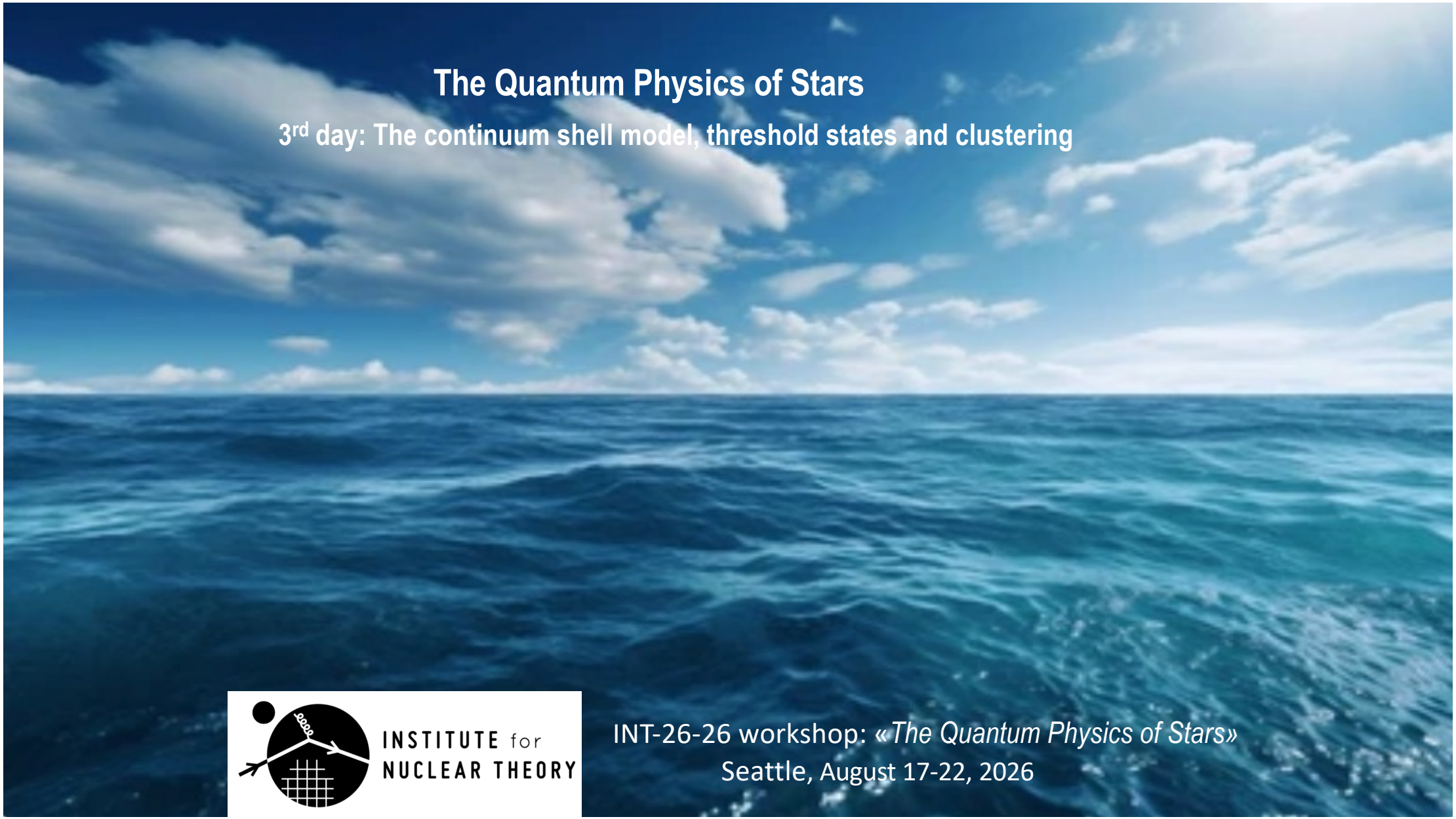


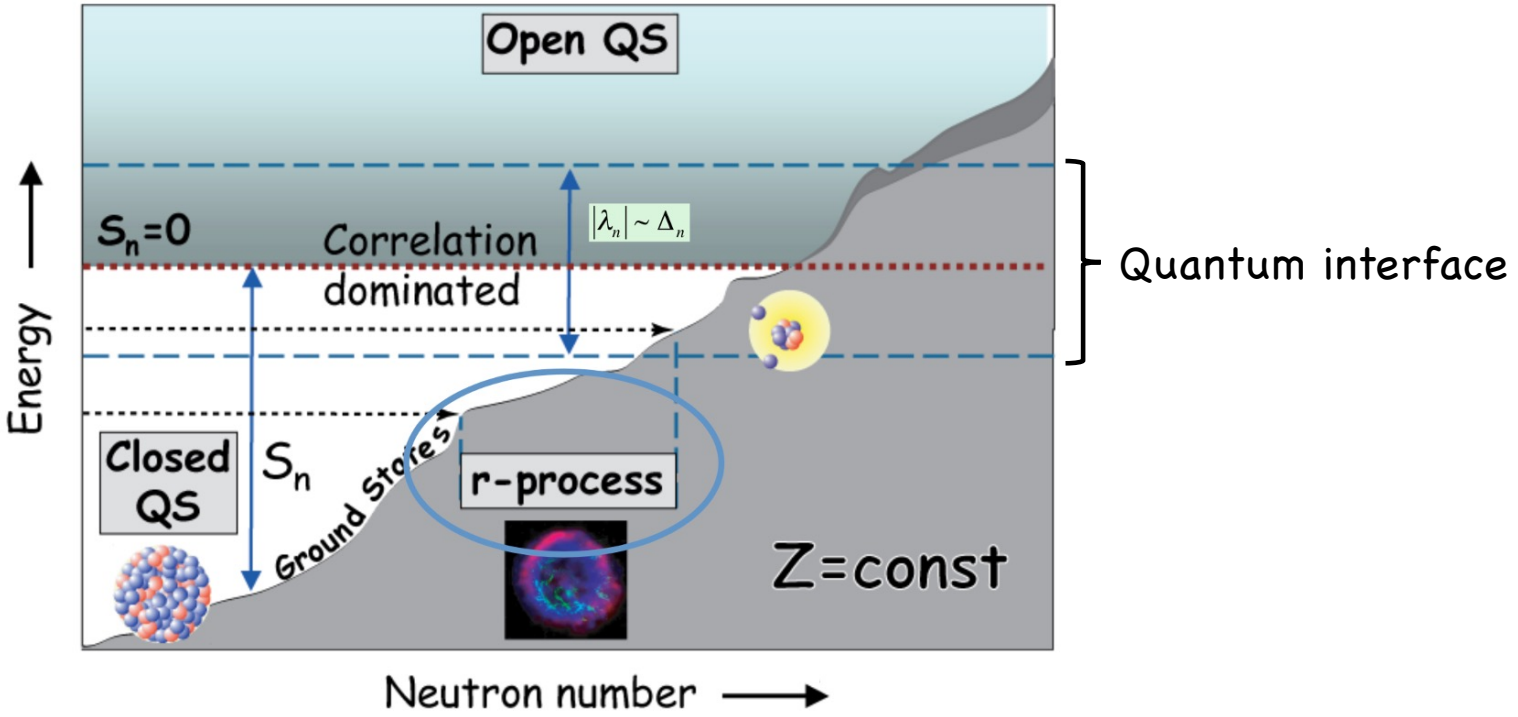
The Quantum Physics of Stars

3rd day: The continuum shell model, threshold states and clustering

INSTITUTE for NUCLEAR THEORY

INT-26-26 workshop: «*The Quantum Physics of Stars*»
