Non-monotonic specific entropy along a first-order line

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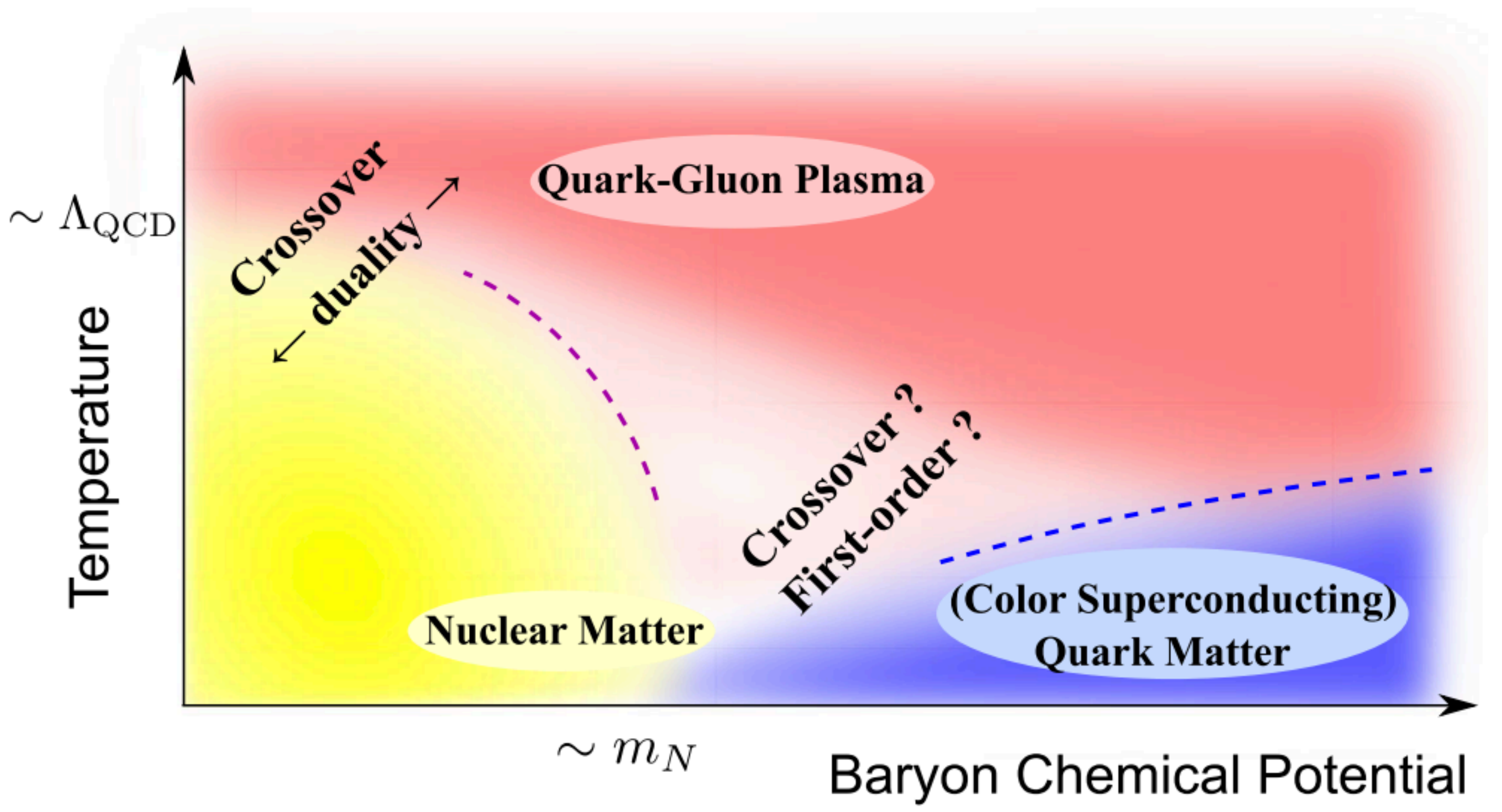
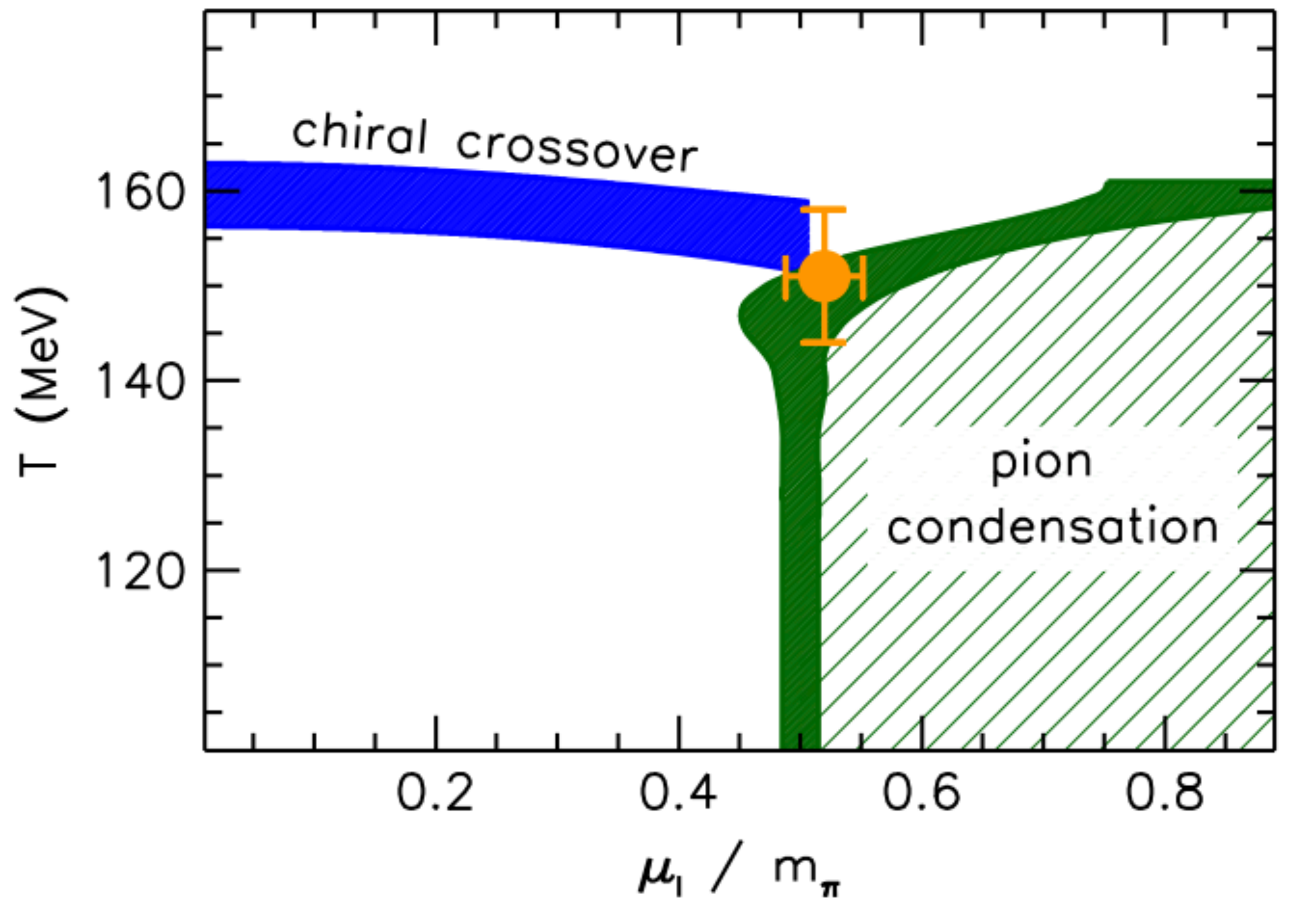
Based on *Phys.Rev.C* 109 (2024) 6, 064905 with Noriyuki Sogabe, Misha Stephanov and Ho-Ung Yee
+ More

INT Workshop on EoS Measurements with Next-Generation Gravitational wave detectors

