INT WORKSHOP INT-20R-77

Origin of the Visible Universe: Unraveling the Proton Mass

June 13, 2022 – June 17, 2022

# Hadron mass decomposition and the role of renormalization

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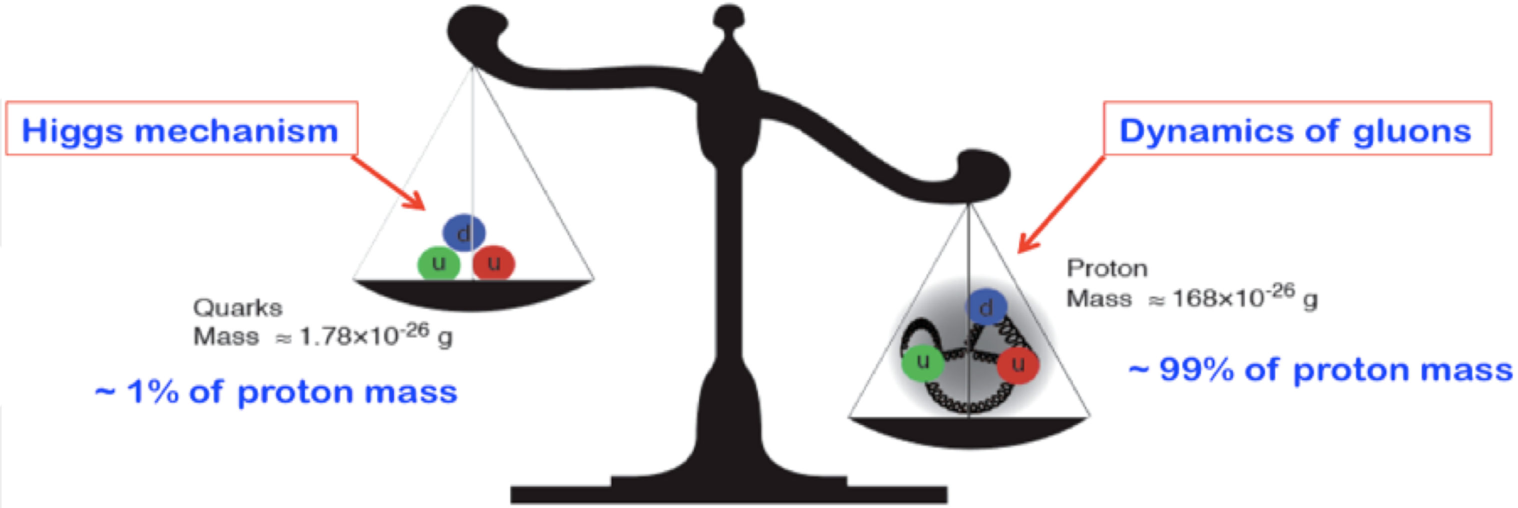


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# Mass of Nucleon?

## ❑ Nucleon Mass:

$m = E/c^2$ from the A. Einstein's famous equation E = mc²

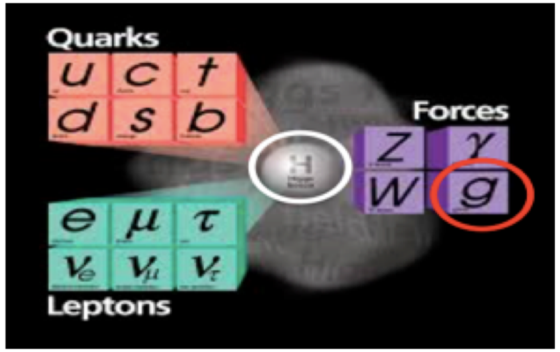
**Mass is the Energy of the nucleon when it is at Rest!**

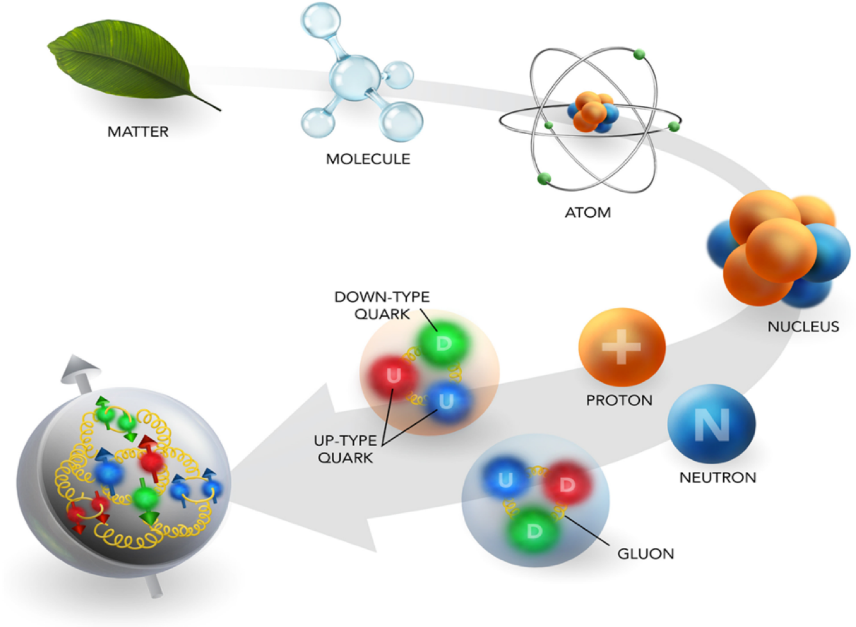
$$M_n = \left.\frac{\langle P|H_{\mathrm{QCD}}(\psi, A)|P\rangle}{\langle P|P\rangle}\right|_{\mathrm{at\ rest}}$$

## ❑ Nucleon is not elementary:

**Nucleon is a strongly interacting, relativistic bound state of quarks and gluons of QCD**

**Our understanding of the nucleon has been evolving, and will continue to evolve, …**



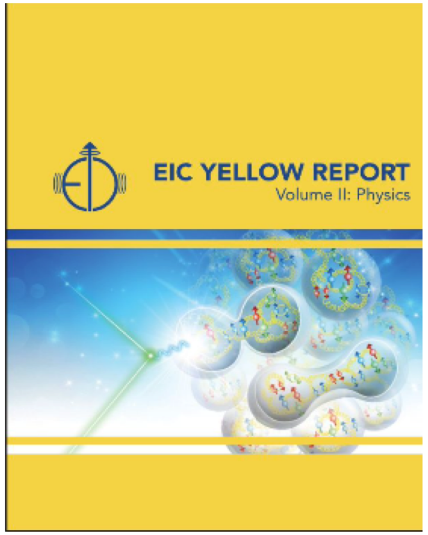

***Understanding it fully is still beyond the best mind that we have!***

$H_{\mathrm{QCD}}(\psi, A)$ **is known, but not** $|P\rangle = ?$

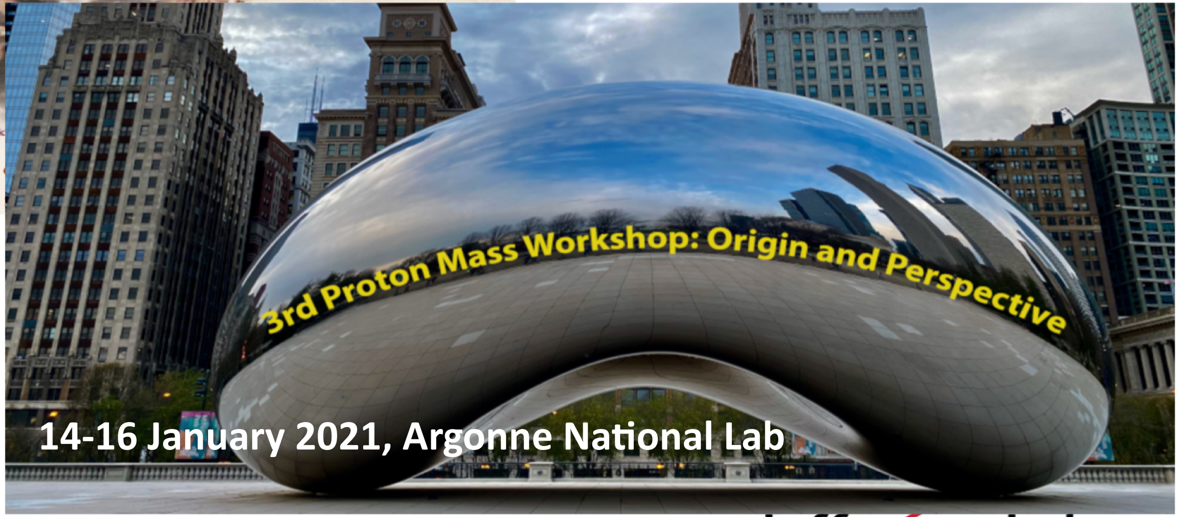
# How does Nucleon Mass arise?

- ❑ **A true international interest and devoted effort:**






(> 200 participants!)




*One of the key questions that EIC is built to address!*



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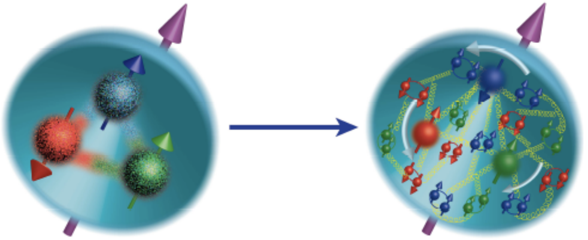
# Mass of Nucleon in QCD

## ❑ Mass without mass:

- QCD Lagrangian does not have mass dimension parameters, other than the masses of current quarks, $m_q \ll M_p$
- Asymptotic freedom ⟷ confinement:



➡ A dynamical scale, $\Lambda_{\rm QCD}$ , consistent with $\dfrac{1}{R} \sim 200\ {\rm MeV}$

## ❑ A consistency check:

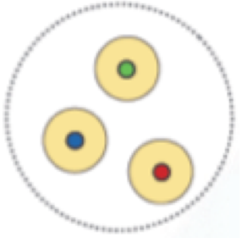
- Bag model:


✧ Kinetic energy of three quarks: $K_q \sim 3/R$

✧ Bag energy (bag constant B): $T_b = \dfrac{4}{3}\pi R^3\, B$

✧ Minimize total energy $K_q + T_b$: $M_p \sim \dfrac{4}{R} \sim \dfrac{4}{0.84 fm} \sim 938\,{\rm MeV}$

- Constituent quark model:



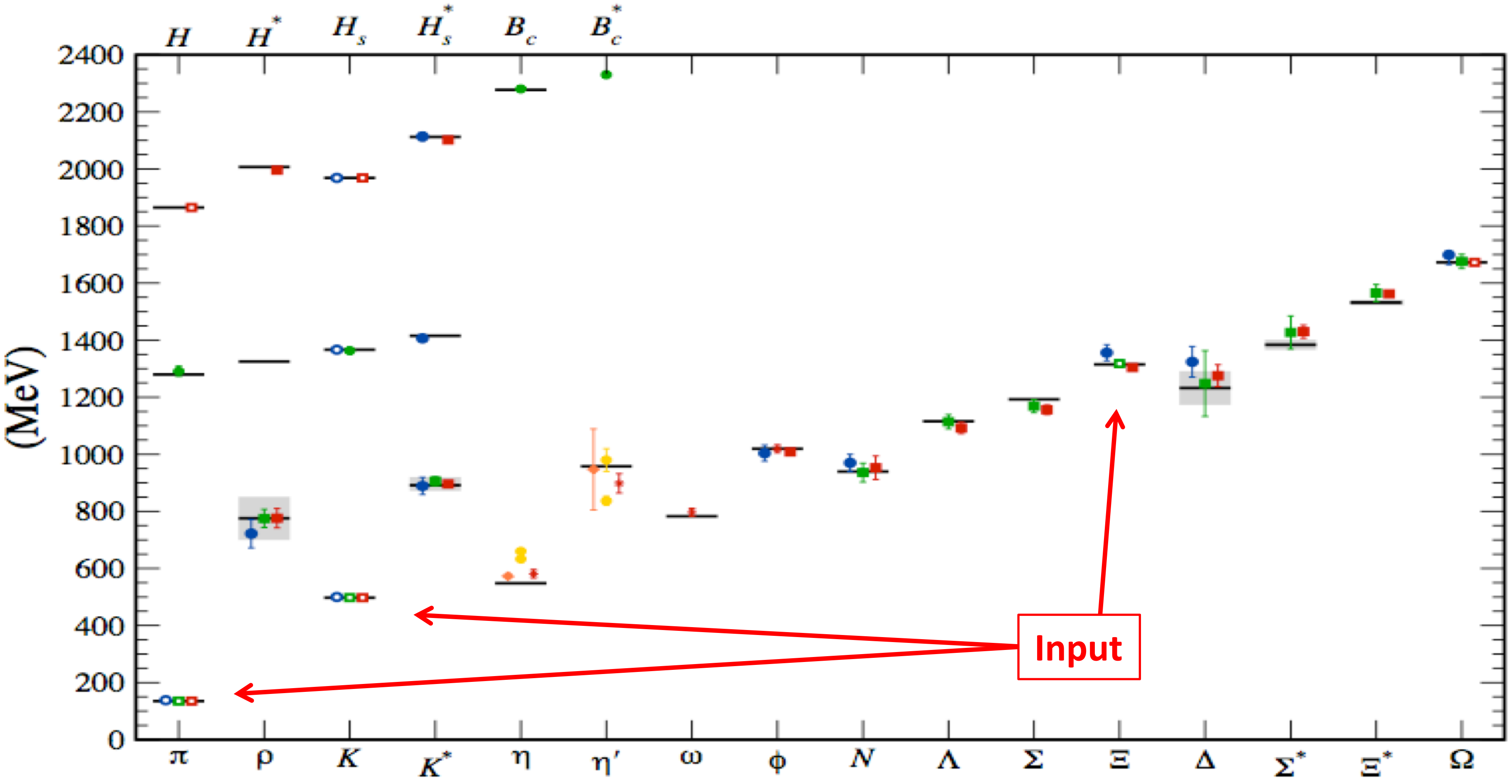
✧ Spontaneous chiral symmetry breaking:

