INSTITUTE for NUCLEAR THEORY

February 12-16, 2024

INT Workshop 24-87W
Electroweak and Beyond the Standard Model
Physics at the EIC

Constraining the Weak Neutral Current Couplings at the EIC

- A note on the success of SM and challenge for finding BSM
- Collision induced radiation for high-energy lepton-hadron scattering
- Inclusive lepton-hadron DIS – one vector boson approximation
- Parity violating lepton-hadron DIS and weak neutral current couplings
- Summary

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Jefferson Lab, Theory Center

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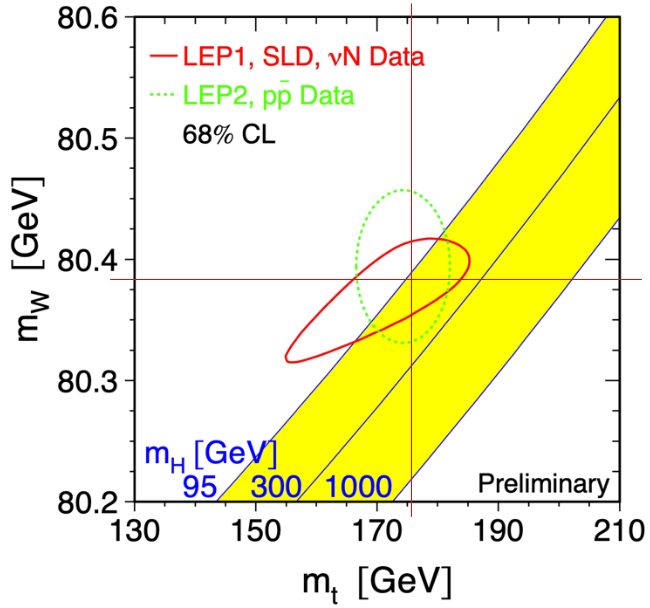
A note on the success of SM and challenge for finding BSM

- **Prediction for P_T distribution for Higgs production at the LHC:**
  - Ten years before Higgs was discovered in 2012 [E. Berger, J. Qiu, arXiv:hep-ph/0210135]
  - Need a Higgs mass: Constraints from precision measurements
  - W mass:

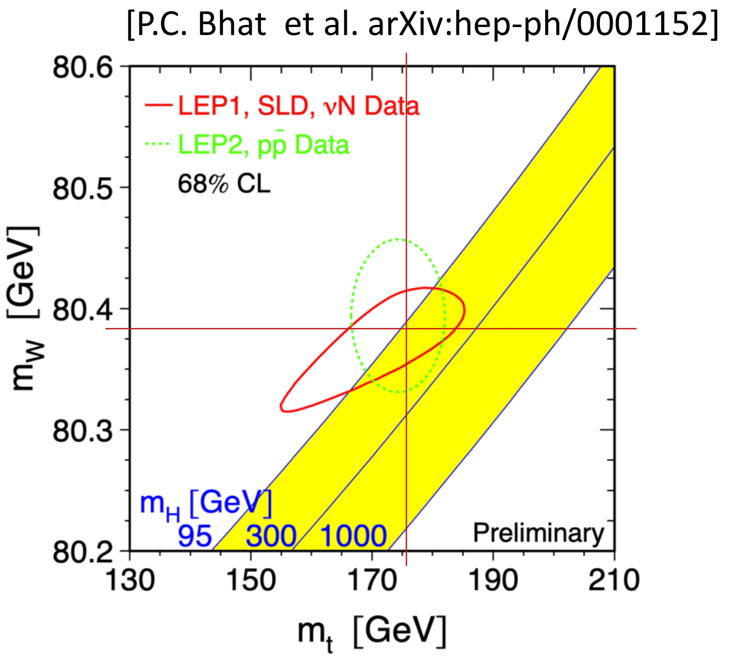
[A. Sirlin 1999]

$$m_W = 80.3827 - 0.0579\ln\left(\frac{M_H}{100}\right) - 0.008\ln^2\left(\frac{M_H}{100}\right) - 0.517\left(\frac{\Delta\alpha_h^{(5)}}{0.0280} - 1\right) + 0.543\left(\left(\frac{m_t}{175}\right)^2 - 1\right) - 0.085\left(\frac{\alpha_s(m_Z)}{0.118} - 1\right), \quad (2)$$

[P.C. Bhat et al. arXiv:hep-ph/0001152]



A note on the success of SM and challenge for finding BSM



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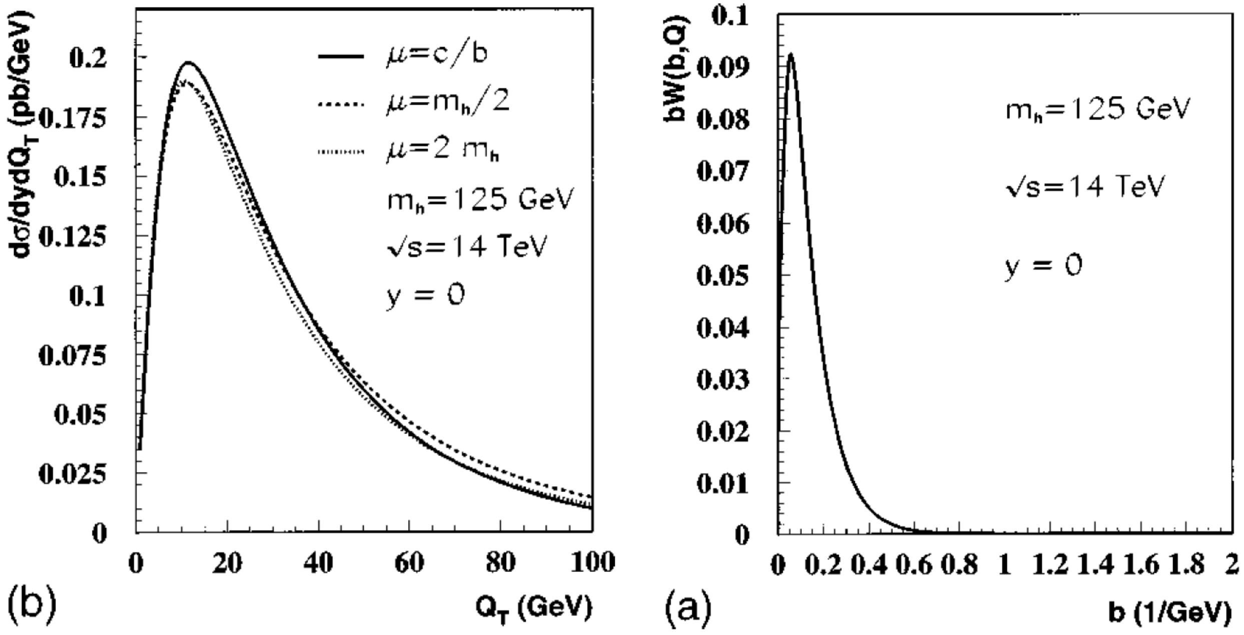
Estimate: $m_H \sim 125 - 126 \text{ GeV}$ Choice: $m_H = 125 \text{ GeV}$



[Berger, Qiu, PRD67, 034026 (2003)]

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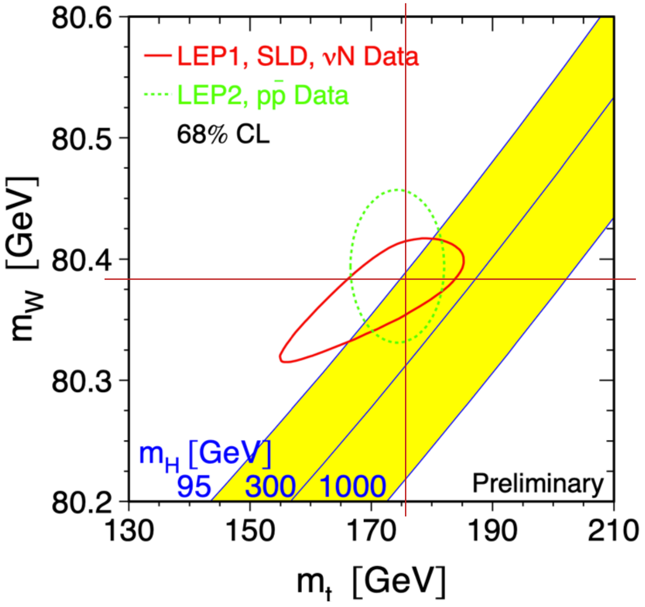
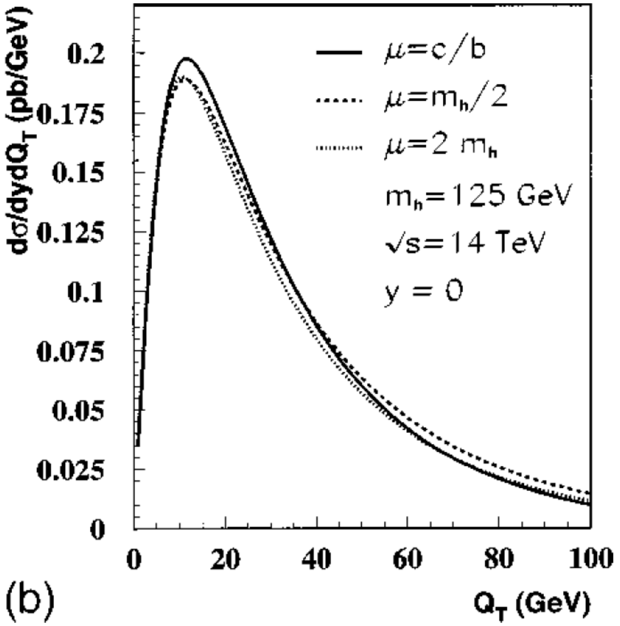
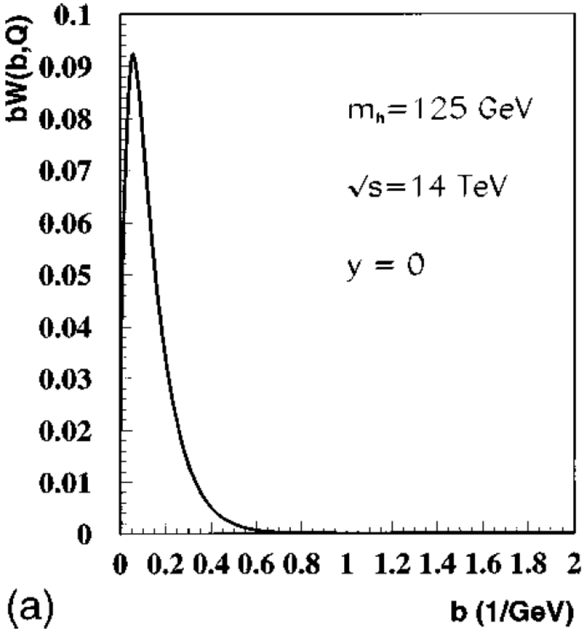


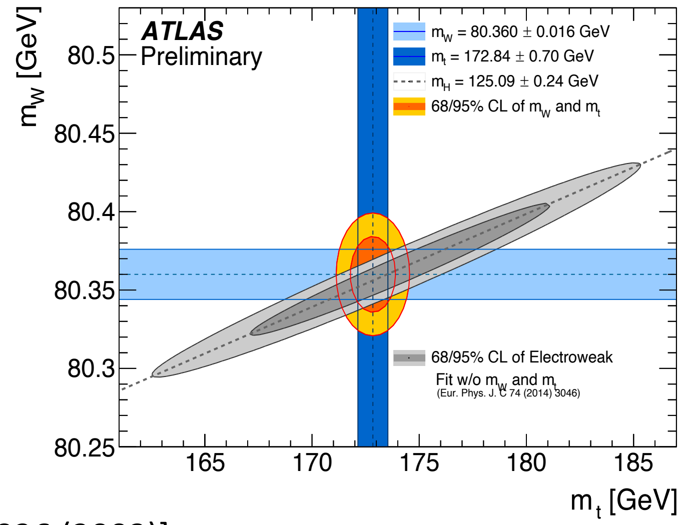


(b)



(a)

- Success of SM
- Challenge for finding BSM
- More precision is needed!
- Role of QED?



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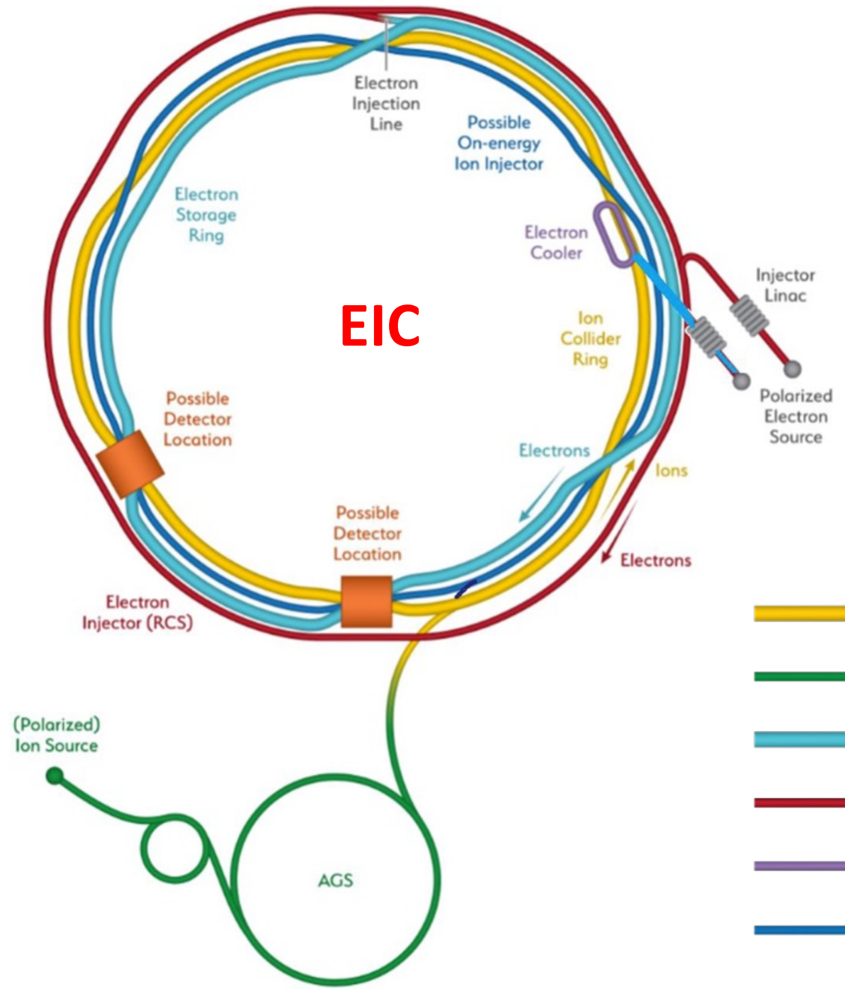
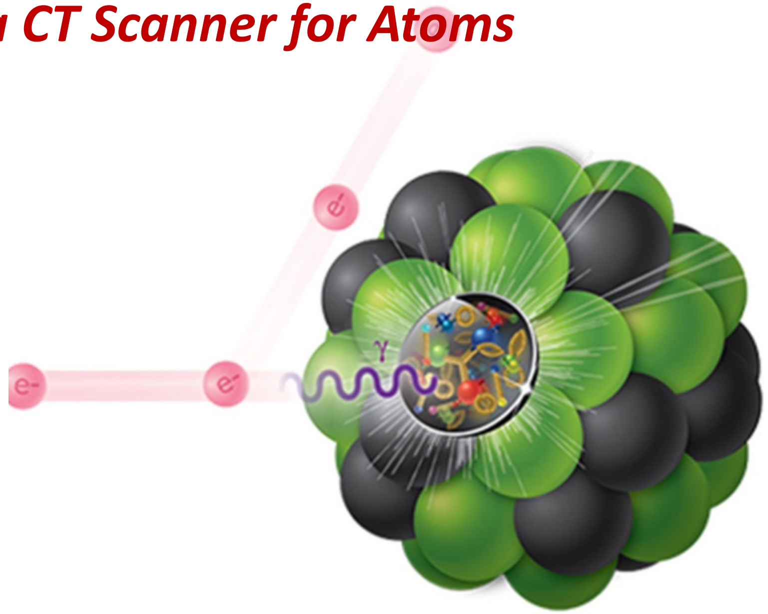
U.S. - based Electron-Ion Collider (EIC)

https://www.bnl.gov/eic/

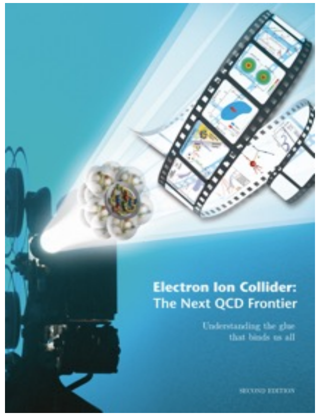
A machine that will unlock the secrets of the strongest force in Nature

