Freeze-out of fluctuations in heavy-ion collisions

arXiv 2211.09142 with Misha Stephanov

Chirality and Criticality: Novel Phenomena in Heavy-Ion Collisions

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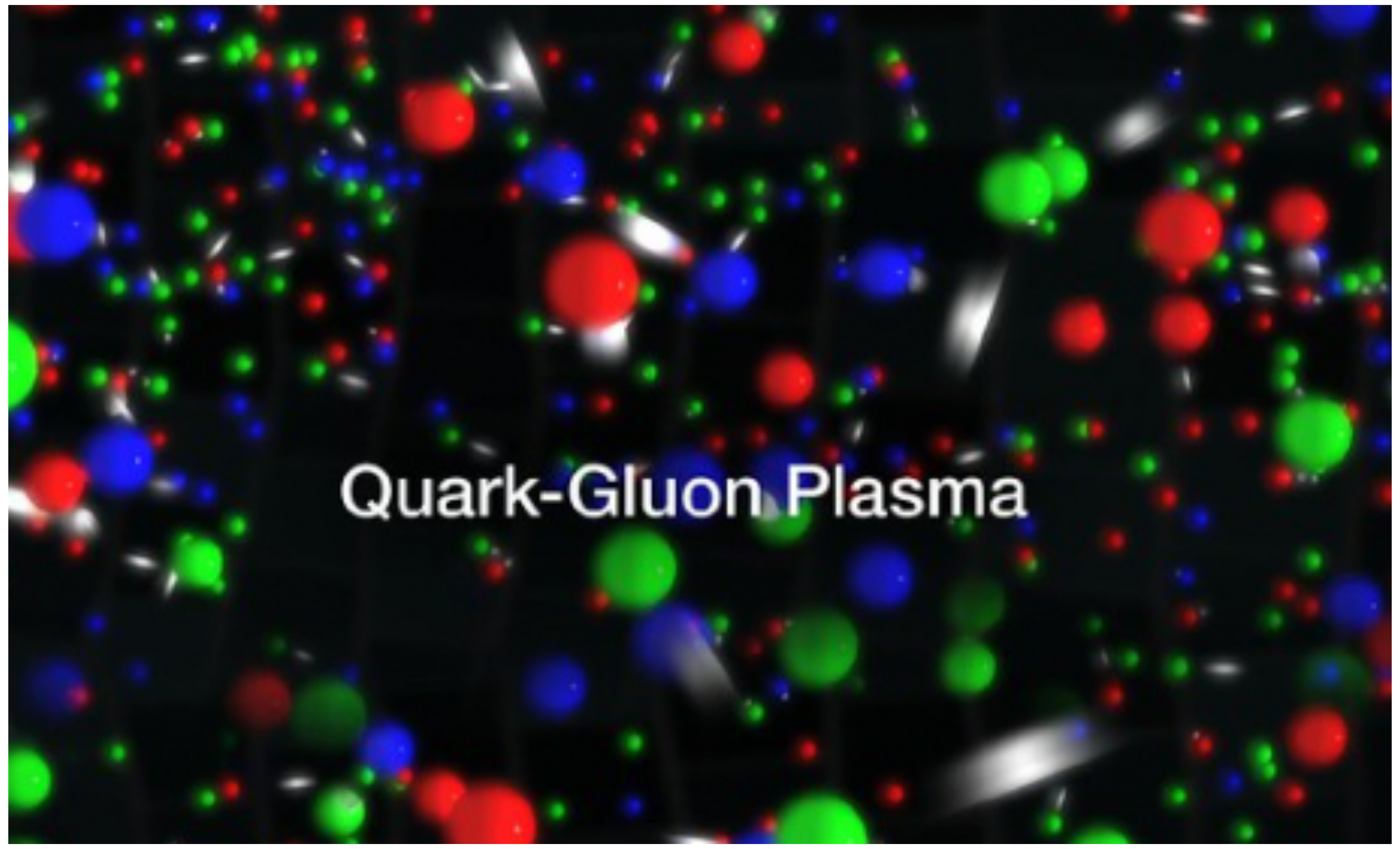
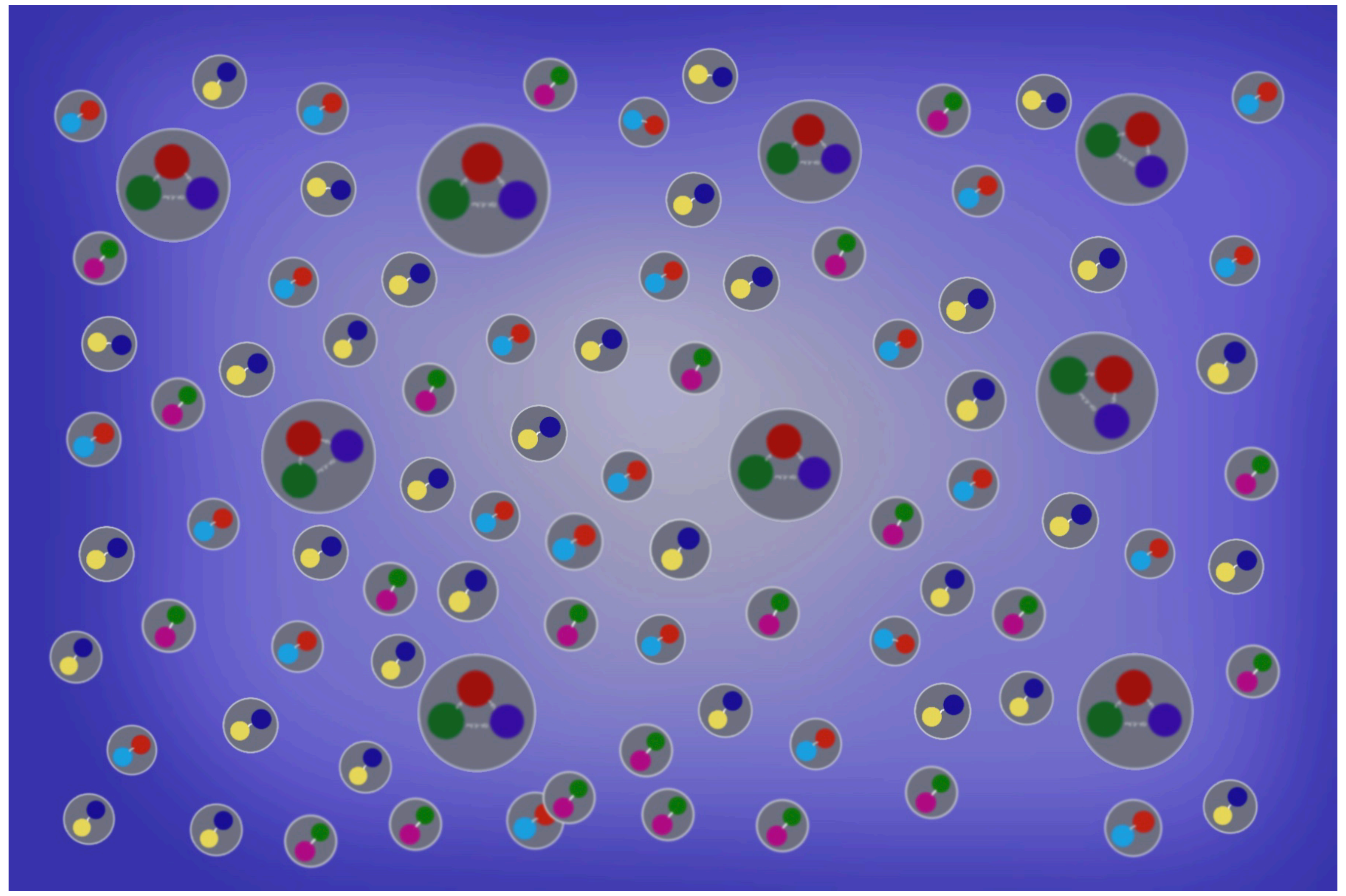
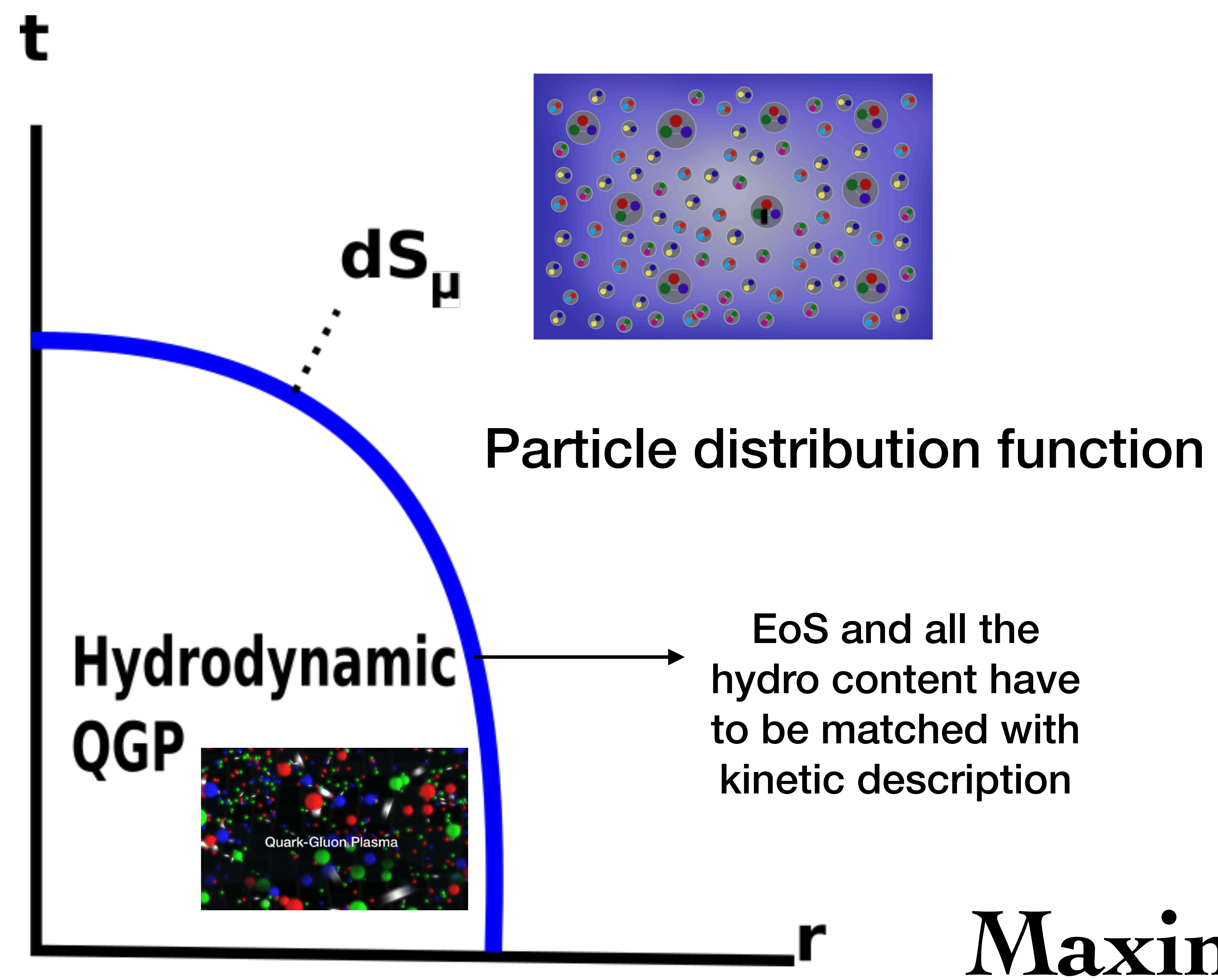
Freeze-out in heavy ion collisions





