

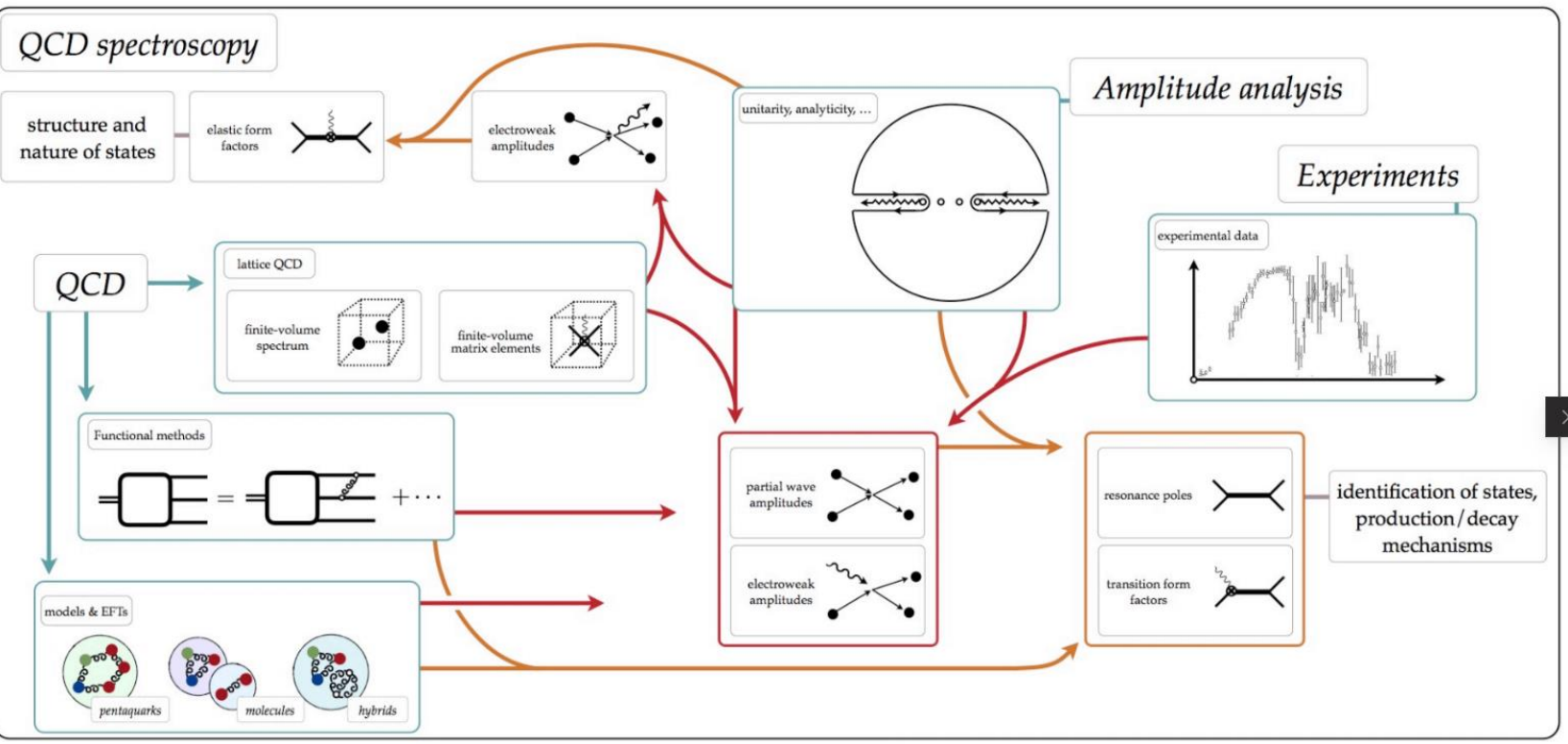
Accessing and Understanding QCD spectrum: Introduction and goals of the workshop

Alessandro Pilloni

INT, March 20th, 2023



Workshop in ~~2020~~ 2023 (finally!)



Raúl Briceño

University of California, Berkeley



Gernot Eichmann

Universität Graz



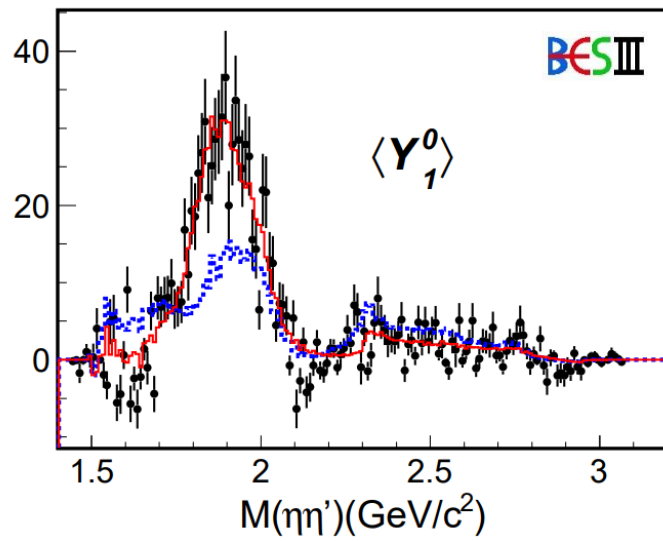
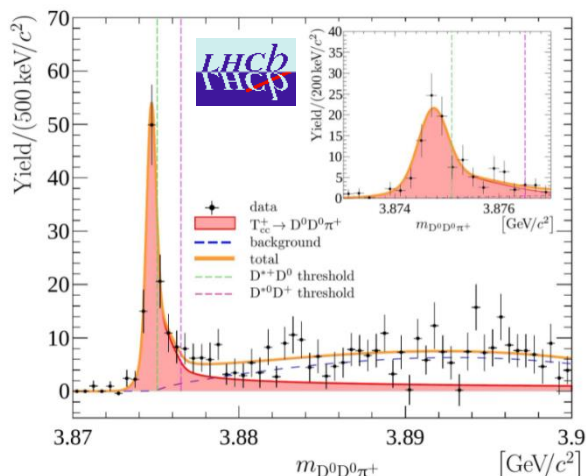
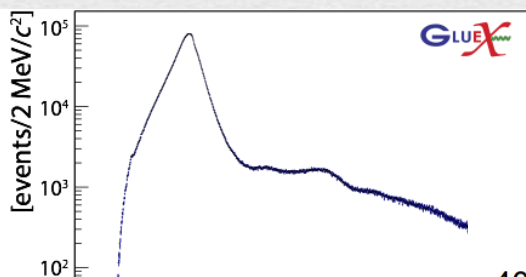
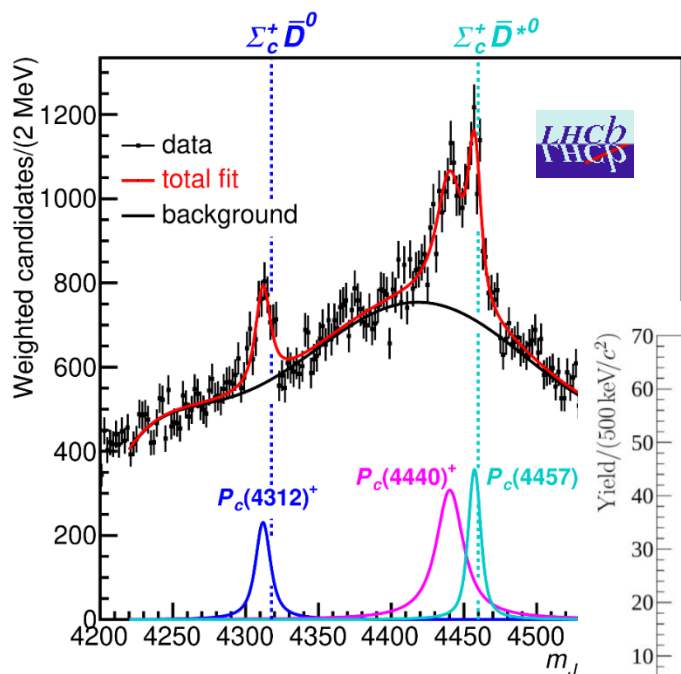
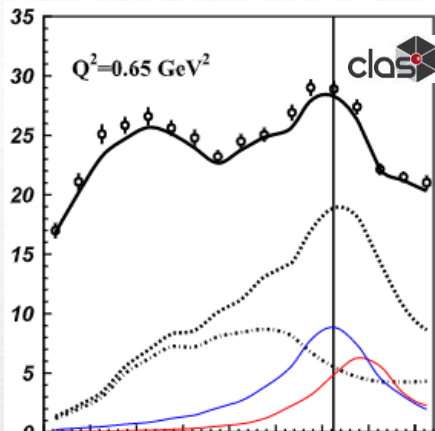
Alessandro Pilloni

Università di Messina



The richness of hadron spectrum

- The structures populating hadron reactions are extremely rich
- Ultimate goal is to understand them in term of QCD degrees of freedom
- To do so, the spectrum of resonances must be correctly reconstructed



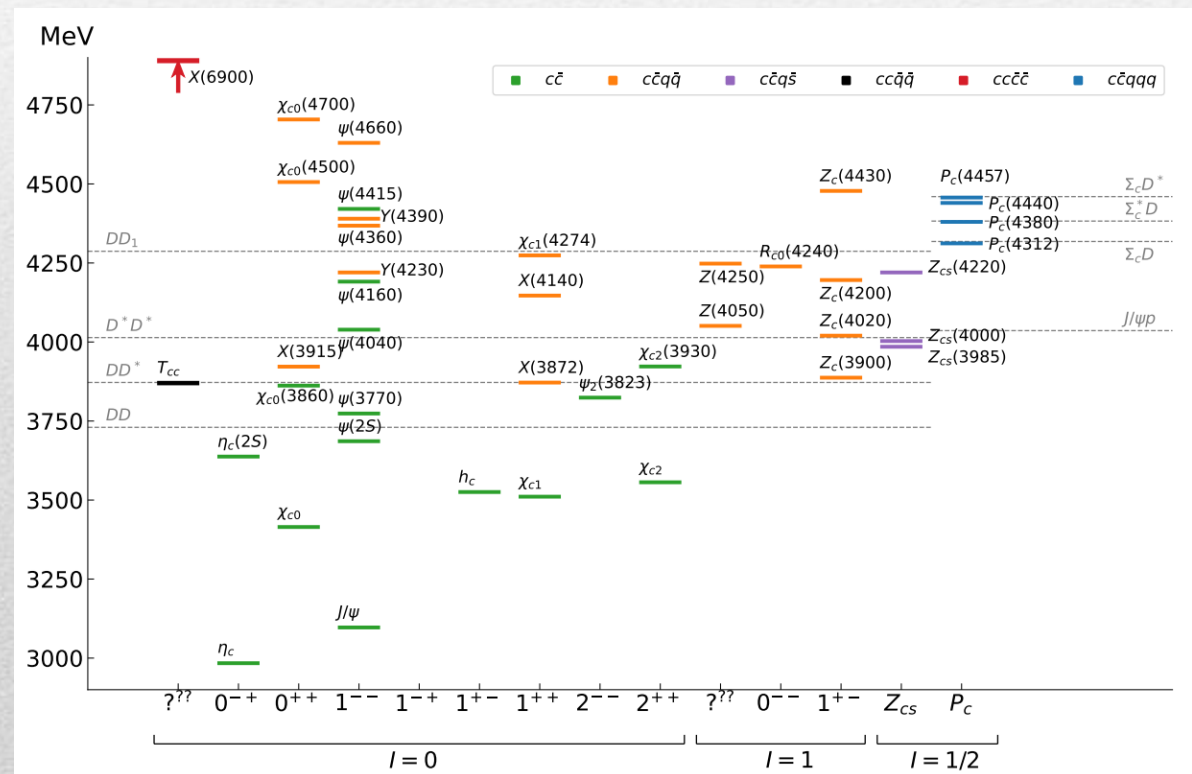
Exotics

The quark model is still the simplest tool to organize the spectrum

JPAC, PPNP 127 (2022), 103981

«Exotic hadrons» are
Beyond the Standard
(quark) Model

Organizing them
teaches us some more
about quark-gluon
interactions

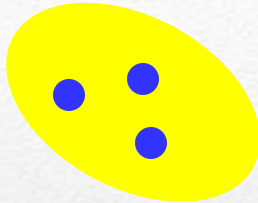


Hadron Spectroscopy

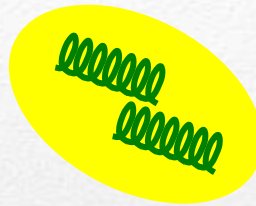
Meson



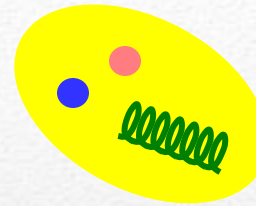
Baryon



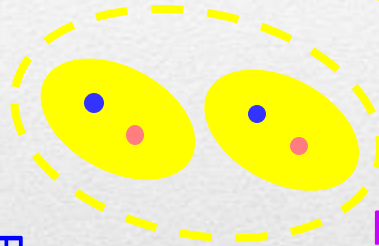
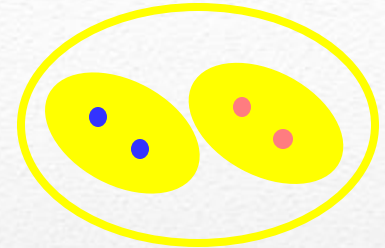
Glueball



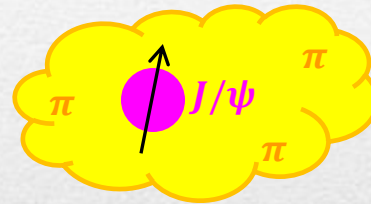
Hybrids



Tetraquark



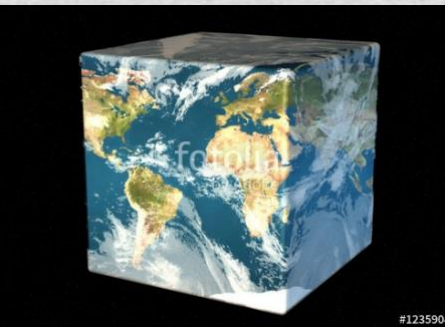
Molecule



Hadroquarkonium



Experiment



Lattice QCD

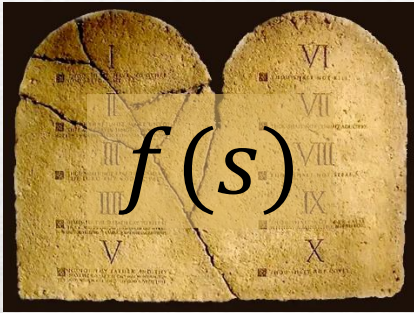
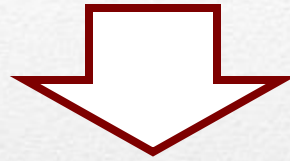


Interpretations on the spectrum leads to understanding fundamental laws of nature

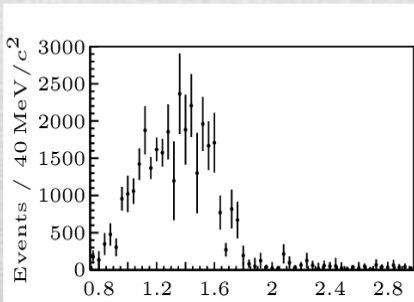
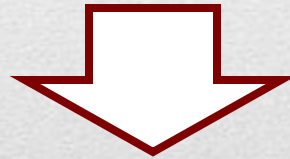
Top-down approach



1) You are given a model/theory



2) You calculate the amplitude



3) You compare with data.
Or you don't.

Predictive power ✓
Physical interpretation ✓
(within the model! ✗)
Biased by the input ✗

Options on the market

Quark-level calculations:

- Quark models, pNRQCD, Functional methods, Coulomb gauge, Holography, large-N QCD, Sum Rules...
- Spectrum generally calculated as bound states of some potential
- Decay rates as «overlap integrals» between two static configurations

Comprehensive picture ✓

Little scattering dynamics ✗

Hadron level calculations:

- Microscopic models inspired to EFT + Unitarization
- Compact states vs. molecules, triangles...
- States as poles of scattering amplitudes
- Couplings as residues at the poles

Scattering dynamics ✓

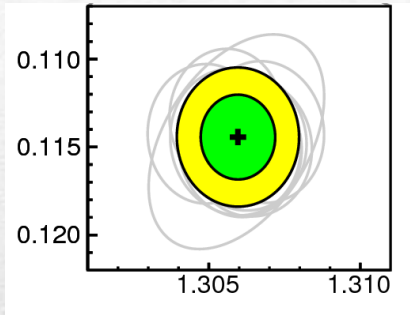
More case-by-case ✓ ✗

Bottom-up approach

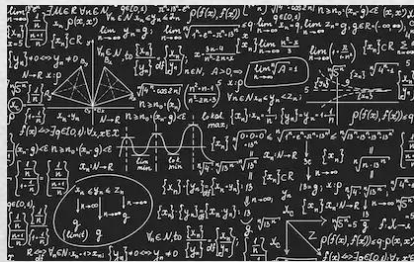
Less predictive power ✗

Some physical interpretation ✗

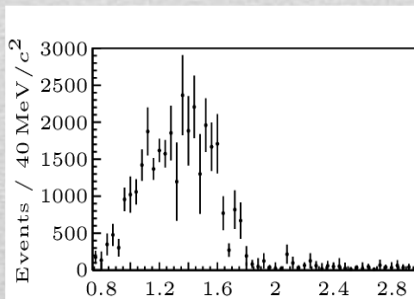
Minimally biased ✓



3) You extract physics

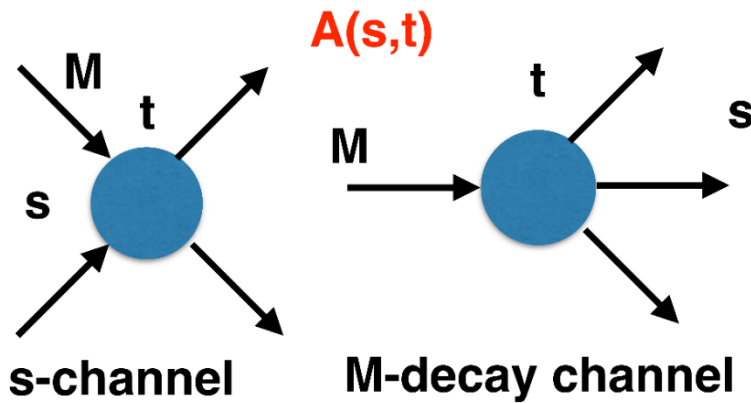


2) You choose a set of generic amplitudes

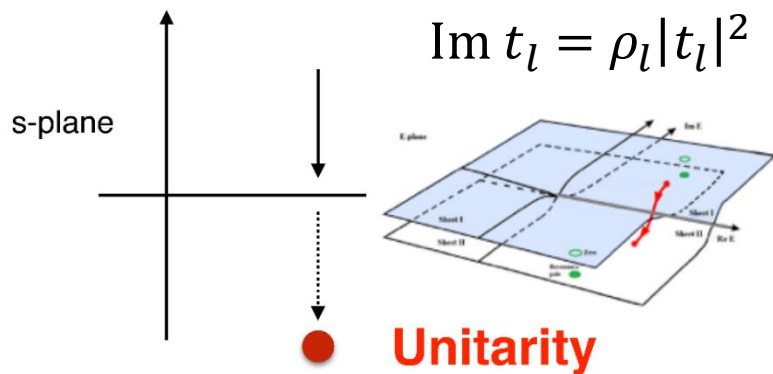


1) You start with data

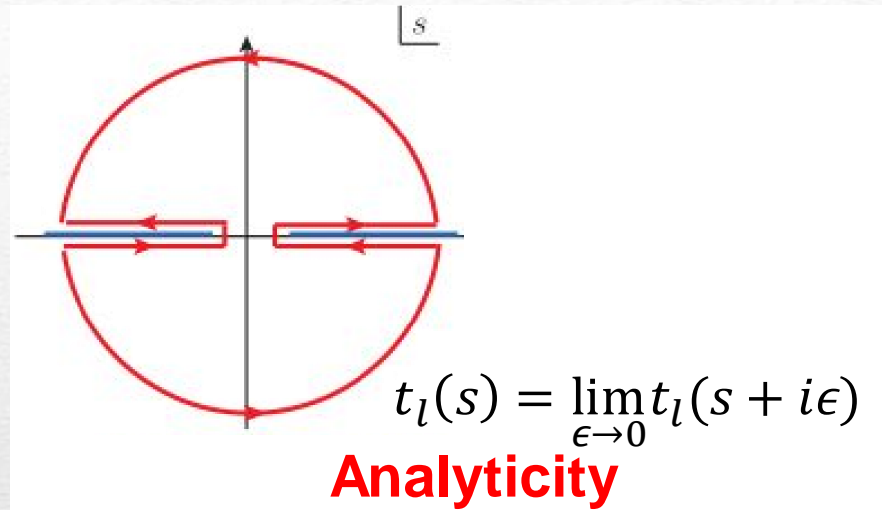
S-Matrix principles



Crossing



+ Lorentz, discrete & global symmetries



These are **constraints** the amplitudes have to satisfy, but **do not fix the dynamics**