Like a CT Scanner for Atoms





- Hadron Storage Ring
- Hadron Injector Complex
- Electron Storage Ring
- Electron Injector Synchrotron
- Electron Cooler
- Possible On-energy Hadron Injector Ring



Basic Tech Requirements

- Center of Mass Energies:
  20 GeV – 141 GeV
- Required Luminosity:
  10^{33} - 10^{34} $cm^{-2}s^{-1}$
- Hadron Beam Polarization:
  80%
- Electron Beam Polarization:
  80%
- Ion Species Range:
  p to Uranium
- Number of interaction regions:
  up to two

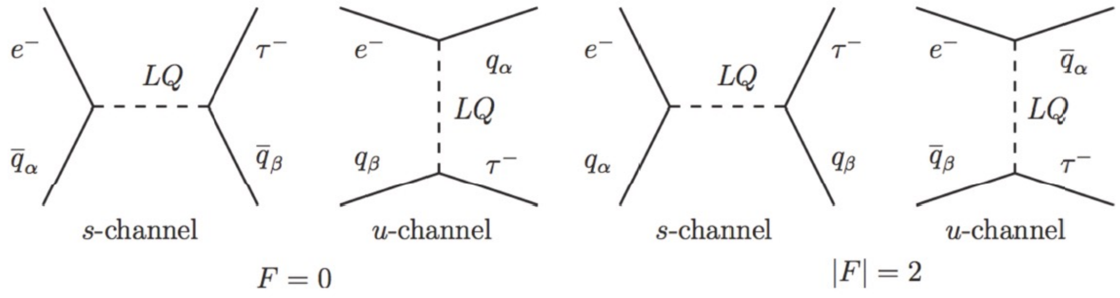
Physics of BSM in the EIC White Paper & the Yellow Report

Chapter 4

Possibilities at the Luminosity Frontier: Physics Beyond the Standard Model

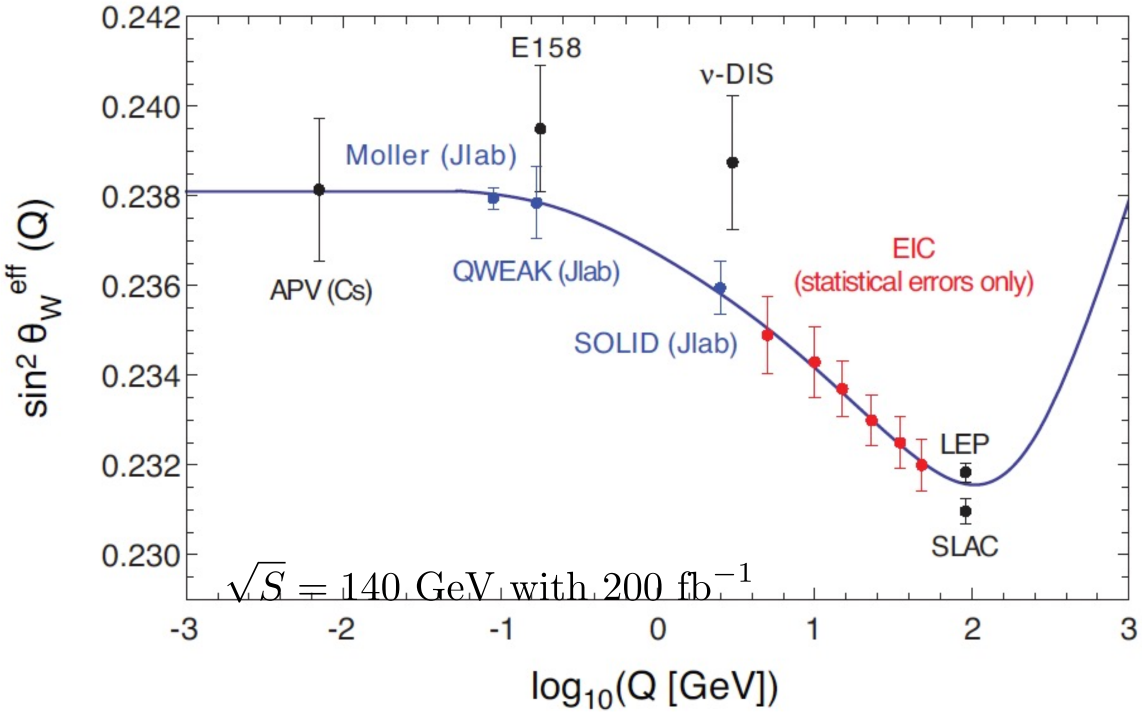
Conveners: Krishna Kumar and Michael Ramsey-Musolf

4.2.1 Charged Lepton Flavor Violation



$\tau \to e$ scattering process via leptoquarks

4.2.2 Precision Measurements of Weak Neutral Current Couplings



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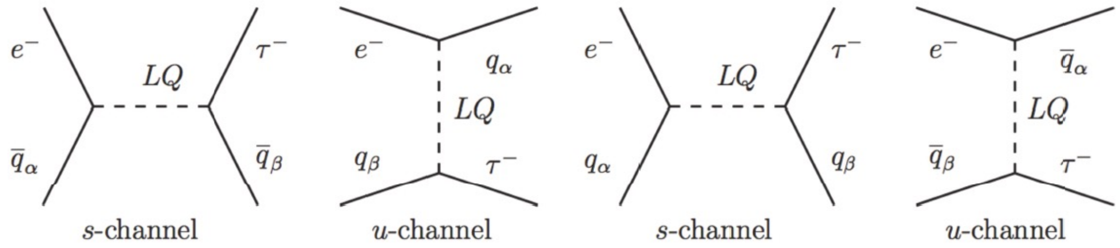

$F = 0$ $\quad$ $|F| = 2$

$\tau \to e$ scattering process via leptoquarks

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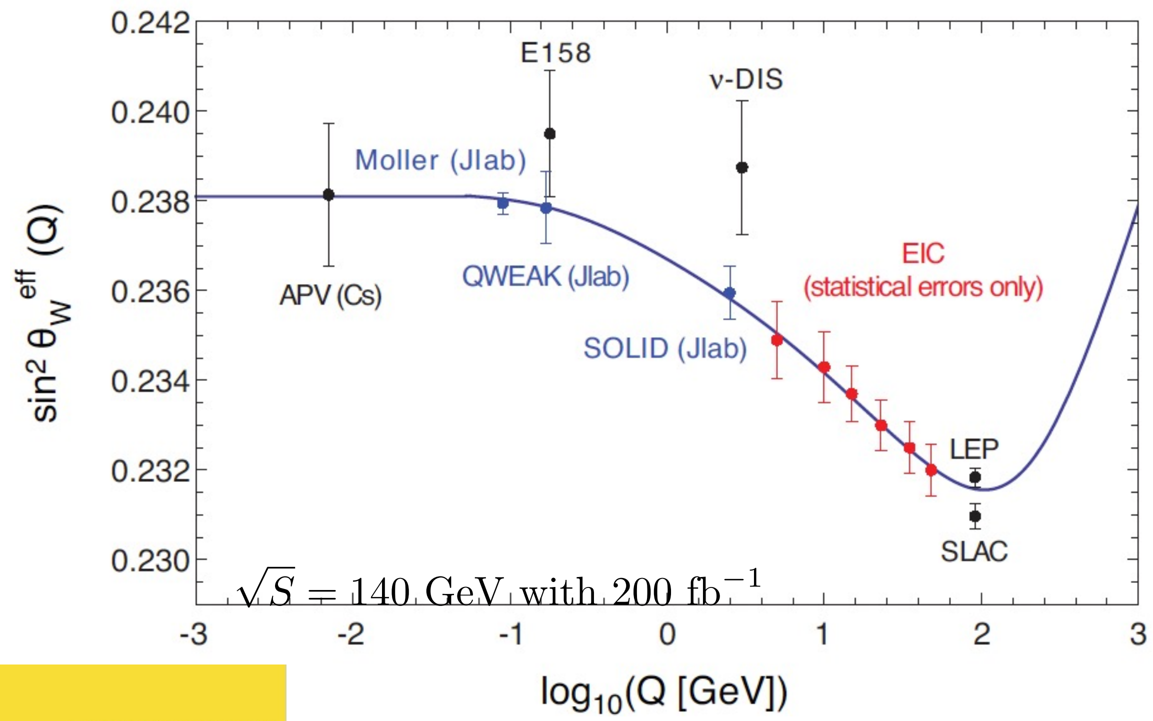
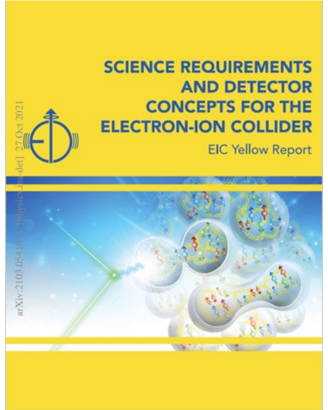




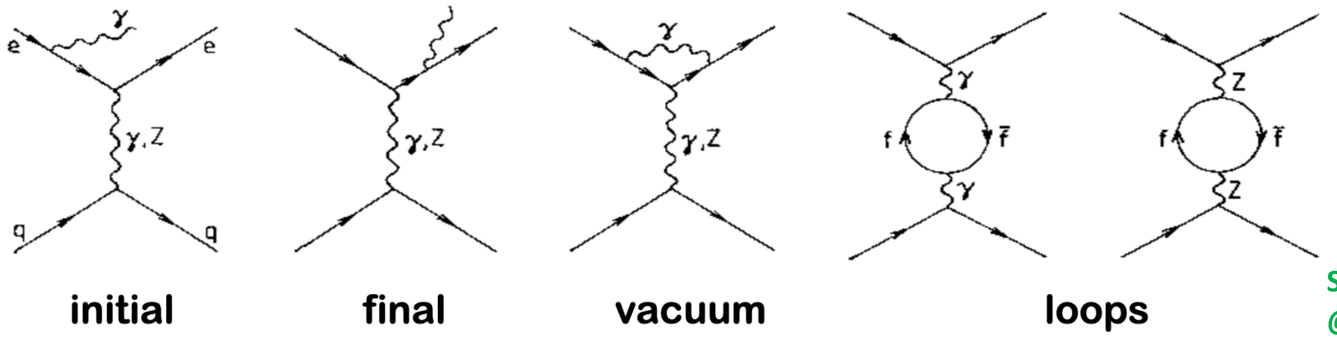
7.5 Connections with Other Fields

7.5.1 Electroweak and BSM physics

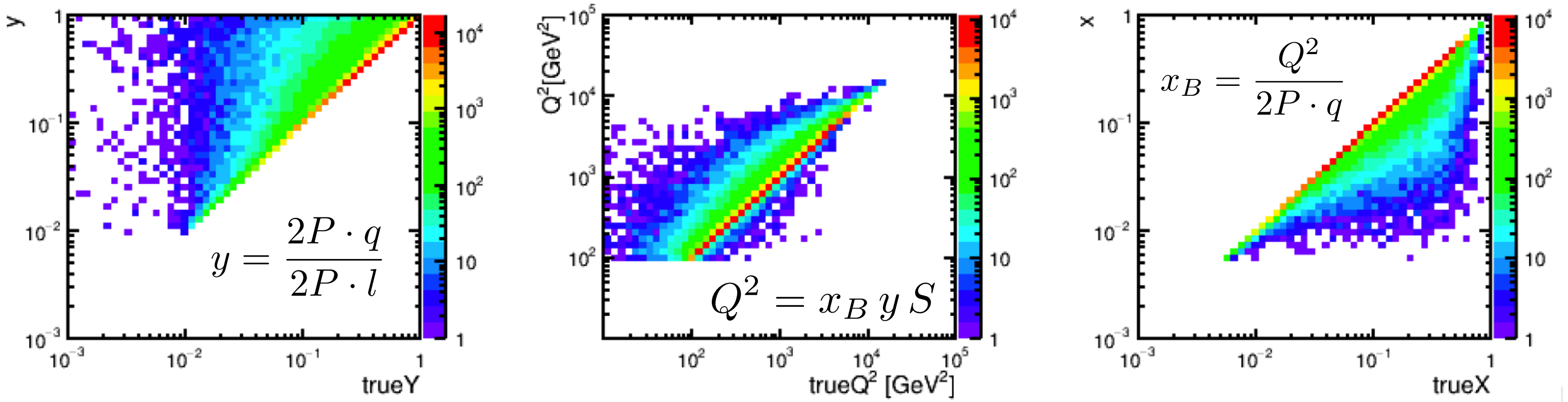
Collision induced radiation for high-energy lepton-hadron scattering

- "Probe" for the hadron is smeared by the induced QED radiation:

Data sample : Int L = 10 fb^{-1}, Kinematics settings: $0.01<y<0.95$, 10^2 GeV^2 $<Q^2<10^5$ GeV^2



See Xiaoxuan Chu
@2nd EIC YR workshop



Instead of a straight line – linear correlation,

the kinematic variables, y, Q^2 , x_B, from the leptons are smeared so much to make them different from what the scattered "quark" experienced!

Ill-defined "photon-hadron" frame?!

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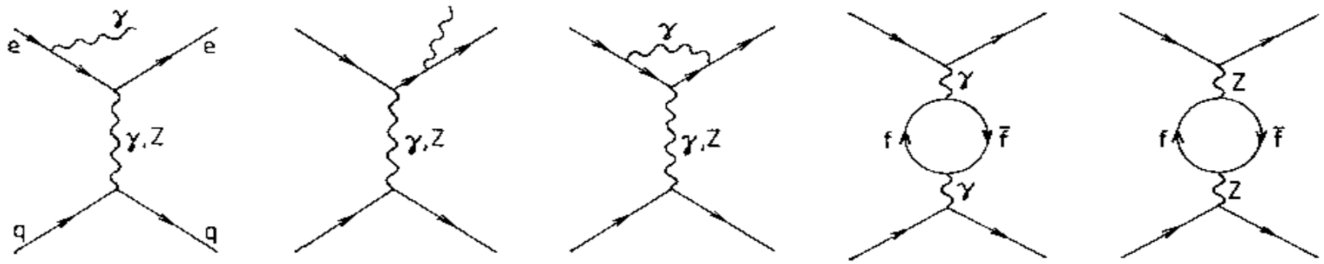

initial **final** **vacuum** **loops**

See Xiaoxuan Chu
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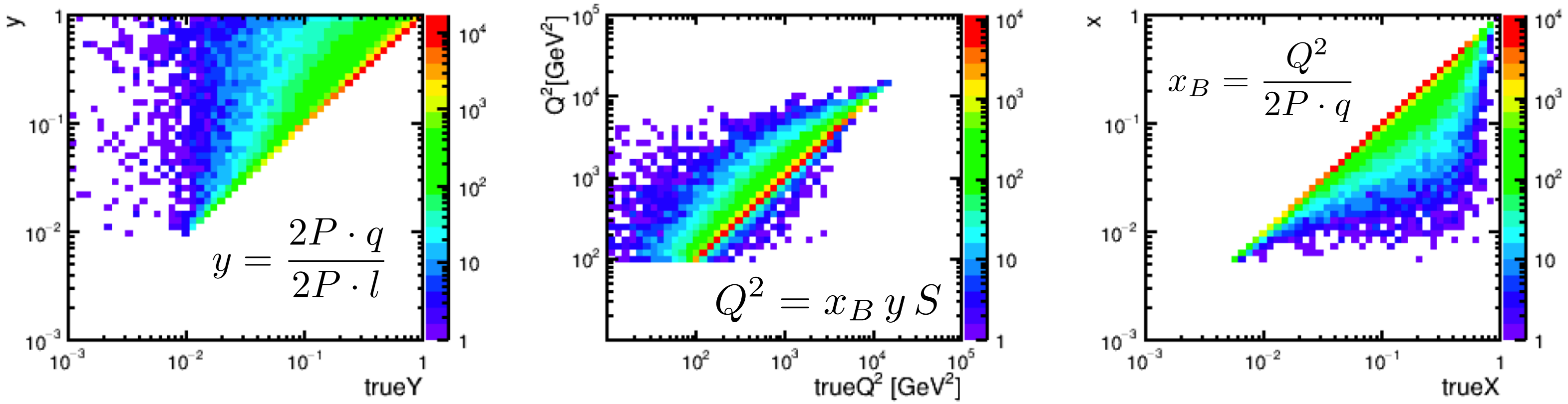

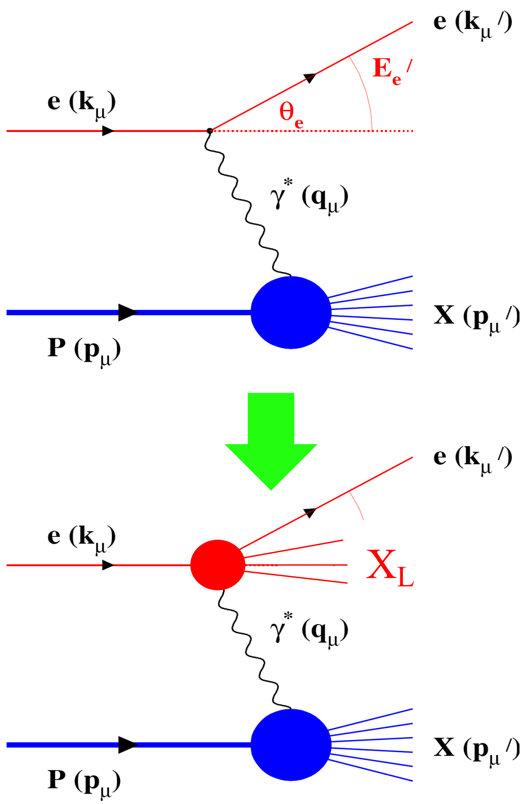

$$q_\mu \;\to\; \hat{q}_\mu$$

$$Q^2 = -q^2 \;\to\; \widehat{Q}^2 = -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \;\to\; \hat{x}_B = \frac{\widehat{Q}^2}{2P\cdot\hat{q}}$$

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the kinematic variables, y, Q^2 , x$_B$, from the leptons are smeared so much to make them different from what the scattered "quark" experienced!

