Relativistic light-front models of hadrons based on QCD degrees of freedom

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Outline

- Motivation - background
- Degrees of freedom
- Model assumptions
- Relativistic invariance
- Sea quarks
- Scattering
- Decays
- Electromagnetic observables
- To do

Goal:

Explore phenomenological (front-form) relativistic models of hadrons based on QCD degrees of freedom. Construct relativistic light-front wave functions of hadrons including sea-quark degrees of freedom.

Elements:

- QCD degrees of freedom (locally and globally SU(3) invariant).
- All scales set by quark masses, 1 coupling constant, CSB scale (π mass).
- Simple enough to treat sea quark degrees of freedom. Charge carriers visible to EM probes.
- Consistent treatment of scattering, decays, spectra and electromagnetic properties?
- Dual QCD - hadronic representations.
- Boosts kinematic.

Inspiration:

- **Structure of the model (degrees of freedom/interactions):**

K. G. Wilson, Phys. Rev. D10, 2445 (1974).

J. B. Kogut and L. Susskind, Phys. Rev. D11, 395 (1975).

E. Seiler, Lecture Notes in Physics, 159, 1 (1982).

- **Treatment of glue DOF:**

O. W. Greenberg and J. Hietarinta, Physics Letters B 86, 309 (1979).

- **Scattering in confined systems:**

R. F. Dashen, J. B. Healy, and I. J. Muzinich, Ann. of Phys. 102, 1 (1976).

Model Hilbert space - motivation - degrees of freedom:

- Kogut and Susskind: (Hamiltonian lattice) degrees of freedom are mutually non-interacting global and local SU(3) color invariant connected networks of quarks, anti-quarks and links:

$$H = H_{\text{static}} + H_{\text{dynamic}}$$

- The static energy of a connected network is equal to the sum of the quark masses and the number of links times the energy per link.

- K & S Hilbert space: Basis = locally and globally gauge invariant eigenstates of H_{static}.

- In the absence of the remaining interactions the static degrees of freedom are confined. Local gauge invariance means separating quarks requires more links.

Model Hilbert space

- Model connected local and global color singlets by confined systems of quarks and anti-quarks. In general there will be towers of excited interactions.

- Greenberg and Hietarinta: Identical quarks in different connected networks behave like distinguishable particles due to the glue (link) degree of freedom.

$$|\downarrow\uparrow\rangle \qquad |\rightleftarrows\rangle \qquad \text{independent}$$

$\rightarrow$ Quarks and anti-quarks confined in different connected singlets are treated as distinguishable. This eliminates Van der Waals forces.

Dynamics

- Covariant derivative and color magnetic interactions allow different connected singlets to move and interact.

- Too many gauge invariant degrees of freedom and too many possible interactions between them to formulate a sensible model of the dynamics.

- Dynamical assumption to test: The physics is dominated by string breaking and the "ground" confining interaction.

- No fundamental QCD justification, except that meson exchange seems to be important in phenomenological hadronic reactions and string breaking is used successfully to model hadronic reactions in PYTHIA.

Question: Does this limited set of model degrees of freedom and interactions result in a consistent picture of spectral properties, lifetimes, cross sections and electromagnetic observables using a limited set of parameters?

Model - meson valence sector - confined singlets:

Mass operator for a quark-anti-quark singlet - scales set by model parameters:

$$M_c = \sqrt{k^2 + V_c + m_q^2} + \sqrt{k^2 + V_c + m_{\bar{q}}^2}$$

$$\boxed{V_c = -\frac{\lambda^2}{4}\nabla_k^2 + V_0}$$

$$M_{nl} \to \sqrt{m_q^2 + \lambda(2n + l + \frac{3}{2}) + V_0} + \sqrt{m_{\bar{q}}^2 + \lambda(2n + l + \frac{3}{2}) + V_0}.$$

π mass and $\pi - \rho$ splitting (sets the CSB scale)

$$V_{csb} := (a + b\mathbf{s}_q \cdot \mathbf{s}_{\bar{q}})\delta_{l0}.$$

V_0 and the quark masses are essentially the same parameter. This is an arbitrary splitting of a single constant. There are no quark mass eigenstates - there is no way to separate what we call a quark mass from what we call a confining interaction.

Assumption: Quarks and anti-quarks transform like discrete mass m_q spin $\frac{1}{2}$ irreducible representations of the Poincaré group (no fundamental justification).