They can be imposed **increasing amount of rigor**, to extract physics information from data

They can be imposed strictly to **browse the «theory space»** and assess where QCD sits among them

The questions we want to address

When cardinales reunite in conclave, they first agree on the characteristics that the new pope should have

Then doors are shut and fights start



The questions we want to address

- What «understanding» mean?
What would be the acceptable end of the hadron quest?
- Once we have determined the spectrum and interactions of hybrids/XYZ/glueballs etc., what do we really want to learn?
- What level of complementarity can we expect between Lattice QCD and experimental data in the next decade?
- Is the present model of collaboration between theory and experiment efficient?
- Could AI technology provide groundbreakingly different tools?

Examples of collaborative efforts



Eric Braaten
Ohio State University



Raúl Briceño
University of California, Berkeley



Michael Döring
George Washington University



Jo Dudek
William & Mary



Robert Edwards
Jefferson Lab



Gernot Eichmann
Universität Graz



César Fernández
Ramírez
UNED/ICN-UNAM



Christian Fischer
JLU Giessen



Rich Lebed
Arizona State University



Jinfeng Liao
Indiana University



Vincent Mathieu
University of Barcelona



Emilie Passemar
Indiana University



Alessandro Pilloni
Università di Messina



Arkaitz Rodas
Jefferson Lab



Stephen Sharpe
University of Washington



Eric Swanson
University of Pittsburgh



Adam Szczepaniak
Indiana University

Funded in 2023 by DoE as a Topical Collaboration in Nuclear Physics

Aims at exploring all aspects of exotics (= hybrids at GlueX)

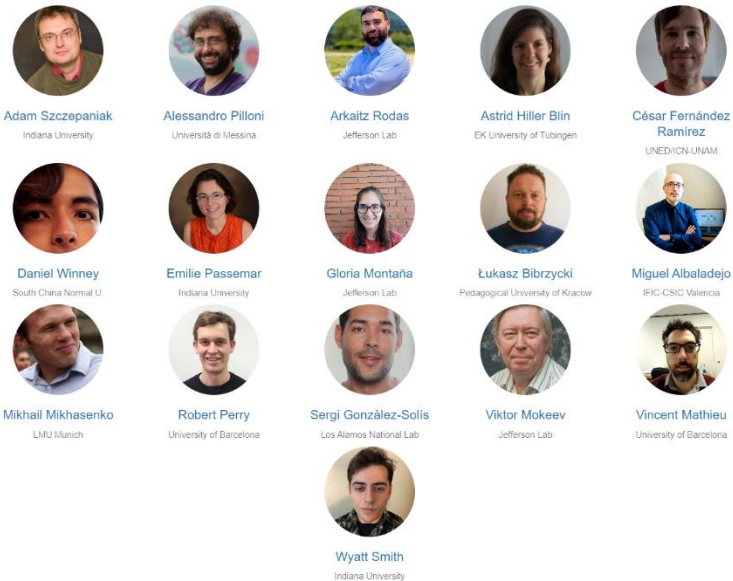
1. from predictions within lattice QCD
2. through reliable extraction of their existence and properties from experimental data,
3. to descriptions of their structure within phenomenological models.

Examples of collaborative efforts

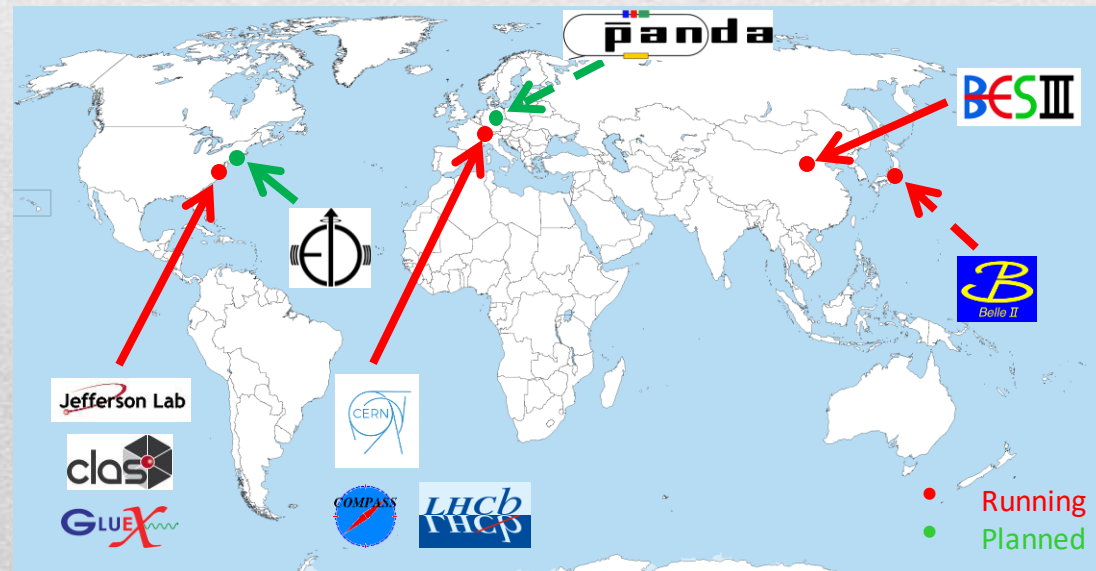
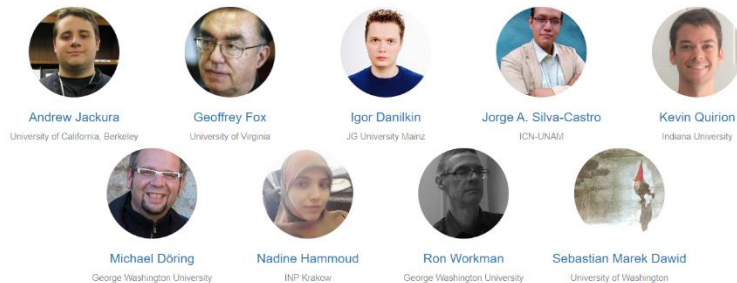


Funded in 2013 as theory support originally for JLab exps, but grew up much larger
Creates bridges between theorists and experimentalists

Full Members



Affiliated Members

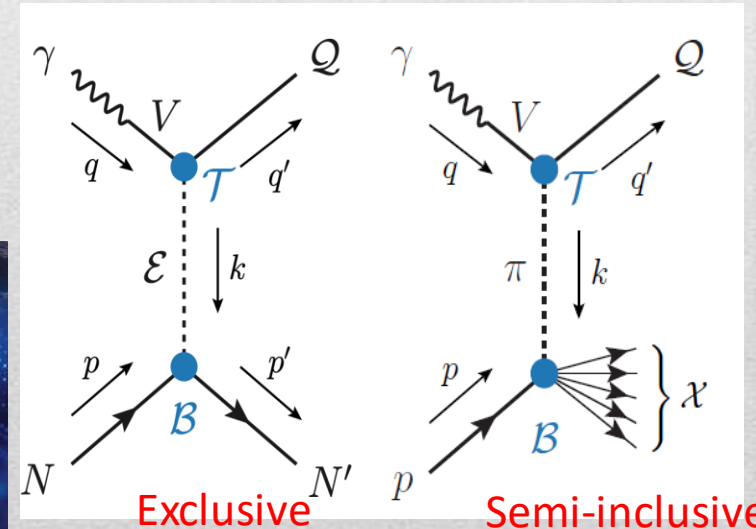


Shaping future experiments



Spectroscopy is one of the main physics cases to push for new facilities, e.g. the energy upgrade at JLab

The community must act coherently to show that such facilities are much needed (\$\$\$!)

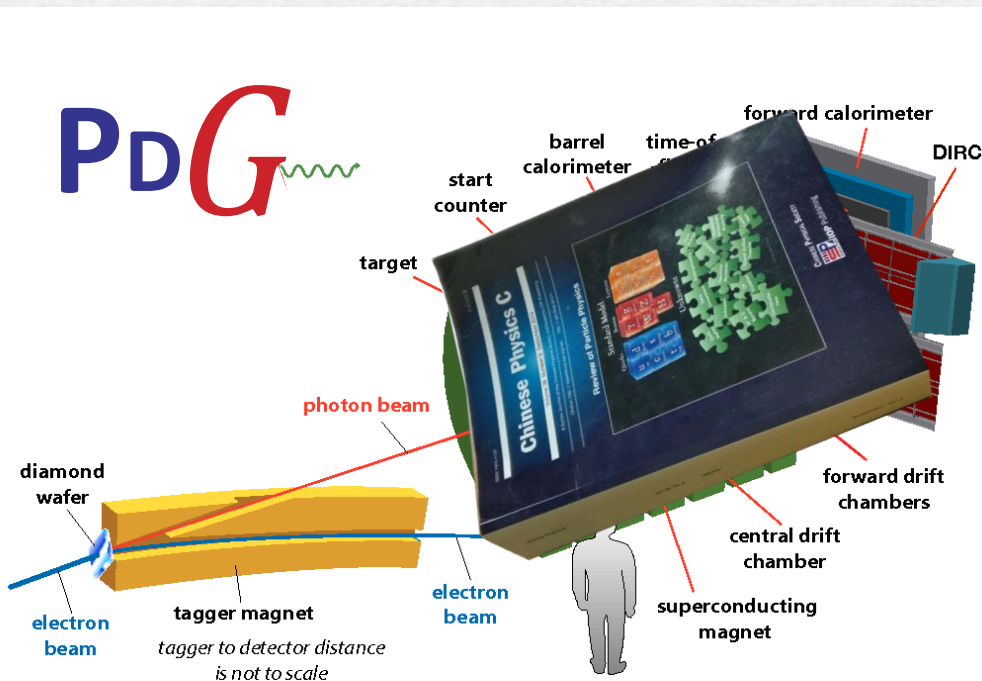


	Monday 03/20	Tuesday 03/21	Wednesday 03/22	Thursday 03/23	Friday 03/24
09:00	Pilloni - Introduction	Glazier - EIC	Free Morning	Shepherd - Light @GlueX	Prelovsek - Tcc in LQCD
09:30	Athenodorou - Glueballs in LQCD	Dobbs - Charmonia @GlueX			Fischer - 4q with DSE
10:00	Markus - Glueballs in DSE	Tripolt - Pole searches		Wunderlich - Bayesian truncation of partial waves	Mohler - $\Lambda(1405)$ in LQCD
10:30	Coffee			Coffee	
11:00	Smith - Coulomb gauge in LQCD	Guerrieri - Bootstrapping QCD		Dawid - Analytic continuation of 3-body amplitudes	Rusetski - 3-body in Finite Volume
11:30	Segovia - Constituent Quark Model			Islam - Analytic continuation of 3-body amplitudes	Ortega-Gama - Inserting currents in Finite Volume
12:00	Swanson - Constituent gluons	Danilkin - Dispersive $\gamma\gamma\rightarrow DD$		Döring - 3-body in LQCD with FVU	Pefkou - 3-body amplitudes with bound states
12:30		Diversity lunch - Sharpe		Outreach lunch - Briceño	
13:00					
13:30					
14:00	Bruschini - Potentials	Pelaez - Dispersive $f_0(1370)$	Nicholson - NN in LQCD	Mai - 3-body in LQCD with FVU	Mikhasenko - Scattering parameters for Tcc
14:30	Lebed - Diquarks vs. molecules	Ruiz de Elvira - Dispersive isospin breaking	Hanlon - NN in LQCD	Shuryak - Light Front	Discussion: Outstanding problems, Eichmann
15:00	Mohapatra - Heavy Hybrids	Rodas - σ in LQCD	Romero-Lopez - maximal isospin 3-meson in LQCD	Brodsky - Light Front	
15:30	Coffee				
16:00	Ikeno - Molecules	Hanhart - Molecules	Walker-Loud - πN in LQCD	Dudek - Radiative decay of K^* in LQCD	Sharpe - 3-body with spin
16:30	Discussion: Models, Swanson	Discussion: Analysis, Szczepaniak	Discussion: Lattice, Jackura	Discussion: Experiments, Austregesilo	
17:00					
17:30					

BACKUP

How theorists think of experiments

All the information about hadrons is collected by the Particle Data Group



What experimentalists present...

Changing beam and target, you can change the font

$a_1(1260)$ [1]			
$J^{PC} = 1^-(1^{++})$			
Mass $m = 1230 \pm 40$ MeV [1]			
Full width $\Gamma = 250$ to 600 MeV			
$a_1(1260)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)	N
3π	seen	577	IP
$(\rho\pi)_{S\text{-wave}}, \rho \rightarrow \pi\pi$	seen	353	
$(\rho\pi)_{D\text{-wave}}, \rho \rightarrow \pi\pi$	seen	353	
$(\rho(1450)\pi)_{S\text{-wave}}, \rho \rightarrow \pi\pi$	seen	†	R
$(\rho(1450)\pi)_{D\text{-wave}}, \rho \rightarrow \pi\pi$	seen	†	
$f_0(500)\pi, f_0 \rightarrow \pi\pi$	seen	—	S
$f_0(980)\pi, f_0 \rightarrow \pi\pi$	not seen	179	
$f_0(1370)\pi, f_0 \rightarrow \pi\pi$	seen	†	E
$f_2(1270)\pi, f_2 \rightarrow \pi\pi$	seen	†	
$\pi^+\pi^-\pi^0$	seen	576	X
$\pi^0\pi^0\pi^0$	not seen	577	
$KK\pi$	seen	250	-
$K^*(892)K$	seen	†	
$\pi\gamma$	seen	608	2

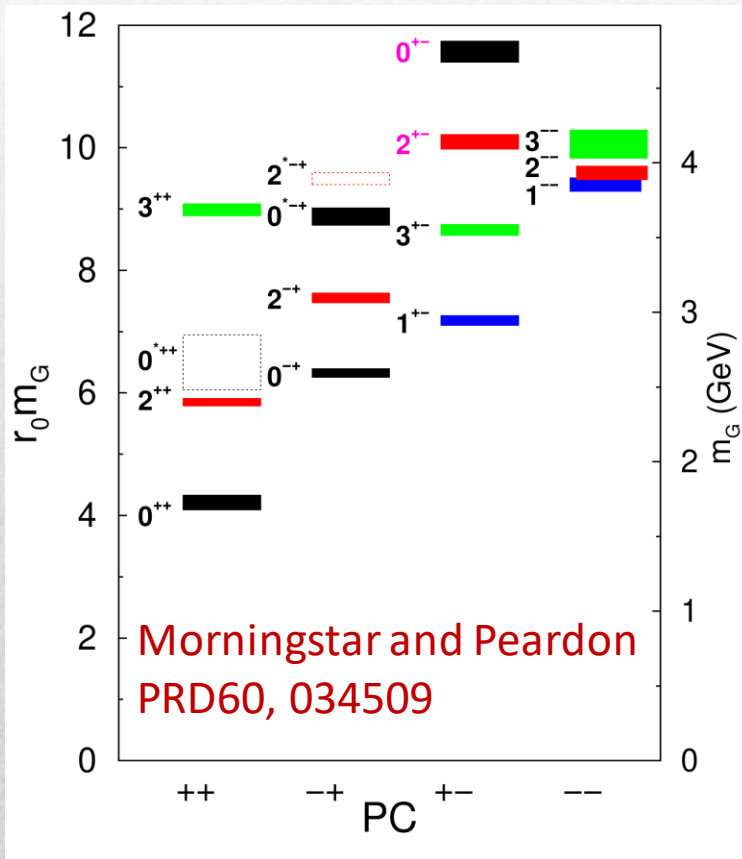
The scalar glueball



0^{++}

Glueballs

The **clearest** sign of confinement in pure Yang-Mills
 The **worst** state to search in real life



J^{PC}	Mass MeV			
	Unquenched This work	Quenched		
		M&P	Ky	Meyer
0^{-++}		2590(40)(130)	2560(35)(120)	2250(60)(100)
2^{-++}	3460(320)	3100(30)(150)	3040(40)(150)	2780(50)(130)
0^{-+}	4490(590)	3640(60)(180)		3370(150)(150)
2^{-+}				3480(140)(160)
5^{-+}				3942(160)(180)
0^{--} (exotic)	5166(1000)			
1^{--}		3850(50)(190)	3830(40)(190)	3240(330)(150)
2^{--}	4590(740)	3930(40)(190)	4010(45)(200)	3660(130)(170)
2^{--}				3.740(200)(170)
3^{--}		4130(90)(200)	4200(45)(200)	4330(260)(200)
1^{+-}	3270(340)	2940(30)(140)	2980(30)(140)	2670(65)(120)
3^{+-}	3850(350)	3550(40)(170)	3600(40)(170)	3270(90)(150)
3^{+-}				3630(140)(160)
2^{+-} (exotic)		4140(50)(200)	4230(50)(200)	
0^{+-} (exotic)	5450(830)	4740(70)(230)	4780(60)(230)	
5^{+-}				4110(170)(190)
0^{++}	1795(60)	1730(50)(80)	1710(50)(80)	1475(30)(65)
2^{++}	2620(50)	2400(25)(120)	2390(30)(120)	2150(30)(100)
0^{++}	3760(240)	2670(180)(130)		2755(30)(120)
3^{++}		3690(40)(180)	3670(50)(180)	3385(90)(150)
0^{++}				3370(100)(150)
0^{++}				3990(210)(180)
2^{++}				2880(100)(130)
4^{++}				3640(90)(160)
6^{++}				4360(260)(200)