Seattle, August 17-22, 2026

Atomic nucleus: the open quantum system

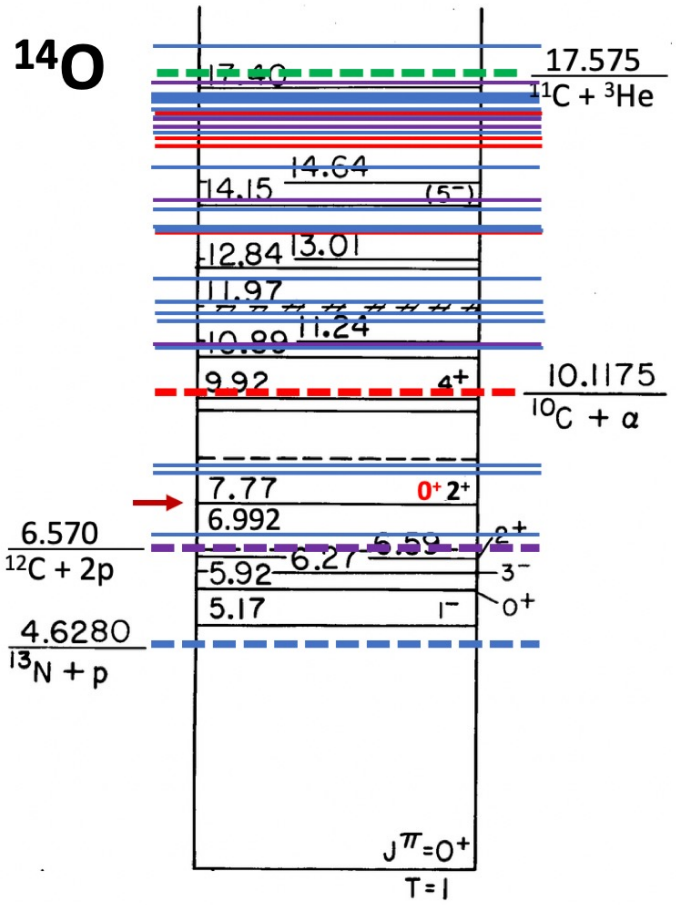


$$S_{1n} \approx -\lambda - \Delta$$

openness mean-field correlations

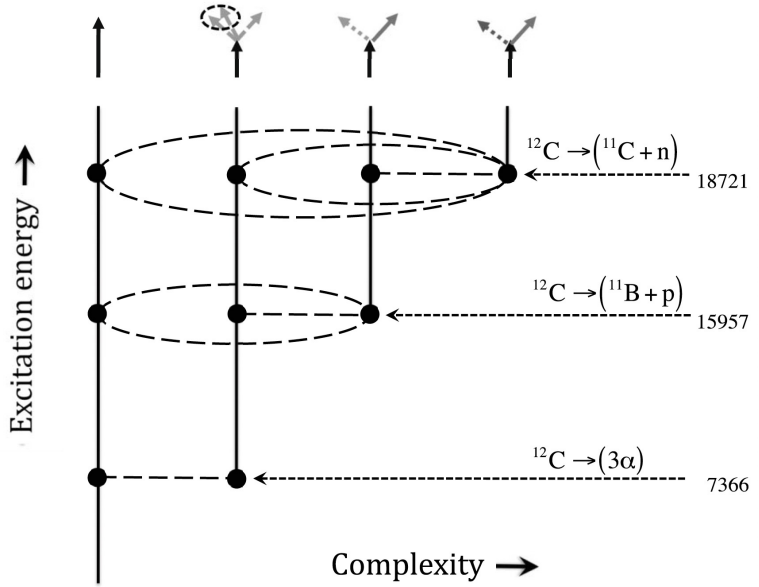
Effect of NN correlations is comparable to the mean field

