Doubly charm ~~and charmonium-like~~ tetraquarks

Sasa Prelovsek

University of Ljubljana & Jozef Stefan Institute, Slovenia

INT Seattle
24th March 2023

outline:

doubly heavy tetraquarks

Padmanath, S.P.: 2202.10110, *Phys.Rev.Lett.* 129 (2022) 032002 (also work with S. Collins)
+ some related lattice QCD studies (not intended as a review talk)

Doubly heavy tetraquarks

$QQ'\bar{q}\bar{q}'$

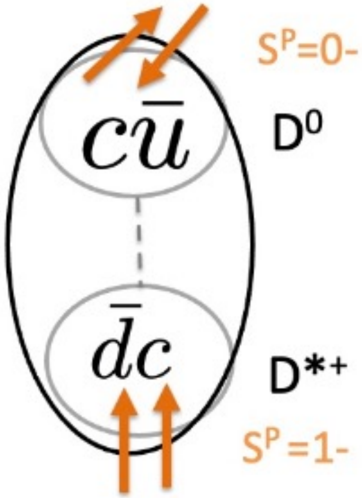
$Q=c,b \quad q=u,d,s$

- Exotic hadrons
- Which states exist? flavor, J^P
- Mass ? Strongly stable ?
- Binding mechanism ?

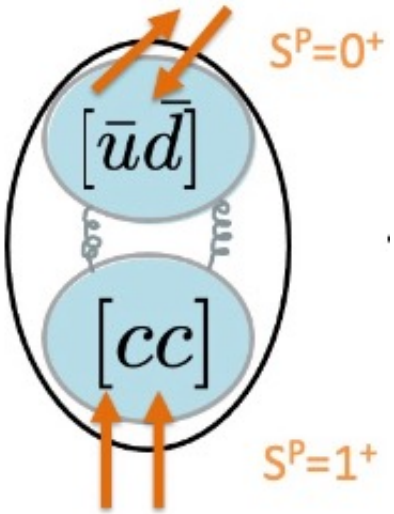
exchange of

light mesons

$\pi, \rho, \pi\pi$?



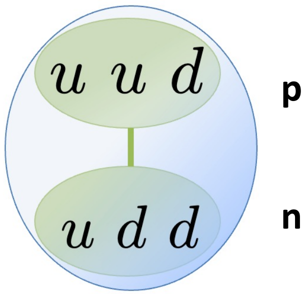
“molecular”



deuterium

$\pi, ..$

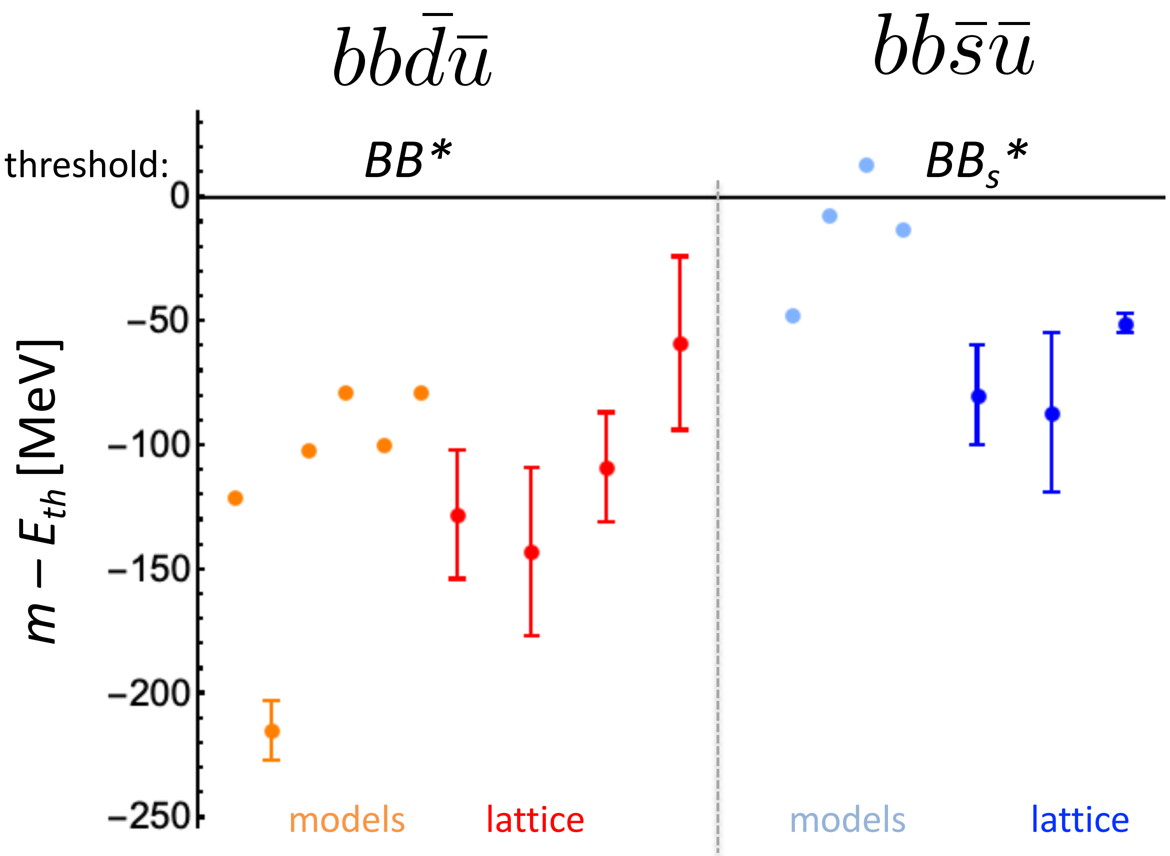
$E_b = -2.2$ MeV



Doubly bottom tetraquarks

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

not found in exp, difficult to find



references from left to right

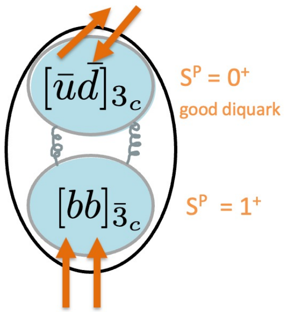
models (many more references):
Eichten and Quigg (2017) 1707.09575 PRL
Karliner and Rosner (2017) 1707.07666 PRL
Ebert et al. (2007) 0706.3853
Silvestre-Brac and Semay (1993)
Janc and Rosina (2004) hep-ph/0405208

lattice: most updated results
Leskovec, Meinel, Pflaumer, Wagner (2019) 1904.04197
Junnarkar, Mathur, Padmanth (2018) 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021) preliminary
Bicudo, Wagner et al. 1612.02758 static potentials

models (many more references)
Eichten and Quigg (2017) 1707.09575 PRL
Parket al. (2018) 1809.05257
Ebert et al. (2007) 0706.3853
Silvestre-Brac and Semay (1993)

lattice: most updated results
Pflaumer, Leskovec,Meinel,Wagner (2021) 2108.10704
Junnarkar, Mathur, Padmanth (2018) 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

likely dominant
(B and B* to close
in BB* molecule
with binding ~0.1 GeV)

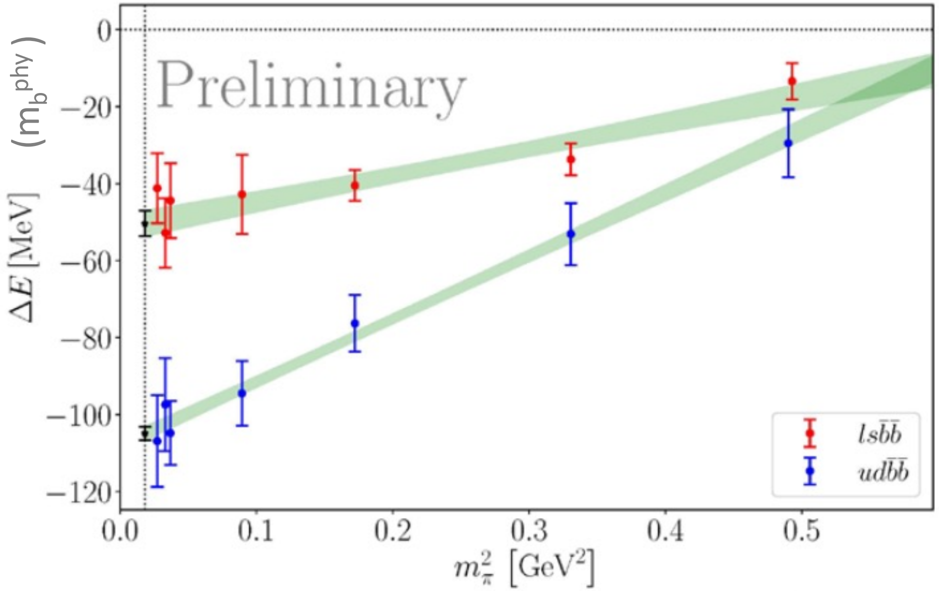


deep binding of bbud also from HALQCD approach: 2212.00202

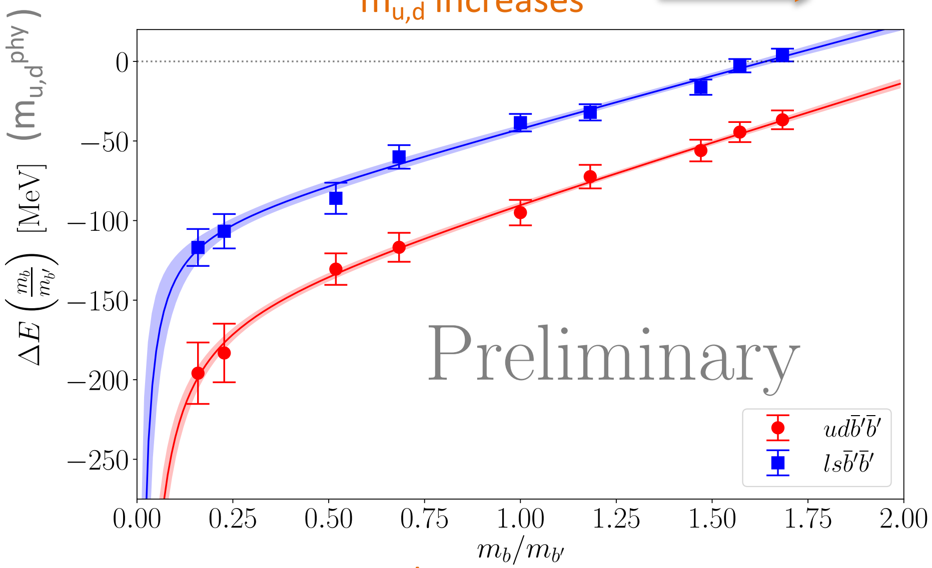
Doubly bottom tetraquarks

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$



$m_{u,d}$ increases →



$m_{b'}$ decreases →

Frances, Colquhoun, Lewis, Maltman (2021)
PoS LATTICE2021 (2022) 144

supports internal structure below

supported also by almost all model studes

Karliner and Rosner (2017), Janc and Rosina (2004), ...

