



UNSW
SYDNEY



NCI
AUSTRALIA

Prospects and challenges for cosmic relics using nuclear spins

With Yeray Garcia del Castillo, Dipan Sengupta, Yvonne Wong @UNSW Sydney

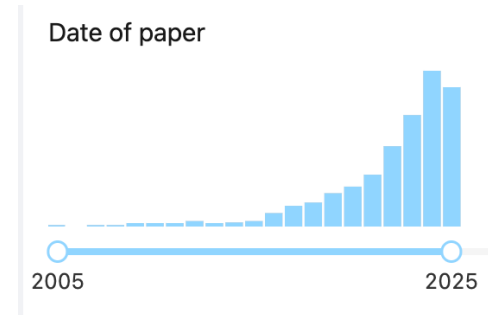
Giovanni Pierobon, UNSW Sydney

2025

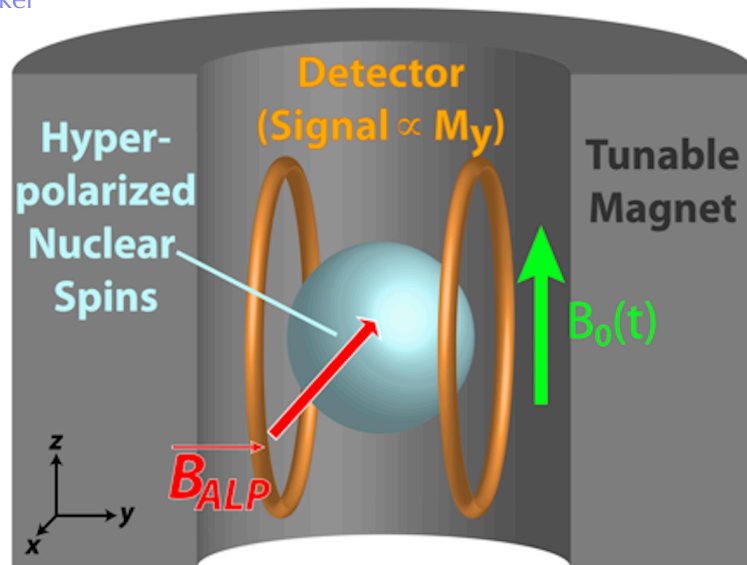
Overview

From inspirehep.net, keyword: quantum sensing

- Quantum sensing for fundamental physics
- Efforts in nuclear magnetic resonance (e.g. CASPER)



D. Budker



What can spin experiments tell us about cosmic relics?

Outline

1. Basics of Nuclear Magnetic Resonance (NMR)

2. Cosmic relics

- Cosmic neutrino background ($C\nu B$)
- Axion dark matter
- "Superradiant" interactions

3. Open Quantum System framework

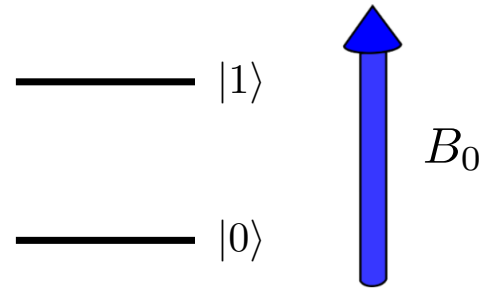
- Lindblad master equation
- Numerical solutions
- Comments on Non-Markovian regime

4. Recent results

- Forecast for $C\nu B$ using CASPEr experiment
- Recent squeezing protocols

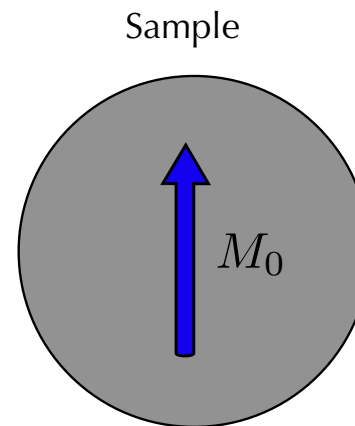
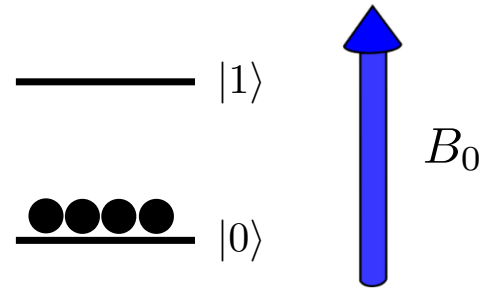
Basics of NMR

- Spin-1/2 into an external magnetic field, splitting is γB_0



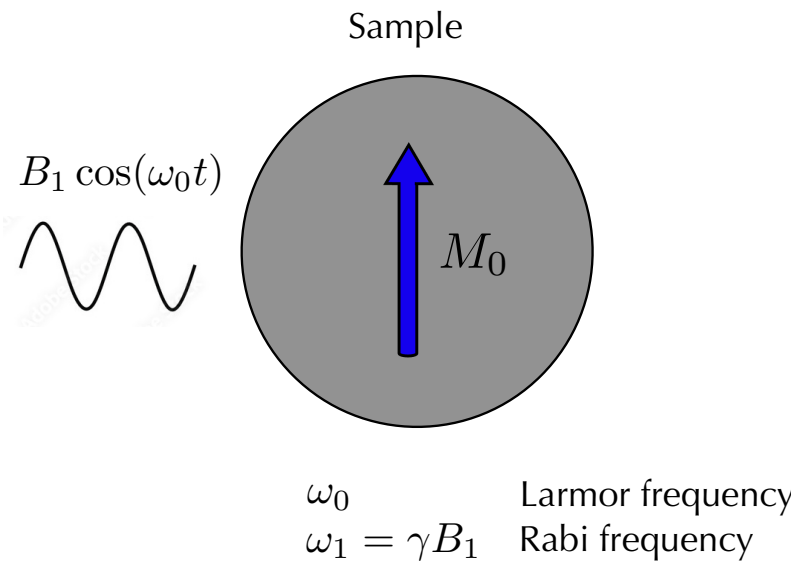
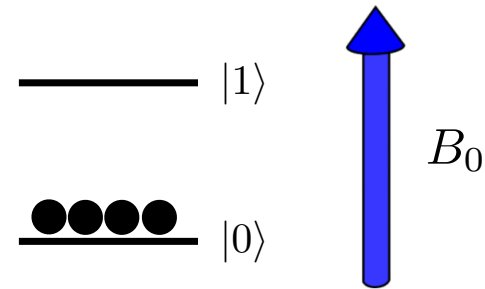
Basics of NMR

- Spin-1/2 into an external magnetic field, splitting is γB_0
- Polarize spins (thermal or optical) in a sample
(e.g. ^{129}Xe , ^3He)



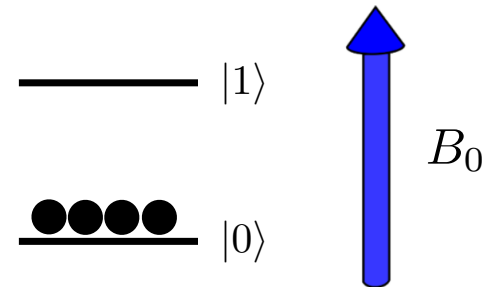
Basics of NMR

- Spin-1/2 into an external magnetic field, splitting is γB_0
- Polarize spins (thermal or optical) in a sample
(e.g. ^{129}Xe , ^3He)
- RF resonance condition $\omega_0 = \gamma B_0$

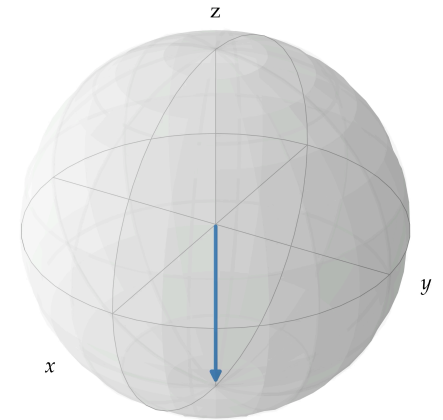


Basics of NMR

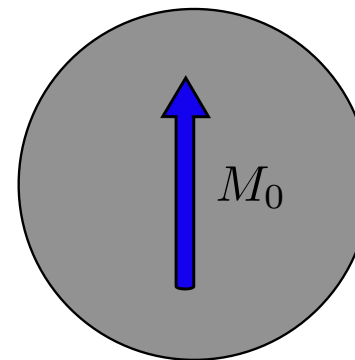
- Spin-1/2 into an external magnetic field, splitting is γB_0
- Polarize spins (thermal or optical) in a sample
(e.g. ^{129}Xe , ^3He)
- RF resonance condition $\omega_0 = \gamma B_0$
 - Pulsed NMR (e.g. MRI)



Pulsed NMR, Bloch sphere visualisation
(B_0 pointing down)



Sample

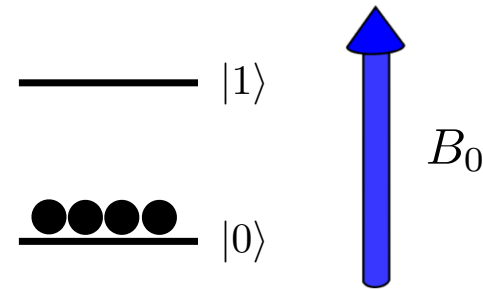


e.g. $\pi/2$ pulse in time $t_{\pi/2} = \frac{\pi}{2\gamma B_1}$

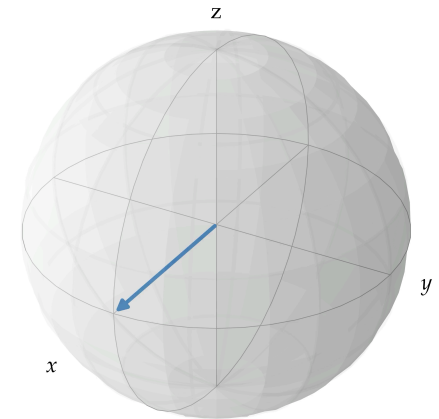
ω_0 Larmor frequency
 $\omega_1 = \gamma B_1$ Rabi frequency

Basics of NMR

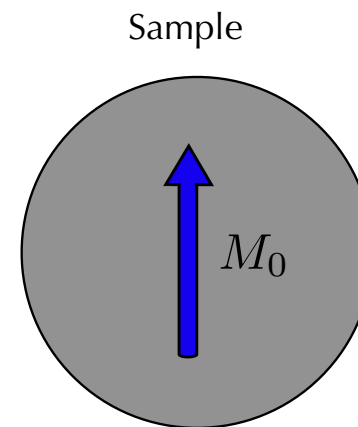
- Spin-1/2 into an external magnetic field, splitting is γB_0
- Polarize spins (thermal or optical) in a sample
(e.g. ^{129}Xe , ^3He)
- RF resonance condition $\omega_0 = \gamma B_0$
 - Pulsed NMR (e.g. MRI)



Pulsed NMR, Bloch sphere visualisation
(B_0 pointing down)



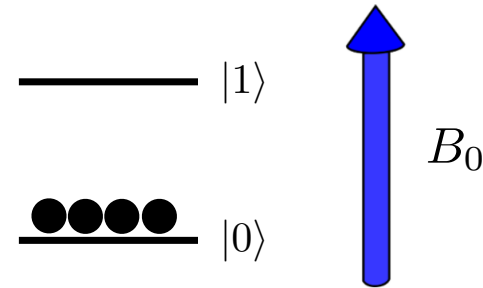
e.g. $\pi/2$ pulse in time $t_{\pi/2} = \frac{\pi}{2\gamma B_1}$



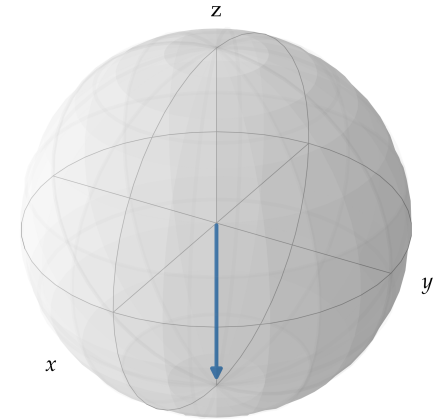
ω_0 Larmor frequency
 $\omega_1 = \gamma B_1$ Rabi frequency

Basics of NMR

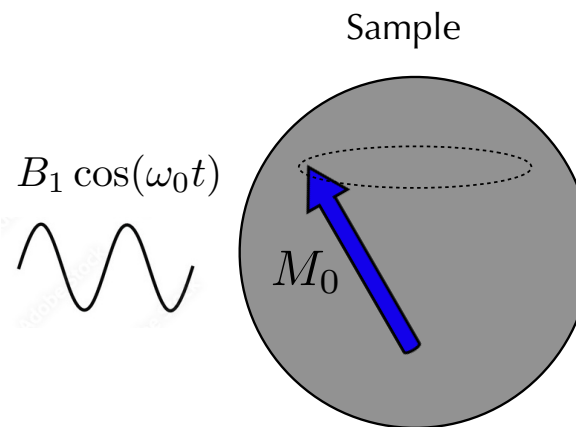
- Spin-1/2 into an external magnetic field, splitting is γB_0
- Polarize spins (thermal or optical) in a sample
(e.g. ^{129}Xe , ^3He)
- RF resonance condition $\omega_0 = \gamma B_0$



Pulsed NMR, Bloch sphere visualisation
(B_0 pointing down)



- Pulsed NMR (e.g. MRI)
- Continuous wave NMR (e.g. CASPER)



e.g. $\pi/2$ pulse in time $t_{\pi/2} = \frac{\pi}{2\gamma B_1}$

ω_0 Larmor frequency
 $\omega_1 = \gamma B_1$ Rabi frequency

Basics of NMR

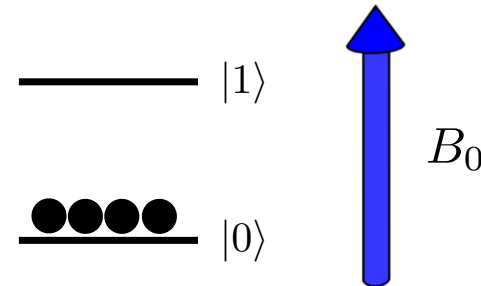
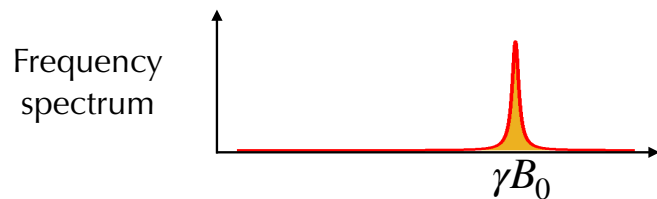
- Spin-1/2 into an external magnetic field, splitting is γB_0

- Polarize spins (thermal or optical) in a sample
(e.g. ^{129}Xe , ^3He)

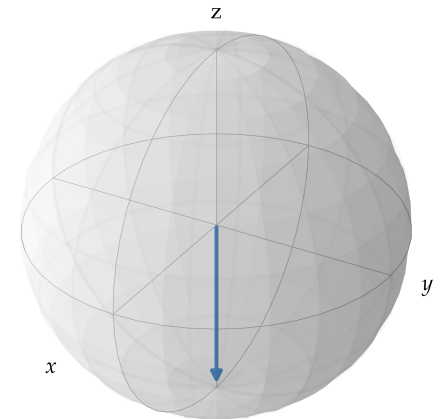
- RF resonance condition $\omega_0 = \gamma B_0$

- Pulsed NMR (e.g. MRI)
- Continuous wave NMR (e.g. CASPER)

