Heavy quark production and TMD physics at the EIC

Cristian Pisano

University and INFN Cagliari

BNL-INT Joint Workshop:
Bridging Theory and Experiment
at the Electron-Ion Collider

INSTITUTE for NUCLEAR THEORY

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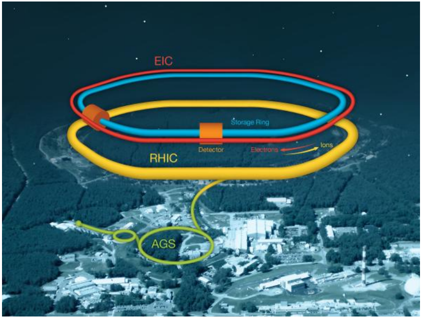
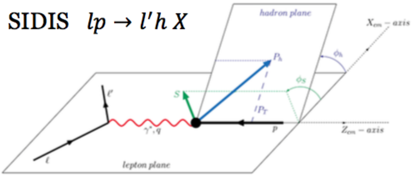
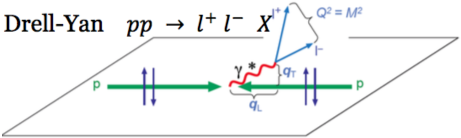


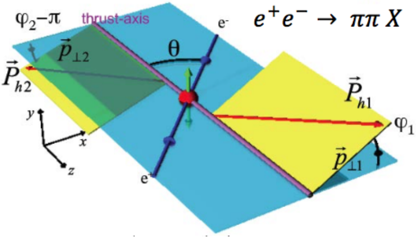
INFN CAGLIARI

Quark TMDs

Two scale processes $Q^2 \gg q_T^2$







Factorization proven

All orders in α_s

Leading order in powers of $1/Q$ (twist)

Collins, Cambridge University Press (2011)
Boussarie et al, TMD handbook 2304.03302

QUARKS	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T}, $h_{1T}^\perp$

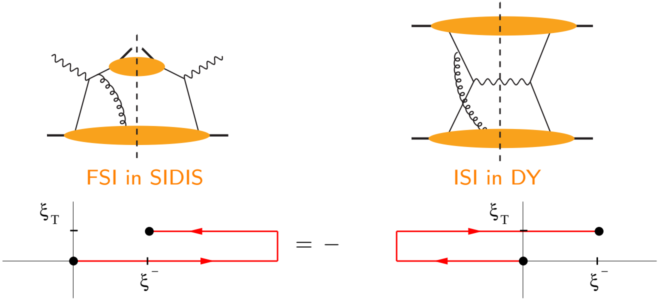
Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Tangerman, NPB 461 (1996)
Boer, Mulders, PRD 57 (1998)

- $h_1^{\perp\, q}$: T-odd distribution of transversely polarized quarks inside an unp. hadron
- $f_{1T}^{\perp\, g}$: T-odd distributions of unp. gluons inside a transversely pol. hadron
- h_{1T}^q, $h_{1T}^{\perp\, q}$: helicity flip distributions: T-even and chiral odd
- Transversity $h_1^q \equiv h_{1T}^q + \frac{\boldsymbol{p}_T^2}{2M_p^2}\, h_{1T}^{\perp\, q}$ survives under p_T integration

They are all known and can all be accessed in SIDIS (mostly COMPASS data)

Gauge invariant definition of Φ (not unique)

$$\Phi^{[\mathcal{U}]} \propto \left\langle P, S \left| \overline{\psi}(0)\, \mathcal{U}^{\mathcal{C}}_{[0,\xi]}\, \psi(\xi) \right| P, S \right\rangle$$



Sign change of T-odd distributions: fundamental test, still under experimental scrutiny

ISI/FSI lead to process dependence of TMDs, could even break factorization

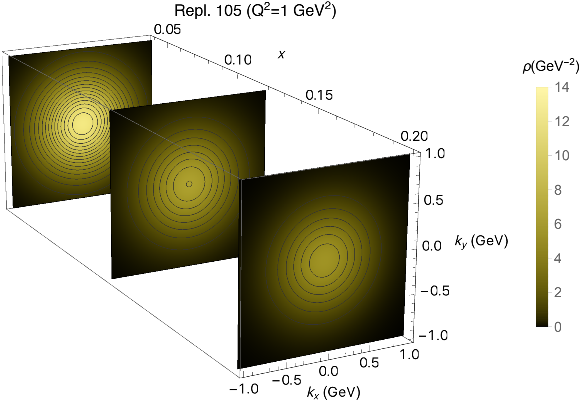
Collins, Qiu, PRD 75 (2007)
Collins, PRD 77 (2007)
Rogers, Mulders, PRD 81 (2010)

Extraction of unpolarized quark TMDs
Selection of recent parameterizations

	Accuracy	SIDIS HERMES	SIDIS COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✓	✓	✓	✓	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✓	✓	✓	✓	2031	1.06
ART23 arXiv:2305.07473	N^4LL	×	×	✓	✓	627	0.96
MAP24 arXiv:2405.13833	N^3LL	✓	✓	✓	✓	2031	1.08

For the complete table and details see talk by M. Radici

Distribution of unpolarized quarks



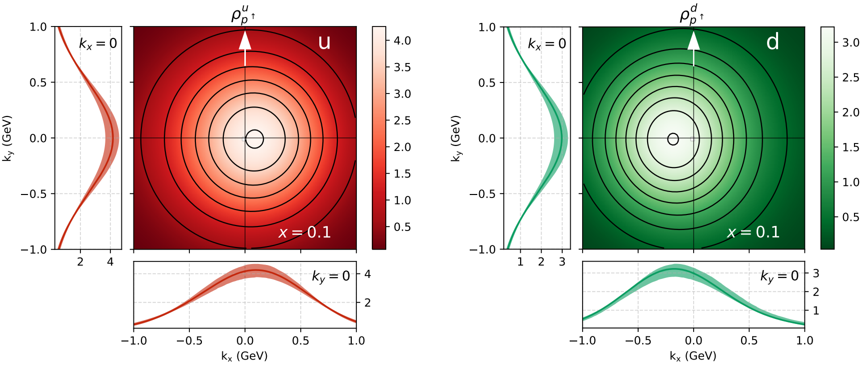
For unpolarized protons, the distribution of unp. quarks is cylindrically symmetric

What happens if the proton is transversely polarized?