Other $QQ'\bar{q}\bar{q}'$ and J^P $bc\bar{q}\bar{q}'$, $cc\bar{q}\bar{q}'$ q=u,d,s

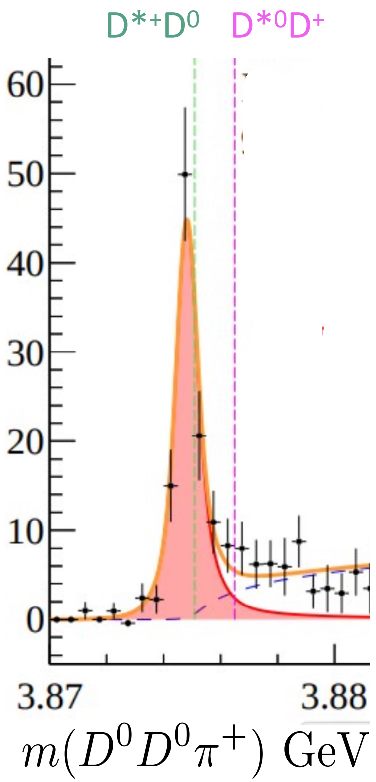
Theoretically expected near or above threshold

States near or above threshold have to be identified as poles in scattering T(E): much more challenging

Experimental discovery of T_{cc} $cc\bar{d}\bar{u}$

The longest lived exotic hadron ever discovered

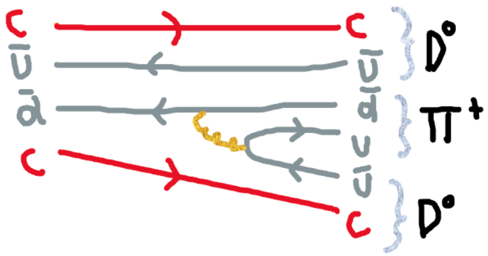
I=0, $J^P=1^+$ (most likely)



$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

LHCb 2109.01038, 2109.01056, Nature Physics

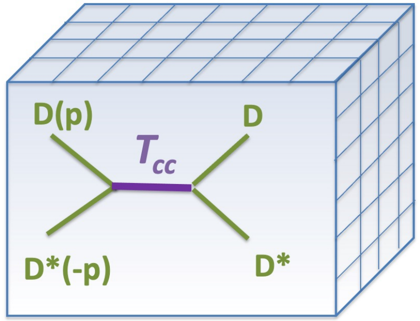


Omitting $D^* \to D\pi$, $T_{cc} \to DD\pi$

T_{cc} would-be a bound state

T_{cc} from lattice

$cc\bar{d}\bar{u}$ I=0, J^P=1+



- pre-2020 simulations extracted E_n , not T(E): Junnarkar et al 1810.12285, HadSpec 1709.01417
- near-threshold states require extraction of scattering amplitude T(E)
- states correspond to poles in T(E)

Our lattice study of T_{cc} channel

Padmanath, S.P.: 2202.10110, *PRL 2022*

m_π =280(3) MeV

$D^* \not\to D\pi,\ T_{cc} \not\to DD\pi$

$DD\pi$ above analyzed region

CLS 2+1 ensembles, a≃0.086 fm, L = 2.1 fm, 2.7 fm

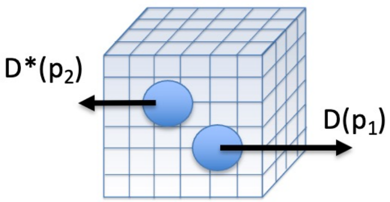
Luscher's eq.

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\ \mathcal{O}_j^+(0)|0\rangle = \sum_n \langle 0|\mathcal{O}_i|n\rangle\ e^{-E_n t}\ \langle n|\mathcal{O}_j^+|0\rangle \quad \to \quad E_n \quad \to \quad T(E)$$

D D*

$$\mathcal{O} = (\bar{u}\gamma_5 c)_{\vec{p}_1}\ (\bar{d}\gamma_i c)_{\vec{p}_2} - (\vec{p}_1 \leftrightarrow \vec{p}_2) \qquad \vec{p}_{1,2} = \vec{n}_{1,2}\ \tfrac{2\pi}{L}$$

$(\bar{u}\gamma_5\gamma_t c)_{\vec{p}_1}\ (\bar{d}\gamma_i\gamma_t c)_{\vec{p}_2}$



P=p1+p2

(0,0,0)

(0,0,1)

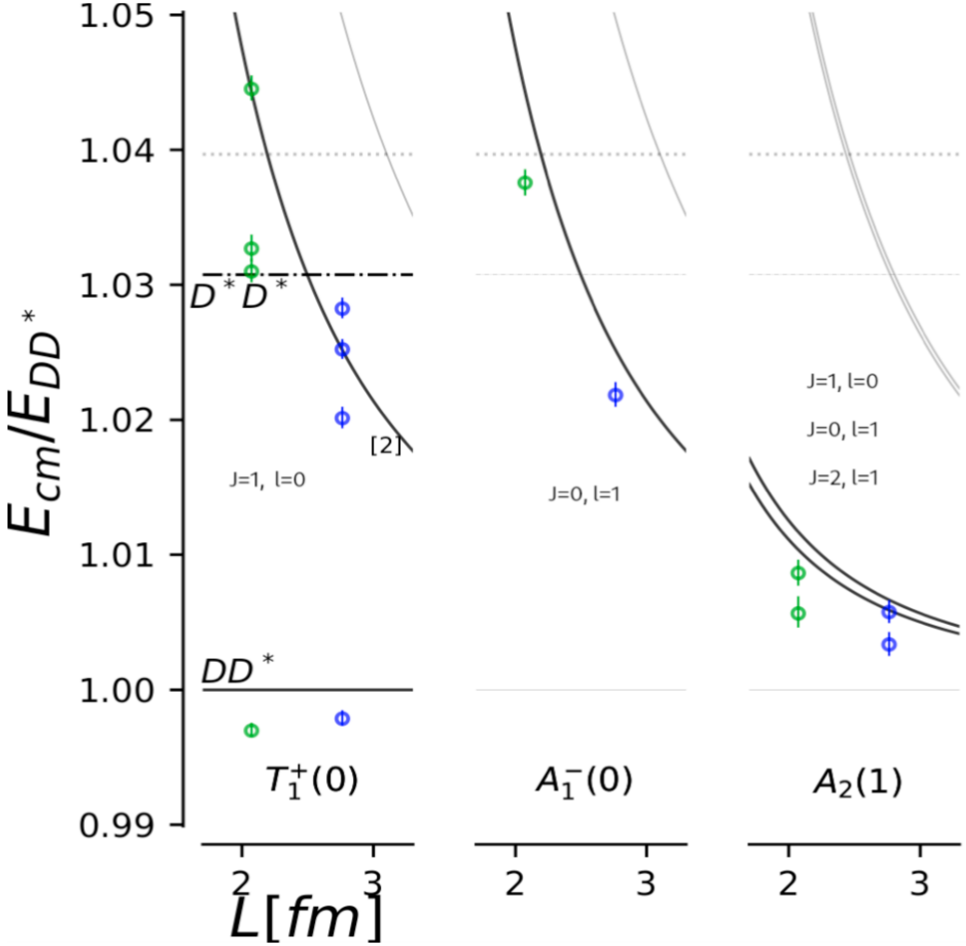
(1,1,0)

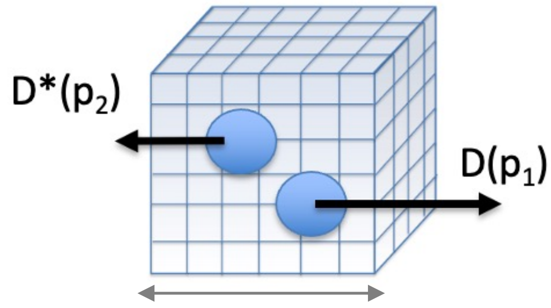
(0,0,2)

Eigen-energies on the lattice

at $m_\pi \approx 280\ MeV$ most informative irreps

$\vec{P}=\vec{0}$ $\vec{P}=\frac{2\pi}{L}\,(0,0,1)$ $\vec{P}=\vec{p}_1+\vec{p}_2$