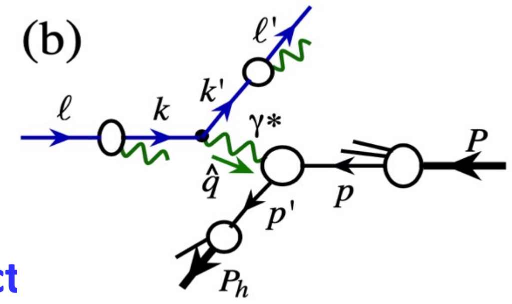
Ill-defined "photon-hadron" frame?!

No simple radiative correction for SIDIS

❑ Radiative correction – Born kinematics:

$$\sigma_{\text{Measured}} \equiv \text{RC} \otimes \sigma_{\text{No QED Radiation}}$$

Necessary requirement: RC – Radiative correction factor
does not depend on the hadronic physics that we want to extract



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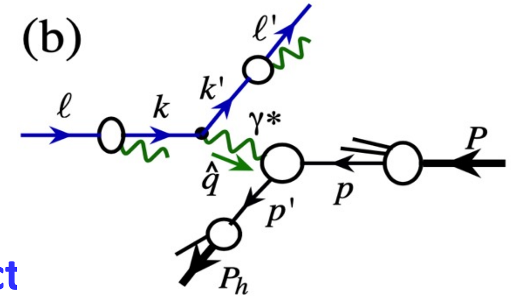
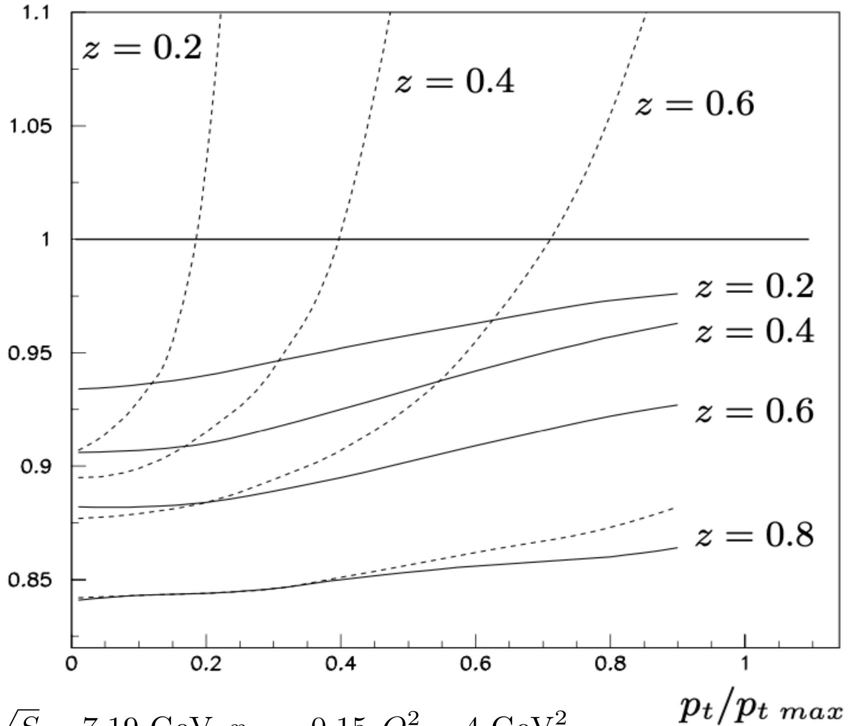


❑ Impact of QED radiation to SIDIS – order of α_{EM}:

$$\bar{\delta} = \sigma_{\text{Measured}} / \sigma_{\text{No QED Radiation}}$$



$\sqrt{S} = 7.19$ GeV, $x_B = 0.15$, $Q^2 = 4$ GeV^2

$$e(l) + N(P) \rightarrow e'(l') + \gamma(k) + h(P_h) + X$$

I. Akushevich et al.
EPJ C10 (1999) 681

Dashed line:

Gaussian pT-dependence

$$b\,\exp(-b\,p_t^2) \qquad \text{where} \quad b = R^2/z^2$$

Solid line:

Power pT-dependence

$$\left[\frac{1}{a + b\,z + p_t^2}\right]^{c+d\,z} \qquad \text{parameters:} \quad R, a, b, c, d$$

$\bar{\delta}$ **depends on physics we want to extract!**

NO simple RC for SIDIS!

QED radiative corrections vs. QED radiative contributions

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

QED radiative corrections:

$$\sigma_{\rm obs}(x_B, Q^2) \overset{?}{=} R_{\rm QED}(x_B, Q^2; x_{B,\rm true}, Q^2_{\rm true}) \times \sigma_{\rm Born}(x_{B,\rm true}, Q^2_{\rm true}) + \sigma_X(x_B, Q^2)$$

- The correction factors R_{QED} and σ_x should not depend on the hadron structure that we wish to extract, and they can be systematically calculated in QED to high precision (not satisfied);
- The effective scale Q^2_{true} for the Born cross section σ_{Born} should be large enough to keep the "true" scattering within the DIS regime (questionable);
- Extraction of $\sigma_{\rm Born}$ is an inverse problem

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QED radiative contributions:

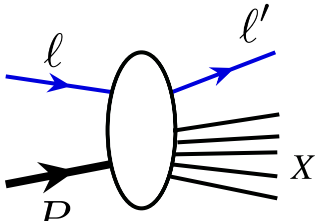
$$\sigma_{\rm obs}(x_B, Q^2) = \sigma^{\rm univ}_{\rm lep}(\mu^2; m_e^2) \otimes \sigma^{\rm univ}_{\rm had}(\mu^2; \Lambda^2_{\rm QCD}) \otimes \widehat{\sigma}_{\rm IR\text{-}safe}(\hat{x}_B, \widehat{Q}^2, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}, \frac{m_e^2}{Q^2}\right)$$

- Infrared sensitive QED contributions – divergent as $m_e/Q \to 0$, are absorbed to universal LDFs and LFFs
- Infrared safe QED contributions – finite as $m_e/Q \to 0$, are calculated order-by-order in power of α
- Power suppressed contributions as $m_e/Q \to 0$, are neglected

Predictive power: Universality of LDFs and LFFs, their evolution, calculable hard parts
Neglect power corrections

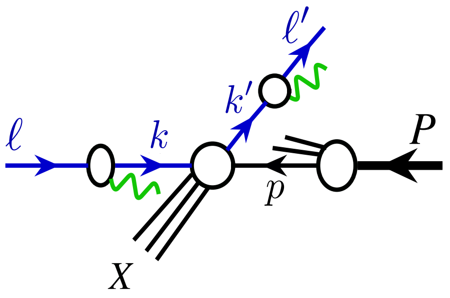
Inclusive lepton-hadron deep inelastic scattering (DIS)

- **Inclusive production of single high p_T lepton in lepton-hadron collision:**



$$e(\ell, \lambda_\ell) + N(P,S) \to e(\ell') + X$$

$$d\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X} = \frac{1}{2s}\left|M_{\ell(\lambda_\ell)P(S)\to\ell' X}\right|^2 d\mathrm{PS}$$



Collinear QED & QCD factorization

$$E'\frac{d\sigma_{\ell P\to\ell' X}}{d^3\ell'} \approx \frac{1}{2s}\sum_{ija}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi\ell, xP, \ell/\zeta, \mu^2) + \cdots$$

Lepton distribution functions (LDFs): $f_{i/e}(\xi,\mu^2)$

Lepton fragmentation functions (LFFs): $D_{e/j}(\zeta,\mu^2)$ $\quad i.j = e, \gamma, \bar{e}, ..., q, g, ...$

Parton distribution functions (PDFs): $f_{a/N}(x,\mu^2)$ $\quad a = q, g, \bar{q}, e, \gamma, \bar{e}, ...$

Short-distance hard coefficients: $\widehat{H}_{ia\to jX}(\xi\ell, xP, \ell/\zeta, \mu^2)$

$$\approx \widehat{H}^{(m,n)}_{ia\to jX}(\xi\ell, xP, \ell/\zeta, \mu^2) \approx \mathcal{O}(\alpha^m\alpha_s^n)$$

Photon is charge neutral
QED factorization works

Nayak, Qiu, Sterman, Phys.Rev.D 72 (2005) 114012

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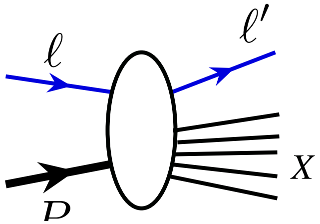
❑ Inclusive production of single high p_T lepton in lepton-hadron collision:



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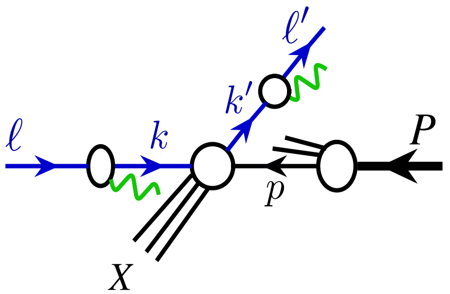

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- **No DIS "Structure Functions"!**

  Concept of one-photon exchange

- **QED & QCD contribution are factorized at the same scale: μ**

  $$(x_B,Q^2)\;\to\;(y,\ell'_T)$$

- **Corrections suppressed by power**

  $$(1/\ell'_T)^\alpha$$

Nayak, Qiu, Sterman, Phys.Rev.D 72 (2005) 114012

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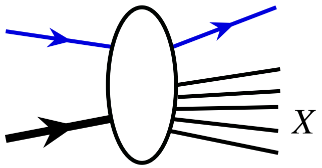

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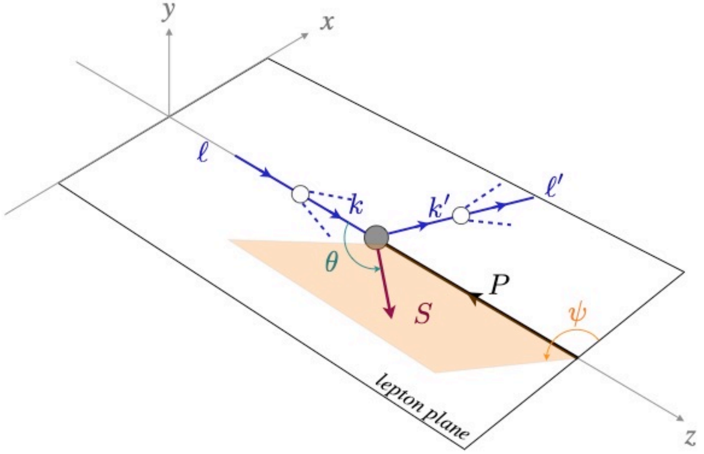
$$d\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X}=\frac{1}{2s}\left|M_{\ell(\lambda_\ell)P(S)\to\ell' X}\right|^2 d\mathrm{PS}$$

- **Recover the concept of structure functions?** $\quad i=e,\quad j=e$



$$E_{\ell'}\frac{d^3\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X}}{d^3\ell'}\approx\sum_{\lambda_k}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}D_{e/e}(\zeta,\mu^2)\int_{\xi_{\min}}^{1}d\xi\, f_{e(\lambda_k)/e(\lambda_\ell)}(\xi,\mu^2)\times\left[E_{k'}\frac{d^3\hat{\sigma}_{k(\lambda_k)P(S)\to k' X}}{d^3k'}\right]_{k=\xi\ell,\,k'=\ell'/\zeta},$$

$$E_{k'}\frac{d^3\hat{\sigma}_{k(\lambda_k)P(S)\to k' X}}{d^3k'}\approx\frac{2\alpha^2}{\hat{s}\,\widehat{Q}^4}L^{(0)}_{\mu\nu}(k,k',\lambda_k)\,W^{\mu\nu}(\hat{q},P,S)$$

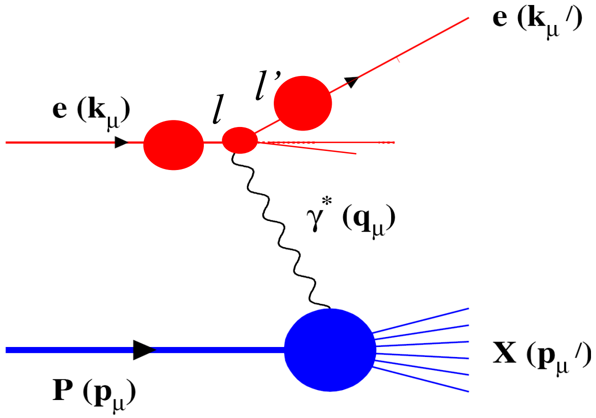
$$W^{\mu\nu}(\hat{q},P,S)=-\widetilde{g}^{\mu\nu}(\hat{q})\,F_1(\hat{x}_B,\widehat{Q}^2)+\frac{1}{P\cdot\hat{q}}\widetilde{P}^\mu(\hat{q})\widetilde{P}^\nu(\hat{q})F_2(\hat{x}_B,\widehat{Q}^2)+\ldots$$

Structure functions are evaluated at $(\hat{x}_B,\widehat{Q}^2)$ **instead of** (x_B,Q^2) **!**

Collinear factorization for QED radiative contribution

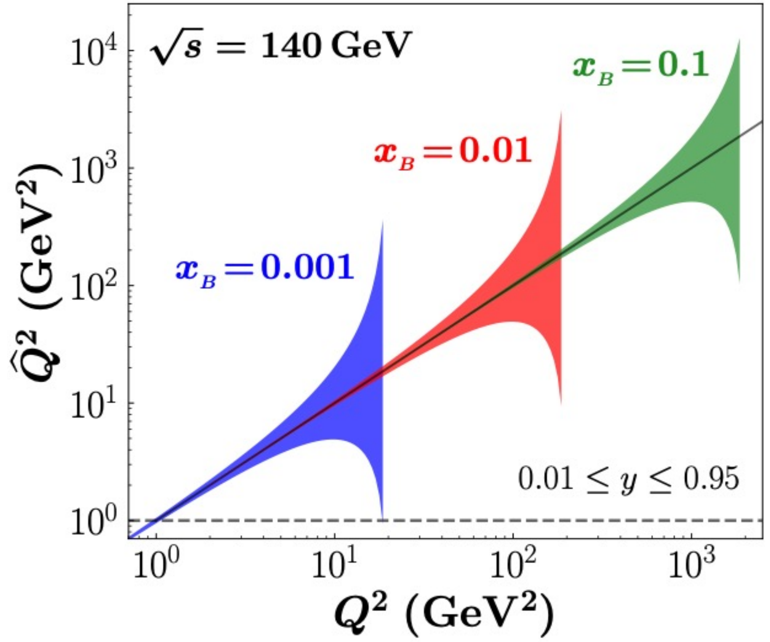
❑ Collinear factorization with the "one-photon" approximation:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B\,dy} \approx \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^{1} d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\,\frac{\hat{x}_B}{\widehat{Q}^2}\right] \times \frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2 F_1(\hat{x}_B,\widehat{Q}^2) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right) F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