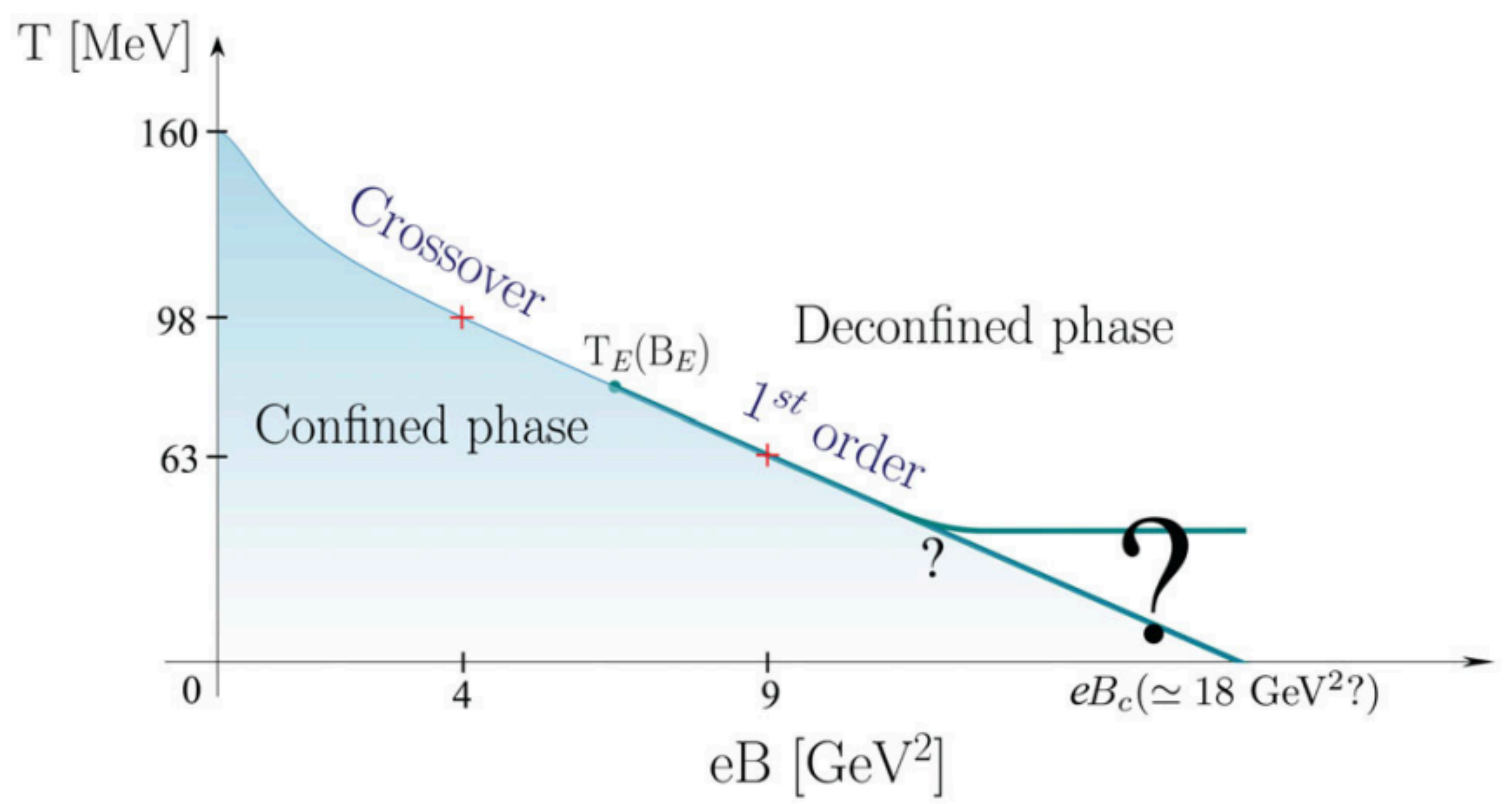
Phase transitions in QCD



Fujimoto, Fukushima, Hotokezaka, and Kyutoku, 23



B. B. Brandt, G. Endrődi, and S. Schmalzbauer,18



Elia, Maio,Sanfillipo,Stanzione, 22

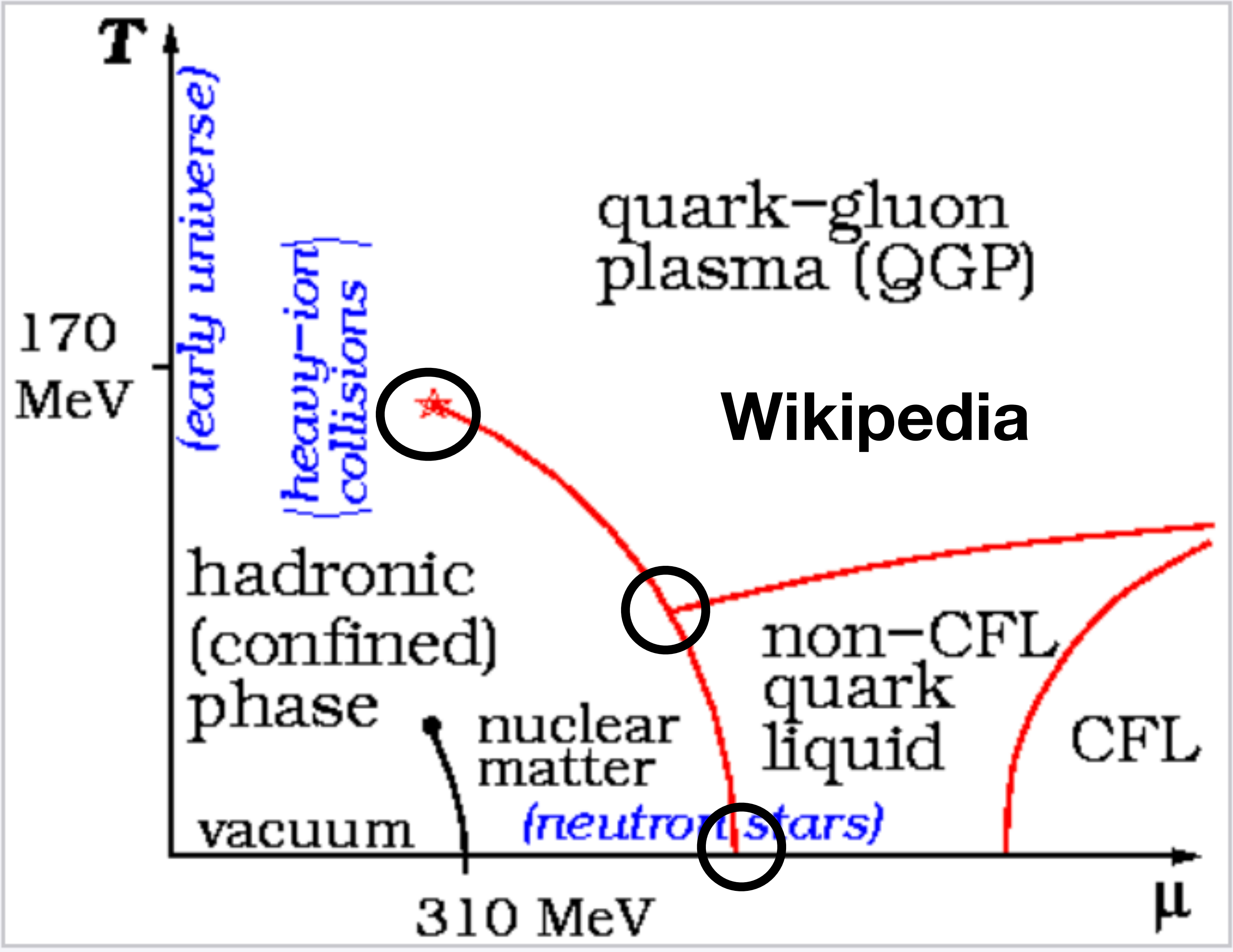
- Various singularities in EoS as the temperature, magnetic field, chemical potentials are varied
- Much of which is a speculation due to sign problem (except a few regimes : finite isospin, zero baryon Cps ,non-zero B etc..)
- Discovering the multi-dimensional phase diagram of QCD : One of the visions of the multi-messenger era