⇒ New regime of the nuclear many-body problem



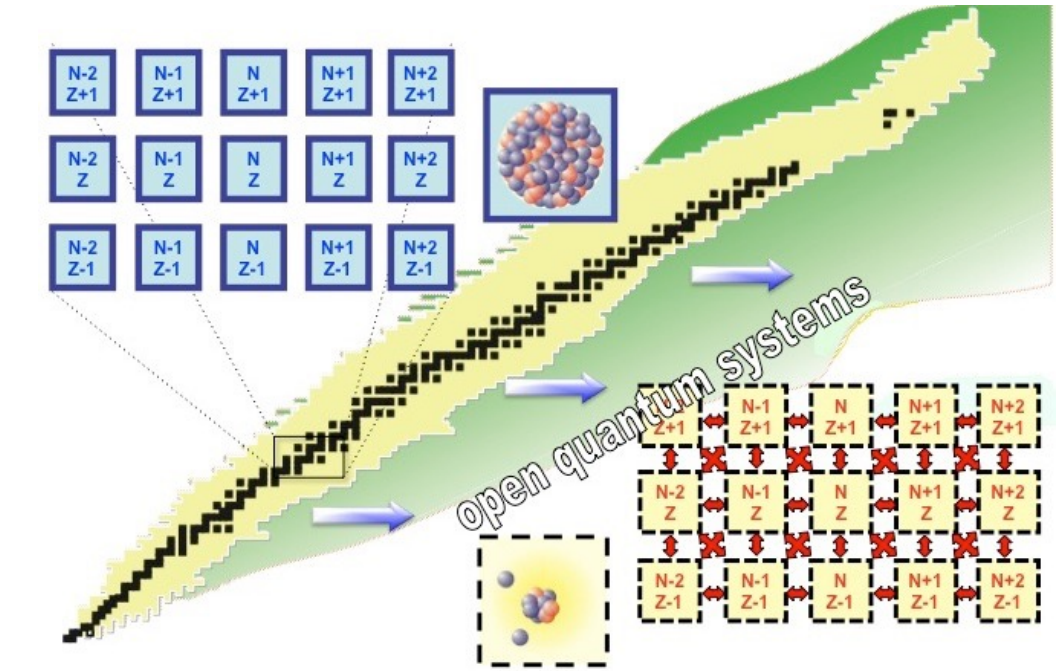
- Nuclear states are embedded in the scattering continuum
  They belong to multidimensional network of correlated states
- Virtual excitations to continuum modify effective interaction
  → effective interaction becomes state/energy dependent

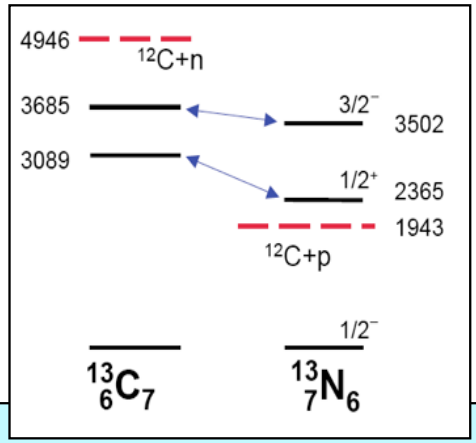
The multichannel representation of ^{12}C

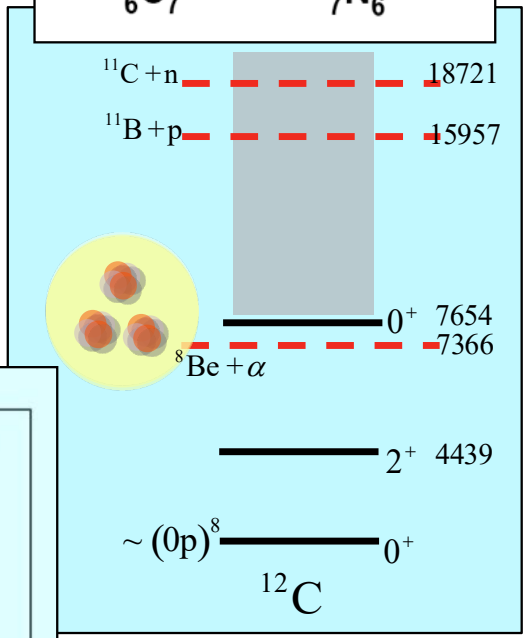


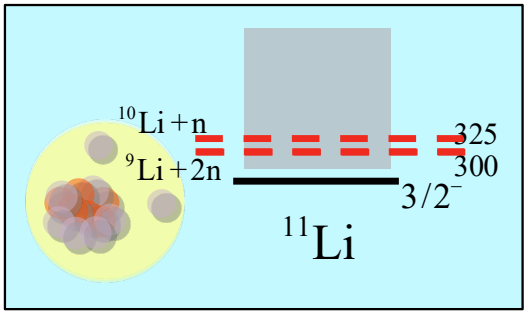
- Thresholds are **branching points** ⟹ *nonanalytic behavior*
  E.P. Wigner, Phys. Rev. 73, 1002 (1948)
- Thresholds effects are rooted in the *unitarity of the scattering matrix* and the resulting flux conservation
- When a new channel opens, the unitarity imposes new channel couplings; hence, a *modification of all eigenfunctions*

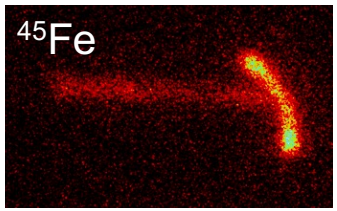
Threshold physics can be defined as the study of physical interactions that occur at interface(s)

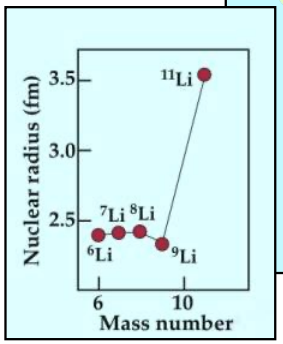

N-2 Z+1
N-1 Z+1
N Z+1
N+1 Z+1
N+2 Z+1
N-2 Z
N-1 Z
N Z
N+1 Z
N+2 Z
N-2 Z-1
N-1 Z-1
N Z-1
N+1 Z-1
N+2 Z-1
open quantum systems