- QED radiation prevents a well-defined "photon-hadron" frame
- Radiation is CO sensitive as $m_e/Q \to 0$, factorized into LDFs & LFFs
- Hadron is probed by $(x_B, Q^2) \to (\hat{x}_B, \widehat{Q}^2)$



$$x_B \to \hat{x}_B \in [x_B, 1] \qquad \widehat{Q}^2_{\min} = Q^2\,\frac{(1-y)}{(1-x_B\,y)} \qquad \widehat{Q}^2_{\max} = Q^2\,\frac{1}{(1-y+x_B\,y)}$$

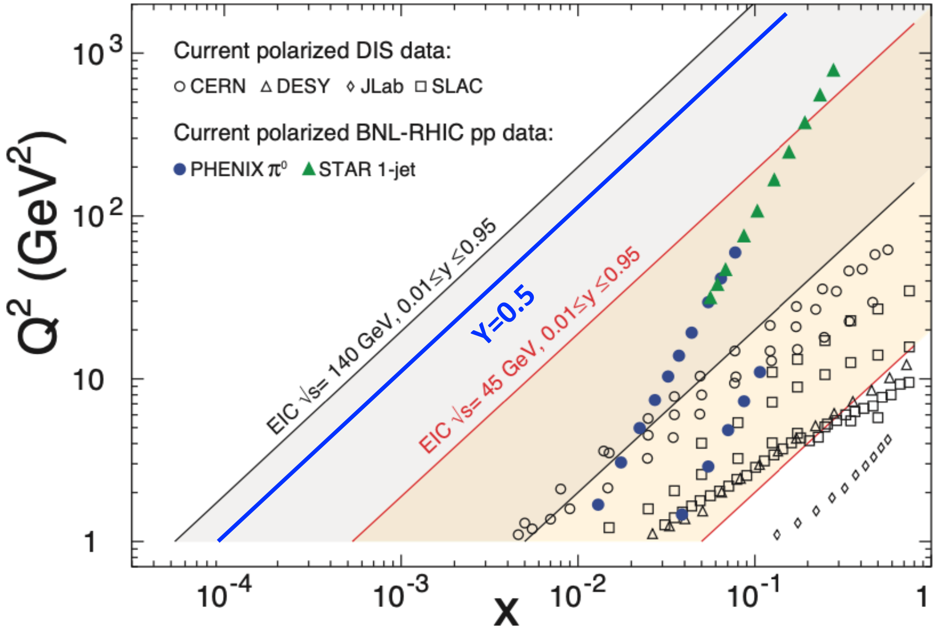
A simple RC factor at x_B is necessarily sensitive to hadronic information from $[x_B, 1]$!

Inclusive lepton-hadron deep inelastic scattering (DIS)

❑ Numerical impact of QED contribution at EIC ($\sqrt{S} = 140\ \mathrm{GeV}$):

$$\frac{\sigma_{\mathrm{noRC}}}{\sigma_{\mathrm{RC}}} \leftrightarrow \frac{\sigma_{1\gamma}}{\sigma_{\mathrm{measured}}} = \eta(x_B, y)$$

B. Badelek et al.
Z Phys C 66 (1995) 591



At $\sqrt{S} = 140\ \mathrm{GeV}$

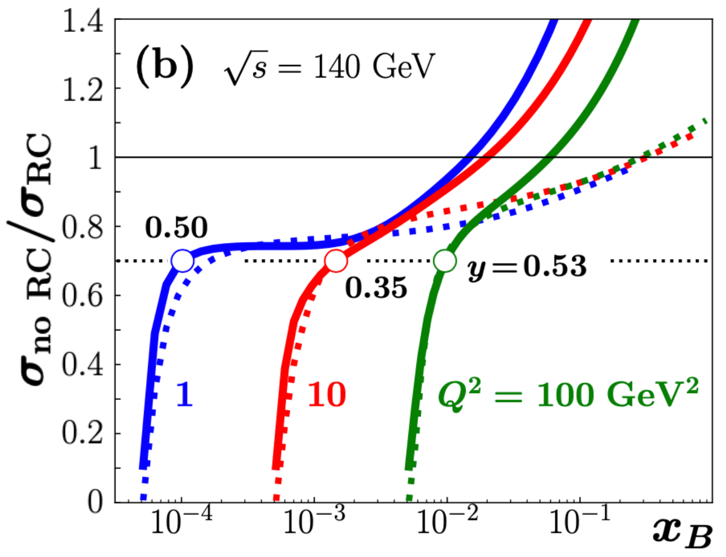
$Q^2 = 1\ \mathrm{GeV}^2$

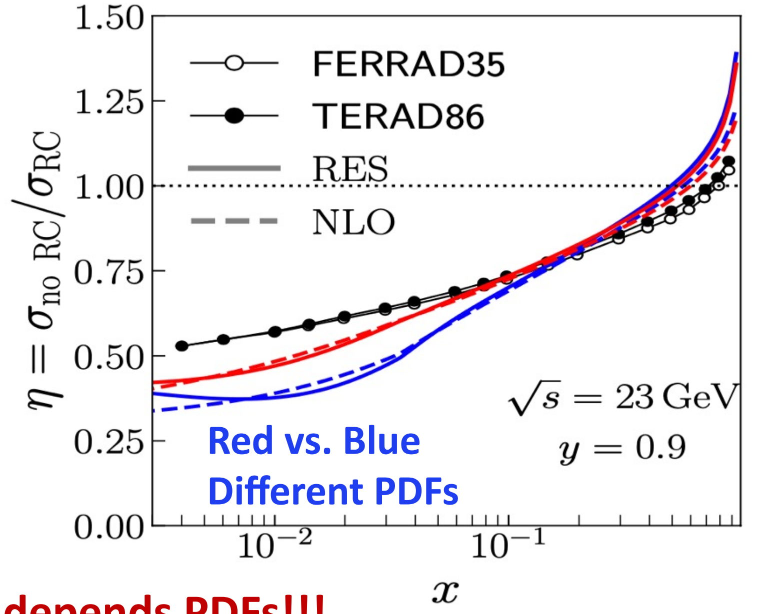
$y = 0.95$

EIC eP could reach:

$x_{\min} \sim 5 \times 10^{-5}$

$Q^2 = x_B\, y\, S$





If we do not have confidence for y > 0.5, due to QED radiation,

EIC's eP reach to small-x could be reduced to $x_{\min} \sim 1 \times 10^{-4}$

or effectively, $\boxed{\sqrt{S} = 140\ \mathrm{GeV} \to 102\ \mathrm{GeV\ at}\ y = 0.95}$

"RC" depends PDFs!!!

Jefferson Lab

Inclusive lepton-hadron deep inelastic scattering (DIS)

❑ Numerical impact of QED contribution at EIC ($\sqrt{S} = 140\ \mathrm{GeV}$):

$$\frac{\sigma_{\mathrm{noRC}}}{\sigma_{\mathrm{RC}}} \leftrightarrow \frac{\sigma_{1\gamma}}{\sigma_{\mathrm{measured}}} = \eta(x_B, y)$$

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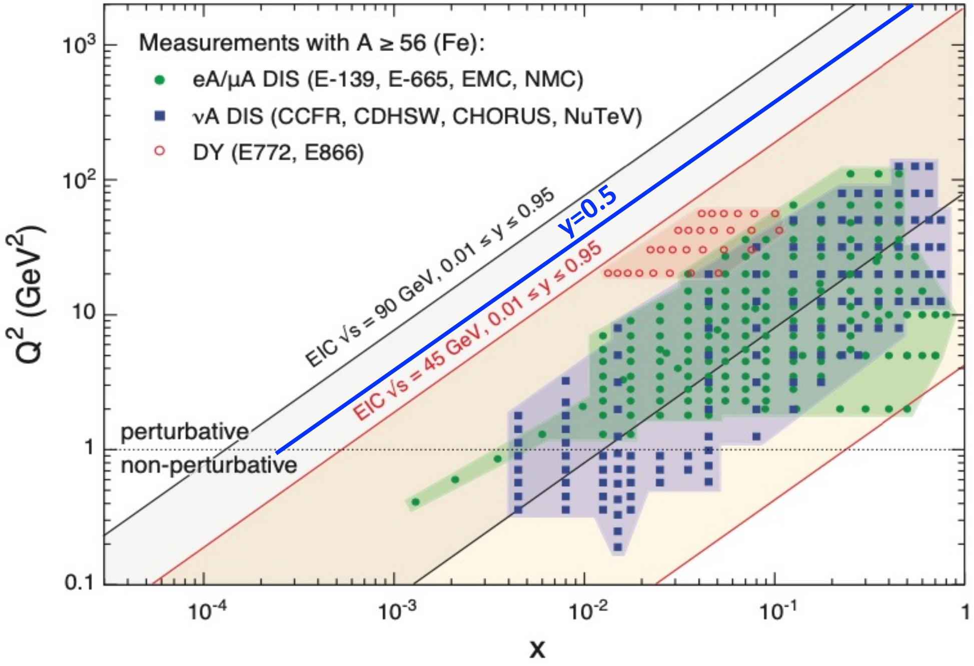
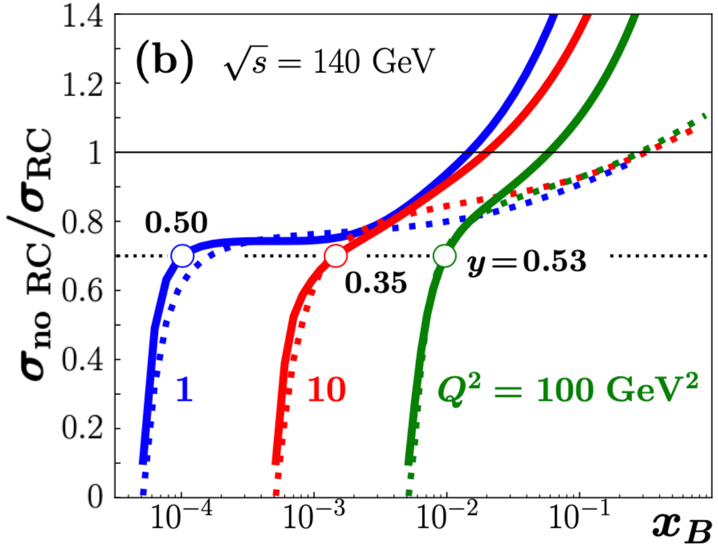




At $\sqrt{S} = 100\ \mathrm{GeV}$

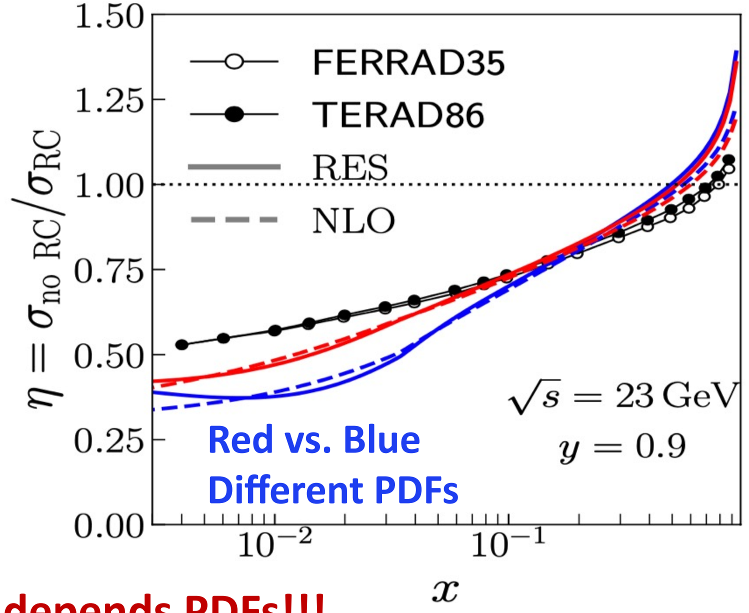
$Q^2 = 1\ \mathrm{GeV}^2$

$y = 0.95$

EIC eA could reach:

$x_{\min} \sim 1 \times 10^{-4}$

$Q^2 = x_B\, y\, S$



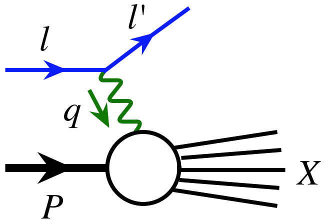
If we do not have confidence for y > 0.5, due to QED radiation,

EIC's eA reach to small-x could be reduced to $x_{\min} \sim 2 \times 10^{-4}$

or effectively, $\boxed{\sqrt{S} = 100\ \mathrm{GeV} \rightarrow 73\ \mathrm{GeV\ at}\ y = 0.95}$

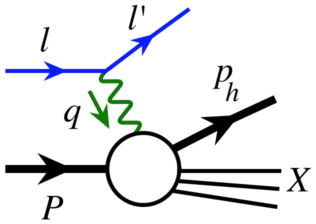
"RC" depends PDFs!!!

What if we measure another particle in the final-state, like SIDIS?



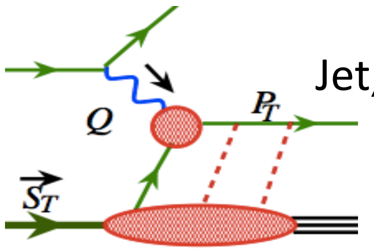
1-Scale: Q^2 **- PDFs**

2-scales



$Q^2 \gg P_{h_T}^2$

In photon-hadron frame!

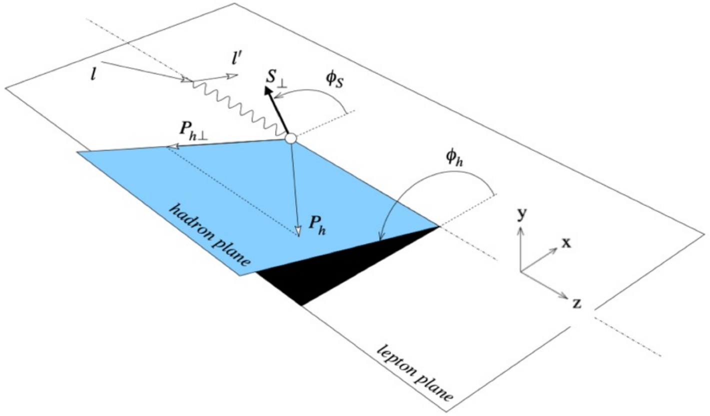


Jet, π, J/ψ, …

$f(x, k_T, Q)$ **- TMDs**

Parton's confined motion, …

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} &= \\
\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{ & F_{UU,T} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&+ \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&+ S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+ S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&+ |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\quad + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&+ |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$



18 SIDIS
Structure Functions

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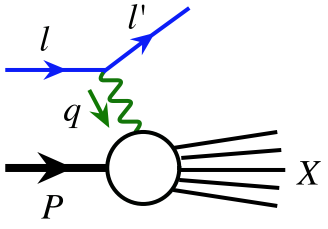
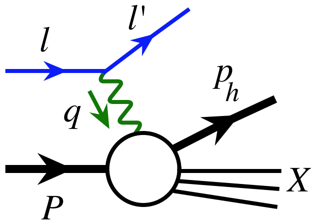
What if we measure another particle in the final-state, like SIDIS?



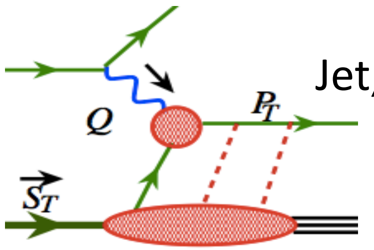
1-Scale: Q^2 **- PDFs**



2-scales

$Q^2 \gg P_{h_T}^2$

In photon-hadron frame!