***Massless quarks gain ~300 MeV mass when traveling in vacuum***

➡ $M_p \sim 3\, m_q^{\rm eff} \sim 900\ {\rm MeV}$

# Mass of Nucleon in QCD

## ❑ From Lattice QCD:




*How does QCD generate this? The role of quarks vs. that of gluons?*

***If we do not understand proton mass, we do not understand QCD!***

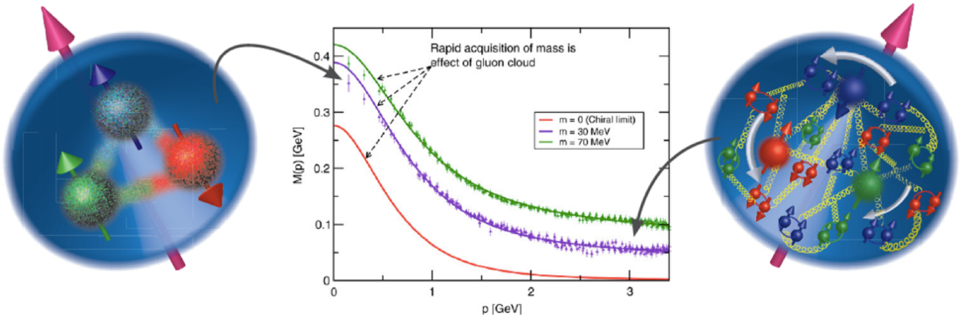
# Beyond Lattice QCD

- **Three-pronged theory approach to explore the origin of nucleon mass:**
  - **Mass decomposition – roles of the constituents – but, not unique!**

    **Matching individual terms to physical observables with controllable approximations – Factorization!**
  - **lattice QCD – calculations of individual terms**
  - **Model calculation – approximated analytical approach**

$$M_n = \sum_{f=q,g} \frac{\langle P|T_f^{00}(0)|P\rangle}{2P^0}\bigg|_{\rm cm}$$

$$= M_q + M_g + M_m + M_a$$




$$M_p = 2m_u^{\rm eff} + m_d^{\rm eff}$$

**Given** $H_{\rm QCD}(\psi, A)$

➡ **QCD Energy-Momentum Tensor (EMT):** $T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$

**With** $T_q^{\mu\nu} = \overline{\psi}\,\gamma^\mu \frac{1}{2}\, i\overleftrightarrow{D}^\nu\,\psi$ **and** $T_g^{\mu\nu} = -F^{\mu\sigma}F^\nu{}_\sigma + \frac{1}{4}g^{\mu\nu}F^2$

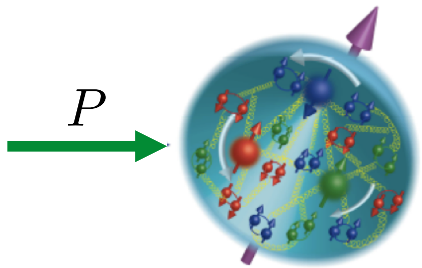
➡ **Decompose** $M_n = \sum_{f=q,g} \frac{\langle P|T_f^{00}(0)|P\rangle}{2P^0}\bigg|_{\rm cm} = \sum_i \langle P|\mathcal{O}_i(0)|P\rangle$

**Not unique, same quantum #, mixed by renormalization!**

***Most important: None of these terms are direct physical observables!***

# Mass of Nucleon in QCD

## ❑ Decomposition of the trace of EMT:




**Trace of the QCD energy-momentum tensor:**

$$T^{\alpha}_{\ \alpha} = \underbrace{\frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s} \underbrace{m_q (1+\gamma_m) \overline{\psi}_q \psi_q}_{\text{Chiral symmetry breaking}}$$

$$\beta(g) = -(11 - 2n_f/3)\, g^3/(4\pi)^2 + \ldots$$

$$\langle P | T^{\alpha}_{\ \alpha}(0) | P \rangle = 2P^2 = 2M_n^2$$

➡ $$\langle T^{\alpha}_{\ \alpha} \rangle = \frac{\langle P | T^{\alpha}_{\ \alpha}(0) | P \rangle}{2P^0} = \frac{M_n^2}{P^0}$$

➡ $$M_n = \langle T^{\alpha}_{\ \alpha} \rangle |_{\text{at reest}}$$

Without separating the quark from gluon contribution to EMT

**In the nucleon's rest frame,** $$\underbrace{\langle \textstyle\int d^3r\, T^{\mu}_{\ \mu} \rangle}_{=M} = \underbrace{\langle \textstyle\int d^3r\, T^{00} \rangle}_{=M} - \sum_i \underbrace{\langle \textstyle\int d^3r\, T^{ii} \rangle}_{=0}$$

**Nucleon mass:** ***Gluon quantum effect + Chiral symmetry breaking!***

**The sigma-term can be calculated in LQCD, Need the trace anomaly to test the sum rule!**

# Mass of Nucleon in QCD

- **Decompositions of $\sum_{f=q,g} T_f^{00}$:**

See talk by Metz and Lorcé

$$M = \underbrace{\left[\langle \int \mathrm{d}^3 r\, \overline{\psi}\gamma^0 iD^0\psi\rangle - \langle \int \mathrm{d}^3 r\, \overline{\psi} m \psi\rangle\right]}_{\text{Quark kinetic and potential energy}} + \underbrace{\langle \int \mathrm{d}^3 r\, \overline{\psi} m \psi\rangle}_{\text{Quark rest mass energy}} + \underbrace{\langle \int \mathrm{d}^3 r\, \tfrac{1}{2}(\vec{E}^2 + \vec{B}^2)\rangle}_{\text{Gluon total energy}}$$

- **Ji's decomposition:**

See talk by Ji

$$T_a^{00} = \underbrace{\bar{T}_a^{00}}_{= \frac{3}{4} T_a^{00} + \frac{1}{4}\sum_i T_a^{ii}} + \underbrace{\hat{T}_a^{00}}_{= \frac{1}{4} T_a^{00} - \frac{1}{4}\sum_i T_a^{ii}} \qquad a = q, g$$

Quark Energy $\langle \bar{T}_q^{00}\rangle$ | Gluon Energy $\langle \bar{T}_g^{00}\rangle$ | Quark Mass $\langle \hat{T}_q^{00}\rangle$ | Trace Anomaly $\langle \hat{T}_g^{00}\rangle$

$$M_n = \sum_{f=q,g} \left.\frac{\langle P|T_f^{00}(0)|P\rangle}{2P^0}\right|_{\mathrm{cm}} = \underbrace{M_q + M_g}_{\text{Relativistic motion}} + M_m + M_a$$

Relativistic motion | χ Symmetry Breaking | Quantum fluctuation

**Different interpretation!**

# Mass of Nucleon in QCD

- Decompositions of $\sum_{f=q,g} T_f^{00}$: 

See talk by Metz and Lorcé

$$M = \underbrace{[\langle \int d^3r\, \overline{\psi}\gamma^0 iD^0\psi\rangle - \langle \int d^3r\, \overline{\psi} m\psi\rangle]}_{\substack{\text{Quark} \\ \text{kinetic and potential energy}}} + \underbrace{\langle \int d^3r\, \overline{\psi} m\psi\rangle}_{\substack{\text{Quark} \\ \text{rest mass energy}}} + \underbrace{\langle \int d^3r\, \tfrac{1}{2}(\vec{E}^2 + \vec{B}^2)\rangle}_{\substack{\text{Gluon} \\ \text{total energy}}}$$

- Relation between two decompositions:

See talk by Ji

$$T_a^{00} = \underbrace{\bar{T}_a^{00}}_{= \frac{3}{4} T_a^{00} + \frac{1}{4}\sum_i T_a^{ii}} + \underbrace{\hat{T}_a^{00}}_{= \frac{1}{4} T_a^{00} - \frac{1}{4}\sum_i T_a^{ii}} \qquad a = q, g$$

$$M_q = \frac{3}{4}\left(a - \frac{b}{1+\gamma_m}\right) M \quad \neq \langle \int d^3r\, \psi^\dagger i\vec{D}\cdot\vec{\alpha}\psi\rangle \qquad M_m = \frac{4+\gamma_m}{4(1+\gamma_m)}\, b\, M \quad = \langle \int d^3r\, (1 + \tfrac{1}{4}\gamma_m)\, \overline{\psi} m\psi\rangle$$

$$M_g = \frac{3}{4}(1-a)\, M \quad \neq \langle \int d^3r\, \tfrac{1}{2}(\vec{E}^2 + \vec{B}^2)\rangle \qquad M_a = \frac{1}{4}(1-b)\, M \quad = \langle \int d^3r\, \tfrac{1}{4}\, \tfrac{\beta(g)}{2g}\, G^2\rangle$$

$a$ and $b$ are defined in terms of $M_g$ and $M_a$, respectively

**But, none of them is a direct physical observable!**

# Mass of Nucleon in QCD

Note: $\langle x \rangle_f$ and $\sigma_q$ are calculable in lattice QCD

## Ji's interpretation:

**Quark Energy** $\langle T_q^{00} \rangle$ : $$M_q = \frac{3}{4}\left( M \sum_q \langle x \rangle_q - \sum_q \sigma_q \right)$$

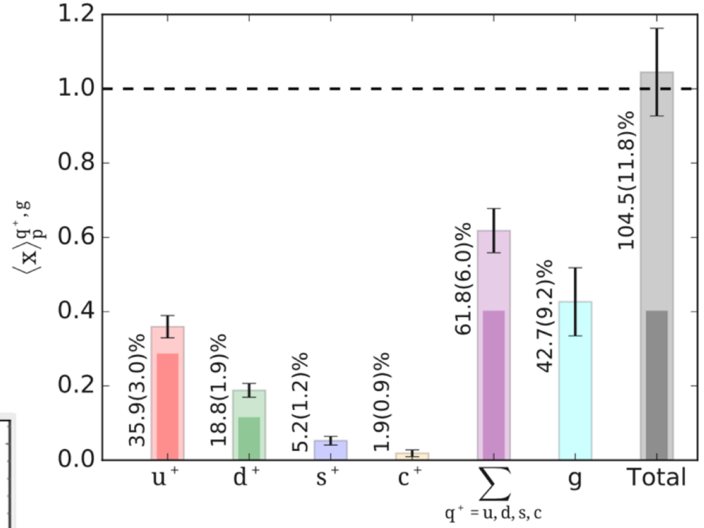
**Gluon Energy** $\langle T_g^{00} \rangle$ : $$M_g = \frac{3}{4} M \langle x \rangle_g$$

**Quark Mass** $\langle \widehat{T}_q^{00} \rangle$ : $$M_m = \sum_q \sigma_q$$

**Trace Anomaly** $\langle \widehat{T}_g^{00} \rangle$ : $$M_a = \frac{\gamma_m}{4} \sum_q \sigma_q - \frac{\beta(g)}{4g}(E^2 + B^2)$$

**Parton momentum fraction:**

$$\langle x \rangle_f = \int_0^1 dx\, x f(x, \mu^2)$$

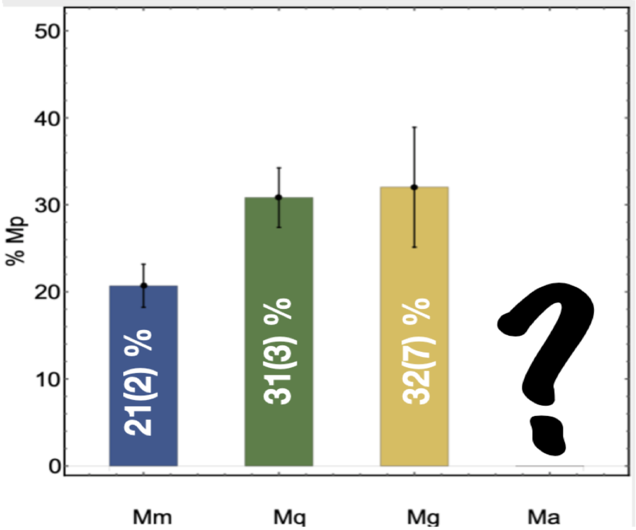



## LQCD calculation:

**Quark sigma-term:**

$$\sigma_q = \frac{\langle P | \overline{\psi}_q(0) m_q \psi_q(0) | P \rangle}{2P^0}$$

| | $u+d$ | $s$ | $c$ |
|---|---|---|---|
| $\sigma$ [MeV] | 41.6(3.8) | 45.6(6.2) | 107(22) |




**Access the trace anomaly Indirectly?**

$$M_a = \frac{M}{4} - \sum_q \frac{\sigma_q}{4}$$

**Or by experiment?**

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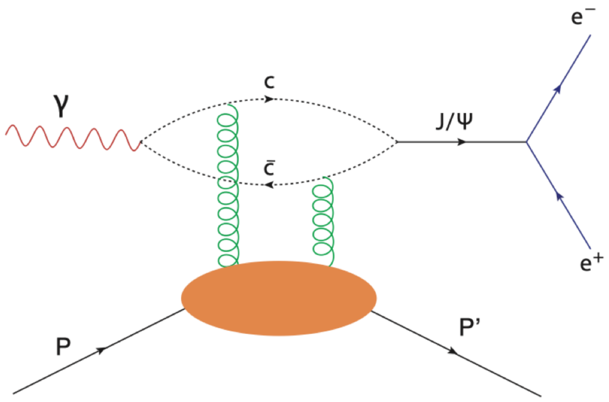
# Extract the Trace Anomaly from Experiments

- **QCD Trace Anomaly:**

$$T^{\alpha}_{\alpha} = \underbrace{\frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s} m_q(1+\gamma_m)\overline{\psi}_q \psi_q$$

$F^{\mu\nu,a} F^{a}_{\mu\nu}$ Is a scalar, and high twist!