First-order phase transitions in QCD

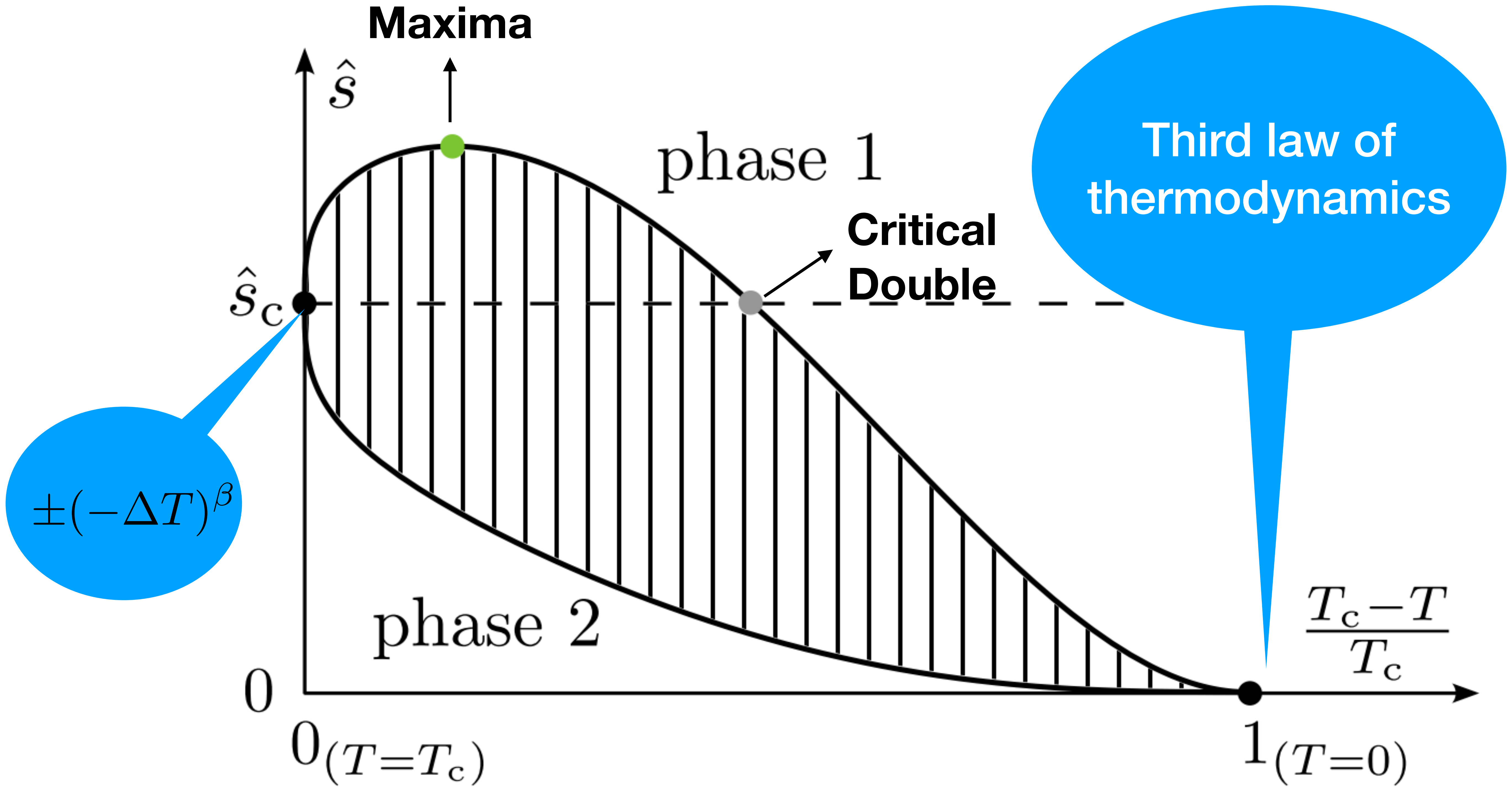
- This talk : Universal features associated with first-order phase transition
- Applicable to any FO phase transition that may be relevant
- How critical point studies related to Beam-Energy Scan program can inform the dynamics of the neutron star mergers?

$\Delta \hat{s} \equiv s/n = 0$ at $T = 0$ and $T = T_c$

Sogabe, MP, Stephanov, Yee, 24



Conjectured form of the phase diagram of QCD matter, with temperature on the vertical axis and quark chemical potential on the horizontal axis, both in mega-electron volts.[15]

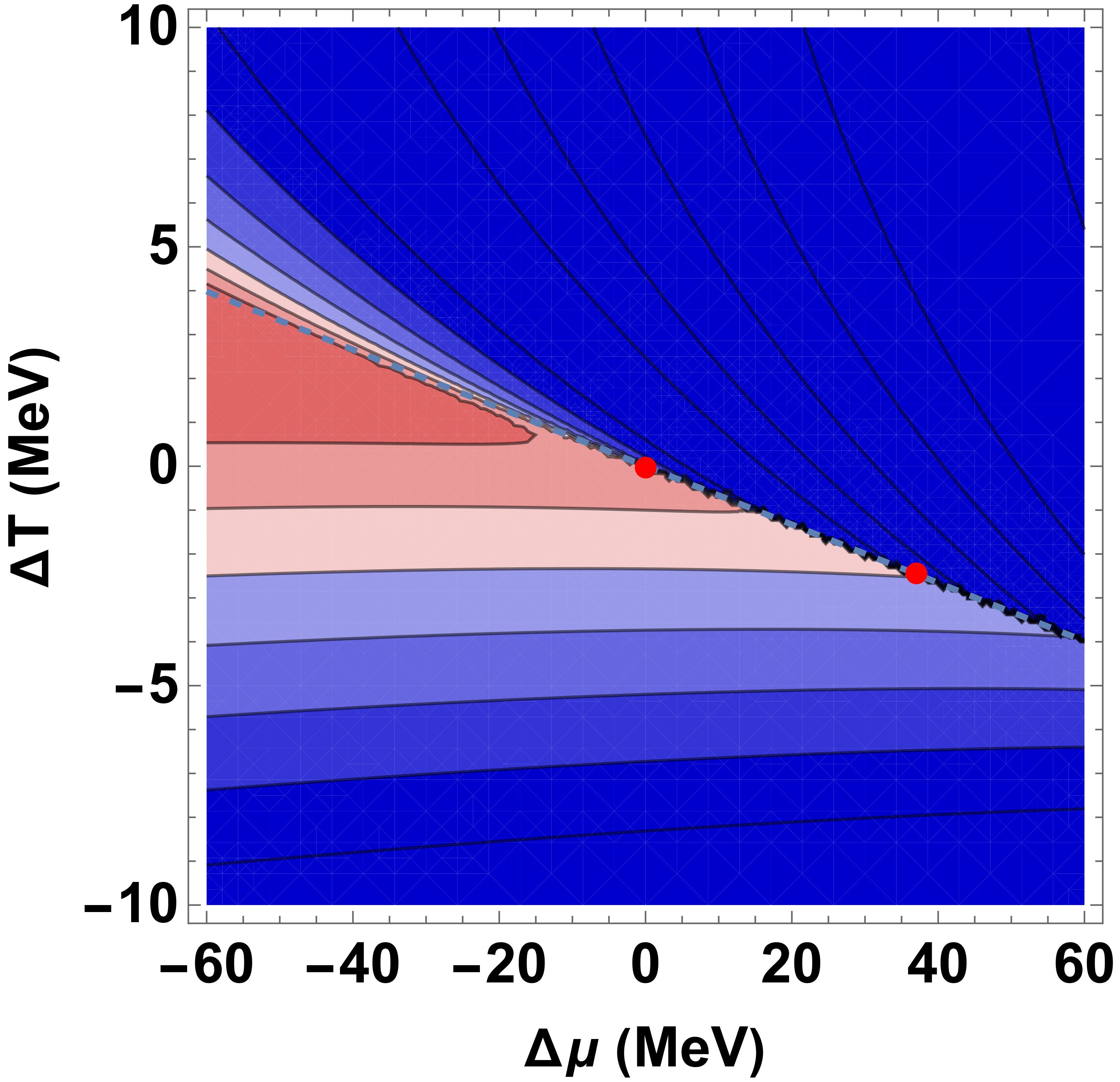


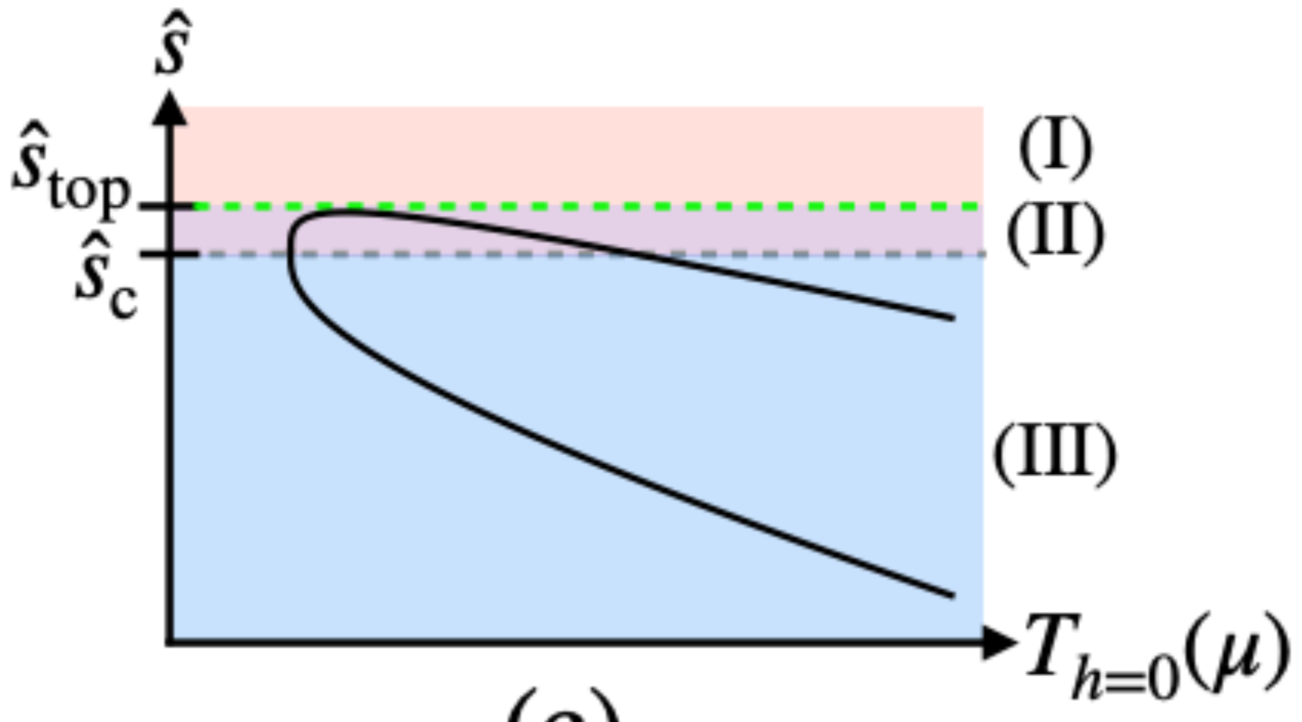
Specific entropy is non-monotonic along at least one of the branches on the first-order line