Jet, π, J/ψ, …

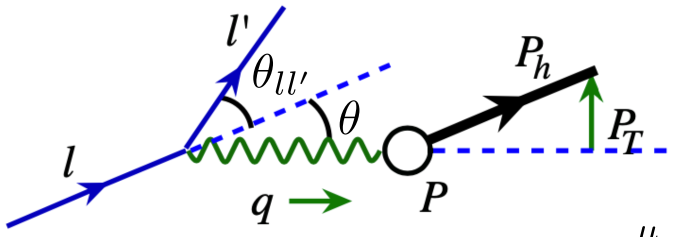
$f(x, k_T, Q)$ **- TMDs**

Parton's confined motion, …

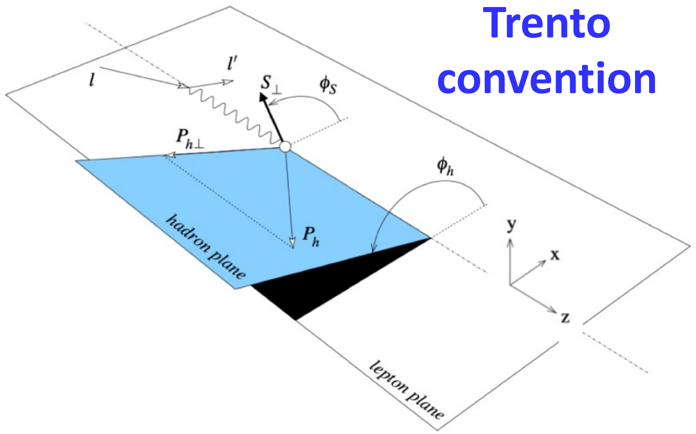
- **Photon-Hadron frame - "Born" kinematic:**

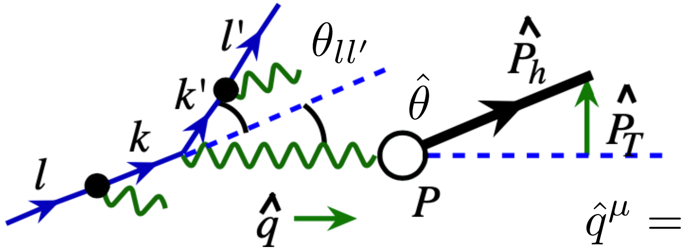
$$E_h \frac{d\sigma}{dx_B dQ^2 d^3 P_h} \approx \frac{\alpha^2}{S} \frac{1+(1-y)^2}{y} \frac{z_h}{Q^2}$$
$$\times \int d^2\mathbf{p}_T d^2\mathbf{p}_{hT} \delta^{(2)}\left(\mathbf{p}_T - \mathbf{p}_{hT} - \mathbf{P}_{hT}/z_h\right)$$
$$\times D_{h/j}(z_h, \mathbf{p}_{hT}) f_{i/h}(x, \mathbf{p}_T) + \ldots$$

- **QED radiation – NO "Born" kinematic:**



Measured: $q^\mu = (l - l')^\mu$





$\hat{q}^\mu = (k - k')^\mu$

True photon momentum – never measured:

$q^\mu \neq \hat{q}^\mu$

$\theta \neq \hat{\theta}$

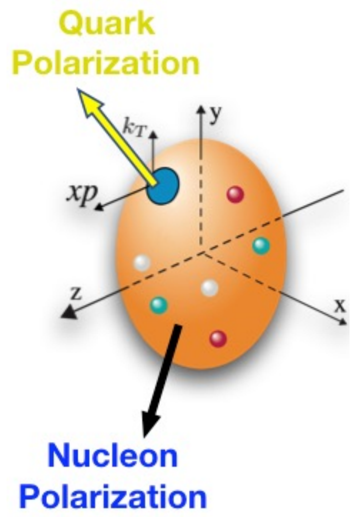
$P_T \neq \hat{P}_T$ ***Trouble!***

Jefferson Lab

Transverse momentum dependent PDFs (TMDs)

❑ Quark TMDs with polarization:

		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$		$h_1^{\perp}(x,k_T^2)$ - Boer-Mulders
	L		$g_1(x,k_T^2)$ - Helicity	$h_{1L}^{\perp}(x,k_T^2)$ - Long-Transversity
	T	$f_{1T}^{\perp}(x,k_T^2)$ - Sivers	$g_{1T}(x,k_T^2)$ - Trans-Helicity	$h_1(x,k_T^2)$ - Transversity $h_{1T}^{\perp}(x,k_T^2)$ - Pretzelosity



Analogous tables for:

- Gluons $f_1 \to f_1^g$ etc
- Fragmentation functions
- Nuclear targets $S \neq \frac{1}{2}$

Transverse momentum dependent PDFs (TMDs)

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		Quark Polarization		
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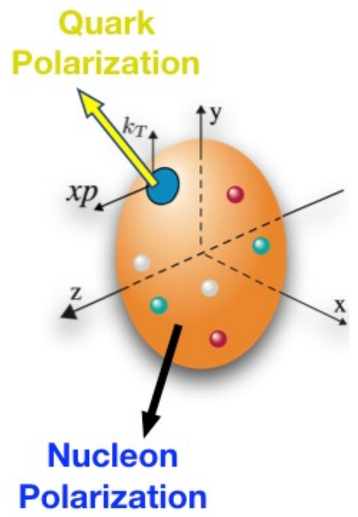
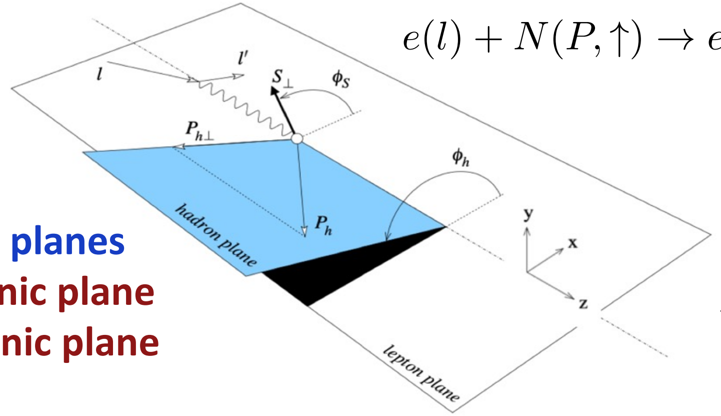
Nucleon Polarization	U	$f_1(x,k_T^2)$		$h_1^{\perp}(x,k_T^2)$ *Boer-Mulders*
	L		$g_1(x,k_T^2)$ *Helicity*	$h_{1L}^{\perp}(x,k_T^2)$ *Long-Transversity*
	T	$f_1^{\perp}(x,k_T^2)$ *Sivers*	$g_{1T}(x,k_T^2)$ *Trans-Helicity*	$h_1(x,k_T^2)$ *Transversity* $h_{1T}^{\perp}(x,k_T^2)$ *Pretzelosity*



Analogous tables for:
- Gluons $f_1 \to f_1^g$ etc
- Fragmentation functions
- Nuclear targets $S \neq \frac{1}{2}$

❑ Polarized SIDIS:



Two planes
Leptonic plane
Hadronic plane

$$e(l) + N(P,\uparrow) \to e(l') + h(P_h) + X$$

Single Transverse-Spin Asymmetry

$$A_{UT} = \frac{1}{P}\frac{\sigma_{lN(\uparrow)} - \sigma_{lN(\downarrow)}}{\sigma_{lN(\uparrow)} + \sigma_{lN(\downarrow)}}$$

In photon-hadron frame:

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^{\perp}$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1$$

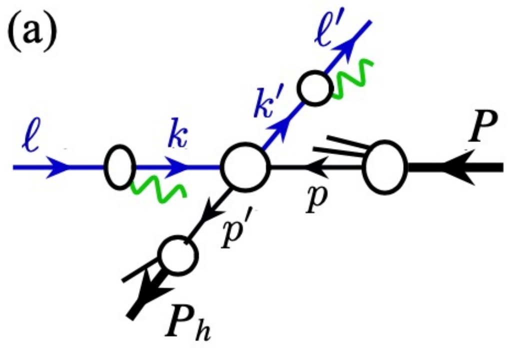
$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$$

Angular modulation provides the best way to separate TMDs

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

❑ Inclusive production of a lepton and a hadron:



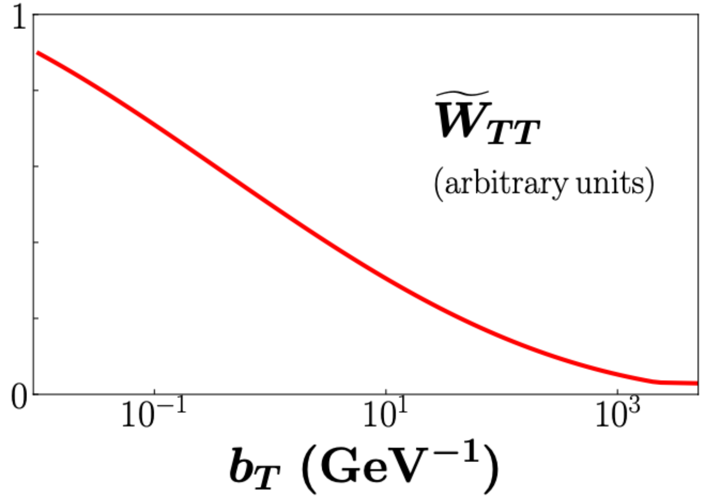
$$e(\ell) + N(P) \to e(\ell') + h(P_h) + X$$

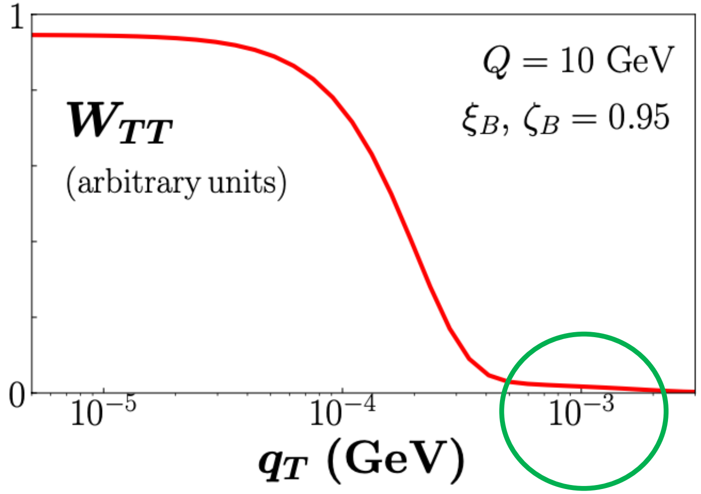
Momentum imbalance between the lepton and the hadron could be sensitive to both parton TMDs and lepton TMDs

Typical parton transverse momentum: $k_T^2 \sim \Lambda_{\text{QCD}}^2 + \langle k_T^2 \rangle_{\text{generated by QCD shower}}$

❑ Estimate of lepton transverse momentum generated by QED shower:

Resummation to lepton TMD





QED broadening for lepton is so much smaller than typical parton kT!

Collinear factorization for high order QED contributions

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

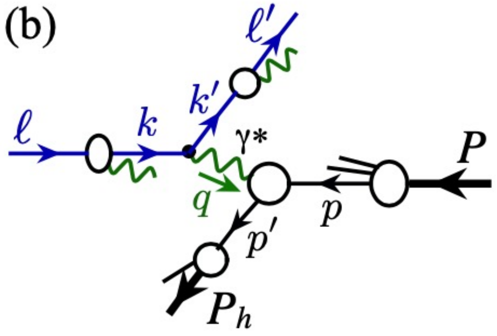
❑ QED factorization of collision-induced radiation – collinear:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$E_{\ell'}E_{P_h}\frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{\mathrm{d}^3\ell'\,\mathrm{d}^3P_h} \approx \sum_{ij\lambda_k}\int_{\zeta_{\min}}^1 \frac{\mathrm{d}\zeta}{\zeta^2} D_{e/j}(\zeta)\int_{\xi_{\min}}^1 \mathrm{d}\xi\, f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\left[E_{k'}E_{P_h}\frac{\mathrm{d}^6\hat{\sigma}_{k(\lambda_k)P(S)\to k' P_h X}}{\mathrm{d}^3k'\,\mathrm{d}^3P_h}\right]_{k=\xi\ell,k'=\ell'/\zeta} + \mathcal{O}\!\left(\frac{m_e^n}{Q^n}\right)$$

- Leading power IR sensitive contribution is universal, as $m_e/Q \to 0$, factorized into LDFs and LFFs
- IR safe contributions are calculated order-by-order in powers of α
- Neglect m_e/Q power suppressed contributions
- Collinear QED factorization for both inclusive DIS and SIDIS, or e^+e^-, … [global fits of LDFs, LFFs]

❑ "One photon"-approximation:



$$\frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{\mathrm{d}x_B\mathrm{d}y\,\mathrm{d}\psi\,\mathrm{d}z_h\,\mathrm{d}\phi_h\mathrm{d}P_{hT}^2} = \sum_{ij\lambda_k}\int_{\zeta_{\min}}^1\frac{\mathrm{d}\zeta}{\zeta^2}\int_{\xi_{\min}}^1\frac{\mathrm{d}\xi}{\xi} f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\,D_{e/j}(\zeta)$$

$$\times\frac{\hat{x}_B}{x_B\xi\zeta}\underbrace{\left[\frac{\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\frac{\hat{y}^2}{2(1-\hat{\varepsilon})}\left(1+\frac{\hat{\gamma}^2}{2\hat{x}_B}\right)\sum_n \hat{w}_n F_n^h(\hat{x}_B,\widehat{Q}^2,\hat{z}_h,\widehat{P}_{hT}^2)\right]}_{\boxed{\text{Evaluated in a “virtual photon-hadron” frame}}}$$

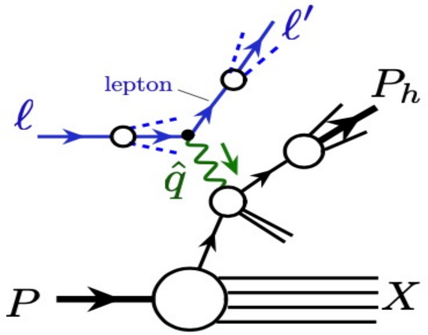
Apply a (ξ,ζ)-dependent Lorentz transformation:

$$\{\hat{q},P,\widehat{P}_h\} \xrightarrow[(\xi,\zeta)]{} \{q,P,P_h\}$$

In a frame to compare with exp. measurements

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

❑ Two-step approach to SIDIS:



One-photon approximation

1) In “virtual-photon” frame, defined by $\hat{q}(\xi, \zeta) - p$

- TMD factorization when $\widehat{P}_T^2 \ll \widehat{Q}^2$
- CO factorization when $\widehat{P}_T^2 \sim \widehat{Q}^2$
- Matching to get the $\widehat{P}_T$-distribution

2) Lorentz transformation from the “virtual-photon” frame to any experimentally defined frame
– lepton-hadron Lab frame, Breit frame (x_B,Q^2), …

QED contribution (not correction) can be systematically improved order-by-order in power α!