lines → non-interacting energies

$$E^{n.i.}=\sqrt{m_D^2+\vec{p}_1^{\,2}}+\sqrt{m_{D^*}^2+\vec{p}_2^{\,2}}$$

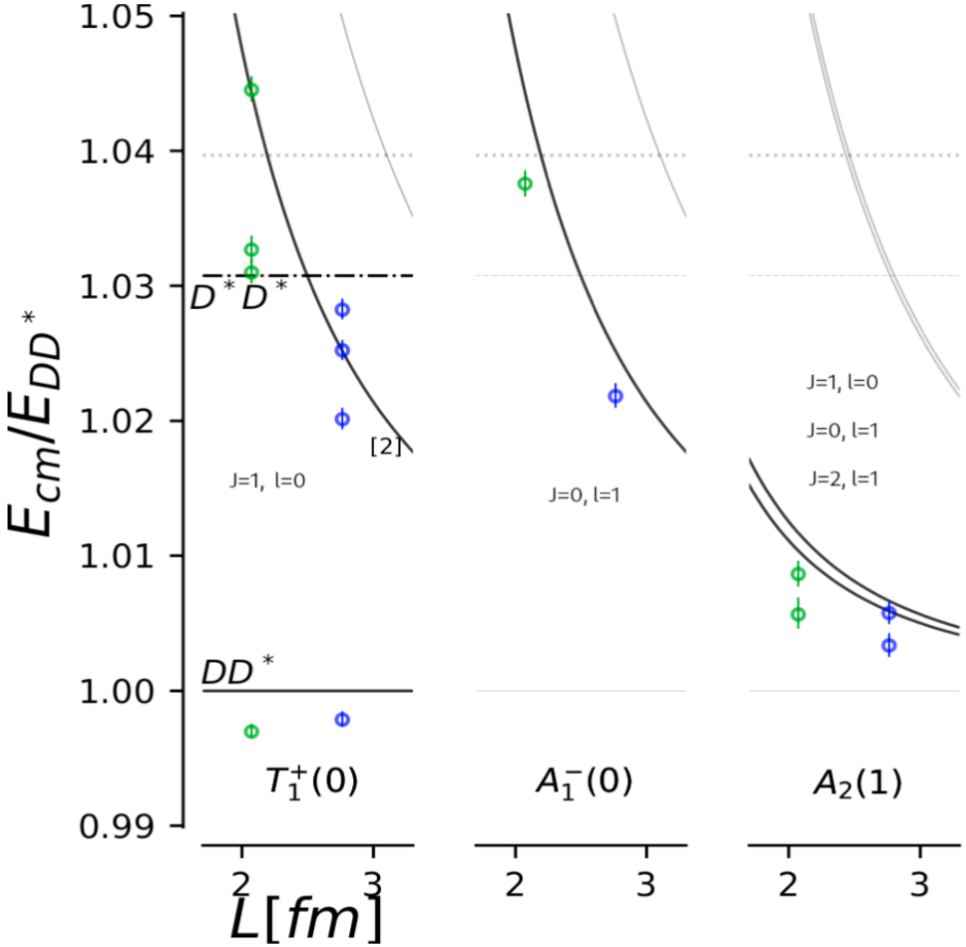
$$\vec{p}_i=\vec{n}_i\frac{2\pi}{L}$$

$$E_{DD^*}\equiv m_D+m_{D^*}$$

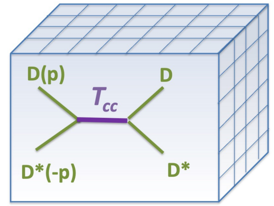
Eigen-energies and scattering amplitude

Padmanath, S.P.: 2202.10110, *PRL 2022*

at $m_\pi \approx 280\ MeV$



$$E_{DD^*} \equiv m_D + m_{D^*}$$

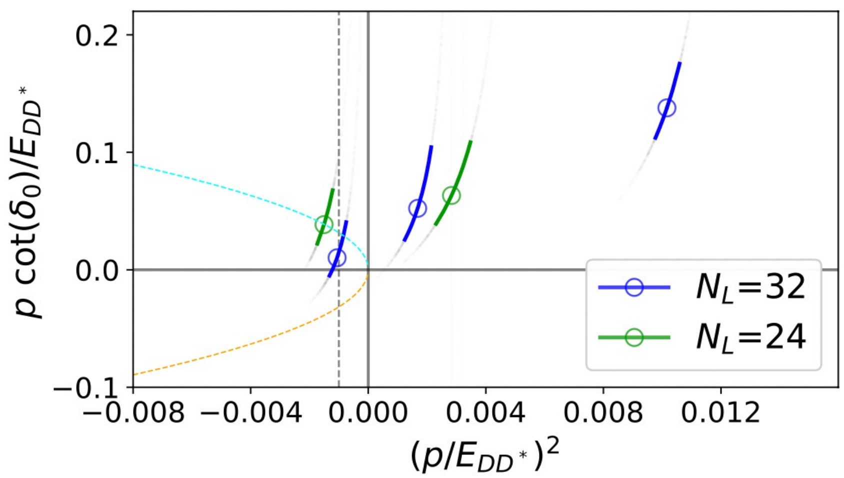


Luscher's relation

E -> T(E), $\delta(E)$

$$T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$

partial wave l=0



effects of left-hand cut (pion exchange in t-channel) omitted,

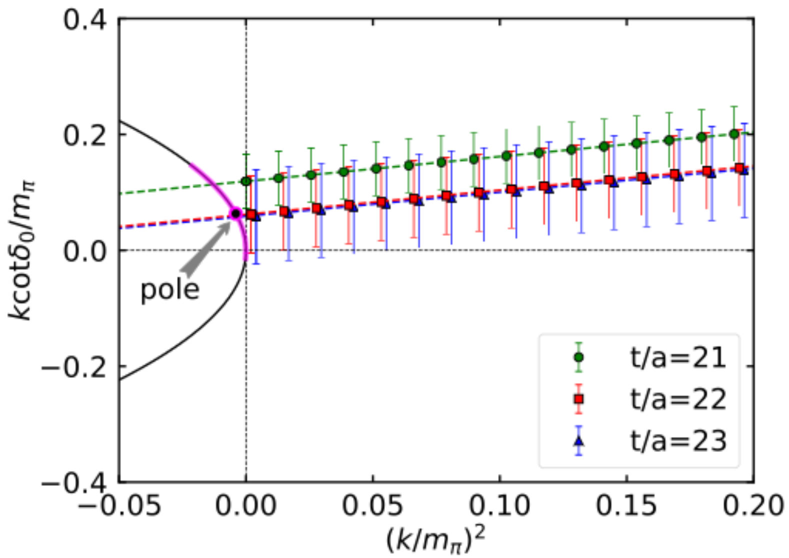
effective range approx. employed

see discussion at the end of the talk and Du et al. 2303.09441

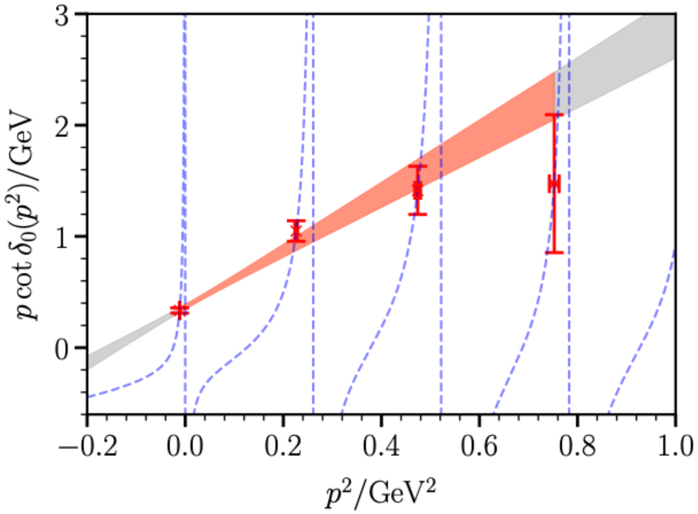
Intermezzo: p cot δ_0 in Tcc chanel from available lattice simulations (other two simulations will be detailed later in the talk)

eff. range approx.: $p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$

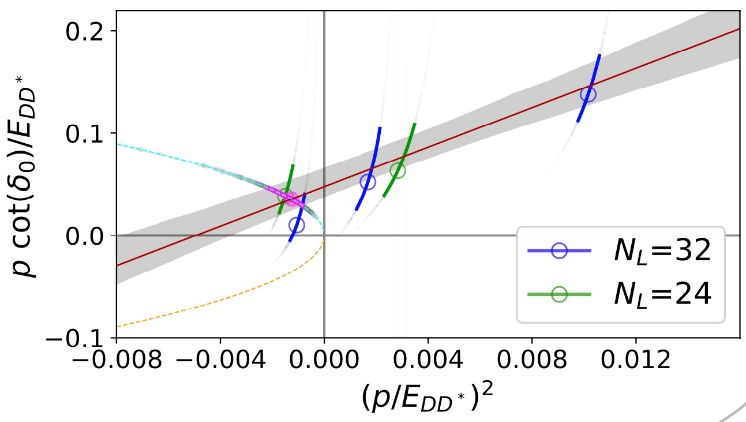
HALQCD, 2302.04505, $m_\pi \approx 146$ MeV



CLQCD 2206.06185, $m_\pi \approx 348$ MeV, PLB



Padmanath, S.P.: 2202.10110, $m_\pi \approx 280$ MeV *PRL*



Eigen-energies and scattering amplitude

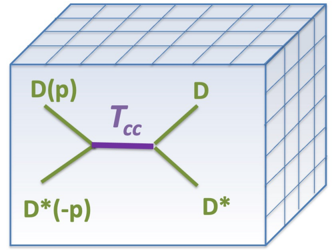
Padmanath, S.P.: 2202.10110, *PRL*

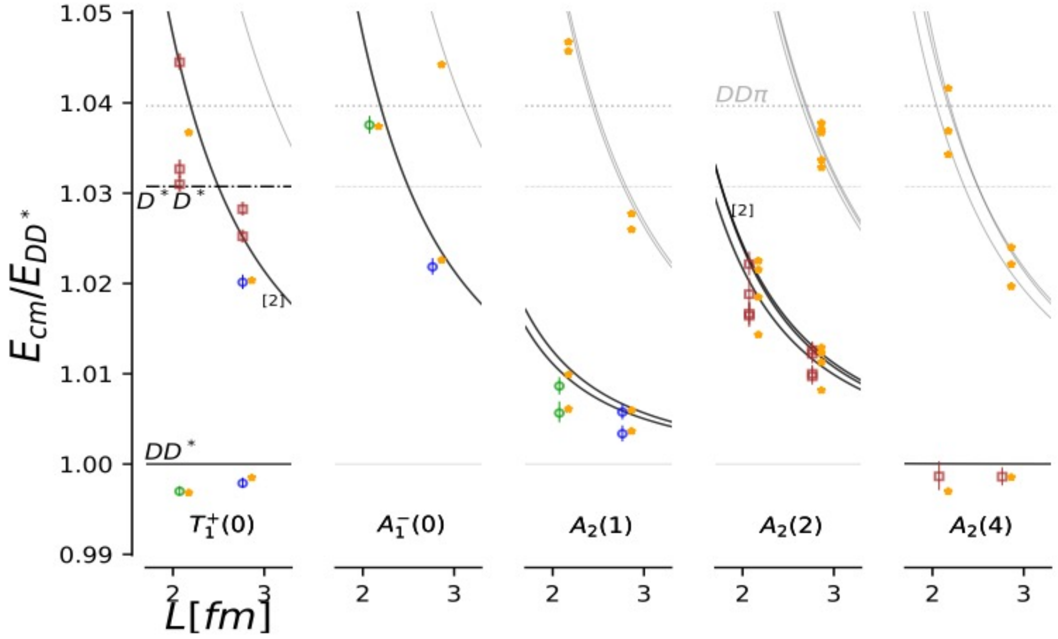
at $m_\pi \approx 280\ MeV$

$$T = \frac{E}{2} \frac{1}{p\cot\delta - ip}$$

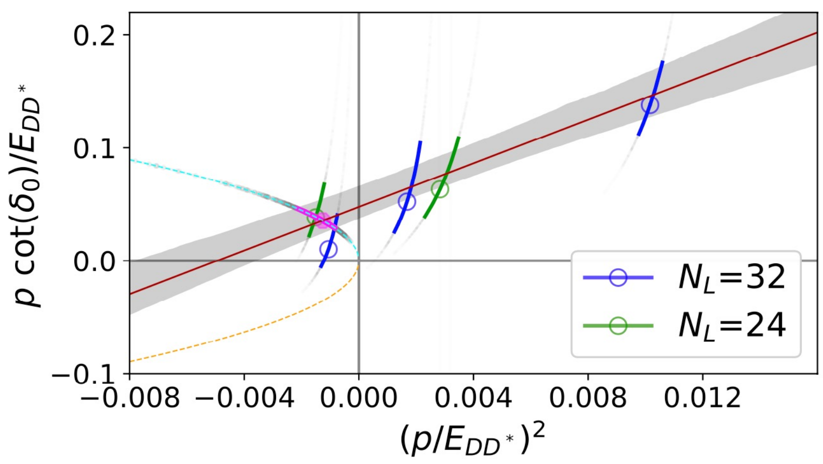
Luscher's relation

E -> T(E), $\delta(E)$





partial wave l=0



$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$$

$a_0 = 1.04(0.29)$ fm & $r_0 = 0.96(^{+0.18}_{-0.20})$ fm

★ reconstructed spectra

■ En not used in the fit

parametrizing l=0,1 with eff. range

$$p^{2l+1}\cot\delta = \frac{1}{a_l} + \frac{1}{2}\, r_l\, p^2$$

virtual bound st. pole

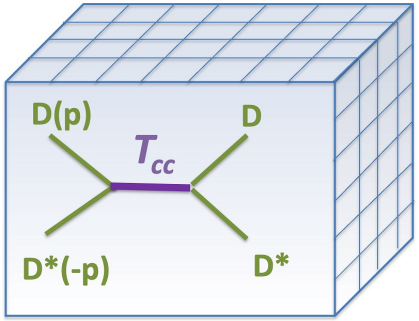
Binding energy:

$\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2})$ MeV

Scattering amplitude and pole

$m_\pi \approx 280$ MeV

$$T = \frac{E}{2} \frac{1}{p \cot \delta - ip}$$



possible effect from left-hand cuts omitted

(see forthcoming paper of Nefediev et al)

Padmanath, S.P.: 2202.10110, *PRL*

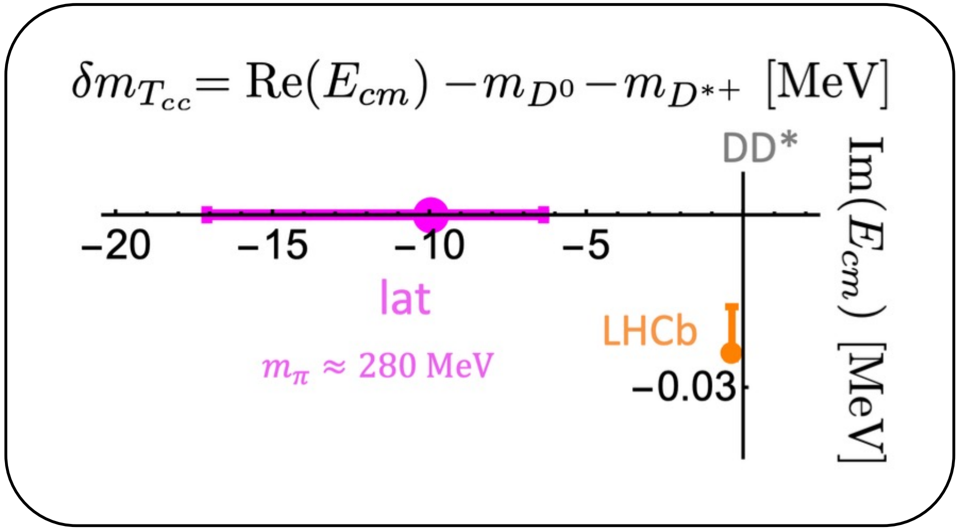
Lattice: virtual bound st. pole

Binding energy:

$\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2})$ MeV

Nature (LHCb): (would-be) bound st. pole

omitting $D^* \to D\pi, \; T_{cc} \to DD\pi$



Dependence on $m_{u/d}$ and m_c

$cc\bar{d}\bar{u}$

Padmanath, S.P.: 2202.10110, *PRL Supplemental material*

in case of molecular binding mechanism

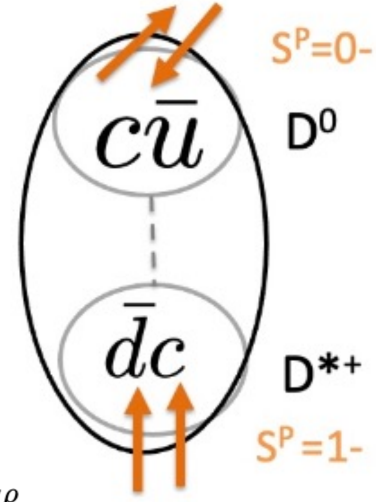
Simple arguments in QM :

$$\pi, \rho, \pi\pi \ ?$$

$m_{u/d}$

$$V(r) \propto -\frac{e^{-m_{ex} r}}{r}$$

$$m_{ex} : \ m_\pi, m_\rho$$



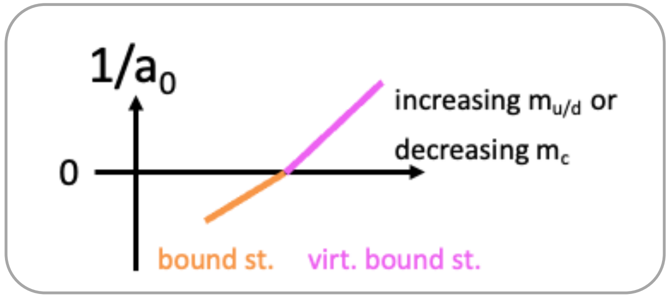
m_c

$$\hat{H}_{kin} = \frac{\hat{p}^2}{2\ m_{red}}$$

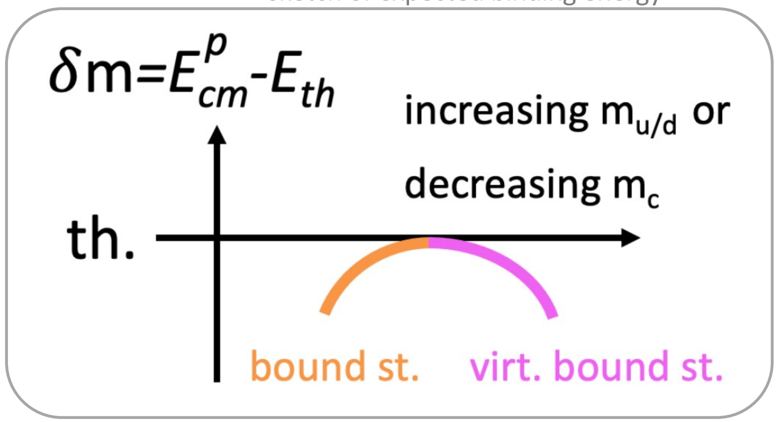
$$m_r \simeq \frac{m_D m_{D^*}}{m_D + m_{D^*}}$$

