

Ground State \rightarrow Ground State Reactions

CEvNS And Coherent π Production

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Caltech

Neutrino Theory Network

PROGRAM 23-86W | INST. NUCL. THEORY | SEATTLE | NOV 2023



PART 1

COHERENCE



PART 2

LOW ENERGY



PART 3

PION PROD.

- Why are coherent reactions simple?
- When do the $GS \rightarrow GS$ transitions dominate?
- How do we profit from kinematics?

- Coherence from limited kinematics.
- Elastic neutrino and electron scattering on nuclei.
- Higher order corrections & their uncertainties.

- Coherent- π production.
- Comparison vs $\nu e \rightarrow \nu e$ & CCQE.
- Proposal for future work + feedback solicitation.



Coherent Scattering

How Coherence Arises

$$\lim_{Q \rightarrow 0} \int dx \, e^{iQ \cdot x} \langle B | J_{\mu}(x) | A \rangle \propto v_{\mu} \times (\text{some } \#)$$

$$v^{\mu} J_{\mu}(x) = \rho(x) \quad \int dx \, \hat{\rho}(x) = \hat{Q}$$

$$(\text{some } \#) = \langle A | \hat{Q} | B \rangle = Q_A \langle B | A \rangle$$

How Coherence Arises

- Matrix elements are proportional to the charge of a particle in the point-like limit.
- An inherently low- q^2 expansion.

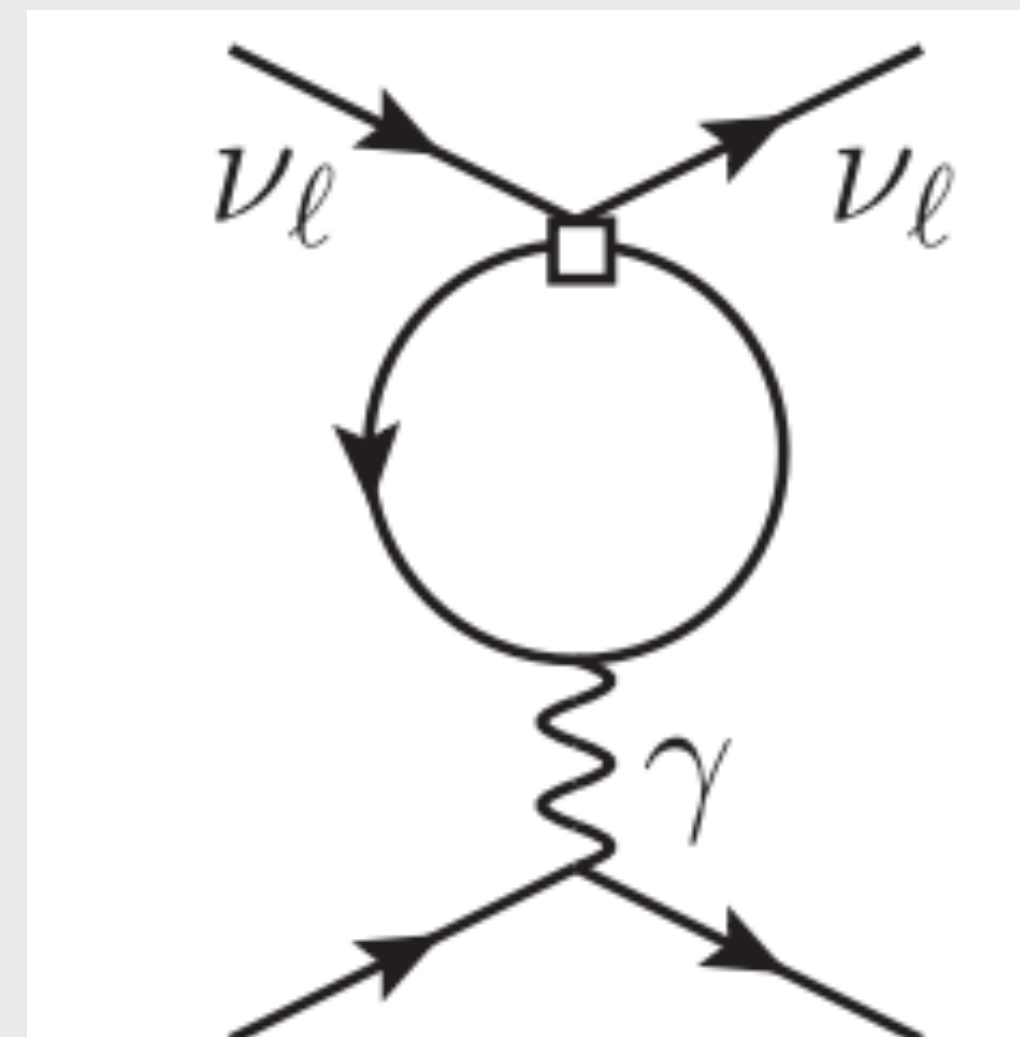
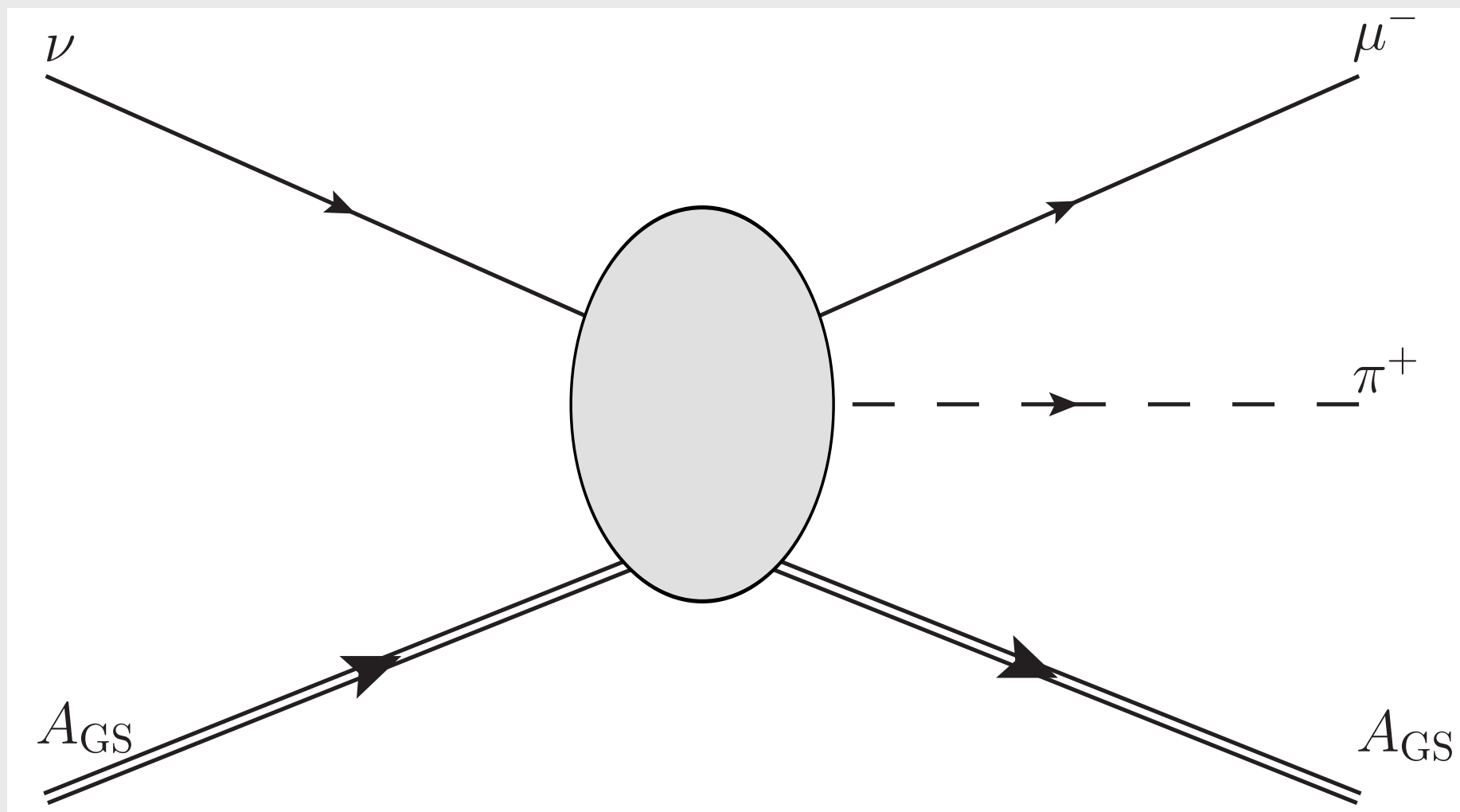


$$Q_A \langle B | \tilde{\rho}(q^2) | A \rangle$$

- "Static" matrix elements

Ground State \rightarrow Ground State

- Matrix elements are simpler in the low- q^2 limit.
- Coherence enhances elastic final states.
Dominates cross section in the relevant phase space.