❑ Case study F_{UU}:

$$
\begin{aligned}
&\frac{d\sigma}{dx\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{\boxed{F_{UU,T}} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} \\
&\quad + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, F_{LU}^{\sin\phi_h} \\
&\quad + S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\; F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\, F_{LL}^{\cos\phi_h}\right] \\
&\quad + |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad + \varepsilon\,\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\, F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&\quad + |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\, F_{LT}^{\cos\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

❑ Case study F_{UU}:

$$\frac{d\sigma^h_{\rm SIDIS}}{dx_B dy\, dz\, dP^2_{hT}} = \int_{\zeta_{\rm min}}^1 d\zeta \int_{\xi_{\rm min(\zeta)}}^1 d\xi\, D_{e/e}(\zeta)\, f_{e/e}(\xi) \times \underbrace{\left[\frac{\hat{x}_B}{x_B\,\xi\zeta}\right]\left[\frac{(2\pi)^2\,\alpha}{\hat{x}_B\hat{y}\,\widehat{Q}^2}\frac{\hat{y}^2}{2(1-\hat{\varepsilon})}F^h_{UU}(\hat{x}_B,\widehat{Q}^2,\hat{z},\widehat{P}_{hT})\right]}_{\text{Evaluated in a “virtual photon-hadron” frame}}$$

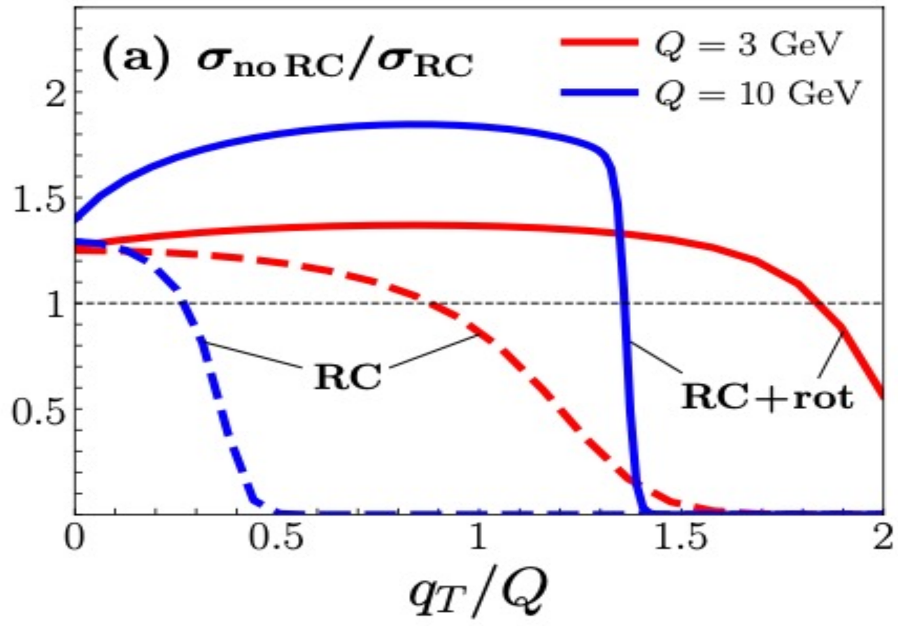
Unpolarized structure function:

$$F^h_{UU} = x_B \sum_q e_q^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{q}_T) \times f_{q/N}(x_B, \boldsymbol{p}_T^2)\, D_{h/q}(z, \boldsymbol{k}_T^2) \qquad \boldsymbol{q}_T = \boldsymbol{P}_{hT}/z$$

(ξ,ζ) - **Dependent Lorentz transformation**

Effectively, a rotation in hadron-rest frame

Solid – with Lorentz transformation
Dashed – without Lorentz transformation



Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

❑ Case study – single transverse spin asymmetry:

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h}$$

$$+\varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}$$

$$+S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\right]$$

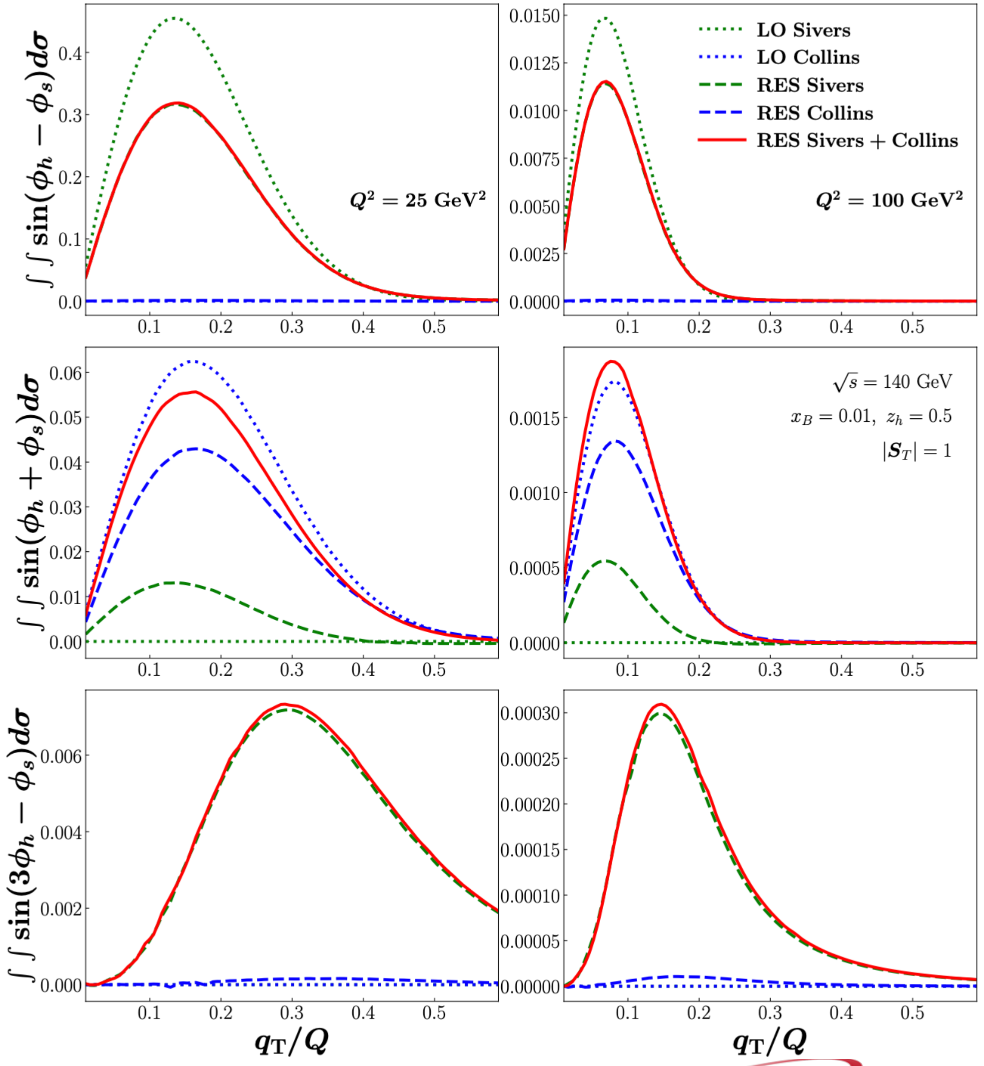
$$+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right]$$

$$\boxed{+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) +\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}}$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+|\boldsymbol{S}_\perp|\lambda_e\left[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}\right.$$

$$\left.+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\right]\Bigg\}$$



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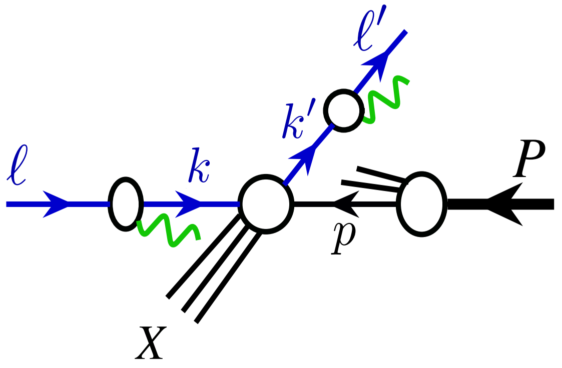
Collinear factorization for QED radiative contribution

- **Without the "one-photon" approximation:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

~ Inclusive single lepton production at high transverse momentum

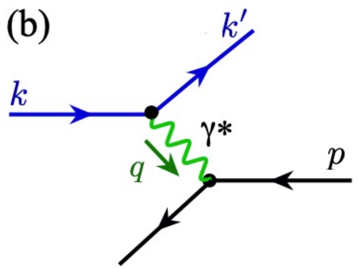


$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2)\;+\;\cdots$$

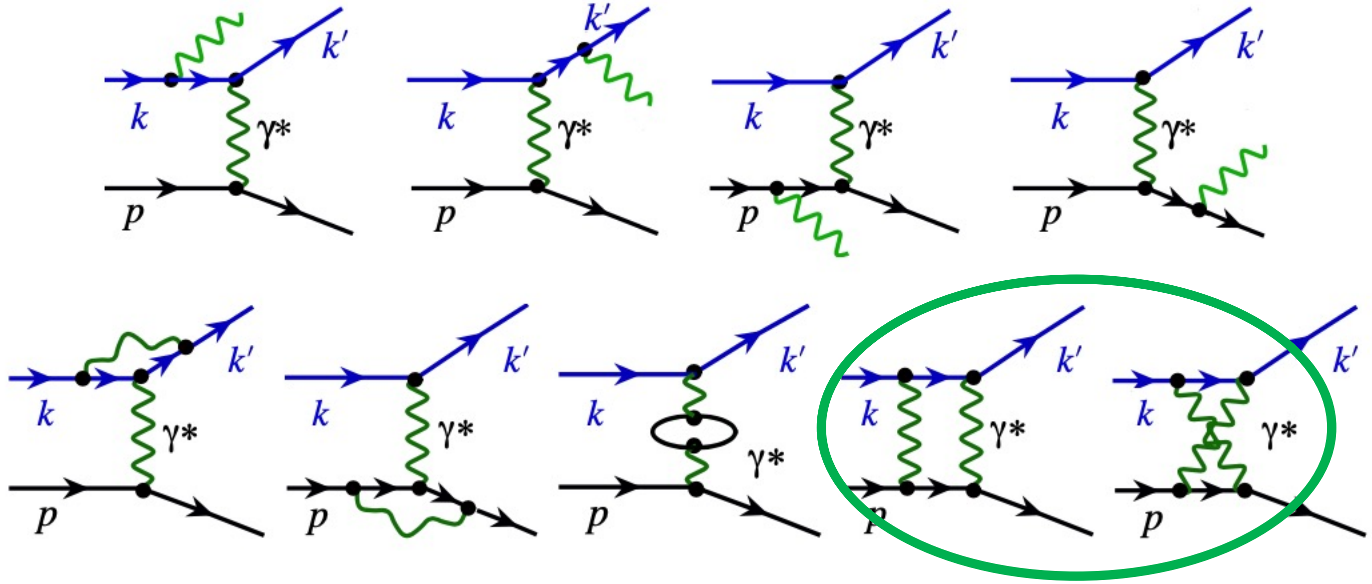
No structure functions, but have PDFs, LDFs, LFFs, …

- **Calculated hard parts in power of $\alpha^m\alpha_s^n$:**

LO



NLO:



More systematic for PVDIS!

Beyond one-photon exchange

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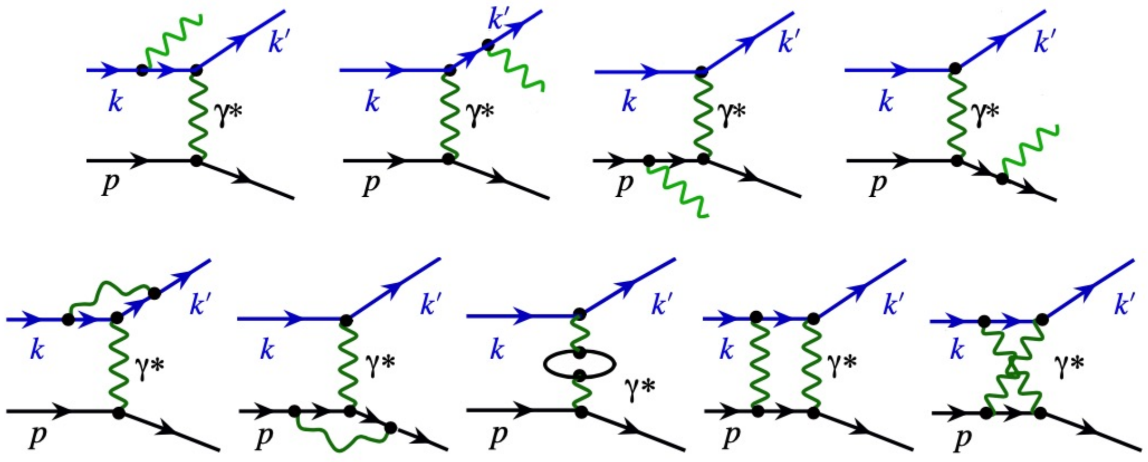
Beyond 1-vector meson exchange: NLO QED contribution

- **Project the external particles to leptons or partons:**

NLO: $e(\ell) \to e(k)\,, \quad e(\ell') \to e(k')\,, \quad h(P) \to q(p)\,.$

$$\begin{aligned}
\sigma^{(1)}_{e(k)+q(p)\to e(k')+X} = & D^{(0)}_{e/e} \otimes f^{(0)}_{e/e} \otimes f^{(0)}_{q/q} \otimes \hat{H}^{(1)}_{e+q\to e+X} \\
& + D^{(1)}_{e/e} \otimes f^{(0)}_{e/e} \otimes f^{(0)}_{q/q} \otimes \hat{H}^{(0)}_{e+q\to e+X} \\
& + D^{(0)}_{e/e} \otimes f^{(1)}_{e/e} \otimes f^{(0)}_{q/q} \otimes \hat{H}^{(0)}_{e+q\to e+X} \\
& + D^{(0)}_{e/e} \otimes f^{(0)}_{e/e} \otimes f^{(1)}_{q/q} \otimes \hat{H}^{(0)}_{e+q\to e+X} \\
& + D^{(0)}_{e/e} \otimes f^{(0)}_{e/e} \otimes f^{(1)}_{\gamma/q} \otimes \hat{H}^{(0)}_{e+\gamma\to e+X}
\end{aligned}$$



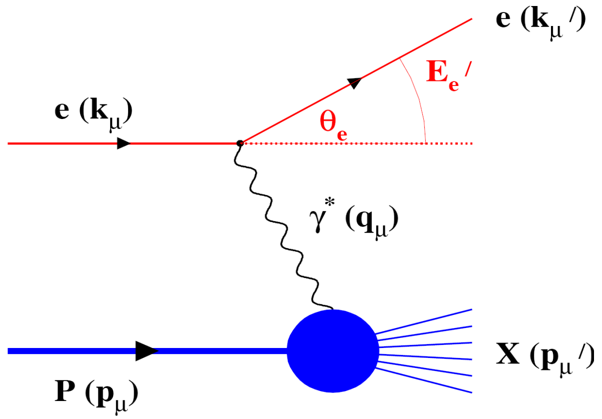
$$\begin{aligned}
\Longrightarrow \quad \hat{H}^{(1)}_{e+q\to e+X} = \sigma^{(1)}_{e+q\to e+X} & - D^{(1)}_{e/e} \otimes \hat{H}^{(0)}_{e+q\to e+X} - f^{(1)}_{e/e} \otimes \hat{H}^{(0)}_{e+q\to e+X} - f^{(1)}_{q/q} \otimes \hat{H}^{(0)}_{e+q\to e+X} \\
& - f^{(1)}_{\gamma/q} \otimes \hat{H}^{(0)}_{e+\gamma\to e+X}
\end{aligned}$$

Completely IR and CO safe! Only depends on factorization scale μ, same in all partonic scattering channels
No need for any "cut-off" parameter(s) in the traditional "Radiative Correction"

➡ **In joint QCD & QED factorization: Lepton-distributions are not pure QED !**
Hadron's parton distributions are not pure QCD !

Separation of LDFs from LFFs – A simpler process

❑ Recall: Photoproduction in ep collision is important & sensitive to how the "photon" is defined



- **Real or quasi-photon is defined by**

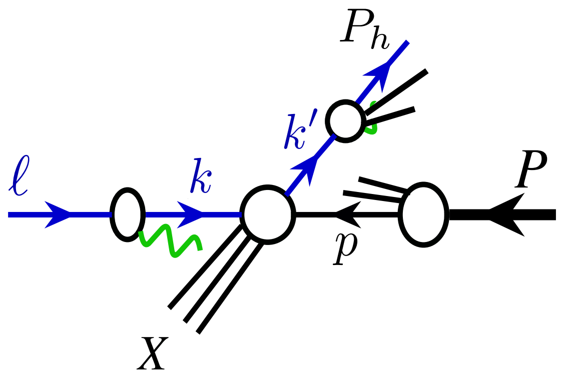
  $k'_T \leq k_{T_{\rm cut}}$ or $\theta_e \leq \theta_{\rm cut}$

- **Photon flux is derived by**

  Evaluating the photon shower with above "cut"
  Weizsaecker-Williiams photon distribution, …

❑ Inclusive single hadron (jet) production in ep collision:

Kang, Meta, Qiu, Zhou, PRD 2011
Hinderer, Schlegel, Vogelsang, PRD 2015, 2016
Abelof, Boughezal, Liu, Petriello, PLB, 2016
Qiu, Wang, Xing, CPL, 2021
Qiu, Watanabe, in preparation



With measuring the scattered electron!