- QGP thermalizes in a relatively short amount of time ~ 1fm/c ~ 10^{-23} s **Hydrodynamics**
- Becomes dilute and hydrodynamics becomes invalid at about 10 fm/s **Kinetic Theory**
- Freeze-out into a gas of hadrons, which free-stream into the detectors.

Freeze-out in heavy ion collisions



- Infinitely many sets of particle distribution functions which match with the a given hydrodynamic description

- What is the most probable freeze-out scenario which matches with all hydrodynamic content?

Maximum entropy freeze-out

Probability that f is realized in one event

$$S = -\int P[f] \log P[f]$$

Overview

- Maximum entropy freeze-out
- ME Freeze-out of fluctuations - Crucial for CP search via HICs
- Determining unknown phenomenological constants from EoS
- Variance of proton multiplicities near CP

Formalism

Applications

Matching ideal hydrodynamics to kinetic description

Hydrodynamic mean densities

$$\{\langle \epsilon u^\mu \rangle \,,\, \langle n \rangle\} \equiv \Psi^a$$

$$\Psi^a = \sum_A \int_{p_A} \bar{f}_A P_A^a$$

Mean particle distribution function at freeze-out

$$\langle f_A \rangle = \bar{f}_A$$

$$\langle \epsilon\, u^\mu \rangle = \sum_A \int_{p_A} \bar{f}_A\, p_A^\mu \,, \quad \langle n \rangle = \sum_A q_A \int_{p_A} \bar{f}_A$$

$$P_A = \begin{bmatrix} p_A^\mu \\ q_A \end{bmatrix}$$

More degrees of freedom on the kinetic side
Infinitely many solutions for these matching conditions

Maximum entropy "freeze-out" of ideal hydrodynamics

Maximizing thermodynamic entropy subject to matching conditions
~ well known Cooper-Frye freeze-out

$$\bar{f}_A \sim e^{-(u^\mu p_\mu - q_A \mu_B)/T}$$ (Classical statistics)

Hydrodynamic system freezes out into ideal hadron-resonance gas with the same values of conserved densities

Extension to freeze-out of viscous hydrodynamics - Everett, Chattopadhyay, Heinz, 21

Matching conditions for correlation functions

Hydrodynamic correlations

$$\Psi^a,\ \left\langle \delta\Psi^a \delta\Psi^b \right\rangle \equiv H^{ab}, \dots\ H^{abc\dots}$$