- **Diffractive heavy quarkonium production:**

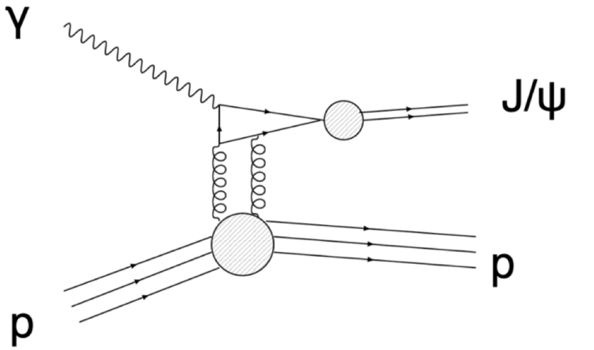



Two gluons may not be factorized into

$$F^{\mu\nu,a} F^{a}_{\mu\nu}$$

**See talk by D. Kharzeev**

**Using a slow-moving dipole approximation to measure the scale field response** $\langle P|F^2|P\rangle$




Near threshold, dominance of

$$g^2\mathbf{E}^{a2} = \frac{8\pi^2}{b}\theta^{\mu}_{\mu} + g^2\theta^{(G)}_{00}$$

Assuming the validity of vector meson dominance, can relate photoproduction to quarkonium scattering amplitude and probe the mass of the proton

DK, Satz, Syamtomov, Zinovjev '99

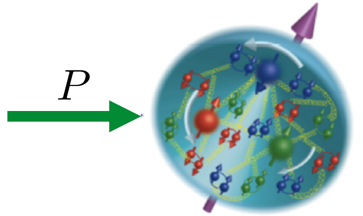
Other approaches to threshold photoproduction:

Hatta, Yang '18; Hatta, Rajan, Yang '19; Mamo, Zahed '19

# Extracting Two-gluon Correlation from Experiments

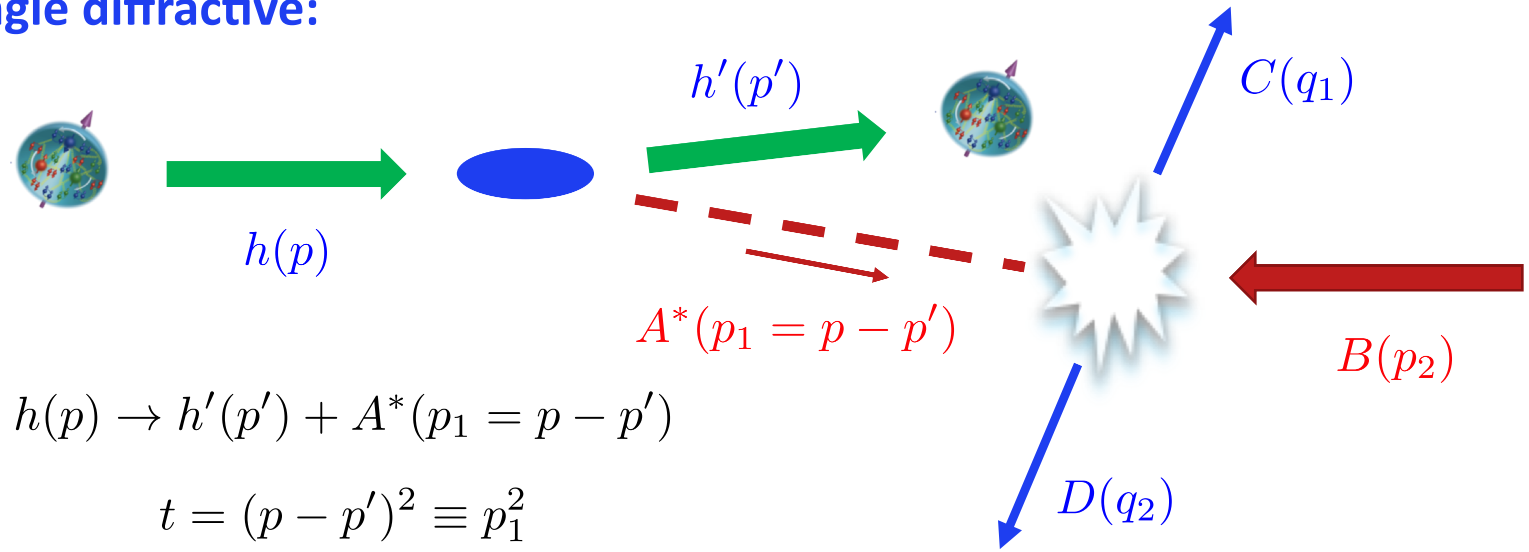
Qiu & Yu, 2022

## ❑ QCD factorization of diffractive $2 \to 3$ exclusive hard processes:




Probing its structure without breaking it!

- **Single diffractive:**




$$h(p) \to h'(p') + A^*(p_1 = p - p')$$

$$t = (p - p')^2 \equiv p_1^2$$

- **Hard probe:** $2 \to 2$ **high** $q_T$ **exclusive process**

$$A^*(p_1) + B(p_2) \to C(q_1) + D(q_2)$$

$$|q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$$

- **The single diffractive** $2 \to 3$ **exclusive hard processes:**

$$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$$

- **Necessary condition for QCD factorization:**

$$|q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$$

**The state $A^*(p_1)$ lives much longer than $2 \to 2$ hard exclusive collision!**

**Not necessarily sufficient!**



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# Extracting Two-gluon Correlation from Experiments

- **Challenge for QCD factorization:** $h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$

Qiu & Yu, 2022

**Gluons in the Glauber region:** $k_s = (\lambda^2, \lambda^2, \lambda)\, Q$

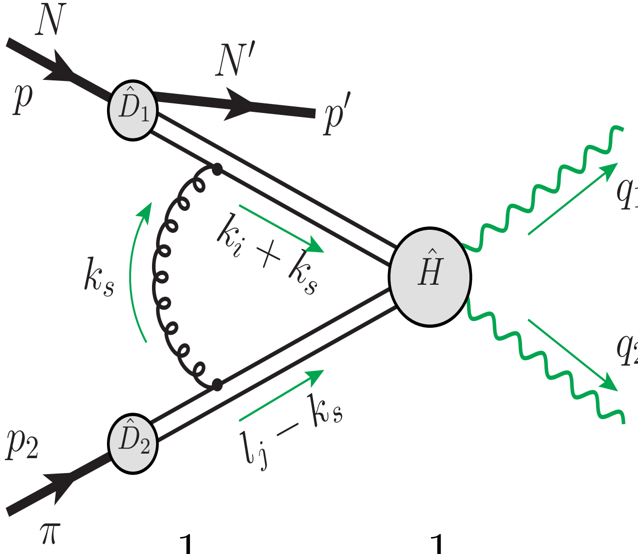
$k_i = (1, \lambda^2, \lambda)\, Q$

$l_j = (\lambda^2, 1, \lambda)\, Q$

***Transverse component contribute to the leading region!***

**ERBL region**

**(Efremov, Radyushkin, Brodsky, Lepage)**



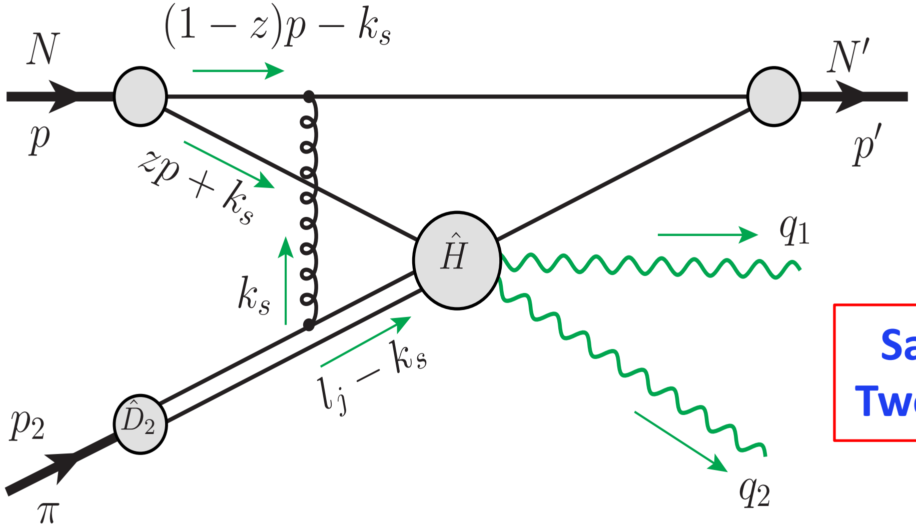

$$\frac{1}{k_s^2 + i\varepsilon} \to \frac{1}{-\mathbf{k}_s^2 + i\varepsilon}$$

$$\frac{1}{(k_i + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

$$\frac{1}{(l_j - k_s)^2 + i\varepsilon} \to \frac{1}{-k_s^+ + i\varepsilon}$$

**No pinch!**

**DGLAP region**




**Same for Two-gluons**

$$\frac{1}{((1-z)p - k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- - i\varepsilon}$$

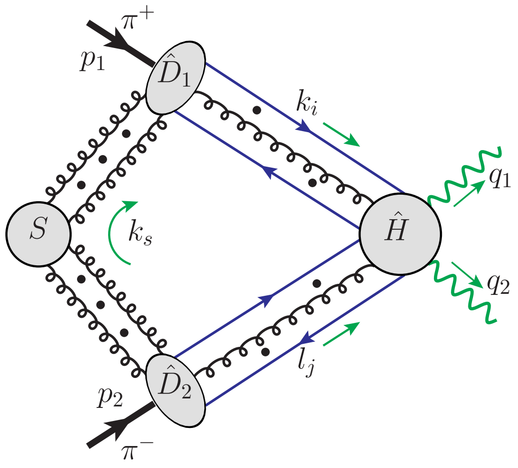
$$\frac{1}{(zp + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

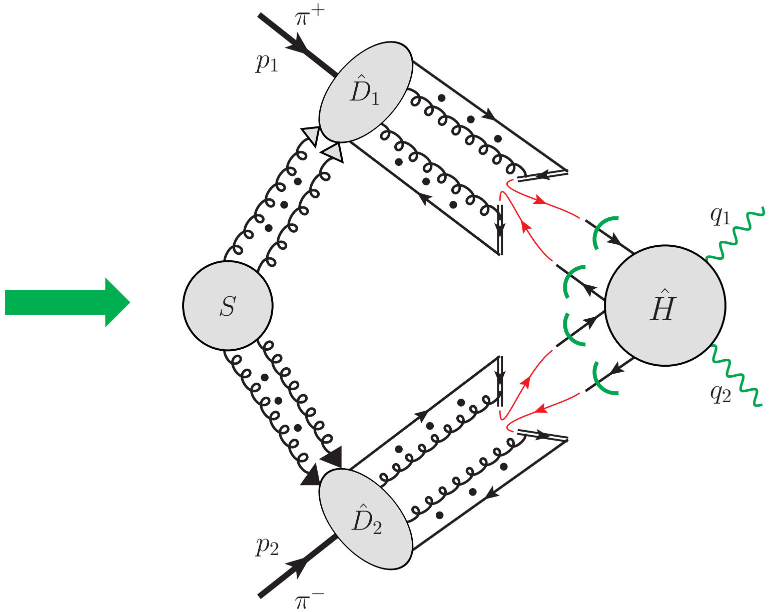
**Pinched!**

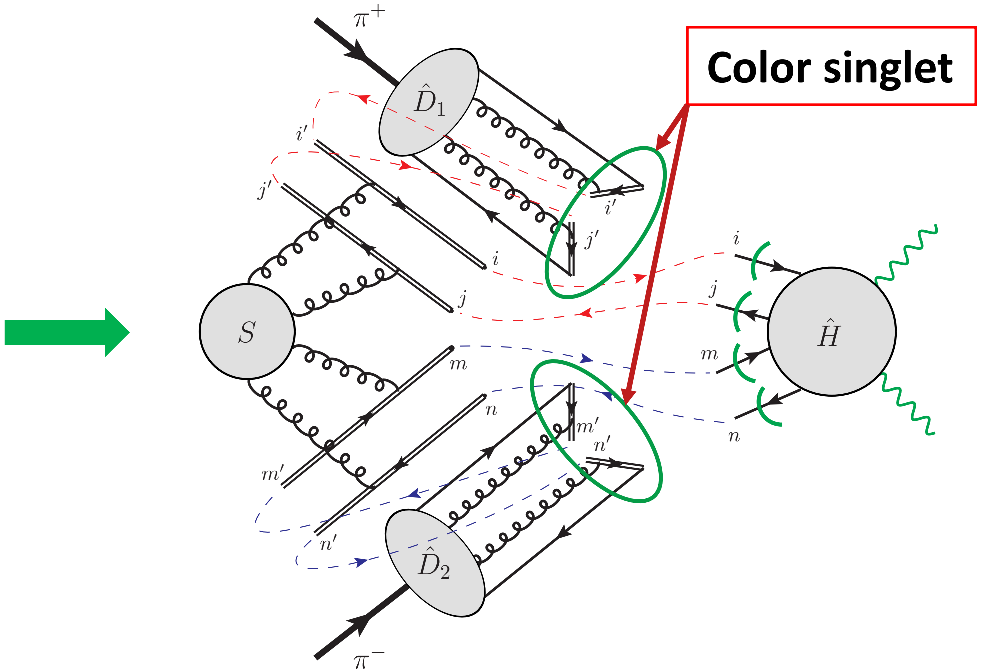
**Same conclusion if $k_s$ flows Through N'!**

# Extracting Two-gluon Correlation from Experiments

- **Factorization – warm up:** $\pi(p_1) + \pi(p_2) \to \gamma(q_1) + \gamma(q_2)$ **Model for ERBL region**




$$\hat{D}_{1i'j'} \propto \delta_{i'j'}, \quad \hat{D}_{2m'n'} \propto \delta_{m'n'}$$

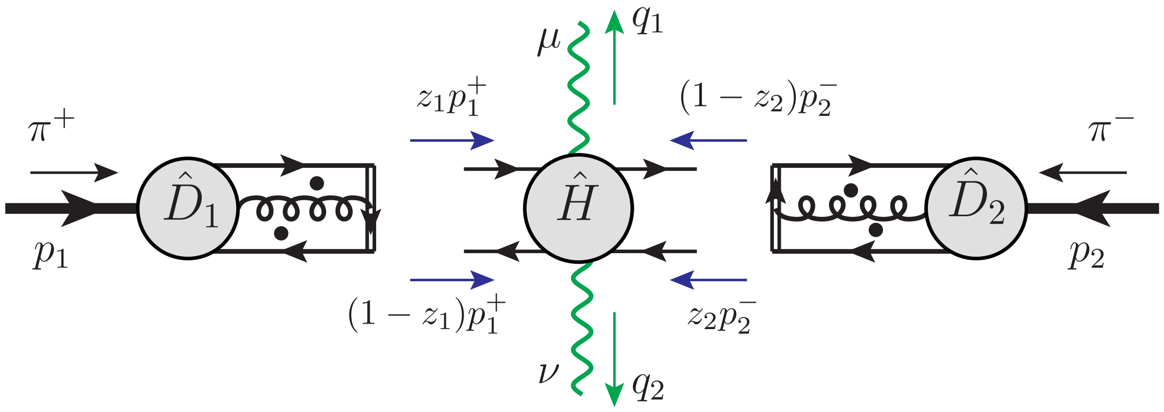
$$\begin{aligned}
\Rightarrow \quad & \delta_{i'j'}\delta_{m'n'} S_{ij,i'j';mn,m'n'} \\
&= \langle 0| \left[\Phi(0,-\infty;n_1)\Phi^\dagger(0,-\infty;n_1)\right]_{ij} \left[\Phi(0,-\infty;n_2)\Phi^\dagger(0,-\infty;n_2)\right]_{mn} |0\rangle \\
&= \delta_{ij}\delta_{mn}
\end{aligned}$$

**Removal of soft interaction - different from inclusive case!**

# Extracting Two-gluon Correlation from Experiments

- **Factorization – warm up:** $\pi(p_1) + \pi(p_2) \to \gamma(q_1) + \gamma(q_2)$ **Model for ERBL region**

$$\mathcal{M}^{\mu\nu} = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2\, D_{\pi^+}(z_1)\, D_{\pi^-}(z_2)\, C^{\mu\nu}(z_1, z_2; p_1^+, p_2^-, q_T) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}/q_T\right)$$