4946
^{12}C+n
3685
3/2$^-$
3502
3089
1/2$^+$
2365
1943
^{12}C+p
1/2$^-$
$^{13}_{6}C_{7}$
$^{13}_{7}N_{6}$


^{11}C +n
18721
^{11}B +p
15957
0$^+$ 7654
7366
^{8}Be +α
2$^+$ 4439
~ (0p)^8
0$^+$
^{12}C


^{10}Li +n
325
^{9}Li +2n
300
3/2$^-$
^{11}Li


^{45}Fe


Nuclear radius (fm)
3.5
3.0
2.5
^{11}Li
^{7}Li
^{8}Li
^{6}Li
^{9}Li
6
10
Mass number

Shell model for open quantum systems

Shell model embedded in the continuum (SMEC)

$$H^{(SM)} \rightarrow \mathcal{H}^{eff}(E) = H'(E) - (i/2)V(E)V^{T}(E)$$

[N×N] [N×N] [N×N] [N×k] [k×N]

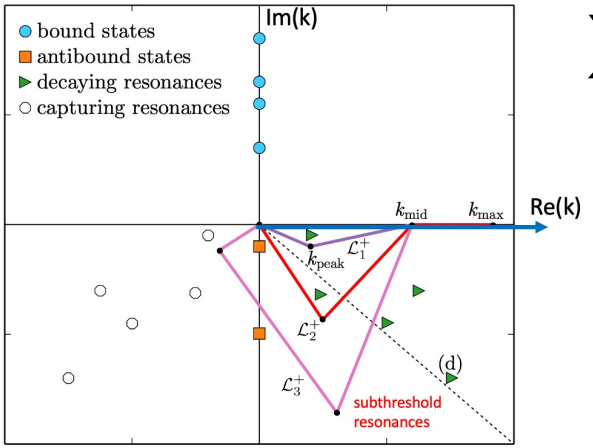
$$= H^{(SM)} + u(E) - (i/2)w(E)$$

Hermitian anti-Hermitian

- Coupling to the environment of scattering continuum and decay channels cannot be reduced to refitting the Hermitian Hamiltonian
- Entrance and exit reaction channels defined

J. Okołowicz, M. Płoszajczak, I. Rotter, Physics Reports 374, 271 (2003)

Gamow Shell Model



$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1$$

T. Berggren, Nucl. Phys. A 109, 265 (1968)

- *Unitary formulation* of the nuclear Shell Model
- *Complex-symmetric* eigenvalue problem for the *Hermitian* Hamiltonian

$$|\Psi_M^J\rangle = \sum_c \int_0^{+\infty} |(c,r)_M^J\rangle \frac{u_c^{JM}(r)}{r} r^2\, dr$$

$$|(c,r)\rangle = \hat{\mathcal{A}}[|\Psi_T^{J_T}; N_T, Z_T\rangle \otimes |r\ L_{CM}\ J_{int}\ J_P; n, z\rangle]_M^J$$

$$H|\Psi_M^J\rangle = E|\Psi_M^J\rangle \rightarrow \sum_c \int_0^{\infty} r^2 \left(H_{cc'}(r,r') - EN_{cc'}(r,r')\right) \frac{u_c(r)}{r} = 0$$

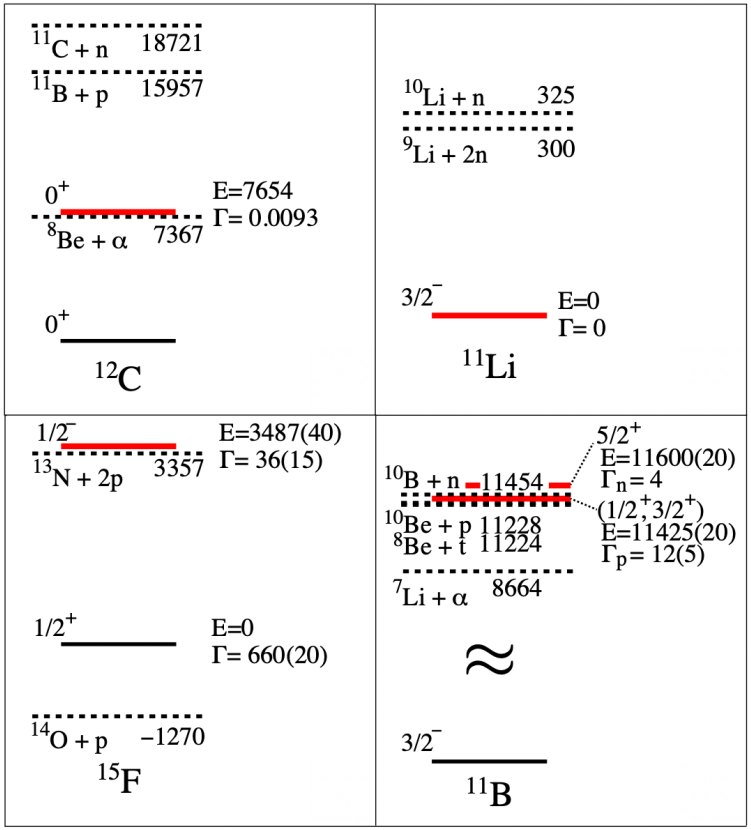
$$H_{cc'}(r,r') = \langle(c,r)|\hat{H}|(c',r')\rangle$$

$$N_{cc'}(r,r') = \langle(c,r)|(c',r')\rangle$$

- Calculation in relative coordinates
- Entrance and exit reaction channels defined

N. Michel, M.Płoszajczak, «*Gamow Shell Model: The Unified Theory of Nuclear Structure and Reactions* » Lecture Notes in Physics, Vol. 983, (Springer Verlag, 2021

Near-threshold phenomena



- Other cases: ^{6}He, ^{6}Li, ^{7}Be, ^{7}Li, ^{11}O, ^{11}C, ^{12}C, ^{17}O, ^{20}Ne, ^{26}O, ^{24}Mg, …
- Various clusterings: ^{2}H, ^{3}He, ^{3}H, 2p, 2n, …
- Threshold effects in hadrons, hadronic molecules, multiquark systems

- *'Fortuitous'* appearance of correlated states close to open channels?