Bare mesons:

Approximate linear confinement

$$\langle r_{nls}^2\rangle^{1/2} = \sqrt{\frac{2}{\lambda}(2n+l+\frac{3}{2})} \qquad M_{nls} \approx \sqrt{2}\lambda\langle r_{RMS}^2\rangle^{\frac{1}{2}}.$$

Approximate Regge behavior

$$l \approx \frac{1}{4\lambda}M_{nls}^2$$

The oscillator parameter is chosen to fit the Regge slope of the $\rho - a$ mesons.

Table: Regge trajectories, $J = L+1, S = 1$ $m_q = \frac{m_\rho}{2} = .385$, $\lambda = .282$

meson	L	exp mass	exp $(\text{mass})^2$	J	calc mass	calc $(\text{mass})^2$
ρ	0	.770	.593	1	.770	.593
a_2	1	1.320	1.742	2	1.311	1.719
ρ_3	2	1.690	2.856	3	1.687	2.846
a_4	3	2.040	4.162	4	1.994	3.976
ρ_5	4	2.350	5.522	5	2.259	5.103
a_6	5	2.450	6.000	6	2.497	6.335

$\langle r_\pi^2\rangle^{1/2} = .64$ fm

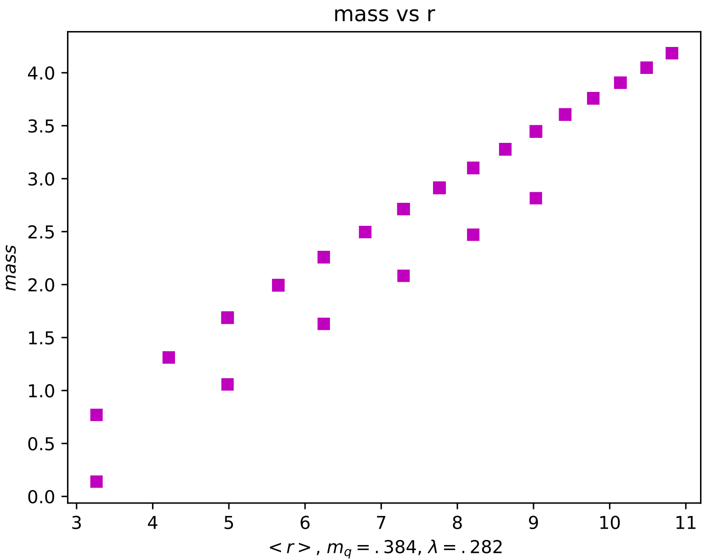


Figure: mass vs $\langle r^2 \rangle^{1/2}$

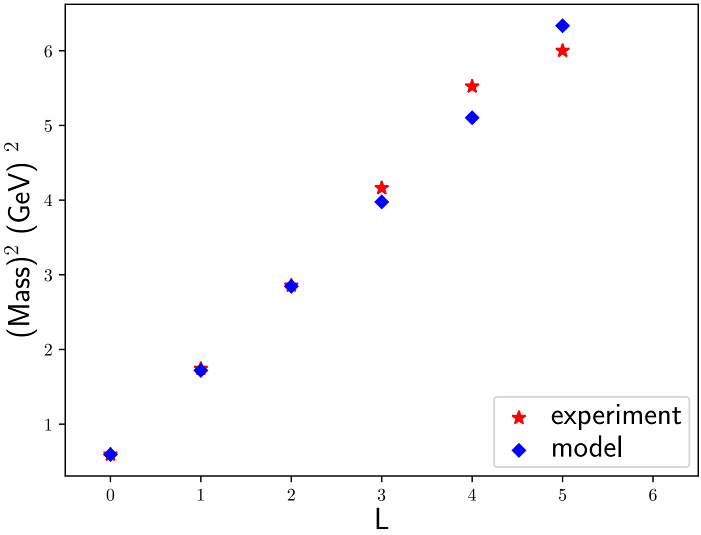


Figure: Regge trajectory for ρ and a mesons

Relativity (unitary representation of the Poincaré group)

CM momentum relativistic:

$$\langle k^2_{nls} \rangle^{1/2} = \sqrt{\frac{\lambda}{2}(2n + l + \frac{3}{2})}. \qquad \sqrt{\frac{3\lambda}{4}} \approx .46(GeV)$$