- Magnetometer detects the flux created by this precessing magnetization

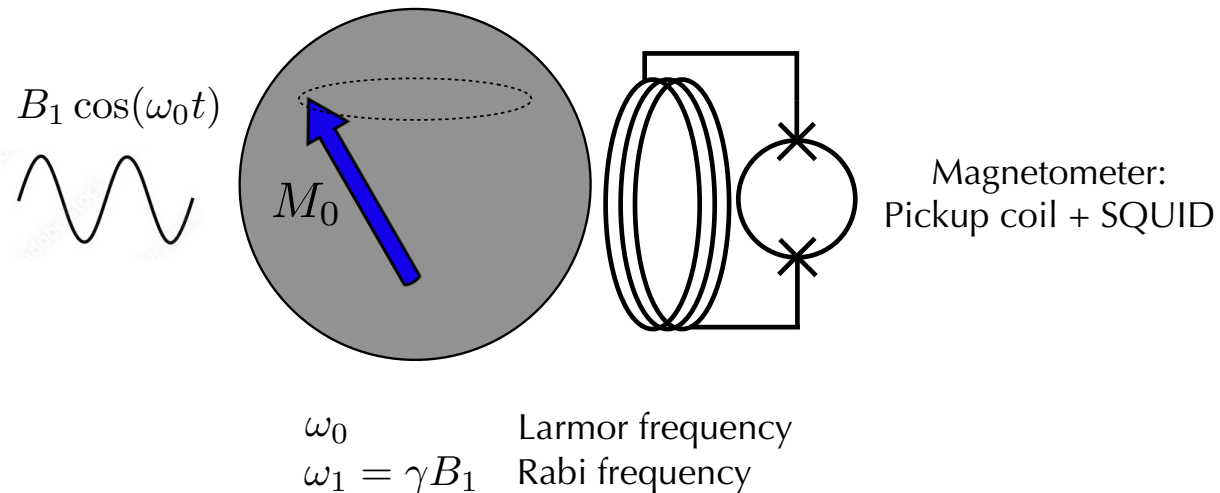


Pulsed NMR, Bloch sphere visualisation
(B_0 pointing down)



Sample

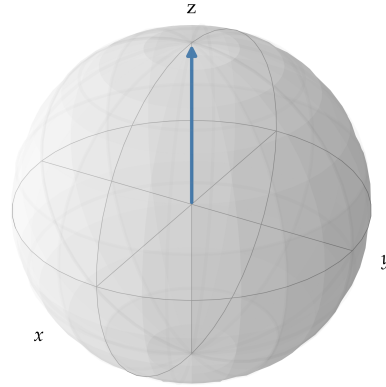
e.g. $\pi/2$ pulse in time $t_{\pi/2} = \frac{\pi}{2\gamma B_1}$



Spin projection noise

Spin-1/2 in magnetic field $H = \omega_0 J_z$

$$\mathbf{J} = \boldsymbol{\sigma}/2$$

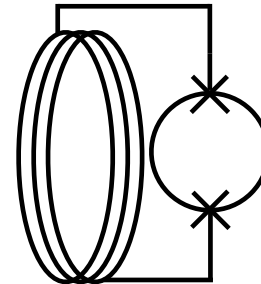
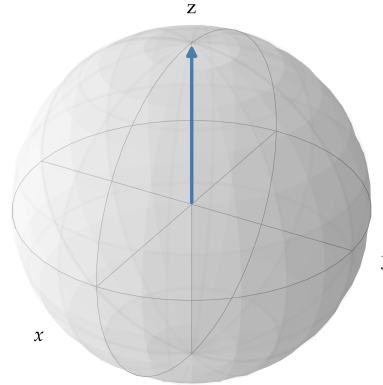


Spin projection noise

Spin-1/2 in magnetic field $H = \omega_0 J_z$

$$\mathbf{J} = \boldsymbol{\sigma}/2$$

$$[J_z, J_x] = iJ_y$$



$$\mu_x = \frac{\gamma}{2} \sigma_x$$

————— $|\downarrow\rangle$

————— $|\uparrow\rangle$

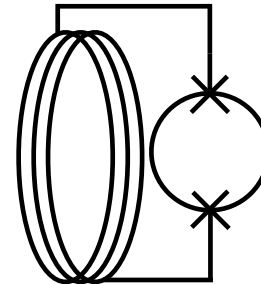
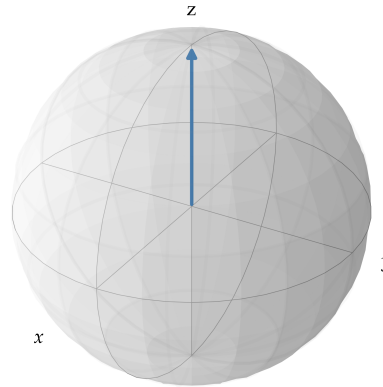
The spin has non-zero
projection along the x -axis

Spin projection noise

Spin-1/2 in magnetic field $H = \omega_0 J_z$

$$\mathbf{J} = \boldsymbol{\sigma}/2$$

$$[J_z, J_x] = iJ_y$$



$$\mu_x = \frac{\gamma}{2} \sigma_x$$

————— $|\downarrow\rangle$

————— $|\uparrow\rangle$

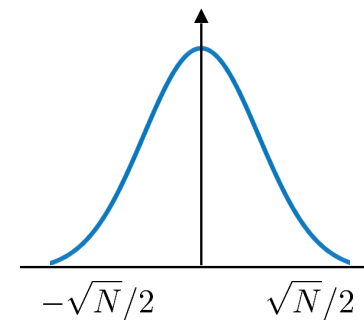
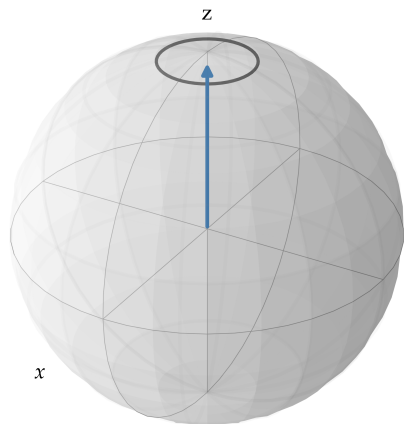
The spin has non-zero projection along the x -axis

In a macroscopic setup, we can perform a macroscopic measurement of the total magnetic moment

$$\mu_x = \frac{\gamma}{2} \sum_i \sigma_x^{(i)}$$

$$\langle J_x \rangle = 0$$

$$\Delta^2 J_x = \langle J_x^2 \rangle - \langle J_x \rangle^2 = \frac{N}{4}$$

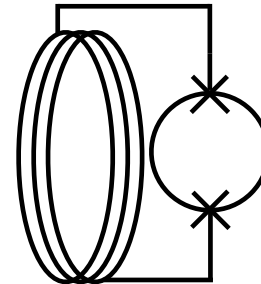
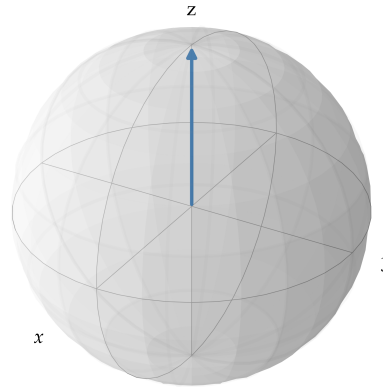


Spin projection noise

Spin-1/2 in magnetic field $H = \omega_0 J_z$

$$\mathbf{J} = \boldsymbol{\sigma}/2$$

$$[J_z, J_x] = iJ_y$$



$$\mu_x = \frac{\gamma}{2} \sigma_x$$

————— $|\downarrow\rangle$

————— $|\uparrow\rangle$

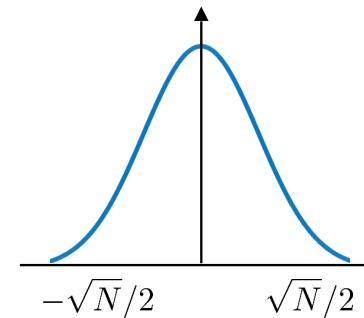
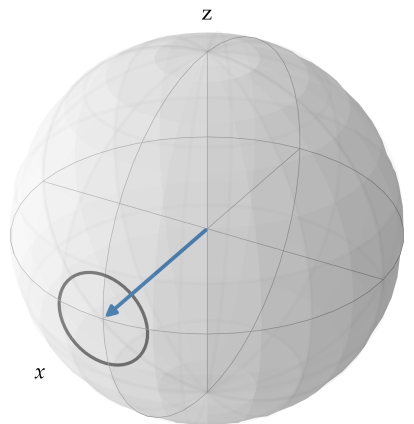
The spin has non-zero projection along the x -axis

In a macroscopic setup, we can perform a macroscopic measurement of the total magnetic moment

$$\mu_x = \frac{\gamma}{2} \sum_i \sigma_x^{(i)}$$

$$\langle J_x \rangle = 0$$

$$\Delta^2 J_x = \langle J_x^2 \rangle - \langle J_x \rangle^2 = \frac{N}{4}$$



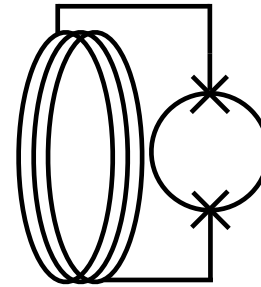
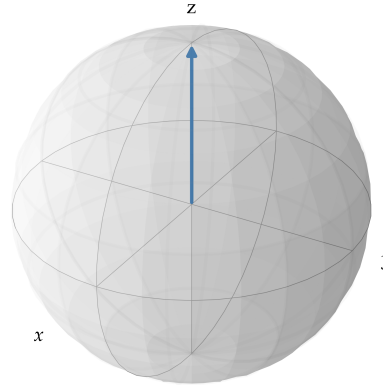
Coherent spin states (CSS) $|\theta, \varphi\rangle_{\text{CSS}} = \otimes^N \left(\cos \frac{\theta}{2} |\uparrow\rangle + e^{i\varphi} \sin \frac{\theta}{2} |\downarrow\rangle \right)$

Spin projection noise

Spin-1/2 in magnetic field $H = \omega_0 J_z$

$$\mathbf{J} = \boldsymbol{\sigma}/2$$

$$[J_z, J_x] = iJ_y$$



$$\mu_x = \frac{\gamma}{2} \sigma_x$$

————— $|\downarrow\rangle$

————— $|\uparrow\rangle$

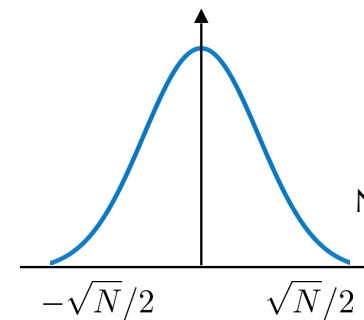
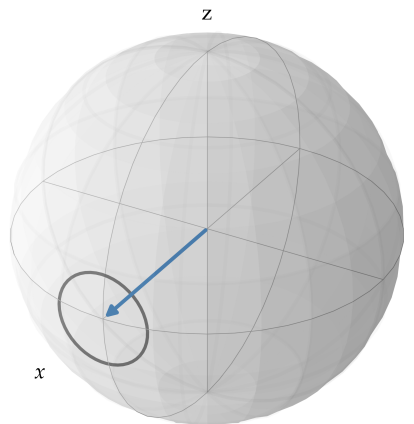
The spin has non-zero projection along the x -axis

In a macroscopic setup, we can perform a macroscopic measurement of the total magnetic moment

$$\mu_x = \frac{\gamma}{2} \sum_i \sigma_x^{(i)}$$

$$\langle J_x \rangle = 0$$

$$\Delta^2 J_x = \langle J_x^2 \rangle - \langle J_x \rangle^2 = \frac{N}{4}$$



Transverse magnetisation error

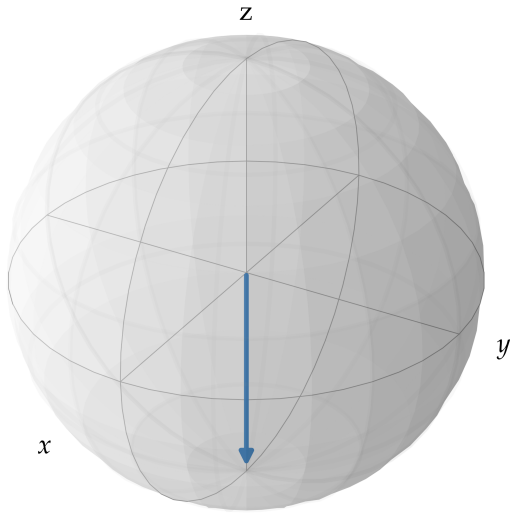
$$\frac{\delta M_{\perp}}{M_0} \approx \frac{1}{\sqrt{N}}$$

Net magnetisation

Standard Quantum Limit (SQL)

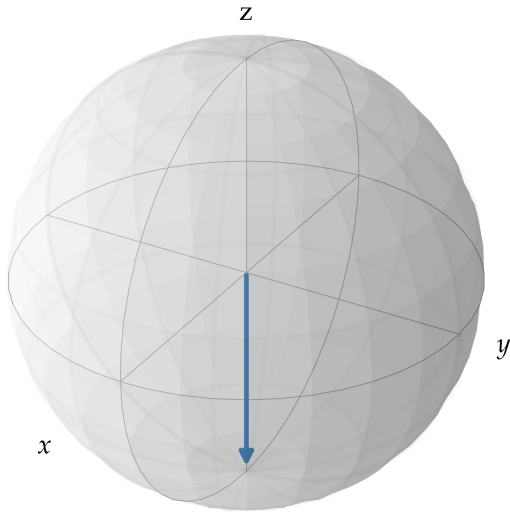
Coherent spin states (CSS) $|\theta, \varphi\rangle_{\text{CSS}} = \otimes^N \left(\cos \frac{\theta}{2} |\uparrow\rangle + e^{i\varphi} \sin \frac{\theta}{2} |\downarrow\rangle \right)$

Polarisation



$$p = \tanh\left(\frac{\hbar\gamma B}{2k_B T}\right) \approx \frac{\hbar\gamma B}{2k_B T}$$

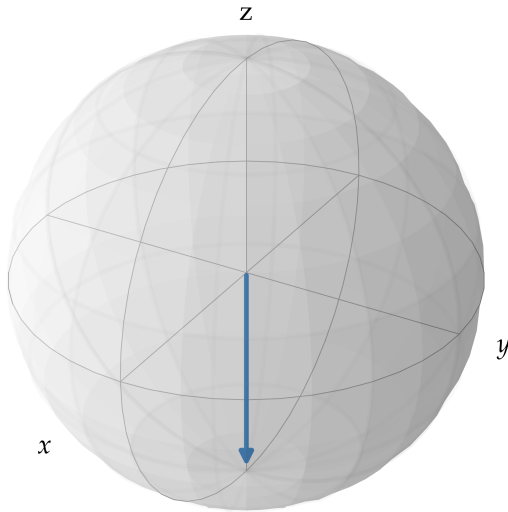
Polarisation



$$p = \tanh\left(\frac{\hbar\gamma B}{2k_B T}\right) \approx \frac{\hbar\gamma B}{2k_B T}$$
$$\simeq 10^{-3} \left(\frac{B}{10 \text{ T}}\right) \left(\frac{4 \text{ K}}{T}\right)$$

Thermal polarisation is too weak,
even at cryogenic temperature and
high field

Polarisation



$$p = \tanh\left(\frac{\hbar\gamma B}{2k_B T}\right) \approx \frac{\hbar\gamma B}{2k_B T}$$
$$\simeq 10^{-3} \left(\frac{B}{10 \text{ T}}\right) \left(\frac{4 \text{ K}}{T}\right)$$