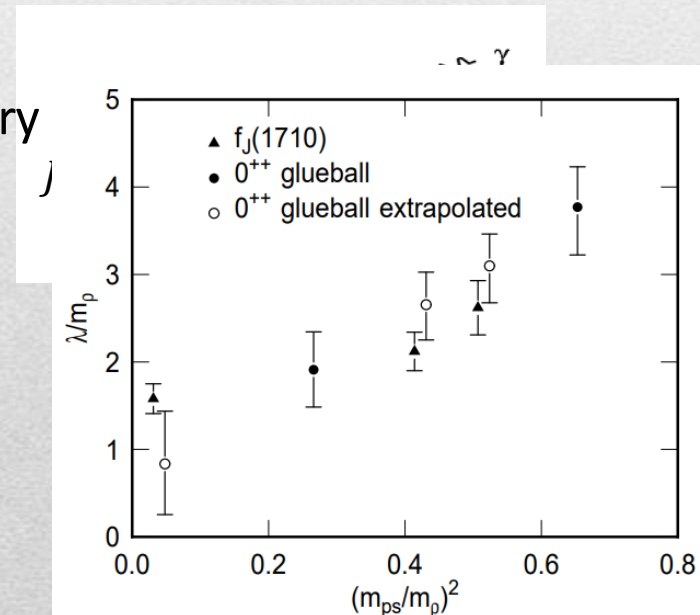
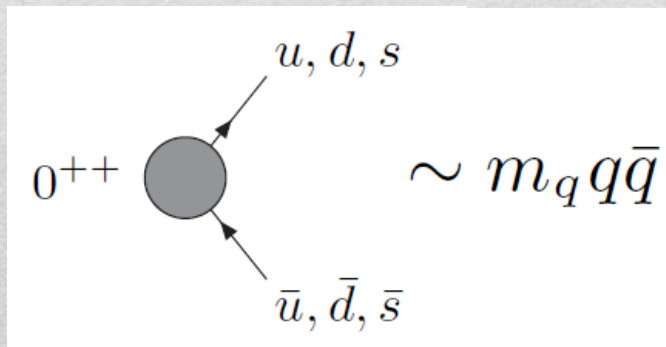
Gregory *et al.*
 JHEP1210, 170

How to identify a glueball

You don't. Since it mixes with light isoscalars, there is no model-independent way of saying which state is (mostly) the glueball. Only suggestions:

- There is one too many wrt QM. Indeed, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$
- A glueball couples to photons only throughout mixing, so radiative widths should be small
- Their production is enhanced in gluon-rich processes, as J/ψ radiative decays

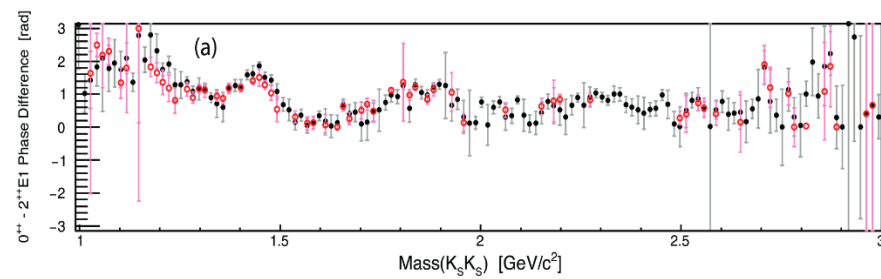
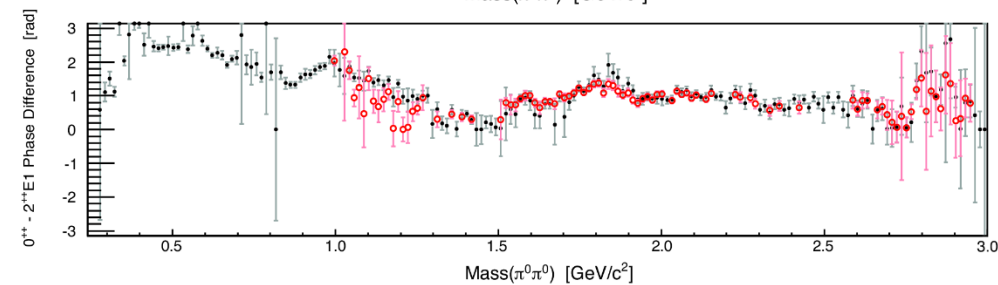
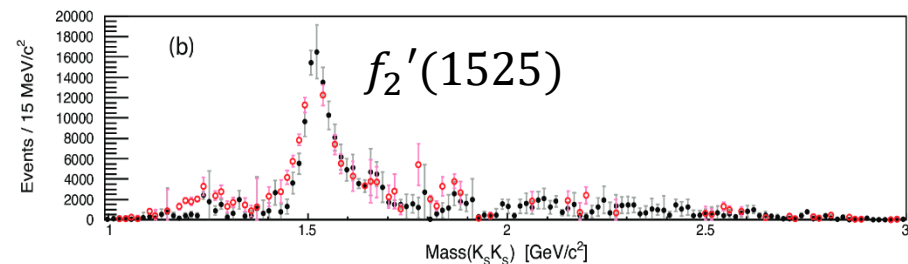
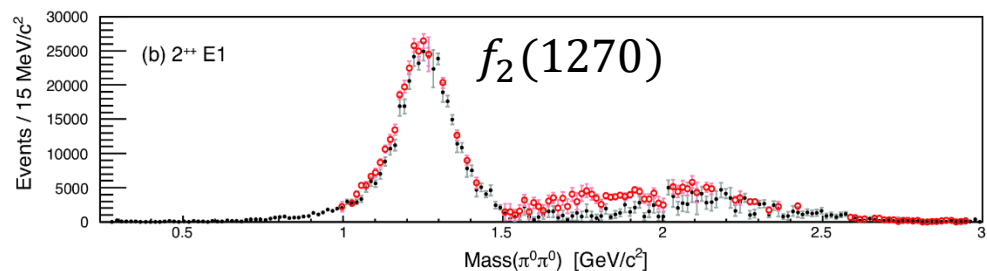
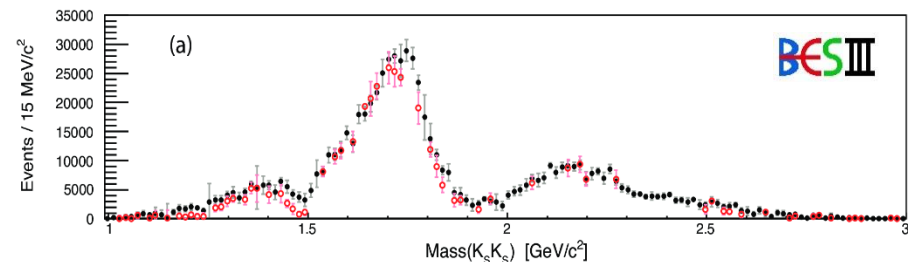
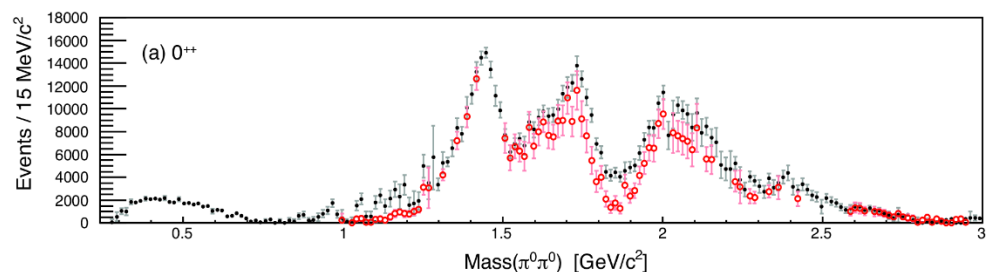
- It couples equally to mesons of all flavors (?)
However, an argument based on chiral symmetry claims the coupling proportional to quark mass



$$J/\psi \rightarrow \gamma \pi^0 \pi^0 \text{ and } \rightarrow \gamma K_S^0 K_S^0$$

We consider the S and D wave by BESIII to use the information about their relative phase.

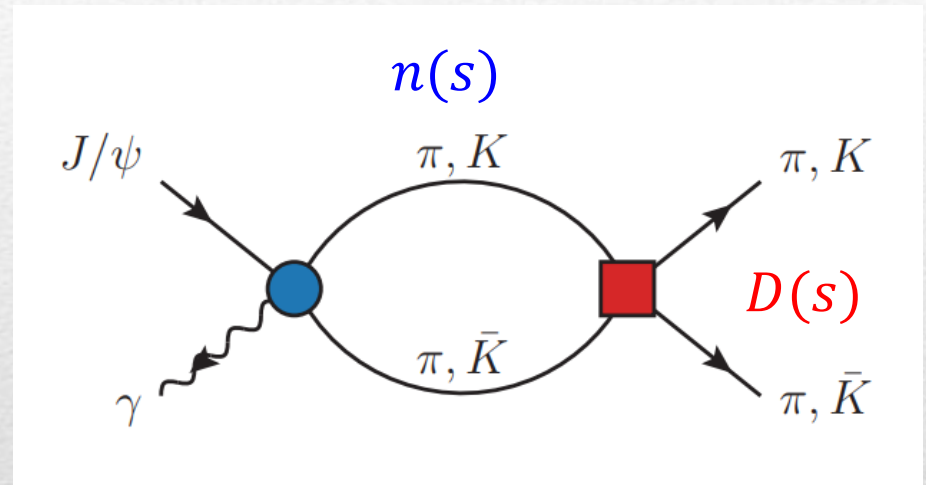
The D -wave is populated by two almost elastic resonances: the $f_2(1270)$ and $f_2'(1525)$



Amplitudes for $J/\psi \rightarrow \gamma PP$

We build the partial wave amplitudes according to the N/D method

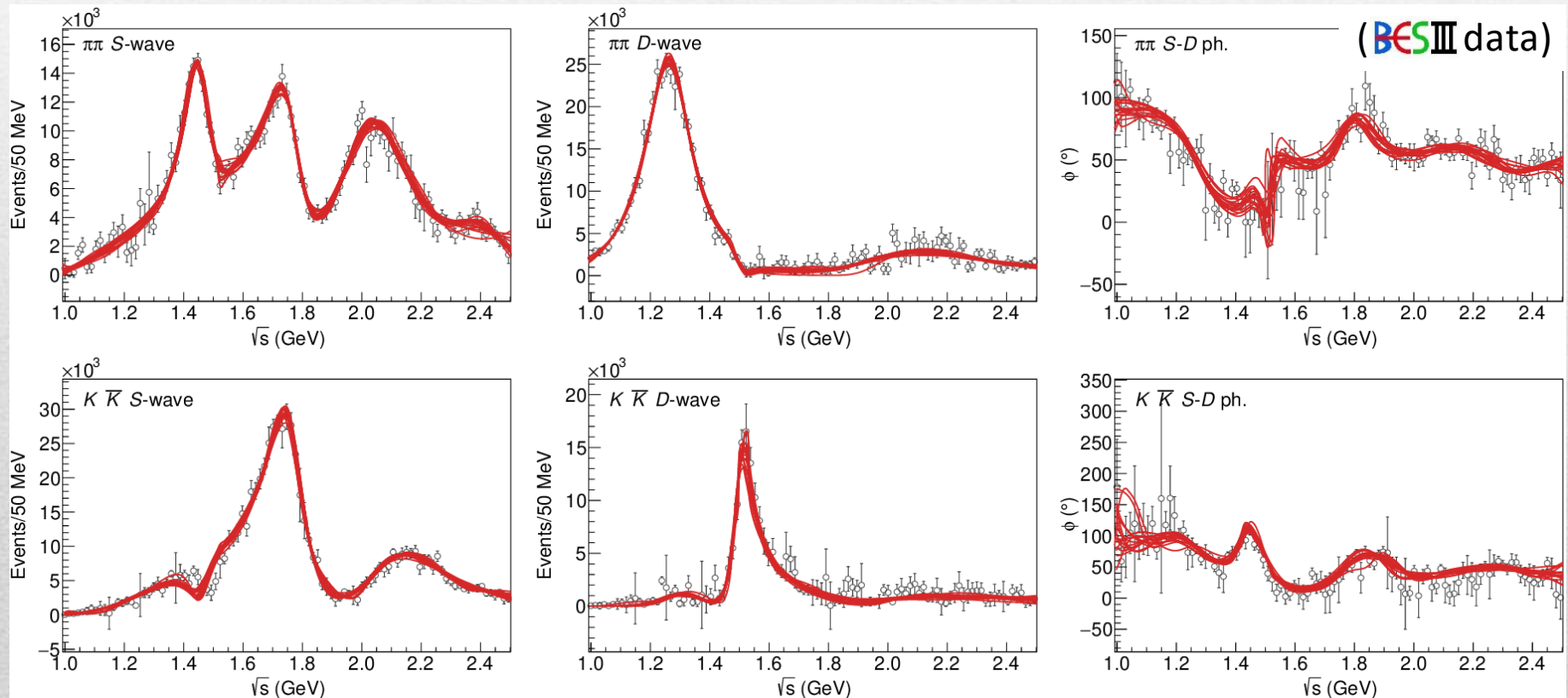
$$a(s) = \frac{n(s)}{D(s)}$$



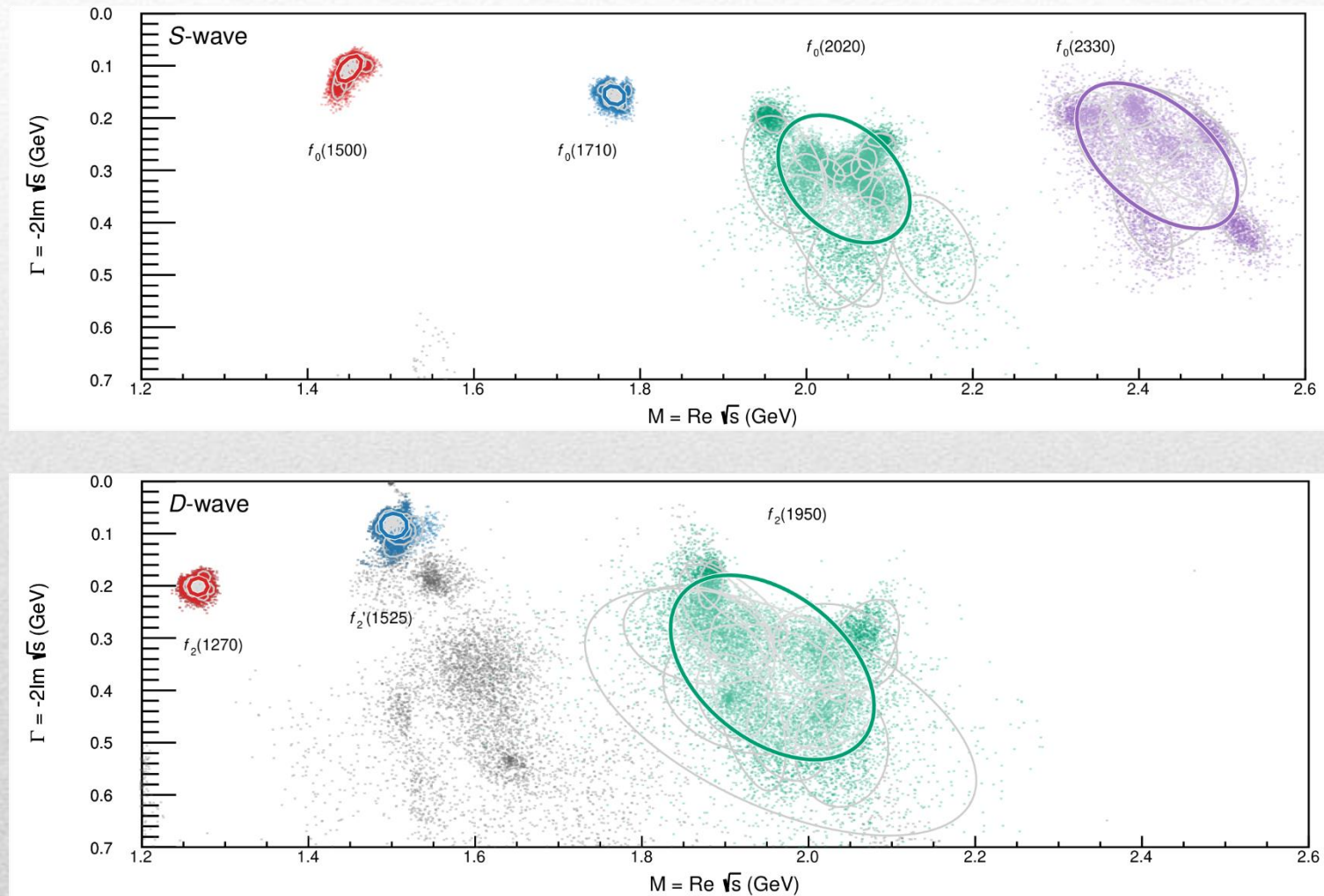
The $n(s)$ \rightarrow background physics, process-dependent, smooth
The $D(s)$ contains all the Final State Interactions
constrained by unitarity \rightarrow universal

$$J/\psi \rightarrow \gamma \pi^0 \pi^0 \text{ and } \rightarrow \gamma K_S^0 K_S^0$$

The final state is not fully described by getting the π_2 right
Being quasi-elastic, it's too constrained



Pole extraction



Looking at the residues

Caveats:

- When extracting the residues, we know we are neglecting some of the relevant channels. This should not affect the ratios seriously, but this is not a rigorous statement

- There is a contribution to the residues from the left-hand cut of the scattering process. To do that, a more realistic model for $N(s')$ is required. Again, this should not affect the ratios seriously

Despite the large systematics, the $f_0(1710)$ couples to kaons more than the $f_0(1500)$

Also, the $f_0(1710)$ couples to the initial gluon-rich state more than the $f_0(1500)$

Both these facts suggest a sizeable glueball component for the $f_0(1710)$

Conclusions

Bottom-up approaches are important!