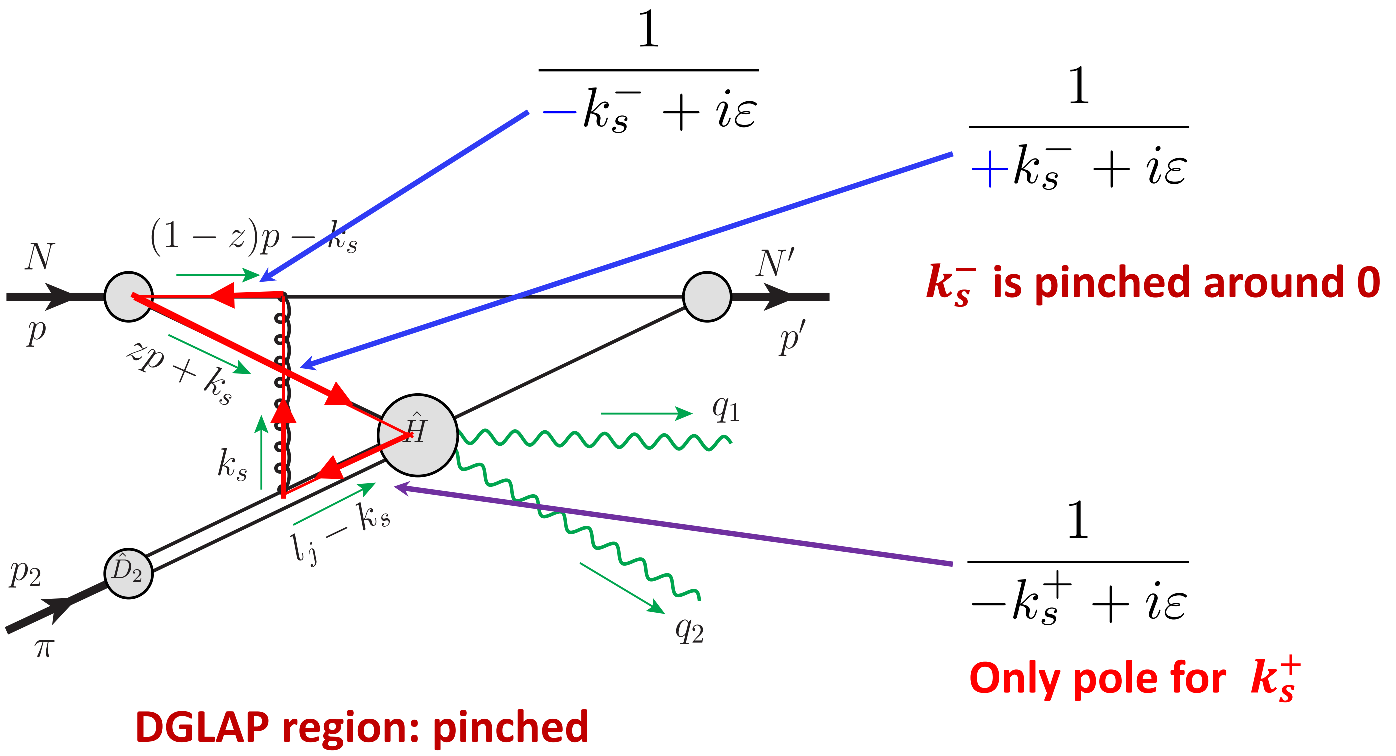

$$D_{\pi^+}(z_1) = \int \frac{\mathrm{d}\xi^-}{4\pi} e^{iz_1 p_1^+ \xi^-} \langle 0|\bar{d}(0)\gamma^+\gamma_5\, \Phi(0,\xi^-; w_2)u(\xi^-)|\pi^+(p_1)\rangle$$

$$D_{\pi^-}(z_2) = \int \frac{\mathrm{d}\zeta^+}{4\pi} e^{iz_2 p_2^- \zeta^+} \langle 0|\bar{u}(0)\gamma^-\gamma_5\, \Phi(0,\zeta^+; w_1)d(y^+)|\pi^-(p_1)\rangle$$

$$C^{\mu\nu}(z_1, z_2; p_1^+, p_2^-, q_T) \equiv \left[\frac{\gamma_5(p_1^+\gamma^-)}{2}\right]_{\alpha\beta} \hat{H}^{\mu\nu}_{\beta\alpha;\sigma\rho}(\hat{k}_1 = z_1 p_1^+, \hat{k}_2 = z_2 p_2^-; q_T) \left[\frac{\gamma_5(p_2^-\gamma^+)}{2}\right]_{\rho\sigma}$$

# Extracting Two-gluon Correlation from Experiments

- **Factorization Glauber gluons:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$ as an example



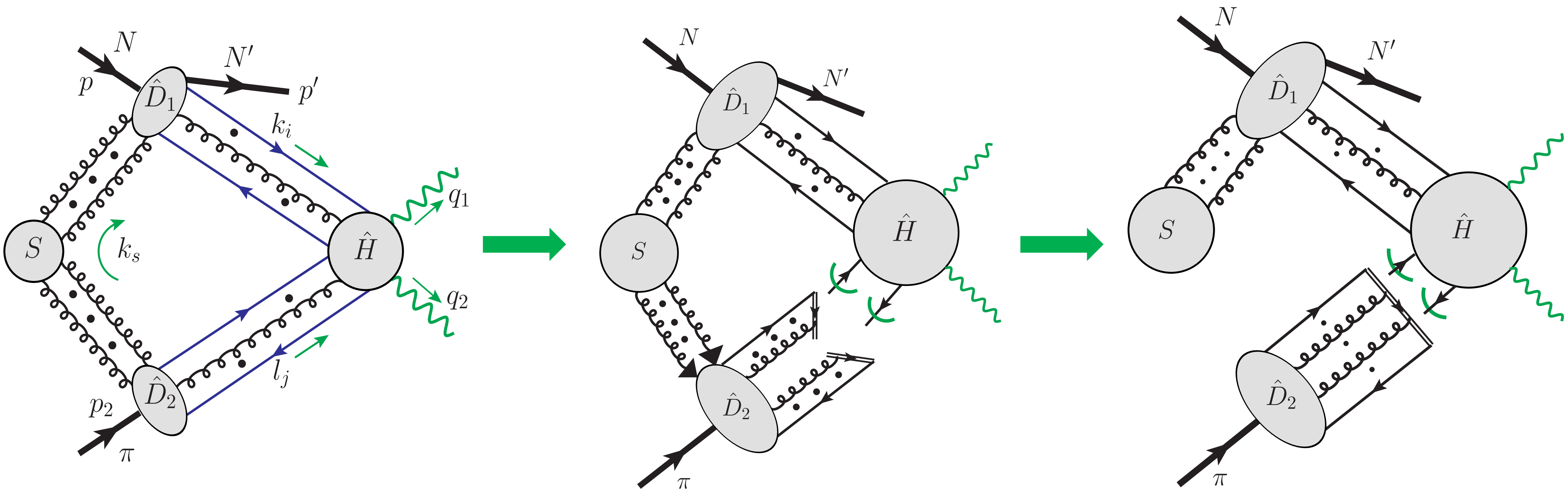

**Deformation out of the Glauber region:**

$$k_s^+ \to k_s^+ - i\mathcal{O}(Q) \quad \Longrightarrow \quad k_s \sim (1, \lambda^2, \lambda)Q$$ **Collinear region**

**Works for both ERBL and DGLAP regions!**

# Extracting Two-gluon Correlation from Experiments

- **Factorization:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$




- Same strategy for proving the factorization of first sub-leading power for inclusive processes

Qiu, Sterman, 1991

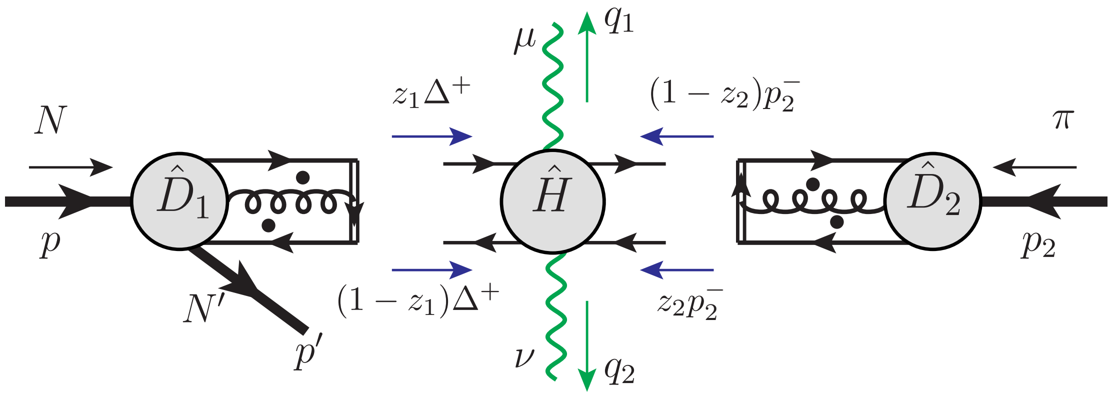
- No QCD factorization for double diffractive hadronic scattering!

$$p + n \to p + n + \gamma + \gamma, \quad p + n \to p + n + \text{jet} + \text{jet}, \quad p + p \to p + p + \text{jet} + \text{jet}, \quad p + \bar{p} \to p + \bar{p} + \text{jet} + \text{jet}, \ldots$$

# Extracting Two-gluon Correlation from Experiments

- **Factorization formula:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$

$$\mathcal{M}^{\mu\nu} = \int \mathrm{d}z_1 \mathrm{d}z_2 \left[ \widetilde{\mathcal{F}}^{ud}_{NN'}(z_1,\xi,t) D(z_2) C^{\mu\nu}(z_1,z_2) + \mathcal{F}^{ud}_{NN'}(z_1,\xi,t) D(z_2) \widetilde{C}^{\mu\nu}(z_1,z_2) \right] + \mathcal{O}(\Lambda_{\rm QCD}/q_T)$$




$$\mathcal{F}^{ud}_{NN'}(z_1,\xi,t) = \int \frac{\mathrm{d}y^-}{4\pi} e^{iz_1\Delta^+ y^-} \langle N'(p')|\bar{d}(0)\gamma^+ \Phi(0,y^-;w_2)\, u(y^-)|N(p)\rangle$$

$$= \frac{1}{2P^+}\left[ H^{ud}_{NN'}(z_1,\xi,t)\,\bar{u}(p')\gamma^+ u(p) - E^{ud}_{NN'}(z_1,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_p}u(p)\right],$$

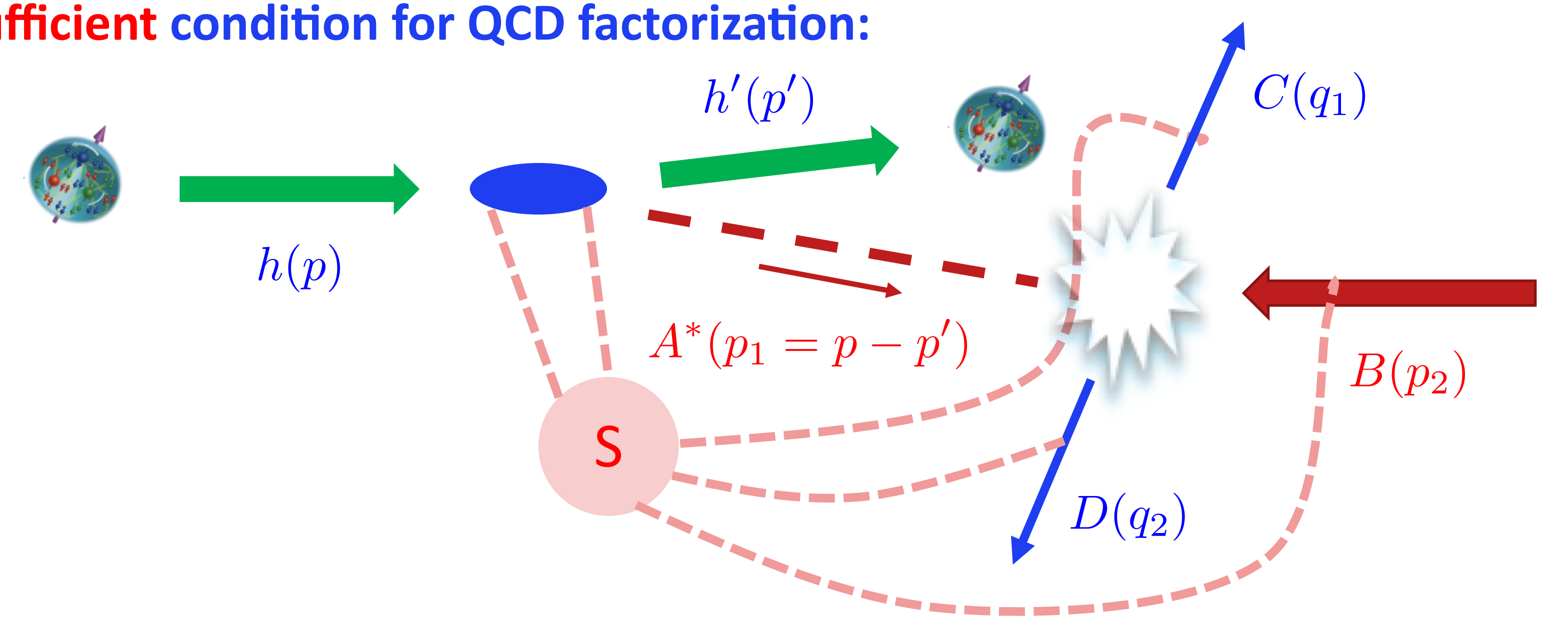
$$\widetilde{\mathcal{F}}^{ud}_{NN'}(z_1,\xi,t) = \int \frac{\mathrm{d}y^-}{4\pi} e^{iz_1\Delta^+ y^-} \langle N'(p')|\bar{d}(y^-)\gamma^+\gamma_5 \Phi(0,y^-;w_2)\, u(0)|N(p)\rangle$$

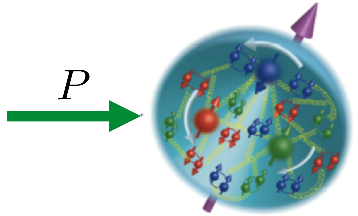
$$= \frac{1}{2P^+}\left[ \widetilde{H}^{ud}_{NN'}(z_1,\xi,t)\,\bar{u}(p')\gamma^+\gamma_5 u(p) - \widetilde{E}^{ud}_{NN'}(z_1,\xi,t)\,\bar{u}(p')\frac{i\gamma_5\sigma^{+\alpha}\Delta_\alpha}{2m_p}u(p)\right]$$