Thermal polarisation is too weak,
even at cryogenic temperature and
high field

Hyperpolarisation

$$p \sim \mathcal{O}(1)$$

Spin exchange optical pumping (SEOP)

1. Optically polarise alkali metal atoms (e.g., Rb) with circularly polarized light
2. Transfer polarization to the noble gas nuclei (Xenon-129) via spin-exchange collisions between atoms

Outline

1. Basics of Nuclear Magnetic Resonance (NMR)

2. Cosmic relics

- Cosmic neutrino background ($C\nu B$)
- Axion dark matter
- "Superradiant" interactions

3. Open Quantum System framework

- Lindblad master equation
- Numerical solutions
- Comments on Non-Markovian regime

4. Recent results

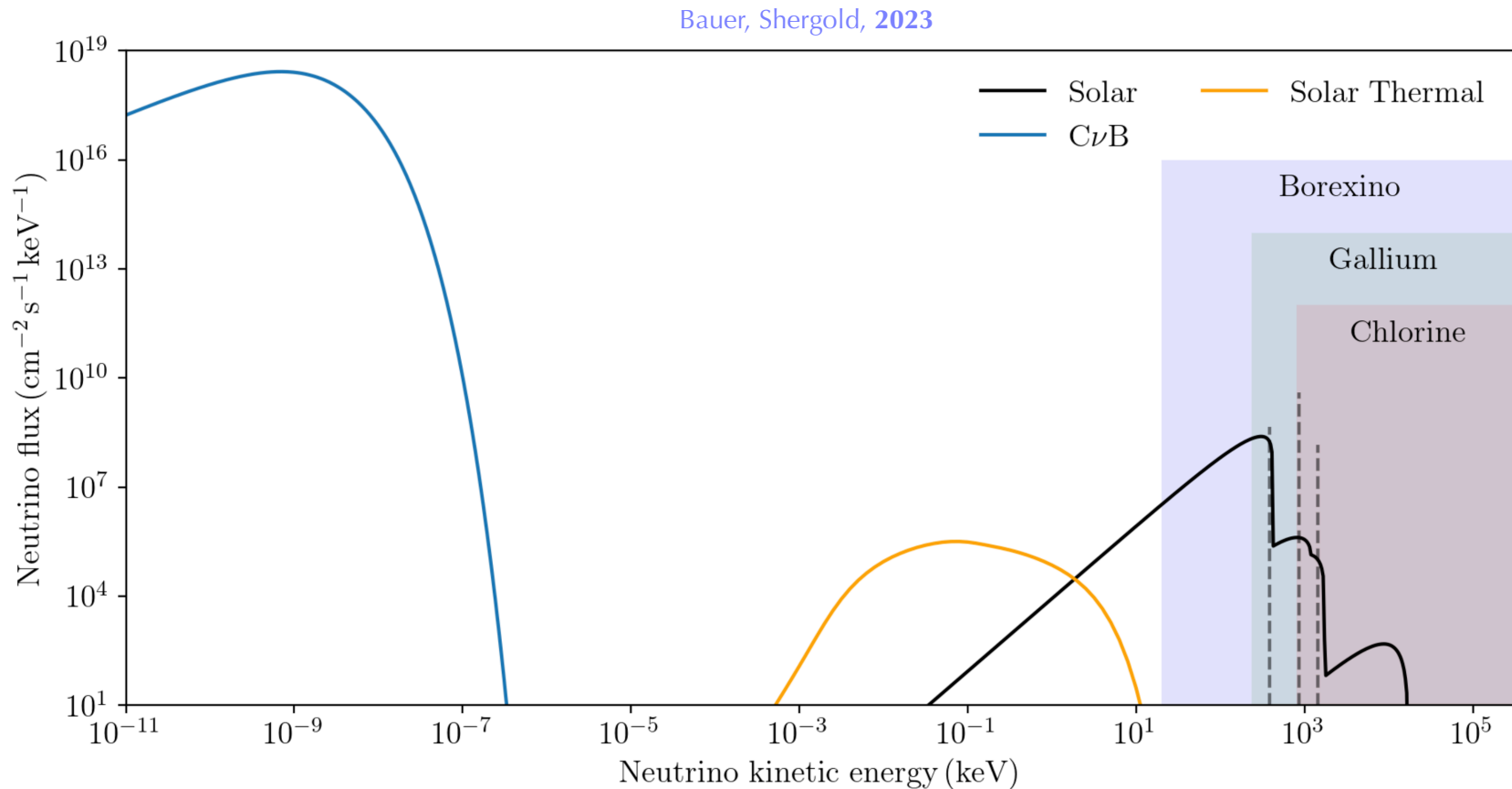
- Forecast for $C\nu B$ using CASPEr experiment
- Recent squeezing protocols

Cosmic neutrino background (C ν B)

- Produced at neutrino decoupling $\tau \sim 1$ s
- Fermi-Dirac with $1.95 \text{ K} \sim 10^{-4} \text{ eV}$
- Cluster via gravity $\delta_\nu \sim \mathcal{O}(1)$
- Number density per flavour $n_\nu \simeq 56 \text{ cm}^{-3}$, $\lambda_\nu \simeq 2 \text{ mm}$
- Only source which is non-relativistic

Cosmic neutrino background (C ν B)

- Produced at neutrino decoupling $\tau \sim 1$ s
- Fermi-Dirac with $1.95 \text{ K} \sim 10^{-4} \text{ eV}$
- Cluster via gravity $\delta_\nu \sim \mathcal{O}(1)$
- Number density per flavour $n_\nu \simeq 56 \text{ cm}^{-3}$, $\lambda_\nu \simeq 2 \text{ mm}$
- Only source which is non-relativistic



$C\nu B$ detection and limits

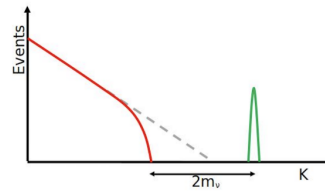
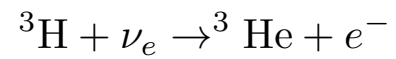
1. Thresholdless probes or very small energies
2. Enhance interaction rates

C ν B detection and limits

1. Thresholdless probes or very small energies
2. Enhance interaction rates

- **PTOLEMY**: detect neutrinos by capturing electron neutrinos on a tritium target (no energy threshold)

Weinberg, 1962



- **KATRIN**: neutrino mass measurement

$$m_\nu < 0.45 \text{ eV}$$

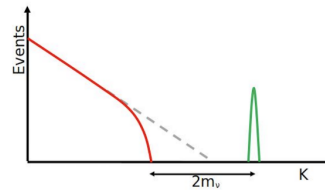
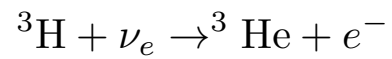
$$\delta_\nu \lesssim 10^{11}$$

$C\nu B$ detection and limits

1. Thresholdless probes or very small energies
2. Enhance interaction rates

- **PTOLEMY**: detect neutrinos by capturing electron neutrinos on a tritium target (no energy threshold)

Weinberg, 1962



- **KATRIN**: neutrino mass measurement

$$m_\nu < 0.45 \text{ eV}$$

$$\delta_\nu \lesssim 10^{11}$$

Coupling known (G_F)

δ_ν

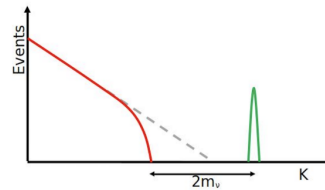
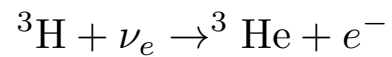
m_ν

$C\nu B$ detection and limits

1. Thresholdless probes or very small energies
2. Enhance interaction rates

- **PTOLEMY**: detect neutrinos by capturing electron neutrinos on a tritium target (no energy threshold)

Weinberg, 1962



- **KATRIN**: neutrino mass measurement

$$m_\nu < 0.45 \text{ eV}$$

$$\delta_\nu \lesssim 10^{11}$$

- **COHERENT ELASTIC SCATTERING**

Opher, 1974

$$\nu + X \rightarrow \nu + X \quad \Gamma \sim N^2$$

$$a_\nu \simeq 10^{-28} \left(\frac{m_\nu}{0.1 \text{ eV}} \right)^2 \text{ cm/s}^2$$

$$a_{\text{lab}} \simeq 10^{-15} \text{ cm/s}^2$$

$$\delta_\nu \sim 10^{13}$$

Coupling known (G_F)

δ_ν

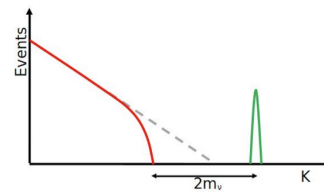
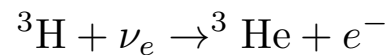
m_ν

CνB detection and limits

1. Thresholdless probes or very small energies
2. Enhance interaction rates

- **PTOLEMY**: detect neutrinos by capturing electron neutrinos on a tritium target (no energy threshold)

Weinberg, 1962



- **KATRIN**: neutrino mass measurement

$$m_\nu < 0.45 \text{ eV}$$

$$\delta_\nu \lesssim 10^{11}$$

- **COHERENT ELASTIC SCATTERING**

Opher, 1974

$$\nu + X \rightarrow \nu + X \quad \Gamma \sim N^2$$

$$a_\nu \simeq 10^{-28} \left(\frac{m_\nu}{0.1 \text{ eV}} \right)^2 \text{ cm/s}^2$$

$$a_{\text{lab}} \simeq 10^{-15} \text{ cm/s}^2$$

$$\delta_\nu \sim 10^{13}$$

- **COHERENT INELASTIC SCATTERING**

Arvanitaki, Dimopoulos, Galanis, 2024

$$\nu + X^\uparrow \rightarrow \nu + X^\downarrow \quad \Gamma \sim N^2$$

Coupling known (G_F)

δ_ν

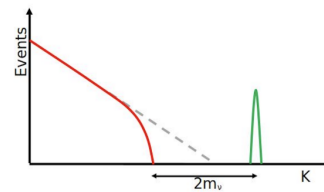
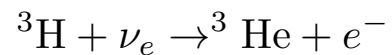
m_ν

CνB detection and limits

1. Thresholdless probes or very small energies
2. Enhance interaction rates

- **PTOLEMY**: detect neutrinos by capturing electron neutrinos on a tritium target (no energy threshold)

Weinberg, 1962



- **KATRIN**: neutrino mass measurement

$$m_\nu < 0.45 \text{ eV}$$

$$\delta_\nu \lesssim 10^{11}$$

- **COHERENT ELASTIC SCATTERING**

Opher, 1974

$$\nu + X \rightarrow \nu + X \quad \Gamma \sim N^2$$

$$a_\nu \simeq 10^{-28} \left(\frac{m_\nu}{0.1 \text{ eV}} \right)^2 \text{ cm/s}^2$$

$$a_{\text{lab}} \simeq 10^{-15} \text{ cm/s}^2$$

$$\delta_\nu \sim 10^{13}$$

- **COHERENT INELASTIC SCATTERING**

Arvanitaki, Dimopoulos, Galanis, 2024

$$\nu + X^\uparrow \rightarrow \nu + X^\downarrow \quad \Gamma \sim N^2$$

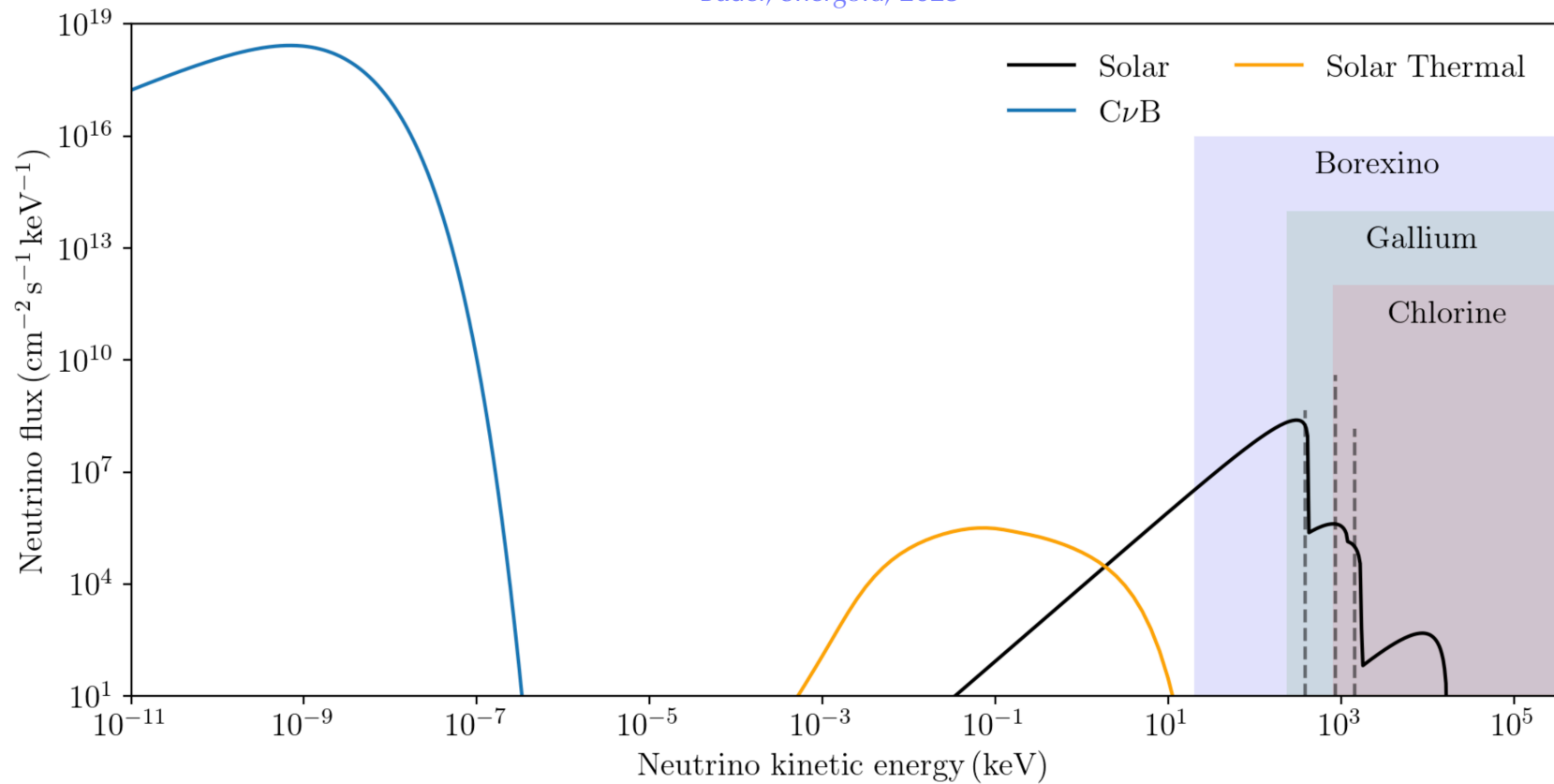
Coupling known (G_F)

δ_ν

m_ν

Cosmic neutrino background (C ν B)