Implications for hydrodynamic trajectories

To zeroth order, hydrodynamic trajectories are isentropes

Sogabe, MP, Stephanov, Yee, 24



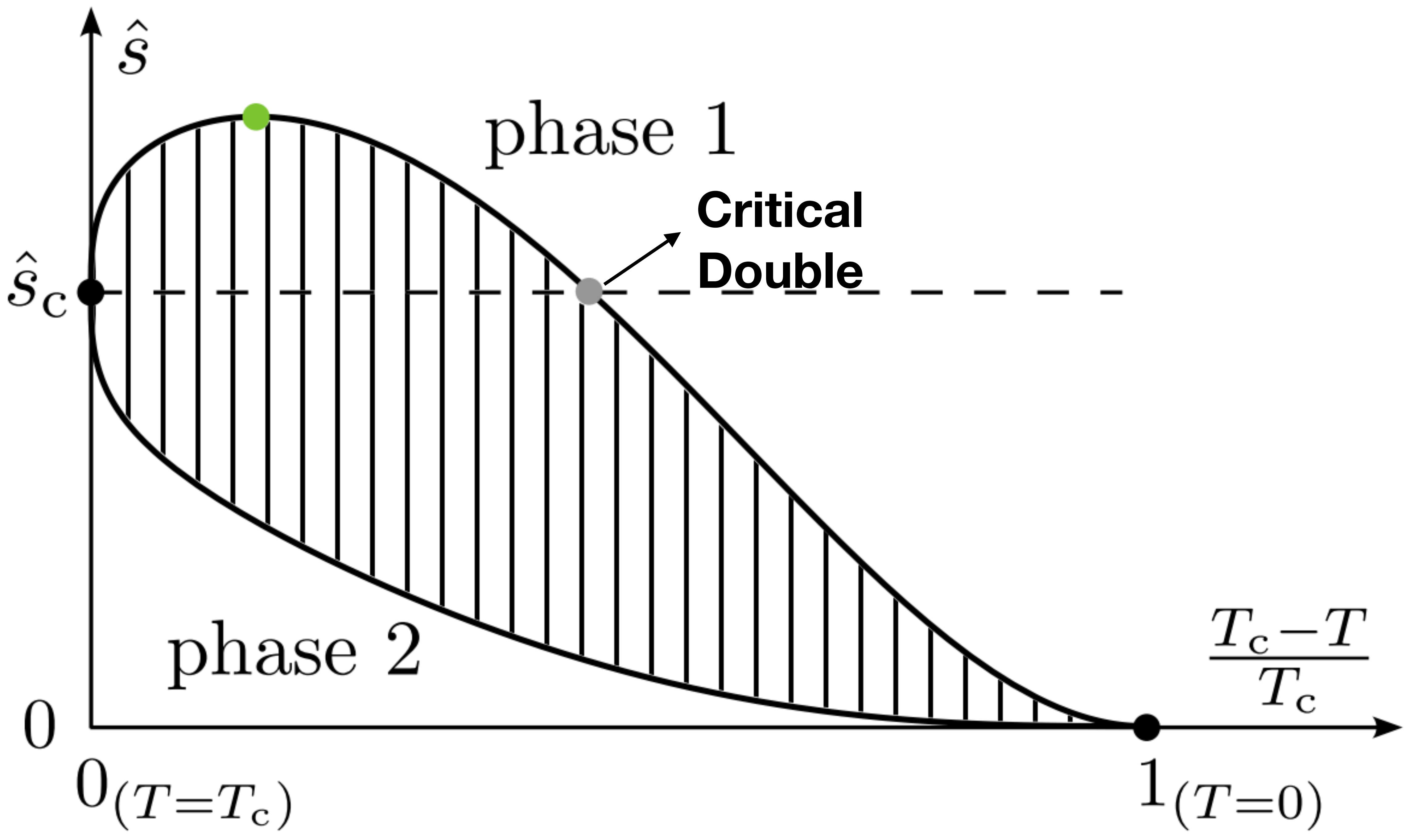


(c)

	(a)	(b)
(I)	Crossover	
(II)	HRG → HRG	QGP → QGP
(III)	QGP → HRG	

(d)

- **Cross-over side : Critical lensing or focussing towards CP**
- **Two different evolution scenarios on the first order side**



- Which phase does critical double lie in?
- Can we estimate $\Delta T_{cd} = T_c - T_{cd}$?

Overview of the talk

- s/n has a maxima on the first-order curve along one of the branches
- Inferences from universality near CP
- Consequences for hydrodynamic evolution during a neutron star merger
- Towards more realistic investigations

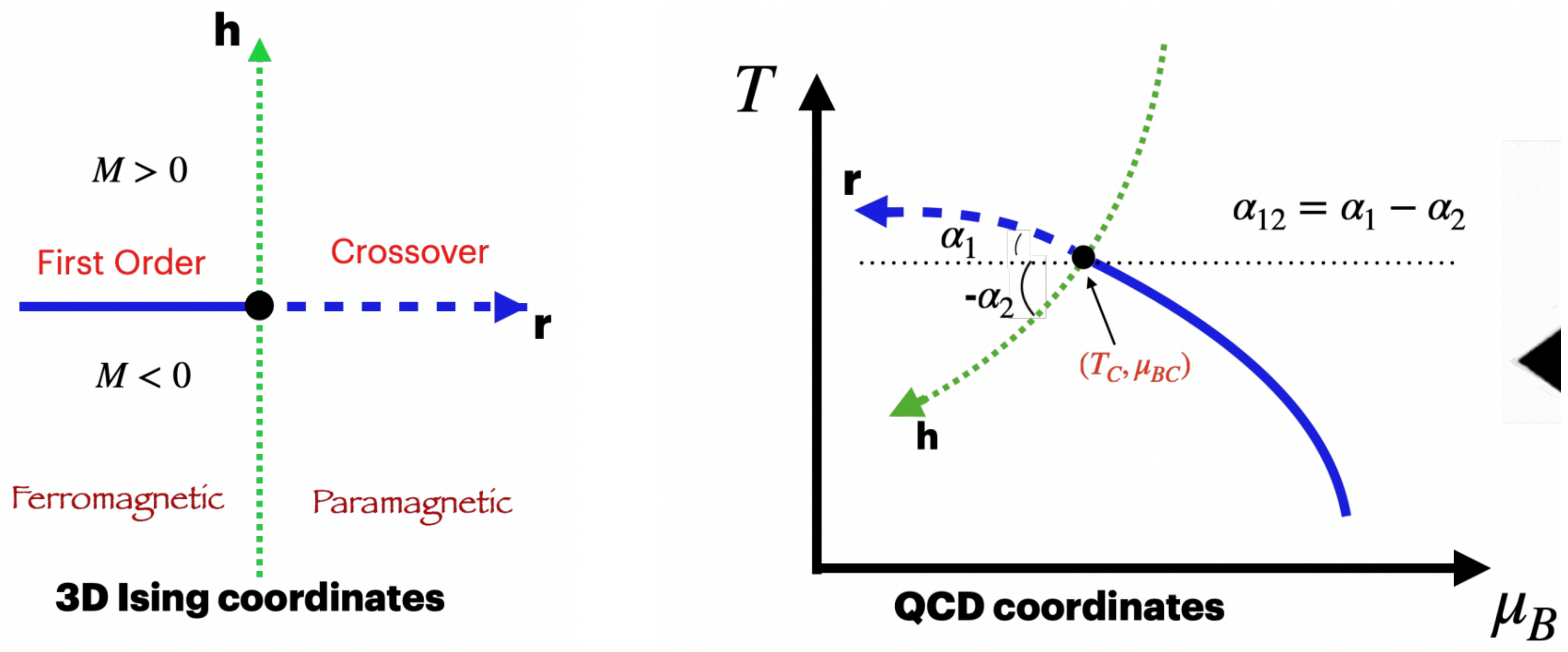
I'll use the example of "QGP-HRG" phase transition and the associated CP to demonstrate the points as this is relevant for neutron star mergers and heavy-ion collisions