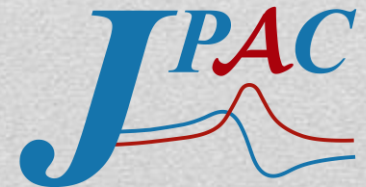
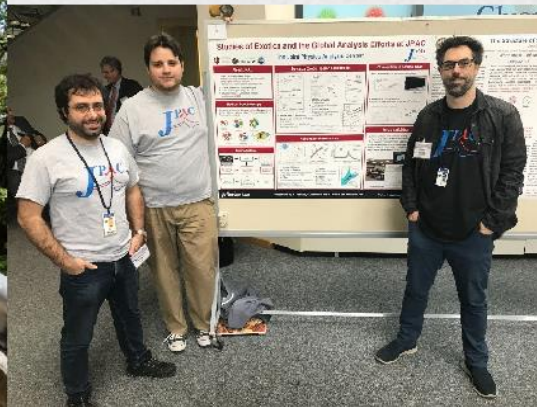
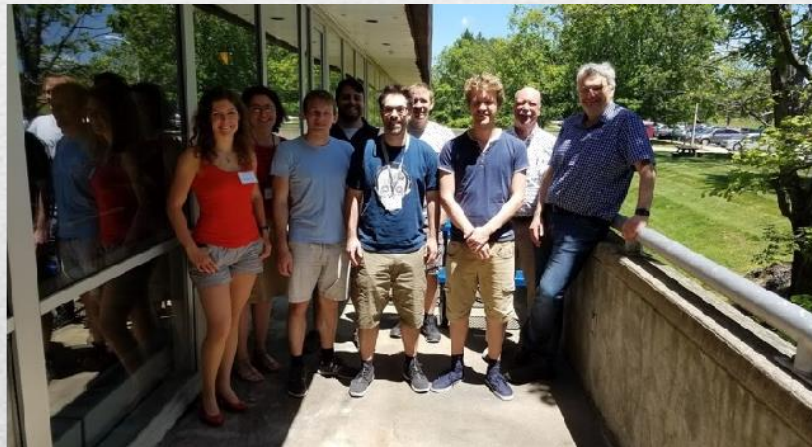
- They allow us to get the most out of high statistics data!
- The study of analytic structures offer insights into the nature of resonances
- Dispersive methods can improve the rigour and robustness in the extraction of the spectrum

Thank you!

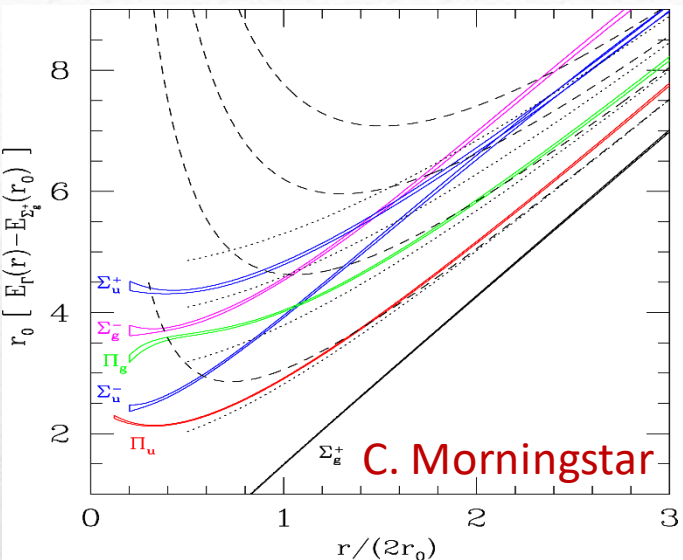
Joint Physics Analysis Center

- Joint effort between theorists and experimentalists in support of experimental data from JLab12 and other accelerator laboratories
- Cooperation between JPAC and experiments: co-authoring papers

<https://www.jpac-physics.org>



Learning about confinement

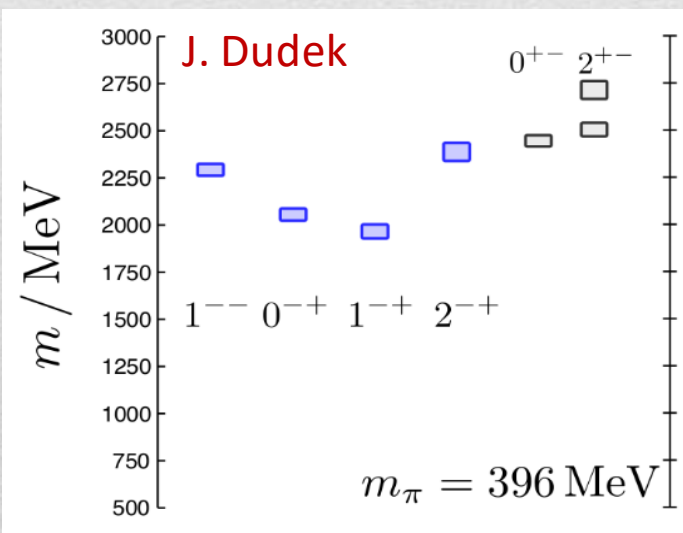


If quarks were infinitely heavy,
gluonic field is confined in a **string**

What is a **constituent gluon**?

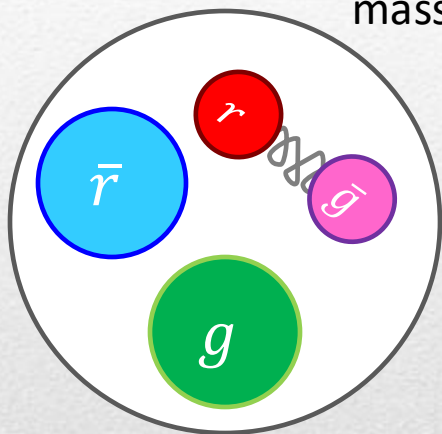
- vibration of the string?
- ~~excitation of a quasiparticle?~~
→ degenerate $J^{PC} = (0, \mathbf{1}, 2)^{-+}, (\mathbf{0}, 1, \mathbf{2})^{+-}$
- ~~excitation of a quasiparticle?~~
→ degenerate $J^{PC} = (0, \mathbf{1}, 2)^{-+}, 1^{--}$
- ~~says lattice QCD...~~
→ degenerate $J^{PC} = (0, \mathbf{1}, 2)^{-+}, 1^{--}$

What about real data?



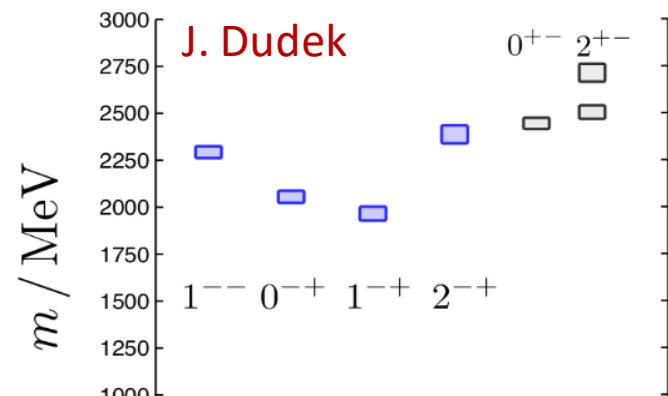
Hybrid hunting

Excited gluon,
 $J^{PC} = 1^{+-}$
 mass $\sim 1.0\text{--}1.5\text{ GeV}$



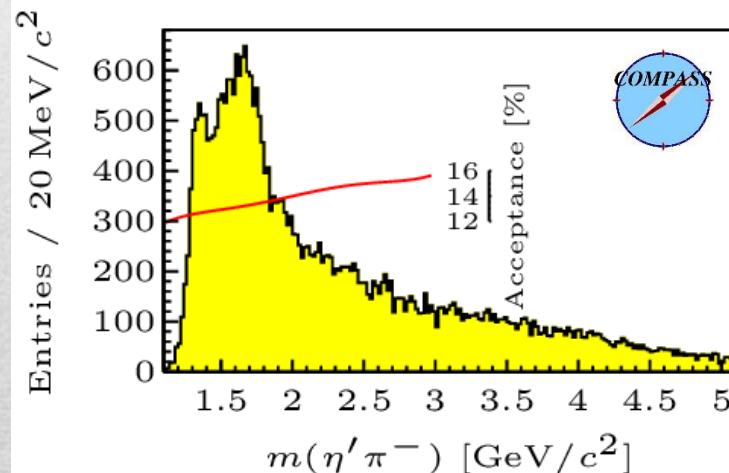
Look for a π_1 state with $J^{PC} = 1^{-+}$

decaying into $\begin{cases} \eta \pi \text{ and } \eta' \pi \\ \rho \pi \rightarrow 3\pi \\ b_1 \pi \rightarrow 5\pi \end{cases}$



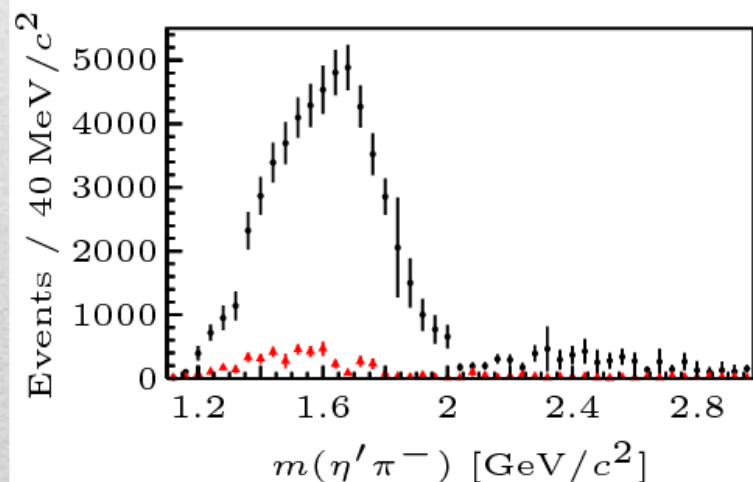
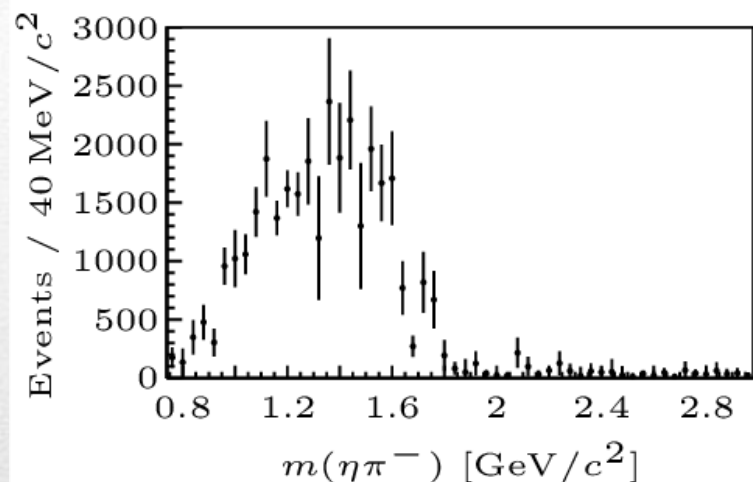
excitation of a quasiparticle?

\rightarrow degenerate $J^{PC} = (0, 1, 2)^{-+}, 1^{--}$



Small signal in data

Two hybrid states???



$\pi_1(1400) \quad I^G(J^{PC}) = 1^-(1^{-+})$

See also the mini-review under non- $q\bar{q}$ candidates in PDG 2006, Journal of Physics G33 1 (2006).

$\pi_1(1400)$ MASS	$1354 \pm 25 \text{ MeV (S = 1.8)}$
$\pi_1(1400)$ WIDTH	$330 \pm 35 \text{ MeV}$

Decay Modes

Mode	Fraction (Γ_i / Γ)	Scale Factor/ Conf. Level
$\Gamma_1 \quad \eta\pi^0$	seen	
$\Gamma_2 \quad \eta\pi^-$	seen	
$\Gamma_3 \quad \eta' \pi$		

Neither lattice nor models predict two 1^{-+} states in this region!

$\pi_1(1600) \quad I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1600)$ MASS	$1662^{+8}_{-9} \text{ MeV}$
$\pi_1(1600)$ WIDTH	$241 \pm 40 \text{ MeV (S = 1.4)}$

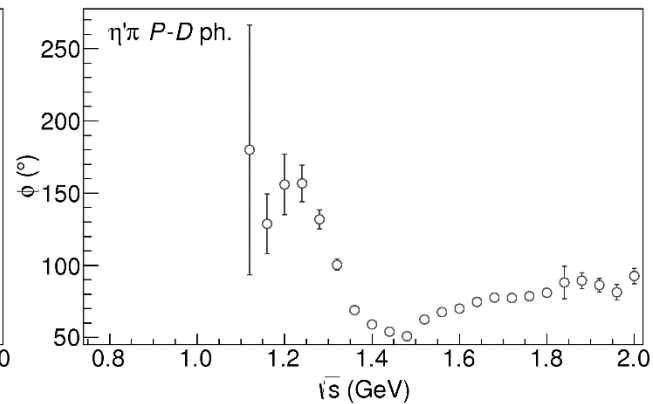
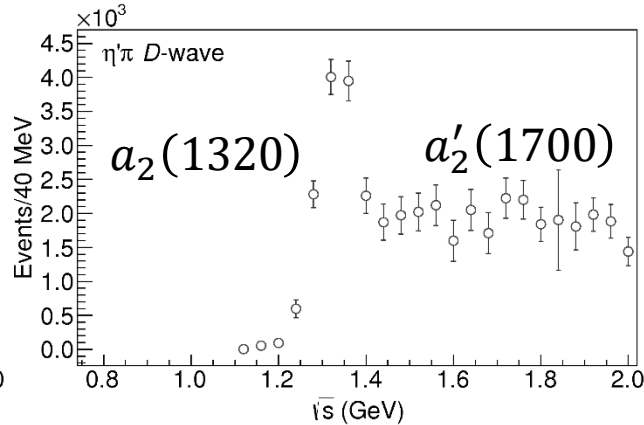
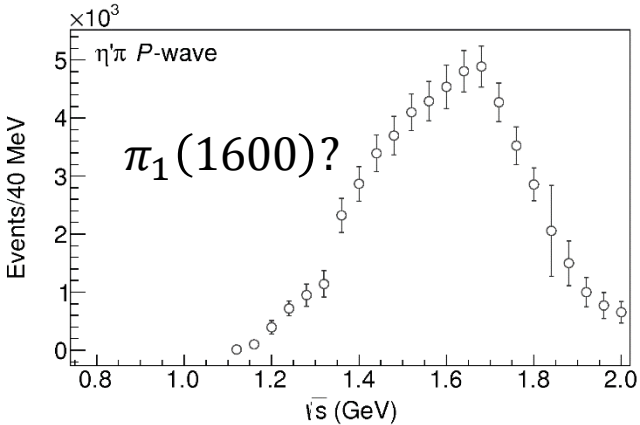
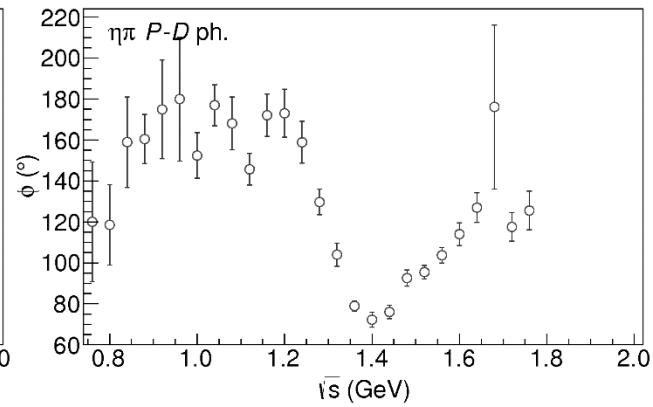
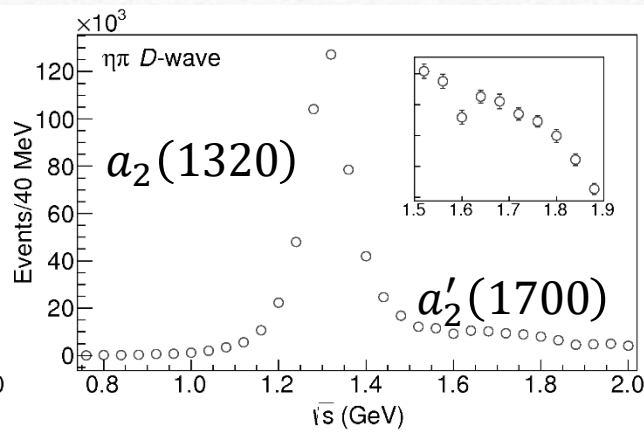
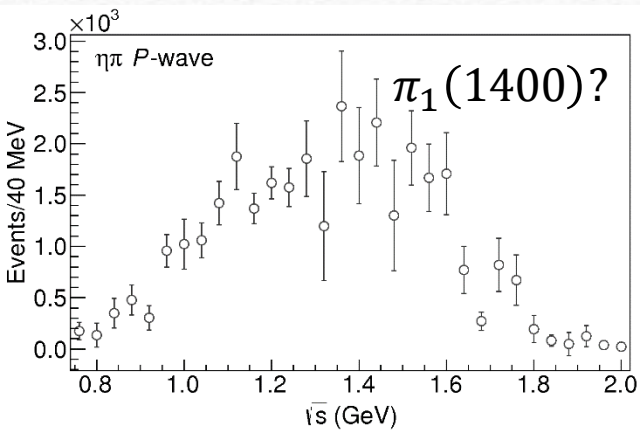
A hybrid meson (**8**) cannot decay into $\eta\pi$ in the SU(3) limit

Tetraquark ($\mathbf{10} \oplus \overline{\mathbf{10}}$)? Requires doubly charged

Mode	Fraction (Γ_i / Γ)	Scale Factor/ Conf. Level
$\Gamma_1 \quad \pi\pi\pi$	seen	
$\Gamma_2 \quad \rho^0\pi^-$	seen	
$\Gamma_3 \quad f_2(1270)\pi^-$	not seen	
$\Gamma_4 \quad \eta(548)\pi^-$	seen	
$\Gamma_5 \quad \eta'(958)\pi^-$	seen	
$\Gamma_6 \quad f_1(1285)\pi$	seen	

Data in $\eta^{(\prime)}\pi$

(COMPASS data)



$$J^{PC} = 1^{-+}$$

$$J^{PC} = 2^{++}$$

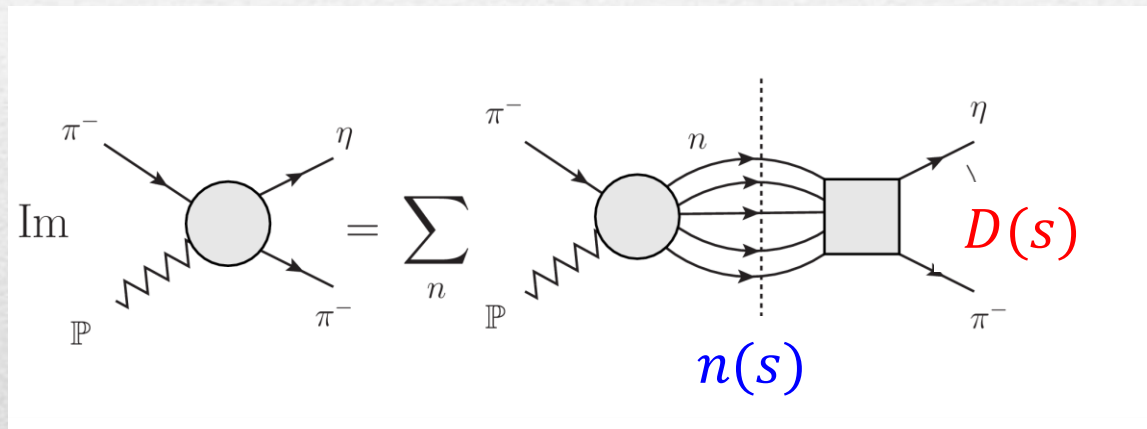
Amplitudes for $\eta^{(\prime)}\pi$

We build the partial wave amplitudes according to the **N/D method**

Jackura, Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB

Rodas, AP *et al.* (JPAC), PRL

$$a(s) = \frac{n(s)}{D(s)}$$



The $n(s)$ \rightarrow background physics, process-dependent, smooth
 The $D(s)$ contains all the Final State Interactions
 constrained by unitarity \rightarrow universal

