dm_{\text{eff};L}/dn_B$
 [Constantinou+, PRC (2014); PRC (2015)]

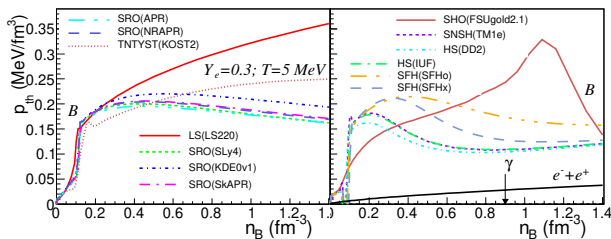
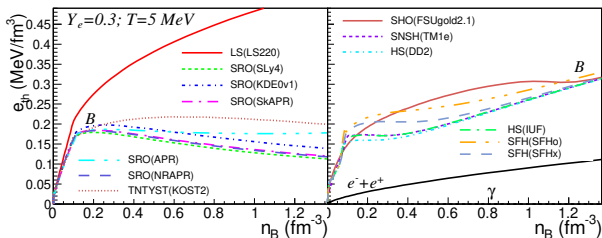


Theoretical framework dep. (I): e_{th} , p_{th}

non-rel. models

RMF models

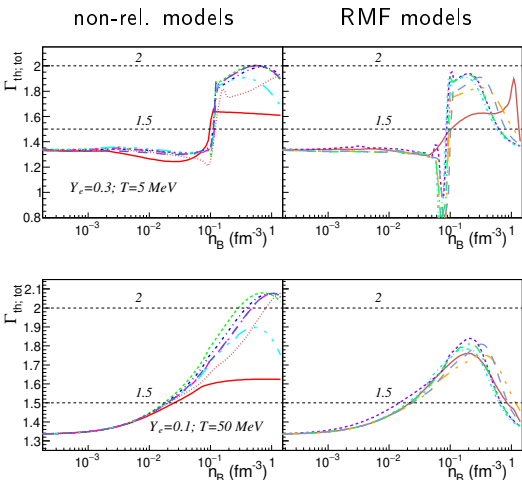
qualitative differences



[Raduta+, EPJA (2021)]

$$\epsilon(k) = k^2/2m_L^* + V(n) \quad \epsilon(k) = \sqrt{k^2 + m_D^{*2}(T)} + \Sigma_V;$$

Thermal index $\Gamma_{th} = 1 + p_{th}/e_{th}$



low n_B : $\Gamma_{th} < 1.5$, no EOS-dep.

high n_B : n_B -, T - and EOS-dep.
 non-rel.: $\Gamma_{th} > 2$
 RMF: $\Gamma_{th} < 1.5$

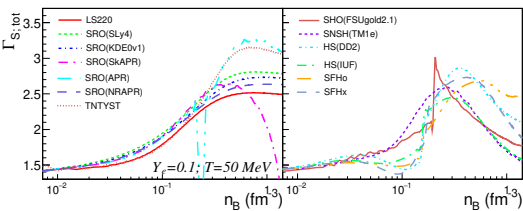
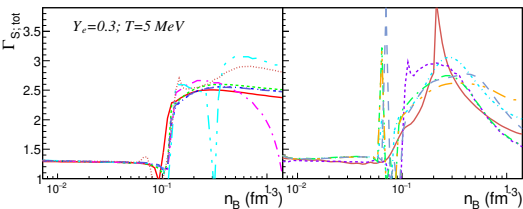
Γ -law is a crude approx.

$$P(n_B, e) = P_{\text{cold}}(n_B) + (\Gamma_{th} - 1)e_{th}(n_B)$$

[Raduta+, EPJA (2021)]

Adiabatic index Γ_S

$$\Gamma_S = (\partial \ln P / \partial \ln n_B) |_S = C_P / C_V \cdot n_B / P \cdot (\partial P / \partial n_B) |_T$$

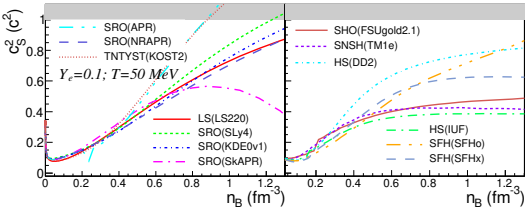
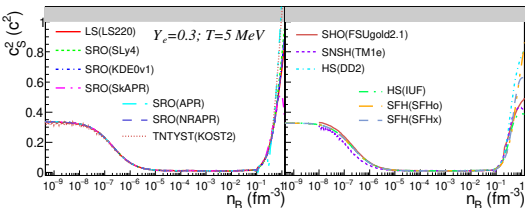