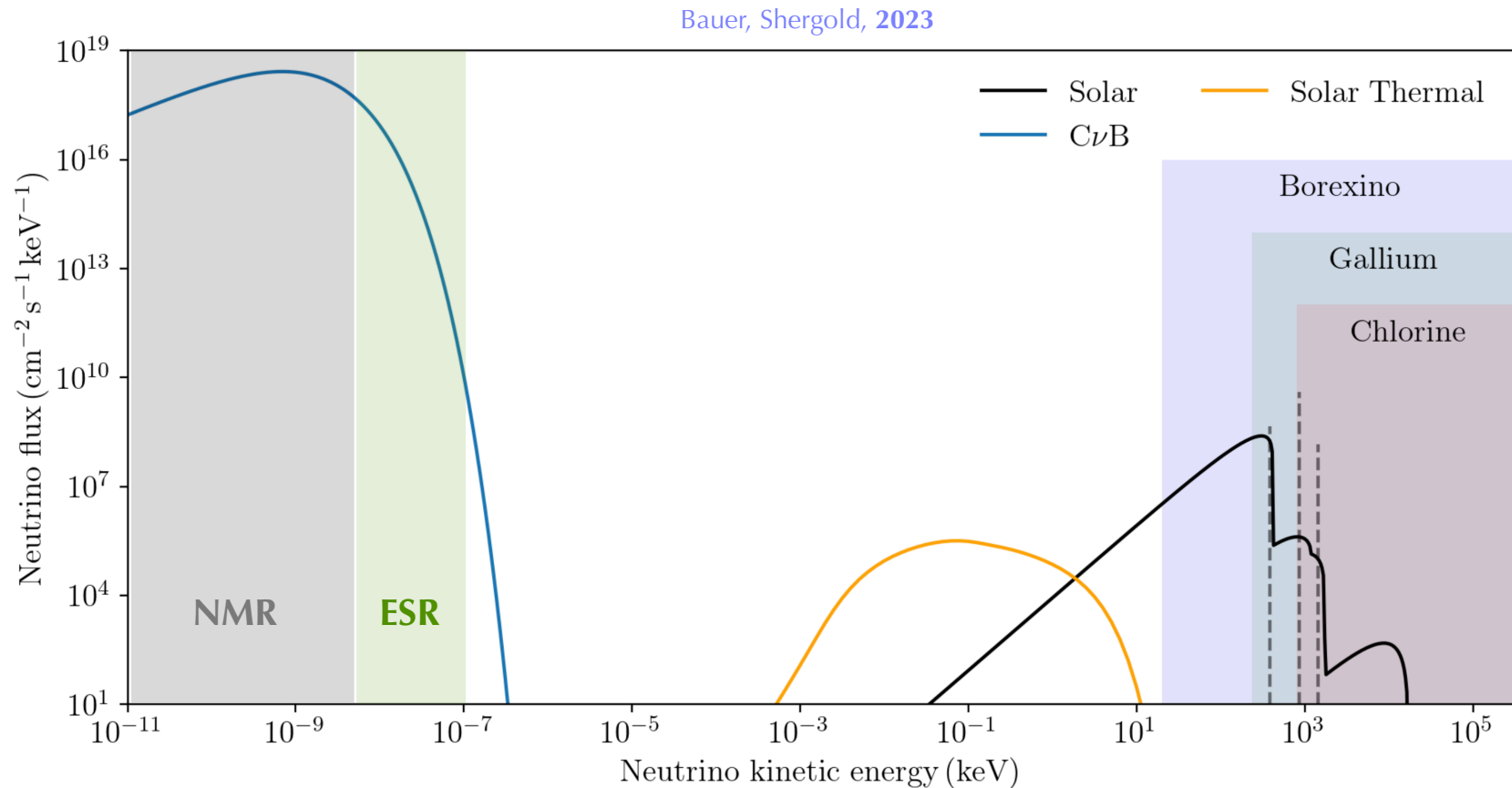
Bauer, Shergold, 2023



Cosmic neutrino background (CνB)

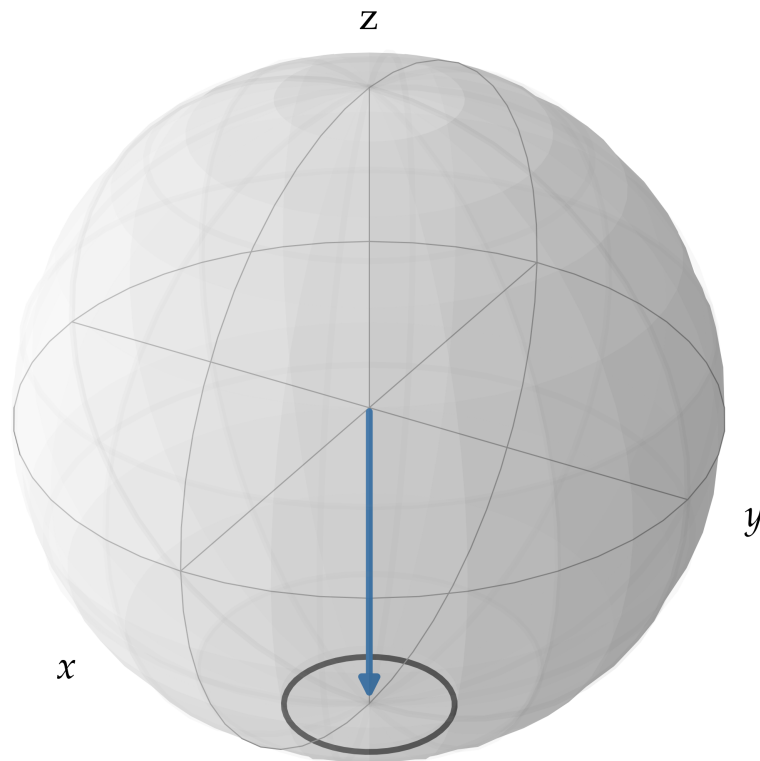
NMR: Nuclear spin splittings have low energies $10^{-8} - 10^{-6}\text{eV}$

ESR: Electron spin splittings $10^{-6} - 10^{-4}\text{eV}$



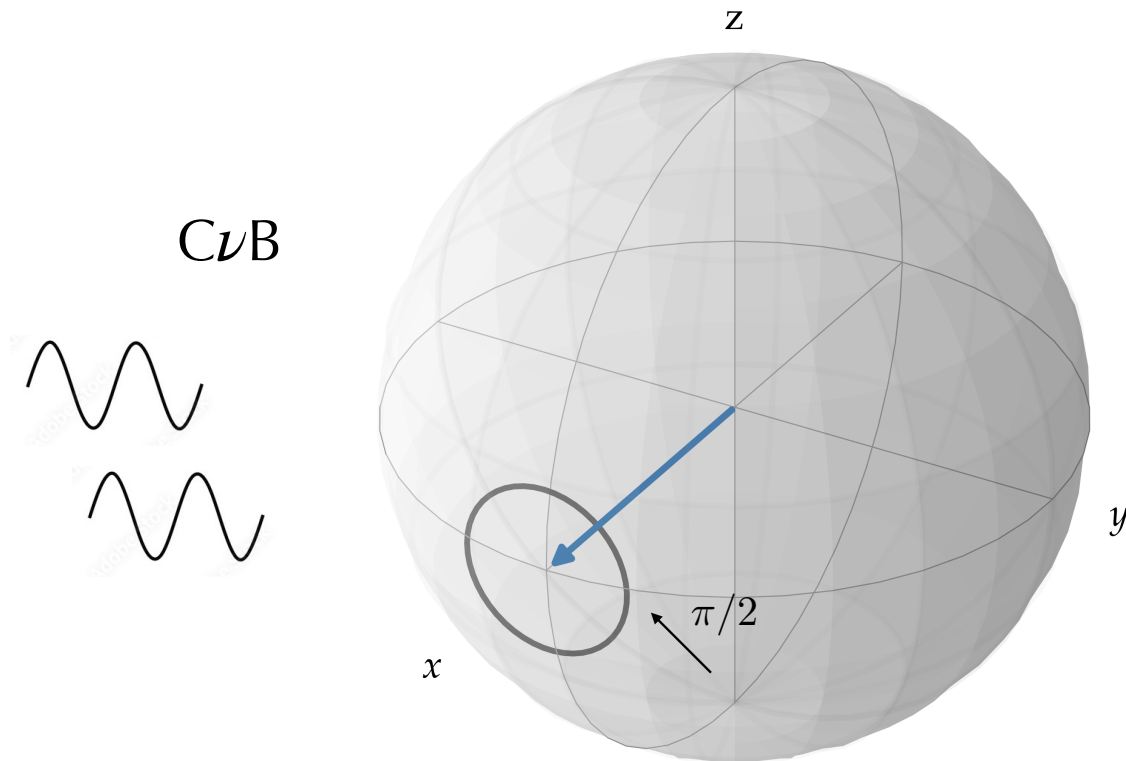
$C\mathcal{L}B$ and NMR

Strategy: pulsed NMR in hyper polarised samples



$C\mathcal{L}B$ and NMR

Strategy: pulsed NMR in hyper polarised samples



Axions

- Well motivated in some cases (strong CP)

$$\begin{aligned} d_n &= (2.4 \pm 1.0) \bar{\theta} \times 10^{-16} e \text{ cm} \\ &\simeq 1.8 \times 10^{-26} e \text{ cm} \quad \bar{\theta} \lesssim 10^{-10} \end{aligned}$$

Axions

- Well motivated in some cases (strong CP)

$$\begin{aligned} d_n &= (2.4 \pm 1.0) \bar{\theta} \times 10^{-16} e \text{ cm} \\ &\simeq 1.8 \times 10^{-26} e \text{ cm} \end{aligned} \quad \bar{\theta} \lesssim 10^{-10}$$

$$\begin{aligned} m_a &\propto 1/f_a \\ g &\propto 1/f_a \end{aligned} \quad \begin{array}{l} \mathbf{1\text{-parameter}} \\ \text{model} \end{array}$$

Axions

- Well motivated in some cases (strong CP)

$$d_n = (2.4 \pm 1.0) \bar{\theta} \times 10^{-16} e \text{ cm}$$

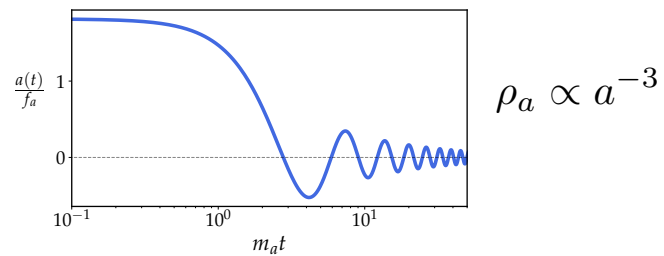
$$\simeq 1.8 \times 10^{-26} e \text{ cm} \quad \bar{\theta} \lesssim 10^{-10}$$

$$m_a \propto 1/f_a$$

$$g \propto 1/f_a$$

1-parameter
model

- Natural dark matter candidates (very cold)
e.g., **misalignment** mechanism



Axions

AxionLimits (Ciaran O'Hare)

- Well motivated in some cases (strong CP)

$$d_n = (2.4 \pm 1.0) \bar{\theta} \times 10^{-16} e \text{ cm}$$

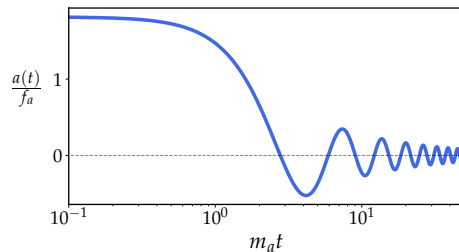
$$\simeq 1.8 \times 10^{-26} e \text{ cm} \quad \bar{\theta} \lesssim 10^{-10}$$

$$m_a \propto 1/f_a$$

$$g \propto 1/f_a$$

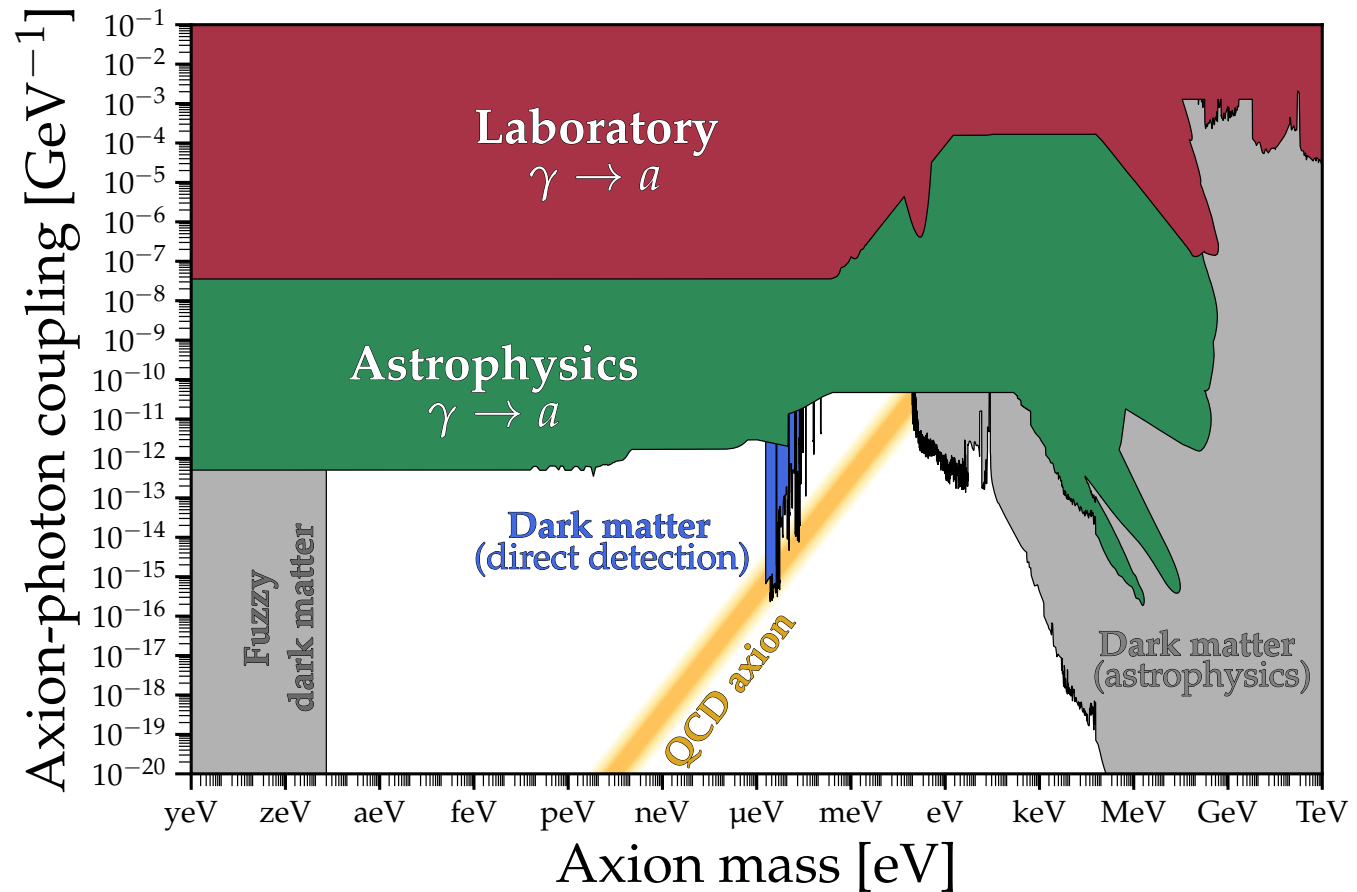
1-parameter model

- Natural dark matter candidates (very cold)
e.g., **misalignment** mechanism



$$\rho_a \propto a^{-3}$$

- Couplings to photons and matter



Axions

AxionLimits (Ciaran O'Hare)