Particle distribution function at freeze-out

$$\langle f_A \rangle = \bar{f}_A\,,\ \langle \delta f_A \delta f_B \rangle = G_{AB}\,,\ \langle \delta f_A \delta f_B \delta f_C \rangle = G_{ABC}\ \dots$$

$$H^{abc\dots} = \sum_{A,B,C,\dots} \int_{p_A p_B p_C \dots} G_{ABC\dots} P_A^a P_B^b P_C^c \dots$$

$$P_A = \begin{bmatrix} p_A^\mu \\ q_A \end{bmatrix}$$

More degrees of freedom on the kinetic side
Infinitely many solutions for these equations

Maximum entropy approach to freeze-out fluctuations

MP, Stephanov, 22

- We *maximize the entropy* associated with the fluctuations of the particle distribution function f, subject to the *constraints of matching conditions.*

Probability distribution functional of f

$$S[\bar{f}, G_2, G_3, G_4, \dots] = -\int P[f; \bar{f}, G_2, \dots] \log P[f; \bar{f}, G_2, \dots]$$

Similar to n-PI action in QFT

Berges, 04, Stephanov, Yin, 17…

Depends on $\bar{f}, G_2, G_3, \dots$

Entropy associated with fluctuations out of equilibrium

Let P_{eq} be the probability distribution function to describe the fluctuations in equilibrium

When two point correlations are out of equilibrium:

$$P[f] = P_{\text{eq}}[f]\, e^{K_{AB} f_A f_B}$$

Compute G_{AB} using P[f] to obtain Gs in terms of Ks

$$G^{-1}_{AB} = -\frac{\delta^2 S_0}{\delta f_A \delta f_B} - K_{AB}$$

Now , we have $P[f; \bar{f}, G_2]$

Maximum entropy freeze-out of two point fluctuations

Entropy to describe off-equilibrium two-point correlations : $S[\bar{f}, G_2] = -\int_f P[f; \bar{f}, G_2] \log P[f; \bar{f}, G_2]$

$$S_2 = S + \frac{1}{2}\mathrm{Tr}\left[\log G\bar{G}^{-1} - G\bar{G}^{-1} + 1\right] \, ,$$

2-PI entropy

Berges, 04, Stephanov, Yin, 17…

Matching conditions :

$$G_{AB} = \bar{G}_{AB} + \Delta G_{AB}$$

$$H^{ab} = \sum_{A,B} \int_{p_A, p_B} G_{AB} P^a_A P^b_B = \bar{H}^{ab} + \Delta H^{ab}$$

Out of equilibrium correlations in HRG

Out of equilibrium correlations in the fluid

Maximize entropy : $G^{-1}_{AB} = \bar{G}^{-1}_{AB} - (\bar{H}^{-1} - H^{-1})_{ab} P^a_A P^b_B$

Entropy associated with fluctuations out of equilibrium

When higher point correlations are out of equilibrium:

$$P[f] \,=\, P_{\rm eq}[f]\, e^{K_{AB} f_A f_B + K_{ABC} f_A f_B f_C + \dots}$$

Re-express Ks in terms of Gs

$$S[\bar{f}, G_2, G_3, G_4, \dots] = -\int P[f; \bar{f}, G_2, \dots] \log P[f; \bar{f}, G_2, \dots]$$

Freeze-out of higher point fluctuations

In the hydrodynamic limit, when the Knudsen number is small :

General freeze-out prescription (linearized)

$$P_A = \begin{bmatrix} p^\mu_A \\ q_A \end{bmatrix}$$

$$\widehat{\Delta} G_{AB\ldots} = \widehat{\Delta} H_{ab\ldots} (\bar{H}^{-1} P \bar{G})^a_A (\bar{H}^{-1} P \bar{G})^b_B \ldots,$$

Irreducible relative cumulants (IRCs)

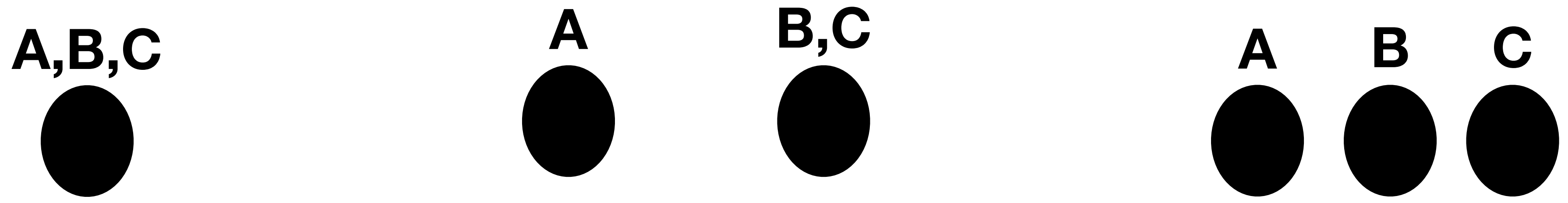
Polynomial in P_{A} expressible in terms of quantities known from EoS

For general non-linear freeze-out prescription, refer MP, Stephanov, 22

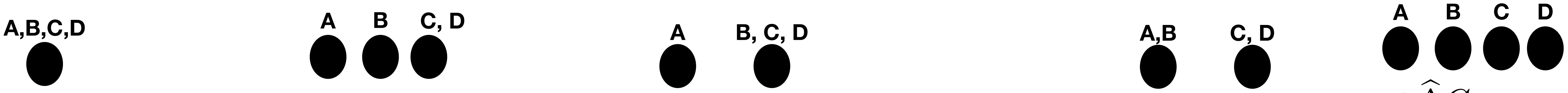
Correlations from maximum entropy freeze-out

$$G_{AB} = \bar{G}_{AB} + \widehat{\Delta} G_{AB}$$

Systematically subtracts terms containing self (equilibrium) correlations



$$G_{ABC} = \bar{G}_{ABC} + 3\widehat{\Delta} G_{AD} \left(\bar{G}_2^{-1} \bar{G}_3\right)_{DBC} + \widehat{\Delta} G_{ABC}$$



$$G_{ABCD} = \bar{G}_{ABCD} + 6\widehat{\Delta} G_{ABF} \left(\bar{G}_2^{-1} \bar{G}_3\right)_{FCD} + 4\widehat{\Delta} G_{AF} \left(\bar{G}_2^{-1} \bar{G}_4\right)_{FBCD} + 3\widehat{\Delta} G_{EF} \left(\bar{G}_2^{-1} \bar{G}_3\right)_{FCD} \left(\bar{G}_2^{-1} \bar{G}_3\right)_{EAB} + \widehat{\Delta} G_{ABCD}$$

For classical gas, phase space integrals of IRCs reduce to factorial cumulants.