[Raduta+, EPJA (2021)]

- $\lim_{n_B \rightarrow 0} \Gamma_S \rightarrow 1.5$
- high n_B : Γ_S decreases
- for NSE models: irregularities at $n_{\text{sat}}/2$ for $T < T_C$
- for $T < T_C$ and $n_B < n_t$, $\Gamma_S \neq 0$
- APR: π condensed phase at $1.5n_{\text{sat}}$
- strong EOS- and n_B dep.

Speed of sound c_s^2

$$c_s^2 = (dP/de)|_{S,A,Y_e} = \Gamma_S P / (e + P)$$



[Raduta+, EPJA (2021)]

◦ strong n_B , T and EOS-dep.

◦ at $T = 5 \text{ MeV}$, $10^{-5} \text{ fm}^{-3} \lesssim n_B \lesssim n_t$:
 $c_s^2 \neq 0$: quenching of the transition or technicalities

◦ APR: π condensed phase at 0.2 fm^{-3}

◦ except SkAPR, for non-rel. models:
 c_s^2 increases with n_B

◦ for most CDFT models: c_s^2 saturates

Models with exotica

- 8 models based on DD2:

BHB(DD2L), BHB(DD2Lphi),

OMHN(DD2Y),

MBB(DD2K),

R(DD2YD)1 ($U_{\Delta}^{(N)} = -83$ MeV), R(DD2YD)2 ($U_{\Delta}^{(N)} = -124$ MeV),

R(DD2YD)3 ($U_{\Delta}^{(N)} = -57$ MeV) [Raduta, EPJA (2022)],

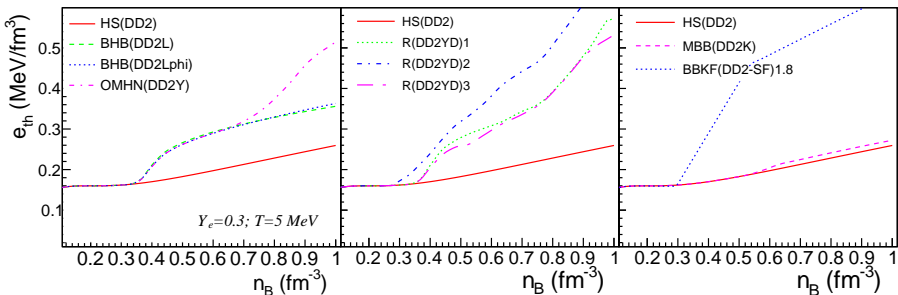
BBKF(DD2-SF)1.8

- models are available on ComPOSE*

* <https://compose.obspm.fr/>

Thermal energy

$$e_{th} = e(n_B, Y_e, T) - e(n_B, Y_e, 0)$$

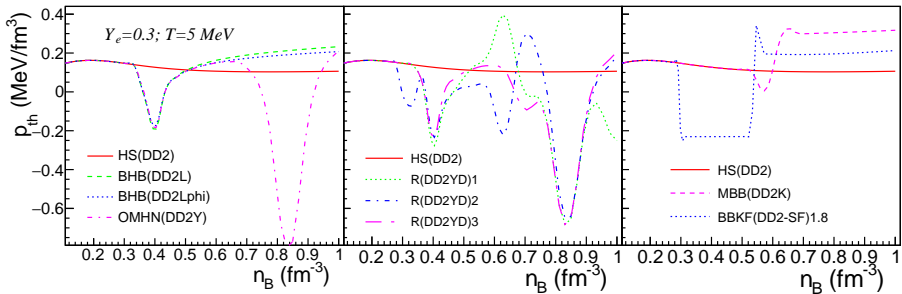


[Raduta, EPJA (2022)]

- e_{th} increases with n_B and T
- the larger the number of particle d.o.f. the larger e_{th}
- $e_{th}^{R(DD2YD)} > e_{th}^{OMHN(DD2Y)} > e_{th}^{BHB(DD2L)} > e_{th}^{HS(DD2)}$