Coherent Scattering

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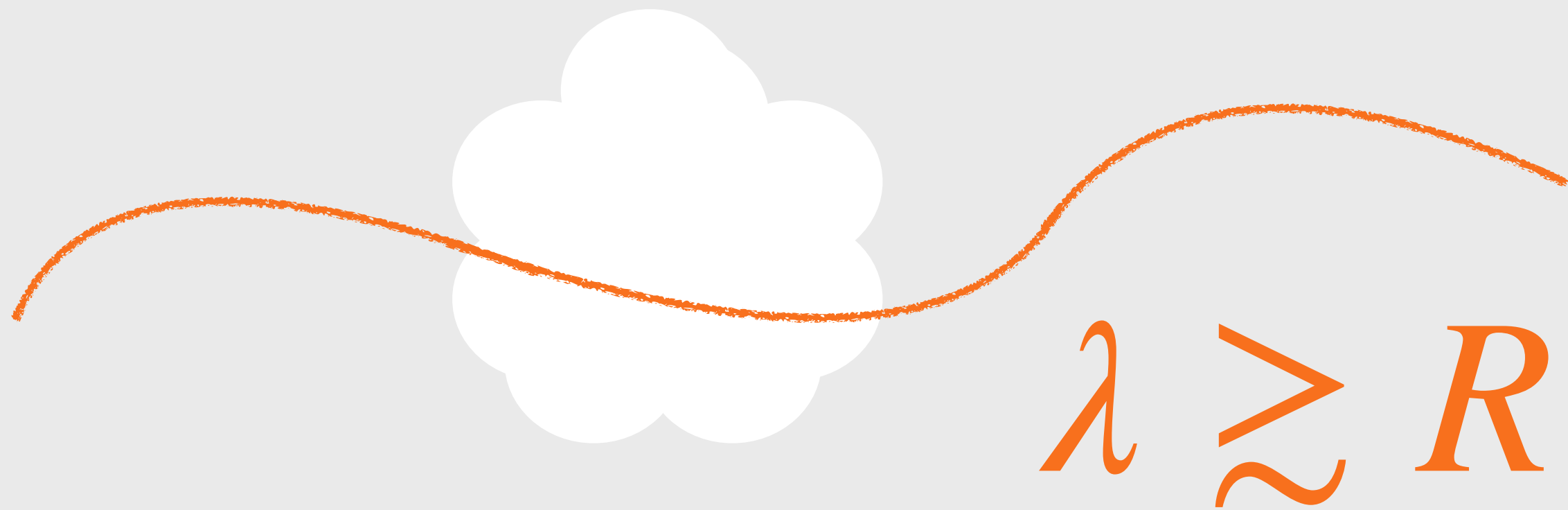


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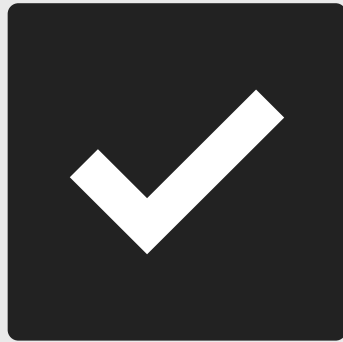
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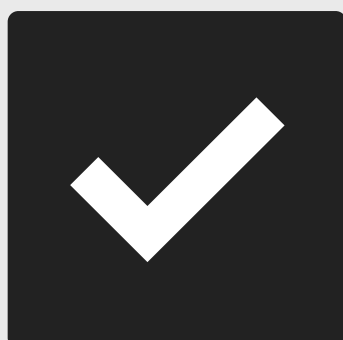
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QED Corrections To CEvNS At $O(\alpha)$

ARXIV:2011.05960

Simplest Coherent Process

$$\nu A_{\text{GS}} \rightarrow \nu A_{\text{GS}}$$

$$\frac{d\sigma_{\nu\ell}}{dT} = \frac{G_{\text{F}}^2 M_{\text{A}}}{4\pi} \left(1 - \frac{T}{E_{\nu}} - \frac{M_{\text{A}} T}{2E_{\nu}^2} \right) F_{\text{W}}(Q^2)$$

TREE LEVEL RESULT

STATIC MATRIX
ELEMENT

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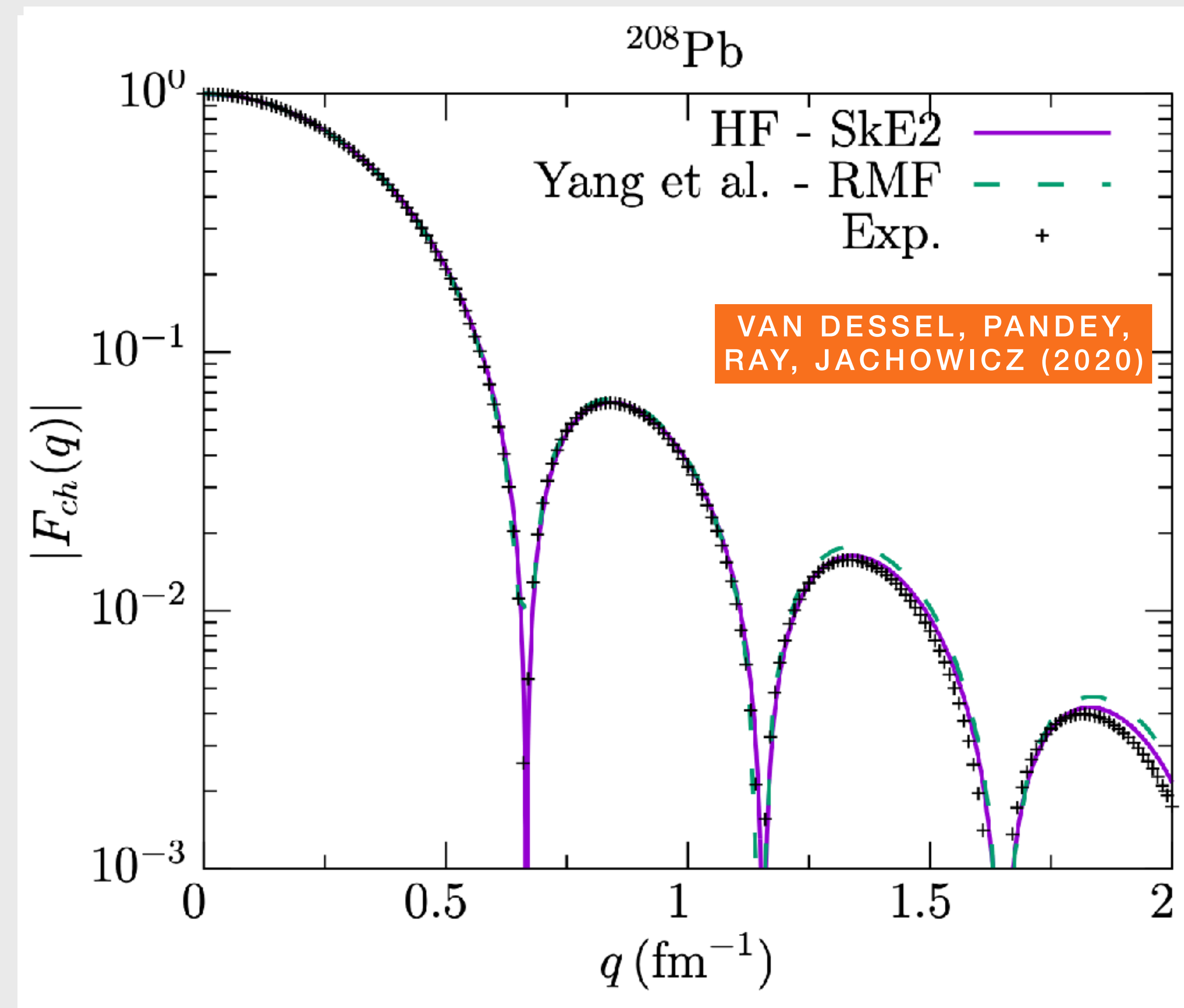
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Weak Form Factor

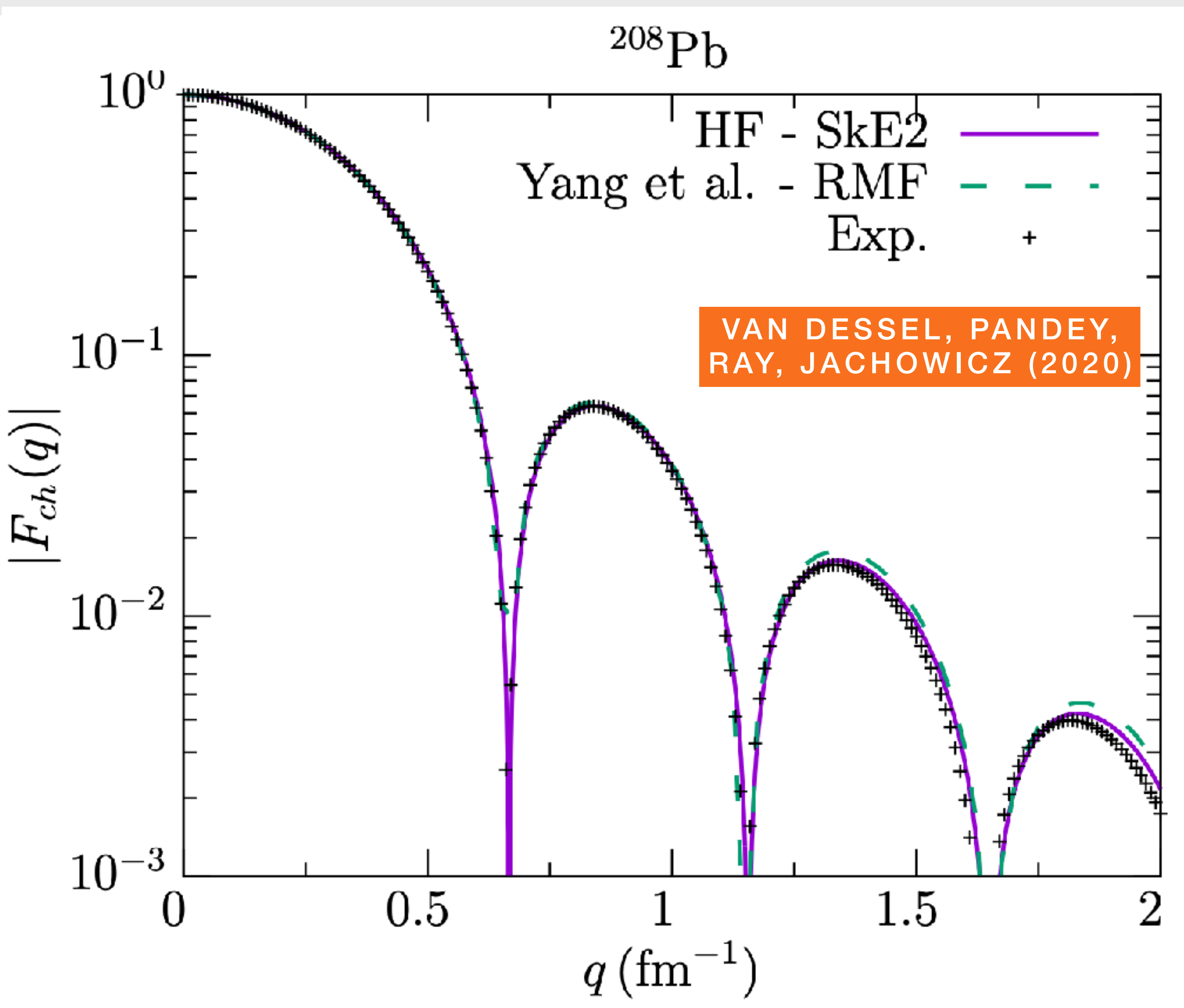
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- Precise predictions available in literature.
- Charge form factor data reproduced extremely well.