Summary - Maximum Entropy Freeze-out

- Allows a natural generalization to freeze-out 'n' point correlation functions of all hydrodynamic variables

- At freeze-out, the out-of equilibrium correlations will serve as initial correlations in the gas which are out-of equilibrium

- Consistent with the commonly employed picture of an ideal HRG with interactions

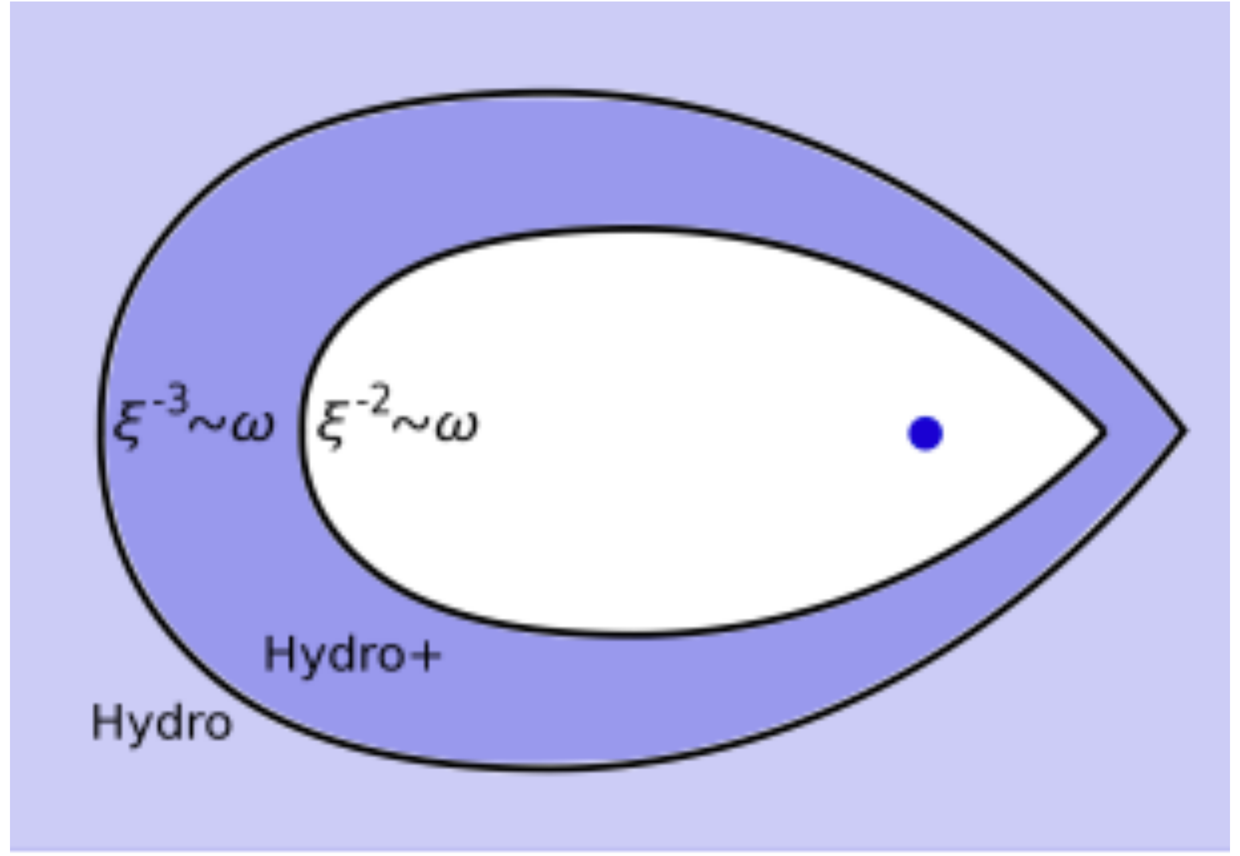
Formally, we have determined the freeze-out prescription. Applications…

Application : Freeze-out near the critical point

- Near the CP : Critical slowing down -> Relaxation to equilibrium is infinitely slow.

- The fluctuations of $\hat{s} \equiv s/n$ which relaxes parametrically as $\Gamma \sim \xi^{-3}$ is the slowest non-hydrodynamic mode

- Focus on a regime where only correlations of $\hat{s}$ are out of equilibrium - Hydro+



Application to Hydro+

Applying *maximum-entropy freeze-out* to a *Hydro+* simulation

where there is only one mode which is singular and out of equilibrium:

$$\Delta G_{AB} = \left(\frac{n_c}{\bar{c}_p T_c}\right)^2 \left[E_A - \frac{w_c}{n_c} q_A\right] \left[E_B - \frac{w_c}{n_c} q_B\right] f_A f_B \Delta \langle \delta\hat{s}\delta\hat{s}\rangle$$

$$\Delta G_{AB} = \widehat{\Delta} H_{ab} (\bar{H}^{-1} P \bar{G})^a_A (\bar{H}^{-1} P \bar{G})^b_B$$

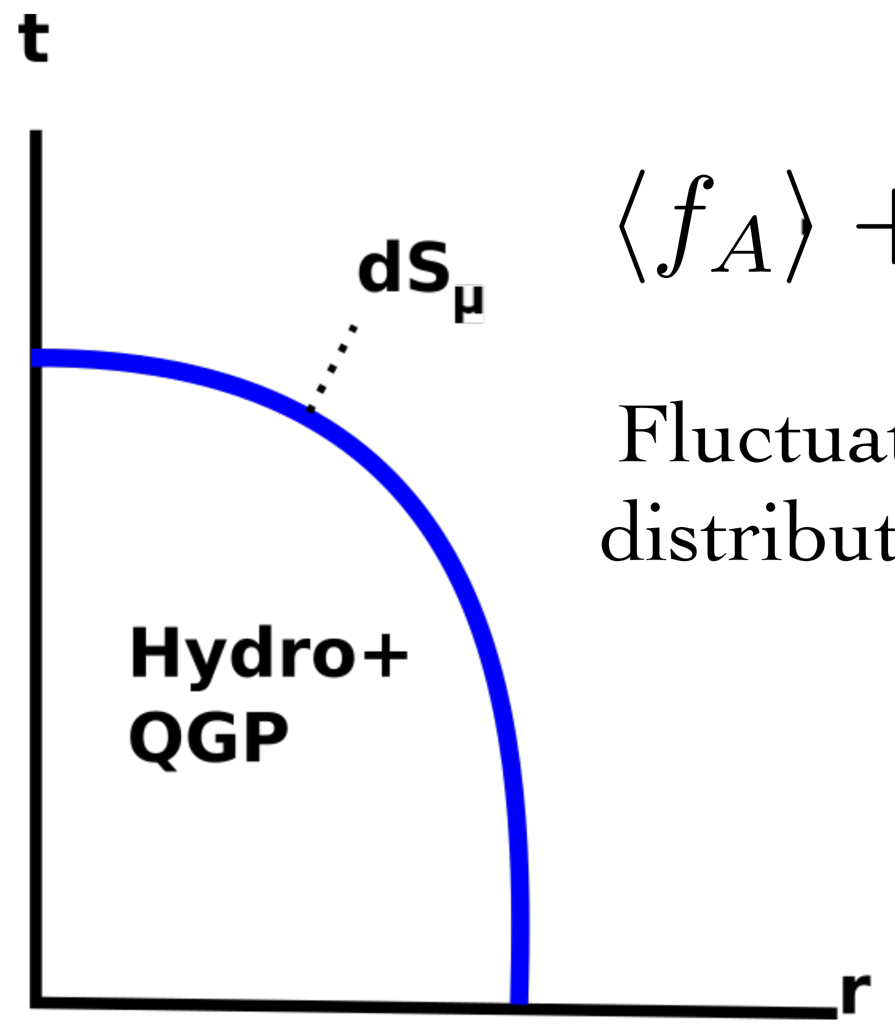
$$\widehat{\Delta} H_{\hat{s}\hat{s}} = \Delta \langle \delta\hat{s}\delta\hat{s}\rangle \ , \ \widehat{\Delta} H_{pp} = \widehat{\Delta} H_{p\hat{s}} = \widehat{\Delta} H_{pu^\mu} = \widehat{\Delta} H_{\hat{s}u^\mu} = \widehat{\Delta} H_{u^\nu u^\mu} = 0$$