Same formalism can be used to have a consistent picture (125 data points)

Distortion in the transverse plane of the TMD quark distribution in a $p^{\uparrow}$

$$\Phi_{q/p^{\uparrow}}^{[\gamma^+]}(x, k_x, k_y) = f_1^q(x, k_T^2) - \frac{k_x}{M} f_{1T}^{\perp q}(x, k_T^2) \qquad [Q^2 = 4 \text{ GeV}^2]$$

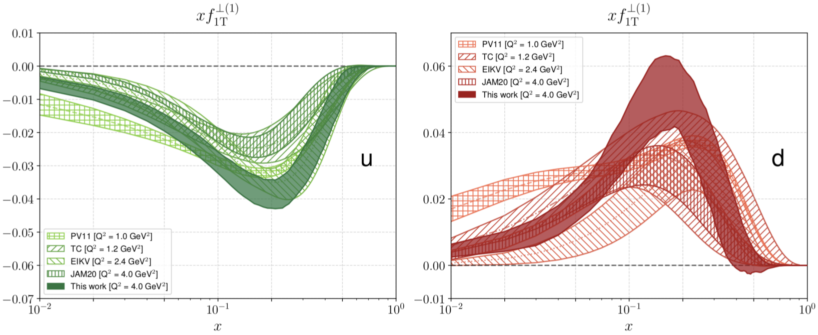


Bacchetta, Delcarro, Pisano, Radici, CP, PLB 827 (2022)

Non zero Sivers effect related to parton orbital angular momentum

The Sivers function
First transverse moments

$$f_{1T}^{\perp(1)\,q}(x) = \int \mathrm{d}^2\boldsymbol{k}_T \, \frac{k_T^2}{2M_p^2} \, f_{1T}^{\perp\,q}(x, \boldsymbol{k}_T^2)$$



Bacchetta, Delcarro, Pisano, Radici, CP, PLB 827 (2022)

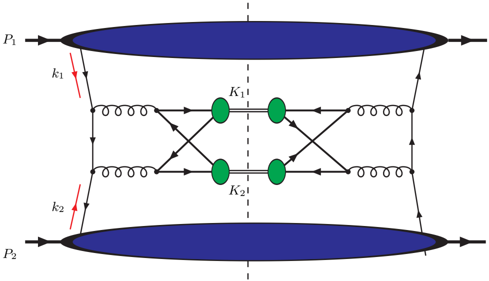
More data from CERN, JLab, EIC will help to reduce error bands and extend the ranges in x and Q^2

J/ψ-pair production at COMPASS

COMPASS Collaboration, PLB 838 (2023)

J/ψ's are relatively easy to detect. Studied at LHCb, CMS & ATLAS
gg fusion dominant, negligible $q\bar{q}$ contributions

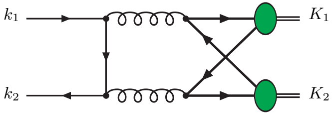
At COMPASS $\bar{u}u$ channel dominant, allowing for flavor separation

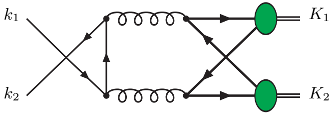


No final state gluon needed for the Born contribution in the Color Singlet Model.
Pure colorless final state, hence simple color structure because one has only ISI

Negligible Color Octet contributions, in particular at low $P_T^{\Psi\Psi}$

At LO pQCD in the Color Singlet Model, relevant diagrams for $q\bar{q}$ annihilation:





C. Flore, CP, in preparation

$\phi_T, \phi_\perp, \phi_S$: azimuthal angles of $\boldsymbol{q}_T \equiv \boldsymbol{K}_{1\perp} + \boldsymbol{K}_{2\perp}$, $\boldsymbol{K}_\perp \equiv (\boldsymbol{K}_{1\perp} - \boldsymbol{K}_{2\perp})/2$ and S_T

Angular modulations for the process $\pi^- p \to J/\psi\, J/\psi\, X$ ($q\bar{q}$ channel)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}^2\boldsymbol{K}_\perp\,\mathrm{d}^2\boldsymbol{q}_T} \propto \frac{\alpha_s^4}{M_\psi^2\, M_\perp^6\, s}\,|R_\psi(0)|^4 \Big\{ F_{UU} + F_{UU}^{\cos 2(\phi_T-\phi_\perp)} \cos 2(\phi_T - \phi_\perp)$$
$$+ F_{UL}^{\sin 2(\phi_T-\phi_\perp)} \sin 2(\phi_T-\phi_\perp) + |\boldsymbol{S}_T| F_{UT}^{\sin(\phi_T-\phi_S)} \sin(\phi_T-\phi_S)$$
$$+ |\boldsymbol{S}_T| F_{UT}^{\sin(\phi_T+\phi_S-2\phi_\perp)} \sin(\phi_T+\phi_S-2\phi_\perp)$$
$$+ |\boldsymbol{S}_T| F_{UT}^{\sin(3\phi_T-\phi_S-2\phi_\perp)} \sin(3\phi_T-\phi_S-2\phi_\perp) \Big\}$$

Exactly as for the DY process!