# Extracting Two-gluon Correlation from Experiments

Qiu & Yu, 2022

- **QCD factorization of diffractive $2 \to 3$ exclusive hard processes:**
  - **Sufficient condition for QCD factorization:**




**Probing its structure without breaking it!**

- **Soft gluon interaction is proved to be factorized from the exclusive $2 \to 2$ sub-processes**
- **Analytic continuation the soft momenta to collinear momenta from $h(p)$**

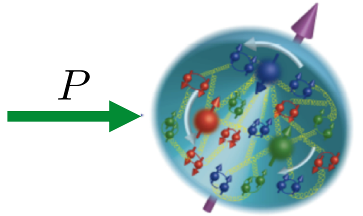
$$\mathcal{M}_{h+B\to h'+C+D} = \sum_{A^*} \mathcal{M}_{h\to h'+A^*} \otimes \mathcal{M}_{A^*+B\to C+D} \otimes \mathcal{S}_{hh'BCD}$$

$$\approx \sum_{A^*} \left[\mathcal{M}_{h\to h'+A^*} \otimes \mathcal{S}_{hh'}\right] \otimes \left[\sum_{bcd} \mathcal{H}_{A^*+b\to c+d} \otimes \mathcal{D}_{B\to b} \otimes \mathcal{D}_{c\to C} \otimes \mathcal{D}_{d\to D}\right] + \mathcal{O}\left(\left[\frac{\sqrt{-t}}{q_T}\right]^{\alpha}\right)$$

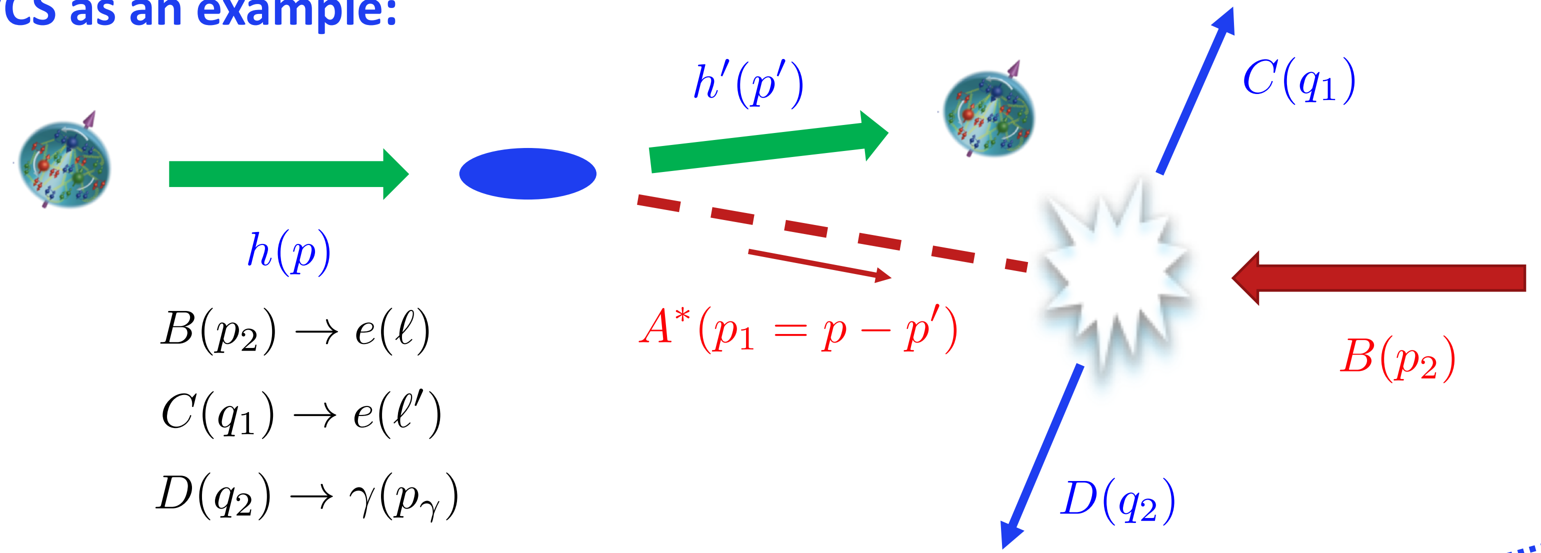
# Extracting Two-gluon Correlation from Experiments

Qiu & Yu, 2022

## ❑ QCD factorization of diffractive $2 \to 3$ exclusive hard processes:




- DVCS as an example:




$B(p_2) \to e(\ell)$

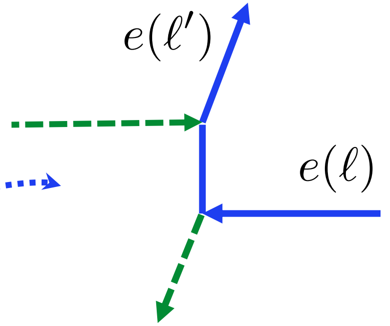
$C(q_1) \to e(\ell')$

$D(q_2) \to \gamma(p_\gamma)$

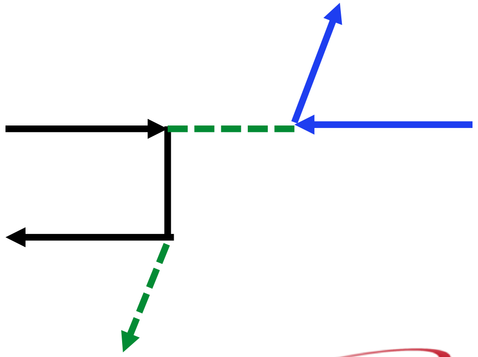
$A^*(p - p') \to \gamma^*(p - p')$

or $A^*(p - p') \to [q\bar{q}]^*(p - p')$

or $A^*(p - p') \to \ \ldots$




$+ \ \ldots$



$+ \ \ldots$

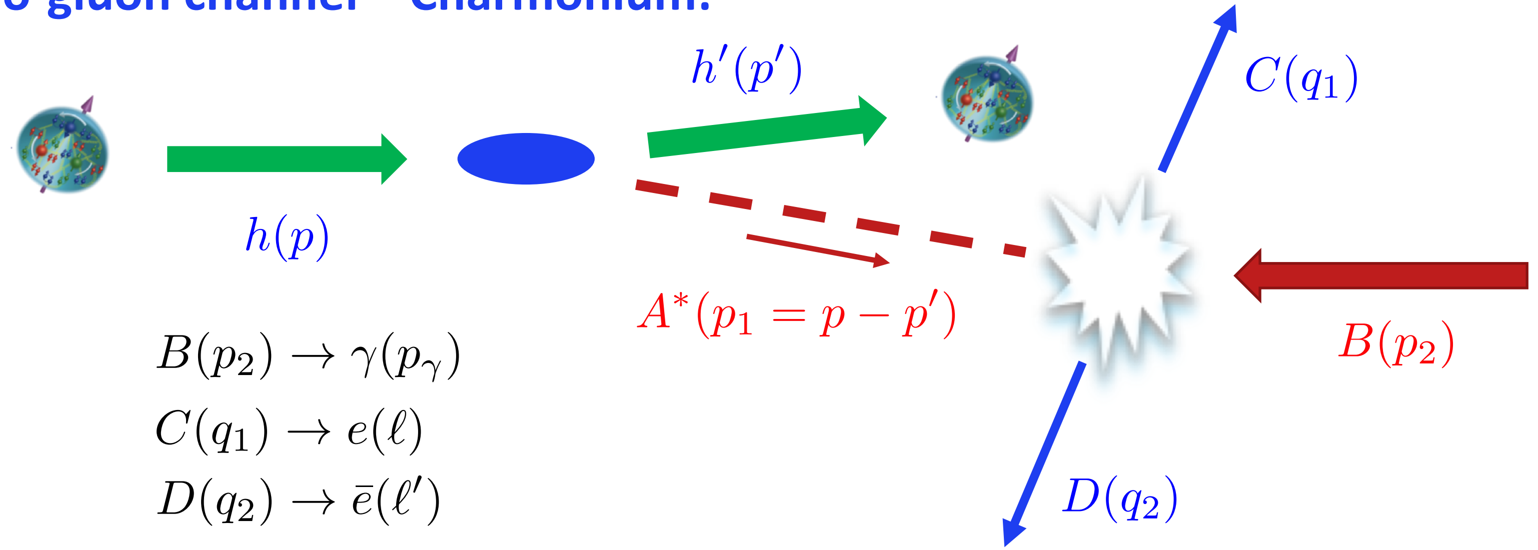
$$\mathcal{M}_{h+\ell\to h'+\ell+\gamma} \approx \mathcal{M}_{h\to h'+\gamma^*} \otimes \mathcal{H}_{\gamma^*+\ell\to\ell'+\gamma}$$
$$+\mathcal{M}_{h\to h'+[q\bar{q}]^*} \otimes \mathcal{H}_{[q\bar{q}]^*+\ell\to\ell'+\gamma}$$
$$+ \ \ldots$$

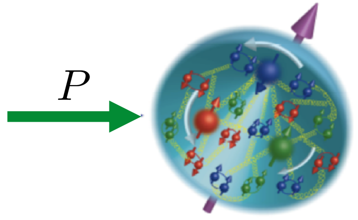
# Extracting Two-gluon Correlation from Experiments

❑ **QCD factorization of diffractive $2 \to 3$ exclusive hard processes:**

Qiu & Yu, 2022

- **Two-gluon channel – Charmonium:**



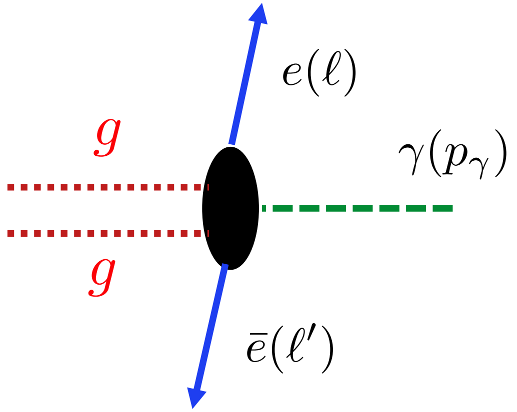




**Probing its structure without breaking it!**

$$B(p_2) \to \gamma(p_\gamma)$$
$$C(q_1) \to e(\ell)$$
$$D(q_2) \to \bar{e}(\ell')$$

**or** $A^*(p-p') \to [gg]^*(p-p')$

**or** $A^*(p-p') \to \ \ldots$




$$\mathcal{M}_{h+\gamma\to h'+e+\bar{e}} \approx \mathcal{M}_{h\to h'+[gg]^*} \otimes \mathcal{H}_{[gg]^*+\gamma\to e\bar{e}(M)}$$

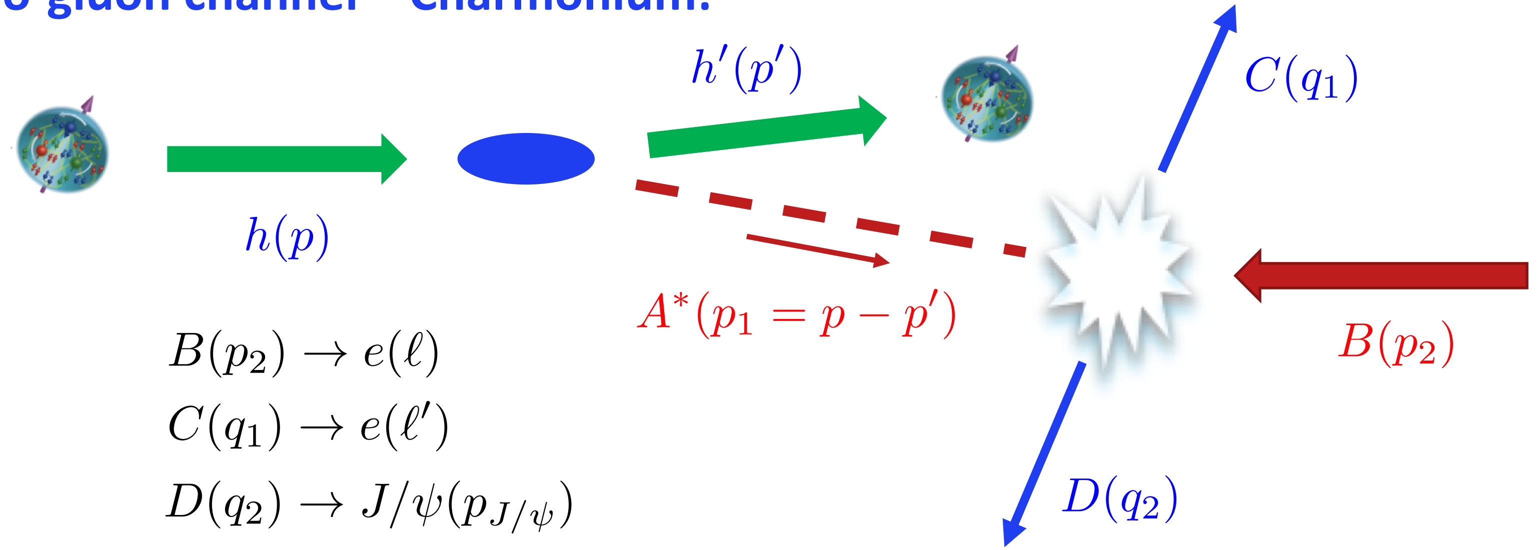
$$+ \ldots$$

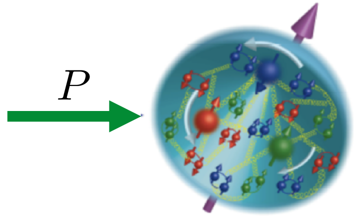
# Extracting Two-gluon Correlation from Experiments