n_c & w_c Baryon density and enthalpy at the critical point

$\bar{c}_p$ Specific heat of HRG in equilibrium

Now, we compare this to a previously used freeze-out prescription for critical fluctuations

Freeze-out prescription based on EFT near critical point



$$\langle f_A \rangle + \delta f_A$$

Fluctuating particle distribution function

We incorporate the effects of critical fluctuations via the modification of particle masses due to their interaction with the critical sigma field

$$\delta m_A \approx g_A \sigma$$ **Stephanov, Rajagopal, Shuryak, 1999**

Fluctuating particle distribution function

$$f_A = \langle f_A \rangle + g_A \frac{\partial \langle f_A \rangle}{\partial m_A} \sigma$$

$$\langle \sigma \rangle = 0, \; \langle \sigma(x_+) \sigma(x_-) \rangle = Z^{-1} \langle \delta \hat{s}(x_+) \delta \hat{s}(x_-) \rangle$$

MP, Rajagopal, Stephanov, Yin, 22

Freeze-out of Gaussian fluctuations near the critical point

Unknowns!

$$\Delta G_{AB} \equiv \langle \delta f_A \delta f_B \rangle = \frac{g_A g_B}{Z T^2} \frac{m_A}{E_A} \frac{m_B}{E_B} f_A f_B \langle \delta \hat{s} \delta \hat{s} \rangle$$

$$\Delta \langle \delta N_A \delta N_B \rangle_\sigma = d_A \, d_B \int Dp_A \int Dp_B \int (dS \cdot p_A) \int (dS \cdot p_B) \Delta G_{AB}$$

Deviations from baseline (critical+dynamical effects)

$$\langle \delta N_A \delta N_B \rangle = \langle N_A \rangle \, \delta_{AB} + \Delta \langle \delta N_A \delta N_B \rangle_\sigma$$

Poisson (or more generally, baseline) contribution

MP, Rajagopal, Stephanov, Yin, 22

Maximum-entropy freeze-out

$$\Delta G_{AB} = \left(\frac{n_c}{\bar{c}_p T_c}\right)^2 \left[E_A - \frac{w_c}{n_c} q_A\right] \left[E_B - \frac{w_c}{n_c} q_B\right] f_A f_B \Delta \langle \delta\hat{s}\delta\hat{s} \rangle$$

Agrees with the prescription obtained using the *EFT with sigma field*:

$$\Delta G_{AB} = \frac{g_A g_B}{Z T^2} \frac{m_A}{E_A} \frac{m_B}{E_B} f_A f_B \Delta \langle \delta\hat{s}\delta\hat{s} \rangle$$

if g_As have a specific energy dependence

Phenomenological implications

Depends on non-critical information from the QCD EoS

Measure of the size of the critical region

$$g_A \equiv \hat{g}_A \, \frac{\sin\alpha_1}{w\sin(\alpha_1-\alpha_2)}$$

BEST EoS parameters

$$\hat{g}_A(E_A) \propto \frac{E_A}{m_A}\left(\frac{E_A}{w_c}-\frac{q_A}{n_c}\right)$$