S. Arnold, A. Metz, M. Schlegel, PRD 79 (2009)

The structure functions are given by products of a hard part and a convolution of TMDs

$$F_{UU} = H_U(z, M_\psi, K_\perp)\, f_1^q \otimes f_1^{\bar q}$$
$$F_{UU}^{\sin 2(\phi_T-\phi_\perp)} = H_P(z, M_\psi, K_\perp)\, h_1^{\perp\, q} \otimes h_{1L}^{\bar q}$$
$$F_{UU}^{\cos 2(\phi_T-\phi_\perp)} = H_P(z, M_\psi, K_\perp)\, h_1^{\perp\, q} \otimes h_1^{\perp\, \bar q}$$
$$F_{UT}^{\sin(\phi_T-\phi_S)} = H_U(z, M_\psi, K_\perp)\, f_1^q \otimes f_{1T}^{\perp\, \bar q}$$
$$F_{UT}^{\sin(\phi_T+\phi_S-2\phi_\perp)} = H_P(z, M_\psi, K_\perp)\, h_1^{\perp\, q} \otimes h_1^{\bar q}\,,$$
$$F_{UT}^{\sin(3\phi_T-\phi_S-2\phi_\perp)} = H_P(z, M_\psi, K_\perp)\, h_1^{\perp\, q} \otimes h_{1T}^{\perp\, \bar q}$$

Hard functions:

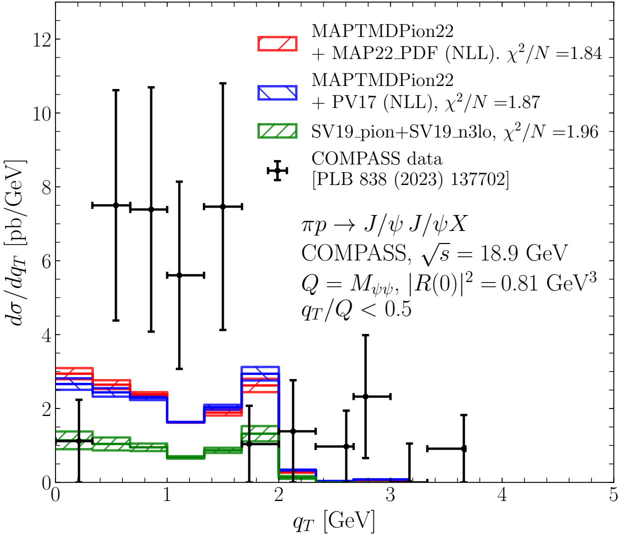
$$H_U = \left[5 - 12\, z(1-z) \left(1 - \frac{M_\psi^2}{M_\perp^2}\right) - \frac{M_\psi^2}{M_\perp^2}\right] \qquad H_P = -\left(1 - \frac{M_\psi^2}{M_\perp^2}\right) [1 - 12\, z(1-z)]$$

Kinematic variables:

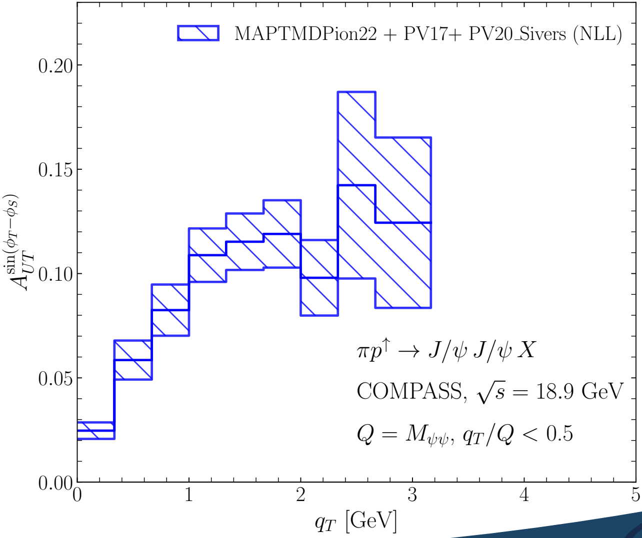
$$M_\perp^2 = M_\psi^2 + K_\perp^2\,, \qquad z = \frac{K_1 \cdot k_1}{k_1 \cdot k_2} = \frac{1}{1 + e^{y_1 - y_2}}$$

In the collinear limit: agreement with existing results

V.G Kartvelishvilil, S.M. Ésakiya, SJP 3 (1983)


MAPTMDPion22
+ MAP22_PDF (NLL). χ^2/N =1.84
MAPTMDPion22
+ PV17 (NLL), χ^2/N =1.87
SV19_pion+SV19_n3lo, χ^2/N =1.96
COMPASS data
[PLB 838 (2023) 137702]
$\pi p \to J/\psi\ J/\psi X$
COMPASS, $\sqrt{s}$ = 18.9 GeV
$Q = M_{\psi\psi}$, $|R(0)|^2$ = 0.81 GeV^3
$q_T/Q < 0.5$
$d\sigma/dq_T$ [pb/GeV]
q_T [GeV]
0
2
4
6
8
10
12
0
1
2
3
4
5

$$A_{UT}^{\sin(\phi_T-\phi_S)} = \frac{f_1^q \otimes f_{1T}^{\perp\,\bar{q}}}{f_1^q \otimes f_1^{\bar{q}}}$$



Gluon TMDs

Gluon TMDs

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Rodrigues, PRD 63 (2001)
Meissner, Metz, Goeke, PRD 76 (2007)

- $h_1^{\perp\, g}$: T-even distribution of linearly polarized gluons inside an unp. hadron
- h_{1T}^g, $h_{1T}^{\perp\, g}$: helicity flip distributions like h_{1T}^q, $h_{1T}^{\perp\, q}$, but T-odd, chiral even!
- $h_1^g \equiv h_{1T}^g + \frac{\boldsymbol{p}_T^2}{2M_p^2}\, h_{1T}^{\perp\, g}$ does not survive under p_T integration, unlike transversity

In contrast to quark TMDs, gluon TMDs are almost unknown
However models have been proposed:

Bacchetta, Celiberto, Radici, Taels, EPJC 80 (2020)
Chakrabarti, Choudhary, Gurjar, Kishore, Maji, Mondal, Mukherjee, PRD 108 (2023)

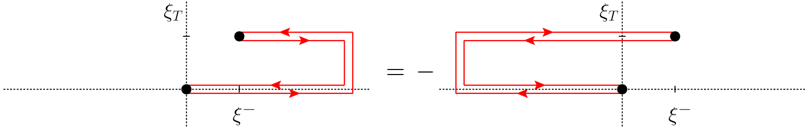
Related Processes

$ep^{\uparrow} \to e' Q\overline{Q} X$, $ep^{\uparrow} \to e'$ jet jet X probe GSF with $[++]$ gauge links (WW)

$p^{\uparrow} p \to \gamma\gamma X$ (and/or other CS final state) probe GSF with $[--]$ gauge links

Analogue of the sign change of $f_{1T}^{\perp\, q}$ between SIDIS and DY (true also for h_1^g and $h_{1T}^{\perp\, g}$)

$$f_{1T}^{\perp\, g\, [e\, p^{\uparrow} \to e'\, Q\overline{Q}\, X]} = -f_{1T}^{\perp\, g\, [p^{\uparrow}\, p \to \gamma\, \gamma\, X]}$$



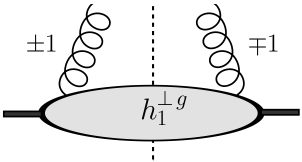
Boer, Mulders, CP, Zhou (2016)

Motivation to study gluon Sivers effects at both RHIC and the EIC

Linearly polarized gluons

Gluons inside an unpolarized hadron can be linearly polarized

It requires nonzero transverse momentum



Interference between ± 1 gluon helicity states

Like the unpolarized gluon TMD, it is T-even and exists in different versions:

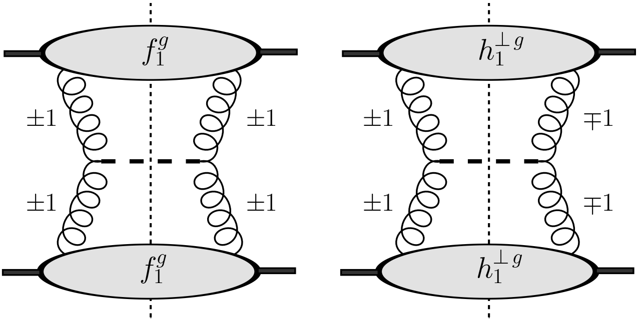
- $[++] = [--]$ (WW) (SIDIS and DY-like process)

Gluons can be probed in heavy quark production in both *ep* and *pp* scattering

Higgs boson production happens mainly via $gg \to H$

Pol. gluons affect the Higgs transverse spectrum at NNLO pQCD

Catani, Grazzini, NPB 845 (2011)
Sun, Xiao, Yuan, PRD 84 (2011)



The nonperturbative distribution can be present at tree level and would contribute to Higgs production at low q_T

Boer, den Dunnen, CP, Schlegel, Vogelsang, PRL 108 (2012)
Boer, den Dunnen, CP, Schlegel, PRL 111 (2013)
Echevarria, Kasemets, Mulders, CP, JHEP 1507 (2015)

$C = +1$ quarkonium production

q_T-distribution of η_Q and χ_{QJ} $(Q = c, b)$ in the kinematic region $q_T \ll 2M_Q$

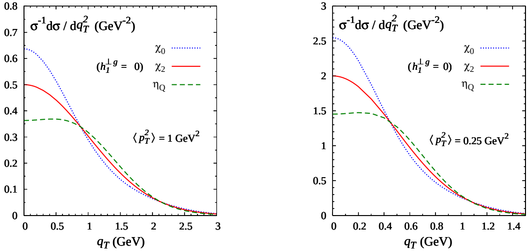
$$\frac{1}{\sigma(\eta_Q)}\frac{d\sigma(\eta_Q)}{d\boldsymbol{q}_T^2} \propto f_1^g \otimes f_1^g\,[1 - R(\boldsymbol{q}_T^2)] \qquad \text{[pseudoscalar]}$$

$$R(\boldsymbol{q}_T^2) = \frac{h_1^{\perp\, g} \otimes h_1^{\perp\, g}}{f_1^g \otimes f_1^g}$$

$$\frac{1}{\sigma(\chi_{Q0})}\frac{d\sigma(\chi_{Q0})}{d\boldsymbol{q}_T^2} \propto f_1^g \otimes f_1^g\,[1 + R(\boldsymbol{q}_T^2)] \qquad \text{[scalar]}$$

$$\frac{1}{\sigma(\chi_{Q2})}\frac{d\sigma(\chi_{Q2})}{d\boldsymbol{q}_T^2} \propto f_1^g \otimes f_1^g$$

Boer, CP, PRD 86 (2012)



Proof of factorization at NLO for $p\,p \to \eta_Q\, X$ in the Color Singlet Model (CSM)

Ma, Wang, Zhao, PRD 88 (2013); PLB 737 (2014)
Echevarria, JHEP 1910 (2019)