- **QCD factorization of diffractive $2 \to 3$ exclusive hard processes:**

Qiu & Yu, 2022

  - **Two-gluon channel – Charmonium:**



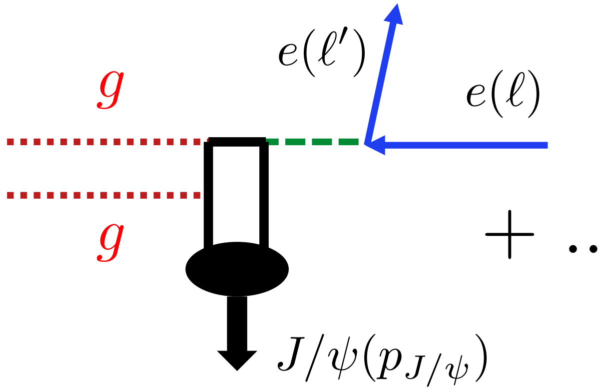




**Probing its structure without breaking it!**

$B(p_2) \to e(\ell)$
$C(q_1) \to e(\ell')$
$D(q_2) \to J/\psi(p_{J/\psi})$

**or** $A^*(p-p') \to [gg]^*(p-p')$

**or** $A^*(p-p') \to \ ...$

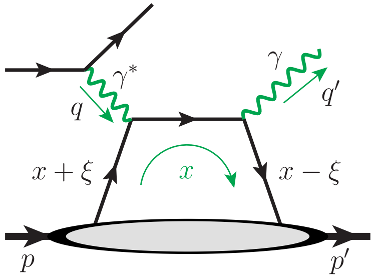



$$\mathcal{M}_{h+e(\ell)\to h'+e+J/\psi} \approx \mathcal{M}_{h\to h'+[gg]^*} \otimes \left[\mathcal{H}_{[gg]^*+e\to e+[c\bar{c}]} \otimes \mathcal{D}_{[c\bar{c}]\to J/\psi}\right]$$

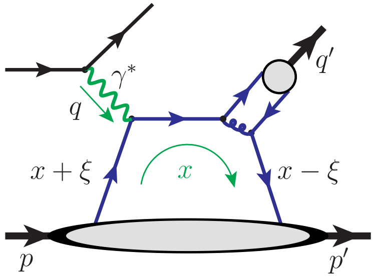
$$+ \ ...$$

# GPDs – QCD Tomography

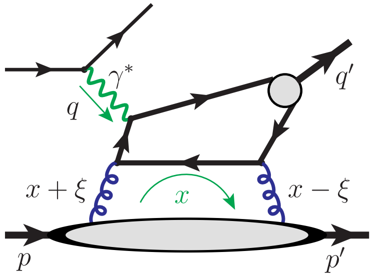
- **Imagining spatial distribution of quarks and gluons:**




**DVCS: Q² >> |t|**




**DVMP**




**DVQP**

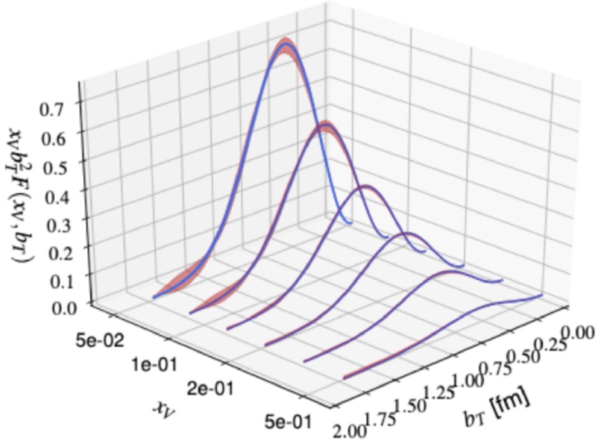
$\dfrac{d\sigma}{dt}$ $\quad t = (p - p')^2$

**Factorization** $\quad Q^2 \gg |t|$

**GPDs:** $\quad f_{i/h}(x, \xi, t; \mu)$

**F.T.** $t_T$ **to** $b_T$

**at** $\xi \propto (p - p')^+ \to 0$



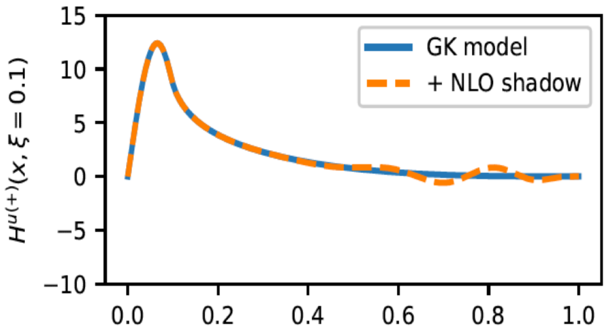

- **Proton radii of quark and gluon spatial distribution,** $r_q(x)$ **&** $r_g(x)$

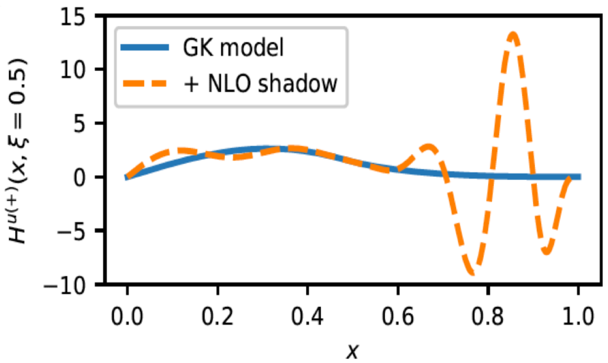
  **Should** $r_q(x)$ **>** $r_g(x)$**, or vice versa? Could** $r_g(x)$ **saturate as** $x \to 0$ **?**

  **…**

- **But, all these observables are not very sensitive to the *x*-dependence!**

  **Sensitive to the total momentum of the pair, not the relative momentum**



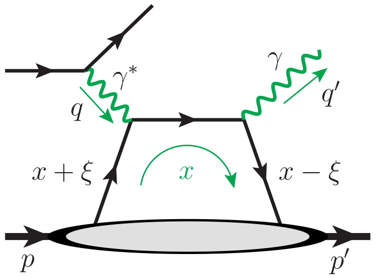



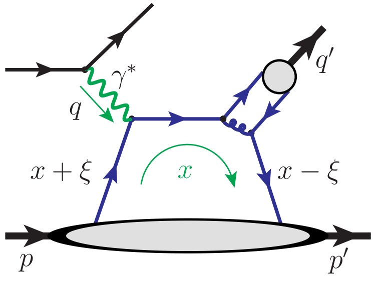

***Blue and dashed***
***Fit the same CFFs !***

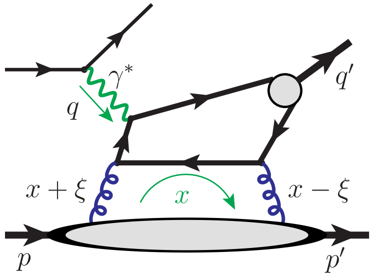
**Phys.Rev. D103 (2021) 114019**

# GPDs – QCD Tomography

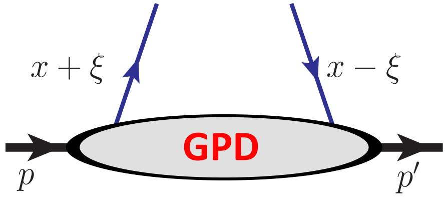
- **Imagining spatial distribution of quarks and gluons:**




**Factorization**




$$i\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t) \cdot C(x,\xi;Q/\mu)$$

- **Sensitivity to $x$ comes from $C(x,\xi;Q/\mu)$:**

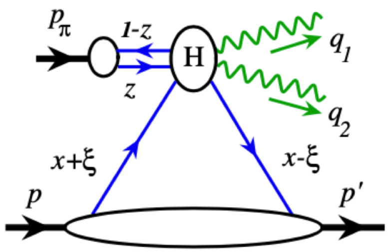
***x*** ~ **loop momentum**

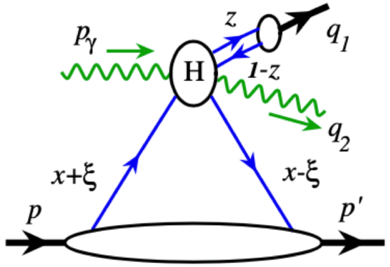
***t*, ξ** ~ **directly meassured**

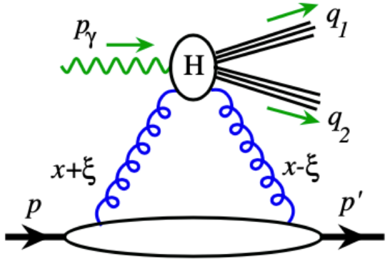
**DVCS:** $C(x,\xi;Q/\mu) \propto \dfrac{1}{x-\xi+i\varepsilon}$ ⟹ $i\mathcal{M} \propto \displaystyle\int_{-1}^{1} \mathrm{d}x\, \frac{F(x,\xi,t)}{x-\xi+i\varepsilon} \equiv \text{“}F_0(\xi,t)\text{”}$

**Not ideal if:** $C(x,\xi;Q/\mu) \Rightarrow C_Q(Q/\mu)\cdot C_x(x)\cdot C_\xi(\xi)$

- **Exclusive massive pair production with high-$P_T$ (two-scale observables):**




+ ...

**Hard scale:** $q_T \gg \Lambda_{\mathrm{QCD}}$

in $p_\pi - (p-p')$ frame

**Soft scale:** $t = (p-p')^2$

**Factorization:** $q_T \gg \sqrt{|t|}$

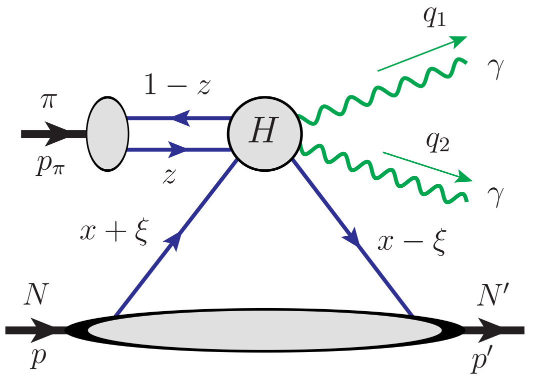
Introduced by G. Duplancic et al.
JHEP 11 (2018) 179

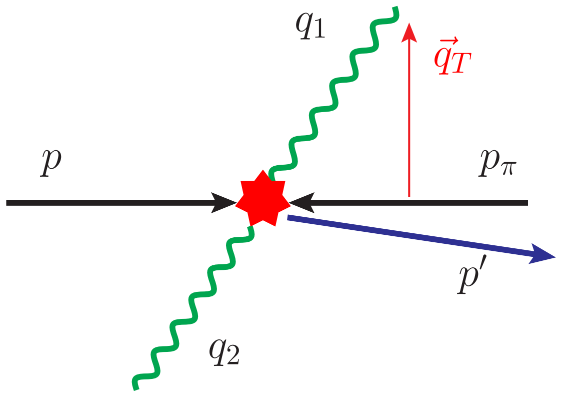
Introduced by Y. Hatta et al.
Phys.Rev.Lett. 116 (2016) 202301

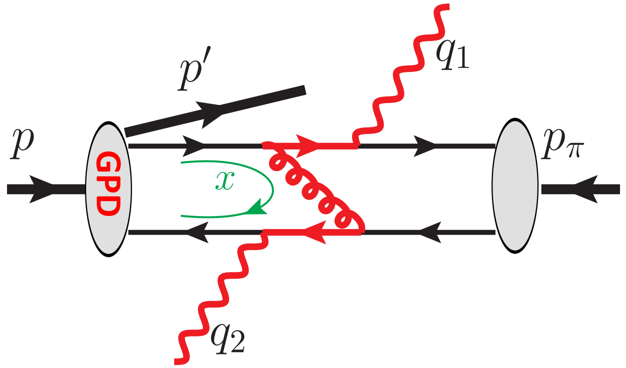
# Exclusive Massive Photon-Pair Production in Meson-Baryon Collision

## ❑ Process:

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$




## ❑ Kinematical observables:

$\boldsymbol{t, \xi, q_T}$

- $\boldsymbol{t = (p - p')^2}$
- $\boldsymbol{\xi = (p^+ - p'^+)/(p^+ + p'^+)}$

**Hard scale:** $\boldsymbol{q_T \gg \Lambda_{\rm QCD}}$

**Soft scale:** $\boldsymbol{t \sim \Lambda^2_{\rm QCD}}$

## ❑ Factorization:

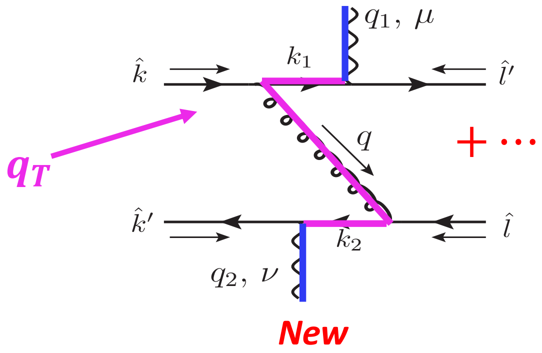
**[*suppressing DA factor*]**

$$\mathcal{M}(t, \xi, q_T) = \int_{-1}^{1} \mathrm{d}x\, F(x, \xi, t; \mu) \cdot C(x, \xi; q_T/\mu) + \mathcal{O}(\Lambda_{\rm QCD}/q_T) \quad \Longrightarrow \quad \frac{\mathrm{d}\sigma}{\mathrm{d}t\ \mathrm{d}\xi\ \mathrm{d}q_T} \sim |\mathcal{M}(t, \xi, q_T)|^2$$

$$x \leftrightarrow q_T$$

# Exclusive Massive Photon-Pair Production in Meson-Meson Collision

## ❑ Hard part for A-type:



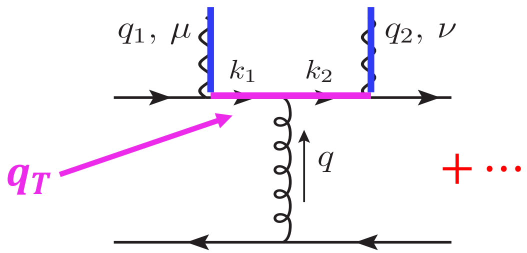

$\kappa = 4\, q_T^2/\hat{s}$

- **Gluon propagator** $q^2 = -\frac{\hat{s}}{4}\left[\left(2z_1 - 1 - \sqrt{1-\kappa}\right)\left(2z_2 - 1 - \sqrt{1-\kappa}\right) + \kappa\right]$

$$\Rightarrow \quad \mathcal{M} \propto \int_0^1 \mathrm{d}z_1\, \mathrm{d}z_2\, \frac{\phi(z_1)\phi(z_2)}{(1-z_1)\,(1-z_2)\,\left[\left(2z_1 - 1 - \sqrt{1-\kappa}\right)\left(2z_2 - 1 - \sqrt{1-\kappa}\right) + \kappa\right]}$$

- **Change $q_T$ changes the $z_1$-$z_2$ integral.**
- **$\mathrm{d}\sigma/\mathrm{d}q_T^2$ provides sensitivity to the DA's functional form of $z$.**