n_c & w_c Baryon density and enthalpy at the critical point

Estimates using BEST EoS

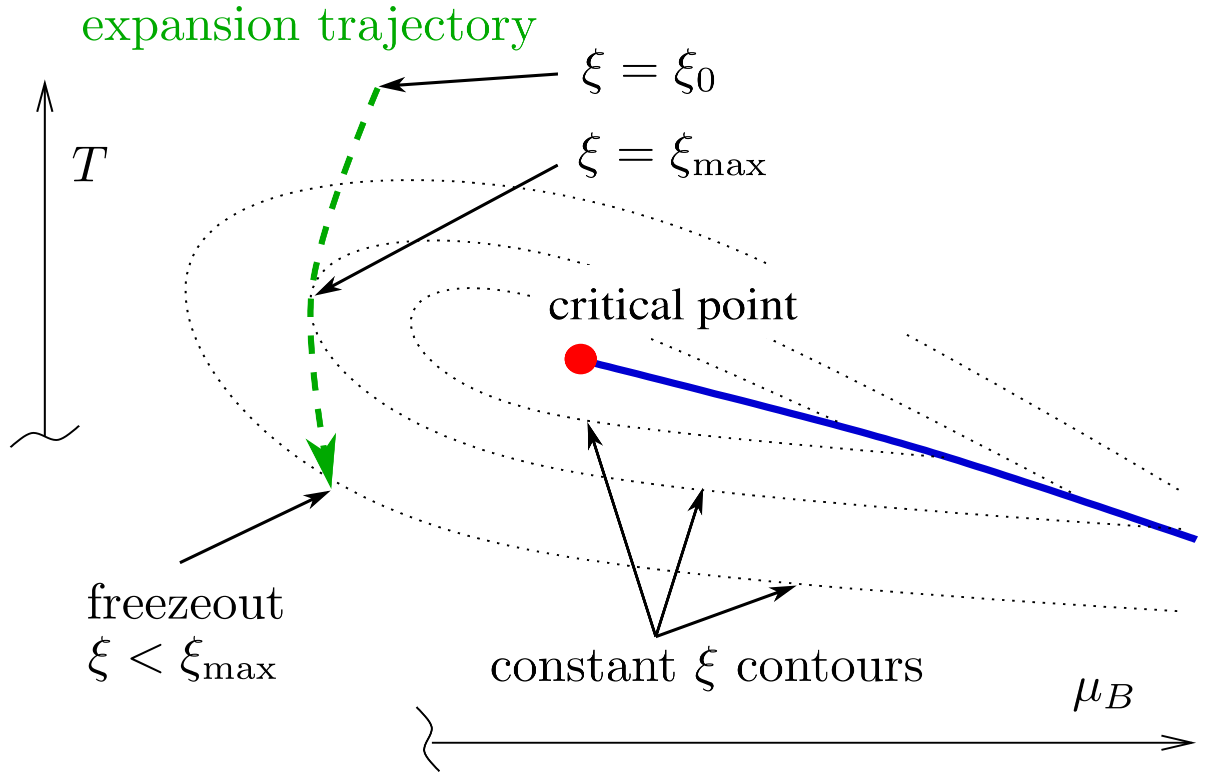
$\mu_c = 350\,\mathrm{MeV}$

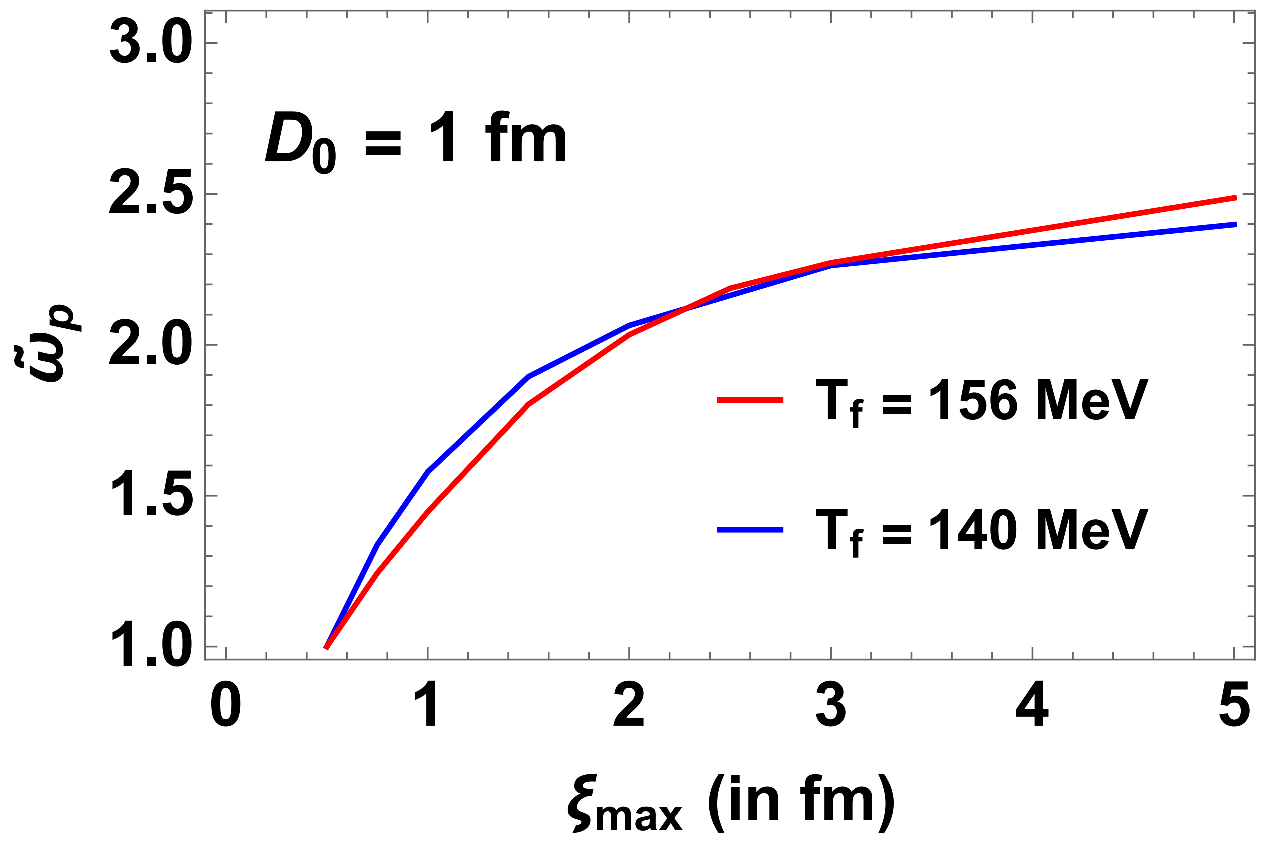
$$\hat{g}_{p,\mathbf{0}} \approx -3.1,\ \hat{g}_{\pi,\mathbf{0}} \approx 0.18,\ \hat{g}_{\bar{p},\mathbf{0}} \approx 5.5$$

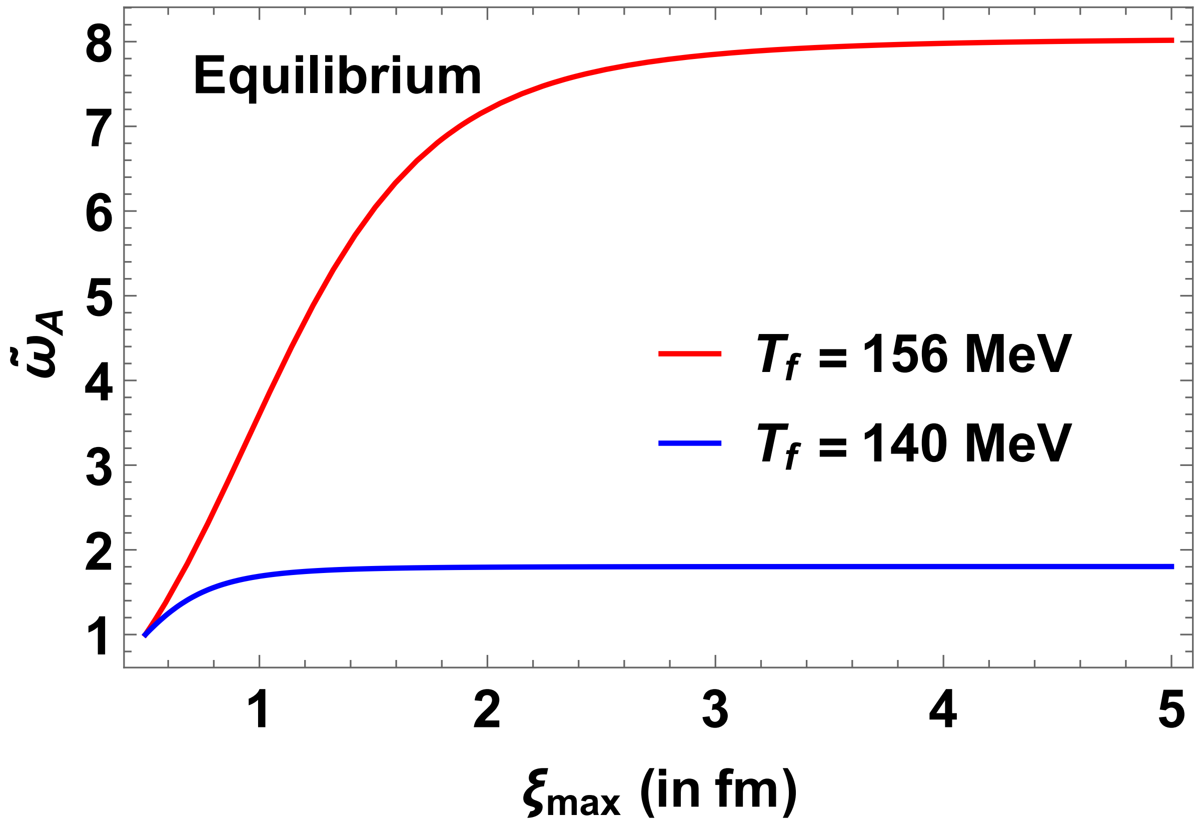
$$q_p = 1,\ q_\pi = 0$$

Mixed correlations of protons and pions can have negative sign

Variance of proton multiplicity near a critical point







$\xi_{\max}$ Proximity of the trajectory to critical point

T_f Proximity of freeze-out point to critical region

MP, Rajagopal, Stephanov, Yin, 22

$T_c = 160$ MeV (T when $\xi = \xi_{max}$)