  ➡ They cannot result from any particular feature of the NN interaction or any dynamical symmetry of the nuclear many-body problem

- The correlated (cluster) states in a vicinity of reaction channel thresholds are the manifestation of *quantum openness* of a many-body system related to the *collective rearrangement* of shell model wave functions due to their mutual coupling via the continuum

- Energetic order of particle emission thresholds which depends on nuclear Hamiltonian

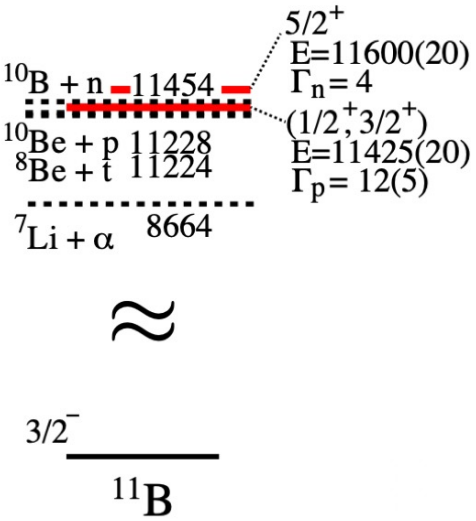
- Absence of stable cluster entirely composed of like nucleons

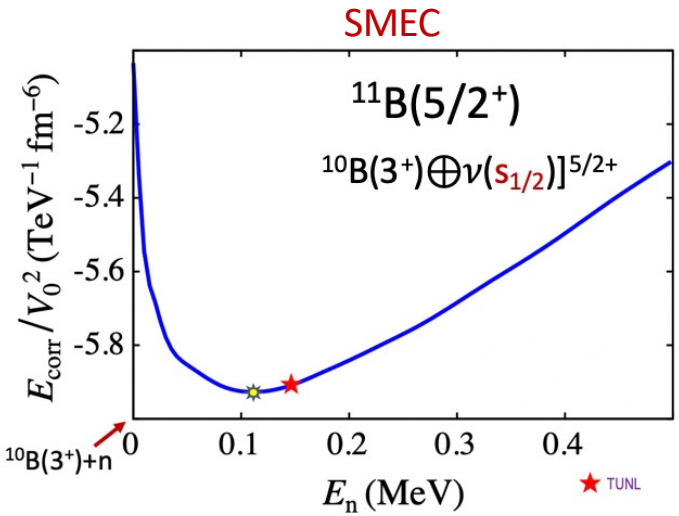
J. Okołowicz, M. Płoszajczak, W. Nazarewicz, Prog. Theor. Phys. Suppl. 196, 230 (2012); Fortschr. Phys. 61, 66 (2013)

NN interaction in different regimes of binding

- *Strong reduction of np interaction* in weakly bound/unbound nuclei
- Strong asymmetry of V_{nn} and V_{pp} *for large* $|S_n - S_p|$ *and low* ℓ_j
- If $S_n \ll S_p$, then $V_{pp} > V_{nn}$, i.e. protons in the neutron-rich environment interact stronger than neutrons J. Wylie et al, Phys. Rev. C 104, L061301 (2021)
- **Warning:** Near and beyond the drip lines, different reactions that lead to the same nucleus/final states may give inconsistent results due to the different *environment* of the continuum states and the particle emission thresholds (*spectroscopic factors*)

Near threshold collectivization – neutron poison





- Key resonance in the absorption of thermal neutrons (σ ~ 3800 barn)

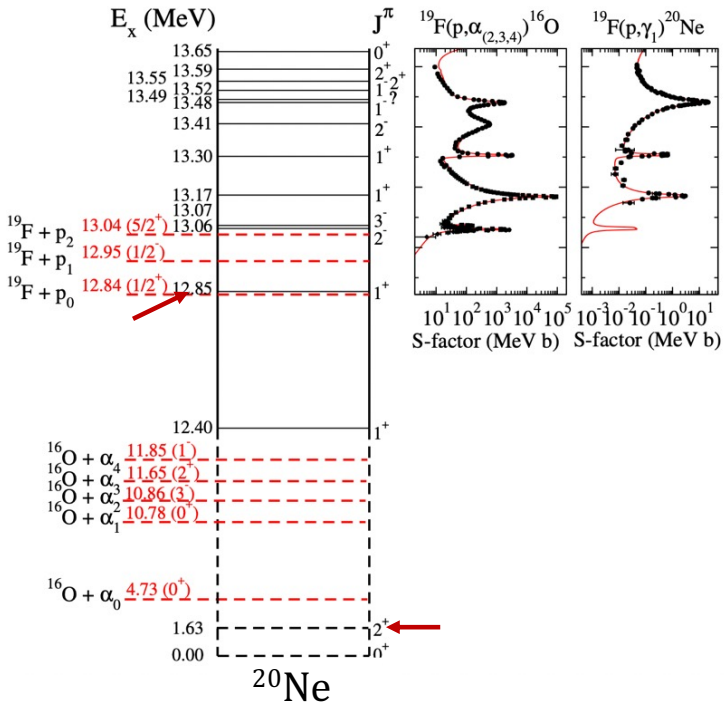
$$E_{corr;\alpha}(E) = \langle \Psi_\alpha(E) | \mathcal{H}_{QQ}(E) - H_{QQ} | \Psi_\alpha(E) \rangle$$