Coupled channel: the model

A. Rodas, AP *et al.* (JPAC) PRL122, 042002

Two channels, $i, k = \eta\pi, \eta'\pi$

Two waves, $J = P, D$

37 fit parameters

$$D_{ki}^J(s) = \left[K^J(s)^{-1} \right]_{ki} - \frac{s}{\pi} \int_{s_k}^{\infty} ds' \frac{\rho N_{ki}^J(s')}{s'(s' - s - i\epsilon)}$$

$$K_{ki}^J(s) = \sum_R \frac{g_k^{(R)} g_i^{(R)}}{m_R^2 - s} + c_{ki}^J + d_{ki}^J s$$

1 K-matrix pole for the P-wave
2 K-matrix poles for the D-wave

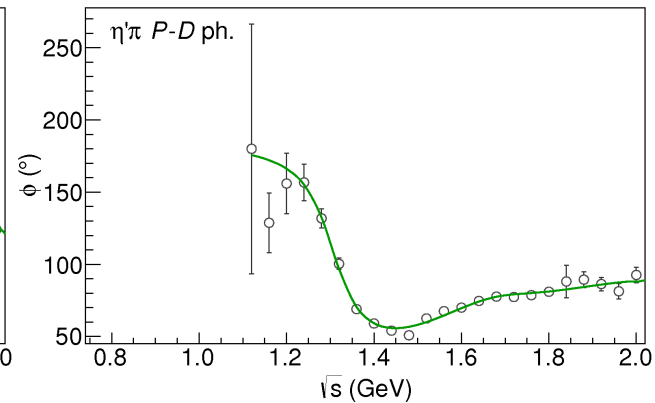
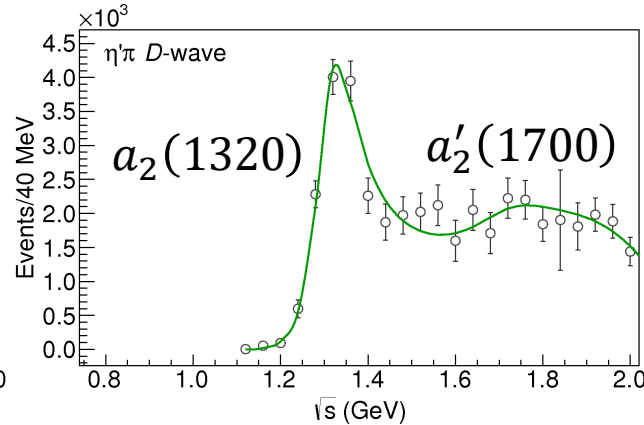
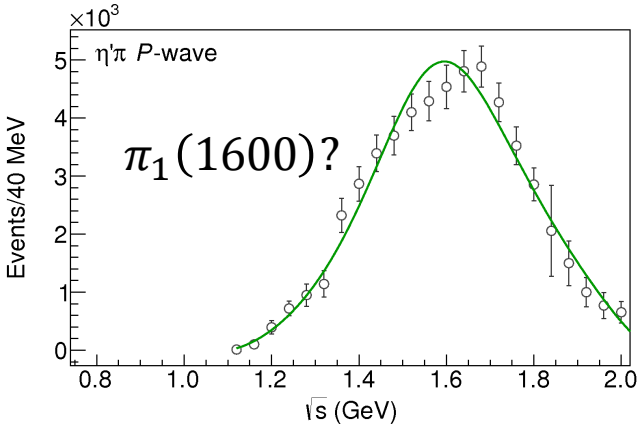
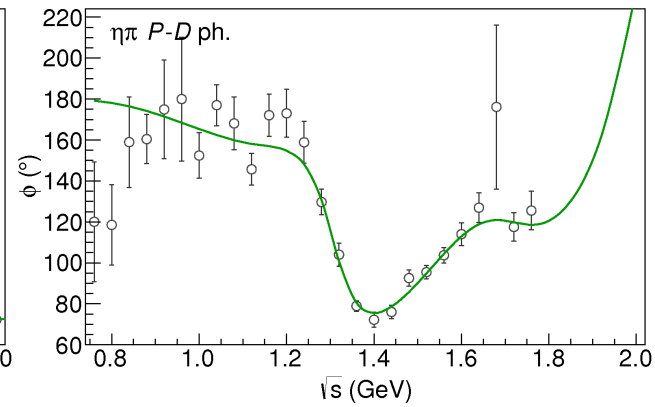
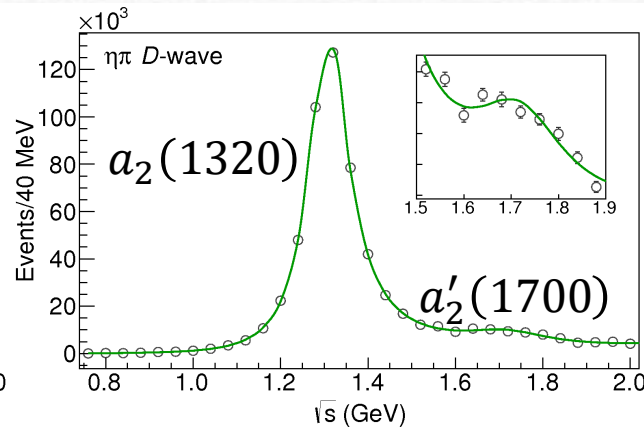
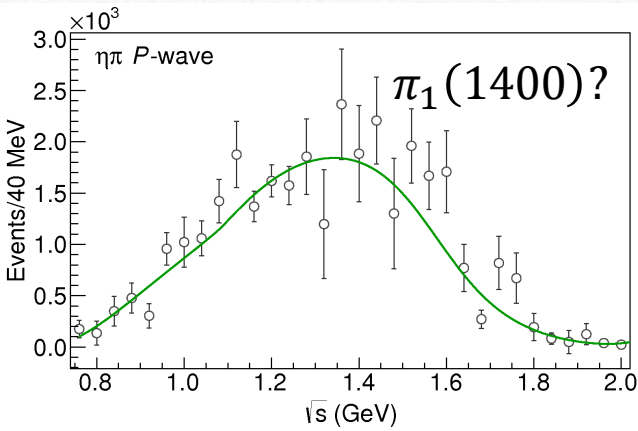
$$\rho N_{ki}^J(s') = \delta_{ki} \frac{\lambda^{J+1/2} \left(s', m_{\eta^{(\prime)}}^2, m_{\pi}^2 \right)}{(s' + s_R)^{2J+1+\alpha}}$$

$$n_k^J(s) = \sum_{n=0}^3 a_n^{J,k} T_n \left(\frac{s}{s + s_0} \right)$$

Left-hand scale (Blatt-Weisskopf radius) $s_R = s_0 = 1 \text{ GeV}^2$
 $\alpha = 2$, 3rd order polynomial for $n_k^J(s)$

Fit to $\eta^{(\prime)}\pi$

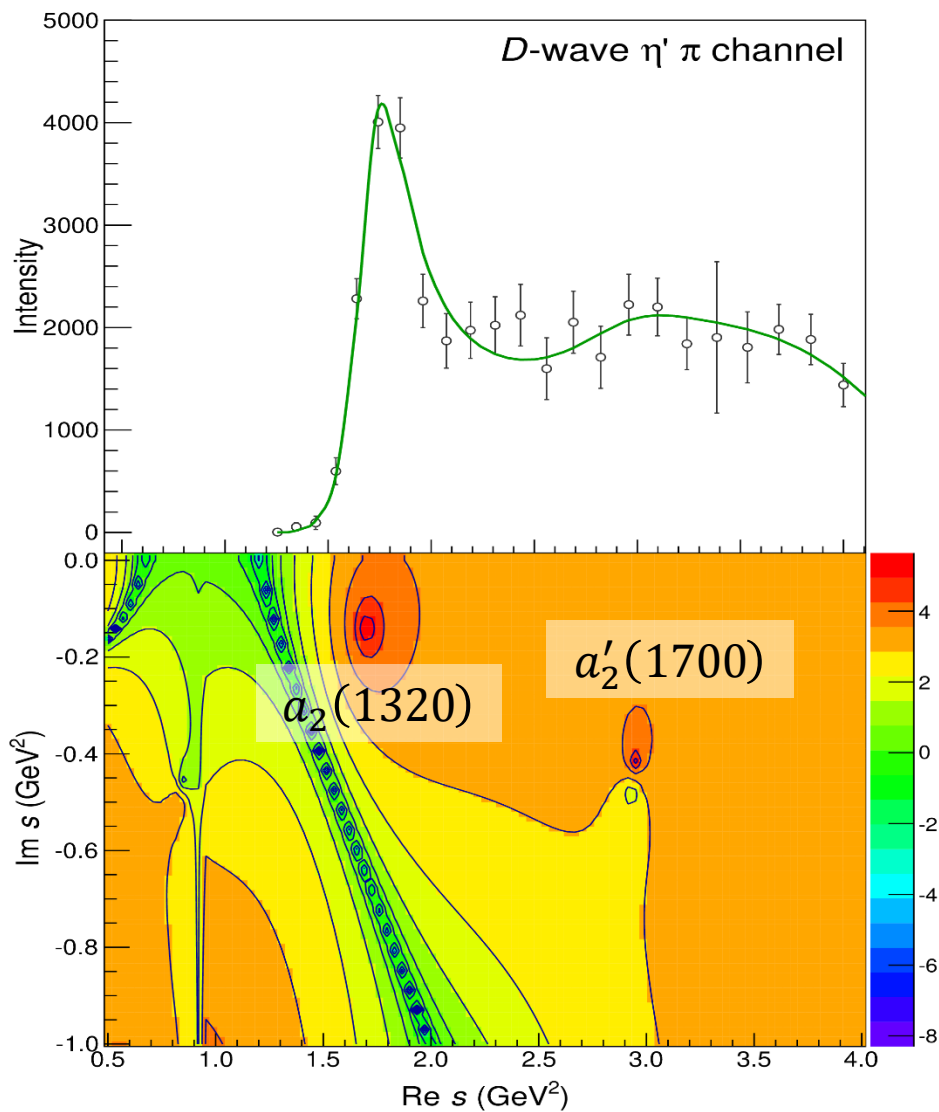
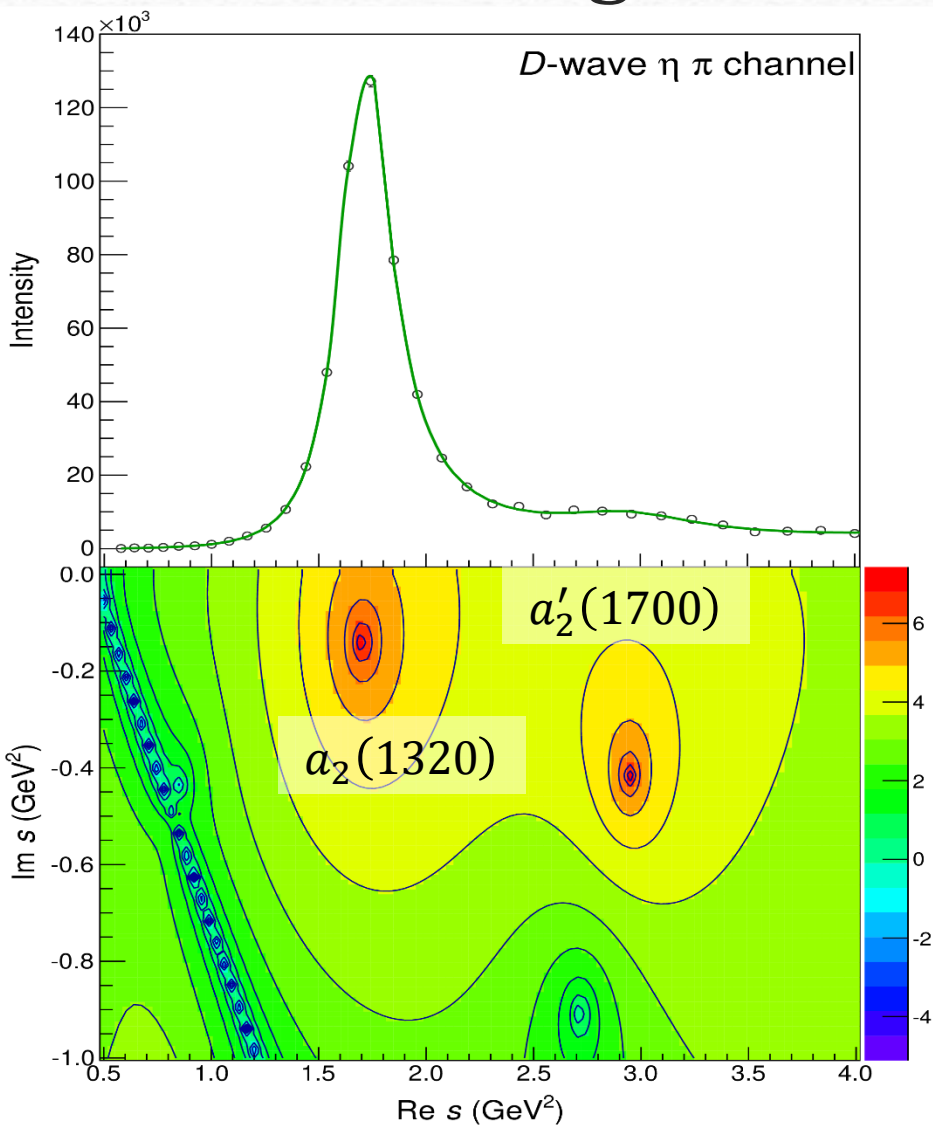
(COMPASS data)



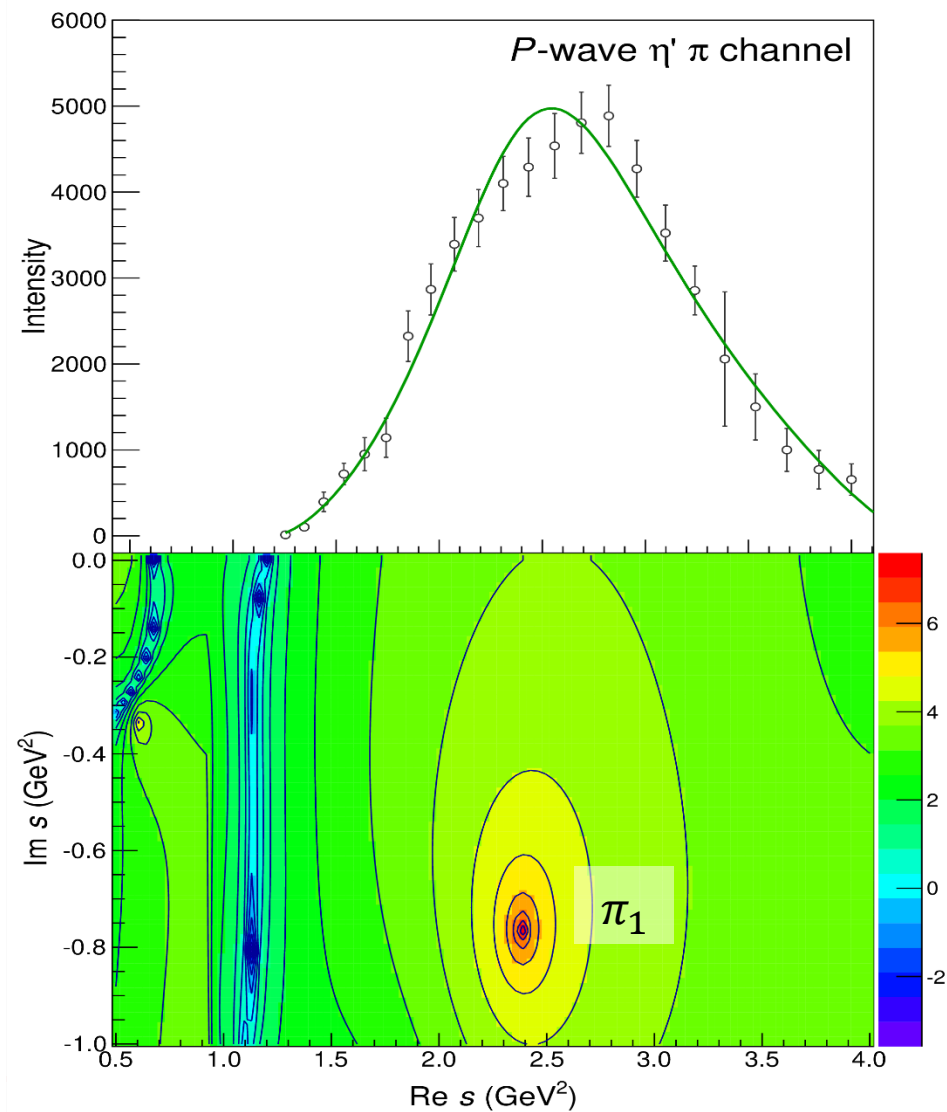
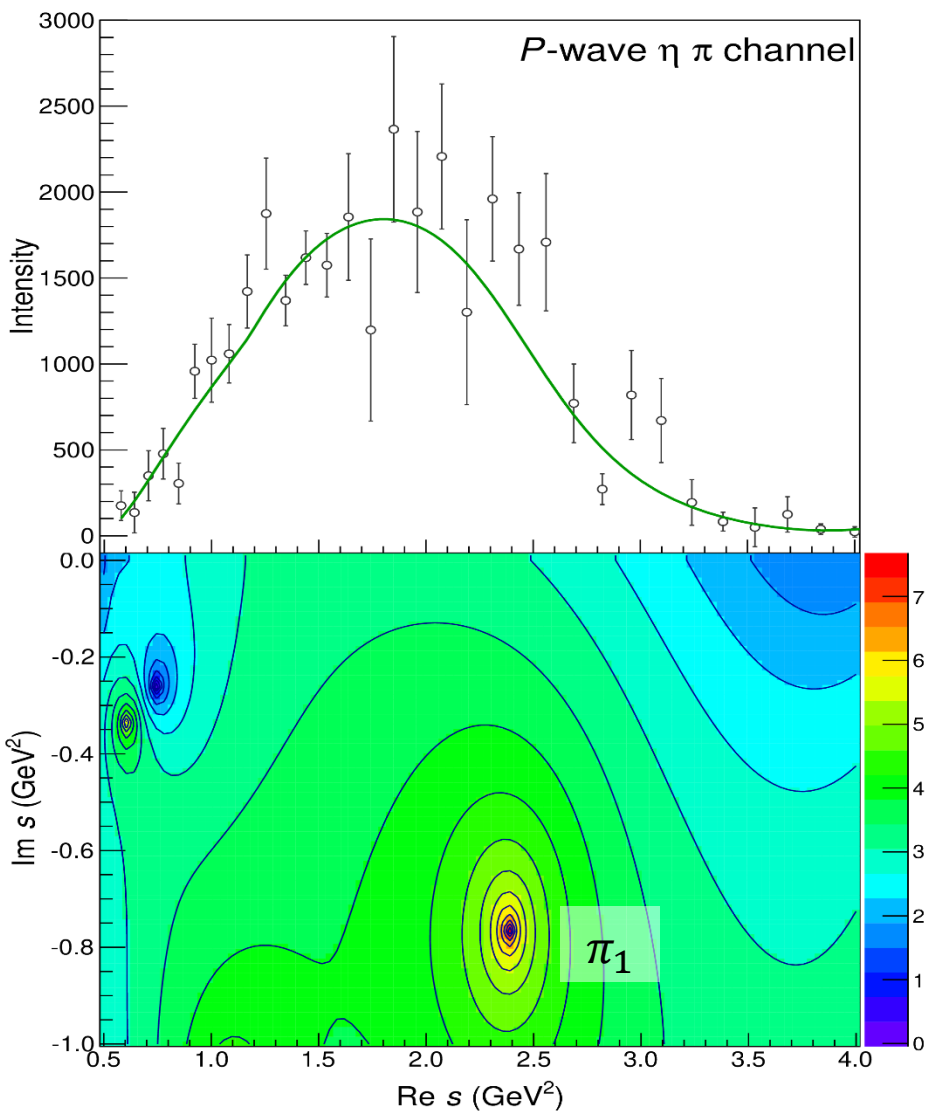
$$J^{PC} = 1^{-+}$$