Future fixed target experiments at LHC

Structure of the cross section for the doubly polarized process $p(S_A) + p(S_B) \to \mathcal{Q}\, X$

$$\frac{\mathrm{d}\sigma[\mathcal{Q}]}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{q}_T} = F_{UU}^{\mathcal{Q}} + F_{UL}^{\mathcal{Q}}\, S_{BL} + F_{LU}^{\mathcal{Q}}\, S_{AL} + F_{UT}^{\mathcal{Q},\sin\phi_{S_B}}\, |\boldsymbol{S}_{BT}| \sin\phi_{S_B} + F_{TU}^{\mathcal{Q},\sin\phi_{S_A}}\, |\boldsymbol{S}_{AT}| \sin\phi_{S_A}$$
$$+ F_{LL}^{\mathcal{Q}}\, S_{AL}\, S_{BL} + F_{LT}^{\mathcal{Q},\cos\phi_{S_B}}\, S_{AL}\, |\boldsymbol{S}_{BT}| \cos\phi_{S_B} + F_{TL}^{\mathcal{Q},\cos\phi_{S_A}}\, |\boldsymbol{S}_{AT}|\, S_{BL} \cos\phi_{S_A}$$
$$+ |\boldsymbol{S}_{AT}||\boldsymbol{S}_{BT}| \left[F_{TT}^{\mathcal{Q},\cos(\phi_{S_A}-\phi_{S_B})} \cos(\phi_{S_A}-\phi_{S_B}) + F_{TT}^{\mathcal{Q},\cos(\phi_{S_A}+\phi_{S_B})} \cos(\phi_{S_A}+\phi_{S_B}) \right]$$

Kato, Maxia, CP, PRD 110 (2024)

Single spin asymmetries for different quarkonia are sensitive to different TMDs

$$F_{UT}^{\eta_Q,\sin\phi_{S_B}} \propto -f_1^g \otimes f_{1T}^{\perp g} + h_1^{\perp g} \otimes h_1^g - h_1^{\perp g} \otimes h_{1T}^{\perp g}$$
$$F_{UT}^{\chi_{Q0},\sin\phi_{S_B}} \propto -f_1^g \otimes f_{1T}^{\perp g} - h_1^{\perp g} \otimes h_1^g + h_1^{\perp g} \otimes h_{1T}^{\perp g}$$
$$F_{UT}^{\chi_{Q2},\sin\phi_{S_B}} \propto -f_1^g \otimes f_{1T}^{\perp g}$$

Such observables are in principle measurable at the planned LHCspin experiment

J/ψ-pair production at the LHC

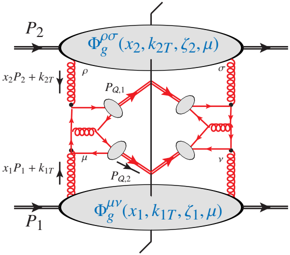
J/ψ-pair production at the LHC

J/ψ's are relatively easy to detect. Accessible at the LHC: already studied by LHCb, CMS & ATLAS

LHCb PLB 707 (2012)
CMS JHEP 1409 (2014)
ATLAS EPJC 77 (2017)

gg fusion dominant, negligible $q\bar{q}$ contributions even at fixed target energies

Lansberg, Shao, NPB 900 (2015)

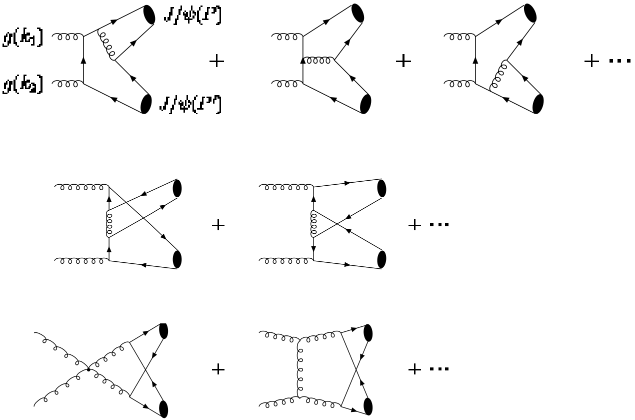


No final state gluon needed for the Born contribution in the Color Singlet Model. Pure colorless final state, hence simple color structure because one has only ISI

Lansberg, Shao, PRL 111 (2013)

Negligible Color Octet contributions, in particular at low $P_T^{\Psi\Psi}$

At LO pQCD in the Color Singlet Model, one needs to consider 36 diagrams



Qiao, Sun, Sun, JPG 37 (2010)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2\boldsymbol{q}_T\mathrm{d}\Omega} \approx A\, f_1^g \otimes f_1^g + B\, f_1^g \otimes h_1^{\perp g} \cos(2\phi_{CS}) + C\, h_1^{\perp g} \otimes h_1^{\perp g} \cos(4\phi_{CS})$$

Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)

- valid up to corrections $\mathcal{O}(q_T/Q)$
- Y: rapidity of the J/ψ-pair, along the beam in the hadronic c.m. frame
- $\mathrm{d}\Omega = \mathrm{d}\cos\theta_{CS}\,\mathrm{d}\phi_{CS}$: solid angle for J/ψ-pair in the Collins-Soper frame

Analysis similar to the one for $pp \to \gamma\gamma X$, $pp \to J\psi\,\gamma^{(*)}\,X$, $pp \to H\,\mathrm{jet}\,X$

Qiu, Schlegel, Vogelsang, PRL 107 (2011)
den Dunnen, Lansberg, CP, Schlegel, PRL 112 (2014)
Lansberg, CP, Schlegel, NPB 920 (2017)
Boer, CP, PRD 91 (2015)

The three contributions can be disentangled by defining the transverse moments

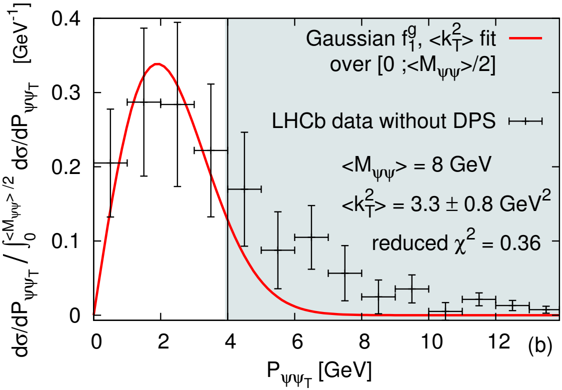
$$\langle \cos n\phi_{CS} \rangle \equiv \frac{\int_0^{2\pi} \mathrm{d}\phi_{CS}\, \cos(n\,\phi_{CS})\, \frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2\boldsymbol{q}_T\mathrm{d}\Omega}}{\int_0^{2\pi} \mathrm{d}\phi_{CS}\, \frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2\boldsymbol{q}_T\mathrm{d}\Omega}} \qquad (n = 2, 4)$$