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Radiative Corrections

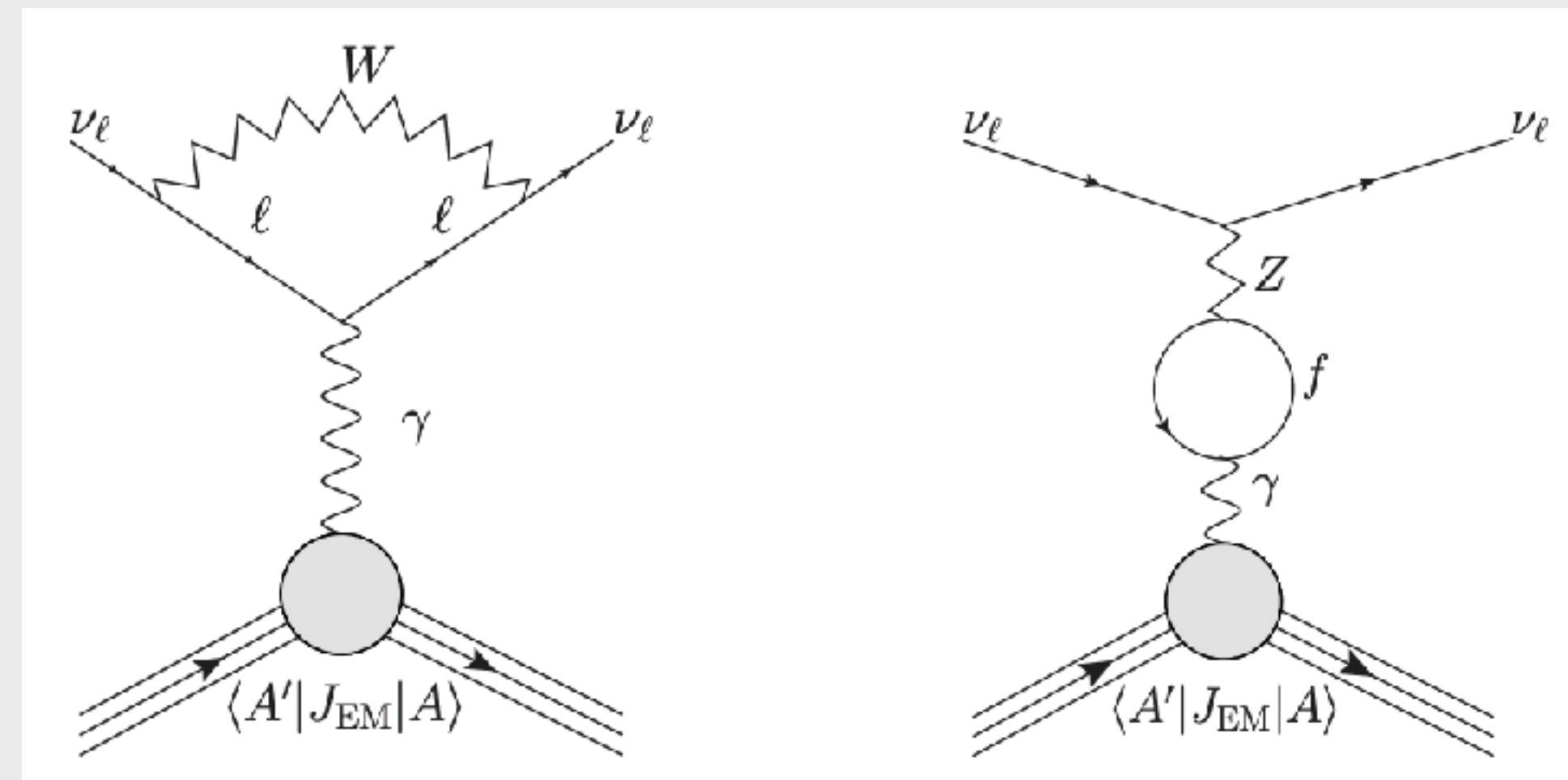
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1-LOOP RESULT

- New matrix element needed at 1-loop.
- But it's the nuclear charge form factor. ✓