Universal EoS near the HRG-QGP Critical Point

3D Ising Universality Class

Ising Gibbs free energy in r and h variables of Ising model

$$P_{\mathrm{QCD}}(\mu,T) = P_{\mathrm{BG}}(\mu,T) + A\,G(r(\mu,T),h(\mu,T))$$



Parotto et al, 18, Kahangirwe et al., 24

Mapping parameters which depend on the microscopics of QCD which are not known

$$\mu_c,\ T_c,\ \alpha_1,\ \alpha_2,\ \rho,\ w$$

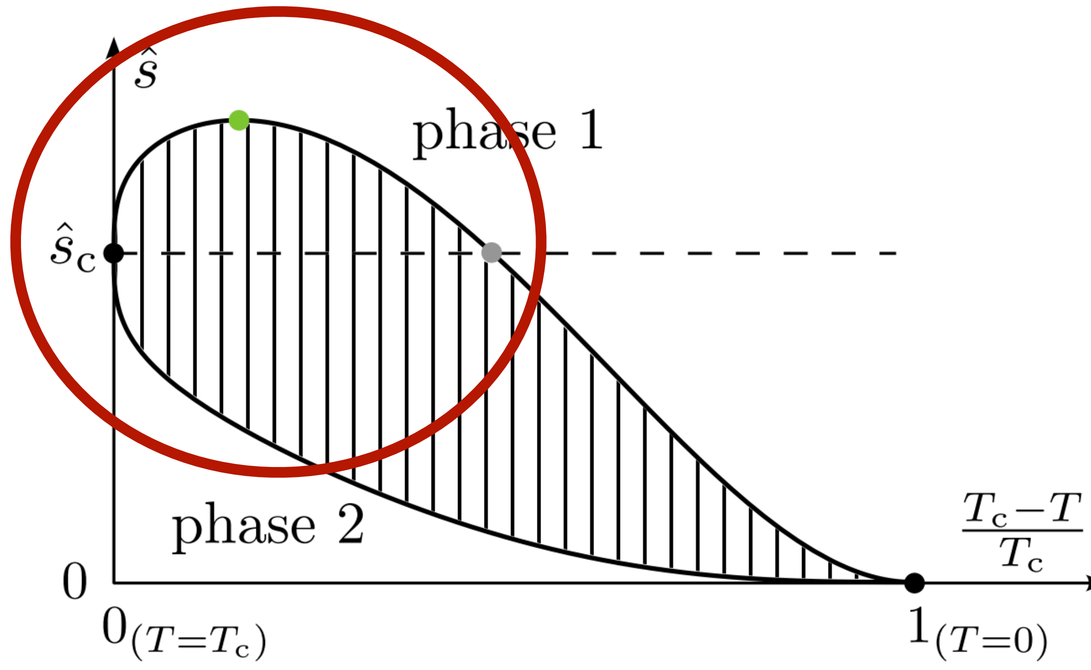
One of the theoretical goals of the critical point search programs (BES) is to estimate these parameters by data-model comparisons by Bayesian techniques

When critical double falls within the "critical region"

- "Critical region" - Critical contribution dominates over regular part

$$\Delta\hat{s} \equiv \hat{s} - \hat{s}_c = \hat{s}_m G_h + \hat{s}_\epsilon G_r + \text{regular/sub-leading terms}$$

Odd - opposite sign for both branches **Even - same functional form for both branches**

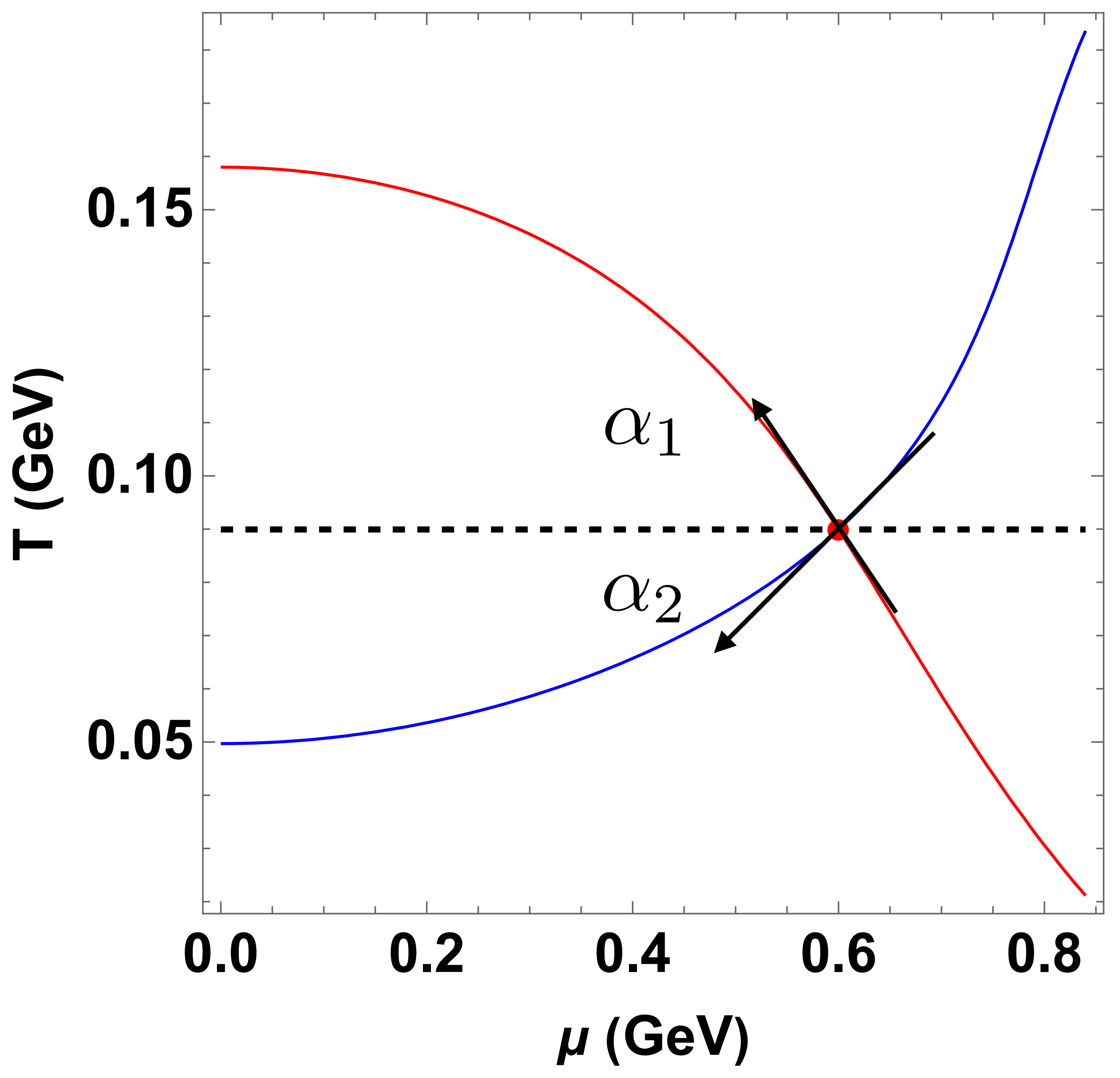


- Leading odd contribution causes the discontinuity between both the branches
- Sub-leading even contribution causes the entropy per baryon to decrease (typically) along both the branches