$$\int \mathrm{d}\phi_{CS}\,\mathrm{d}\sigma \implies f_1^g \otimes f_1^g$$

$$\langle \cos 2\phi_{CS} \rangle \implies f_1^g \otimes h_1^{\perp g}$$

$$\langle \cos 4\phi_{CS} \rangle \implies h_1^{\perp g} \otimes h_1^{\perp g}$$

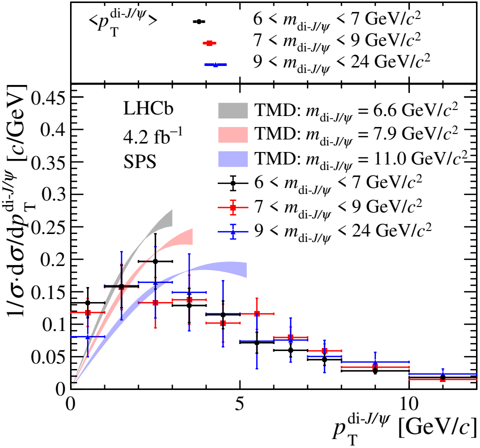
We consider $q_T = P_T^{\Psi\Psi} \leq M_{\Psi\Psi}/2$ in order to have two different scales



Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
LHCb Coll., JHEP 06 (2017)

Gaussian model:
$$f_1^g(x, \boldsymbol{k}_T^2) = \frac{f_1^g(x)}{\pi \langle k_T^2 \rangle} \exp\left(-\frac{\boldsymbol{k}_T^2}{\langle k_T^2 \rangle} \right)$$

No obvious broadening can be seen due to the large uncertainties



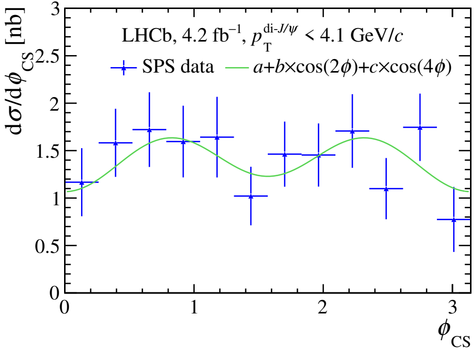
Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
LHCb Coll., 2311.14085

The average values of the p_T distributions slightly increase with mass

$$\langle \cos 2\phi \rangle = -0.029 \pm 0.050\,(\text{stat}) \pm 0.009\,(\text{syst})$$
$$\langle \cos 4\phi \rangle = -0.087 \pm 0.052\,(\text{stat}) \pm 0.013\,(\text{syst})$$

Theoretical predictions consistent with measureaments

Scarpa, Boer, Echevarria, Lansberg, CP, Schlegel EPJC 80 (2020)



LHCb Coll., 2311.14085

The results are consistent with zero, but the presence of an azimuthal asymmetry at a few percent level is allowed

Quarkonium production at the EIC

Charm and J/ψ production at the EIC

$e\,p \to e\,J/\psi\,X$ (with the inclusion of TMD shape functions)

Mukherjee, Rajesh, EPJ.C 77 (2017)
Kishore, Mukherjee, PRD 99 (2019)
Bacchetta, Boer, CP, Taels, EPJ.C 80 (2020)
Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

$e\,p \to e\,J/\psi\,\text{jet}\,X$

D'Alesio, Murgia, CP, Taels, PRD 100 (2019)
Kishore, Mukherjee, Pawar, Siddiqah, PRD 106 (2022)
R. Kishore, A. Mukherjee, A. Pawar, S. Rajesh, M. Siddiqah, PRD 111 (2025)

$e\,p \to e\,J/\psi\,\gamma\,X$

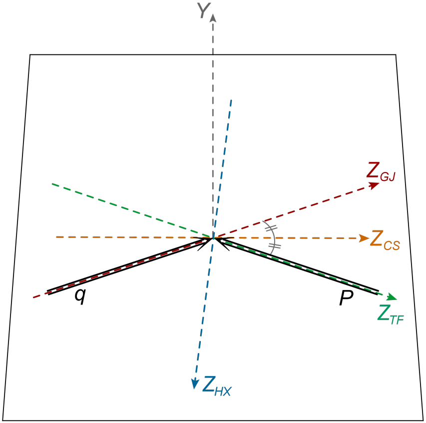
Chakrabarti, Kishore, Mukherjee, Rajesh, PRD 107 (2023)

$e\,p \to e\,D\,\text{jet}\,X$

Banu, Mukherjee, Pawar, Rajesh, PRD 108 (2023)

See talks by D. Boer and A. Mukherjee

We study $\gamma^*(q) + p(P) \to J/\psi(P_\psi) + X$ in the J/ψ rest frame



HX: Helicity
TF: Target
CS: Collins-Soper
GJ: Gottfried-Jackson

The frames are related to each other by a rotation around the Y axis

Model-independent arguments (gauge invariance, hermiticity, parity conservation) lead to eight independent helicity structure functions:

Lam, Tung, PRD 18 (1978)
Boer, Vogelsang, PRD 74 (2006)

$$\begin{aligned}
\mathcal{W}_T^{\mathcal{P}} &\equiv \mathcal{W}_{11}^{\mathcal{P}} = \mathcal{W}_{-1-1}^{\mathcal{P}} \\
\mathcal{W}_L^{\mathcal{P}} &\equiv \mathcal{W}_{00}^{\mathcal{P}} \\
\mathcal{W}_\Delta^{\mathcal{P}} &\equiv \sqrt{2}\,\mathrm{Re}\,\mathcal{W}_{10}^{\mathcal{P}} \\
\mathcal{W}_{\Delta\Delta}^{\mathcal{P}} &\equiv \mathcal{W}_{1-1}^{\mathcal{P}} = \mathcal{W}_{-11}^{\mathcal{P}}
\end{aligned}$$