STATIC MATRIX ELEMENTS



Radiative Corrections

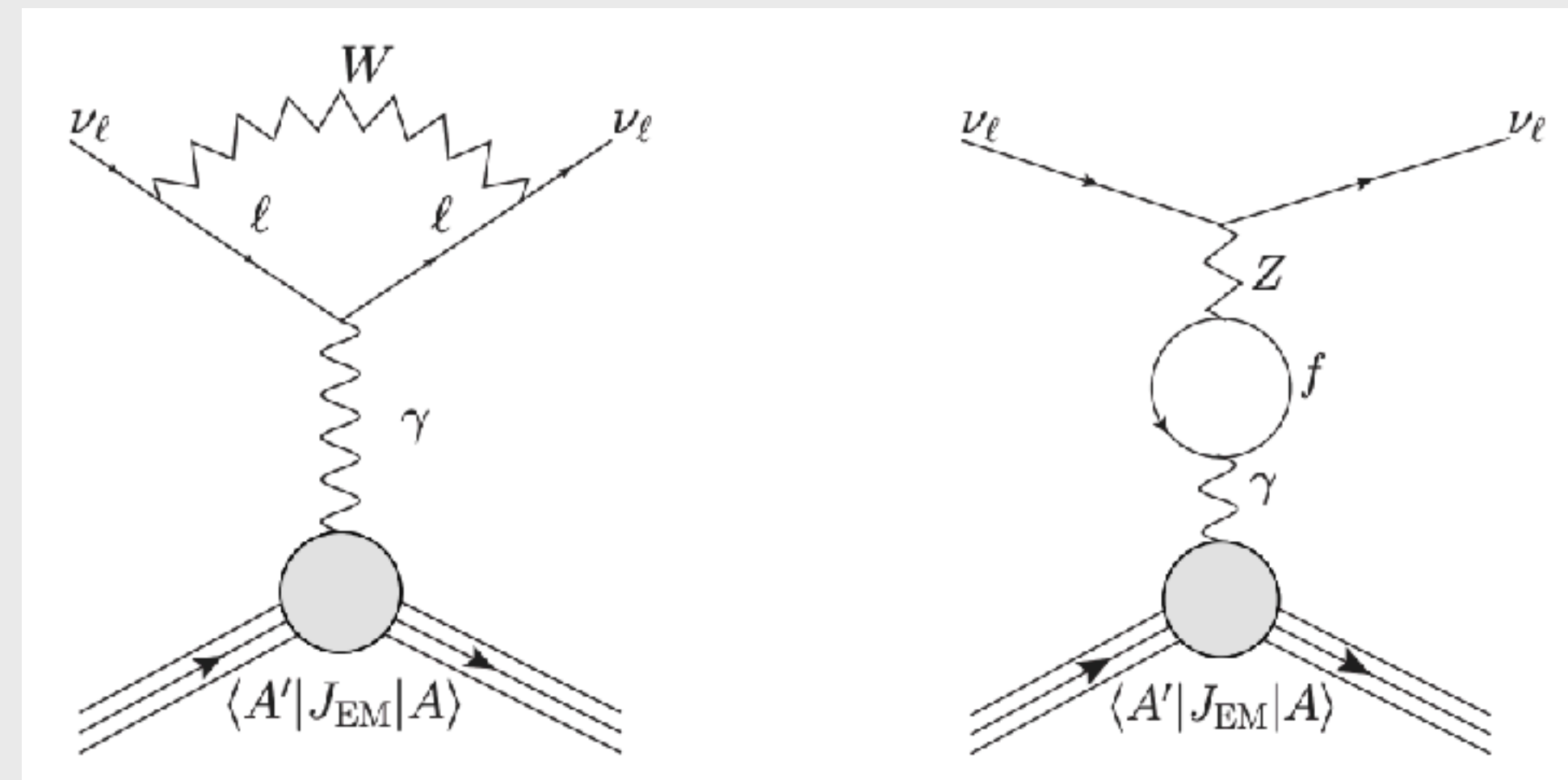
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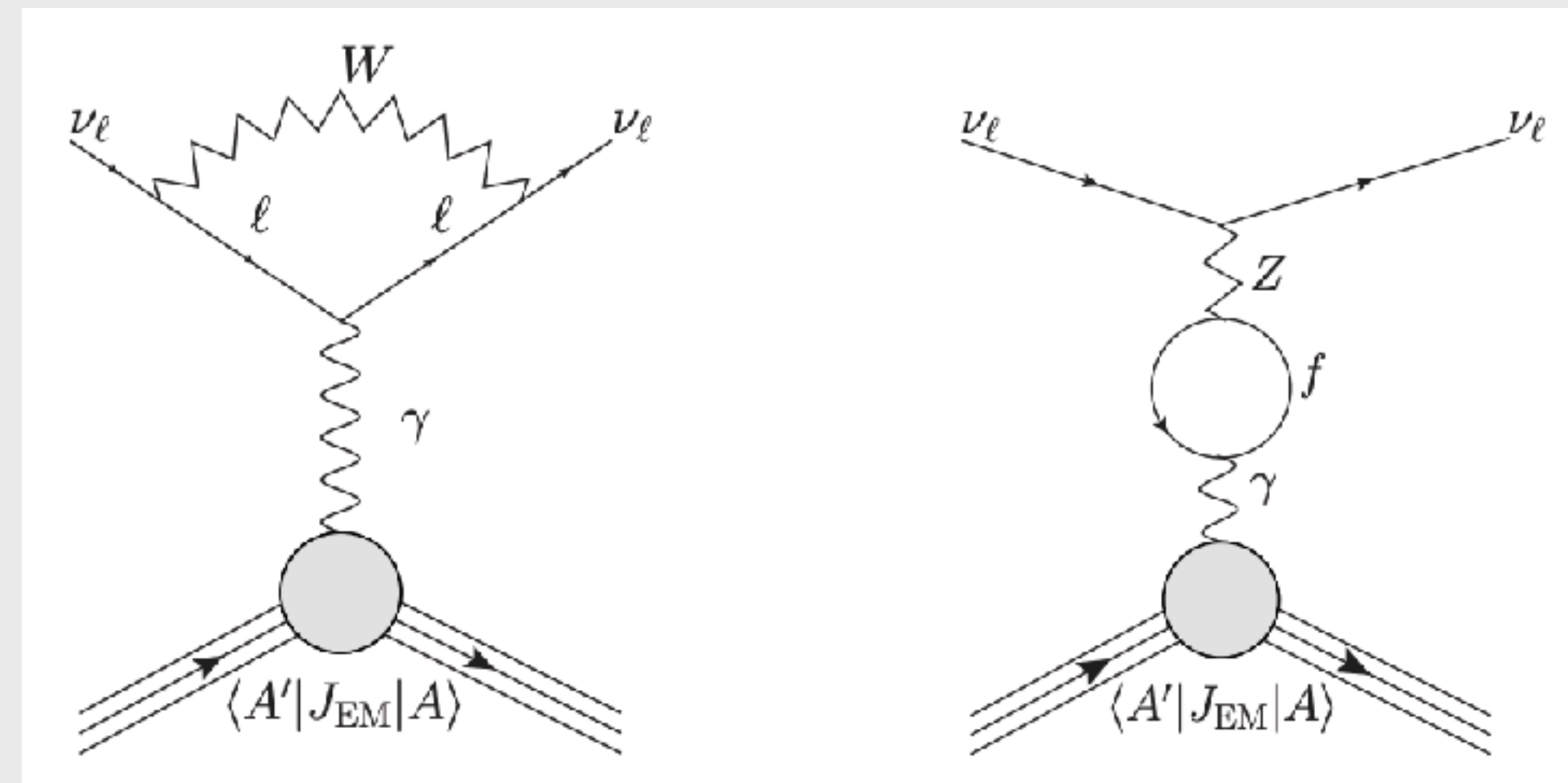
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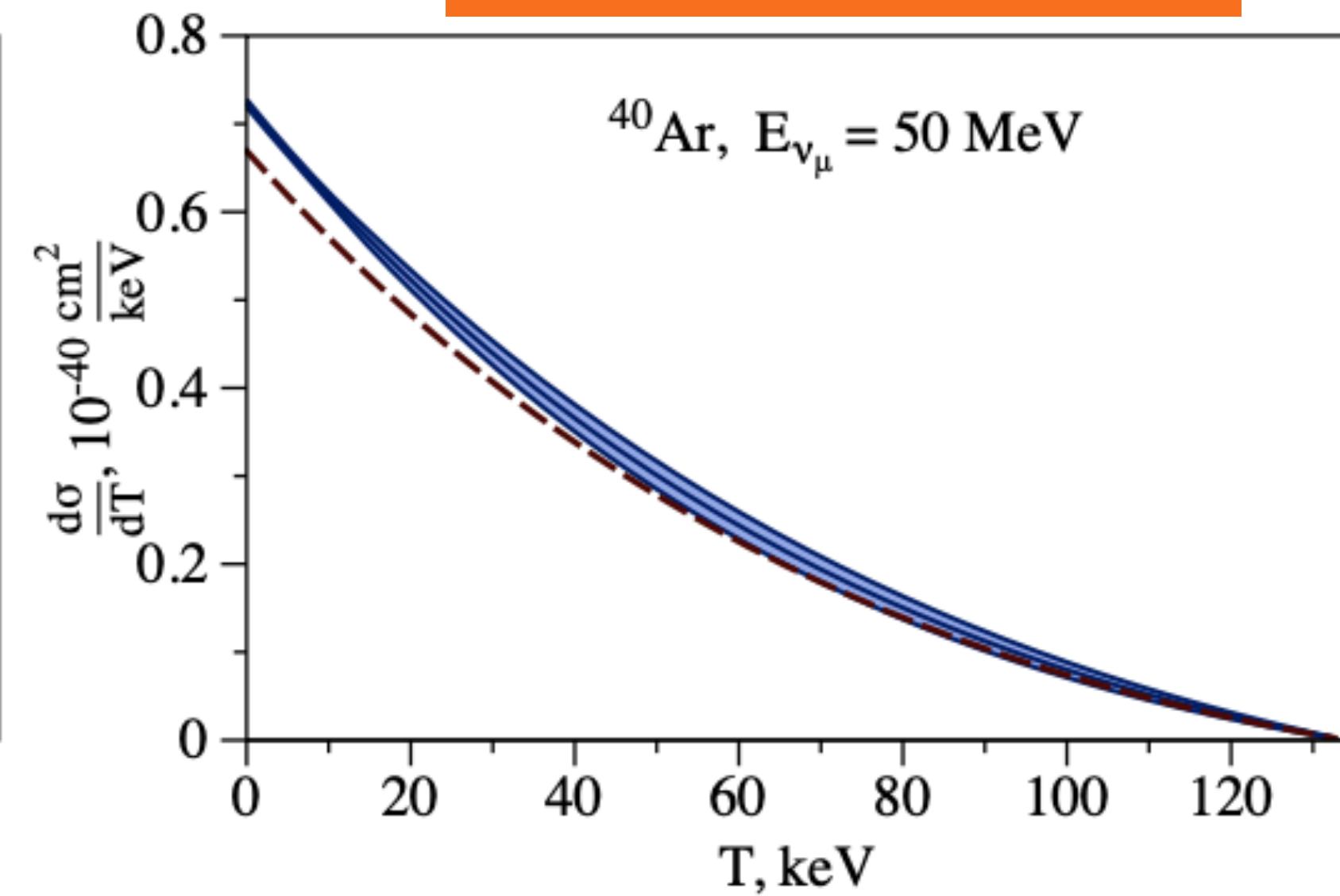
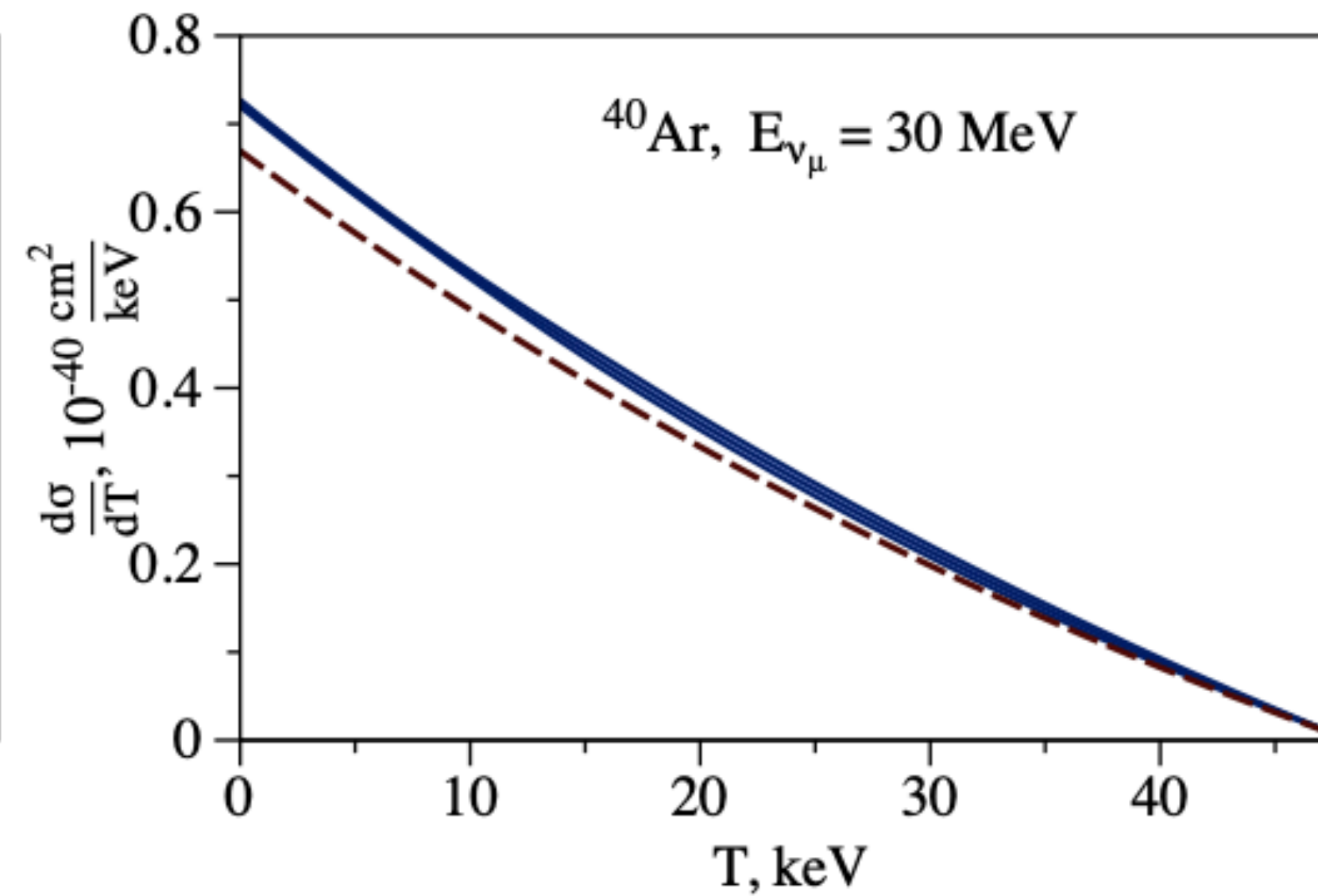
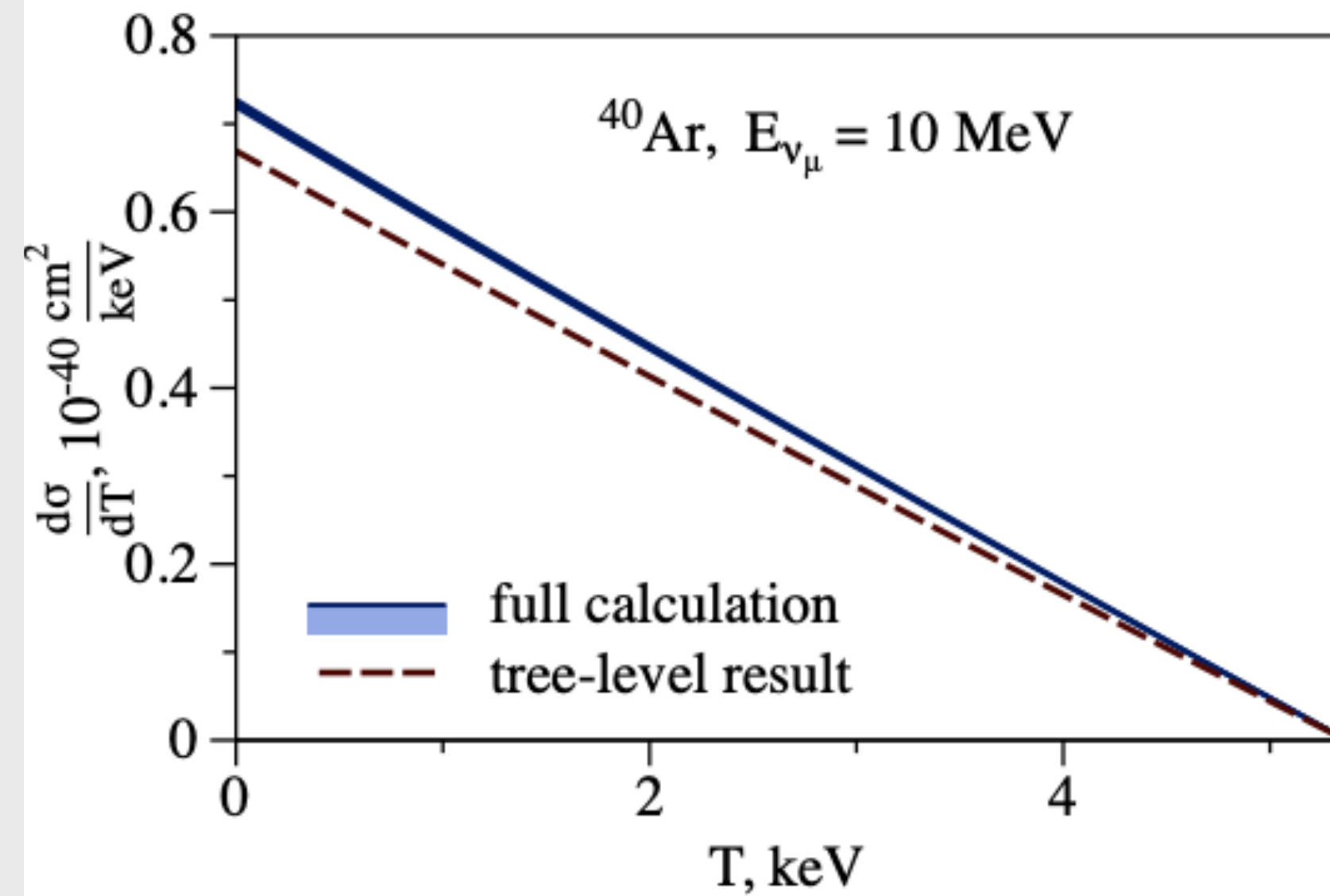
STATIC MATRIX ELEMENTS