Bare hadron wave functions, $\tilde{\mathbf{p}} = (p^+, \mathbf{p}_\perp)$, $\mu = \mathbf{s}_f \cdot \hat{\mathbf{z}}$:

$$\underbrace{\langle \tilde{\mathbf{P}}, j, \tilde{\mu}, k, l, s}_{\mathcal{H}_{q\bar{q}}} | \underbrace{\tilde{\mathbf{P}}', j', \tilde{\mu}', n', l', s'}_{\mathcal{H}_{njls}} \rangle = \delta(\tilde{\mathbf{P}} - \tilde{\mathbf{P}}') \delta_{\tilde{\mu}\tilde{\mu}'} \delta_{j'j} \delta_{s's} \delta_{l'l} \tilde{R}_{n'l'}(k).$$

Dual representation of the hadronic Hilbert space:$(k \leftrightarrow n)$

$$\mathcal{H}_{q\bar{q}} \sim \mathcal{H}_H := \oplus \mathcal{H}_{njls}. \qquad U_{q\bar{q}}(\Lambda, a) = \sum_{njls} U_{njls}(\Lambda, a)$$

Unitary representation of the Poincaré group on $\mathcal{H}_{njls}$:

$$U_{njls}(\Lambda, a) | \tilde{\mathbf{P}}, j, \tilde{\mu}, n, l, s \rangle =$$

$$e^{ia \cdot \Lambda P_{nls}} \sum_{\tilde{\nu}} | \tilde{\mathbf{\Lambda}} P_{nls}, j, \tilde{\nu}, n, l, s \rangle \sqrt{\frac{(\Lambda P_{nls})^+}{P^+}} D^j_{\tilde{\nu}\tilde{\mu}} [B_f^{-1}(\Lambda P_{nls}) \Lambda B_f(P_{nls})]$$

Summary - bare mesons:

Wave functions are known analytically (harmonic oscillator).

Exact unitary light-front representation of the Poincaré group - including transverse rotations - composite systems have a well-defined spin.

Approximate linear confinement.

Approximate linear Regge trajectory - slope fixes λ.

Only flavor dependence is quark masses at this point.

Gauge invariant basis.

String breaking - model assumptions

A quark-anti-quark pair is produced with equal probability at any point on the line between the original quark-anti-quark pair.

Delta functions are replaced by delta-function normalized Gaussians with the width of oscillator ground state (replaces line by a "flux tube" with width determined by oscillator parameter λ).

Spin independent vertex:

$$\langle \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{12} | v_{2:1} | \mathbf{r} \rangle := g\sqrt{\lambda}\delta(\mathbf{r} - 2\mathbf{r}_{12}) \int_0^1 d\eta \delta_{\sqrt{\frac{\lambda}{2}}}(\mathbf{r}_1 - \eta\mathbf{r})\delta_{\sqrt{\frac{\lambda}{2}}}(\mathbf{r}_2 - (1-\eta)\mathbf{r})$$

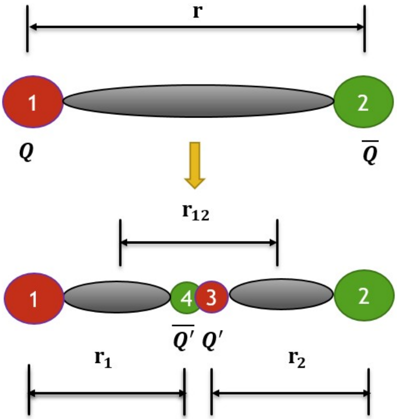
where the Gaussian approximate delta function is

$$\delta_{\sqrt{\frac{\lambda}{2}}}(\mathbf{r}) := (\frac{\lambda}{4\pi})^{3/2} e^{\frac{-\lambda r^2}{4}} \qquad \int \delta_{\sqrt{\frac{\lambda}{2}}}(\mathbf{r}) d\mathbf{r} = 1.$$

The dimensionless coupling constant g must be a constant of order unity. Fixed by ρ lifetime.