- Well motivated in some cases (strong CP)

$$d_n = (2.4 \pm 1.0) \bar{\theta} \times 10^{-16} e \text{ cm}$$

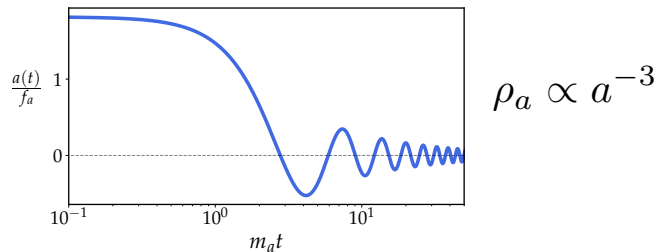
$$\simeq 1.8 \times 10^{-26} e \text{ cm} \quad \bar{\theta} \lesssim 10^{-10}$$

$$m_a \propto 1/f_a$$

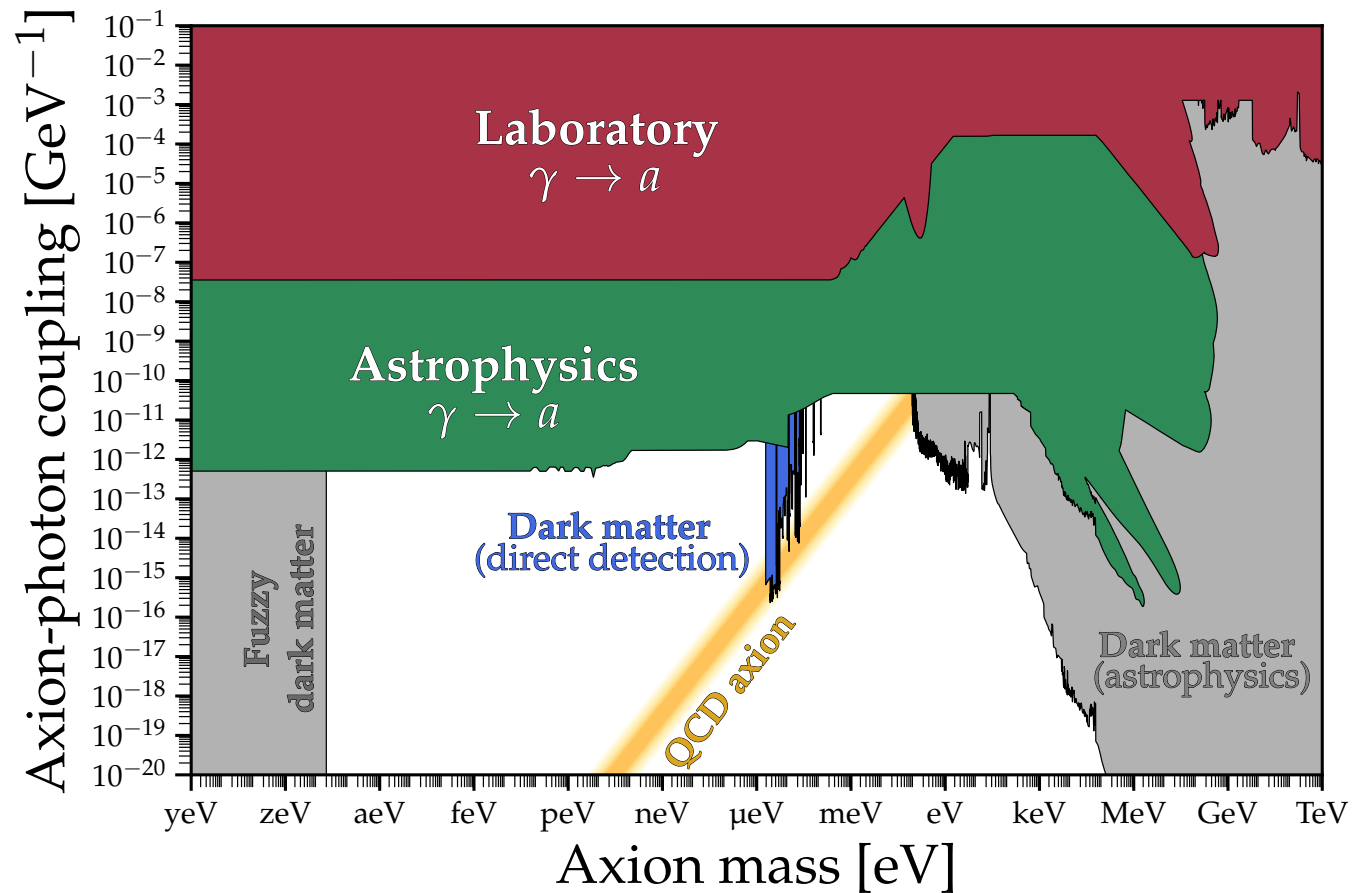
$$g \propto 1/f_a$$

1-parameter model

- Natural dark matter candidates (very cold)
e.g., **misalignment** mechanism

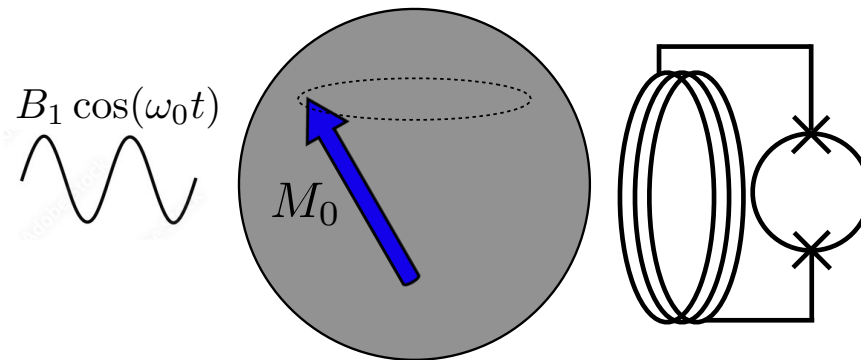


- Couplings to photons and matter
- Competitive searches need to “scan”



Axions and NMR

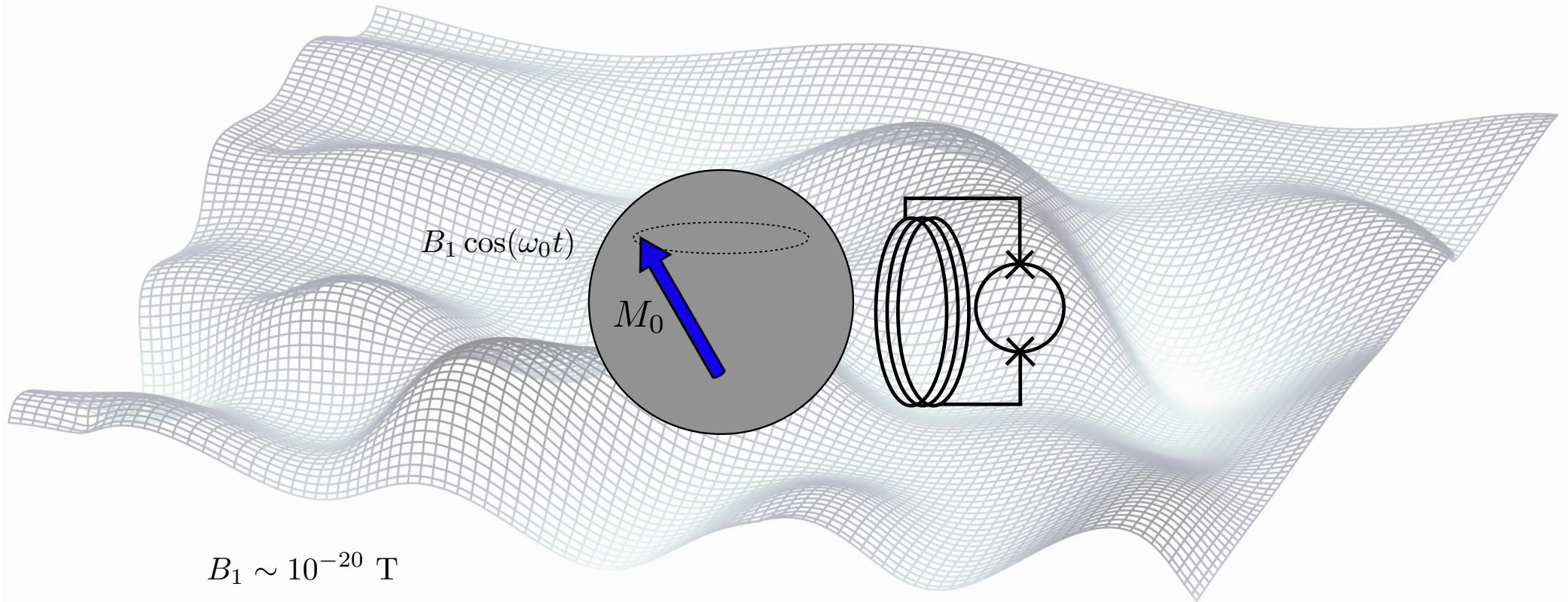
Strategy: resonant continuous NMR in hyper polarised samples



Axions and NMR

Strategy: resonant continuous NMR in hyper polarised samples

$$H_{\text{grad}} = -g \nabla a \cdot \boldsymbol{\sigma}_\psi$$



$$B_1 \sim 10^{-20} \text{ T}$$

$$\omega_1 = \gamma B_1 \sim 10^{-13} \text{ rad/s}$$

Superradiant interactions

We can enhance scattering coherently

$$\lambda \gg R, \quad \Gamma \sim N^2 \sim R^6$$

Superradiant interactions

We can enhance scattering coherently

$$\lambda \gg R, \quad \Gamma \sim N^2 \sim R^6$$

CνB (scatterings)

$$\begin{aligned} \Gamma_{\text{total}}^{\text{M}}(R, m_\nu) &= 2 \Gamma_{\text{total}}^{\text{D}}(R, m_\nu) \\ &\approx 0.2 \text{ Hz} \left(\frac{n_s}{3 \cdot 10^{22} \text{ cm}^{-3}} \right)^2 \\ &\quad \times \left(\frac{R}{10 \text{ cm}} \right)^4 \left(\frac{m_\nu}{0.1 \text{ eV}} \right) \end{aligned}$$

e.g., liquid He densities

2408.04021

Superradiant interactions

We can enhance scattering coherently

$$\lambda \gg R, \quad \Gamma \sim N^2 \sim R^6$$

C ν B (scatterings)

$$\begin{aligned} \Gamma_{\text{total}}^{\text{M}}(R, m_\nu) &= 2 \Gamma_{\text{total}}^{\text{D}}(R, m_\nu) \\ &\approx 0.2 \text{ Hz} \left(\frac{n_s}{3 \cdot 10^{22} \text{ cm}^{-3}} \right)^2 \\ &\quad \times \left(\frac{R}{10 \text{ cm}} \right)^4 \left(\frac{m_\nu}{0.1 \text{ eV}} \right) \end{aligned}$$

e.g., liquid He densities

2408.04021

Axions (absorption/emission)

$$\begin{aligned} \Gamma_{\pm} &= \frac{N}{2} \left(\frac{N}{2} + 1 \right) \frac{k_0^3}{3\pi f_a^2} |\tilde{a}(k_0)|^2 \\ &\approx 1 \text{ Hz} \left(\frac{N}{10^{15}} \right)^2 \left(\frac{m}{2 \cdot 10^{-8} \text{ eV}} \right) \end{aligned}$$

2408.04021

Superradiant interactions

We can enhance scattering coherently

$$\lambda \gg R, \quad \Gamma \sim N^2 \sim R^6$$

C ν B (scatterings)

$$\begin{aligned} \Gamma_{\text{total}}^{\text{M}}(R, m_\nu) &= 2 \Gamma_{\text{total}}^{\text{D}}(R, m_\nu) \quad \swarrow \text{e.g., liquid He densities} \\ &\approx 0.2 \text{ Hz} \left(\frac{n_s}{3 \cdot 10^{22} \text{ cm}^{-3}} \right)^2 \\ &\quad \times \left(\frac{R}{10 \text{ cm}} \right)^4 \left(\frac{m_\nu}{0.1 \text{ eV}} \right) \end{aligned}$$

2408.04021

Net effect $\gamma_{\text{net}} = \gamma_- - \gamma_+ > 0$

Axions (absorption/emission)

$$\begin{aligned} \Gamma_{\pm} &= \frac{N}{2} \left(\frac{N}{2} + 1 \right) \frac{k_0^3}{3\pi f_a^2} |\tilde{a}(k_0)|^2 \\ &\approx 1 \text{ Hz} \left(\frac{N}{10^{15}} \right)^2 \left(\frac{m}{2 \cdot 10^{-8} \text{ eV}} \right) \end{aligned}$$

2408.04021

No net effect $\gamma_- = \gamma_+$

Superradiant interactions

We can enhance scattering coherently

$$\lambda \gg R, \quad \Gamma \sim N^2 \sim R^6$$

CνB (scatterings)

$$\begin{aligned} \Gamma_{\text{total}}^{\text{M}}(R, m_\nu) &= 2 \Gamma_{\text{total}}^{\text{D}}(R, m_\nu) \quad \swarrow \text{e.g., liquid He densities} \\ &\approx 0.2 \text{ Hz} \left(\frac{n_s}{3 \cdot 10^{22} \text{ cm}^{-3}} \right)^2 \\ &\quad \times \left(\frac{R}{10 \text{ cm}} \right)^4 \left(\frac{m_\nu}{0.1 \text{ eV}} \right) \end{aligned}$$

2408.04021

$$\text{Net effect } \gamma_{\text{net}} = \gamma_- - \gamma_+ > 0$$

Axions (absorption/emission)

$$\begin{aligned} \Gamma_{\pm} &= \frac{N}{2} \left(\frac{N}{2} + 1 \right) \frac{k_0^3}{3\pi f_a^2} |\tilde{a}(k_0)|^2 \\ &\approx 1 \text{ Hz} \left(\frac{N}{10^{15}} \right)^2 \left(\frac{m}{2 \cdot 10^{-8} \text{ eV}} \right) \end{aligned}$$

2408.04021

$$\text{No net effect } \gamma_- = \gamma_+$$

Caveats

1. "Quantum coherence" needed
2. Coherence is only partial if $\lambda_{\text{in}} \lesssim R$

Relic neutrino interaction

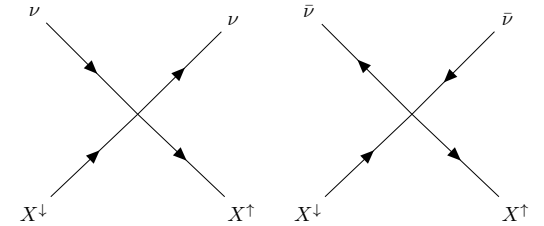
$$H = \omega_0 J_z + \Sigma_z J_z + \Sigma_- J_- + \Sigma_+ J_+$$

↑
Spin raising and
lowering operators

$$\Sigma_{\pm} = \sum_i \frac{G_F}{\sqrt{2}} g_A \bar{\nu}_i (\gamma^1 \mp i\gamma^2)(1 - \gamma^5) \nu_i$$

Inelastic

4-Fermi vertex



Relic neutrino interaction

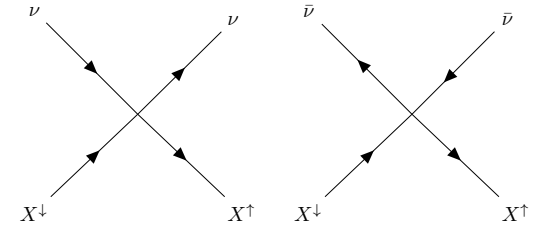
$$H = \omega_0 J_z + \Sigma_z J_z + \Sigma_- J_- + \Sigma_+ J_+$$

↑
Spin raising and
lowering operators

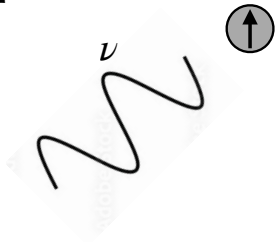
$$\Sigma_{\pm} = \sum_i \frac{G_F}{\sqrt{2}} g_A \bar{\nu}_i (\gamma^1 \mp i\gamma^2) (1 - \gamma^5) \nu_i$$