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Results For Monoenergetic Neutrino

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$E_\nu, \text{ MeV}$	$10^{40} \cdot \sigma_{\nu\mu}, \text{ cm}^2$	$10^{40} \cdot \sigma_{\nu\mu}^0, \text{ cm}^2$
50	34.64(1.36)	32.05
30	15.37(0.25)	14.23
10	1.91(0.01)	1.77

- Big $\sim 10\%$ corrections.
- Dominated by RG-running.


Construction Of Error Budget

E_ν , MeV	Nuclear	Nucleon	Hadronic	Quark	Pert.	Total	$10^{40} \cdot \sigma_{\nu\mu}$, cm ²	$10^{40} \cdot \sigma_{\nu\mu}^0$, cm ²
50	4.	0.06	0.56	0.13	0.08	4.05	34.64(1.36)	32.05
30	1.5	0.014	0.56	0.13	0.03	1.65	15.37(0.25)	14.23
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- At small Q^2 we use nuclear radii & Taylor series.
- At larger Q^2 we take the largest spread between 8 calculations

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$$G_{E,M}^i(Q^2) = G_{E,M}^i(0) \left[1 - \frac{(r_{E,M}^i)^2}{6} Q^2 + O(Q^4) \right]$$

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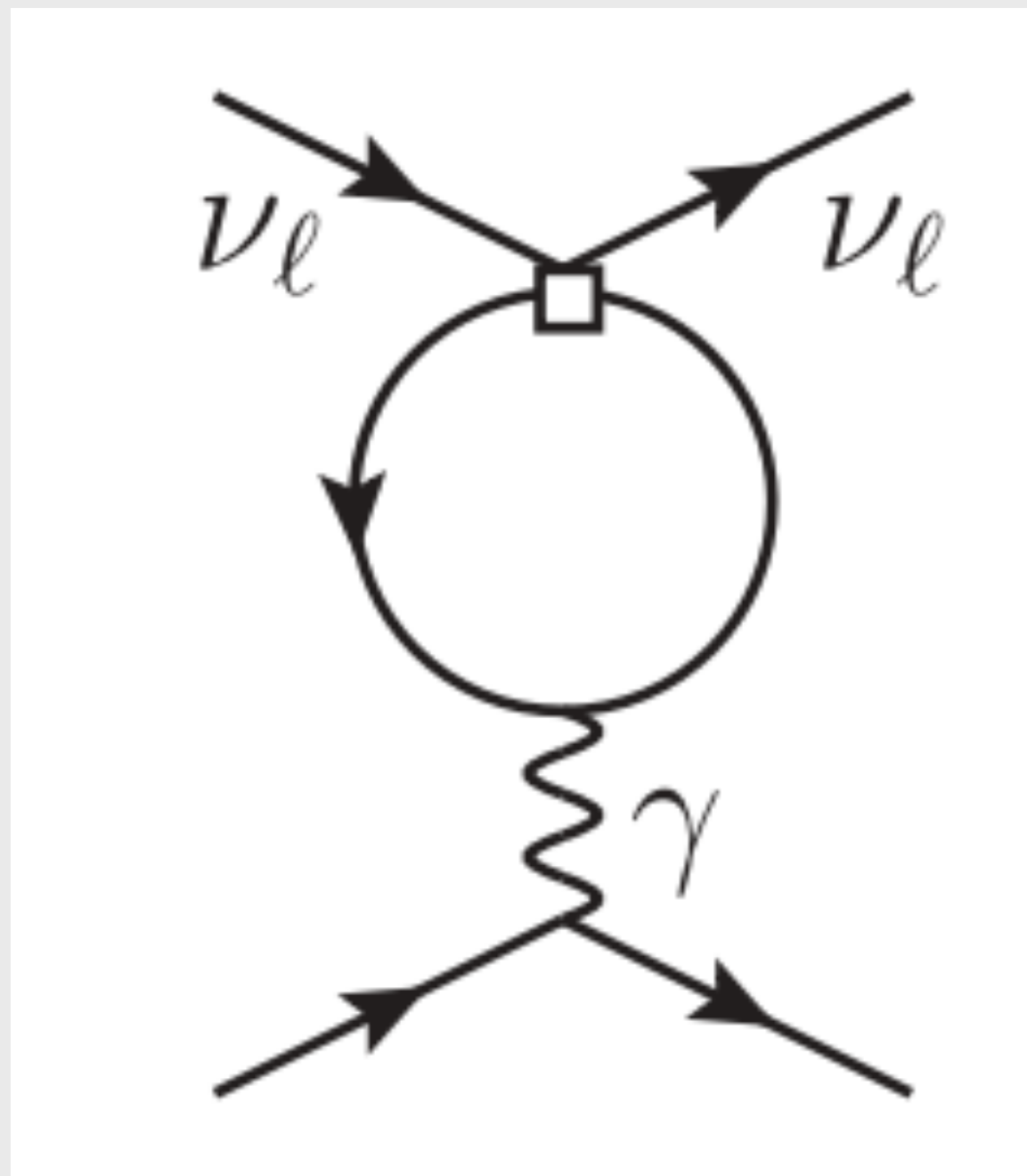