Spin dependent part (q,$\bar{q}$ have opposite parity):

$$Y_{1m}(\hat{\mathbf{r}}_{12})\langle s_3, \mu_3, s_4, \mu_4 | 1, \mu_s \rangle \langle 1, m_l, 1, \mu_s | 0, 0 \rangle.$$


r
1
2
Q
$\overline{Q}$
r_{12}
1
4
3
2
$\overline{Q'}$ Q'
r_1
r_2

Hadronic representation of vertex: (spin-independent part)

The 9 dimensional integral over the initial and final bare meson states can be computed analytically for any three bare meson states

The string-breaking vertex fixes all hadronic production vertices:

$$\langle n_1, l_1, m_1, n_2, l_2, m_2, \mathbf{r}_{12} | v_{2:1} | n, l, m \rangle =$$

$$\frac{g}{\sqrt{\lambda}} R_{nl}(2r_{12})(2\lambda)^{3/2} \frac{(\sqrt{\frac{\lambda}{2}} r_{12})^{2n_1+l_1+2n_2+l_2}}{\sqrt{2n_1!\Gamma(n_1+l_1+\frac{3}{2})}\sqrt{2n_2!\Gamma(n_2+l_2+\frac{3}{2})}} \times$$

$$e^{-\frac{\lambda}{4}r_{12}^2} \sum_{k_1+k_2=2r} \frac{(l_1+2n_1)!(l_2+2n_2)!}{k_1!k_2!(l_1+2n_1-k_1)!(l_2+2n_2-k_2)!}(-)^{k_2}(\frac{1}{2})^{l_1+2n_1+l_2+2n_2} \times$$

$$\frac{1}{2r+1} M(\frac{1}{2}+r, \frac{3}{2}+r, -\frac{\lambda r_{12}^2}{4}) Y_{lm}(\hat{\mathbf{r}}_{12}) Y^*_{l_1 m_1}(\hat{\mathbf{r}}_{12}) Y^*_{l_2 m_2}(\hat{\mathbf{r}}_{12}).$$

Momentum space requires a one-dimensional Fourier Bessel transform of r_{12}.

The full vertex is defined by including the spin dependent part and embedding it in the full Hilbert space so it commutes with and is independent of P^+, $\mathbf{P}_\perp$ and $\mathbf{s}_f$.

Tweaks:

The structure of the model is constrained because the scales are fixed by the same number of parameters as QCD.

Unable to get a consistent picture of scattering, lifetimes, bare meson spectra due to these constraints.

This was fixed by applying a unitary scale transformation to the vertex that reduced the width of the flux tube by a factor of 2.

$$\langle n_1, l_1, m_1, n_2, l_2, m_2, \mathbf{r}_{12}|v_{2:1}|n, l, m\rangle \to (2)^{3/2}\langle n_1, l_1, m_1 n_2, l_2, m_2, 2r_{12}|v_{2:1}|n, l, m\rangle$$

This is still consistent with the scale set by the confining interaction.

The up and down quark masses were taken to be half of the ρ mass. The only calculations sensitive to the quark masses were the form factor calculations. Pion form factor calculations ignoring sea quarks were closer to data using m_q : .385 GeV$\to$ .2 GeV. These calculations did not include sea quark contributions.

Sea quarks - truncation to 1+2 bare meson subspace:

Model Hilbert space

$$\mathcal{H} = \mathcal{H}_H \oplus (\mathcal{H}_H \otimes \mathcal{H}_H) \qquad \text{Hadronic representation.}$$

$$\mathcal{H} = \mathcal{H}_{q\bar{q}} \oplus (\mathcal{H}_{q\bar{q}} \otimes \mathcal{H}_{q\bar{q}}) \qquad \text{Dual QCD DOF representation.}$$

Bare meson unitary representation of the Poincaré group

$$U_0(\Lambda, a) = \begin{pmatrix} U_{q\bar{q}}(\Lambda, a) & 0 \\ 0 & U_{q\bar{q}}(\Lambda, a) \otimes U_{q\bar{q}}(\Lambda, a) \end{pmatrix}.$$

String breaking dynamics

$$M = M_0 + V = \underbrace{\begin{pmatrix} M_c & 0 \\ 0 & \sqrt{M_{c1}^2 + \mathbf{q}^2} + \sqrt{M_{c2}^2 + \mathbf{q}^2} \end{pmatrix}}_{M_0} + \underbrace{\begin{pmatrix} 0 & v_{1:2} \\ v_{2:1} & 0 \end{pmatrix}}_{V},$$

$v_{i:j}$ is the string breaking vertex.

Relativistic dynamics including string breaking:

The string breaking vertex is constructed to commute with light-front kinematic subgroup and $\mathbf{s}_{f0}$ (not $\mathbf{J}_0$!).