Thermal pressure

$$p_{th} = p(n_B, Y_e, T) - p(n_B, Y_e, 0)$$

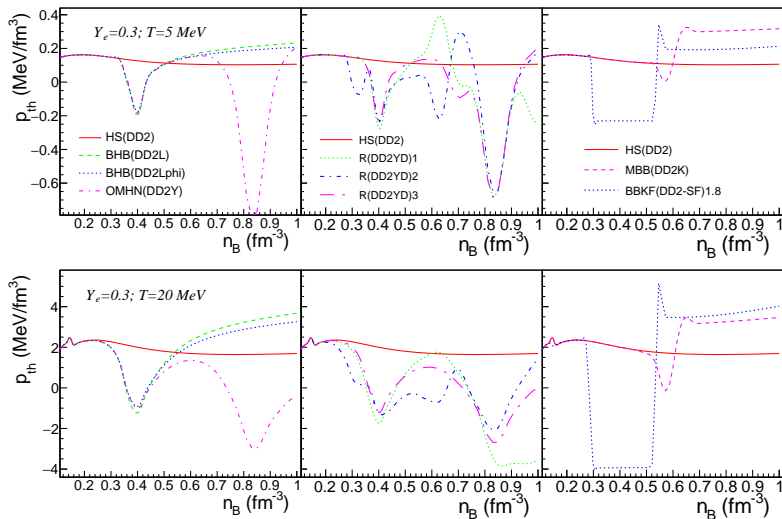


[Raduta, EPJA (2022)]

- nucleation of exotic d.o.f. diminishes p_{th}
- under specific conditions $p_{th} < 0$
- overall a strong n_B - and EOS- dependence; also T -dependence
- $p_{th} = \text{const.}$ in the coex. phase for (approximate) Maxwell construction;
 $p_{th} \neq \text{const.}$ if the hadron-quark phase trans. is done following Glendenning

Thermal pressure

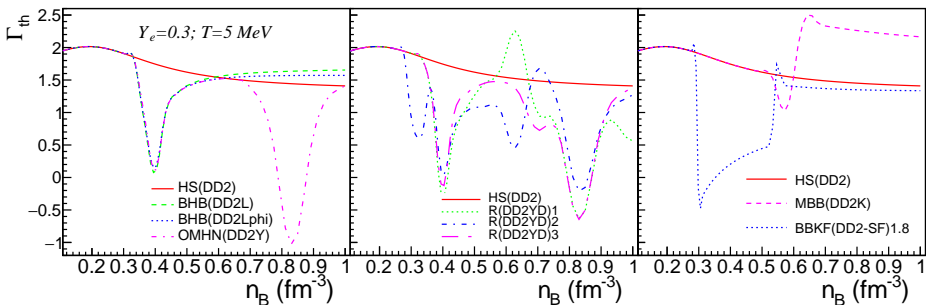
$$p_{th} = p(n_B, Y_e, T) - p(n_B, Y_e, 0)$$



- also T -dependence

[Raduta, EPJA (2022)]

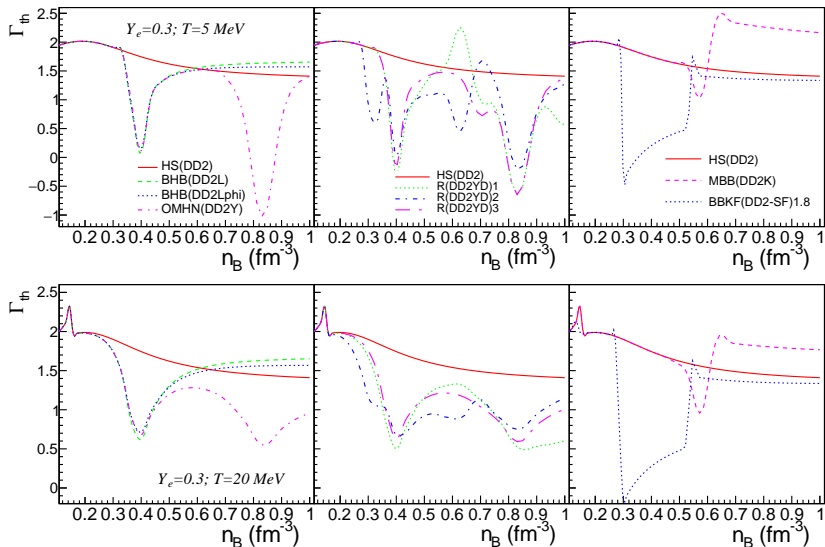
Thermal index $\Gamma_{\text{th}} = 1 + p_{\text{th}}/e_{\text{th}}$