$$J^{PC} = 2^{++}$$

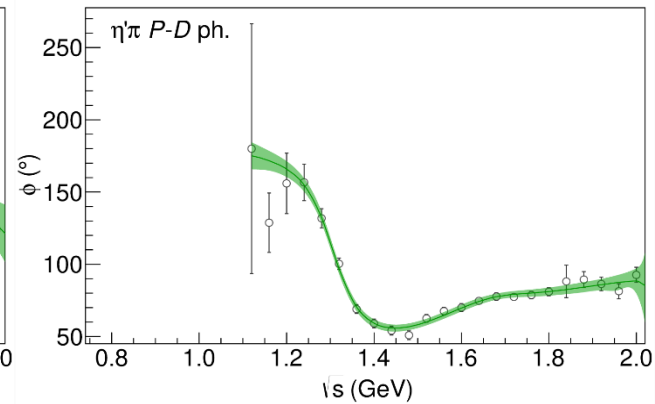
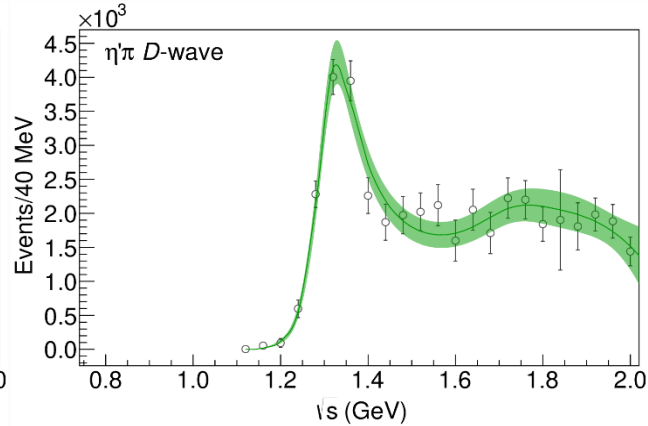
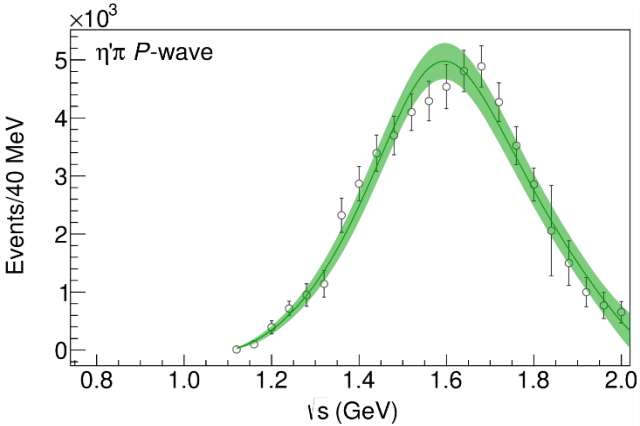
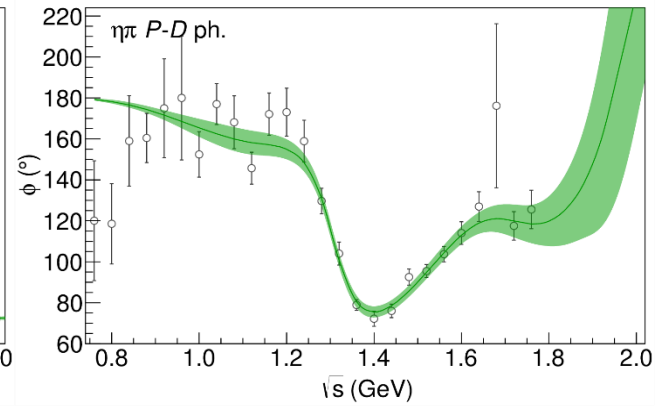
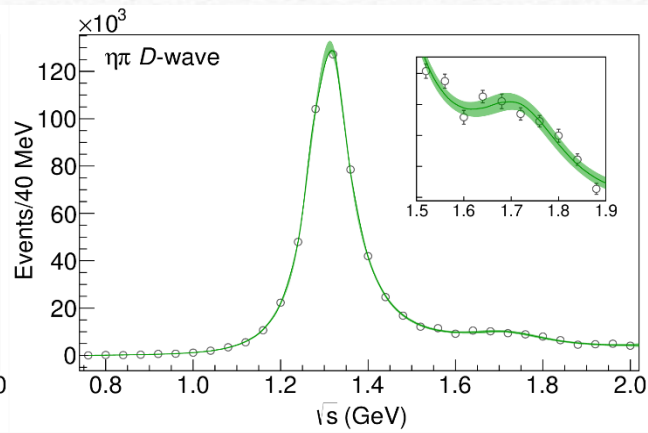
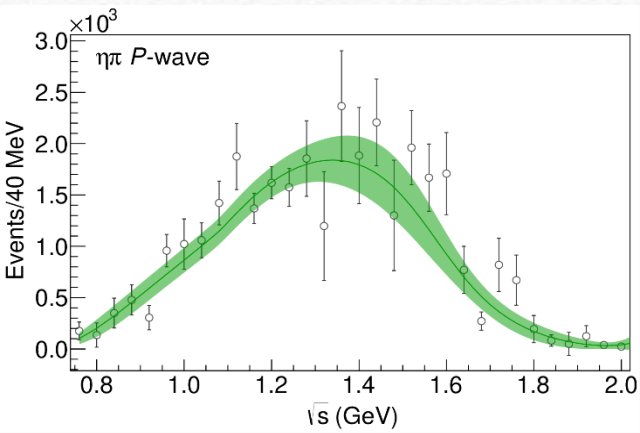
Pole hunting



Pole hunting



Statistical Bootstrap



Correlations

Denominator parameters uncorrelated with the numerator ones ✓

Production (numerator) parameters

$m_{D,2}^2$	16	-16	16	-15	-20	16	-19	1	-5	5	-5	6	15	-19	6	-7
$g_{\eta'\pi}^{D,2}$	45	-45	44	-43	-8	-3	-5	-8	-40	41	-41	41	-4	-2	-4	-6
$g_{\eta\pi}^{D,2}$	13	-13	13	-13	-3	-8	-2	-8	-1	2	-3	7	-10	5	-8	-4
$m_{D,1}^2$	24	-23	21	-15	-4	5	-15	1	-25	20	-9	-12	5	-4	-13	2
$g_{\eta'\pi}^{D,1}$	9	-9	10	-12	18	4	-27	32	-5	7	-11	19	9	10	-24	35
$g_{\eta\pi}^{D,1}$	23	-22	20	-15	-0	1	-13	1	-24	20	-9	-12	1	-0	-16	4
$m_{P,1}^2$	25	-24	24	-23	-21	12	-9	-6	-26	28	-31	36	2	-10	7	-12
$g_{\eta'\pi}^{P,1}$	-12	11	-11	9	8	-7	7	-0	14	-13	12	-9	5	-5	7	-2
$g_{\eta\pi}^{P,1}$	-6	6	-7	10	-6	10	-3	3	5	-5	8	-11	11	-11	11	-4
Γ_{π_1}	22	-23	23	-25	-4	5	-9	3	-3	2	1	-5	6	-6	0	-1
m_{π_1}	-10	9	-8	4	12	-8	3	3	6	-6	7	-7	-6	11	-9	8
$\Gamma_{a_2'}$	-17	17	-16	14	-21	25	-8	6	17	-15	8	4	26	-27	28	-10
$m_{a_2'}$	8	-9	9	-11	-17	21	-13	8	-3	4	-8	16	19	-21	17	-7
Γ_{a_2}	-3	3	-4	4	-6	4	1	-4	2	-3	4	-7	5	-7	6	-4
m_{a_2}	-6	6	-5	5	-12	14	-5	3	7	-6	4	-0	13	-15	15	-7
	$P_{a_0}^{\pi\pi}$	$P_{a_1}^{\pi\pi}$	$P_{a_2}^{\pi\pi}$	$P_{a_3}^{\pi\pi}$	$D_{a_0}^{\pi\pi}$	$D_{a_1}^{\pi\pi}$	$D_{a_2}^{\pi\pi}$	$D_{a_3}^{\pi\pi}$	$P_{a_0}^{\eta\pi}$	$P_{a_1}^{\eta\pi}$	$P_{a_2}^{\eta\pi}$	$P_{a_3}^{\eta\pi}$	$D_{a_0}^{\eta\pi}$	$D_{a_1}^{\eta\pi}$	$D_{a_2}^{\eta\pi}$	$D_{a_3}^{\eta\pi}$

K-matrix «pole» parameters

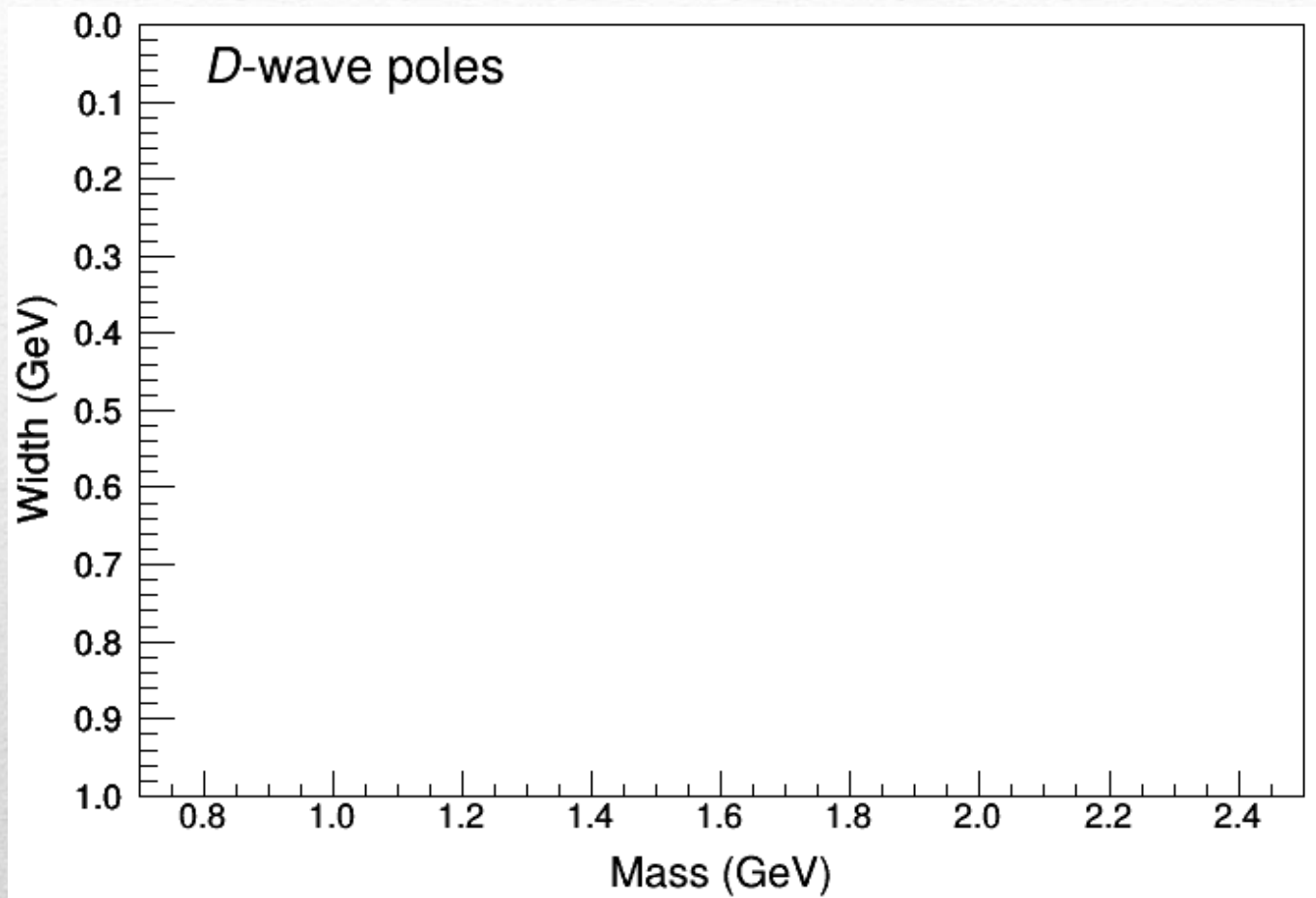
Denominator parameters uncorrelated between P - and D -wave ✓

K-matrix «bkg» parameters

$d_{\eta'\pi,\eta'\pi}^D$	-27	0	-22	-57	-53	84	22	-3	20	-20	-91	100
$d_{\eta\pi,\eta'\pi}^D$	32	-4	22	32	63	-76	-29	4	-18	-18	100	-91
$d_{\eta\pi,\eta\pi}^D$	-22	11	1	67	-15	-17	24	-4	-4	100	-18	-20
$d_{\eta'\pi,\eta'\pi}^P$	-28	-74	-90	-8	-17	20	23	72	100	-4	-18	20
$d_{\eta\pi,\eta'\pi}^P$	-45	-94	-60	2	-5	2	45	100	72	-4	4	-3
$d_{\eta\pi,\eta\pi}^P$	-92	-30	-24	-8	-9	13	100	45	23	24	-29	22
$c_{\eta'\pi,\eta'\pi}^D$	-18	-7	-19	-26	-84	100	13	2	20	-17	-76	84
$c_{\eta\pi,\eta'\pi}^D$	10	6	16	-19	100	-84	-9	-5	-17	-15	63	-53
$c_{\eta\pi,\eta\pi}^D$	14	3	10	100	-19	-26	-8	2	-8	67	32	-57
$c_{\eta'\pi,\eta'\pi}^P$	31	67	100	10	16	-19	-24	-60	-90	1	22	-22
$c_{\eta\pi,\eta'\pi}^P$	41	100	67	3	6	-7	-30	-94	-74	11	-4	0
$c_{\eta\pi,\eta\pi}^P$	100	41	31	14	10	-18	-92	-45	-28	-22	32	-27
	$P_{\eta\pi,\eta\pi}^P$	$P_{\eta\pi,\eta'\pi}^P$	$P_{\eta'\pi,\eta'\pi}^P$	$D_{\eta\pi,\eta\pi}^D$	$D_{\eta\pi,\eta'\pi}^D$	$D_{\eta'\pi,\eta'\pi}^D$	$D_{\eta\pi,\eta\pi}^P$	$D_{\eta\pi,\eta'\pi}^P$	$D_{\eta'\pi,\eta'\pi}^P$	$D_{\eta\pi,\eta\pi}^D$	$D_{\eta\pi,\eta'\pi}^D$	$D_{\eta'\pi,\eta'\pi}^D$