$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}\ r_0\ p^2$$

sketch of expected scattering lenght a0



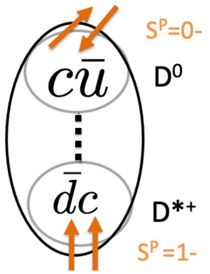
sketch of expected binding energy

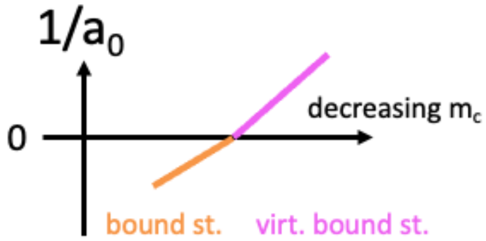


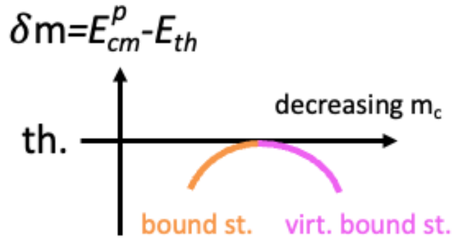
Hypothesis partly already verified and

to be furthertested by future simulations

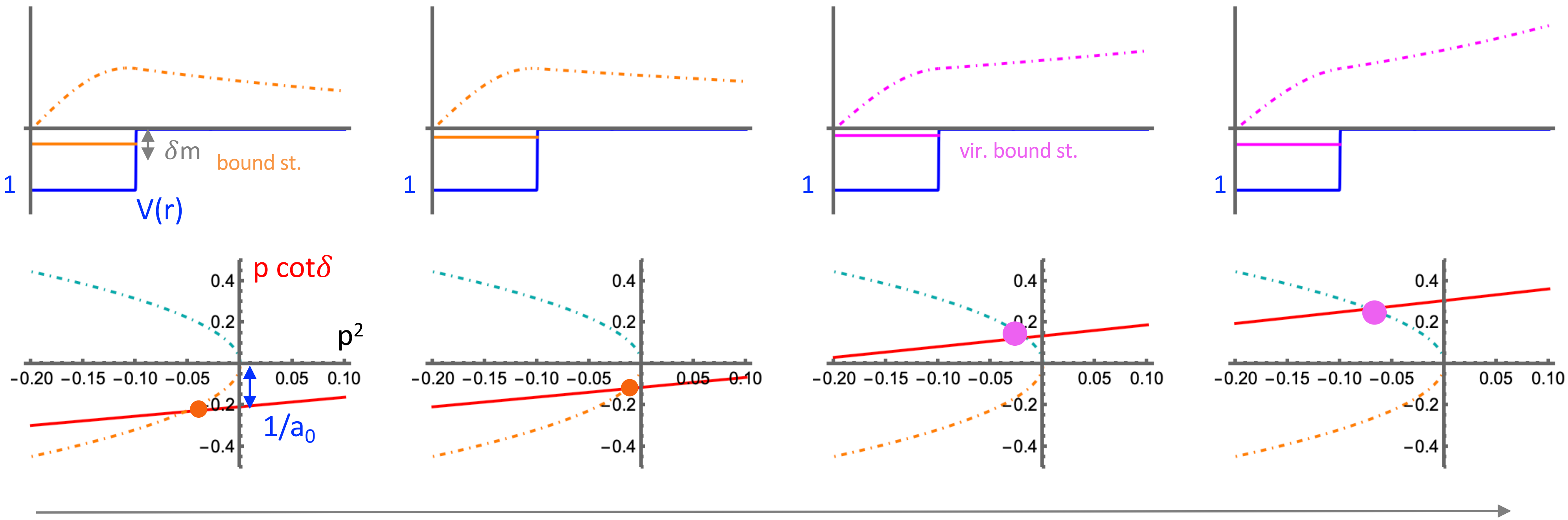
Dependence on m_c





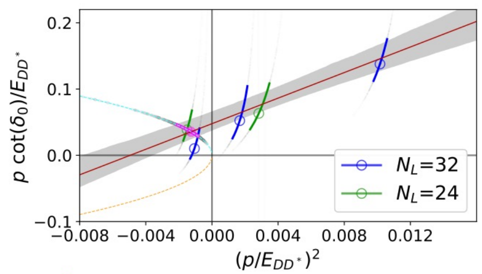


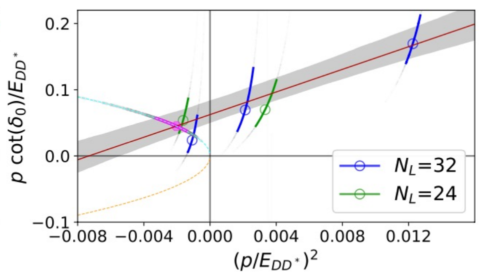
Square well potential (analogous conclusion for other fully attractive shapes)



decreasing m_c and m_r

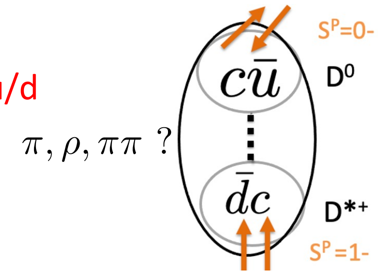
	m_D [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
$m_c^{(h)}$	1927(1)	1.04(29)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
$m_c^{(l)}$	1762(1)	0.86(0.22)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.

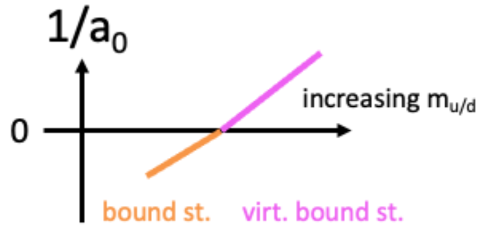


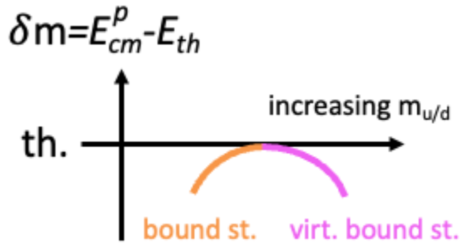


Padmanath, S.P.: 2202.10110, *PRL*

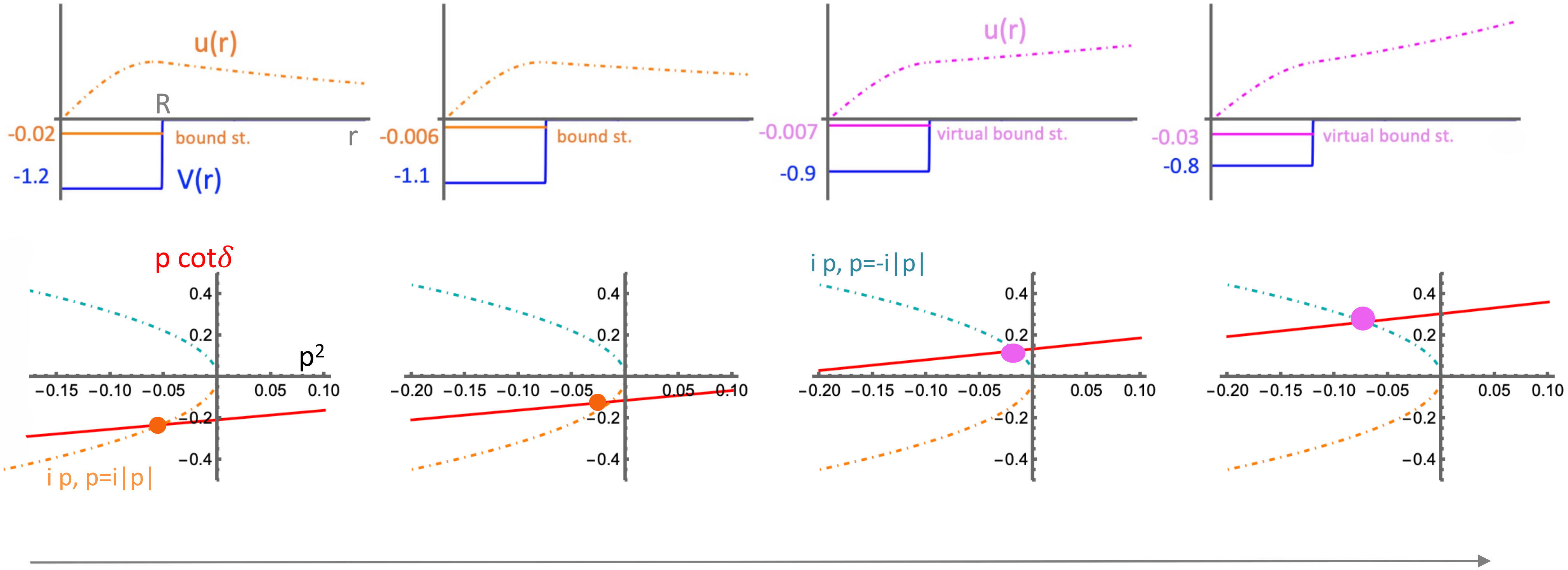
Dependence on $m_{u/d}$







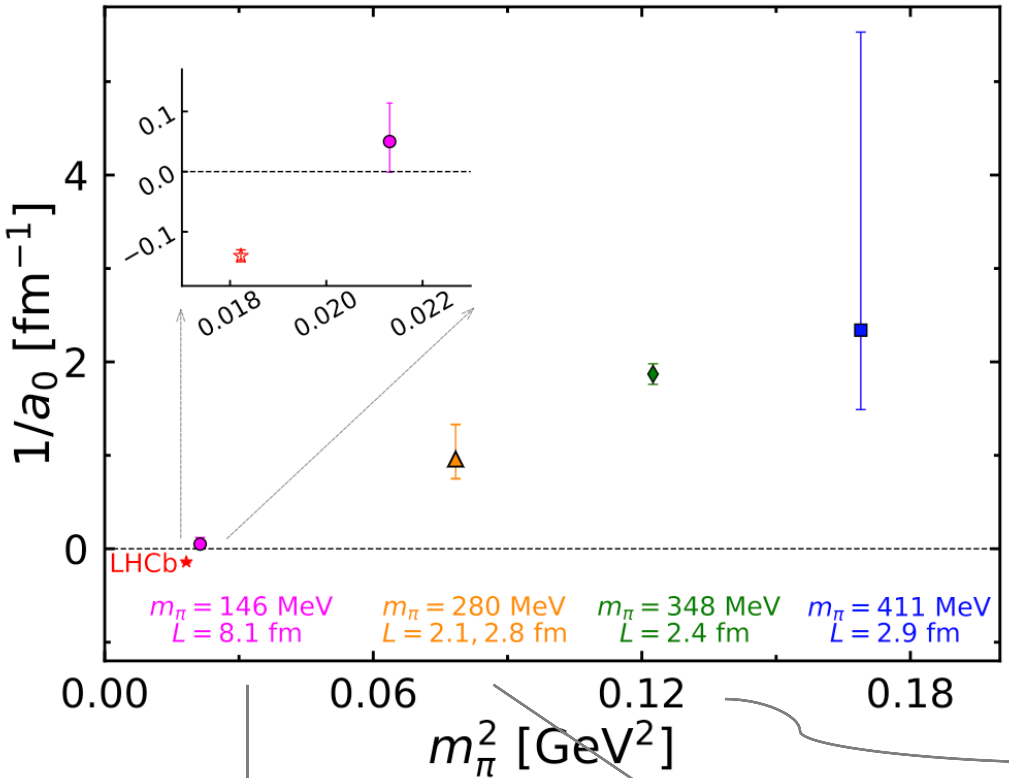
Square well potential (analogous conclusion for other shapes)