- $\mathcal{P} = \perp, \parallel$: γ^* polarization (w.r.t. P, q)
- $\Lambda = T, L, \Delta, \Delta\Delta$: J/ψ helicity

However, by looking at the angular dependence of the decaying leptons only four linear combinations can be disentangled

$$\mathcal{W}_\Lambda \equiv \left[1 + (1-y)^2\right] \mathcal{W}_\Lambda^{\perp} + (1-y)\mathcal{W}_\Lambda^{\parallel} \qquad \text{with} \qquad \Lambda = T, L, \Delta, \Delta\Delta$$

Usual SIDIS variables:

$$Q^2 = -q^2\,, \qquad x_B = \frac{Q^2}{2P\cdot q}\,, \qquad y = \frac{P\cdot q}{P\cdot \ell}\,, \qquad z = \frac{P\cdot P_\psi}{P\cdot q}$$

Cross section differential in $\Omega = (\theta, \varphi)$, solid angle of the decaying lepton ℓ^+

$$\mathrm{d}\sigma \equiv \frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}^4 P_\psi\,\mathrm{d}\Omega}$$

$$\mathrm{d}\sigma \propto \frac{\alpha^2}{yQ^2}\left[\mathcal{W}_T(1+\cos^2\theta) + \mathcal{W}_L(1-\cos^2\theta) + \mathcal{W}_\Delta \sin 2\theta \cos\varphi + \mathcal{W}_{\Delta\Delta}\sin^2\theta\cos 2\varphi\right]$$

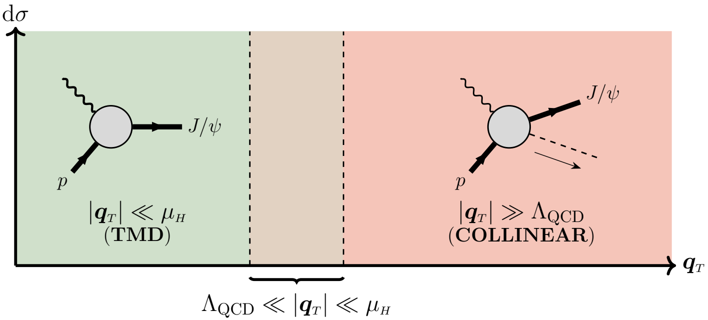
Alternatively, in terms of the polarization parameters λ, μ, ν :

$$\mathrm{d}\sigma \propto \frac{\alpha^2}{yQ^2}(\mathcal{W}_T+\mathcal{W}_L)\left[1+\lambda\cos^2\theta + \mu\sin 2\theta\cos\varphi + \frac{1}{2}\,\nu\,\sin^2\theta\cos 2\varphi\right]$$

$$\lambda = \frac{\mathcal{W}_T-\mathcal{W}_L}{\mathcal{W}_T+\mathcal{W}_L}\,, \qquad \mu = \frac{\mathcal{W}_\Delta}{\mathcal{W}_T+\mathcal{W}_L}\,, \qquad \nu = \frac{2\mathcal{W}_{\Delta\Delta}}{\mathcal{W}_T+\mathcal{W}_L}$$

Three physical scales, two theoretical tools

Bacchetta, Boer, Diehl, Mulders, JHEP 08 (2008)
Boer, D'Alesio, Murgia, CP, Taels, JHEP 09 (2020)
D'Alesio, Maxia, Murgia, CP, Rajesh, JHEP 037 (2022)
Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)



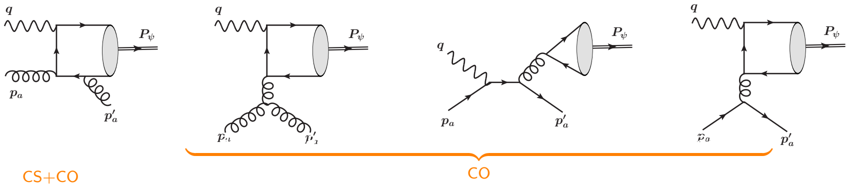
TMD factorization proven only for light hadron production in SIDIS

Matching in the intermediate region: a test of TMD factorization

The helicity structure functions can be calculated within the NRQCD framework

Contributing partonic subprocesses at the orders α_s^2 and v^4

$$\gamma^*(q) + a(p_a) \to J/\psi(P_\psi) + a(p_a') \qquad a = g, q, \bar{q}$$



Fock states included in the calculation: ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^3P_0^{[8]}$

($Q^2 = 0$) Beneke, Kramer, Vanttinen, PRD 57 (1998)
(W_T, W_L) Yuan, Chao, PRD 63 (2001)
(unpolarized) Kniehl, Zwirner, NPB 621 (2002)

q_T: transverse momentum of the photon w.r.t. P_ψ, P

Frame-independent leading power behavior of the structure functions up to corrections of $\mathcal{O}(\Lambda_{\rm QCD}/|\boldsymbol{q}_T|)$, $\mathcal{O}(|\boldsymbol{q}_T|/Q)$ in the region $\Lambda_{\rm QCD}^2 \ll \boldsymbol{q}_T^2 \ll Q^2$

$$\mathcal{W}_T^{\perp} = \widehat{w}_T^{\perp} \, \frac{1}{\boldsymbol{q}_T^2} \left[L\left(\frac{Q^2+M_\psi^2}{\boldsymbol{q}_T^2}\right) f_1^g(x,\mu^2) + \left(P_{gg}\otimes f_1^g + P_{gi}\otimes f_1^i\right)(x,\mu^2)\right]$$

$$\mathcal{W}_L^{\perp} = \widehat{w}_L^{\perp} \, \frac{1}{\boldsymbol{q}_T^2} \left[L\left(\frac{Q^2+M_\psi^2}{\boldsymbol{q}_T^2}\right) f_1^g(x,\mu^2) + \left(P_{gg}\otimes f_1^g + P_{gi}\otimes f_1^i\right)(x,\mu^2)\right]$$