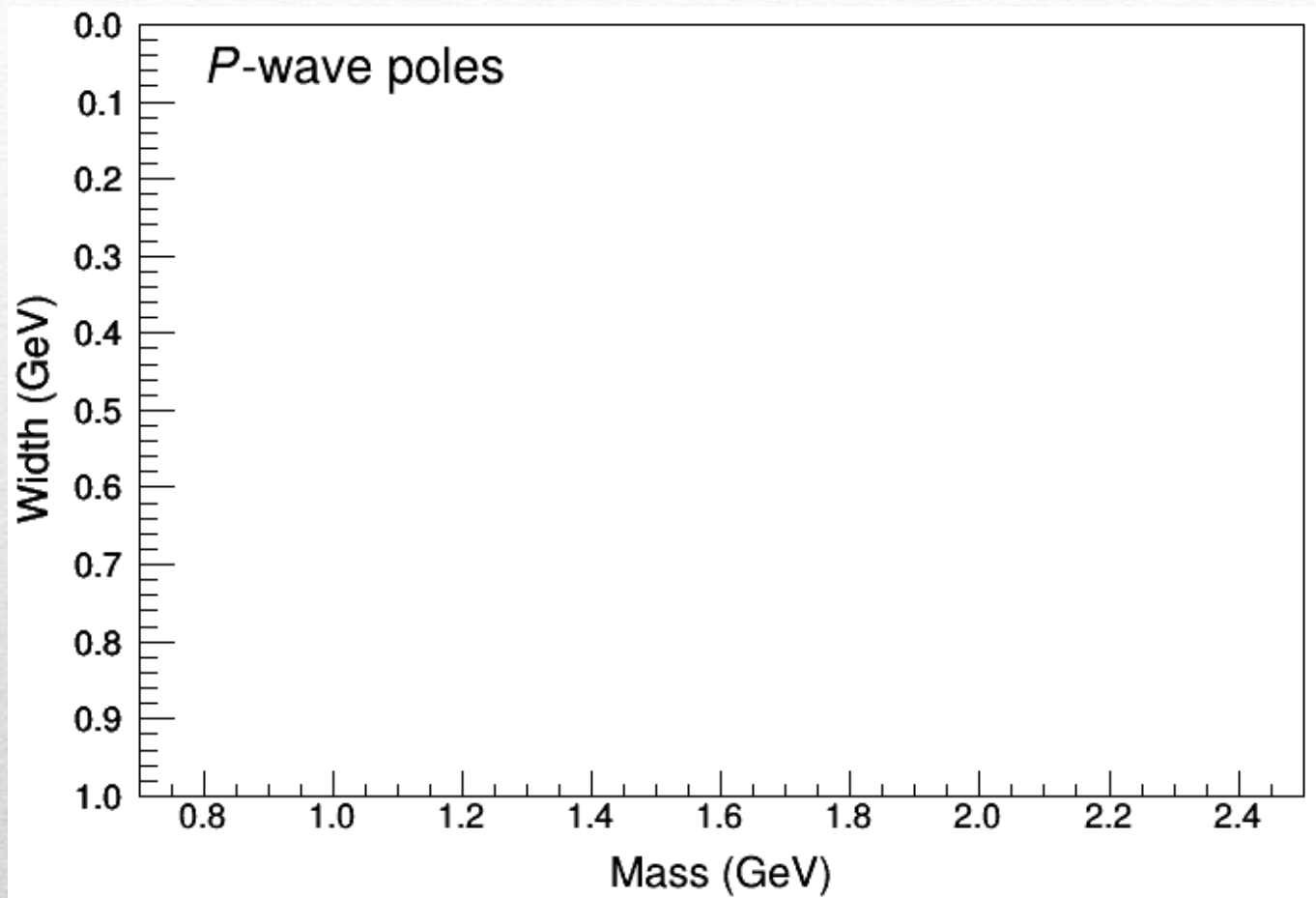
K-matrix «bkg» parameters

Statistical Bootstrap



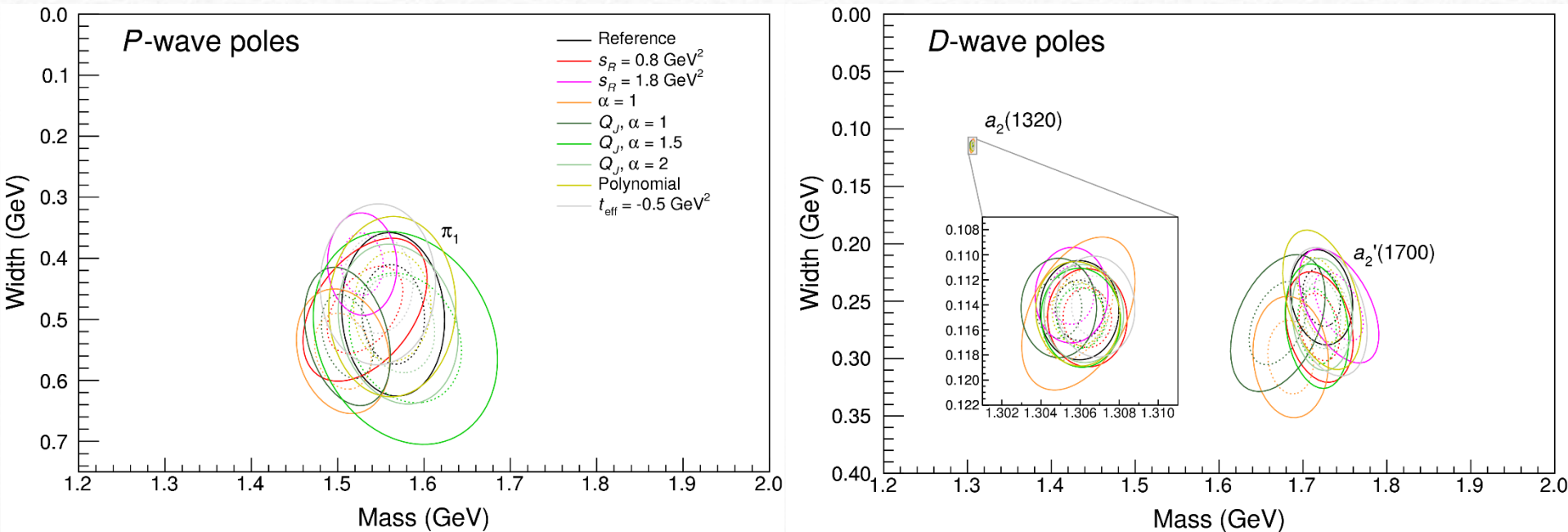
For each fit, we search poles: two clusters in D -wave: $a_2(1320)$ and $a'_2(1700)$

Statistical Bootstrap



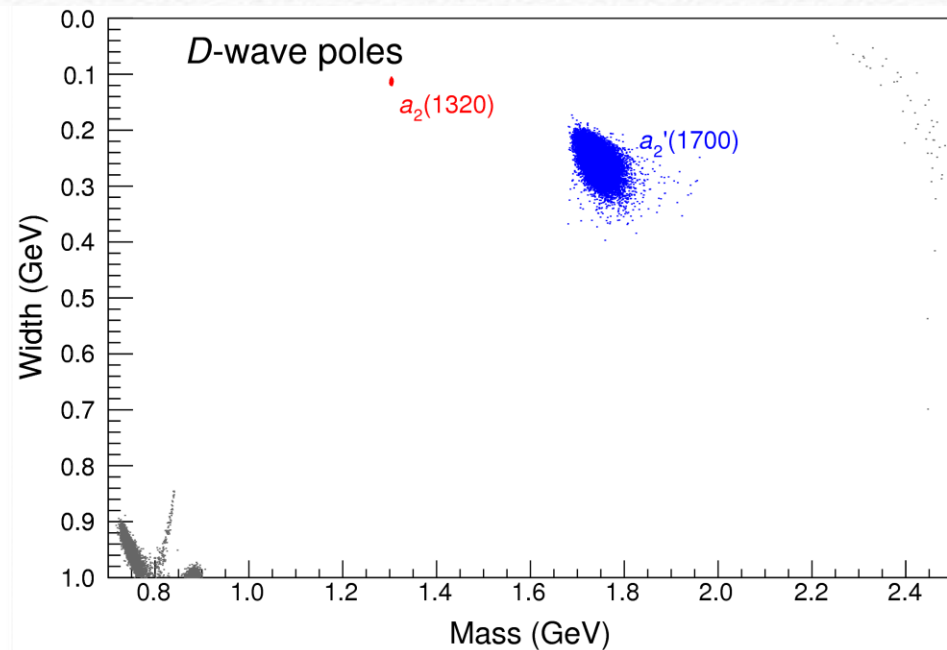
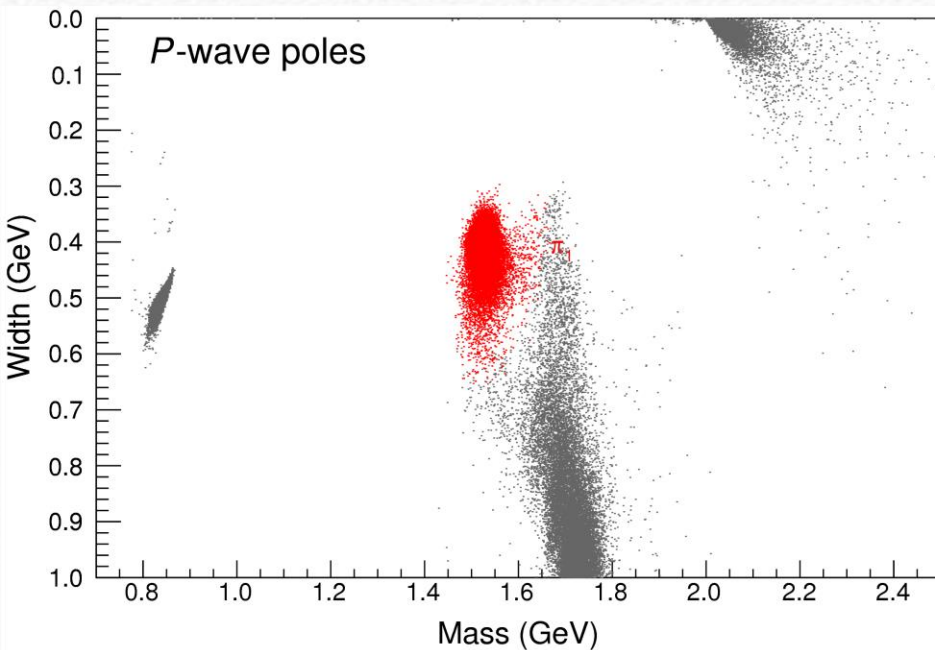
Only one stable cluster in P -wave: a single π_1

Systematic studies



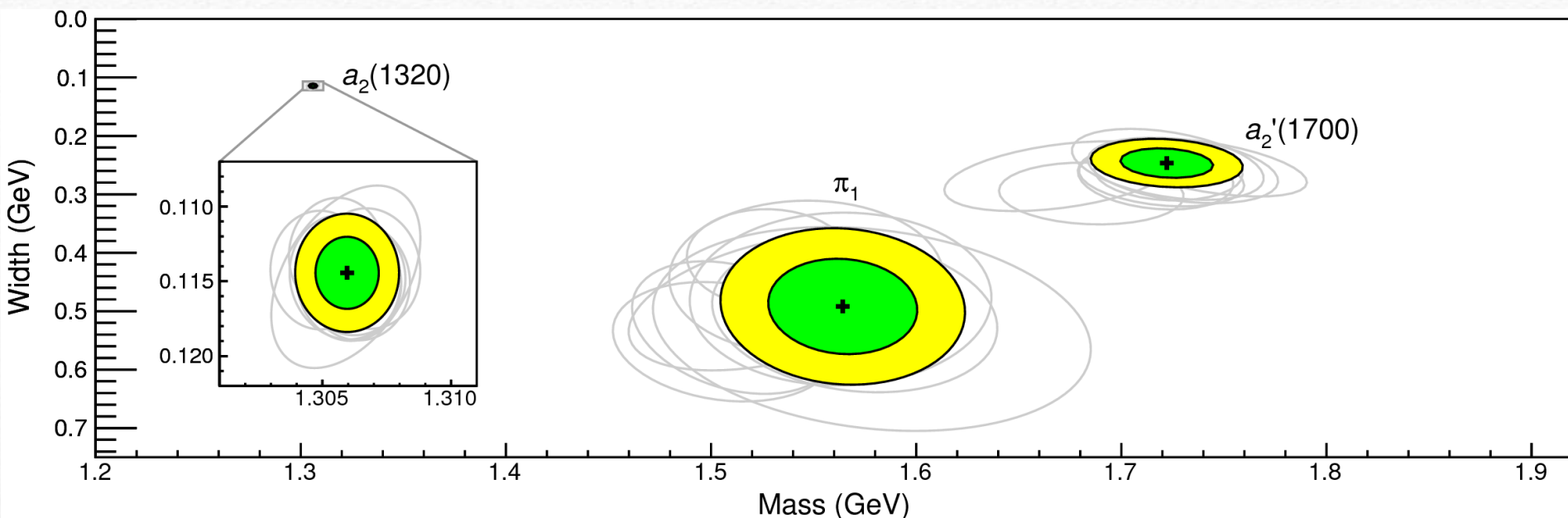
For each class, the maximum deviation of mass and width is taken as a systematic error
 Deviation smaller than the statistical error are neglected
 Systematic of different classes are summed in quadrature

Bootstrap for $s_R = 1.8 \text{ GeV}^2$



Our skepticism about a second pole in the relevant region is confirmed:
It is unstable and not trustable

Final results



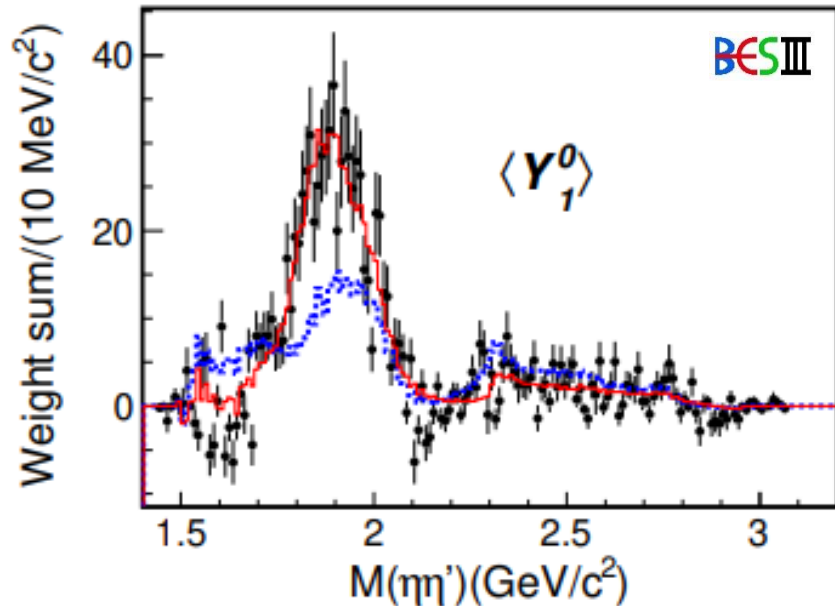
Poles	Mass (MeV)	Width (MeV)
$a_2(1320)$	$1306.0 \pm 0.8 \pm 1.3$	$114.4 \pm 1.6 \pm 0.0$
$a_2'(1700)$	$1722 \pm 15 \pm 67$	$247 \pm 17 \pm 63$
π_1	$1564 \pm 24 \pm 86$	$492 \pm 54 \pm 102$

Agreement with Lattice is restored

That's the **most rigorous** extraction of an exotic meson available so far!

An isoscalar η_1 ?

There is a recent claim by BESIII in $J/\psi \rightarrow \gamma \eta \eta'$
of resonant activity in P-wave

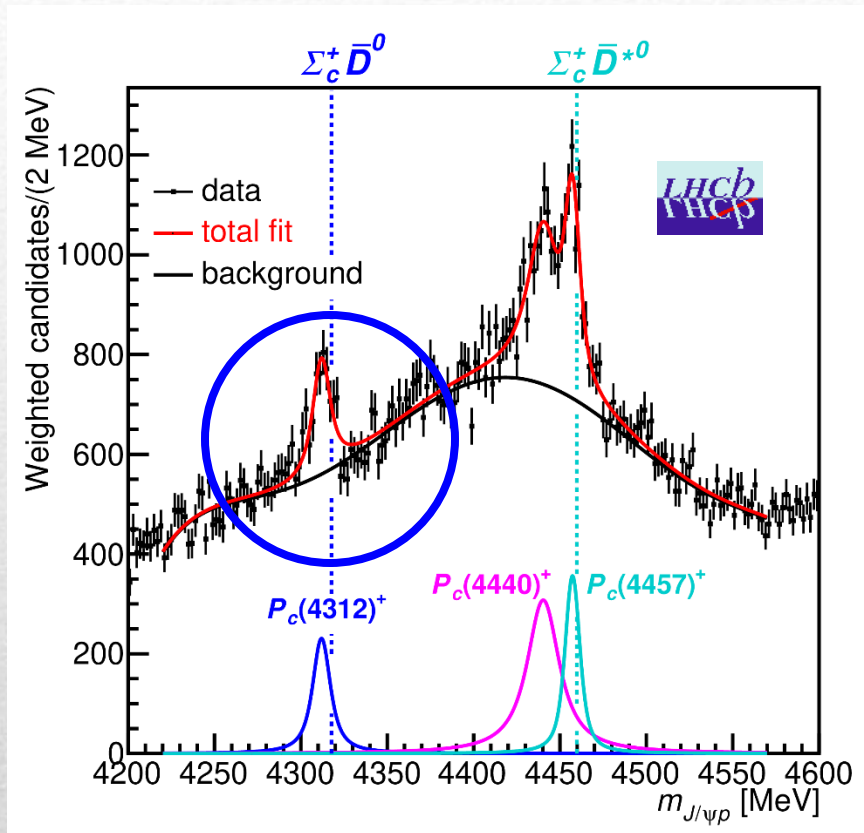


Not enough information
to perform a similar analysis but...
stay tuned!

BW parameters:

$$M = 1855 \pm 9_{-1}^{+6} \text{ MeV},$$
$$\Gamma = 188 \pm 18_{-8}^{+3} \text{ MeV}$$

New pentaquarks discovered



The lowest $P_c(4312)$ appears as an **isolated peak** at the $\Sigma_c^+ \bar{D}^0$ threshold

A detailed study of the lineshape provides insight on its nature

Bottom-up:

DON'T YOU DARE describing everything!!!

Focus on the peak region

Minimal(istic) model for $P_c(4312)$

( data)

$$\frac{dN}{d\sqrt{s}} = \rho(s) [|F(s)|^2 + b_0 + b_1 s]$$

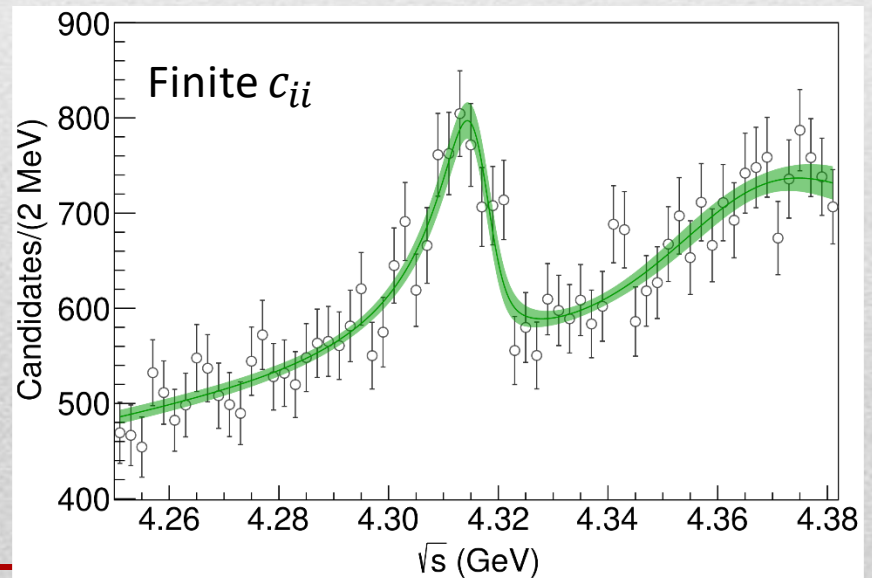
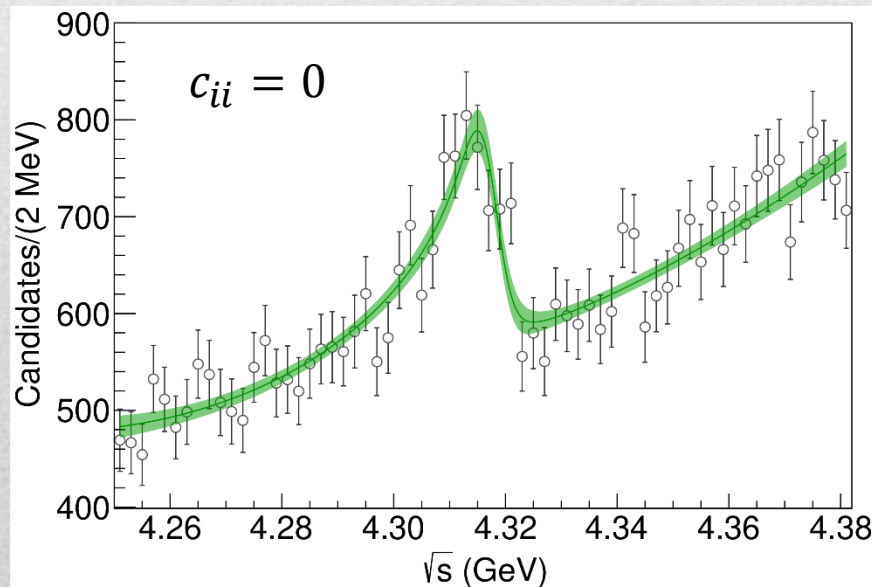
Fernandez-Ramirez, AP *et al.* (JPAC), PRL 123, 092001