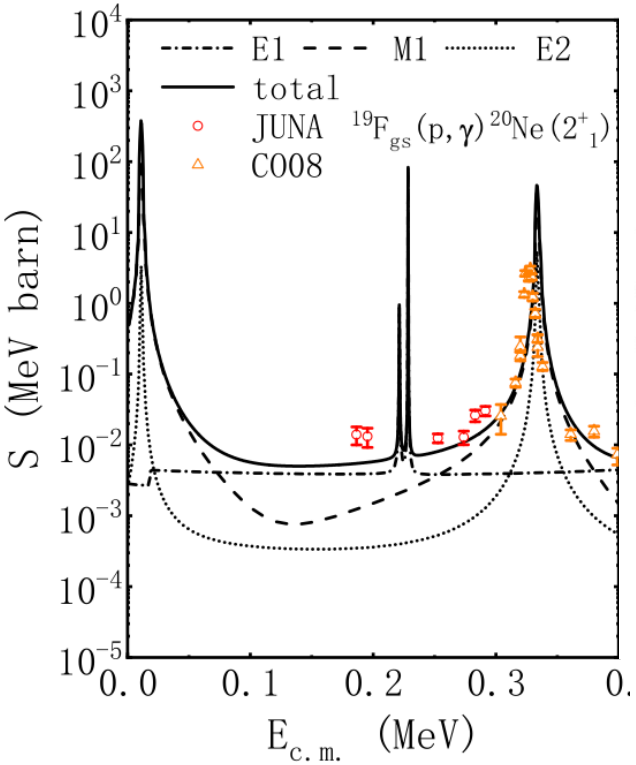
- Interaction through the continuum leads to the *collectivization* of shell model eigenstates and formation of the *aligned eigenstate* which couples strongly to the decay channel(s) and carries many of its characteristics

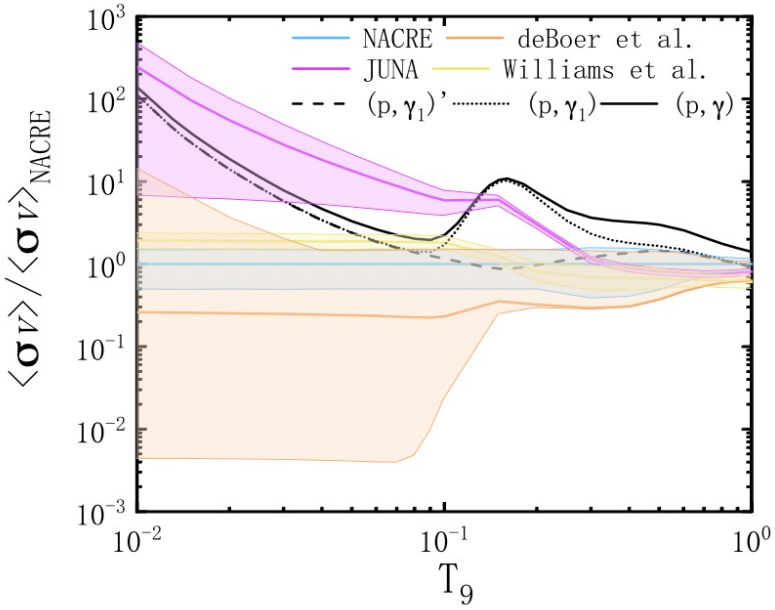
Role of near-threshold resonances in nucleosynthesis

Does $^{19}F(p,\gamma)^{20}Ne$ breakout reaction from the CNO cycle overcomes $^{19}F(p,\alpha)^{16}O$ back-process reaction cross section becoming a source of the Ca abondance in the first generation stars?

R.J. DeBoer et al, Nature 610, 656 (2022)