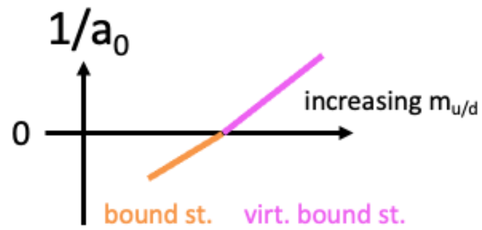


increasing $m_{u/d}$, decreasing attraction

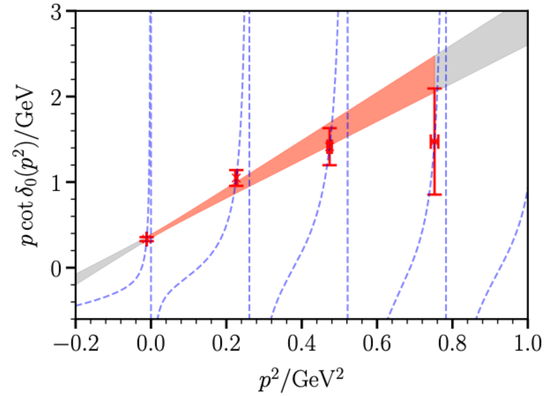
Tcc from lattice: dependence of 1/a0 on $m_{u/d}$

summary from 2302.04505

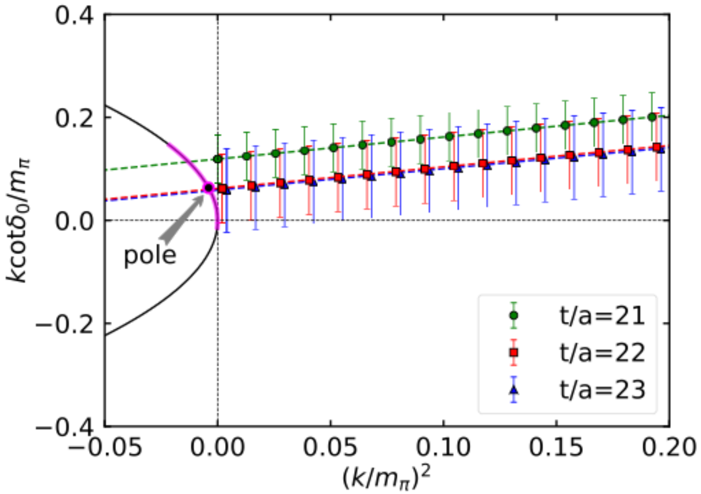




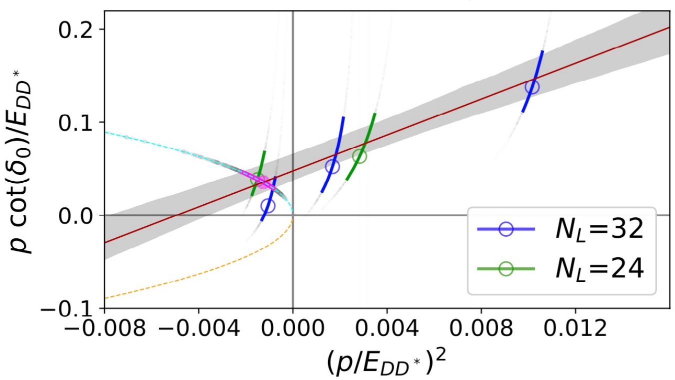
CLQCD 2206.06185, $m_\pi \approx 348$ MeV, PLB



HALQCD, 2302.04505, $m_\pi \approx 146$ MeV

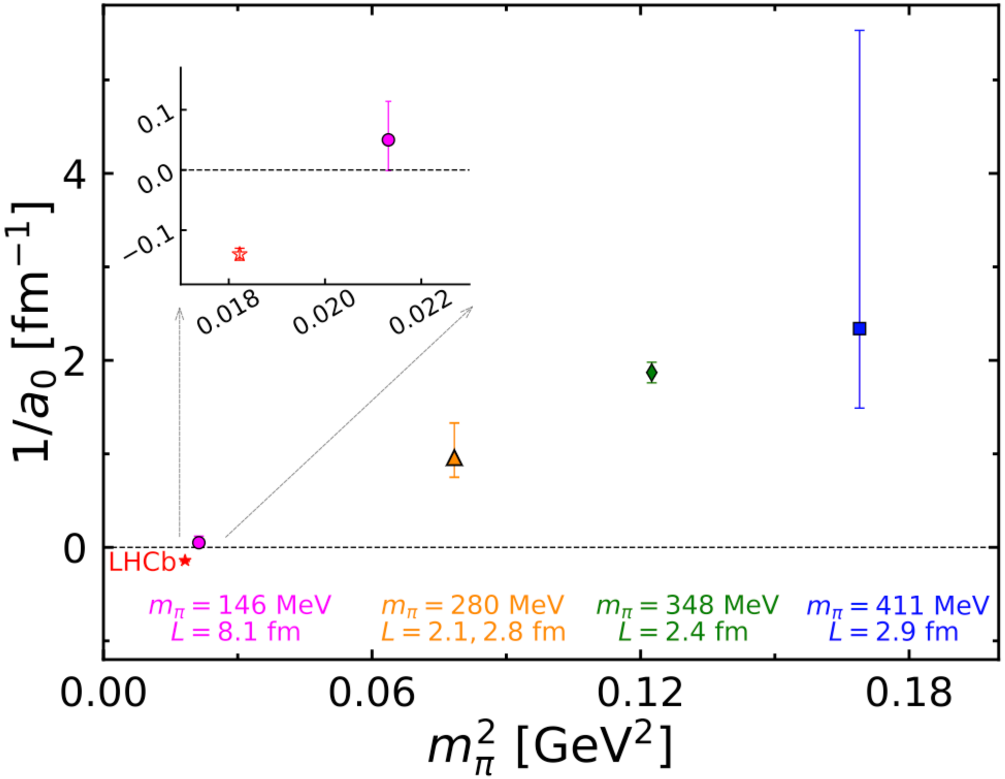


Padmanath, S.P.: 2202.10110, $m_\pi \approx 280$ MeV *PRL*

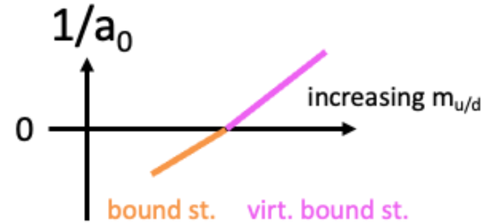


T_{cc} from lattice: dependence on $m_{u/d}$

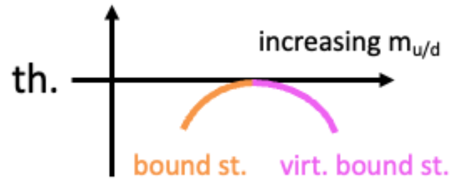
summary from 2302.04505



$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$$



$\delta m = E_{cm}^p - E_{th}$

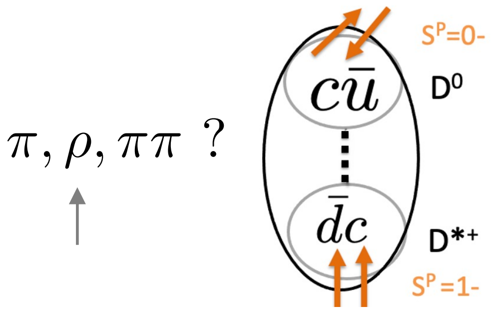


	LHCb	subsequent lattice sim. HALQCD coll, 2302.04505 HALQCD method	our lattice sim. Luscher's method	subsequent lat. sim. CLQCD 2206.06185 Luscher's method
δm :	- 0.36(4) MeV	- 0.045(77) MeV	$-9.9(^{+3.6}_{-7.2})$ MeV.	
	bound st.	virtual bound st.		

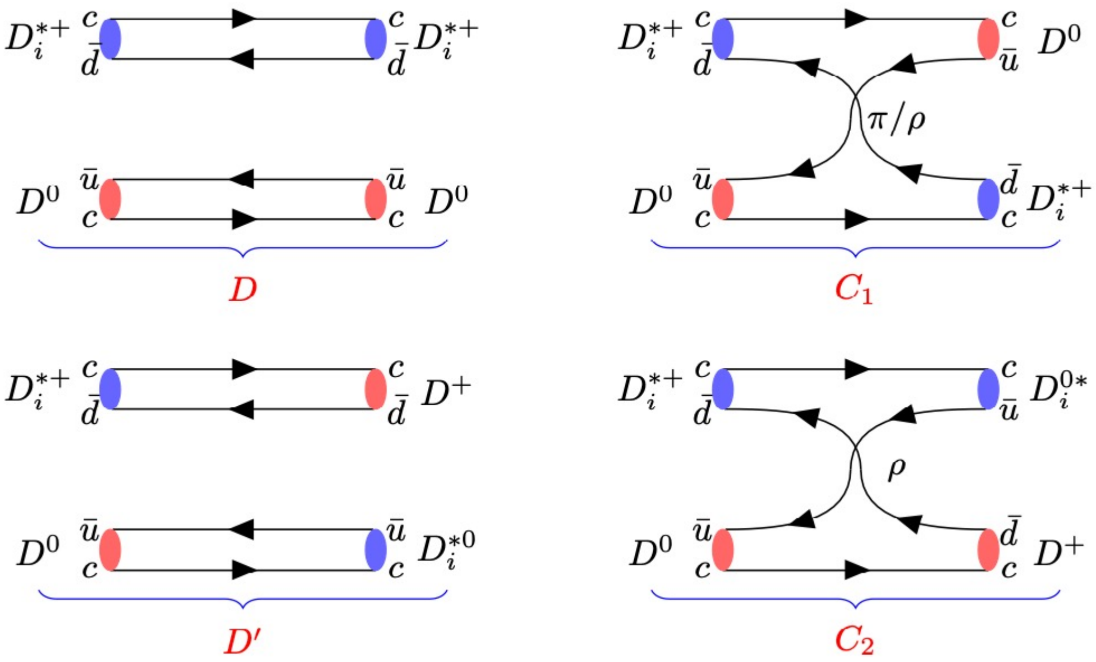
Dominant exchanged particles at $m_\pi \approx 348$ MeV

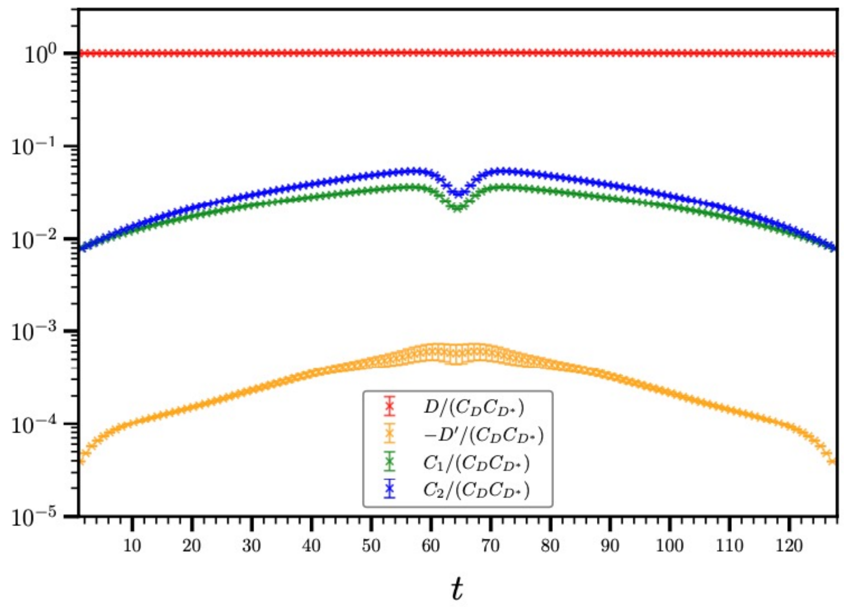
subsequent lattice study via Luscher's method
CLQCD, Chen et al. 2206.06185, PLB

comparison of I=0,1 :
attraction in I=0 channel arises
mainly from ρ exchange



$$C^{(I)}(p,t) = D - C_1(\pi/\rho) + (-)^{I+1}\,(D' - C_2(\rho))$$





Dominant exchanged particles

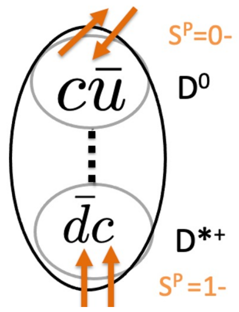
at $m_\pi \approx 146$ MeV, $L \approx 8.1$ fm !!

subsequent lattice sim.