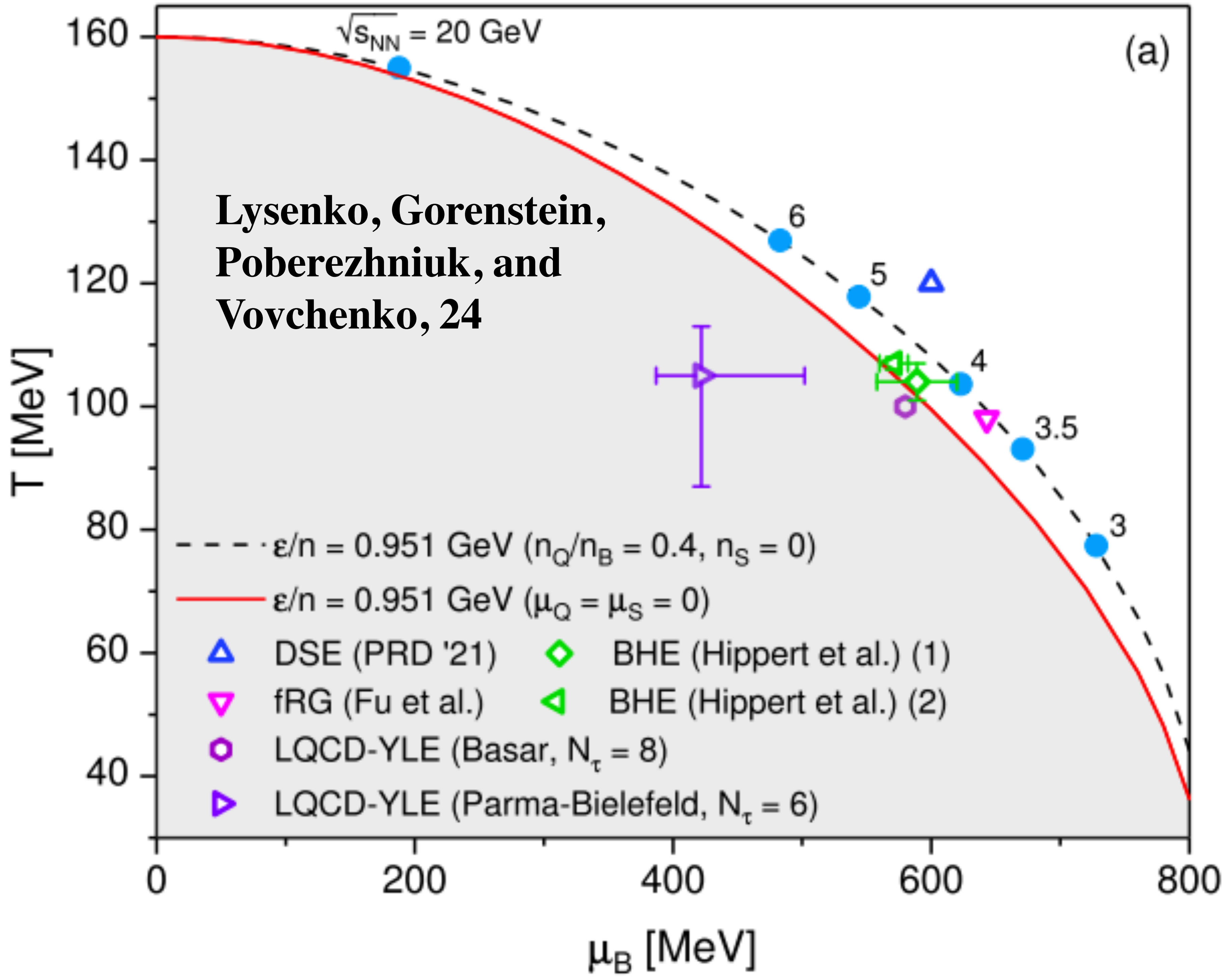
Location of critical double can be estimated from universality if the mapping to the universal EoS is known

We'll first assume that the critical double lies within the critical region and make some estimates

Some educated estimates of mapping parameters



- **Reasonable estimates of** $T_c(\mu)$ and α_1 **available from lattice QCD extrapolations at** $\mu_B = 0$ **Kahangirwe et al, 24**

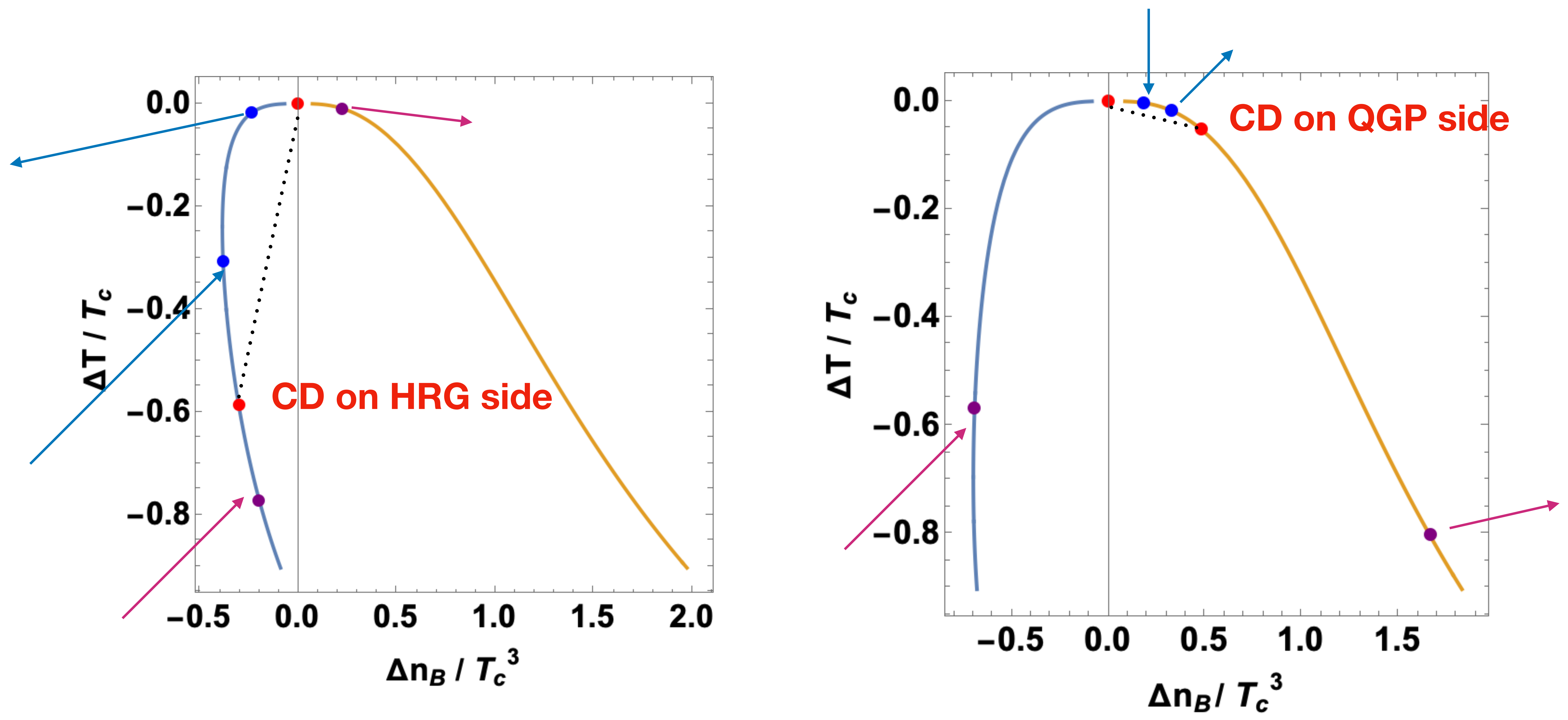


Choice : $\mu_c = 600\,\mathrm{MeV}\,, \rho\ =\ 1\,, w\ =\ 1$

$$T_c\ \approx\ 90\,\mathrm{MeV}\,,\ \alpha_1\ \approx 16.6^\circ$$

Lattice studies at imaginary chemical potential, improved Pade approximants, FRG, Holographic EoS matched to lattice QCD and data-driven-analysis suggest a CP between 400 to 700 MeV