## ❑ Hard part for B-type:




*Like "time-like" form factor*

- **Gluon propagator** $q^2 = z_2(1-z_1)\hat{s}$

$$\Rightarrow \quad \mathcal{M} \propto \int_0^1 \mathrm{d}z_1\, \mathrm{d}z_1\, \frac{\phi(z_1)\phi(z_2)}{z_1\,(1-z_1)\,z_2\,(1-z_2)} \sim \left[\int_0^1 \mathrm{d}z\, \frac{\phi(z)}{z\,(1-z)}\right]^2$$

- ***Not* sensitive to DA functional form.**
- **Relies on $\phi(z) = 0$ at end points.**
- **Sudakov resummation could suppress the end-point sensitivity.**

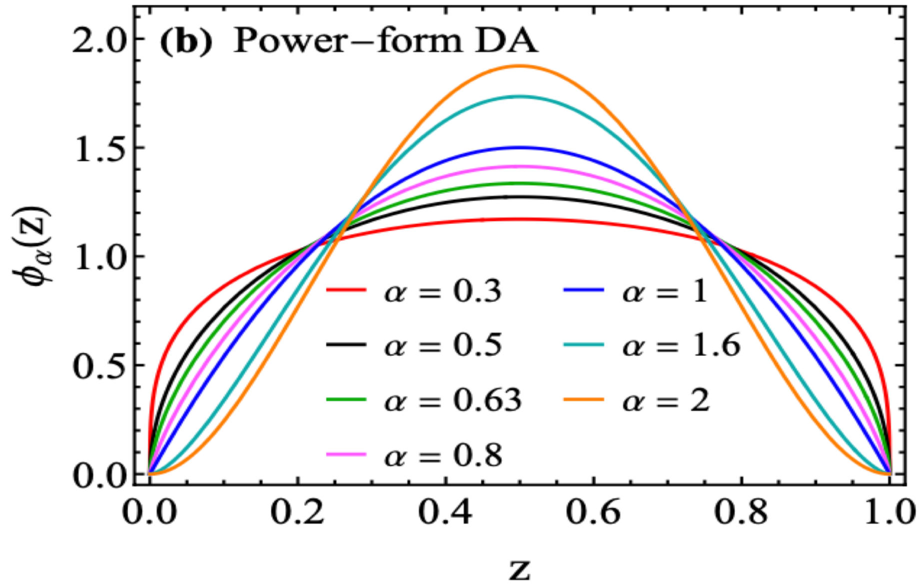
Li, Sterman, 1992

# Numerical results

- DA parametrization:

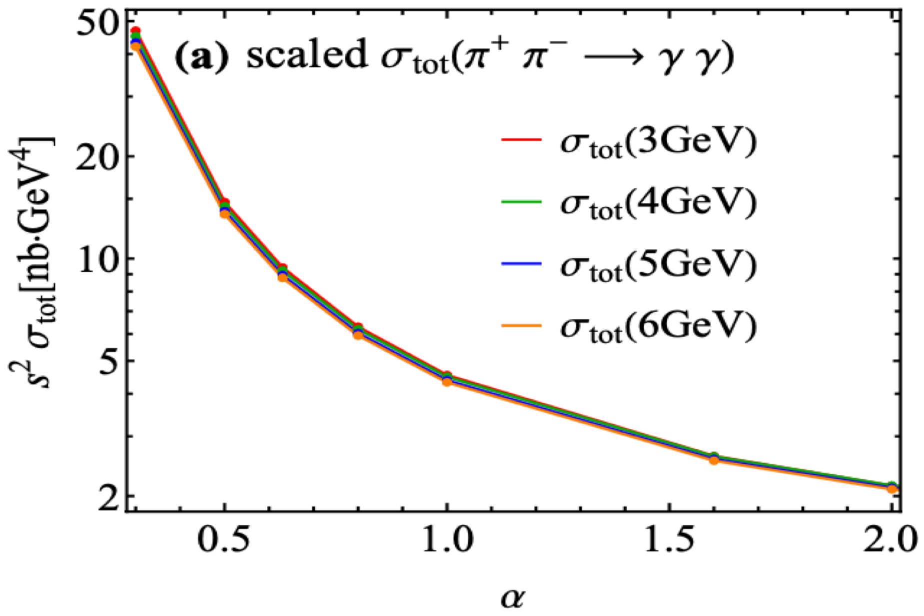
$$\phi_\alpha(z) = \frac{i\, f_\pi}{2} \cdot \left[ \frac{z^\alpha (1-z)^\alpha}{B(1+\alpha, 1+\alpha)} \right]$$

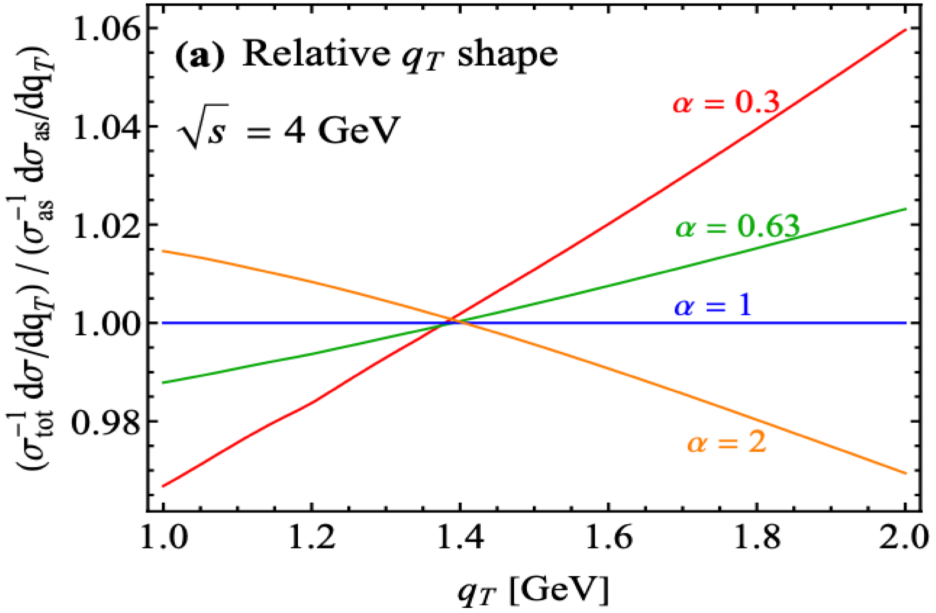
Change $\alpha \Longrightarrow$ Change $z$ dependence




- “Total” cross section:

$$\sigma_{\rm tot} = \int_{1\ \rm GeV}^{\sqrt{s}/2} \mathrm{d}q_T\, \frac{\mathrm{d}\sigma}{\mathrm{d}q_T}$$



# Numerical results

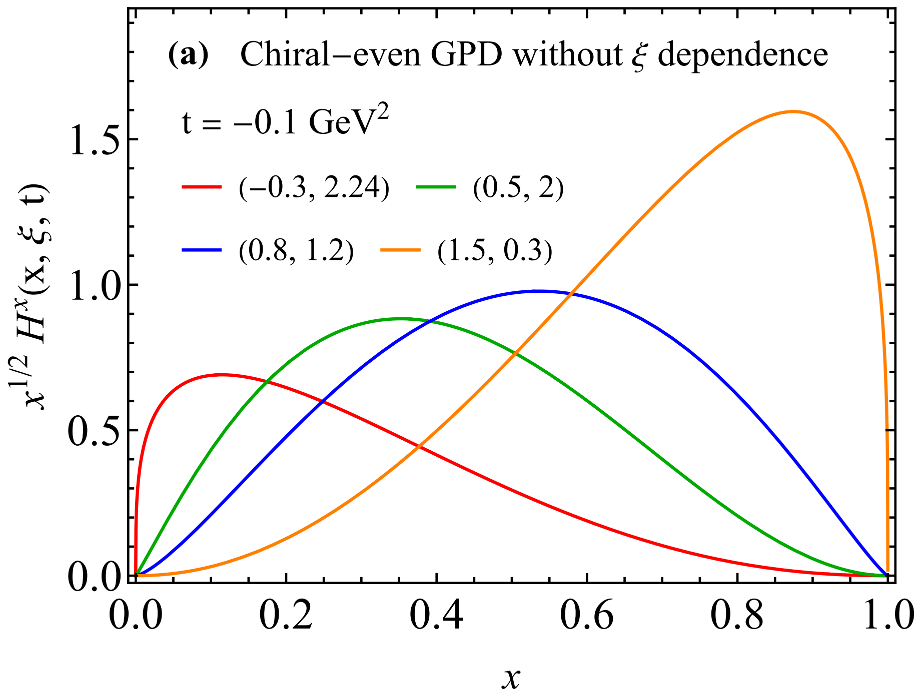
- GPD models – simplified GK model:

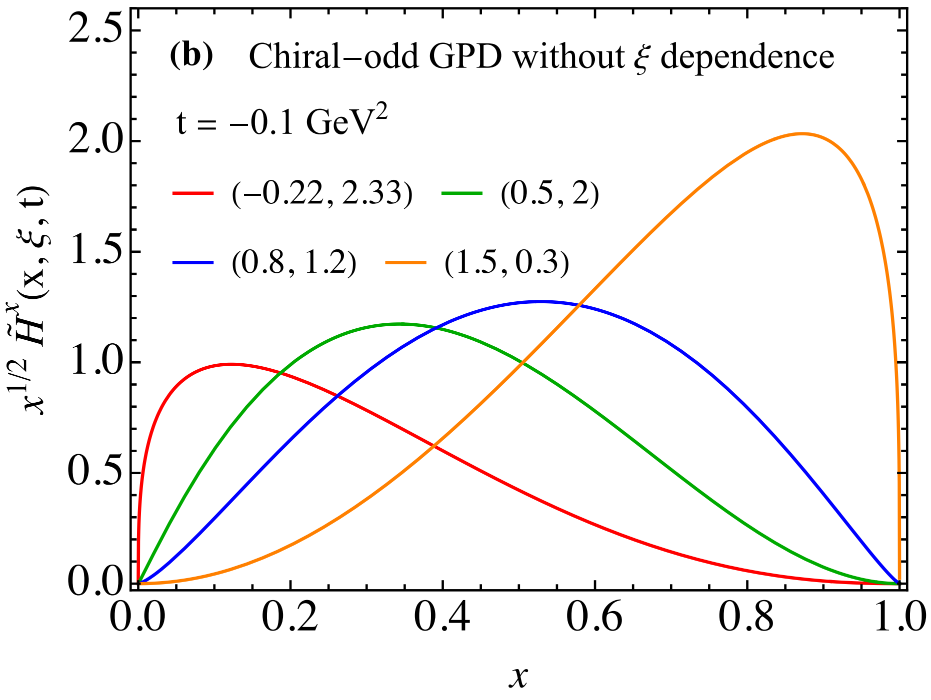
Goloskokov, Kroll
hep-ph/0501242
arXiv: 0708.3569
arXiv: 0906.0460

$$H_{pn}(x,\xi,t) = \theta(x)\, x^{-0.9\,(t/\mathrm{GeV}^2)}\, \frac{x^{\rho}(1-x)^{\tau}}{B(1+\rho, 1+\tau)}$$

$$\widetilde{H}_{pn}(x,\xi,t) = \theta(x)\, x^{-0.45\,(t/\mathrm{GeV}^2)}\, \frac{1.267\, x^{\rho}(1-x)^{\tau}}{B(1+\rho, 1+\tau)}$$

- Neglect $E, \widetilde{E}$. Neglect evolution effect.
- Tune $(\rho, \tau)$ to control $x$ shape.
- Fix DA: $D(z) = N\, z^{0.63}(1-z)^{0.63}$



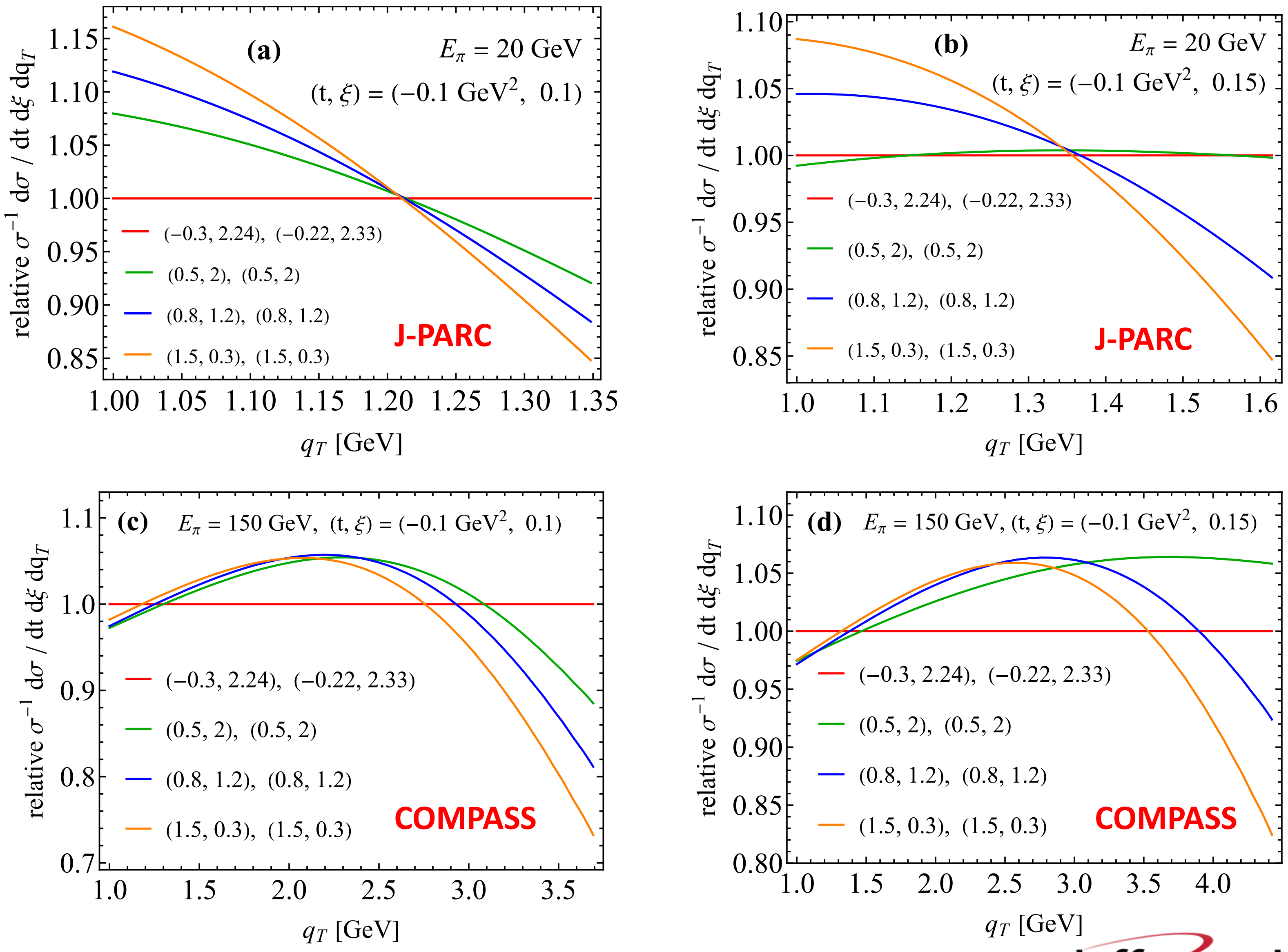
# Numerical results

$$\frac{d\sigma}{dt\, d\xi\ dq_T} \sim |H(x,\xi,t)|^2$$

**Relative $q_T$ shape**

$$\frac{\sigma_{\rm tot}^{-1} d\sigma/dq_T}{\text{some shape func}}$$

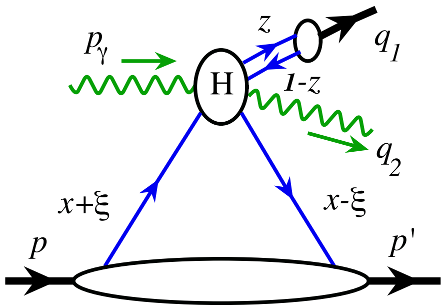
$$\sigma_{\rm tot} = \int_{1\ \rm GeV}^{\sqrt{\hat{s}}/2} dq_T \frac{d\sigma}{dt d\xi dq_T}$$