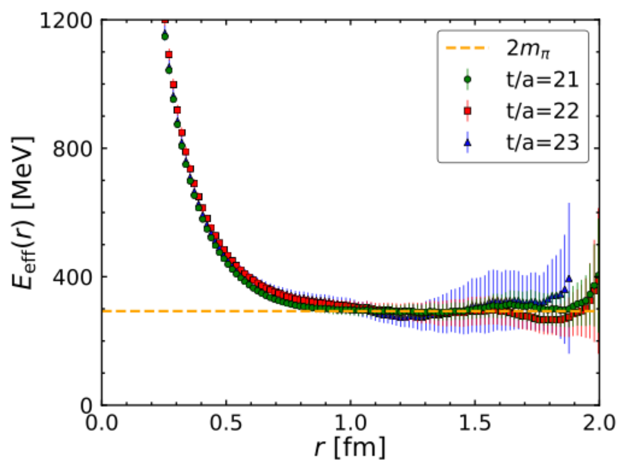
HALQCD coll, 2302.04505

HALQCD method

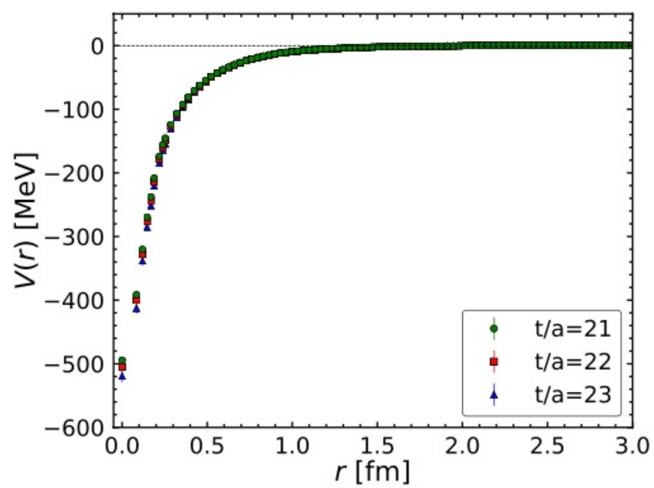
$\pi, \rho, \pi\pi$?

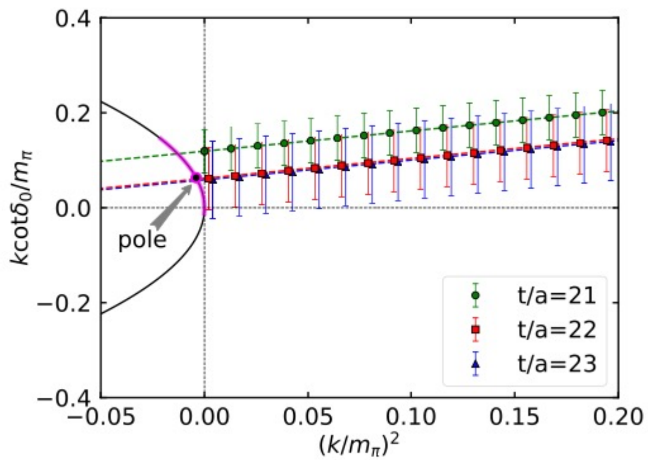


$$E_{\rm eff}(r) = -\frac{\ln[V(r)r^2/a_3]}{r}$$

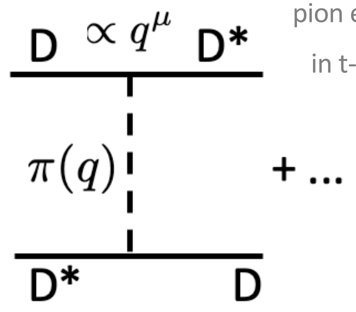


$$V(r) \sim - \frac{e^{-2m_\pi r}}{r^2} \quad r > 1 \text{ fm}$$





Pion exchange and left-hand cut



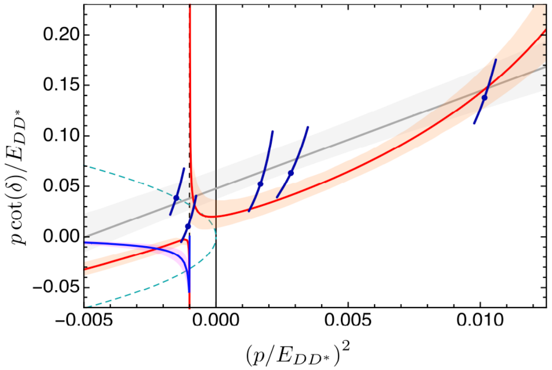
pion exchange in t-channel

- possible effects from left-hand cut : requires further work
- pion exchange: suppressed near threshold due to derivative coupling
- pheno studies: one-pion exchange not dominant
- CLQCD, HALQCD lattice studes: one-pion exchanges not dominant
- generalization of Luscher's relation on left-hand cut: 2301.03981, Raposo& Hansen @ lat22

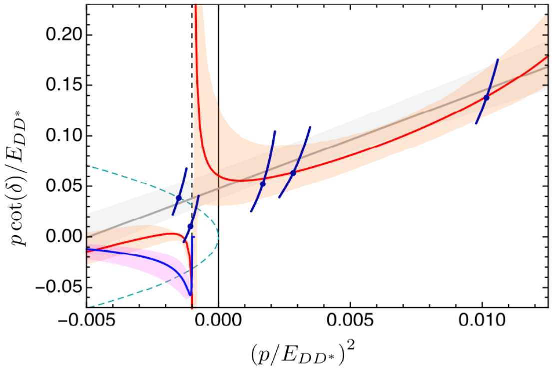
- reanalysis our Tcc data incorporating left-hand cut

  Du et al. 2303.09441 @ talk by Christof Hanhart

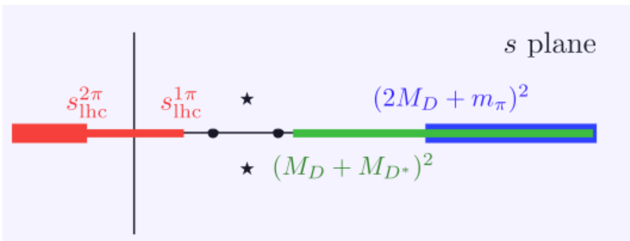
".. The appearance of a pair of virtual states is indeed natural near the point where they are about to turn to a narrow resonance ... "



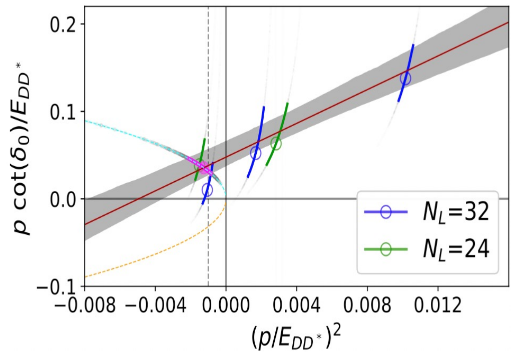
● two virtual bound states



★ two narrow resonances



Padmanath, S.P.: 2202.10110, *PRL*



one virtual bound state

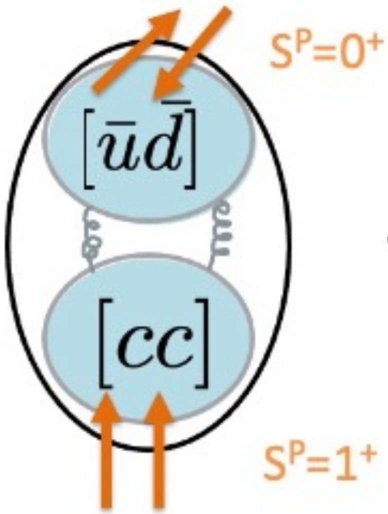
both conclusions support the presence of significant attraction and poles, likely due to Tcc

Dependence on $m_{u/d}$ and m_c

$cc\bar{d}\bar{u}$

in case of diquark antidiquark binding mechanism

- [QQ] binding increases with increasing mQ
- [<u>ud</u>] binding decreases with decreasing $m_{u/d}$
- [QQ][<u>ud</u>] binding with respect to DD* threshold,
  dependence on m_Q , $m_{u/d}$: ??

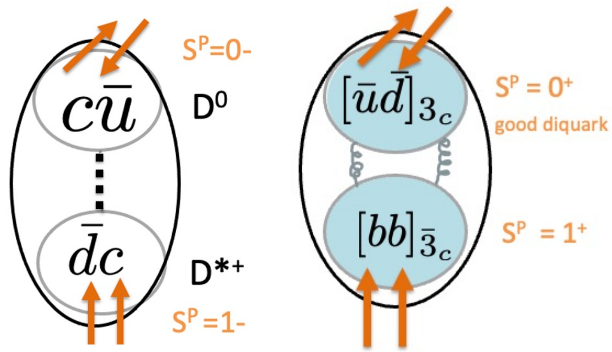


generic/robust predictions for δm and 1/a0 needed:

looking forwatd to test those from lattice

Conclusions

likely dominant



- Tcc=ccud is the longest-lived exotic hadron ever discovered
- doubly heavy tetraquarks are good probes for binding mechanisms
- valuable theoretical probe: explore states as a function of quark masses
- theory: states near or above threshold have to be extracted from T(E)
- progress on lattice, many challenges left
- excited to see whether more states get discovered in exp or theory

$$cc\bar{u}\bar{d},\ \ bb\bar{u}\bar{d},\ \ bc\bar{u}\bar{d},\ \ cc\bar{u}\bar{s}, \ldots$$

Backup

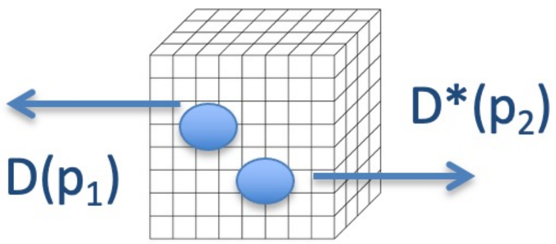
Interpolators for Tcc

Example: P=0

$J^P=1^+$ -> cubic irrep T_1^+

P=D, V=D*

$$
\begin{aligned}
O^{l=0} &= P(\{0,0,0\})V_z(\{0,0,0\}) \\
O^{l=0} &= P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\
&\quad + P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\
&\quad + P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \\
O^{l=2} &= P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\
&\quad + P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\
&\quad -2[P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \\
O^{l=0} &= V_{1x}[0,0,0]V_{2_y}[0,0,0] - V_{1y}[0,0,0]V_{2x}[0,0,0]
\end{aligned}
$$



$$\chi^2(\{a\}) = \sum_L \sum_{\vec{P}\Lambda n} \sum_{\vec{P}'\Lambda' n'} dE_{cm}(L, \vec{P}\Lambda n; \{a\}) \qquad (1)$$
$$\mathcal{C}^{-1}(L; \vec{P}\Lambda n; \vec{P}'\Lambda' n') dE_{cm}(L, \vec{P}'\Lambda' n'; \{a\}) \ .$$

Here

$$dE_{cm}(L, \vec{P}\Lambda n; \{a\}) = E_{cm}(L, \vec{P}\Lambda n) - E_{cm}^{an.}(L, \vec{P}\Lambda n; \{a\})$$

$$(t_l^{(J)})^{-1} = \frac{2(\tilde{K}_l^{(J)})^{-1}}{E_{cm} p^{2l}} - i \frac{2p}{E_{cm}}, \quad (\tilde{K}_l^{(J)})^{-1} = p^{2l+1} \cot \delta_l^{(J)} \qquad (5)$$