The branch where the critical double lies



$\alpha_2 = -10^\circ , \quad \hat{s}_c = 19.3$

$\hat{s}_c > \cot \alpha_1$

$\alpha_2 = -5^\circ , \hat{s}_c = 2.3$

$\hat{s}_c < \cot \alpha_1$

$\mu_c = 600\,\mathrm{MeV} , \rho = 1 , w = 1$

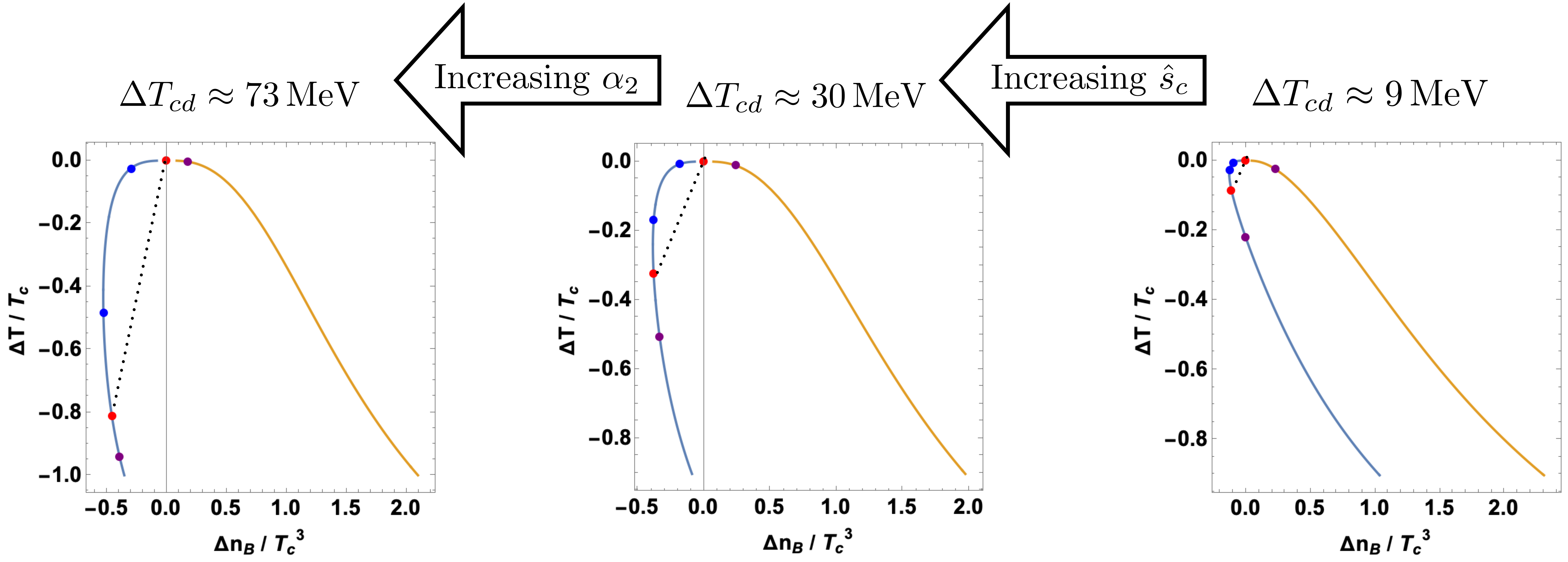
$T_c \approx 90\,\mathrm{MeV} , \alpha_1 \approx 16.6^\circ$

Side determined by sign of

$$\hat{s}_c - \cot \alpha_1$$

Value of entropy/baryon at Critical Point

Separation between critical double and critical point



$\alpha_2 = -7^\circ\,,\ \hat{s}_c = 19.3$ $\alpha_2 = -30^\circ\,,\ \hat{s}_c = 19.3$ $\alpha_2 = -30^\circ,\ \hat{s}_c = 10.3$

- Proportional to $\left(\dfrac{\hat{s}_c \sin\alpha_1 - \cos\alpha_1}{\hat{s}_c \sin\alpha_2 - \cos\alpha_2}\right)^{\frac{1}{\beta\delta-1}}$ $,\ \beta\delta > 1$

In the chiral limit, $\alpha_1 - \alpha_2 \to 0$ **MP, Stephanov, 19**

How would regular/sub-leading parts of EoS affect ΔT_{cd}?

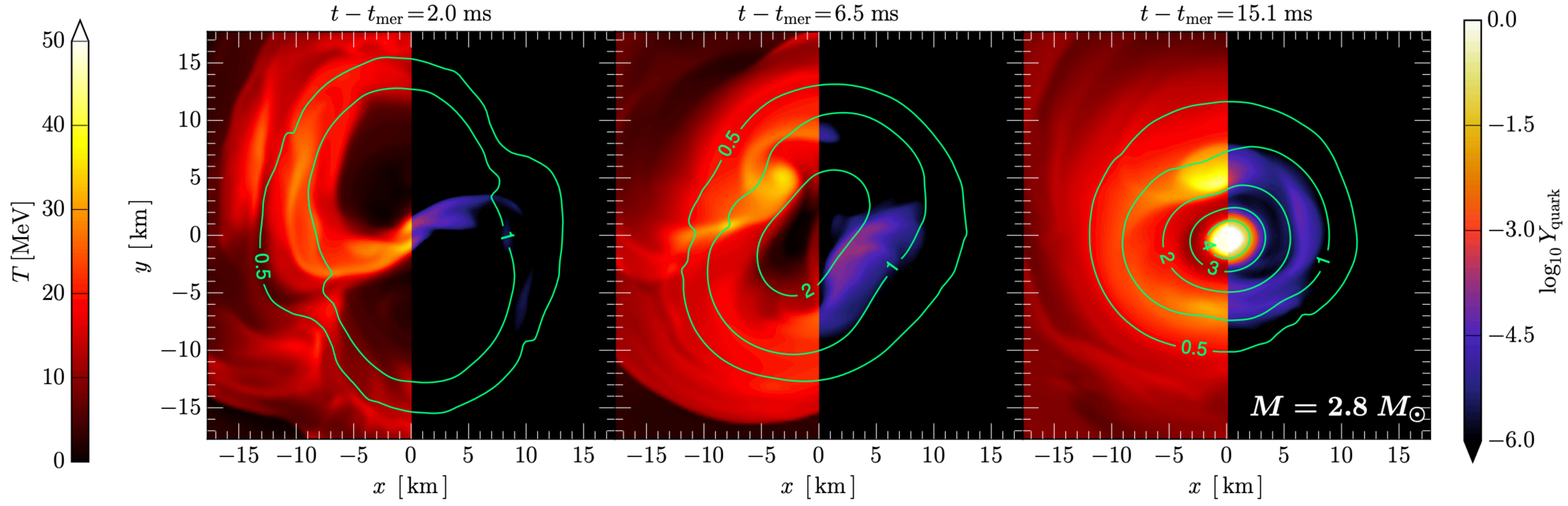
$$\Delta\hat{s} = \hat{\tilde{s}}_m \left(\frac{\mu^2-\mu_c^2}{T_c^2}\right)^{\beta} + \hat{\tilde{s}}_\epsilon \left(\frac{\mu^2-\mu_c^2}{T_c^2}\right)^{\beta+\beta\delta-1} + \hat{s}_{\text{reg},1}\left(\frac{\mu^2-\mu_c^2}{T_c^2}\right) + \hat{s}_5\left(\frac{\mu^2-\mu_c^2}{T_c^2}\right)^{\beta+\Delta_5} + \hat{s}_{\text{reg},2}\left(\frac{\mu^2-\mu_c^2}{T_c^2}\right)^{2} +$$

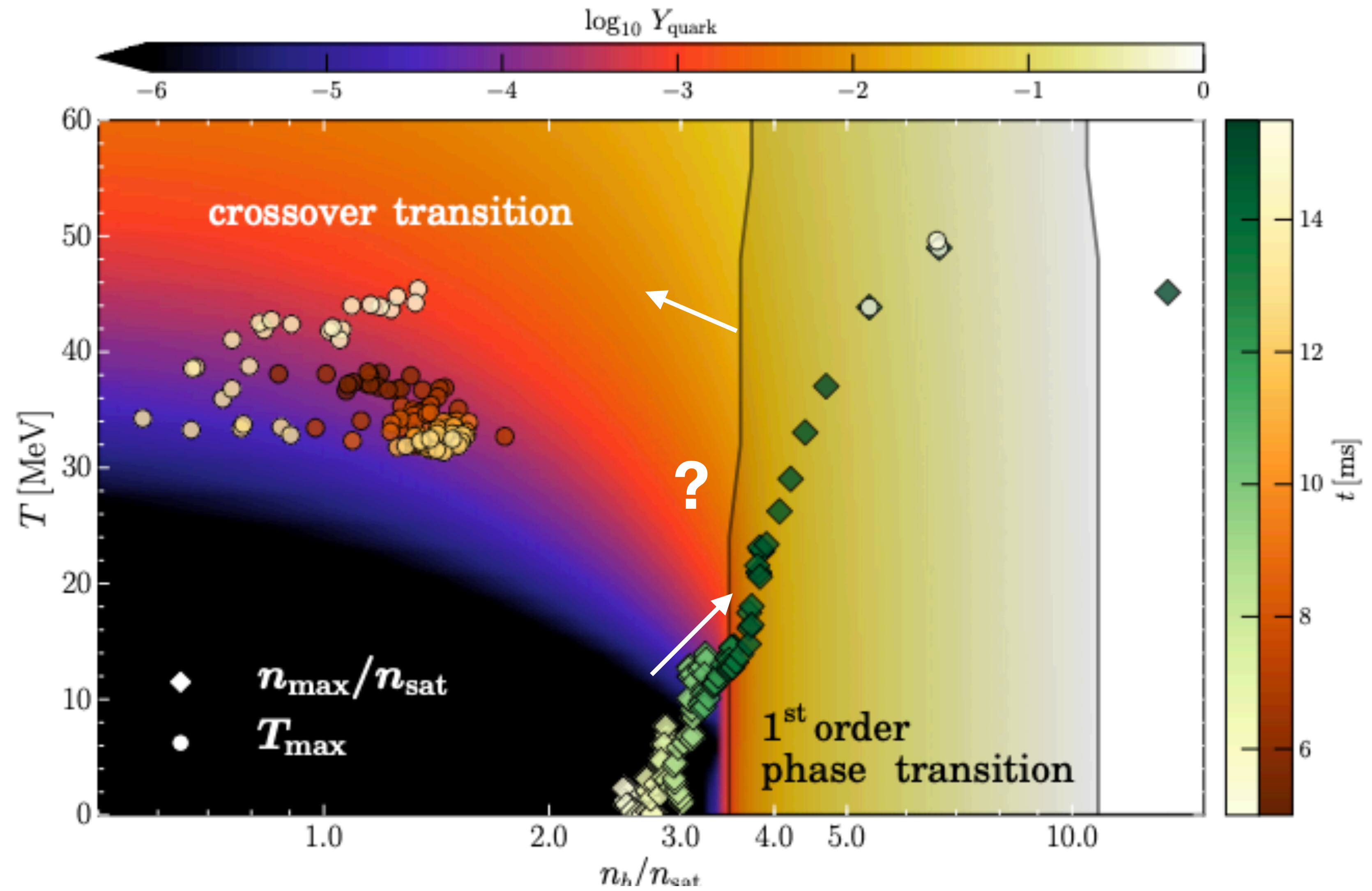
$$+\ldots\,,\ \beta\approx 1/3\,,\ \beta+\beta\delta-1\approx 0.9\,,\ \Delta_5\approx 1.443$$