[Raduta, EPJA (2022)]

- strong n_B - and EOS- dependence
- minima are related to the onset of new species
- $\Gamma_{\text{th}} < 0$
- for (approx.) Maxwell construction: Γ_{th} increases in the phase coex.
- $\Gamma_{\text{th}} < 1.5$ (the low limit of the Γ -law);
use of Γ -law is inappropriate for models with exotica

Thermal index $\Gamma_{\text{th}} = 1 + p_{\text{th}}/e_{\text{th}}$



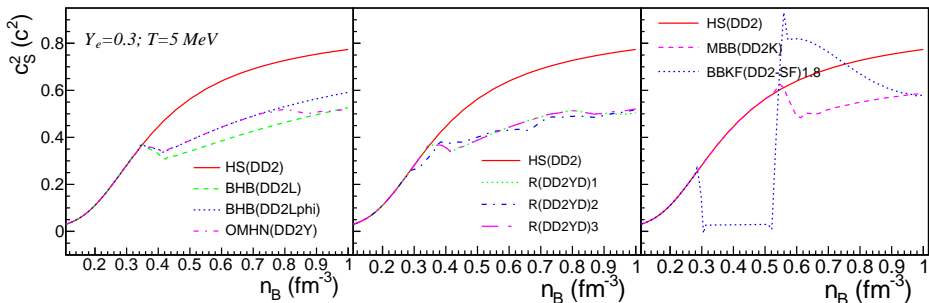
- also T -dependence

[Raduta, EPJA (2022)]

Speed of sound: $c_S^2 = dP/d\epsilon|_{S,A,Y_Q}$

$$c_S^2 = \Gamma_S P / (\epsilon + P)$$

$$\Gamma_S = (\partial \ln P / \partial \ln n_B)_S = (C_P / C_V) (n_B / P) (\partial P / \partial n_B)_T$$



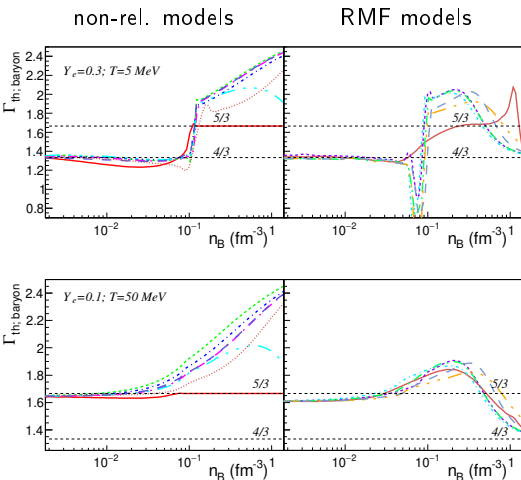
[Raduta, EPJA (2022)]

- strong n_B - and EOS- dependence;
- $c_S^2 = 0$ only for Maxwell treatment of phase coex.
- c_S^2 decreasing with n_B after the onset of a new species

Conclusions

- we have reviewed thermal prop. of nucleonic EOS
 - non-rel. and relativistic models manifest different features
 - the role of single particle energies; Landau versus Dirac effective masses
 - $m_L^*(n_B)$, $dm_L^*(n_B)/dn_B$ affect all thermal properties
- Γ -law is proven not to provide a reliable solution
- we have reviewed thermal prop. of models with exotica
 - e_{th} , p_{th} , Γ_{th} , c_S^2 shown to have strong n_B - and EOS-dependence;
for exotic d.o.f. p_{th} , $\Gamma_{\text{th}} < 0$ at low T

Thermal index $\Gamma_{\text{th}} = 1 + p_{\text{th}}/e_{\text{th}}$: baryonic sector



low n_B and high T :

$\Gamma_{\text{th}} \rightarrow 5/3$, ideal gas

RMF at high n_B :

$\Gamma_{\text{th}} \rightarrow 4/3$, ultra-rel. gas

strong n_B - and EOS-dep.

non-rel. and RMF qualitatively diff.

[Raduta+, EPJA (2021)]