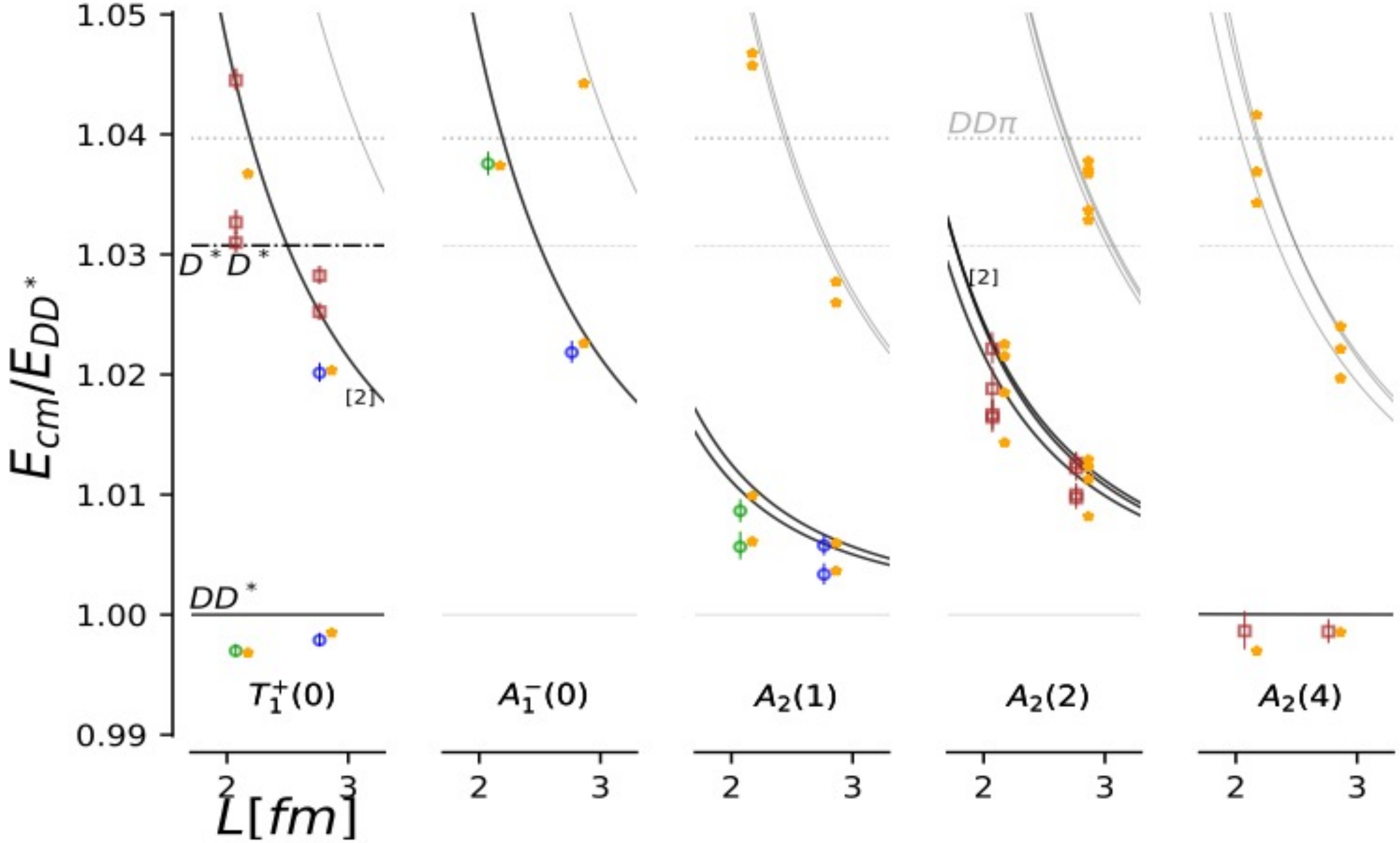
We parametrize it with the effective range expansion

$$\tilde{K}^{-1} = \begin{bmatrix} \frac{1}{a_0^{(1)}} + \frac{r_0^{(1)} p^2}{2} & 0 & 0 \\ 0 & \frac{1}{a_1^{(0)}} + \frac{r_1^{(0)} p^2}{2} & 0 \\ 0 & 0 & \frac{1}{a_1^{(2)}} \end{bmatrix} . \qquad (6)$$

Details on Tcc

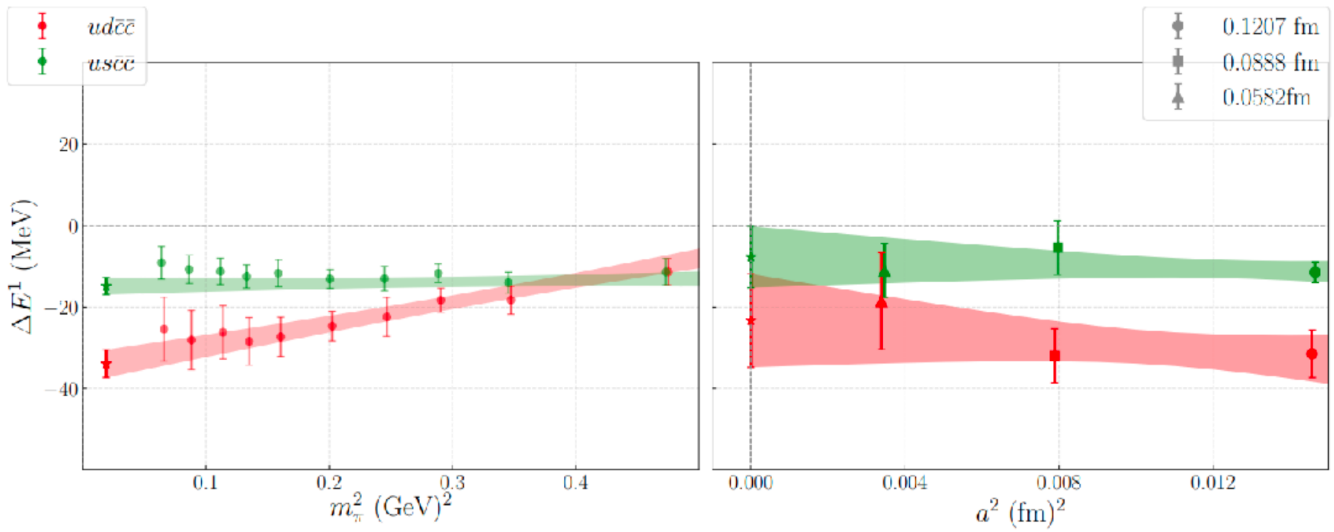
$\vec{P}$	LG	Λ^P	J^P	l	interpolators: $M_1(\vec{p}_1^{\,2})M_2(\vec{p}_2^{\,2})$
$(0,0,0)$	O_h	T_1^+	1^+	$0,2$	$D(0)D^*(0)$, $D(1)D^*(1)$ [2], $D^*(0)D^*(0)$
$(0,0,0)$	O_h	A_1^-	0^-	1	$D(1)D^*(1)$
$(0,0,1)\frac{2\pi}{L}$	Dic_4	A_2	$0^-,1^+,2^-$	$0,1,2$	$D(0)D^*(1)$, $D(1)D^*(0)$
$(1,1,0)\frac{2\pi}{L}$	Dic_2	A_2	$0^-,1^+,2^-,2^+$	$0,1,2$	$D(0)D^*(2)$, $D(1)D^*(1)$ [2], $D(2)D^*(1)$
$(0,0,2)\frac{2\pi}{L}$	Dic_4	A_2	$0^-,1^+,2^-$	$0,1,2$	$D(1)D^*(1)$

	m_D [MeV]	m_{D^*} [MeV]	M_{av} [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$r_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)	1.04(29)	$0.96(^{+0.18}_{-0.20})$	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)	0.86(0.22)	$0.92(^{+0.17}_{-0.19})$	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp. [2, 37]	1864.85(5)	2010.26(5)	3068.6(1)	-7.15(51)	[-11.9(16.9),0]	$-0.36(4)$	bound st.



Previous lattice QCD study of T_{cc} channel

Junnarkar, Mathur, Padmanath, PRD 99, 034507 (2019), 1810.12285



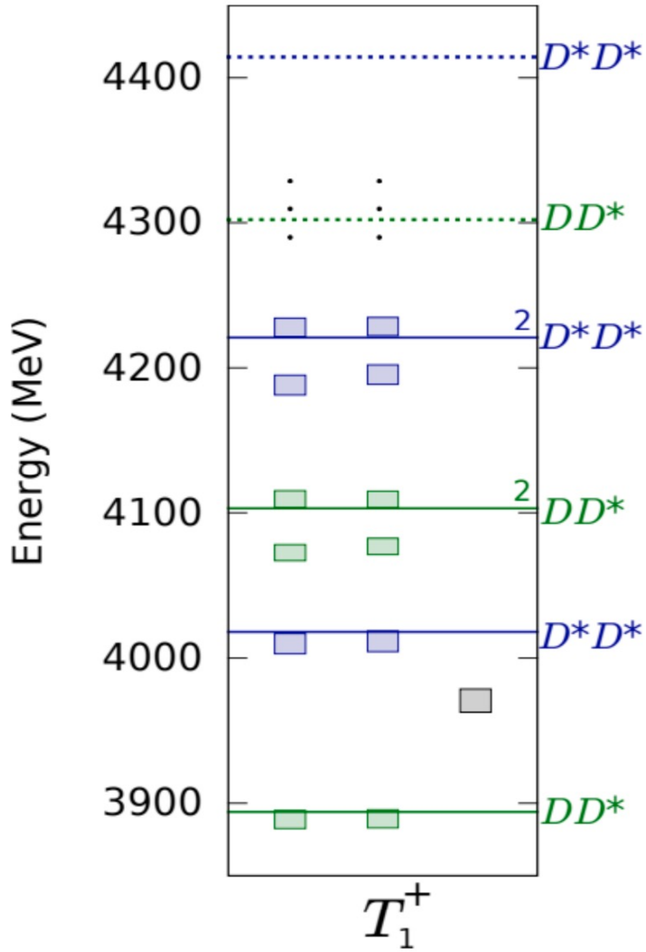
lowest finite-volume eigen-energy for P=0, $J^P=1^+$, I=0

- Study performed on LQCD ensembles with different lattice spacings. Single volume and only rest frame finite-volume irreps considered.
- Including a meson-meson and diquark-antidiquark interpolator. Diquark-antidiquark interpolators do not influence the low energy spectrum.
- The ground state energy subjected to chiral and continuum extrapolations.
- A finite-volume energy level 23(11) MeV below DD^* threshold. No rigorous scattering analysis and no pole structure determined.

Previous lattice QCD study of T_{cc} channel

Hadron Spectrum, JHEP 11, 033 (2017), 1709.01417

finite-volume eigen-energies for P=0, $J^P=1^+$, I=0



- Single volume rest frame study on a relatively coarse lattice ($a_s \sim 0.12$ fm).
- Large basis of meson-meson and diquark-antidiquark interpolators.
- Diquark-antidiquark interpolators do not influence the low energy spectrum.
- No statistically significant energy shifts observed near DD^* threshold. $\Rightarrow$ No scattering amplitude extraction.