Depends on α_2

- Regular parts of EoS contribute equally to both branches
- The leading regular correction can be qualitatively and somewhat quantitatively mimicked by varying α_2 **MP, Stephanov, 19** **Sogabe, MP, Stephanov, Yee, 24**
- In general, the regular part of the EoS can increase or decrease ΔT_{cd}

What significance could this result have for neutron star mergers?





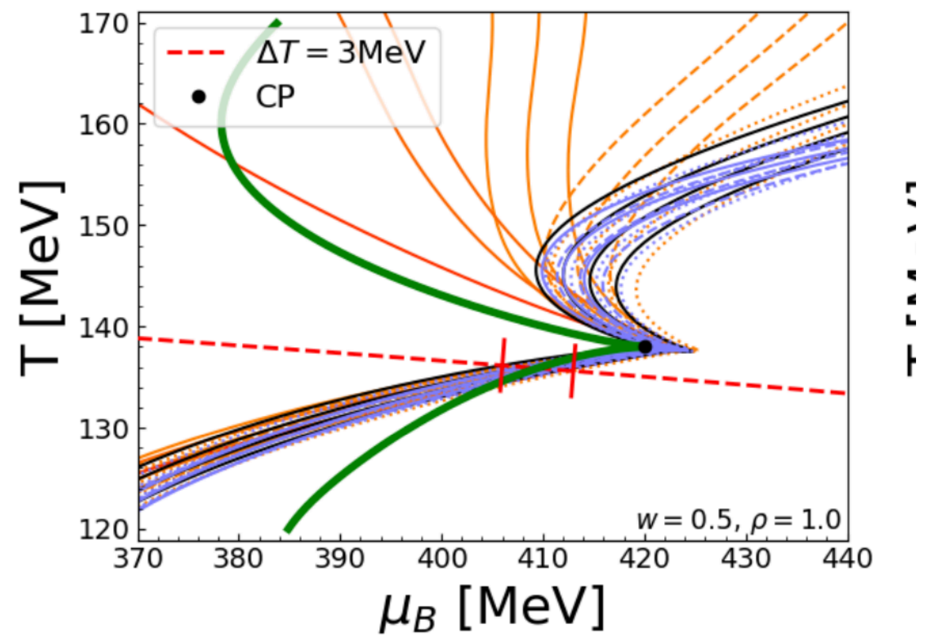
Most et al, PRL 122 (2019) 6, 061101

"We also show that the phase transition leads to a very hot and dense quark core that, when it collapses to a black hole, produces a ringdown signal different from the hadronic one."

The system can end up not having a dense quark despite entering a first-order phase transition regime.

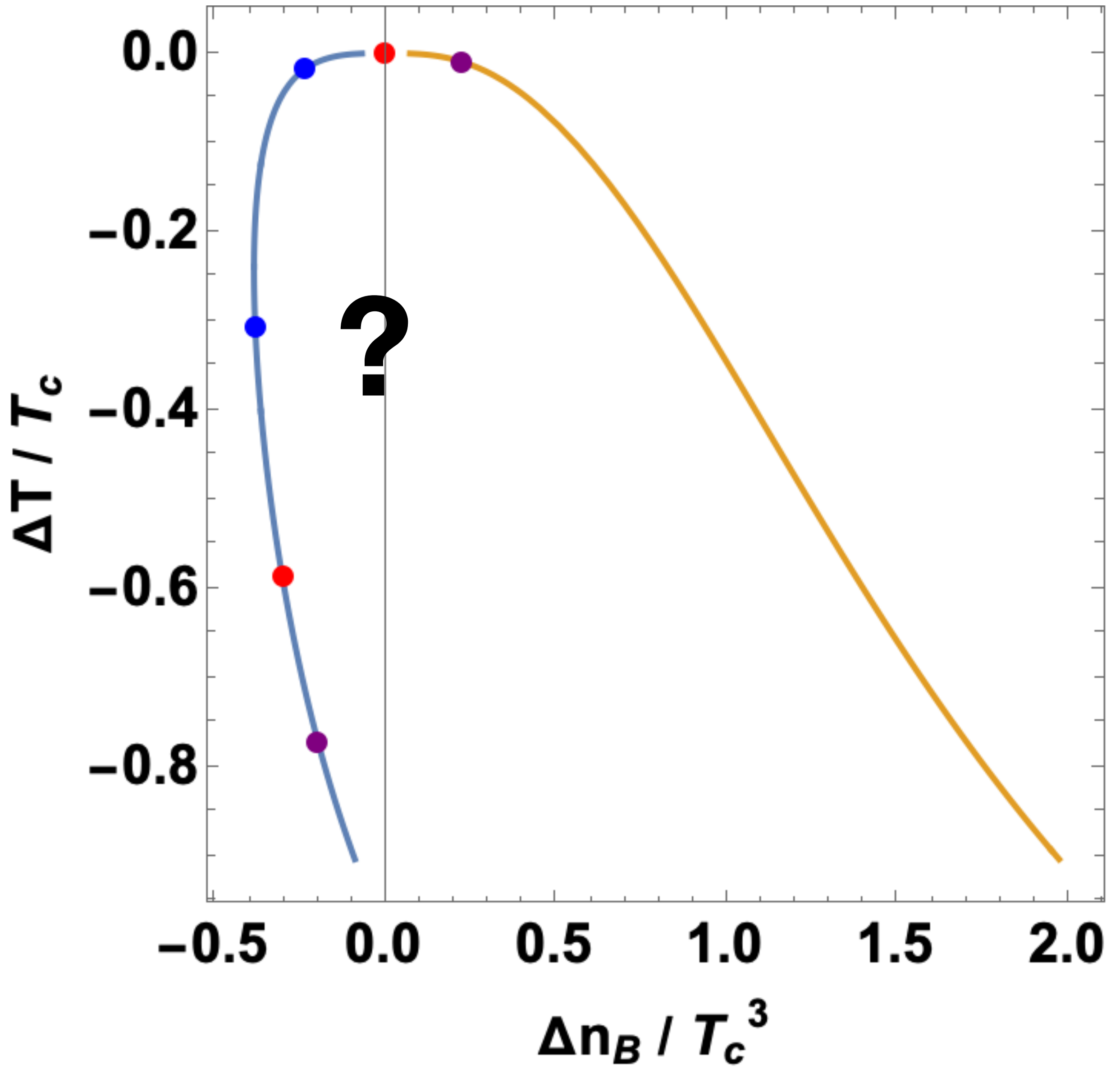
Realistic hydrodynamic trajectories

- Universal features of isentropes near CP have been found to be stable against viscous corrections on the cross-over side



Dore et al, *PRD* 106 (2022) 9, 094024

- Could the dynamics in the co-existence region modify these predictions?



Conclusion & Looking forward

- Non-monotonicity of the specific entropy along one of the branches of the FO curve leads to interesting dynamical consequences.
- Based on the specifics of the critical point of interest, dynamically different scenarios can be realized
- Bounds/estimates on the QCD EoS near CP can give guidance in the nature of dynamics in neutron-star mergers
- We expect that BES data-model comparisons can help us to determine the EoS parameters close to CP.