# Exclusive Photo-Production of a $\pi\gamma$ Pair

- **Process:** $\gamma(p_\gamma) + h(p) \to \pi^{\pm}(q_1) + \gamma(q_2) + h'(p')$

  **Introduced by G. Duplancic et al. [JHEP 11 (2018) 179],**
  **No contribution from gluon GPDs**




Hall D at JLab

$$\frac{d\sigma}{dt\,d\xi\,dq_T^2} \quad \text{or} \quad \frac{d\sigma}{dt\,d\xi\,d\cos\theta}$$

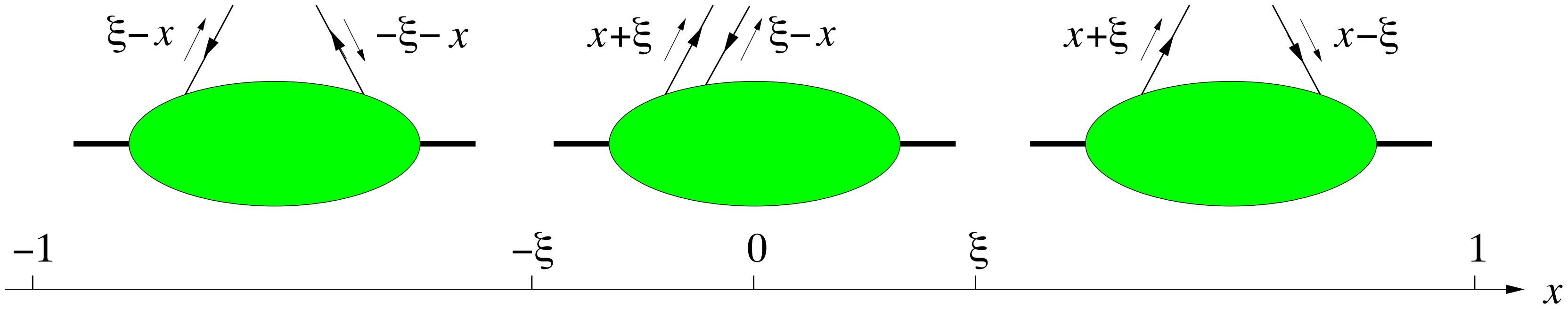
- **Factorization:**

  **Proved to be valid when** $q_T \gg \sqrt{|t|} \gtrsim \Lambda_{\rm QCD}$

- **Cancellation of unwanted propagators & cosθ dependence:**

$$\begin{aligned}\mathrm{Re}\,O_{++} = {} & (e_1 - e_2)^2 \left[\frac{1-\cos\theta}{1+\cos\theta} \cdot P\frac{x+z-2xz}{2\,x\,z\,(1-x)\,(1-z)}\right] + (e_1^2 - e_2^2)\left[\frac{2}{1-\cos\theta} \cdot P\frac{x-z}{x\,z\,(1-x)\,(1-z)}\right] \\ & - e_1 e_2 P\frac{1-\cos\theta}{x\,z\,(1-x)\,(1-z)} \cdot \frac{(xz+(1-x)(1-z))\,(x(1-x)+z(1-z))}{(2(1-x)(1-z)-(1+\cos\theta)xz)\,(2xz-(1+\cos\theta)(1-x)(1-z))}\end{aligned}$$

- **Sensitive to ERBL region (complementary)**




**Also sensitive to DA in the bulk region.**

# Exclusive $\pi^0\gamma$ Pair Production

- **Phenomenology:**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}|t|\,\mathrm{d}\xi\,\mathrm{d}\cos\theta_\pi\,\mathrm{d}\phi_\pi} = \frac{|\mathcal{A}|^2}{32\,s\,(2\pi)^4\,(1+\xi)^2}$$

$$\frac{1}{2}\overline{|\mathcal{A}|^2} = \left(\frac{2\pi\alpha_s}{s}f_\pi\right)^2\left(\frac{C_F}{N_c}\right)^2\left(\frac{1+\xi}{\xi}\right)^2(1-\xi^2) \times \left[|O_{++}^{[\widetilde{H}]}|^2 + |O_{+-}^{[\widetilde{H}]}|^2 + |\widetilde{O}_{++}^{[H]}|^2 + |\widetilde{O}_{+-}^{[H]}|^2\right]$$

- **Factorized helicity amplitude:**

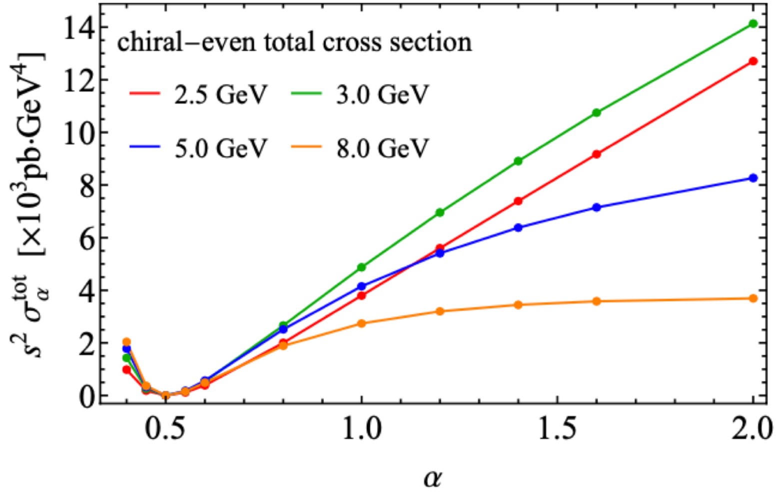
$$O_{\lambda\lambda'}^{[\widetilde{H}]} = \sum_q \int_{x_L}^{x_R}\mathrm{d}x\int_0^1 \mathrm{d}z\,\widetilde{H}^q(x,\xi,t)\,\phi_\pi^q(z)\,O_{\lambda\lambda'}^q(x,z)$$

- **Pion distribution amplitude:**

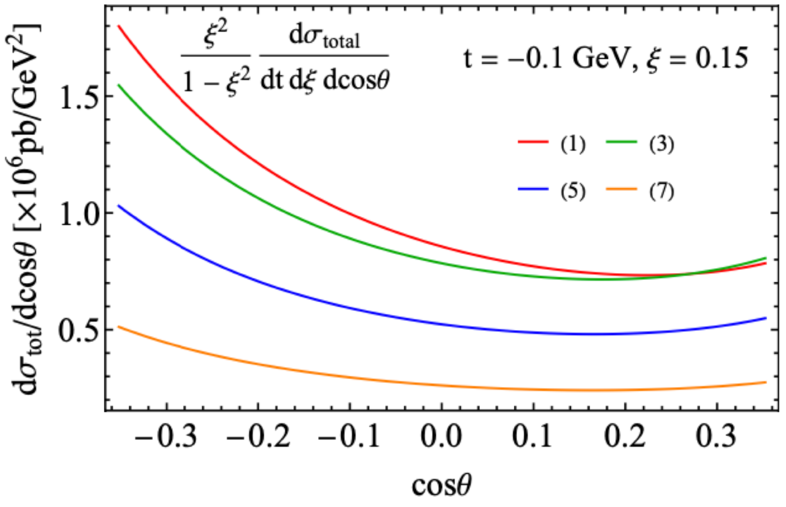
$$\phi_{\pi^0}^d(z) = \phi_{\pi^0}^u(z) = \frac{1}{\sqrt{2}}\frac{z^\alpha(1-z)^\alpha}{\mathrm{B}(1+\alpha,1+\alpha)}, \quad (\alpha > 0)$$

- **Model GPDs = simplified GK model:**
  - Taking $n_i = 0$
  - Parametrizing the forward limit as $x^a(1-x)^b$
  - Neglecting the D-term

- **Sensitivity on DAs (total – $q_T$ > 1 GeV):**




- **Sensitivity on GPDs (α = 0.63):**



# Exclusive $\pi^0\gamma$ Pair Production

## ❑ Phenomenology:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}|t|\,\mathrm{d}\xi\,\mathrm{d}\cos\theta_\pi\,\mathrm{d}\phi_\pi} = \frac{|\mathcal{A}|^2}{32\,s\,(2\pi)^4\,(1+\xi)^2}$$

$$\frac{1}{2}\,\overline{|\mathcal{A}|^2} = \left(\frac{2\pi\alpha_s}{s}f_\pi\right)^2\left(\frac{C_F}{N_c}\right)^2\left(\frac{1+\xi}{\xi}\right)^2(1-\xi^2) \times \left[|O_{++}^{[\widetilde{H}]}|^2 + |O_{+-}^{[\widetilde{H}]}|^2 + |\widetilde{O}_{++}^{[H]}|^2 + |\widetilde{O}_{+-}^{[H]}|^2\right]$$

- **Factorized helicity amplitude:**

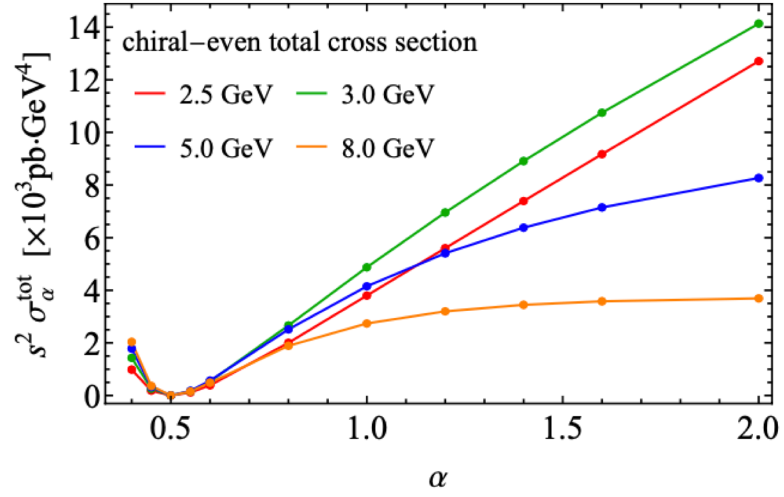
$$O_{\lambda\lambda'}^{[\widetilde{H}]} = \sum_q \int_{x_L}^{x_R} \mathrm{d}x \int_0^1 \mathrm{d}z\,\widetilde{H}^q(x,\xi,t)\,\phi_\pi^q(z)\,O_{\lambda\lambda'}^q(x,z)$$

- **Pion distribution amplitude:**

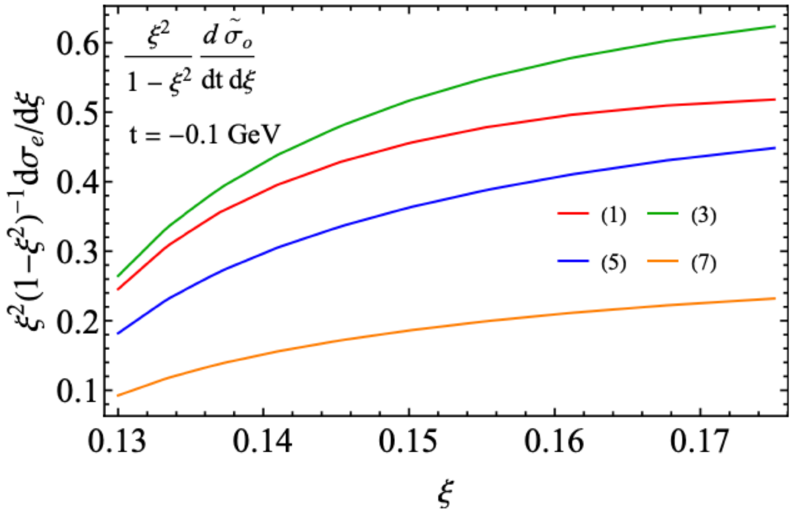
$$\phi_{\pi^0}^d(z) = \phi_{\pi^0}^u(z) = \frac{1}{\sqrt{2}}\frac{z^\alpha(1-z)^\alpha}{\mathrm{B}(1+\alpha,1+\alpha)}, \quad (\alpha > 0)$$

- **Model GPDs = simplified GK model:**
  - Taking $n_i = 0$
  - Parametrizing the forward limit as $x^a(1-x)^b$
  - Neglecting the D-term

## ❑ Sensitivity on DAs (total – $q_T$ > 1 GeV):




## ❑ Sensitivity on GPDs ($\alpha$ = 0.63):



## Summary and Outlook

- ❑ Nucleon mass is the charge of gravitational force, impacts every sectors of our physical world!

- ❑ Understanding the emergency of nucleon mass is a challenge:
  - Mass is closely related to QCD trace anomaly and chiral symmetry breaking
  - Mass decomposition to matrix elements of quark/gluon fields is not unique
  - Single hadron matrix element of quark/gluon field operator is not physical observable

  *Extract each of them from experimental observable(s) with controllable approximation - factorization*

- ❑ Introduced the diffractive $2 \to 3$ exclusive hard processes for extracting GPDs, …
  - Provide both necessary and sufficient conditions for leading power factorization
  - Including existing/known processes for extracting GPDs, ..
  - Identify new factorizable exclusive processes for GPDs more sensitive to x-dependence

  *Introduce a path forward to identify more factorizable exclusive processes for extracting GPDs, and multi-parton correlations, …*

**Thanks!**