Single hard scale, collinear factorization

$$E_h \frac{d\sigma_{\ell P\to P_h x}}{d^3 P_h} = \frac{1}{2s}\sum_{i,a,b}\int_{z_{\rm min}}^1 \frac{dz}{z^2}\int_{\xi_{\rm min}}^1 \frac{d\xi}{\xi} D_{h/b}(z,\mu^2) f_{i/e}(\xi,\mu^2)$$
$$\times \int_{x_{\rm min}}^1 \frac{dx}{x} f_{a/N}(x,\mu^2)\, \widehat{H}_{ia\to bX}(\xi\ell, xP, P_h/z, \mu^2) + \ldots$$

- **Universal lepton distribution functions (LDFs)**
- **No artificial cut to define the "photon"**
- **Single factorization scale: μ**

Evolution of lepton distribution functions (LDFs)

❑ Modified DGLAP equation for LDFs:

Qiu, Watanabe
In preparation

$$\frac{\partial}{\partial \ln\mu^2}\begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix} = \left(\begin{array}{ccc|ccc} P_{ee}^{(1,0)} & P_{e\bar{e}}^{(2,0)} & P_{e\gamma}^{(1,0)} & P_{eq}^{(2,0)} & P_{e\bar{q}}^{(2,0)} & P_{eg}^{(2,1)} \\ P_{\bar{e}e}^{(2,0)} & P_{\bar{e}\bar{e}}^{(1,0)} & P_{\bar{e}\gamma}^{(1,0)} & P_{\bar{e}q}^{(2,0)} & P_{\bar{e}\bar{q}}^{(2,0)} & P_{\bar{e}g}^{(2,1)} \\ P_{\gamma e}^{(1,0)} & P_{\gamma\bar{e}}^{(1,0)} & P_{\gamma\gamma}^{(1,0)} & P_{\gamma q}^{(1,0)} & P_{\gamma\bar{q}}^{(1,0)} & P_{\gamma g}^{(1,1)} \\ \hline P_{qe}^{(2,0)} & P_{q\bar{e}}^{(2,0)} & P_{q\gamma}^{(1,0)} & P_{qq}^{(0,1)} & P_{q\bar{q}}^{(0,2)} & P_{qg}^{(0,1)} \\ P_{\bar{q}e}^{(2,0)} & P_{\bar{q}\bar{e}}^{(2,0)} & P_{\bar{q}\gamma}^{(1,0)} & P_{\bar{q}q}^{(0,2)} & P_{\bar{q}\bar{q}}^{(0,1)} & P_{\bar{q}g}^{(0,1)} \\ P_{ge}^{(2,1)} & P_{g\bar{e}}^{(2,1)} & P_{g\gamma}^{(1,1)} & P_{gq}^{(0,1)} & P_{g\bar{q}}^{(0,1)} & P_{gg}^{(0,1)} \end{array}\right) \otimes \begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix}$$

Evolution kernels in both QCD and QED:

$$P_{ij}(\xi,\mu^2) = \sum_{n,m=0}^{\infty}\left(\frac{\alpha_{em}(\mu^2)}{2\pi}\right)^n \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^m \hat{P}_{ij}^{(n,m)}(\xi) = \sum_{n,m=0}^{\infty} P_{ij}^{(n,m)}(\xi,\mu^2)$$

with $P_{ij}^{(0,0)} = 0\,, \quad N_F\,, \quad N_l$

- **Factorization scale:**

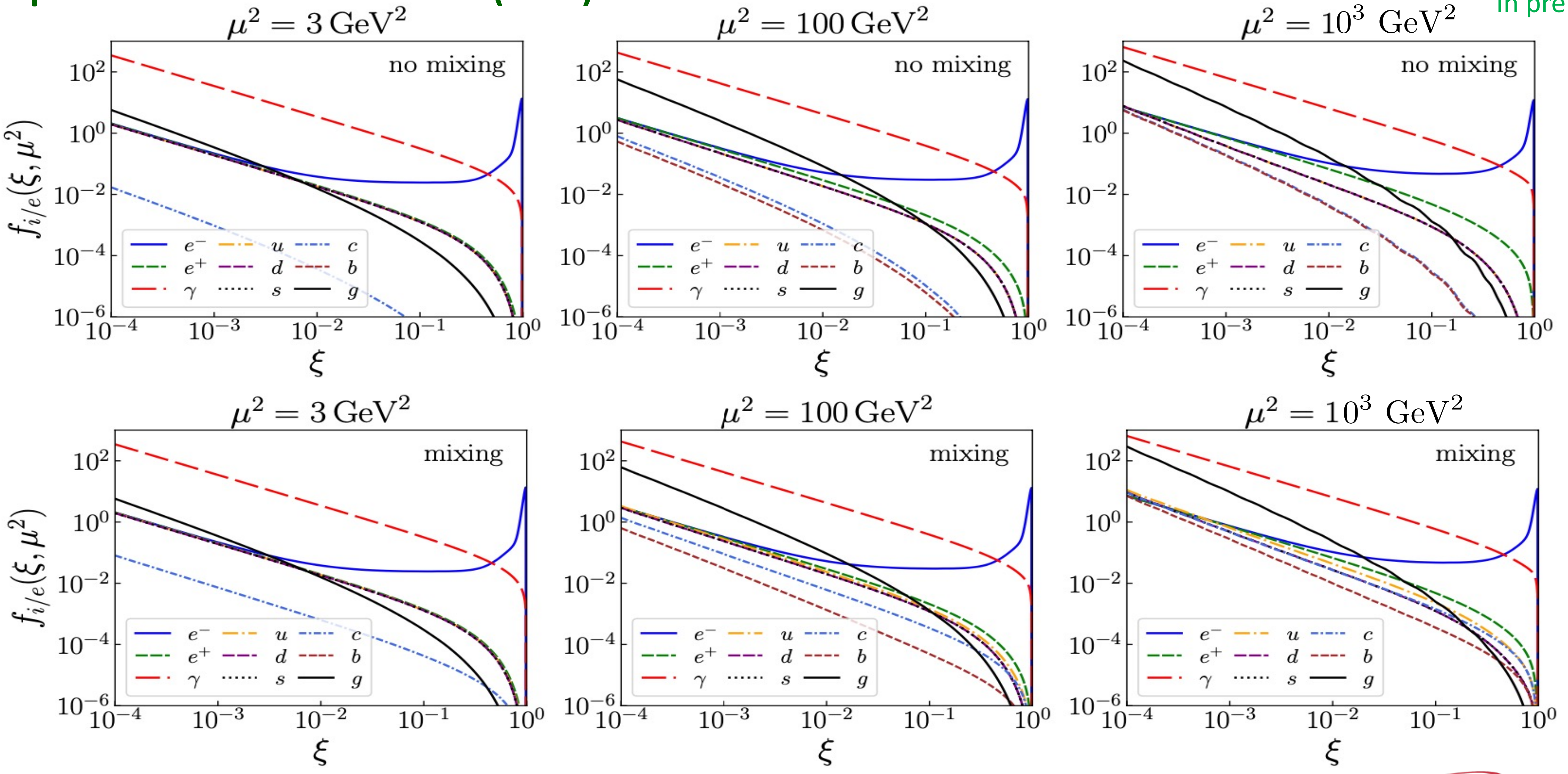
  $\mu^2 \sim m_c^2$

- **Input LDFs at μ^2:**
  - Perturbatively generated by solving QED evolution from lepton mass threshold
  - With perturbatively calculated fixed-order MSbar LDFs
  - Test the size of non-perturbative hadronic contribution
  - …

Evolution of lepton distribution functions (LDFs)

Qiu, Watanabe
In preparation

- **Lepton distribution functions (LDFs):**



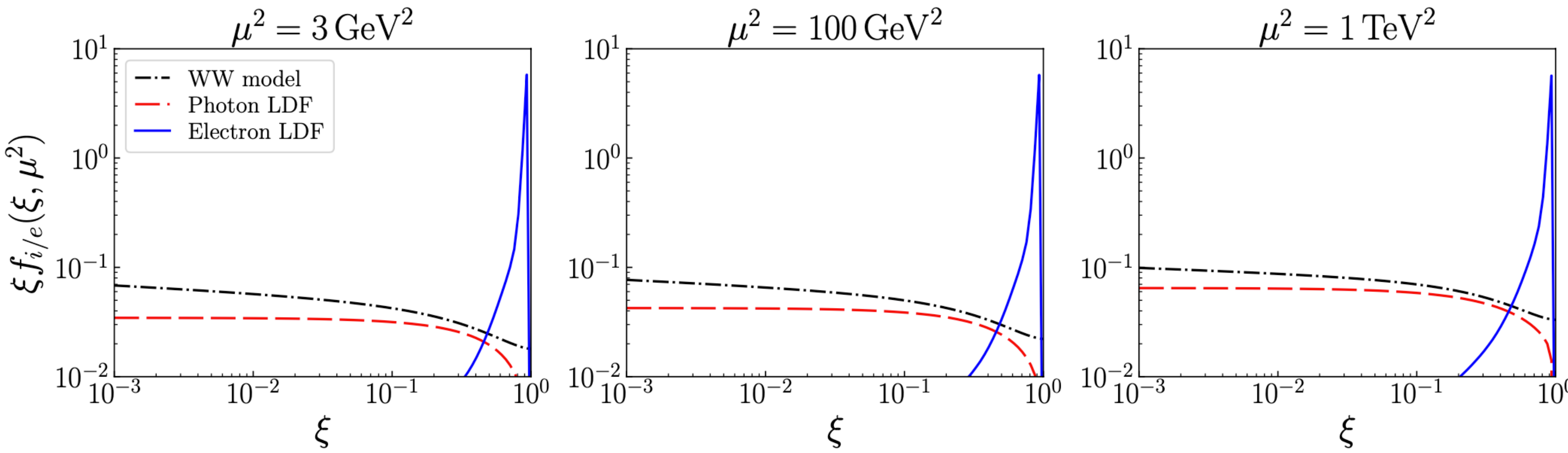
With LDFs, we calculated single hadron production, including J/ψ production at the EIC

Evolution of lepton distribution functions (LDFs)

❑ Photon distribution of the electron:

- **Weizsäcker-William photon distribution:**

$$f_{\gamma/e}^{\mathrm{WW}}(\xi,\mu^2) = \frac{\alpha_{em}(\mu^2)}{2\pi} P_{\gamma e}(\xi)\left[\ln\left(\frac{\mu^2}{\xi^2 m_e^2}\right) - 1\right]$$



- **LDFs are not purely perturbative in QED!**

  - **Precision measurements for BSM physics at the EIC needs reliable lepton distributions**
  - **Joint global analysis of lepton and hadron distribution functions should be carried out.**
  - **Impact on searching BSM at ILC or CEPC, FCC, …**

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Parity violating lepton-hadron DIS

❑ Inclusive single lepton cross sections:

- Still assume one-vector boson exchange – to test the impact of collinear radiation – LDFs
- Finishing up the complete NLO hard part for EW+QCD

Process:

$$l(\ell) + N(P) \to l(\ell') + X$$

Cross section:

$$\frac{d\sigma}{dx_B dy d\psi} = \frac{Q^2}{2x_B}\frac{E_{\ell'} d\sigma}{d^3\ell'} = \frac{\alpha^2 y}{Q^4}\sum_i \eta_i L_i^{\mu\nu} W_{\mu\nu}^i$$

$$\eta_\gamma = 1,$$

$$\eta_{\gamma Z} = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\frac{Q^2}{Q^2+M_Z^2},$$

$$\eta_Z = \eta_{\gamma Z}^2,$$

$$\begin{aligned}
W_{\mu\nu}^i(P,q,S) = & -\tilde{g}_{\mu\nu}\, F_1^i(x_B,Q^2) + \frac{\widetilde{P}_\mu \widetilde{P}_\nu}{P\cdot q} F_2^i(x_B,Q^2) - i\epsilon_{\mu\nu\rho\sigma}\frac{q^\rho P^\sigma}{2P\cdot q} F_3^i(x_B,Q^2) \\
& + i\epsilon_{\mu\nu\rho\sigma}\frac{q^\rho M}{P\cdot q}\left[S^\sigma g_1^i(x_B,Q^2) + \left(S^\sigma - \frac{S\cdot q}{P\cdot q}P^\sigma\right) g_2^i(x_B,Q^2)\right] \\
& + \frac{M}{P\cdot q}\left[\frac{1}{2}\left(\widetilde{P}_\mu \widetilde{S}_\nu + \widetilde{P}_\nu \widetilde{S}_\mu\right) - \frac{S\cdot q}{P\cdot q}\widetilde{P}_\mu \widetilde{P}_\nu\right] g_3^i(x_B,Q^2) \\
& + M\frac{S\cdot q}{P\cdot q}\left[\frac{\widetilde{P}_\mu \widetilde{P}_\nu}{P\cdot q} g_4^i(x_B,Q^2) - \tilde{g}_{\mu\nu}\, g_5^i(x_B,Q^2)\right],
\end{aligned}$$

$$L_\gamma^{\mu\nu} = 2\left(\ell^\mu \ell'^\nu + \ell^\nu \ell'^\mu - \ell\cdot\ell' g^{\mu\nu} - i\lambda_\ell \epsilon^{\mu\nu\rho\sigma}\ell_\rho \ell'_\sigma\right),$$

$$L_{\gamma Z}^{\mu\nu} = (g_V^e + e\lambda_\ell g_A^e) L_\gamma^{\mu\nu},$$

$$L_Z^{\mu\nu} = (g_V^e + e\lambda_\ell g_A^e)^2 L_\gamma^{\mu\nu},$$

Parity violating lepton-hadron DIS

❑ Inclusive single lepton cross sections – one-vector boson approximation:

Taking into account contribution of collinear radiations:

$$\frac{E_{\ell'}\mathrm{d}\sigma_{\ell P\to\ell' X}}{\mathrm{d}^3\ell'} \approx \int_{\zeta_{\min}}^{1} \frac{\mathrm{d}\zeta}{\zeta^2} D_{e/e}(\zeta,\mu^2) \int_{\xi_{\min}}^{1} \mathrm{d}\xi\, f_{e(\lambda_k)/e(\lambda_\ell)}(\xi,\mu^2) \left[\frac{E_{k'}\,\mathrm{d}\hat{\sigma}_{kP\to k'X}}{\mathrm{d}^3k'}\right]_{k=\xi\ell,k'=\ell'/\zeta}$$

Parity violating lepton-spin asymmetry:

$$A_{\rm PVE} = \frac{\sigma_{\ell(\lambda_\ell=1)P\to\ell'X} - \sigma_{\ell(\lambda_\ell=-1)P\to\ell'X}}{\sigma_{\ell(\lambda_\ell=1)P\to\ell'X} + \sigma_{\ell(\lambda_\ell=-1)P\to\ell'X}} = \frac{\Delta\sigma_{\lambda_\ell}}{\sigma_{\ell P\to\ell'X}}$$

Unpolarized cross section:

$$\begin{aligned}\frac{\mathrm{d}\sigma_{\ell P\to\ell'X}}{\mathrm{d}x_B\mathrm{d}y} = &\int_{\zeta_{\min}}^{1} \frac{\mathrm{d}\zeta}{\zeta^2} D_{e/e}(\zeta,\mu^2) \int_{\xi_{\min}}^{1} \mathrm{d}\xi\, f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right] \\
&\times \frac{4\pi\alpha^2}{\hat{x}_B\hat{y}\widehat{Q}^2}\Big[\hat{x}_B\hat{y}^2\,F_1^\gamma(\hat{x}_B,\widehat{Q}^2) + K_{\hat{y}}\,F_2^\gamma(\hat{x}_B,\widehat{Q}^2) \\
&\quad + \eta_{\gamma Z}\,g_V^e\left(\hat{x}_B\hat{y}^2\,F_1^{\gamma Z}(\hat{x}_B,\widehat{Q}^2) + K_{\hat{y}}\,F_2^{\gamma Z}(\hat{x}_B,\widehat{Q}^2)\right) \\
&\quad + \eta_Z\,{g_V^e}^2\left(\hat{x}_B\hat{y}^2\,F_1^{Z}(\hat{x}_B,\widehat{Q}^2) + K_{\hat{y}}\,F_2^{Z}(\hat{x}_B,\widehat{Q}^2)\right)\Big]\end{aligned}$$