$$\omega_A(y_{max}) \equiv \frac{\langle \delta N_A^2(y_{max}) \rangle_\sigma}{\langle N_A(y_{max}) \rangle}$$

$$\tilde{\omega}_A \equiv \frac{\omega_A}{\omega_A^{nc}}$$

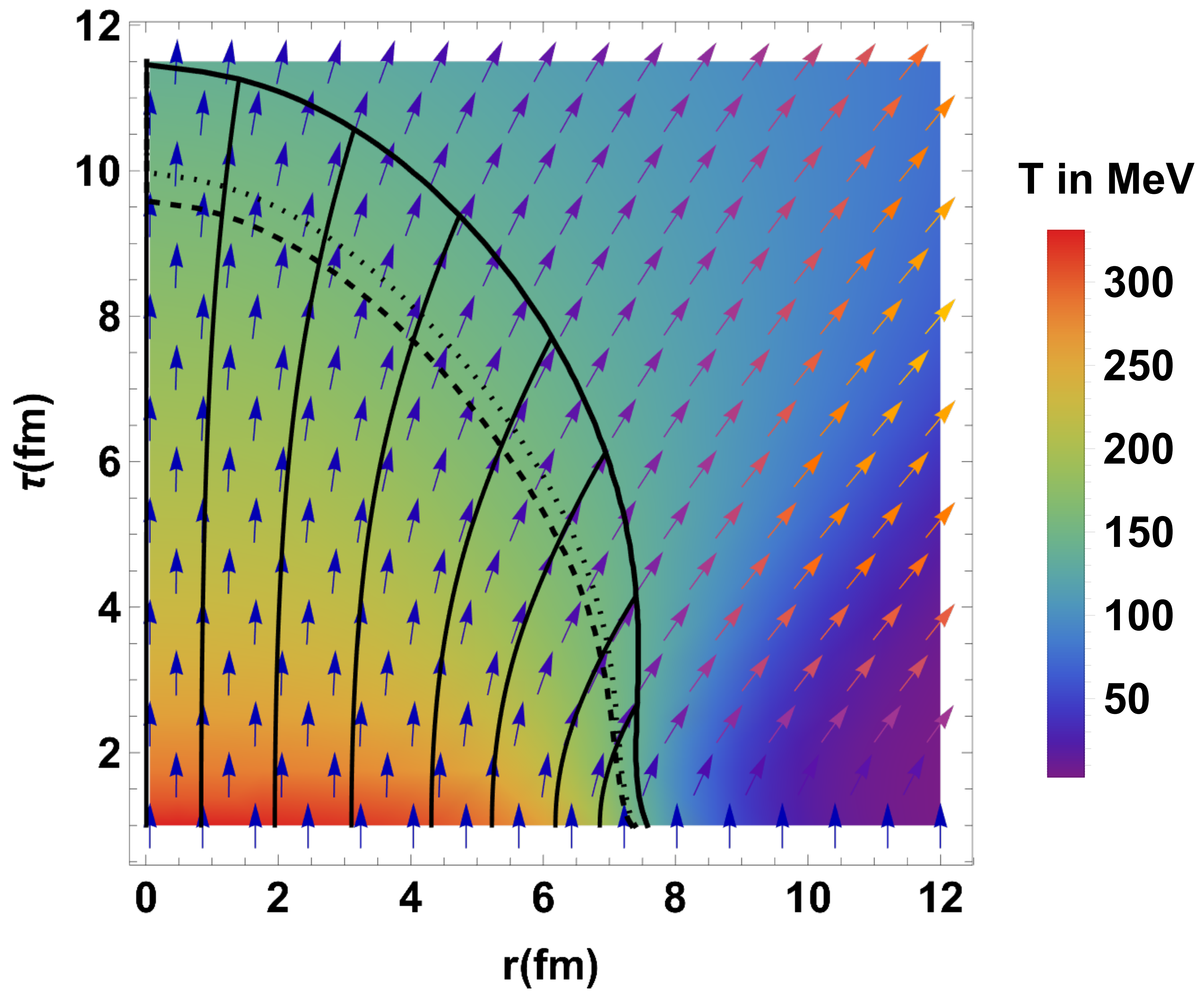
- The fluctuations are **reduced relative to equilibrium** value (conservation laws)
- Compared to the equilibrium scenario, the fluctuations are **less sensitive to freeze-out temperature**

Summary

- Dynamics of fluctuations have important consequences for their magnitude at freeze-out , it also reduces the sensitivity to the freeze-out location
- A general prescription for freeze-out has been recently developed - Maximum entropy approach
- Previously, unknown parameters crucial for the freeze-out of fluctuations near the QCD critical point in terms of the QCD equation of state
- Numerical implementation of freeze-out of higher-point fluctuations needs to be performed..

Thank you!