Diagonalize M in the basis of simultaneous eigenstates of $M_0, P_0^+, \mathbf{P}_{0\perp}, s_0^2, s_{0fz}$ and invariant degeneracy quantum numbers, d.

$U_I(\Lambda, a)$ is defined so these states transform irreducibly

$$U_I(\Lambda, a)|(M, s, d)\tilde{\mathbf{P}}, \tilde{\mu}\rangle :=$$

$$e^{ia\cdot\Lambda P_M} \sum_{\tilde{\nu}} |(M, s, d)\tilde{\mathbf{\Lambda}}, P_M, \tilde{\nu}\rangle \sqrt{\frac{(\Lambda P)^+}{P^+}} D^s_{\tilde{\nu}\tilde{\mu}}[B_f^{-1}(\Lambda P_M)\Lambda B_f(P_M)]$$

This is different than $U_0(\Lambda, a)$. It requires diagonalizing M. The operators $M, P_0^+, \mathbf{P}_{0\perp}, s_0^2, s_{0fz}$ are commuting self-adjoint operators. $U_I(\Lambda, a)$ is defined so simultaneous eigenstates of these operators transform irreducibly.

Hadronic eigenstates can be expressed in terms of current quark spins and momenta using Poincaré Clebsch-Gordon coefficients in a light-front basis.

Mass eigenvalue problem - sea quarks:

$$|\Psi\rangle = \begin{pmatrix} |\Psi_1\rangle \\ |\Psi_2\rangle \end{pmatrix}$$

Coupled eigenvalue equations

$$(\lambda - M_c)|\Psi_1\rangle = v_{1:2}|\Psi_2\rangle$$

$$(\lambda - \sqrt{M_{c1}^2 + \mathbf{q}^2} + \sqrt{M_{c2}^2 + \mathbf{q}^2})|\Psi_2\rangle = v_{2:1}|\Psi_1\rangle$$

These decouple

$$|\Psi_1\rangle = (\lambda - M_c)^{-1} v_{12}(\lambda - \sqrt{M_{c1}^2 + \mathbf{q}^2} + \sqrt{M_{c2}^2 + \mathbf{q}^2}))^{-1} v_{2:1}|\Psi_1\rangle$$

$$|\Psi_2\rangle = (\lambda - \sqrt{M_{c1}^2 + \mathbf{q}^2} + \sqrt{M_{c2}^2 + \mathbf{q}^2}))^{-1}|\Psi_1\rangle$$

Normalization:

$$1 = \langle\Psi_1|\Psi_1\rangle + \langle\Psi_2|\Psi_2\rangle$$

$\langle\Psi_2|\Psi_2\rangle$ = sea quark probability

Equation still has an infinite number of channels - it requires a truncation.

Mass eigenvalues are real zeroes of $F(\lambda)$ between 0 and the two bare meson threshold:

$$F(\lambda) = \det\left(I - (\lambda - M_c)^{-1} v_{1:2}(\lambda - \sqrt{M_{c1}^2 + \mathbf{q}^2} + \sqrt{M_{c2}^2 + \mathbf{q}^2}))^{-1} v_{2:1}\right).$$

Results:

Model calculation keeping 2 $q\bar{q}$ channels with n $\leq$ 4:

Table: Parameters

λ	.282 $(\text{GeV})^2$
g	5.44
$m_q = m_{\bar{q}}$	.385 GeV
m_{π_0}	.160 GeV
$m_{\rho 0}$	.882 GeV

Table: Results

bare pion mass	.1600 GeV
m_π - 2^{nd} order perturbation theory ($n \leq 4$)	.1327 GeV
m_π exact ($n \leq 4$)	.1329 GeV
valence quark probability	82%
sea quark probability	18%

Scattering of bare mesons:(*s*-channel case)

Wave operators exist with infinite number of bare mesons. Time-dependent methods result in coupled equations

$$T^{22}(e+i0^+) = 0 + v_{2:1}(e - M_1 + i0^+)^{-1} T^{12}(e+i0^+)$$

$$T^{12}(e+i0^+) = v_{1:2} + v_{1:2}(e - M_2 + i0^+)^{-1} T^{22}(e+i0^+).$$

These equations can expressed in terms of the solution of

$$T^{12}(e+i0^+) = v_{1:2} + v_{1:2}(e - M_2 + i0^+)^{-1} v_{2:1}(e - M_1 + i0^+)^{-1} T^{12}(e+i0^+).$$