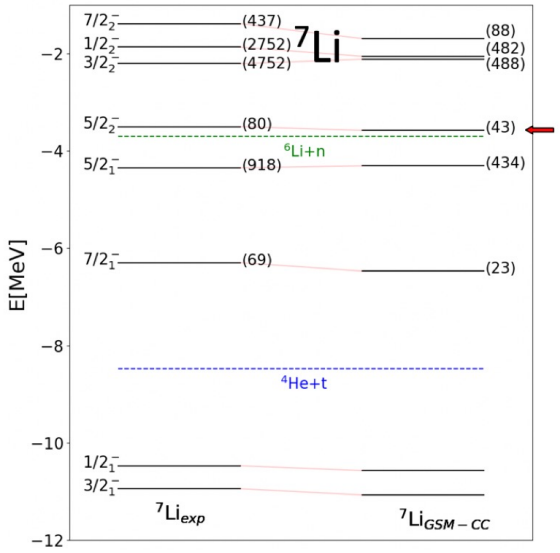




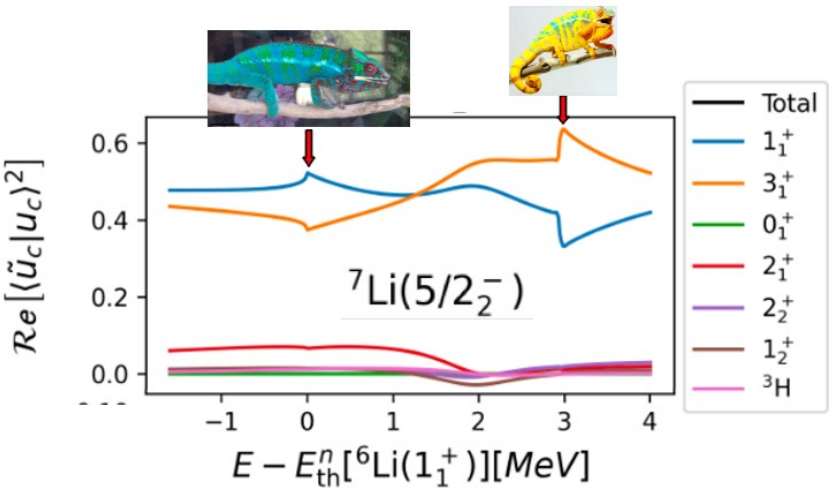


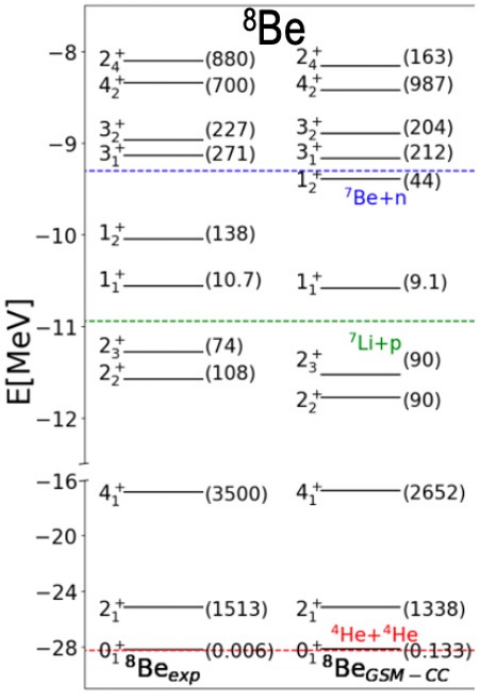
X.B. Wang et al. Phys. Rev C110, L061601 (2024)

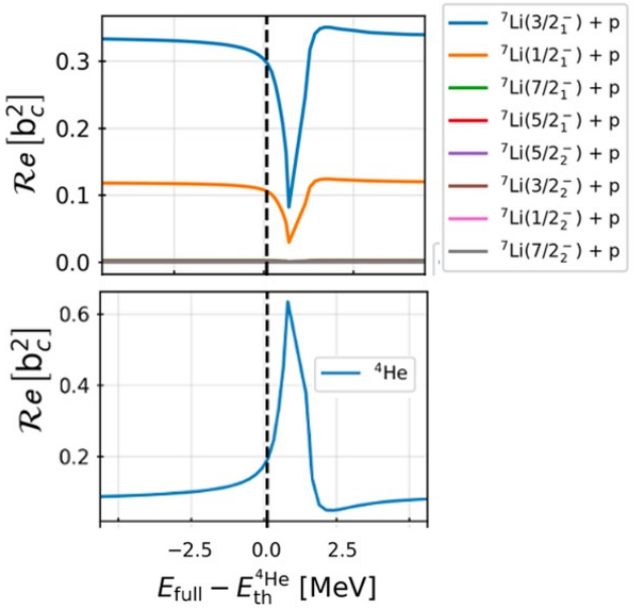
Mimicry mechanism of clusterization



$[|^4\mathrm{He}\rangle \otimes |^3\mathrm{H}\rangle], [|\,^6\mathrm{Li}\,\rangle \otimes |\mathrm{n}\rangle]$







- *Chameleon nature of resonances*
  The resonance changes its structure as a result of the alignment (*mimicry*) with the nearby new reaction channel
- Near-threshold clustering is the *emergent phenomenon*

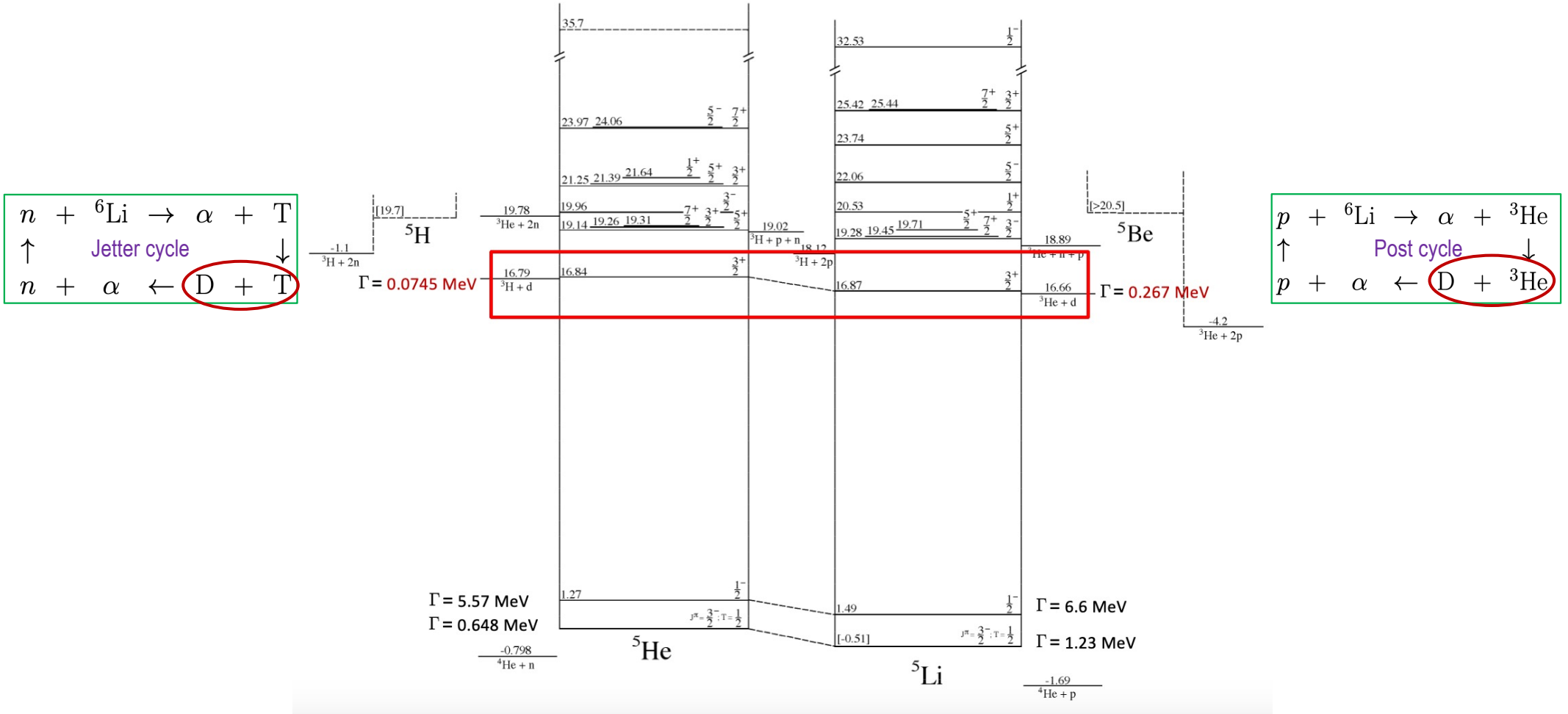
J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