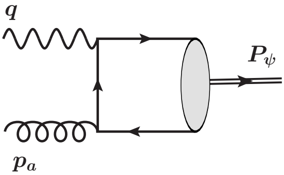
$$\mathcal{W}_L^{\parallel} = \widehat{w}_L^{\parallel} \, \frac{1}{\boldsymbol{q}_T^2} \left[L\left(\frac{Q^2+M_\psi^2}{\boldsymbol{q}_T^2}\right) f_1^g(x,\mu^2) + \left(P_{gg}\otimes f_1^g + P_{gi}\otimes f_1^i\right)(x,\mu^2)\right]$$

$$\mathcal{W}_{\Delta\Delta}^{\perp} = \widehat{w}_{\Delta\Delta}^{\perp} \, \frac{1}{\boldsymbol{q}_T^2} \left(\delta P_{gg}\otimes f_1^g + \delta P_{gi}\otimes f_1^i\right)(x,\mu^2)$$

$\widehat{w}_\Lambda^{\mathcal{P}}$: partonic structure functions for $\gamma^{*\mathcal{P}} g \to J/\psi^{\Lambda}$: depend on NRQCD LDMEs

$$L\left(\frac{Q^2+M_\psi^2}{\boldsymbol{q}_T^2}\right) \equiv C_A\left[\ln\frac{Q^2+M_\psi^2}{\boldsymbol{q}_T^2} - 1 - \ln\frac{M_\psi^2}{M_\psi^2+Q^2}\right] - \frac{11\,C_A - 4 n_f T_R}{6}$$

When $q_T^2 \ll Q^2$ at $\mathcal{O}(\alpha_s)$ only color-octet (CO) production channels dominate



Neglecting smearing effects in quarkonium formation:

$$\mathcal{W}_T^{\perp} = \widehat{w}_T^{\perp}\, f_1^g(x, \boldsymbol{q}_T^2) \qquad \mathcal{W}_T^{\parallel} = \widehat{w}_T^{\parallel}\, f_1^g(x, \boldsymbol{q}_T^2) \qquad \mathcal{W}_L^{\parallel} = \widehat{w}_L^{\parallel}\, f_1^g(x, \boldsymbol{q}_T^2)$$

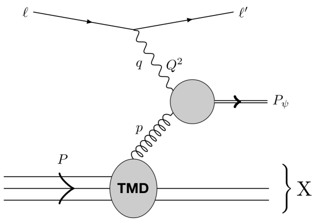
$$\mathcal{W}_{\Delta\Delta}^{\perp} = \widehat{w}_{\Delta\Delta}^{\perp}\, h_1^{\perp\, g}(x, \boldsymbol{q}_T^2)$$

$\mathcal{W}_{\Delta\Delta}^{\perp}$ gives access to $h_1^{\perp\, g}$ and to the poorly known ${}^3P_0$ LDME

Smearing effects encoded in $\Delta^{[n]}$ need to be included to match the result in the intermediate overlapping region $\Lambda_{\rm QCD}^2 \ll \boldsymbol{q}_T^2 \ll Q^2$

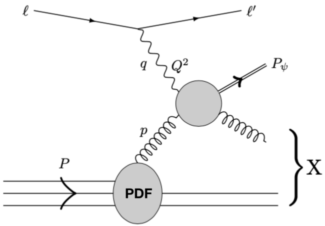
Imposing the matching of the TMD and collinear results in the overlapping region
$\Lambda^2_{\text{QCD}} \ll \boldsymbol{q}_T^2 \ll Q^2$: $f_1^g \longrightarrow \mathcal{C}[f_1^g\, \Delta^{[n]}]$

TMD



$\boldsymbol{q}_T^2 \ll Q^2$

Collinear



$\Lambda^2_{\text{QCD}} \ll \boldsymbol{q}_T^2$

Boer, D'Alesio, Murgia, CP, Taels, JHEP 09 (2020)
D'Alesio, Maxia, Murgia, CP, Rajesh, JHEP 037 (2022)
Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

Knowing the perturbative tail of the gluon TMD, we determine the one of $\Delta^{[n]}$

Factorization scale fixed to be: $\mu^2 = M_\psi^2 + Q^2$

$$\Delta^{[n]}(k_T^2, \mu^2) = -\frac{\alpha_S}{2\pi^2 k_T^2}\, C_A \left(1 + \log \frac{M_\psi^2}{M_\psi^2 + Q^2}\right) \langle \mathcal{O}^{[n]} \rangle \qquad \text{for} \quad k_T \gg \Lambda_{\mathrm{QCD}}$$

Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

Less divergent than fragmentation functions of light quarks $\propto \log Q^2/k_T^2$

Independent of J/ψ polarization and CO quantum numbers

It should not depend on Q^2: hint of process dependence (photoproduction result is obtained by imposing $Q^2 = 0$)

Conclusions

- Quarkonia are good probes for gluon TMDs: first extraction of unpolarized gluon TMD from LHCb data on di-J/ψ production
- Independent information about quark TMDs could come from COMPASS analysis of di-J/ψ-data in pion-proton collisions
- At the EIC: J/ψ production and polarization are useful tools to probe TMD shape functions and gluon distributions
- Azimuthal asymmetries in heavy quark pair and dijet production could probe WW-type gluon TMDs (similar to SIDIS for quark TMDs)

Boer, Brodsky, Mulders, CP, PRL (2011)
CP, Boer, Brodsky, Buffing, Mulders, JHEP 10 (2013)
Boer, Mulders, CP, Zhou, JHEP 08 (2016)