HALQCD study of Tcc

Lyu, Aoki et al, 2302.04505

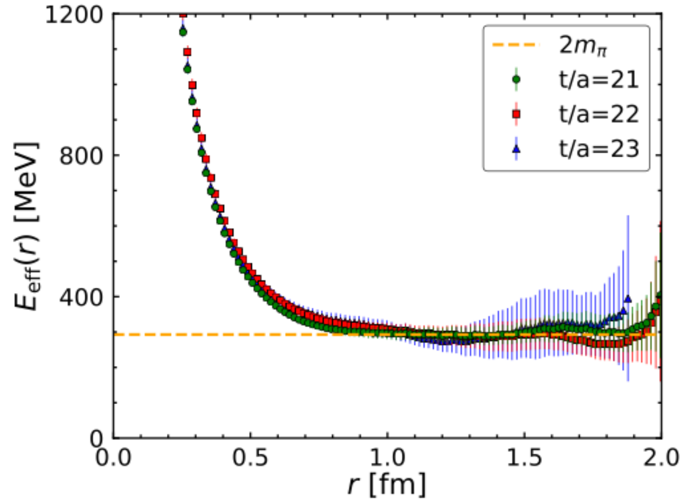
$$R(\boldsymbol{r},t) = \sum_{\boldsymbol{x}} \langle 0|D^*(\boldsymbol{x}+\boldsymbol{r},t)D(\boldsymbol{x},t)\overline{\mathcal{J}}(0)|0\rangle \,/e^{-(m_{D^*}+m_D)t}$$

$$\left[\frac{1+3\delta^2}{8\mu}\partial_t^2 - \partial_t - H_0 + O(\delta^2\partial_t^3)\right] R(\boldsymbol{r},t) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t).$$

$$V(r) = R^{-1}(\boldsymbol{r},t)\left[\frac{1+3\delta^2}{8\mu}\partial_t^2 - \partial_t - H_0\right] R(\boldsymbol{r},t).$$

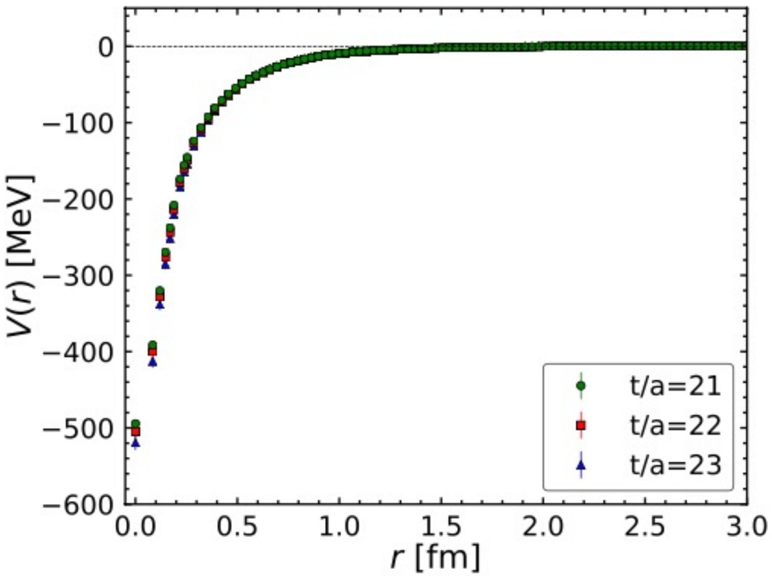
$$V(r) \sim \frac{e^{-2m_\pi r}}{r^2} \quad r > 1\text{ fm}$$

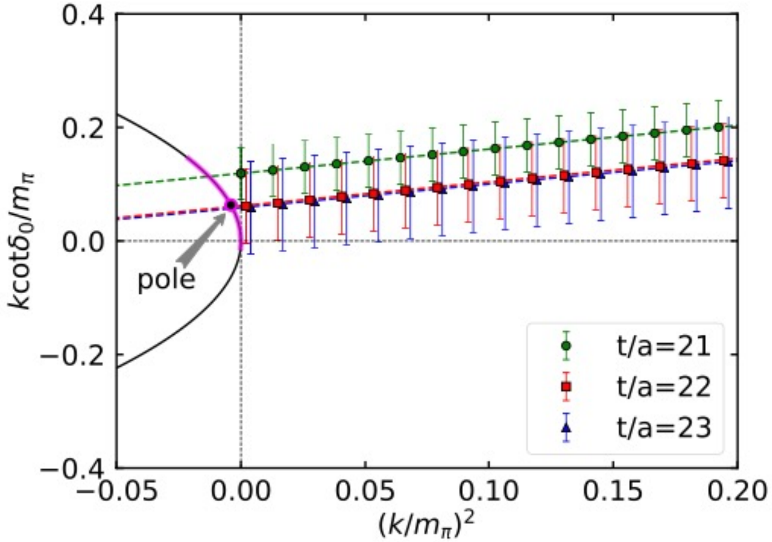
$$E_{\text{eff}}(r) = -\frac{\ln[V(r)r^2/a_3]}{r}$$



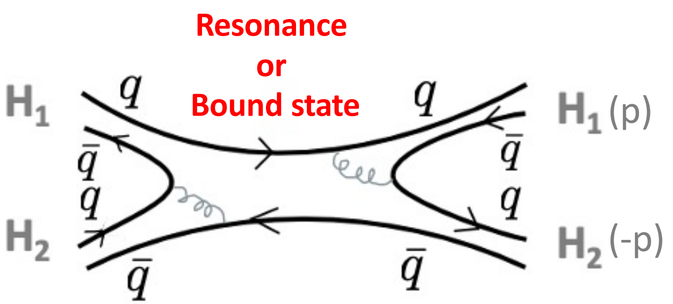
$$V_{\text{fit}}^B(r;m_\pi) = \sum a_i e^{-(r/b_i)^2} + a_3(1-e^{-(r/b_3)^2})^n V_\pi^n$$

parameter set, $(a_1, a_2) = (-284(36), -201(60))$ in MeV, $a_3 = -45(12)$ MeV $\cdot$ fm^2, and $(b_1, b_2, b_3) = (0.15(2), 0.32(12), 0.49(24))$ in fm. Also, we find that

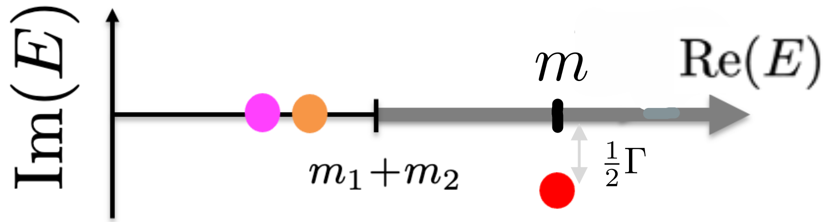




Extract resonances and (virtual) bound states from H_1 H_2 scattering



scattering matrix T(E)

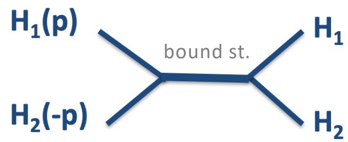


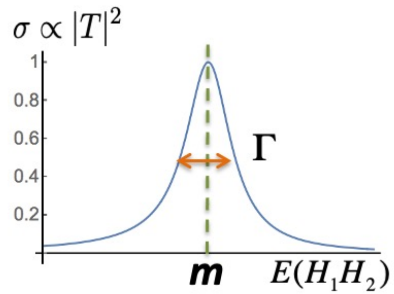
Virtual bound st. | Bound st. | Resonance

p = - i |p| | p = i |p|

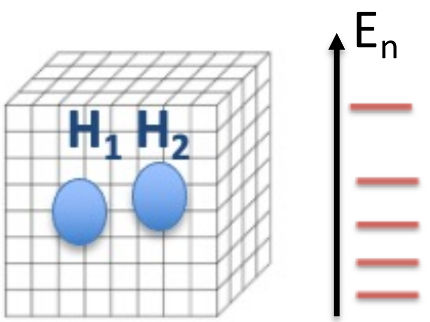
$$T(E) \propto \frac{1}{E^2 - m^2}$$

$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$





Scattering matrix T(E) from lattice QCD



periodic b.c.

real E | for real E | for complex E

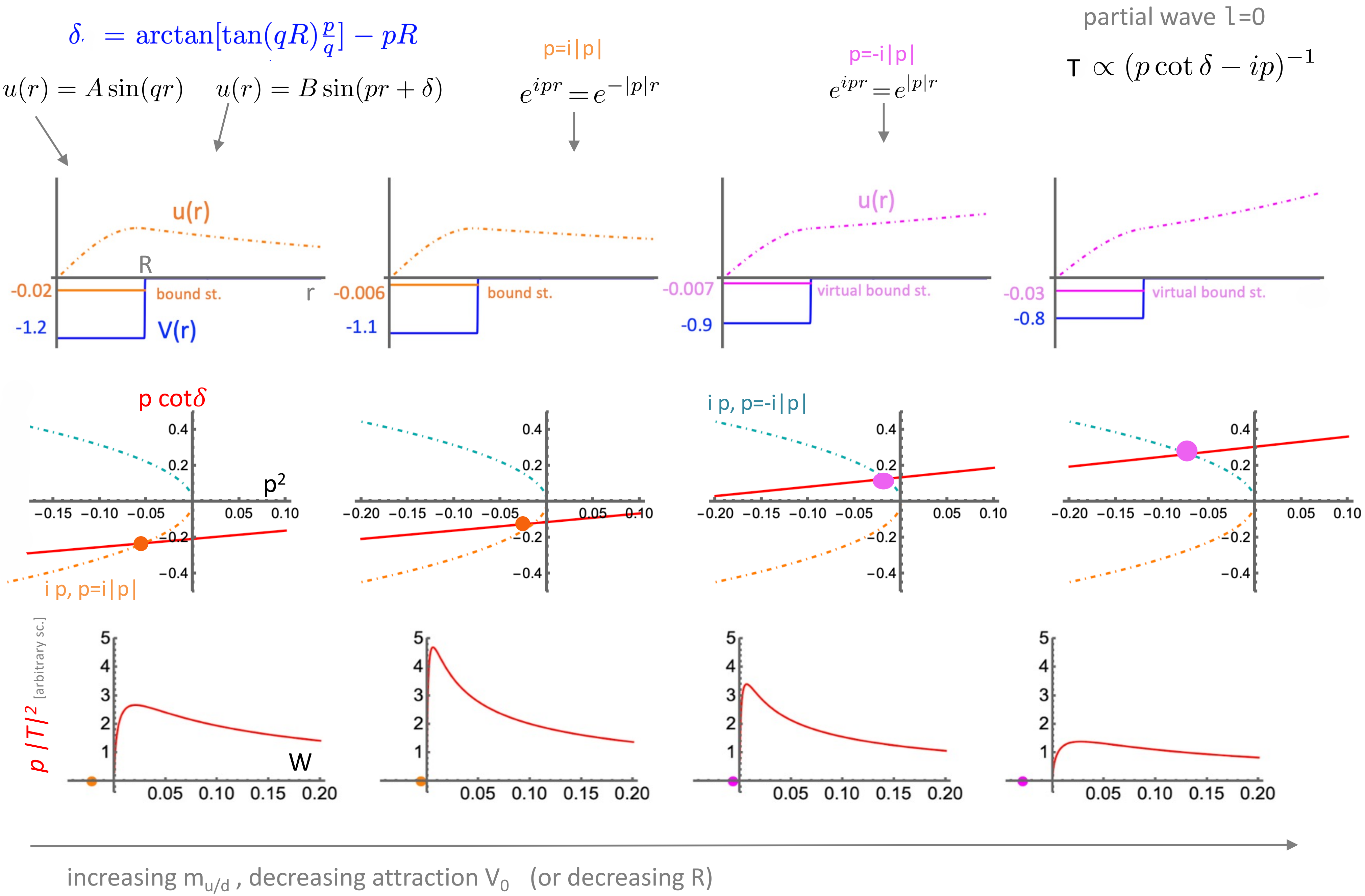
$$E \rightarrow T(E) \rightarrow T(E^c)$$

analytic relation:
Luscher 1991

analytic contin.
to complex E

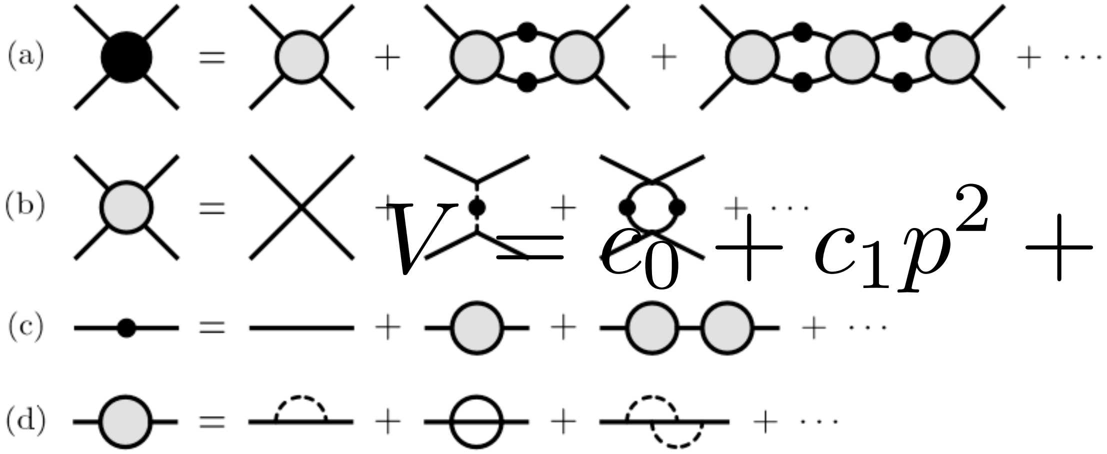
simple argument: next slide

Simplest Example: scattering in square-well potential in QM



Generalization of Luscher's relation on left-hand cut

2301.03981, Raposo& Hansen @ lat22 (ongoing...)



$$V = c_0 + c_1 p^2 + V_{OPE}$$

Du et al. 2303.09441 @ talk by Christof Hanhart

$$V = c_0 + c_2(k^2 + k'^2) + V_{OPE}$$