Inelastic

4-Fermi vertex



○ Single spin



$$\gamma_- \simeq 1.1 \times 10^{-48} \text{ Hz}$$

$$\gamma_{\text{net}} = \gamma_- - \gamma_+ \simeq 6 \times 10^{-51} \text{ Hz}$$

$$\gamma_{\text{net}} \propto \omega_0$$

Relic neutrino interaction

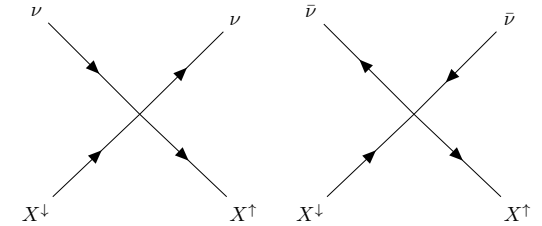
$$H = \omega_0 J_z + \Sigma_z J_z + \Sigma_- J_- + \Sigma_+ J_+$$

↑
Spin raising and
lowering operators

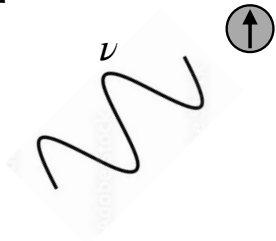
$$\Sigma_{\pm} = \sum_i \frac{G_F}{\sqrt{2}} g_A \bar{\nu}_i (\gamma^1 \mp i\gamma^2) (1 - \gamma^5) \nu_i$$

Inelastic

4-Fermi vertex



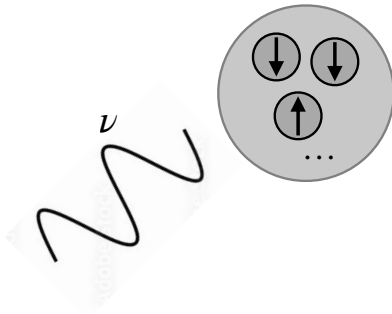
○ Single spin



$$\gamma_- \simeq 1.1 \times 10^{-48} \text{ Hz}$$

$$\gamma_{\text{net}} = \gamma_- - \gamma_+ \simeq 6 \times 10^{-51} \text{ Hz} \quad \gamma_{\text{net}} \propto \omega_0$$

○ N spins



$$H_S = \omega_0 \sum_{\alpha} J_z^{\alpha},$$

$$H_I^I(t) = \sum_{\alpha} \Sigma_z^{\alpha} J_z^{\alpha} + e^{-i\omega_0 t} \sum_{\alpha} \Sigma_-^{\alpha} J_-^{\alpha} + e^{i\omega_0 t} \sum_{\alpha} \Sigma_+^{\alpha} J_+^{\alpha}$$

$$\Gamma_{\pm} \propto G_F^2 \int_{-1}^1 d\mu F_{\pm}(\mathbf{q})$$

Form factor, including
cross terms

Coherence requirements

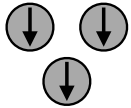
Form factor

$$F_{\pm}(\vec{p}' - \vec{p}) \equiv \sum_{\mathbf{f}_{\text{spin}}} \left| \langle \mathbf{f}_{\text{spin}} | \sum_{\alpha} J_{\pm}^{\alpha} e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}_{\alpha}} | \mathbf{i}_{\text{spin}} \rangle \right|^2$$

Coherence requirements

Form factor $F_{\pm}(\vec{p}' - \vec{p}) \equiv \sum_{\mathbf{f}_{\text{spin}}} \left| \langle \mathbf{f}_{\text{spin}} | \sum_{\alpha} J_{\pm}^{\alpha} e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}_{\alpha}} | \mathbf{i}_{\text{spin}} \rangle \right|^2$

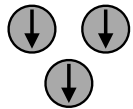
1) $|\mathbf{i}_{\text{spin}}\rangle = |\downarrow_1, \downarrow_2, \dots, \downarrow_{\gamma}, \dots, \downarrow_N\rangle$



Coherence requirements

Form factor $F_{\pm}(\vec{p}' - \vec{p}) \equiv \sum_{\mathbf{f}_{\text{spin}}} \left| \langle \mathbf{f}_{\text{spin}} | \sum_{\alpha} J_{\pm}^{\alpha} e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}_{\alpha}} | \mathbf{i}_{\text{spin}} \rangle \right|^2$

1) $|\mathbf{i}_{\text{spin}}\rangle = |\downarrow_1, \downarrow_2, \dots, \downarrow_{\gamma}, \dots, \downarrow_N\rangle$



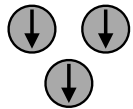
$$F_{+}^G(\vec{p}' - \vec{p}) = N$$

$$\Gamma = \left| \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} + \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} + \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \right|^2$$

Coherence requirements

Form factor $F_{\pm}(\vec{p}' - \vec{p}) \equiv \sum_{\mathbf{f}_{\text{spin}}} \left| \langle \mathbf{f}_{\text{spin}} | \sum_{\alpha} J_{\pm}^{\alpha} e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}_{\alpha}} | \mathbf{i}_{\text{spin}} \rangle \right|^2$

1) $|\mathbf{i}_{\text{spin}}\rangle = |\downarrow_1, \downarrow_2, \dots, \downarrow_{\gamma}, \dots, \downarrow_N\rangle$



$$F_{+}^G(\vec{p}' - \vec{p}) = N$$

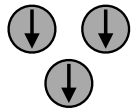
$$\Gamma = \left| \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} + \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} + \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \right|^2$$

$$\left\langle \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} \middle| \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} \right\rangle = \left\langle \begin{array}{cc} \uparrow & \downarrow \end{array} \middle| \begin{array}{cc} \downarrow & \uparrow \end{array} \right\rangle = 0$$

Coherence requirements

Form factor $F_{\pm}(\vec{p}' - \vec{p}) \equiv \sum_{\mathbf{f}_{\text{spin}}} \left| \langle \mathbf{f}_{\text{spin}} | \sum_{\alpha} J_{\pm}^{\alpha} e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}_{\alpha}} | \mathbf{i}_{\text{spin}} \rangle \right|^2$

1) $|\mathbf{i}_{\text{spin}}\rangle = |\downarrow_1, \downarrow_2, \dots, \downarrow_{\gamma}, \dots, \downarrow_N\rangle$



$$F_{+}^G(\vec{p}' - \vec{p}) = N$$

Incoherent, irrespective of the momentum transfer

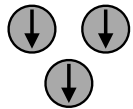
$$\Gamma = \left| \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} + \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} + \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \right|^2$$

$$\left\langle \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} \middle| \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} \right\rangle = \left\langle \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} \middle| \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \right\rangle = 0$$

Coherence requirements

Form factor $F_{\pm}(\vec{p}' - \vec{p}) \equiv \sum_{\mathbf{f}_{\text{spin}}} \left| \langle \mathbf{f}_{\text{spin}} | \sum_{\alpha} J_{\pm}^{\alpha} e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}_{\alpha}} | \mathbf{i}_{\text{spin}} \rangle \right|^2$

1) $|\mathbf{i}_{\text{spin}}\rangle = |\downarrow_1, \downarrow_2, \dots, \downarrow_{\gamma}, \dots, \downarrow_N\rangle$



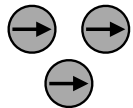
$$F_{+}^G(\vec{p}' - \vec{p}) = N$$

Incoherent, irrespective of the momentum transfer

$$\Gamma = \left| \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} + \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} + \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \right|^2$$

$$\left\langle \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} \middle| \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} \right\rangle = \left\langle \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} \middle| \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \right\rangle = 0$$

2) $|\mathbf{i}_{\text{spin}}\rangle = |\rightarrow_1, \rightarrow_2, \dots, \rightarrow_{\gamma}, \dots, \rightarrow_N\rangle$

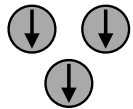


$$|\rightarrow\rangle \equiv (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$$

Coherence requirements

Form factor $F_{\pm}(\vec{p}' - \vec{p}) \equiv \sum_{\mathbf{f}_{\text{spin}}} \left| \langle \mathbf{f}_{\text{spin}} | \sum_{\alpha} J_{\pm}^{\alpha} e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}_{\alpha}} | \mathbf{i}_{\text{spin}} \rangle \right|^2$

1) $|\mathbf{i}_{\text{spin}}\rangle = |\downarrow_1, \downarrow_2, \dots, \downarrow_{\gamma}, \dots, \downarrow_N\rangle$



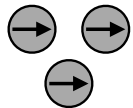
$$F_{+}^G(\vec{p}' - \vec{p}) = N$$

Incoherent, irrespective of the momentum transfer

$$\Gamma = \left| \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} + \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} + \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \right|^2$$

$$\langle \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} | \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} \rangle = \langle \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} | \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \rangle = 0$$

2) $|\mathbf{i}_{\text{spin}}\rangle = |\rightarrow_1, \rightarrow_2, \dots, \rightarrow_{\gamma}, \dots, \rightarrow_N\rangle$ $|\rightarrow\rangle \equiv (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$



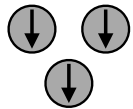
$$F_{\pm}^P(\vec{p}' - \vec{p}) = \frac{N}{2} + \frac{1}{4} \sum_{\alpha, \beta \neq \alpha} e^{-i(\vec{p}' - \vec{p}) \cdot (\vec{x}_{\alpha} - \vec{x}_{\beta})}$$

$$\langle \begin{array}{cc} \uparrow & \rightarrow \\ & \rightarrow \end{array} | \begin{array}{cc} \rightarrow & \uparrow \\ & \rightarrow \end{array} \rangle = \frac{1}{2}$$

Coherence requirements

Form factor $F_{\pm}(\vec{p}' - \vec{p}) \equiv \sum_{\mathbf{f}_{\text{spin}}} \left| \langle \mathbf{f}_{\text{spin}} | \sum_{\alpha} J_{\pm}^{\alpha} e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}_{\alpha}} | \mathbf{i}_{\text{spin}} \rangle \right|^2$

1) $|\mathbf{i}_{\text{spin}}\rangle = |\downarrow_1, \downarrow_2, \dots, \downarrow_{\gamma}, \dots, \downarrow_N\rangle$



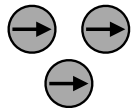
$$F_{+}^G(\vec{p}' - \vec{p}) = N$$

Incoherent, irrespective of the momentum transfer

$$\Gamma = \left| \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} + \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} + \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \right|^2$$

$$\langle \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} | \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} \rangle = \langle \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} | \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \rangle = 0$$

2) $|\mathbf{i}_{\text{spin}}\rangle = |\rightarrow_1, \rightarrow_2, \dots, \rightarrow_{\gamma}, \dots, \rightarrow_N\rangle \quad |\rightarrow\rangle \equiv (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$



$$F_{\pm}^P(\vec{p}' - \vec{p}) = \frac{N}{2} + \frac{1}{4} \sum_{\alpha, \beta \neq \alpha} e^{-i(\vec{p}' - \vec{p}) \cdot (\vec{x}_{\alpha} - \vec{x}_{\beta})}$$

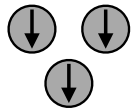
$$\langle \begin{array}{cc} \uparrow & \rightarrow \\ & \rightarrow \end{array} | \begin{array}{cc} \rightarrow & \uparrow \\ & \rightarrow \end{array} \rangle = \frac{1}{2}$$

$$= \frac{N}{2} + \frac{N^2}{4} \frac{9}{\underbrace{|\vec{p}' - \vec{p}|^2}_{q} R^2} |j_1(|\vec{p}' - \vec{p}|R)|^2$$

Coherence requirements

Form factor $F_{\pm}(\vec{p}' - \vec{p}) \equiv \sum_{\mathbf{f}_{\text{spin}}} \left| \langle \mathbf{f}_{\text{spin}} | \sum_{\alpha} J_{\pm}^{\alpha} e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}_{\alpha}} | \mathbf{i}_{\text{spin}} \rangle \right|^2$

1) $|\mathbf{i}_{\text{spin}}\rangle = |\downarrow_1, \downarrow_2, \dots, \downarrow_{\gamma}, \dots, \downarrow_N\rangle$



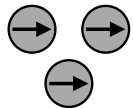
$$F_{+}^G(\vec{p}' - \vec{p}) = N$$

Incoherent, irrespective of the momentum transfer

$$\Gamma = \left| \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} + \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} + \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \right|^2$$

$$\langle \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} | \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} \rangle = \langle \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} | \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \rangle = 0$$

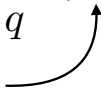
2) $|\mathbf{i}_{\text{spin}}\rangle = |\rightarrow_1, \rightarrow_2, \dots, \rightarrow_{\gamma}, \dots, \rightarrow_N\rangle \quad |\rightarrow\rangle \equiv (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$



$$F_{\pm}^P(\vec{p}' - \vec{p}) = \frac{N}{2} + \frac{1}{4} \sum_{\alpha, \beta \neq \alpha} e^{-i(\vec{p}' - \vec{p}) \cdot (\vec{x}_{\alpha} - \vec{x}_{\beta})}$$

$$= \frac{N}{2} + \frac{N^2}{4} \frac{9}{\underbrace{|\vec{p}' - \vec{p}|^2}_{q} R^2} |j_1(|\vec{p}' - \vec{p}|R)|^2$$

Coherent, provided the small momentum transfer

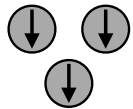


$$\langle \begin{array}{cc} \uparrow & \rightarrow \\ & \rightarrow \end{array} | \begin{array}{cc} \rightarrow & \uparrow \\ & \rightarrow \end{array} \rangle = \frac{1}{2}$$

Coherence requirements

Form factor $F_{\pm}(\vec{p}' - \vec{p}) \equiv \sum_{\mathbf{f}_{\text{spin}}} \left| \langle \mathbf{f}_{\text{spin}} | \sum_{\alpha} J_{\pm}^{\alpha} e^{-i(\vec{p}' - \vec{p}) \cdot \vec{x}_{\alpha}} | \mathbf{i}_{\text{spin}} \rangle \right|^2$

1) $|\mathbf{i}_{\text{spin}}\rangle = |\downarrow_1, \downarrow_2, \dots, \downarrow_{\gamma}, \dots, \downarrow_N\rangle$



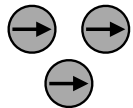
$$F_{+}^G(\vec{p}' - \vec{p}) = N$$

Incoherent, irrespective of the momentum transfer

$$\Gamma = \left| \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} + \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} + \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \right|^2$$

$$\langle \begin{array}{cc} \downarrow & \downarrow \\ & \uparrow \end{array} | \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} \rangle = \langle \begin{array}{cc} \uparrow & \downarrow \\ & \downarrow \end{array} | \begin{array}{cc} \downarrow & \uparrow \\ & \downarrow \end{array} \rangle = 0$$

2) $|\mathbf{i}_{\text{spin}}\rangle = |\rightarrow_1, \rightarrow_2, \dots, \rightarrow_{\gamma}, \dots, \rightarrow_N\rangle \quad |\rightarrow\rangle \equiv (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$



$$F_{\pm}^P(\vec{p}' - \vec{p}) = \frac{N}{2} + \frac{1}{4} \sum_{\alpha, \beta \neq \alpha} e^{-i(\vec{p}' - \vec{p}) \cdot (\vec{x}_{\alpha} - \vec{x}_{\beta})}$$

$$= \frac{N}{2} + \frac{N^2}{4} \frac{9}{\underbrace{|\vec{p}' - \vec{p}|^2}_{q} R^2} |j_1(|\vec{p}' - \vec{p}|R)|^2$$