Dynamics of fluctuations near a critical point



Stephanov, Yin, 17

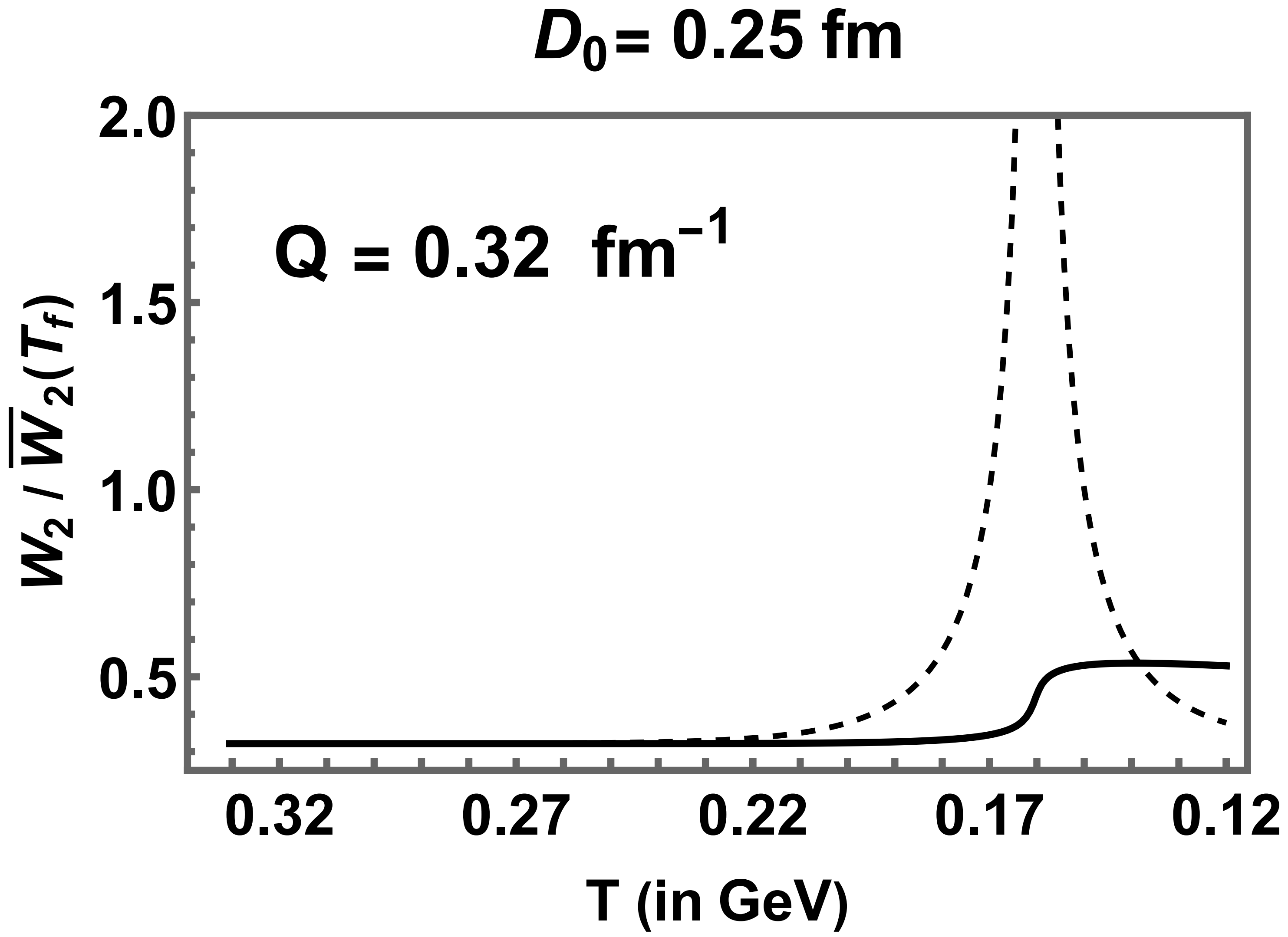
Rajagopal, Ridgway, Weller, Yin, 19

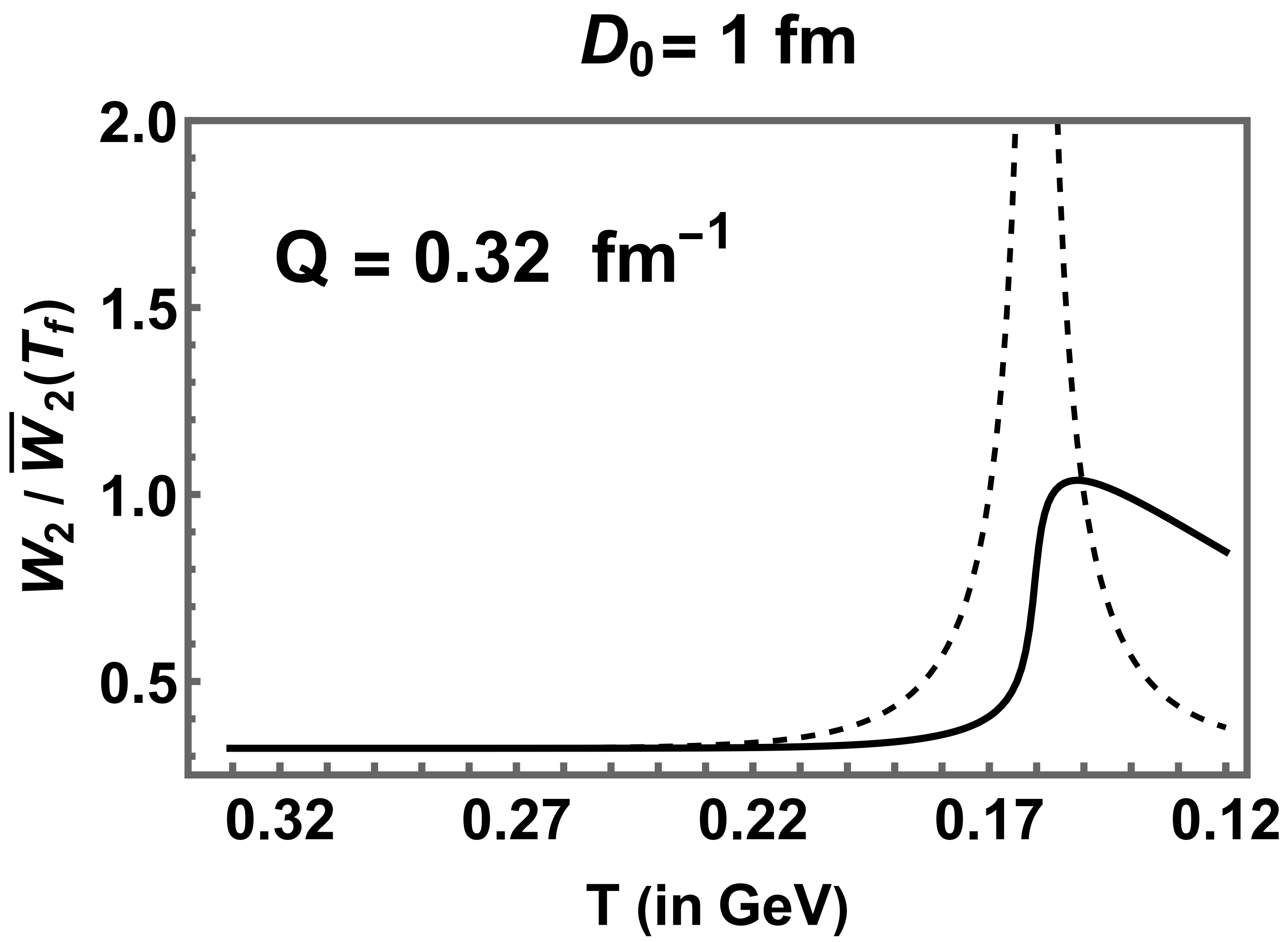
MP, Rajagopal, Stephanov, Yin, 22

$$\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-)\rangle = \int_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\mathbf{\Delta x}} W_2(x,\mathbf{Q})$$

The contribution of low Q modes dominate the particle correlations $Q \leq \tau_f^{-1}\sqrt{m/T}$

$$T_f \;=\; 150\,\mathrm{MeV}$$





$$u\cdot\partial W_2(x,\mathbf{Q}) \;=\; -\Gamma(|\mathbf{Q}|\xi)\left(W_2(x,\mathbf{Q}) - \bar{W}_2(x,\mathbf{Q})\right)$$

$$\Gamma(x) = \frac{D_0\xi_0}{\xi^3}K(x),\; K(x) \sim x^2 \text{ for } x \ll 1$$

Model H

Maximum - entropy freeze-out hasn't been implemented in numerical simulations yet..

However, the EFT based approach has been implemented for freeze-out of Gaussian fluctuations..

We'll use the EFT based freeze-out approach (g_As assumed to be constant) to demonstrate affects of critical slowing down and conservation qualitatively.