Parity violating lepton-hadron DIS

❑ Inclusive single lepton cross sections – one-vector boson approximation:

Taking into account contribution of collinear radiations:

$$\frac{E_{\ell'} \mathrm{d}\sigma_{\ell P\to\ell' X}}{\mathrm{d}^3\ell'} \approx \int_{\zeta_{\min}}^1 \frac{\mathrm{d}\zeta}{\zeta^2} D_{e/e}(\zeta,\mu^2) \int_{\xi_{\min}}^1 \mathrm{d}\xi\, f_{e(\lambda_k)/e(\lambda_\ell)}(\xi,\mu^2) \left[\frac{E_{k'}\, \mathrm{d}\hat\sigma_{kP\to k'X}}{\mathrm{d}^3 k'}\right]_{k=\xi\ell,\, k'=\ell'/\zeta}$$

Parity violating lepton-spin asymmetry:

$$A_{\rm PVE} = \frac{\sigma_{\ell(\lambda_\ell=1)P\to\ell' X} - \sigma_{\ell(\lambda_\ell=-1)P\to\ell' X}}{\sigma_{\ell(\lambda_\ell=1)P\to\ell' X} + \sigma_{\ell(\lambda_\ell=-1)P\to\ell' X}} = \frac{\Delta\sigma_{\lambda_\ell}}{\sigma_{\ell P\to\ell' X}}$$

Polarized cross section:

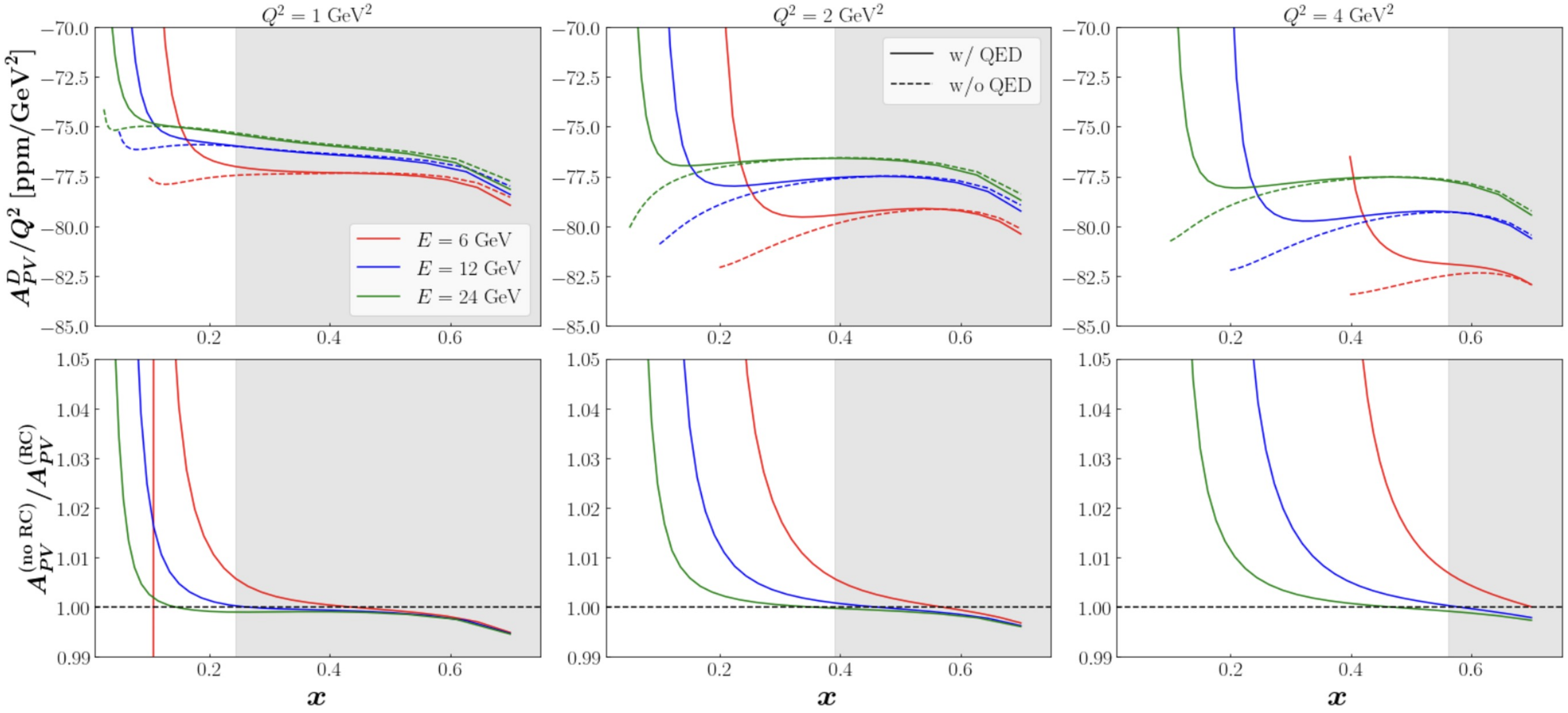
$$\begin{aligned}\frac{\mathrm{d}\Delta\sigma_{\lambda_\ell}}{\mathrm{d}x_B \mathrm{d}y} =& \int_{\zeta_{\min}}^1 \frac{\mathrm{d}\zeta}{\zeta^2} D_{e/e}(\zeta,\mu^2) \int_{\xi_{\min}}^1 \mathrm{d}\xi\, \Delta f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\frac{\hat x_B}{\widehat{Q}^2}\right] \\ &\times \frac{4\pi\alpha^2}{\hat x_B \hat y \widehat{Q}^2}\Big[-\hat x_B\left(\hat y - \tfrac{1}{2}\hat y^2\right) F_3^\gamma(\hat x_B,\widehat{Q}^2) \\ &+ \eta_{\gamma Z}\Big(e g_A^e \hat x_B \hat y^2\, F_1^{\gamma Z}(\hat x_B,\widehat{Q}^2) + e g_A^e K_{\hat y}\, F_2^{\gamma Z}(\hat x_B,\widehat{Q}^2) - g_V^e\left(\hat y - \tfrac{1}{2}\hat y^2\right) F_3^{\gamma Z}(\hat x_B,\widehat{Q}^2)\Big) \\ &+ \eta_Z\Big(2e g_V^e g_A^e \hat x_B \hat y^2\, F_1^Z(\hat x_B,\widehat{Q}^2) + 2e g_V^e g_A^e\, K_{\hat y}\, F_2^Z(\hat x_B,\widehat{Q}^2) \\ &\quad - ({g_V^e}^2 + {g_A^e}^2)\, \hat x_B\left(\hat y - \tfrac{1}{2}\hat y^2\right) F_3^Z(\hat x_B,\widehat{Q}^2)\Big)\Big],\end{aligned}$$

$$\Delta f_{e/e} = f_{e(\lambda_k=1)/e(\lambda_\ell=1)} - f_{e(\lambda_k=-1)/e(\lambda_\ell=1)}$$

A_{PV} at JLab 12 program and beyond – JAM results

❑ Impact of QED radiative contributions:

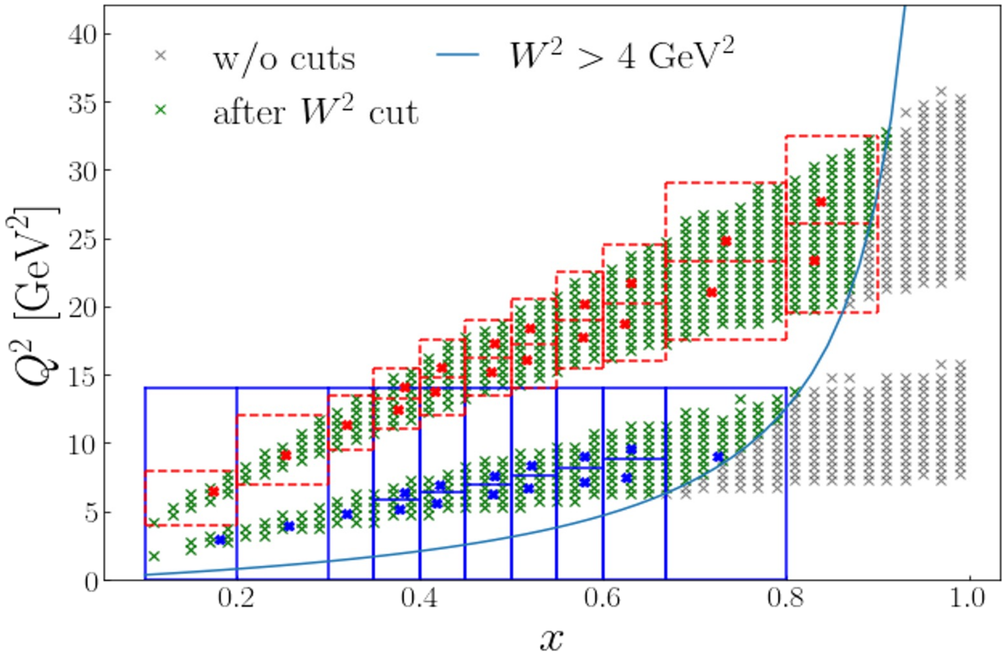
$$\frac{d(\Delta_\ell)\sigma}{dx\,dy} = \int_{\zeta_{\min}}^{1}\int_{\xi_{\min}(\zeta)}^{1}\frac{d\zeta}{\zeta^2}d\xi D_{\ell/\ell}(\zeta)(\Delta_\ell)f_{\ell/\ell}(\xi)\left[\frac{d(\Delta_\ell)\sigma_0}{dx\,dy}\right]_{\ell\to\xi\ell,\ell'\to\ell'/\zeta}$$

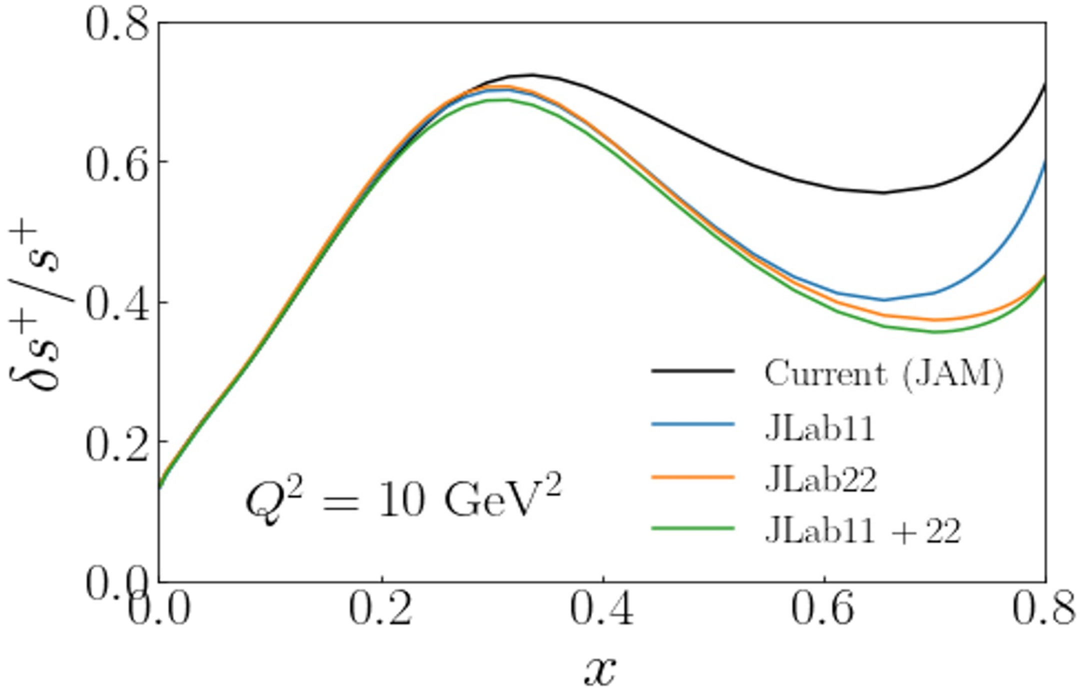


Impact of QED corrections is under control if x is sufficiently large at JLab energy

A_{PV} at JLab 12 program and beyond – JAM results

❑ Impact at JLab 12 and beyond – with SoLID detector:





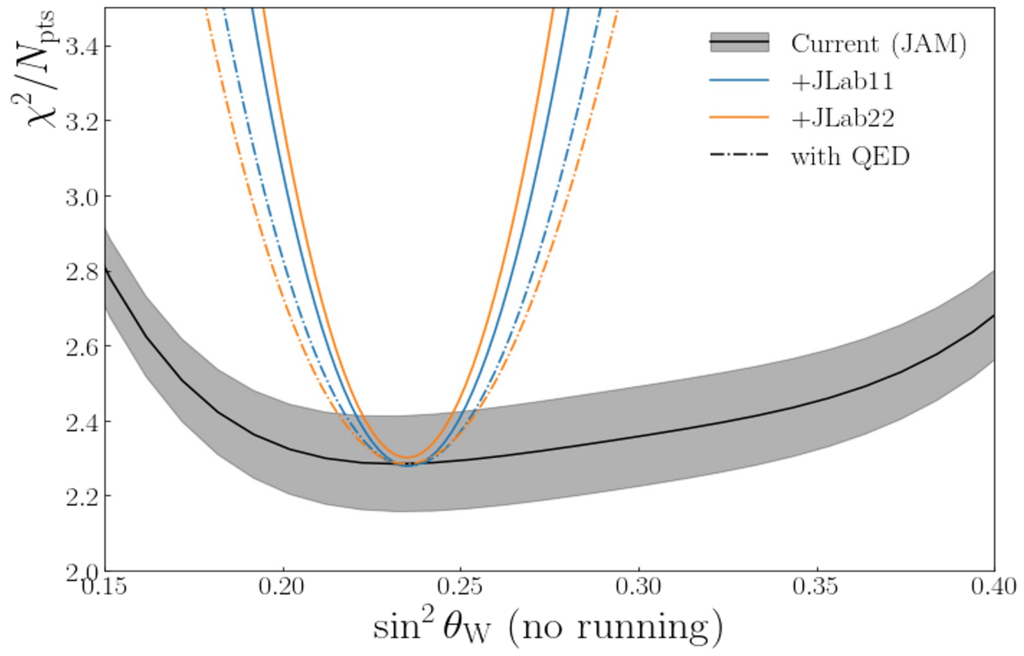
- The JLab 12 program and its upgrade offer competitive constraints compared to the current baseline.
- QED systematics have a minor impact on the projected results.
- The JLab upgrade at 22 GeV offers an opportunity to explore the role of power corrections and test the reliability of extracting the strange quark PDF.

A_{PV} at JLab 12 program and beyond – JAM results

❑ Impact on determining $\sin^2\theta_W$:

- **Apv is a unique class of observables that have not been included in PDF constraints.**
- **It provides a clean signal compared to SIDIS and is complementary to the LHC program.**
- **In particular, the strange quark PDFs in the intermediate to large x region are sensitive to Apv measurements on proton and deuteron targets at JLab kinematics.**
- **It also provides opportunities to constrain the Weinberg angle at lower energies, relevant to BSM searches.**

Impact of A_{PV} deuteron



- Apv provides unique opportunities to constrain the Weinberg angle at scales of Q^2~ 5 GeV^2.
- The reconstruction of the Weinberg angle is largely insensitive to QED effects.
- The impact on Weinberg angle is limited in this analysis by the projected systematic uncertainties.

More work are underway!

Summary and Outlook – Thank you!

- Collision induced QED radiation is an integrated part of the lepton-hadron collision
  - Radiative correction approach is difficult for a consistent treatment beyond the inclusive DIS
  - No well-defined photon-hadron frame, if we cannot recover all QED radiation
  - Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC
- Factorization approach to include both QCD and QED radiative contributions provides a consistent and controllable approximation to high-energy lepton scattering processes
  - QED radiation is a part of production cross sections, treated in the same way as QCD radiation from quarks and gluons (Have not be able to extend this to full EW+QCD factorization!)
  - No artificial and/or process dependent scale(s) introduced for treating QED radiation, other than the standard factorization scale, universal lepton distribution and fragmentation functions
  - All perturbatively calculable hard parts are IR safe for both QCD and QED
  - All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)
- QED collision induced radiation should be treated in terms of factorization approach for lepton scattering processes with a large momentum transfer, in particular, for precision BSM measurements!