Coherent, provided the small momentum transfer

$$\langle \begin{array}{cc} \uparrow & \rightarrow \\ & \rightarrow \end{array} | \begin{array}{cc} \rightarrow & \uparrow \\ & \rightarrow \end{array} \rangle = \frac{1}{2}$$

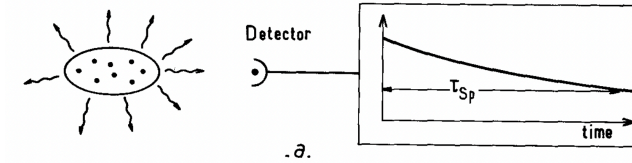
Conditions for coherence in inelastic scatterings

1. Non-orthogonal initial and final states
2. Small momentum transfer, $qR \ll 1$

Dicke superradiance

Gross, Haroche, 1982

Spontaneous emission: probability of detecting the emitted photon decreases exponentially with time passed since its excitation



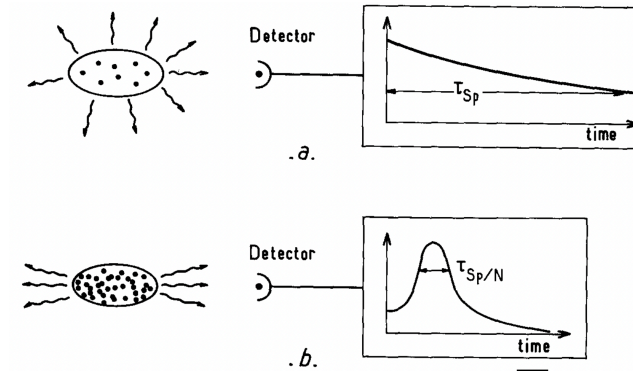
Dicke superradiance

Gross, Haroche, 1982



Spontaneous emission: probability of detecting the emitted photon decreases exponentially with time passed since its excitation

In 1954, Dicke considered the situation in which emitters decay in close proximity to each other



Cooperative spontaneous emission of radiation from an ensemble of identical two-level quantum systems

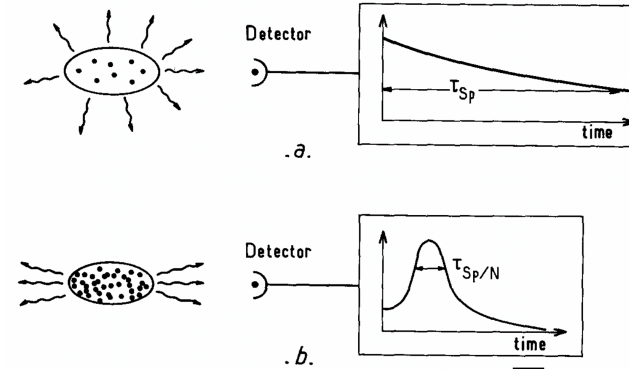
Dicke superradiance

Gross, Haroche, 1982



Spontaneous emission: probability of detecting the emitted photon decreases exponentially with time passed since its excitation

In 1954, Dicke considered the situation in which emitters decay in close proximity to each other



Cooperative spontaneous emission of radiation from an ensemble of identical two-level quantum systems

Dicke basis $|j, m\rangle$

$$j \leq N/2, \quad |m| \leq j$$

Eigenstates of the collective operators J^2, J_z

$$\Gamma_{em} = \gamma_{em} \langle J_+ J_- \rangle = \gamma_{em} (j + m)(j - m + 1)$$

$$\Gamma_{em} = \gamma_{em} N \quad j = \frac{N}{2}, \quad m = \frac{N}{2} \quad \text{Fully excited state}$$

$$\Gamma_{em} = \gamma_{em} \frac{N}{2} \left(\frac{N}{2} + 1 \right) \quad j = \frac{N}{2}, \quad m = 0 \quad \text{Superposition}$$

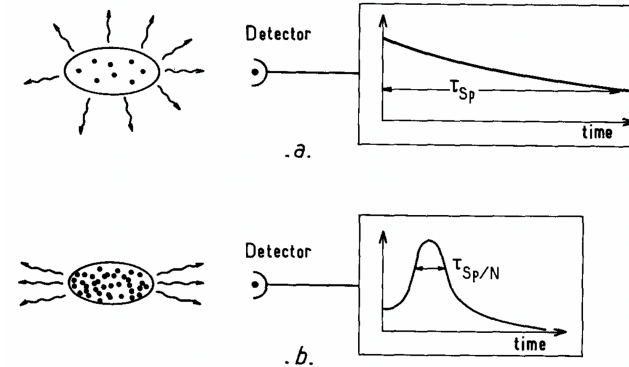
Dicke superradiance

Gross, Haroche, 1982



Spontaneous emission: probability of detecting the emitted photon decreases exponentially with time passed since its excitation

In 1954, Dicke considered the situation in which emitters decay in close proximity to each other



Cooperative spontaneous emission of radiation from an ensemble of identical two-level quantum systems

Dicke basis $|j, m\rangle$

$$j \leq N/2, \quad |m| \leq j$$

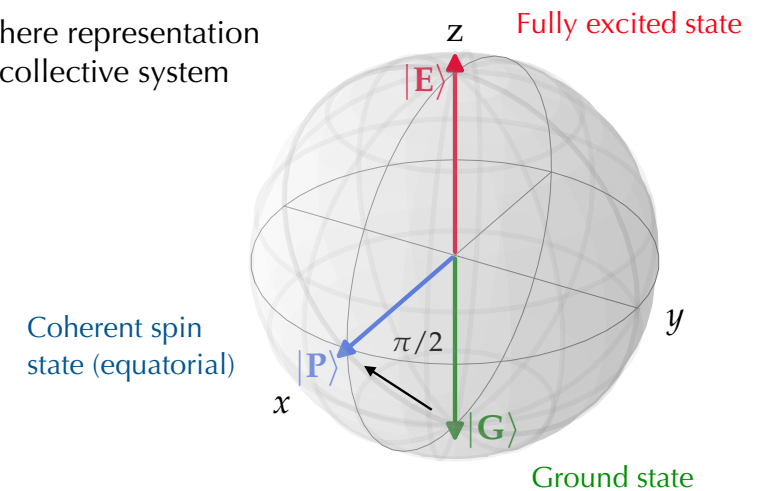
Eigenstates of the collective operators J^2, J_z

$$\Gamma_{\text{em}} = \gamma_{\text{em}} \langle J_+ J_- \rangle = \gamma_{\text{em}} (j + m)(j - m + 1)$$

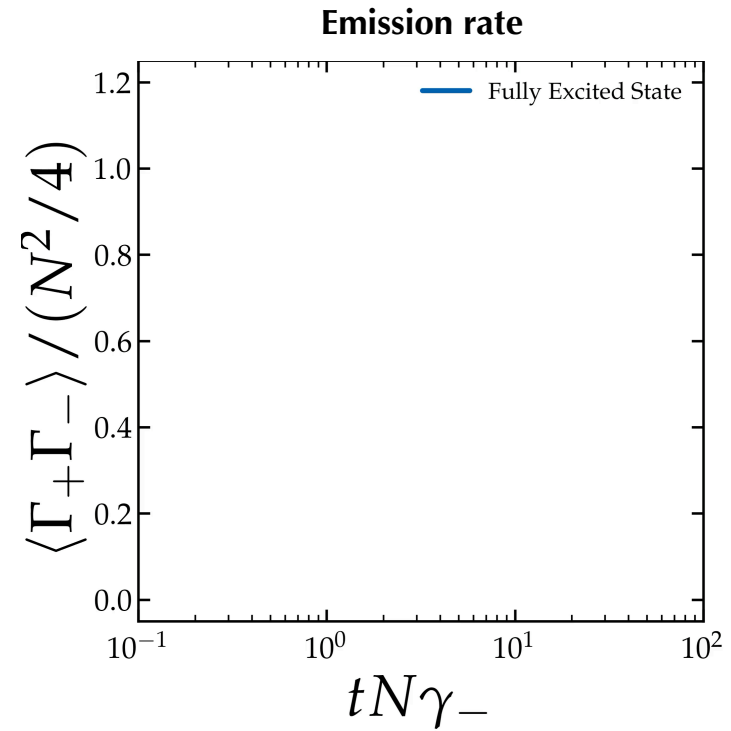
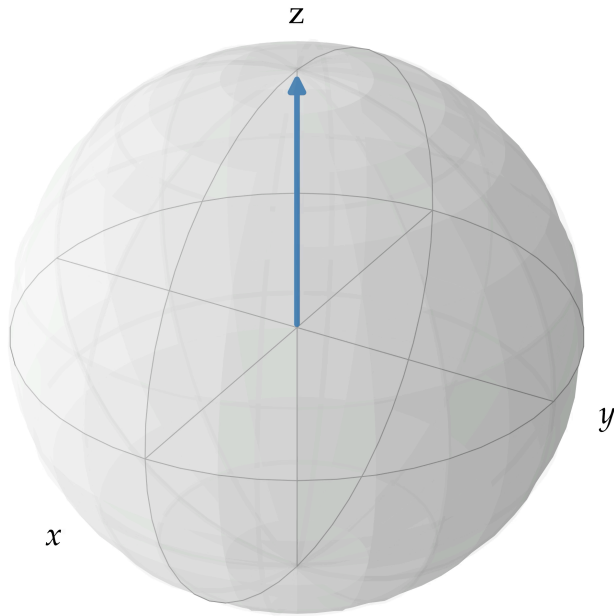
$$\Gamma_{\text{em}} = \gamma_{\text{em}} N \quad j = \frac{N}{2}, \quad m = \frac{N}{2} \quad \text{Fully excited state}$$

$$\Gamma_{\text{em}} = \gamma_{\text{em}} \frac{N}{2} \left(\frac{N}{2} + 1 \right) \quad j = \frac{N}{2}, \quad m = 0 \quad \text{Superposition}$$

Bloch sphere representation of the collective system



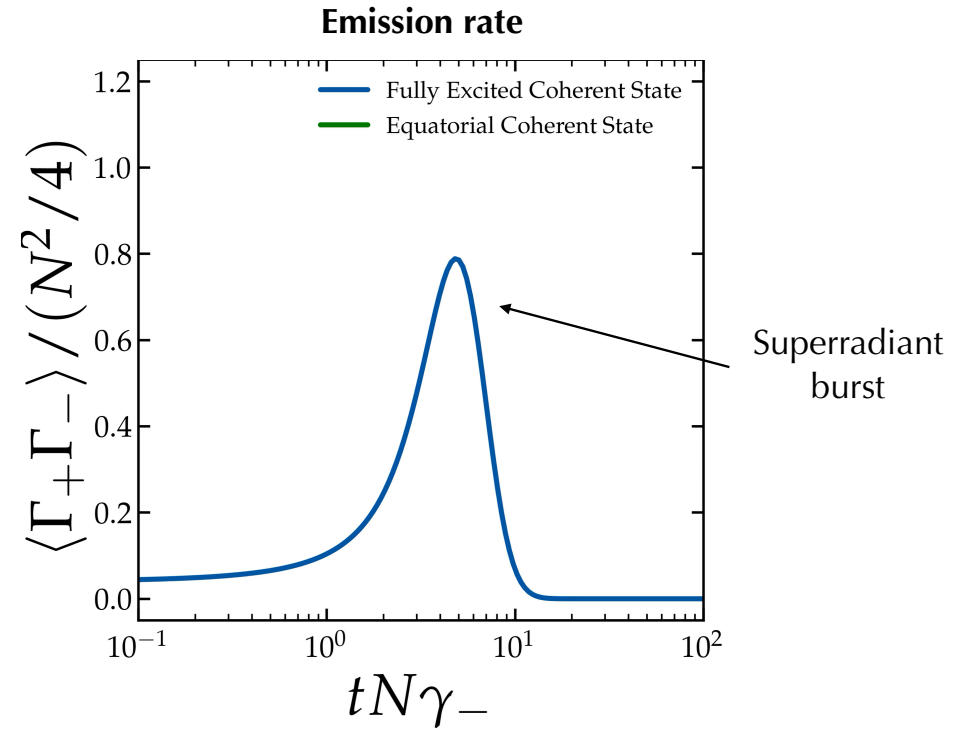
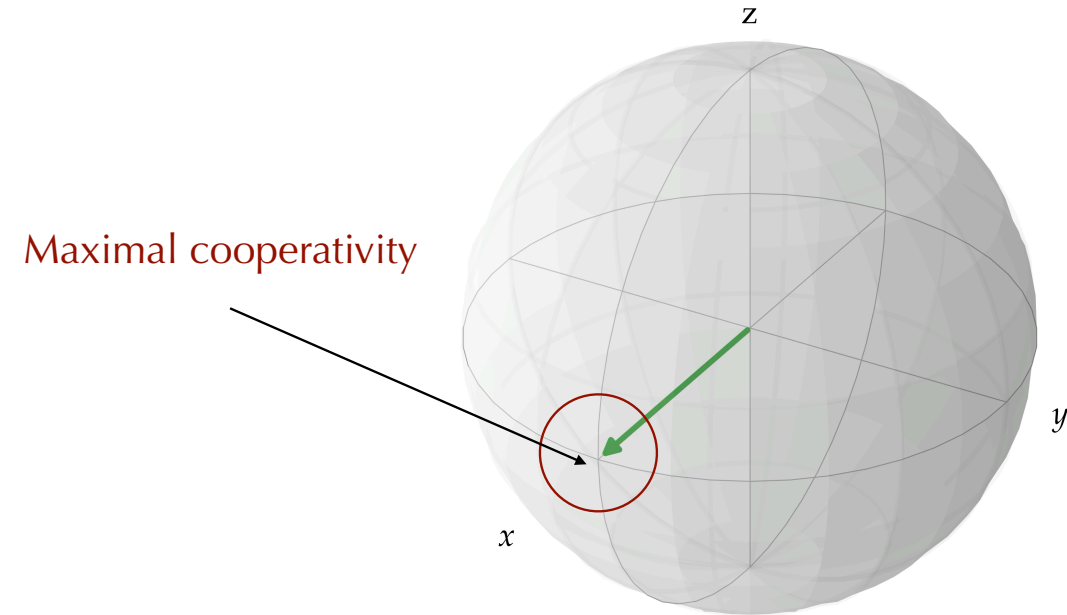
Dicke superradiance



$$\Gamma_{\text{em}} = \gamma_{\text{em}} \langle J_+ J_- \rangle = \gamma_{\text{em}} (j + m)(j - m + 1)$$

$\Gamma_{\text{em}} = \gamma_{\text{em}} N$	$j = \frac{N}{2}, \quad m = \frac{N}{2}$	Fully excited state
$\Gamma_{\text{em}} = \gamma_{\text{em}} \frac{N}{2} \left(\frac{N}{2} + 1 \right)$	$j = \frac{N}{2}, \quad m = 0$	Superposition