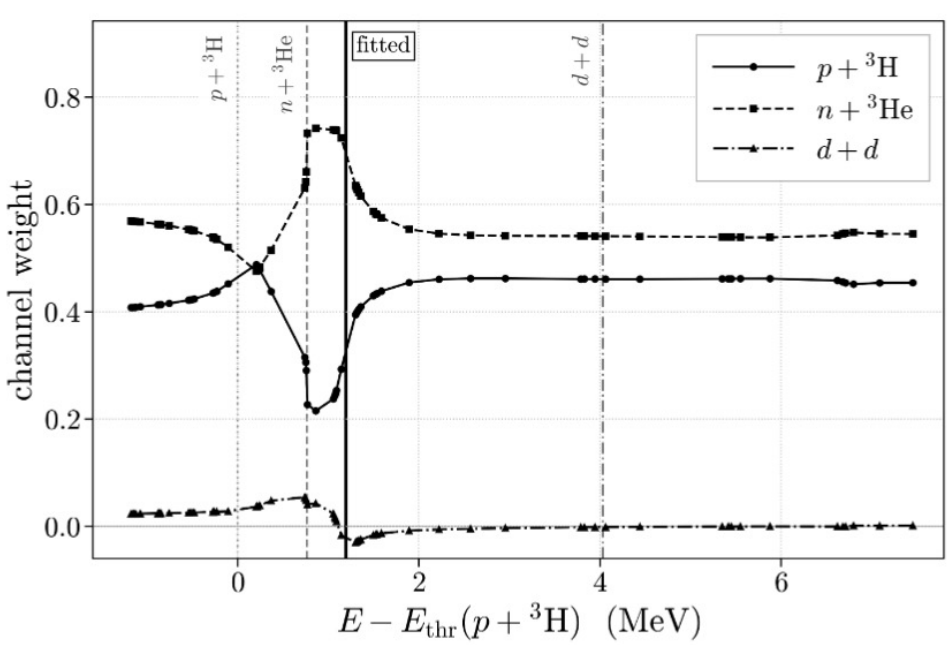
Lectures: 3rd day

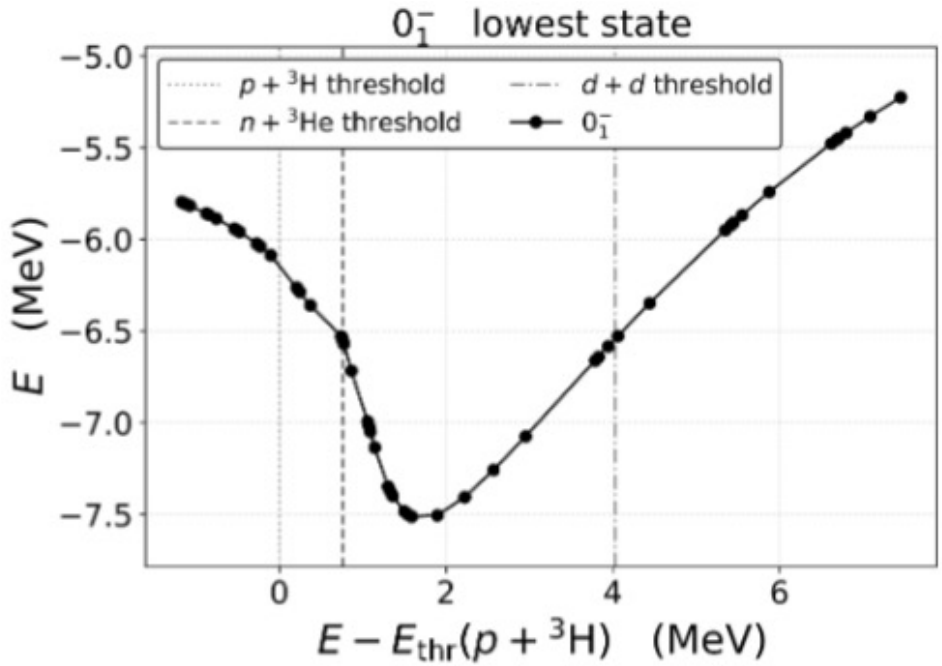
Nicolas Michel	Description of structure and reaction observables of drip line nuclei in Gamow shell model
Lee Sobotka	Threshold phenomena dispassionately divorced from astrophysics
Masaaki Kimura	GCM + Time-Dependent Approach for sub-barrier reaction
Alexis Diaz-Torres	Quantum dynamical modeling of stellar carbon fusion

Role of near-threshold resonances in nucleosynthesis

^{5}He, ^{5}Li and fusion




channel weight
0.0
0.2
0.4
0.6
0.8
0
2
4
6
$E - E_{\mathrm{thr}}(p + {}^3\mathrm{H})$ (MeV)
$p + {}^3\mathrm{H}$
$n + {}^3\mathrm{He}$
$d + d$
fitted
$p + {}^3\mathrm{H}$
$n + {}^3\mathrm{He}$
$d + d$


0_1^- lowest state
E (MeV)
−5.0
−5.5
−6.0
−6.5
−7.0
−7.5
0
2
4
6
$E - E_{\mathrm{thr}}(p + {}^3\mathrm{H})$ (MeV)
$p + {}^3\mathrm{H}$ threshold
$n + {}^3\mathrm{He}$ threshold
$d + d$ threshold
0_1^-