Effective range expansion

$$F(s) = (N_1 + N_2 s) T_{11}(s)$$

$$T(s) = \begin{pmatrix} m_{11} - c_{11}s - i\rho_1(s) & m_{12} \\ m_{12} & m_{22} - c_{22}s - i\rho_2(s) \end{pmatrix}^{-1}$$

We can set $c_{ii} = 0$ to reduce to the scattering length approximation



Minimal(istic) model for $P_c(4312)$

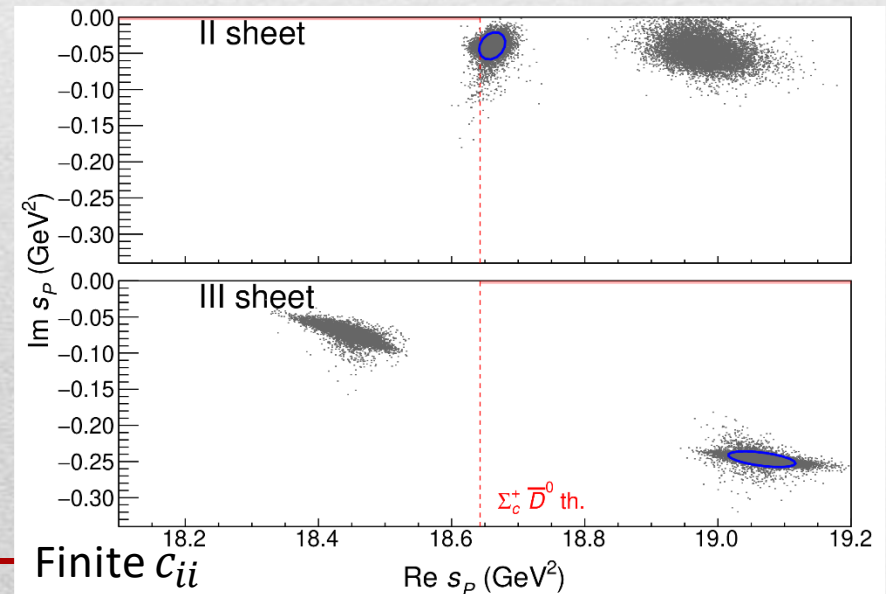
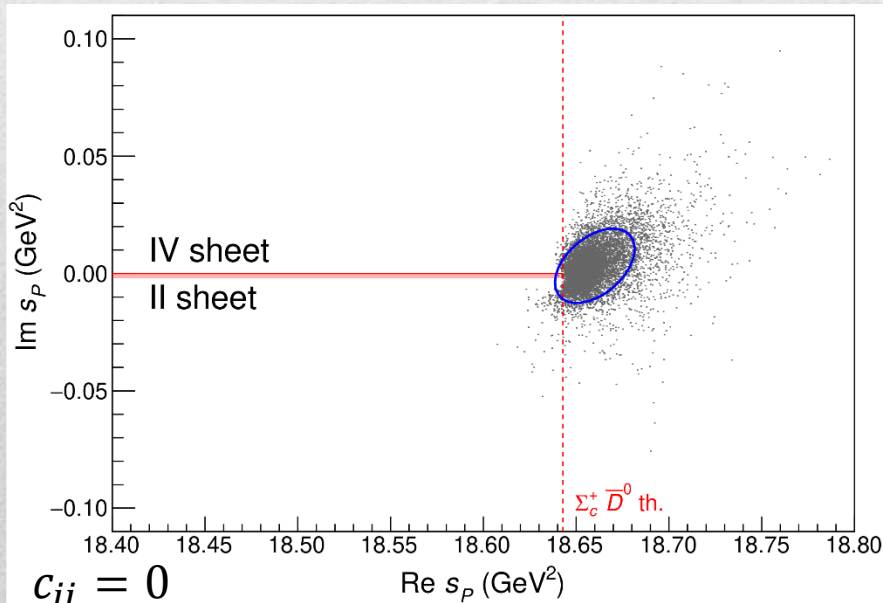
$$\frac{dN}{d\sqrt{s}} = \rho(s) [|F(s)|^2 + b_0 + b_1 s]$$

Effective range expansion

$$F(s) = (N_1 + N_2 s) T_{11}(s)$$

We can set $c_{ii} = 0$ to reduce to the scattering length approximation

$$T(s) = \begin{pmatrix} m_{11} - c_{11}s - i\rho_1(s) & m_{12} \\ m_{12} & m_{22} - c_{22}s - i\rho_2(s) \end{pmatrix}^{-1}$$



Minimal(istic) model for $P_c(4312)$

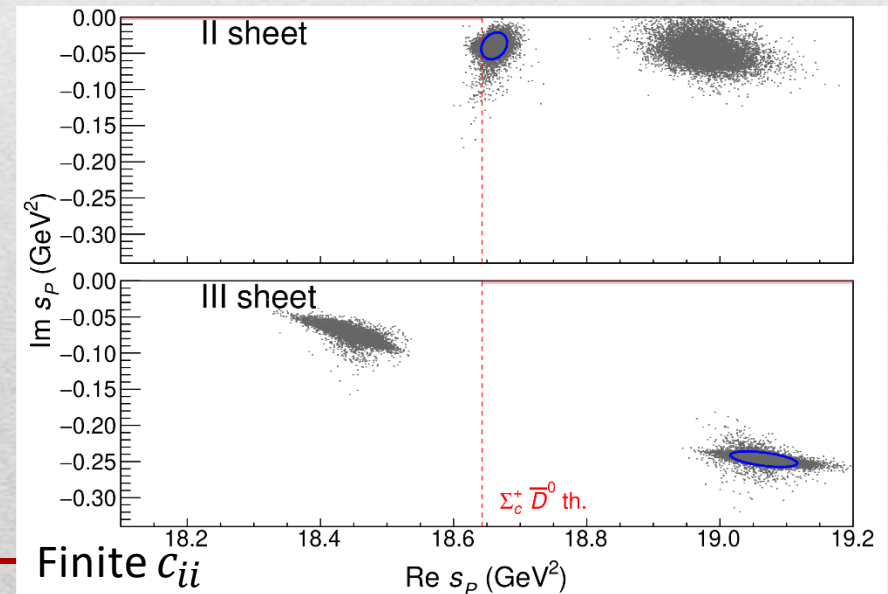
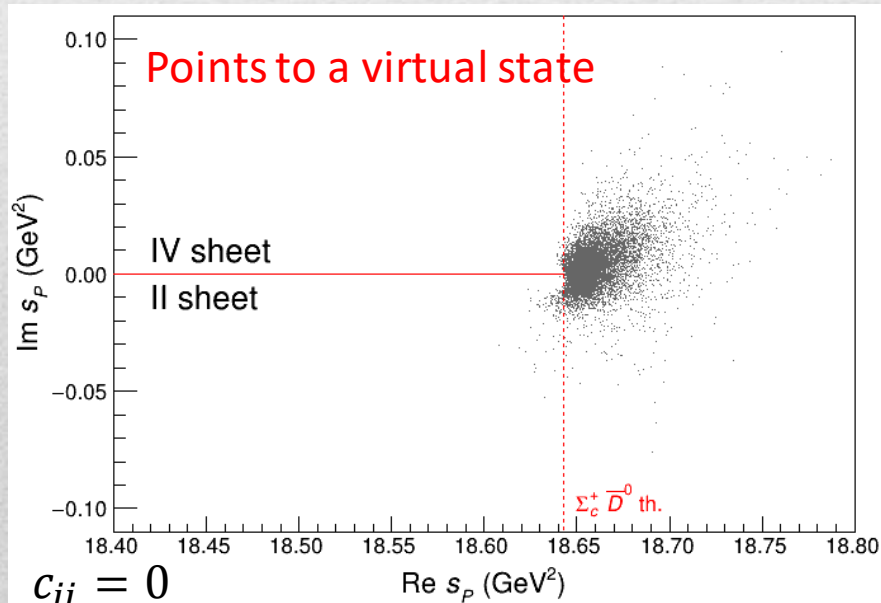
$$\frac{dN}{d\sqrt{s}} = \rho(s) [|F(s)|^2 + b_0 + b_1 s]$$

Effective range expansion

$$F(s) = (N_1 + N_2 s) T_{11}(s)$$

We can set $c_{ii} = 0$ to reduce to the scattering length approximation

$$T(s) = \begin{pmatrix} m_{11} - c_{11}s - i\rho_1(s) & m_{12} \\ m_{12} & m_{22} - c_{22}s - i\rho_2(s) \end{pmatrix}^{-1}$$



Minimal(istic) model for $P_c(4312)$

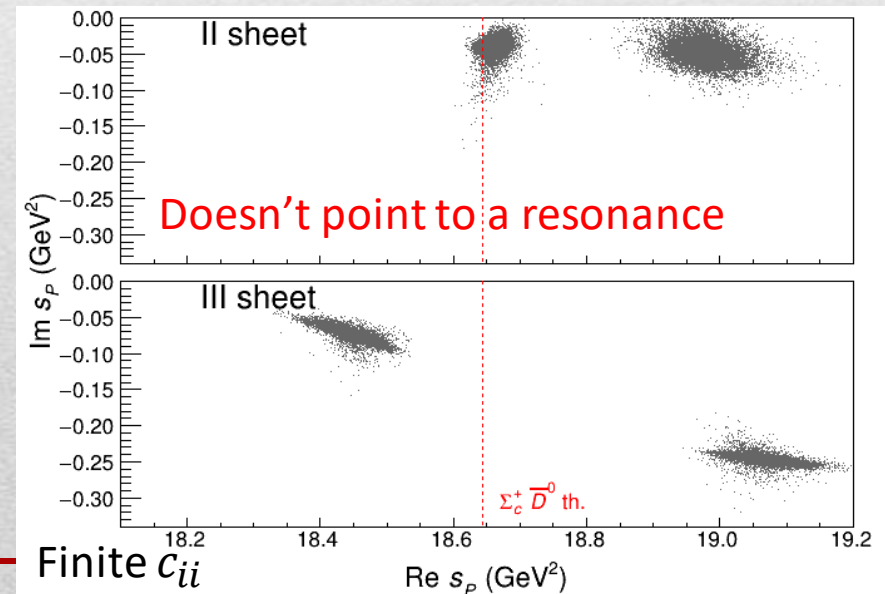
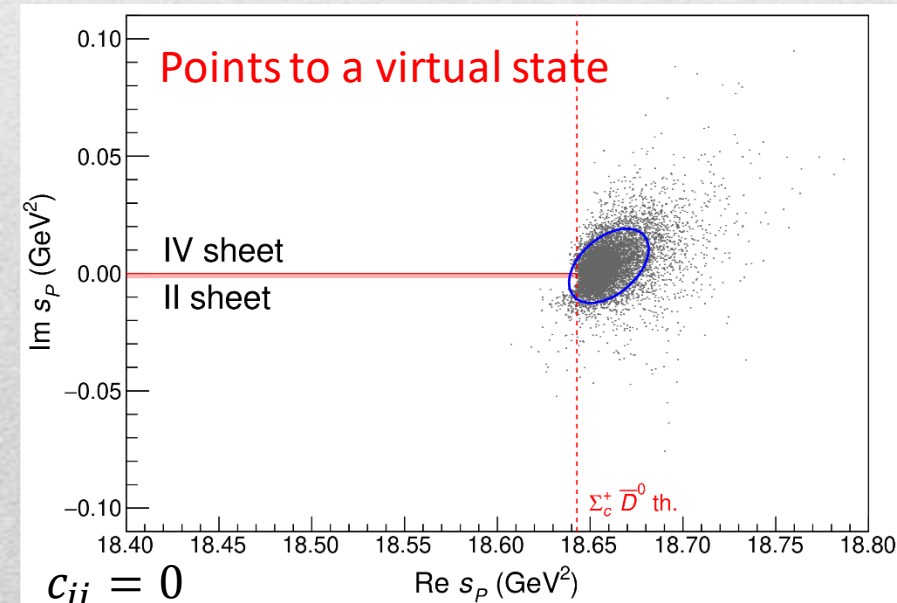
$$\frac{dN}{d\sqrt{s}} = \rho(s) [|F(s)|^2 + b_0 + b_1 s]$$

$$F(s) = (N_1 + N_2 s) T_{11}(s)$$

$$T(s) = \begin{pmatrix} m_{11} - c_{11}s - i\rho_1(s) & m_{12} \\ m_{12} & m_{22} - c_{22}s - i\rho_2(s) \end{pmatrix}^{-1}$$

Effective range expansion

We can set $c_{ii} = 0$ to reduce to the scattering length approximation



Minimal(istic) model for $P_c(4312)$

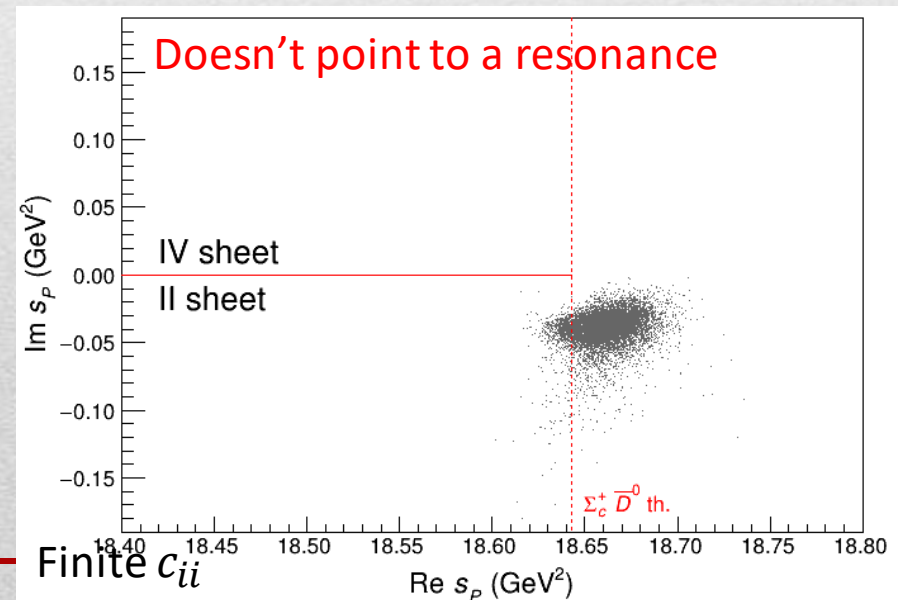
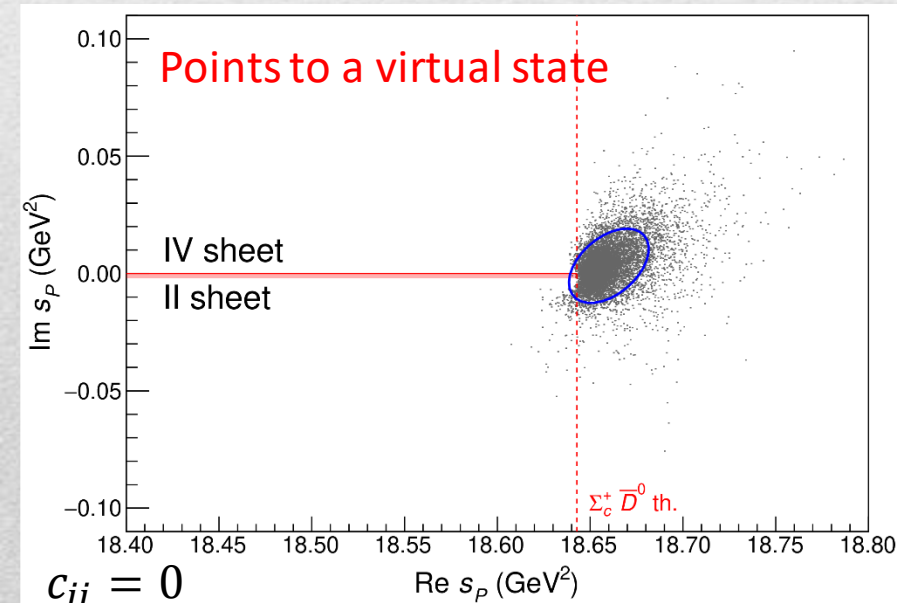
$$\frac{dN}{d\sqrt{s}} = \rho(s) [|F(s)|^2 + b_0 + b_1 s]$$

Effective range expansion

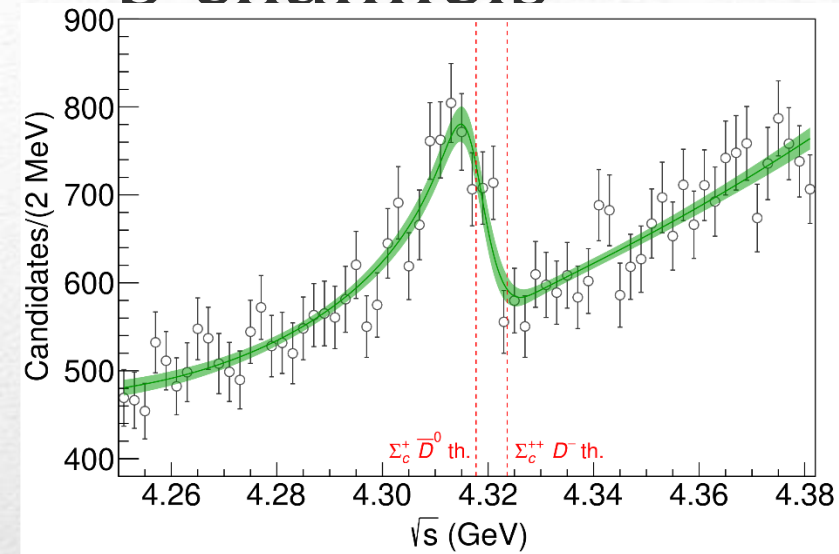
$$F(s) = (N_1 + N_2 s) T_{11}(s)$$

$$T(s) = \begin{pmatrix} m_{11} - c_{11}s - i\rho_1(s) & m_{12} \\ m_{12} & m_{22} - c_{22}s - i\rho_2(s) \end{pmatrix}^{-1}$$

We can set $c_{ii} = 0$ to reduce to the scattering length approximation



3 channels



$$T(s) = \begin{pmatrix} m_{11} - c_{11}s - i\sqrt{s-s_1} & m_{12} & \lambda m_{12} \\ m_{12} & m_{22} - c_{22}s - i\sqrt{s-s_2} & \lambda m_{23} \\ \lambda m_{12} & \lambda m_{23} & 1 + \lambda(-1 + m_{22} - c_{22}s - i\sqrt{s-s_3}) \end{pmatrix}^{-1},$$