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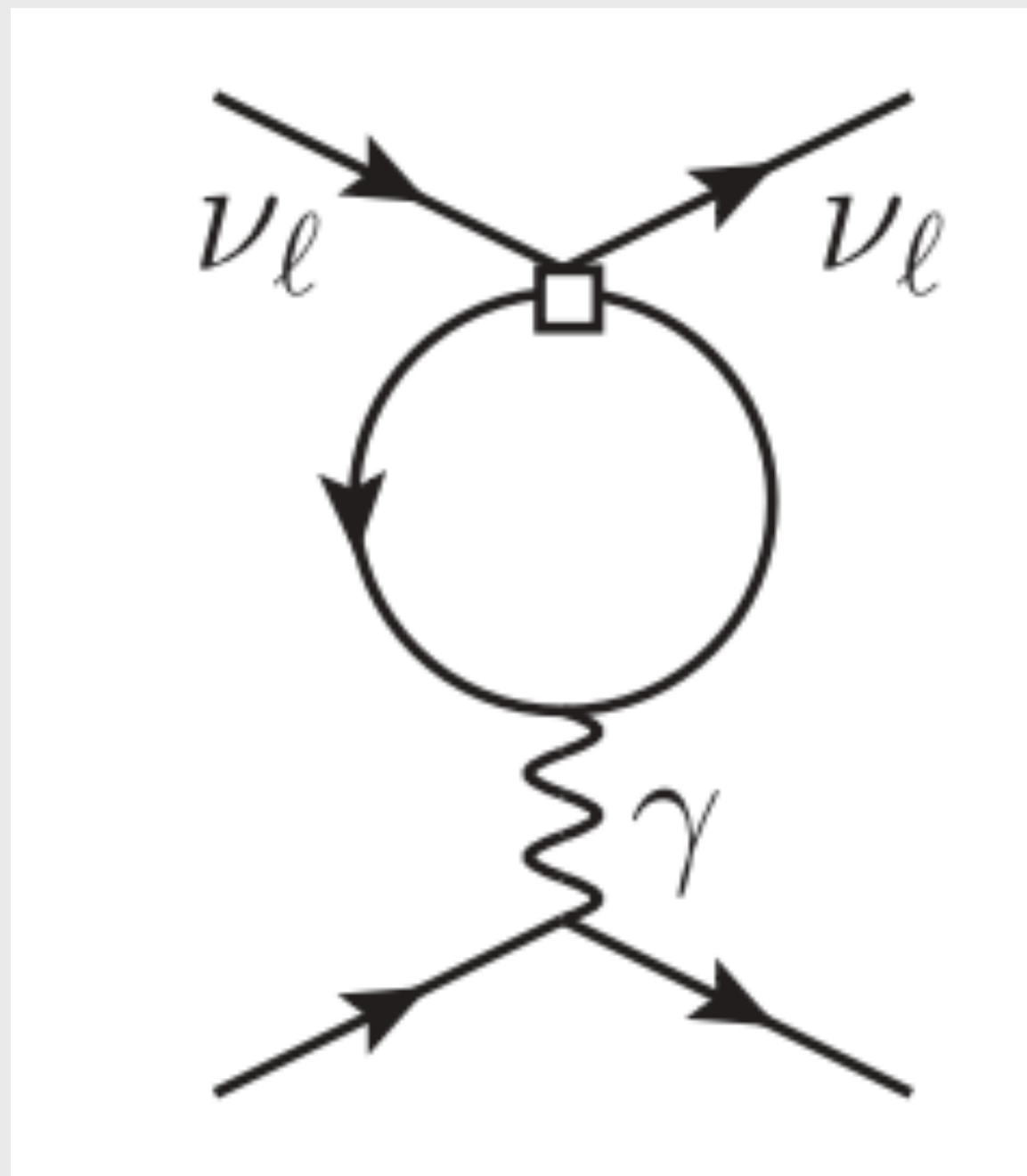
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LOW-SCALE WILSON COEFFICIENTS

- Variation of input parameters.
- Solve RG eq's.
- Add in quadrature.

$c_L^{\nu\ell\ell'}, l = \ell'$	$c_L^{\nu\ell\ell'}, l \neq \ell'$	$c_R^{\nu\ell\ell'}$	c_L^u	c_R^u	c_L^d	c_R^d
2.39818(33)	-0.90084(32)	0.76911(60)	1.14065(13)	-0.51173(38)	-1.41478(12)	0.25617(20)
2.412	-0.887	0.763	1.141	-0.508	-1.395	0.254

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- Vary of renormalization scale.
- Estimate of higher order logarithmic contributions.
- Well established procedure (with good empirical results) in EFT/pQCD literature

$$1 + \alpha(\mu)(\log \mu + \#)$$

$$+ \alpha^2(\mu)(\log^2 \mu + \log \mu + \#)$$


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Coulomb Corrections To CEvNS

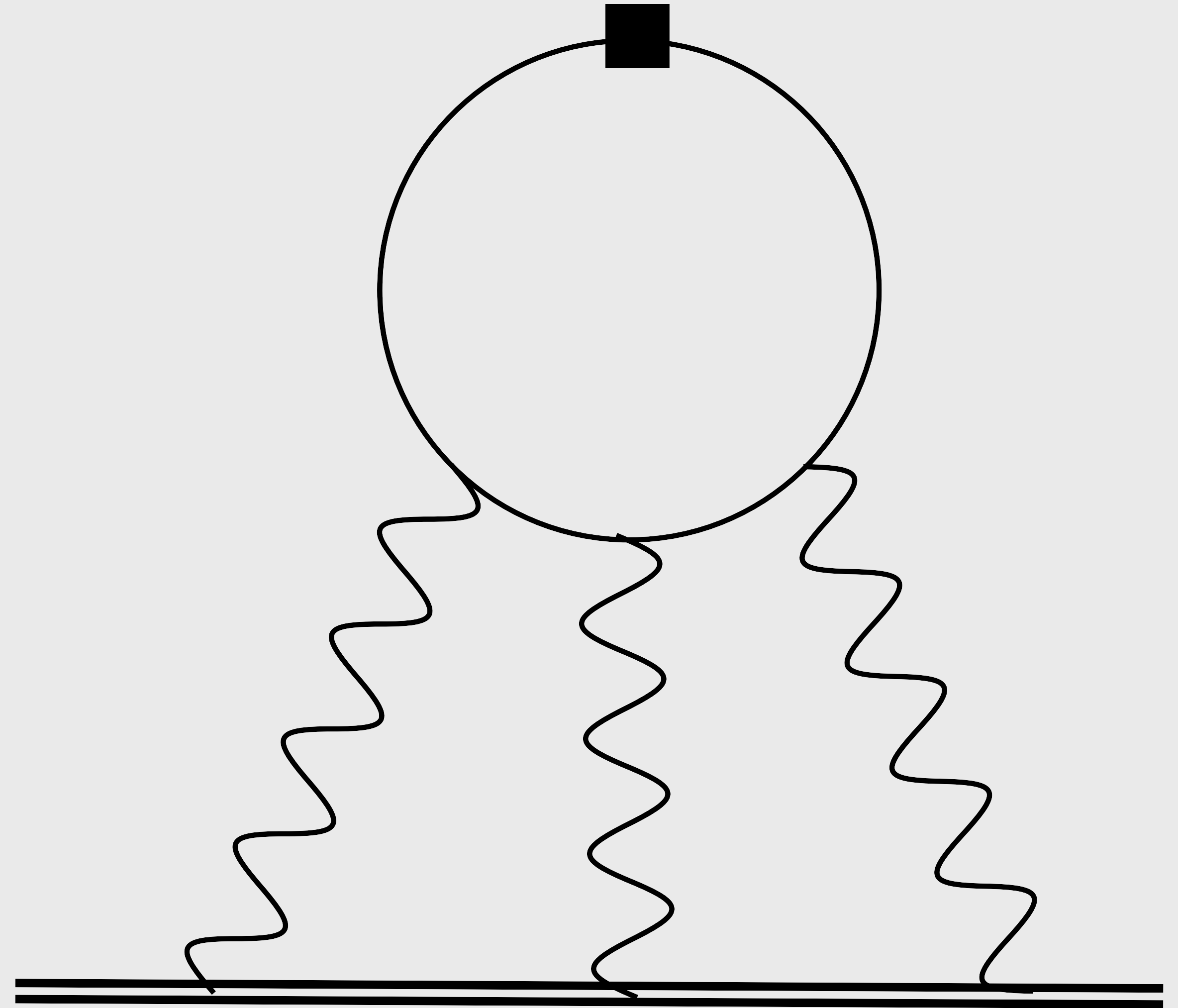
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Radiative Corrections For Precision CEvNS

- Separation of scales and point-like expansion offers clean SM prediction.
- COHERENT aims for a future percent level measurement.
- Natural question: are we missing anything?

Coulomb Corrections & CEvNS

- Nature of observable requires using heavy nuclei with large Z .
- Higher orders in $Z\alpha$ are not necessarily small.
- This is a 3-loop effect. Not included in RG-running etc. of EFT analysis.



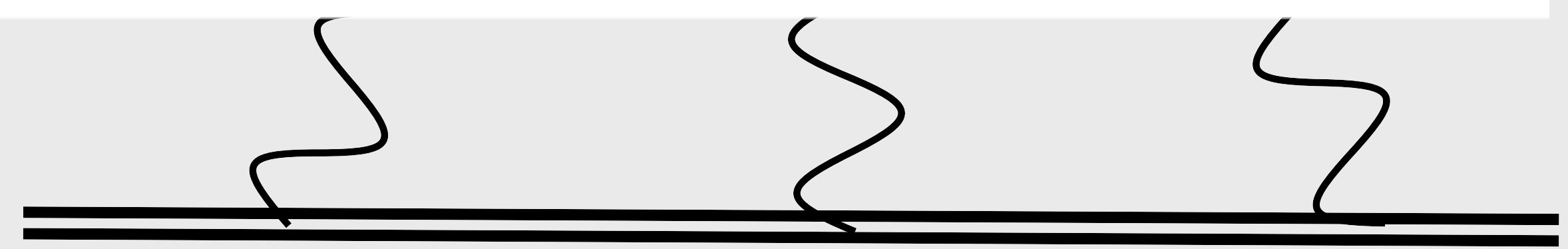
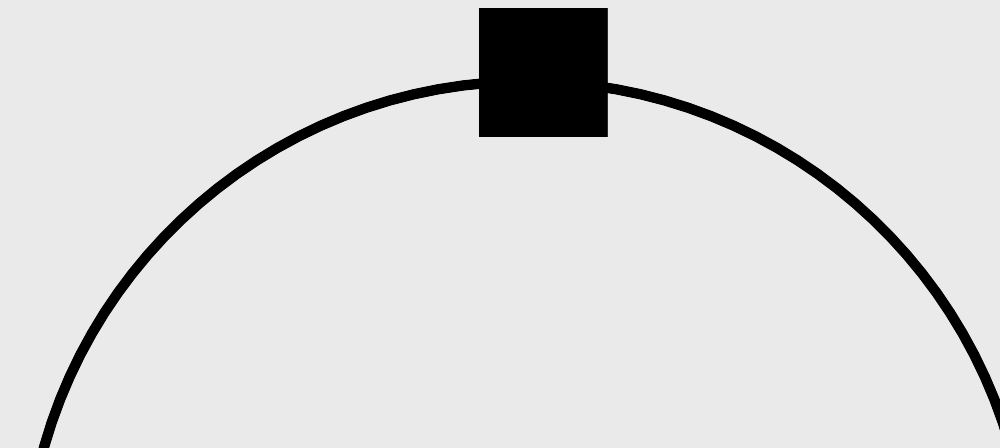
Coulomb Corrections & CEvNS

- Nature of observable

$$\Pi_{\text{WK}}(Q^2, m_f) = \frac{1}{2} \int_0^\infty d\zeta \frac{Q^2}{Q^2 + 4\zeta^2 m_f^2} \frac{1}{\zeta^4}$$