This equation has an infinite number of poles in the continuum. These are spurious and can be eliminated by defining

$$\Gamma_{12}(e+i0^+) := (e - M_1 + i0^+)^{-1} T^{12}(e+i0^+)$$

$$\Gamma_{12}(e+i0^+) = (e - M_1 - v_{1:2}(e - M_2 + i0^+)^{-1} v_{2:1})^{-1} v_{1:2}$$

$$T^{22}(e+i\epsilon^+) = v_{2:1} \frac{1}{e - M_1 - v_{1:2}(e - M_2 + i0^+)^{-1} v_{2:1}} v_{1:2}$$

This has no spurious singularities in the continuum.

Note that there are no long-range Van der Waals forces because the quarks in different singlets are treated as distinguishable.

Data: Phys. Rev. D7,1279(1973), Phys. Rev. D12,681(1975).

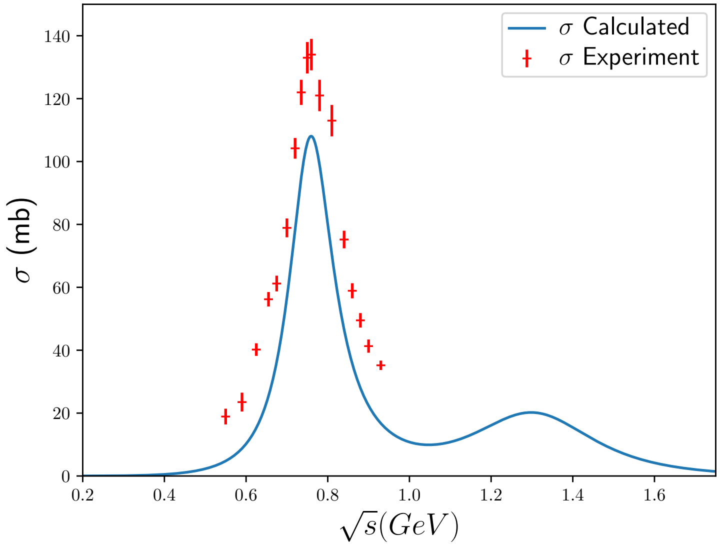


Figure: $\pi - \pi$ scattering cross section (s-channel)

Unstable particles

When

$$M_{n_1,n_2,0} < M_{n_0}$$

$M_{n_1,n_2,q_{120}} = M_{n_0}$ has solutions for real q_{120}^2 that depend on n_1 and n_2:

$$q_{120}^2 = \frac{M_{n_1}^4 + M_{n_2}^4 + M_{n_0}^4 - 2M_{n_1}^2 M_{n_2}^2 - 2M_{n_1}^2 M_{n_0}^2 - 2M_{n_2}^2 M_{n_0}^2}{4M_{n_0}^2}$$

The decay width is

$$\Gamma = \sum_{\mathbf{n}_1 \mathbf{n}_2} 2\pi \frac{q_{120}\omega_{n1}(q_{120})\omega_{n2}(q_{120})}{\omega_{n1}(q_{120}) + \omega_{n2}(q_{120})} |\langle n_1, n_2, q_{120}|v_{21}|n_0\rangle|^2$$

The sum is over the open decay channels.

Table: Results

bare ρ mass	.882 GeV
position ρ resonance (fixes g)	.770 GeV
shift	-.122 GeV
calculated width of ρ resonance	.134 GeV
experimental width of ρ resonance	.150 GeV

Pion Form factor - including sea quark contributions

$$F_\pi(Q^2) = \langle \pi, \tilde{\mathbf{p}}' | I^+(0) | \pi, \tilde{\mathbf{p}} \rangle$$

$$F_\pi(Q^2) =$$
$$_1\langle \pi, \tilde{\mathbf{p}}' | I^\mu(0) | \pi, \tilde{\mathbf{p}} \rangle_1 +$$
$$_1\langle \pi, \tilde{\mathbf{p}}' | I^\mu(0) | \frac{1}{m_\pi - M_2} v_{2:1} \frac{1}{m_\pi - M_1} | \pi, \tilde{\mathbf{p}} \rangle_1 +$$
$$_1\langle \pi, \tilde{\mathbf{p}} | \frac{1}{m_\pi - M_1} v_{12} \frac{1}{m_\pi - M_2} | I^\mu(0) | \pi, \tilde{\mathbf{p}} \rangle_1 +$$
$$_1\langle \pi, \tilde{\mathbf{p}} | \frac{1}{m_\pi - M_1} v_{12} \frac{1}{m_\pi - M_2} | I^\mu(0) | \frac{1}{m_\pi - M_2} v_{2:1} \frac{1}{m_\pi - M_1} | \pi, \tilde{\mathbf{p}}' \rangle_1$$

Calculations below do not include sea quark contribution (m_π = eigenvalue).