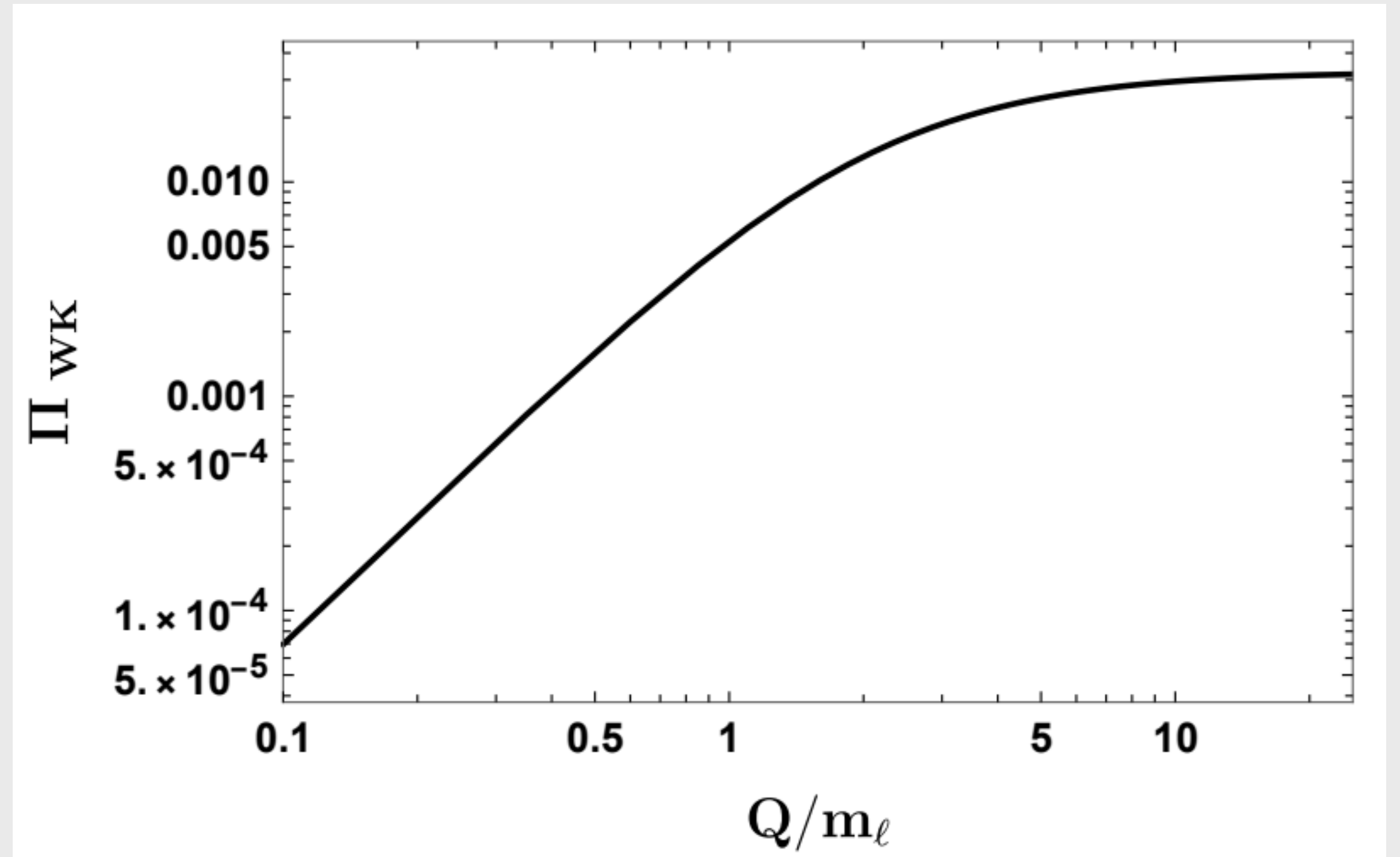
- $\left[F(\zeta) - \frac{\pi^2}{12} \sqrt{\zeta^2 - 1} \Theta(\zeta - 1) \right].$

- included in RG-running etc. of EFT analysis.



Results For CEvNS

- Result is very small.
- Not obvious. Numerical cancellation of large numbers.



$$\lim_{Q \rightarrow \infty} \Pi_{WK}(Q/m_e) = \frac{1}{2} \left[\frac{\pi^2}{6} - \frac{2}{3} \zeta(3) - \frac{7}{9} \right] \approx 0.03 .$$

Results For CEvNS

- Coefficient of $(Z\alpha)^2$ turns out to be ~ 100 times smaller than the 1-loop coefficient.
- Negligible for any realistic analysis.

Q	Asymmetry	$\mathcal{A}_{\ell,\ell'}^{(0)}$	$\mathcal{A}_{\ell,\ell'}^{(1)}$
1 MeV	$\ell = e \quad \ell' = \mu$	3.36	0.02
	$\ell = e \quad \ell' = \tau$	5.24	0.02
	$\ell = \mu \quad \ell' = \tau$	1.88	—
10 MeV	$\ell = e \quad \ell' = \mu$	2.12	0.03
	$\ell = e \quad \ell' = \tau$	4.00	0.03
	$\ell = \mu \quad \ell' = \tau$	1.88	—
30 MeV	$\ell = e \quad \ell' = \mu$	1.40	0.03
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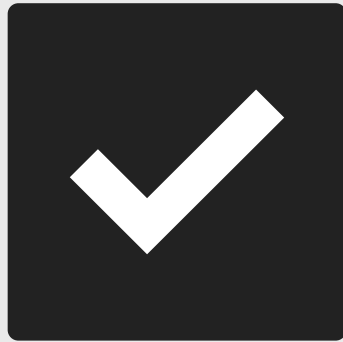
Other Necessities For Precision CEvNS

Radiative Corrections To Neutrino Spectra

- Pion and muon decays also modified by radiative corrections.
- Effects are calculated by O. Tomalak in [arXiv:2112.12395](https://arxiv.org/abs/2112.12395).
- Small effects $\sim 0.5\%$ due to lack of logarithmic enhancements.

Inelastic Backgrounds

- Subdominant due to absence of coherence.
- Need to be "invisible" channels.
- See talk by V. Pandey and [arXiv:2206.08590](https://arxiv.org/abs/2206.08590)



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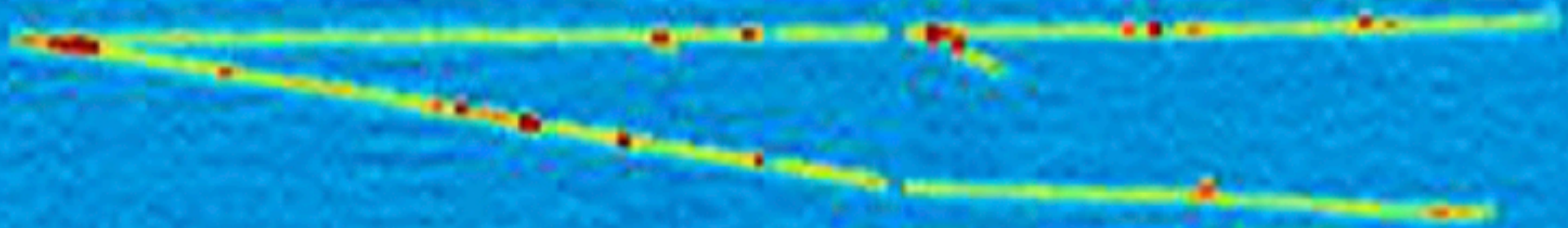
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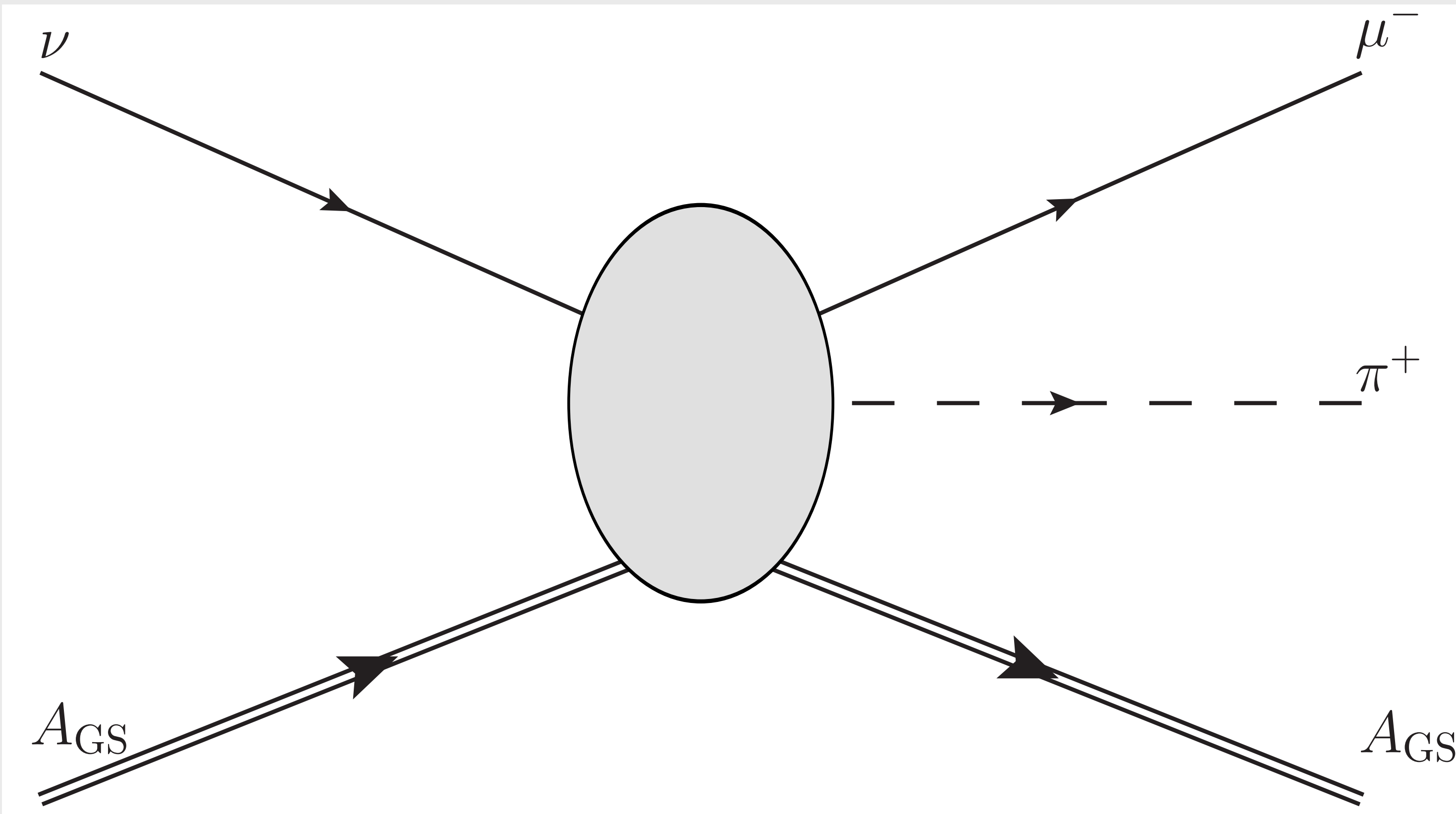
Coherent Pion Production

$$\nu A_{\text{GS}} \rightarrow \ell \pi A_{\text{GS}}$$

$$\nu A_{\text{GS}} \rightarrow \ell \pi A_{\text{GS}}$$

HADRONIC \rightarrow NAIVELY COMPLICATED

COMPELLING FOR E_ν



- Pion and muon reconstructable.
- Minimal energy transfer to nucleus.

This Channel Is Worth Pursuing

- In the chiral limit, $m_\pi^2 = 0$, amplitude is related to $\pi A_{\text{GS}} \rightarrow \pi A_{\text{GS}}$ elastic scattering.
- Corrections to this limit are controlled by small parameters e.g. m_π^2/E_ν^2 .
- Amplitude can be systematically expanded (in terms of hadronic matrix elements).

Disclaimer: Work In Progress. No Warranty Implied

This Part Of The Talk Is Propaganda

GOALS

- Convince you that a *bonafide* prediction of coherent pion production is possible within a systematic framework.
- Motivate experimental interest in:
 1. Do flux measurements with coherent pion production.
 2. Measure elastic pion scattering (differential in Q^2)
- Motivate theory work for necessary hadronic modeling.

1) Kinematics

$$E_\ell + E_\pi + T_A = E_\nu \implies p_\nu = (E_\nu, 0, 0, E_\nu)$$

$$p_{\pi\ell}^\mu = p_\pi^\mu + p_\ell^\mu \implies m_{\pi\ell}^2$$

$$m_{\pi\ell}^2 \ \& \ \mathbf{p}_{\pi\ell} \ \& \ \mathbf{p}_\nu \implies Q^2 = (\mathbf{p}'_A - \mathbf{p}_A)^2$$

FULLY RECONSTRUCTABLE KINEMATICS!

2) Statistics

- A good "standard candle" has two features:
 1. Good statistics.
 2. "Precision" theory.



Number of events

$$\nu e \rightarrow \nu e$$

$$E_\nu \sigma \sim 10^{-42} \text{ cm}^2 \text{ GeV}$$

$$\sim 3 \times 10^3 \text{ /yr}$$

$$\nu A \rightarrow \ell \pi A$$

$$E_\nu \sigma \sim 10^{-40} \text{ cm}^2 \text{ GeV}$$

$$\sim 3 \times 10^5 \text{ /yr}$$

$$\nu A \rightarrow \ell X$$

$$E_\nu \sigma \sim 10^{-38} \text{ cm}^2 \text{ GeV}$$

$$\sim 10^8 \text{ /yr}$$

Events
DUNE

3) Chiral Symmetries

- Matrix element dominated by longitudinal axial current

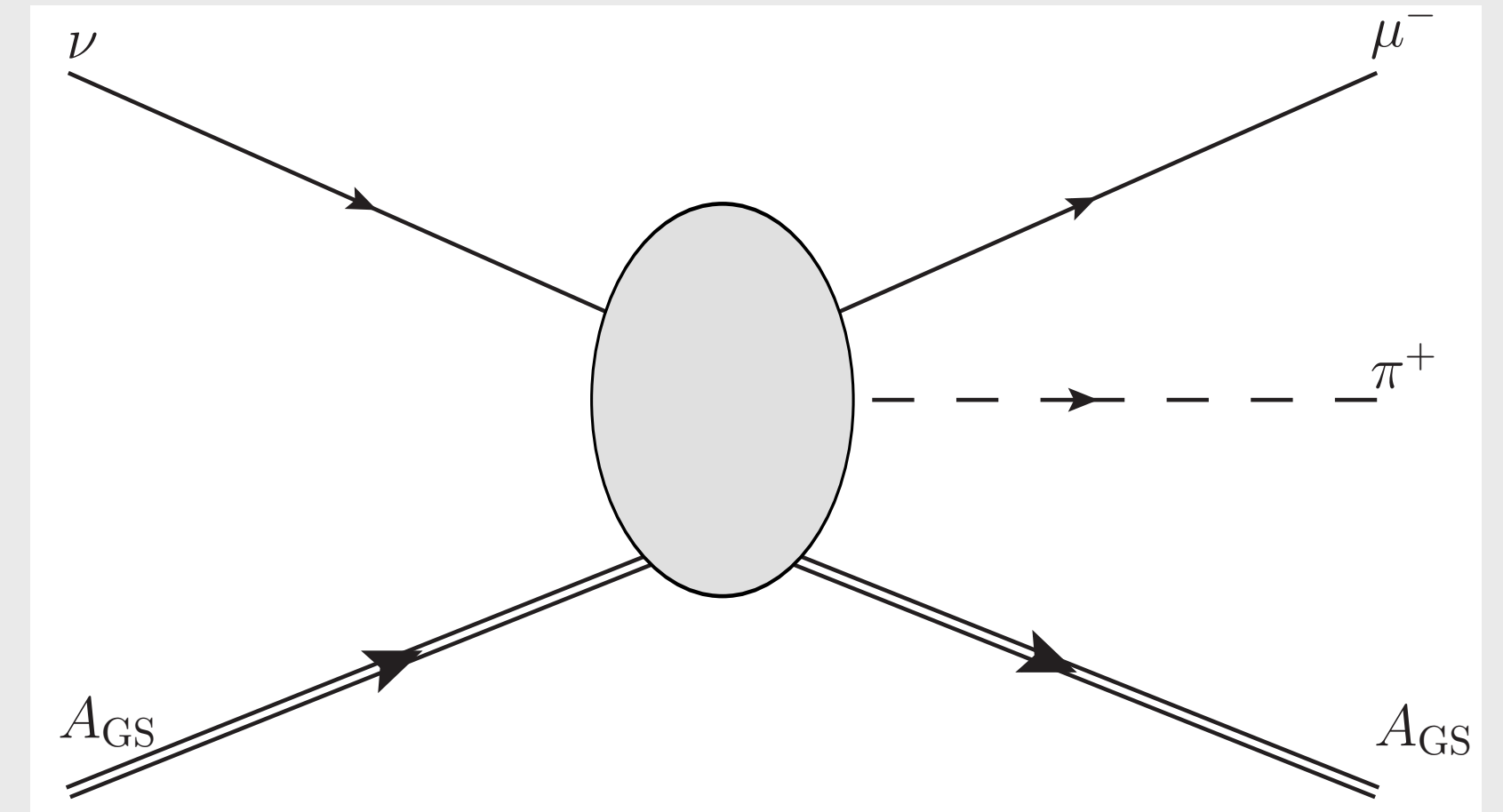
$$\langle \pi A | J_{\mu}^A | A \rangle \epsilon_L^{\mu} = \epsilon_L^{\mu} A_{\mu}^{\text{reg}}$$

- In chiral limit

$$\kappa^{\mu} A_{\mu}^{\text{reg}} = f_{\pi} T_{A\pi}$$

- For $|\mathbf{Q}| \ll |\mathbf{p}_{\pi}|$

$$\epsilon_L^{\mu} A_{\mu}^{\text{reg}} \approx \kappa^{\mu} A_{\mu}^{\text{reg}}$$



$$\langle \pi A | J_{\mu}^A | A \rangle = \frac{f_{\pi} \kappa_{\mu}}{\kappa^2} T_{A\pi} + A_{\mu}^{\text{reg}}$$

$$\epsilon_L^{\mu} \kappa_{\mu} = 0$$

ADLER (1964), RHEIN & SEHGAL (1983),
KARTAVTSEV, PASCHOS & GOUNARIS (2006)

Elastic Pion Nucleus Scattering

- At leading order in m_π^2/E_ν^2 we need data-driven input.
- Therefore need ***pion-nucleus*** elastic scattering data.
- Must match relevant experimental targets.

USE NEW
DATA FROM
EMPHATIC &
NA61/SHINE

EXPRESION OF INTEREST

Measurements of kaon and pion scattering in the WCTE facility at CERN

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Safe Vs Dangerous Phase Space

$$\sigma_{\text{safe}}(E_\nu) = \int_{\text{safe}} d\Pi \sum_{\text{spins}} |\mathcal{M}|^2$$

SAFE := WHERE
APPROX. WORK

$$\sigma_{\text{safe}}(E_\nu) \leq \sigma(E_\nu)$$

$$\Phi(E_\nu) = N_{\text{safe}}(E_\nu) / \sigma_{\text{safe}}(E_\nu)$$

Safe Phase Space

- Both lepton and pion should be forward going.
- Small opening angles.

$$E_{\pi}^2 \gg m_{\ell\pi}^2 \gg Q^2$$

$$\theta_{\ell\pi} \lesssim \frac{(200 \text{ MeV})}{E_{\nu}}$$

- Small momentum transfers.

Summary Of The Propaganda

- A *bonafide* prediction of coherent pion production is possible within a systematic framework.
- I think there should be experimental interest in:
 1. Flux measurements with coherent pion production.
 2. Measure elastic pion scattering (differential in Q^2).
- More theory work is needed.