FF data from: Nuclear Physics B 277, 168 (1986), Phys. Rev. Lett. 86, 1713 (2001), Phys. Rev. D 17, 1693 (1978)

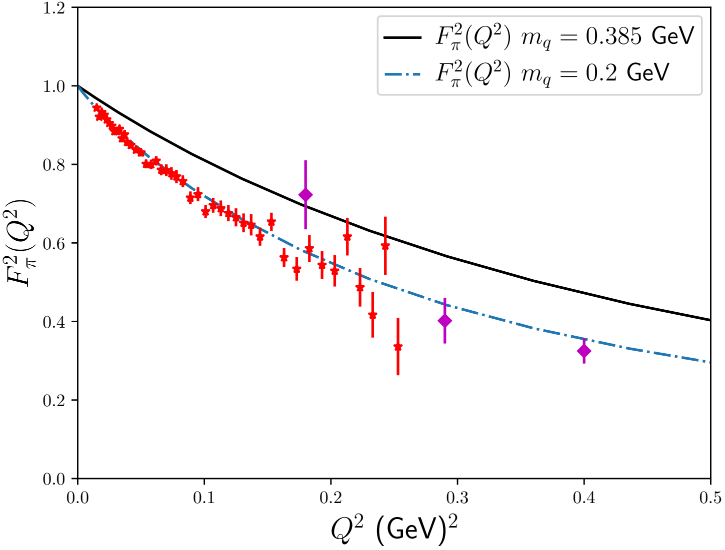


Figure: Pion Form Factor

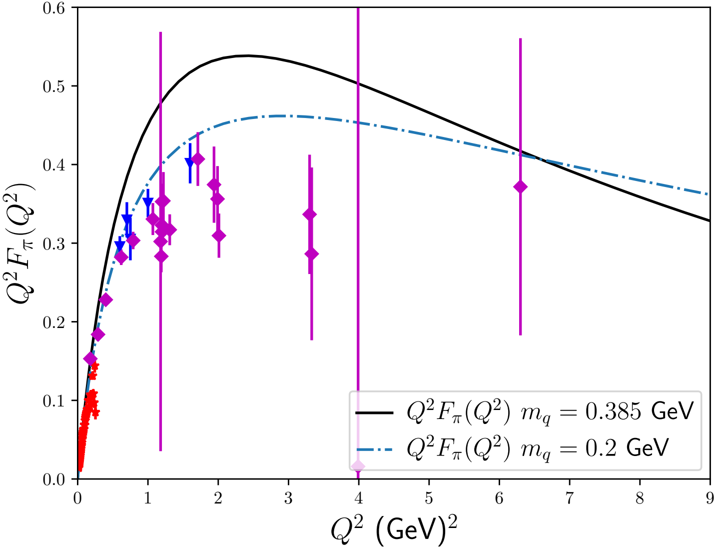


Figure: Pion Form Factor

Conclusions/Outlook:

- Simple models with the same # of parameters as QCD and dynamics given by string breaking gives a qualitatively consistent picture of spectral properties, lifetimes, cross sections and electromagnetic properties.

- Model gives analytic expressions for fully relativistic wave functions, including explicit sea quark degrees of freedom, for any mesons.

- One string breaking vertex gives all $1 \leftrightarrow 2$ meson vertices.

- Boosts kinematic; focus is on charge carriers that are sensitive to E&M probes.

- Method can be directly applied to baryons and exotics assuming that they can be represented using quark-diquark singlet degrees of freedom.

To do:

- Include one-body currents in the sea contribution to the pion form-factor calculations.

- Calculate relativistic proton wave function including sea quark contributions using quark-diquark-singlet degrees of freedom.

- Nucleon-form factors, structure functions including sea quark contributions.

- Mass spectrum and wave functions for exotics.