Dicke superradiance

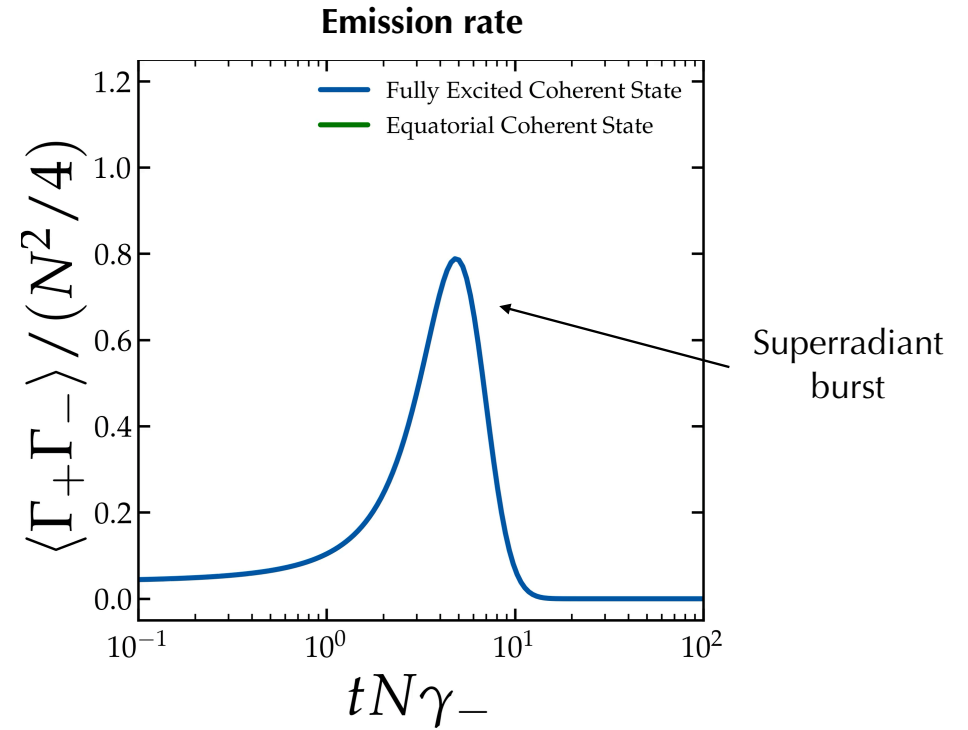
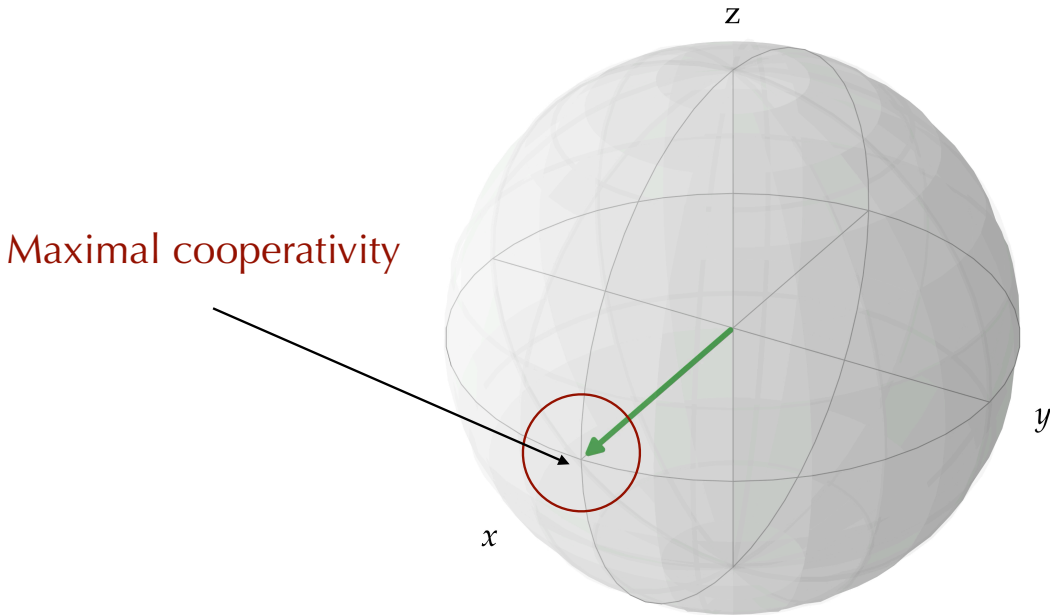


$$\Gamma_{\text{em}} = \gamma_{\text{em}} \langle J_+ J_- \rangle = \gamma_{\text{em}} (j + m)(j - m + 1)$$

$$\Gamma_{\text{em}} = \gamma_{\text{em}} N \quad j = \frac{N}{2}, \quad m = \frac{N}{2} \quad \text{Fully excited state}$$

$$\Gamma_{\text{em}} = \gamma_{\text{em}} \frac{N}{2} \left(\frac{N}{2} + 1 \right) \quad j = \frac{N}{2}, \quad m = 0 \quad \text{Superposition}$$

Dicke superradiance



$$\Gamma_{\text{em}} = \gamma_{\text{em}} \langle J_+ J_- \rangle = \gamma_{\text{em}} (j + m)(j - m + 1)$$

$\Gamma_{\text{em}} = \gamma_{\text{em}} N$	$j = \frac{N}{2}, \quad m = \frac{N}{2}$	Fully excited state
$\Gamma_{\text{em}} = \gamma_{\text{em}} \frac{N}{2} \left(\frac{N}{2} + 1 \right)$	$j = \frac{N}{2}, \quad m = 0$	Superposition

Advantage by starting in the equatorial plane with respect to ground state

Outline

1. Basics of Nuclear Magnetic Resonance (NMR)

2. Cosmic relics

- Cosmic neutrino background ($C\nu B$)
- Axion dark matter
- "Superradiant" interactions

3. Open Quantum System framework

- Lindblad master equation
- Numerical solutions
- Comments on Non-Markovian regime

4. Recent results

- Forecast for $C\nu B$ using CASPEr experiment
- Recent squeezing protocols

An open quantum system

$$H = H_S + H_B + H_{\text{int}} \quad \frac{d\rho}{dt} = -i[H, \rho]$$

The dynamics of the **total** neutrino-spin system is governed by the Liouville-von-Neumann equation

$$\rho = \rho_S \otimes \rho_B \quad \rho_S = \text{Tr}_B(\rho)$$

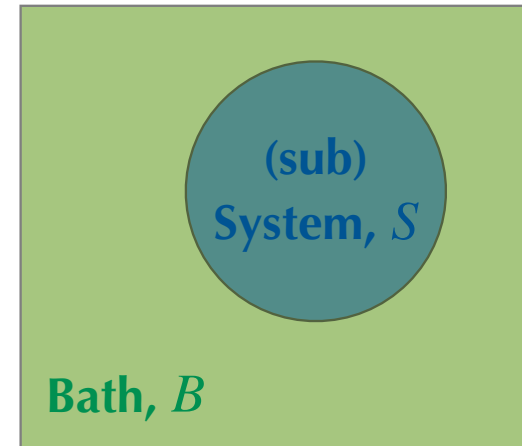
In the Markovian limit, the evolution of the subsystem is governed by the master **Lindblad** equation

$$\frac{d\rho_S}{dt} = -i[H_{\text{int}}^I, \rho_S] + \sum_k \gamma_k \mathcal{D}_{\mathcal{O}_k}[\rho_S(t)] \quad \mathcal{D}_{\mathcal{O}}[\rho_S(t)] = \mathcal{O}\rho_S\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O}, \rho_S\}$$

$$\tau_B \ll \tau_S$$

Environment correlation time is smaller than the subsystem relaxation time

Total system, $S + B$



An open quantum system

$$H = H_S + H_B + H_{\text{int}} \quad \frac{d\rho}{dt} = -i[H, \rho]$$

The dynamics of the **total** neutrino-spin system is governed by the Liouville-von-Neumann equation

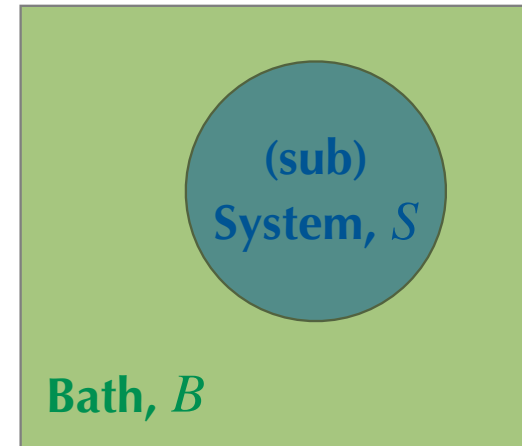
$$\rho = \rho_S \otimes \rho_B \quad \rho_S = \text{Tr}_B(\rho)$$

In the Markovian limit, the evolution of the subsystem is governed by the master **Lindblad** equation

$$\frac{d\rho_S}{dt} = -i[H_{\text{int}}^I, \rho_S] + \sum_k \gamma_k \mathcal{D}_{\mathcal{O}_k}[\rho_S(t)] \quad \mathcal{D}_{\mathcal{O}}[\rho_S(t)] = \mathcal{O}\rho_S\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O}, \rho_S\}$$

- Born approximation: neutrinos have weak interactions
- Markovian limit: relic background
- Secular approximation: NMR frequencies very large

Total system, $S + B$



Environment correlation time is smaller than the subsystem relaxation time

Dissipative channels

$$\frac{d\rho_S}{dt} = -i[H_{\text{int}}^I, \rho_S] + \sum_k \gamma_k \mathcal{D}_{\mathcal{O}_k}[\rho_S(t)] \qquad \mathcal{D}_{\mathcal{O}}[\rho_S(t)] = \mathcal{O}\rho_S\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O}, \rho_S\}$$

Dissipative channels

$$\frac{d\rho_S}{dt} = -i[H_{\text{int}}^I, \rho_S] + \sum_k \gamma_k \mathcal{D}_{\mathcal{O}_k}[\rho_S(t)] \quad \mathcal{D}_{\mathcal{O}}[\rho_S(t)] = \mathcal{O}\rho_S\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O}, \rho_S\}$$

Process

Operator

Rate

**Collective
effects**
 N^2

Neutrino emission	$\mathcal{D}_- = J_- \rho J_+ - \frac{1}{2}\{J_+ J_-, \rho\}$	γ_-
Neutrino absorption	$\mathcal{D}_+ = J_+ \rho J_- - \frac{1}{2}\{J_- J_+, \rho\}$	γ_+

Dissipative channels

$$\frac{d\rho_S}{dt} = -i[H_{\text{int}}^I, \rho_S] + \sum_k \gamma_k \mathcal{D}_{\mathcal{O}_k}[\rho_S(t)] \quad \mathcal{D}_{\mathcal{O}}[\rho_S(t)] = \mathcal{O}\rho_S\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O}, \rho_S\}$$

	Process	Operator		Rate
--	---------	----------	--	------

Collective
effects
 N^2

Neutrino emission	$\mathcal{D}_- = J_- \rho J_+ - \frac{1}{2}\{J_+ J_-, \rho\}$			γ_-
Neutrino absorption	$\mathcal{D}_+ = J_+ \rho J_- - \frac{1}{2}\{J_- J_+, \rho\}$			γ_+

Local
effects
 N

Local emission	$\mathcal{D}_-^{\text{loc}} = \sum_\alpha (J_-^\alpha \rho J_+^\alpha - \frac{1}{2}\{J_+^\alpha J_-^\alpha, \rho\})$			γ_-^{loc}
Local absorption	$\mathcal{D}_+^{\text{loc}} = \sum_\alpha (J_+^\alpha \rho J_-^\alpha - \frac{1}{2}\{J_-^\alpha J_+^\alpha, \rho\})$			γ_+^{loc}
Local dephasing	$\mathcal{D}_\phi^{\text{loc}} = \sum_\alpha (J_z^\alpha \rho J_z^\alpha - \frac{1}{2}\{(J_z^\alpha)^2, \rho\})$			γ_ϕ^{loc}

Dissipative channels

$$\frac{d\rho_S}{dt} = -i[H_{\text{int}}^I, \rho_S] + \sum_k \gamma_k \mathcal{D}_{\mathcal{O}_k}[\rho_S(t)] \quad \mathcal{D}_{\mathcal{O}}[\rho_S(t)] = \mathcal{O}\rho_S\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O}, \rho_S\}$$

Process	Operator	Rate
---------	----------	------

**Collective
effects**
 N^2

Neutrino emission	$\mathcal{D}_- = J_- \rho J_+ - \frac{1}{2}\{J_+ J_-, \rho\}$	γ_-
Neutrino absorption	$\mathcal{D}_+ = J_+ \rho J_- - \frac{1}{2}\{J_- J_+, \rho\}$	γ_+

**Local
effects**
 N

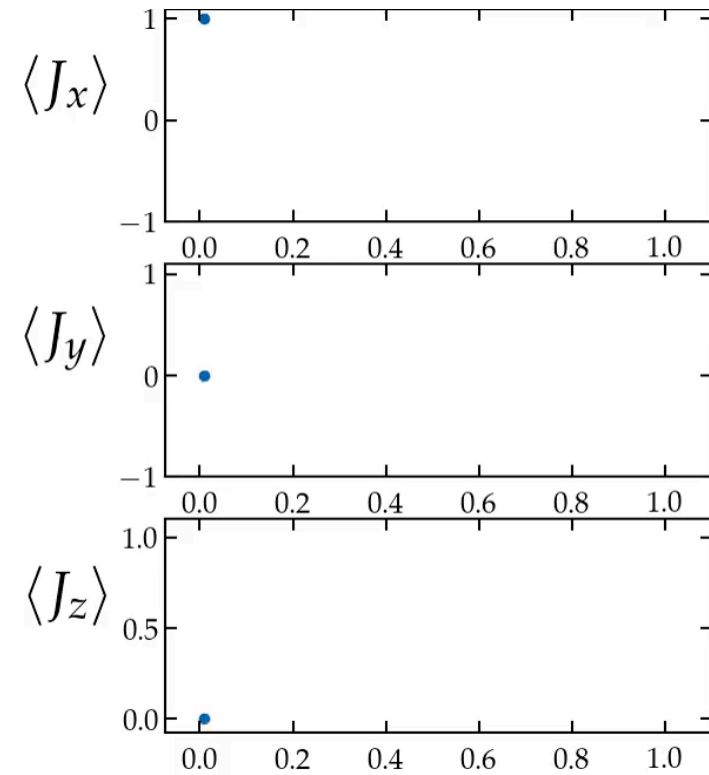
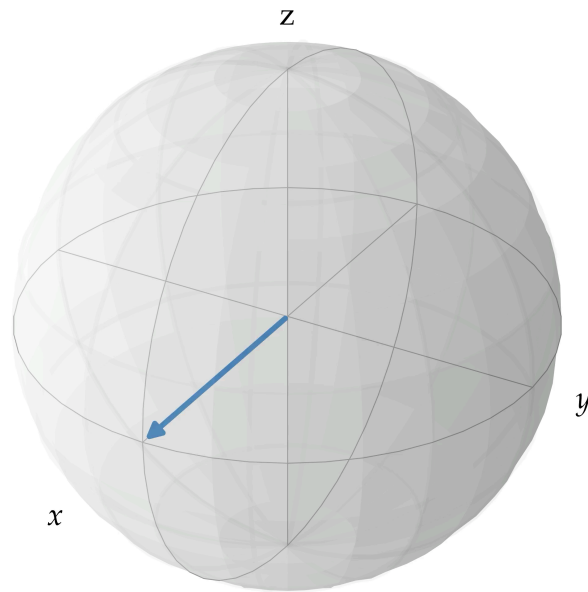
Local emission	$\mathcal{D}_-^{\text{loc}} = \sum_\alpha (J_-^\alpha \rho J_+^\alpha - \frac{1}{2}\{J_+^\alpha J_-^\alpha, \rho\})$	γ_-^{loc}
Local absorption	$\mathcal{D}_+^{\text{loc}} = \sum_\alpha (J_+^\alpha \rho J_-^\alpha - \frac{1}{2}\{J_-^\alpha J_+^\alpha, \rho\})$	γ_+^{loc}
Local dephasing	$\mathcal{D}_\phi^{\text{loc}} = \sum_\alpha (J_z^\alpha \rho J_z^\alpha - \frac{1}{2}\{(J_z^\alpha)^2, \rho\})$	γ_ϕ^{loc}

Dissipative dynamics

Total **neutrino-induced collective** effects $\gamma_{\pm}$

$$\gamma_+/\gamma_- = 0.9$$

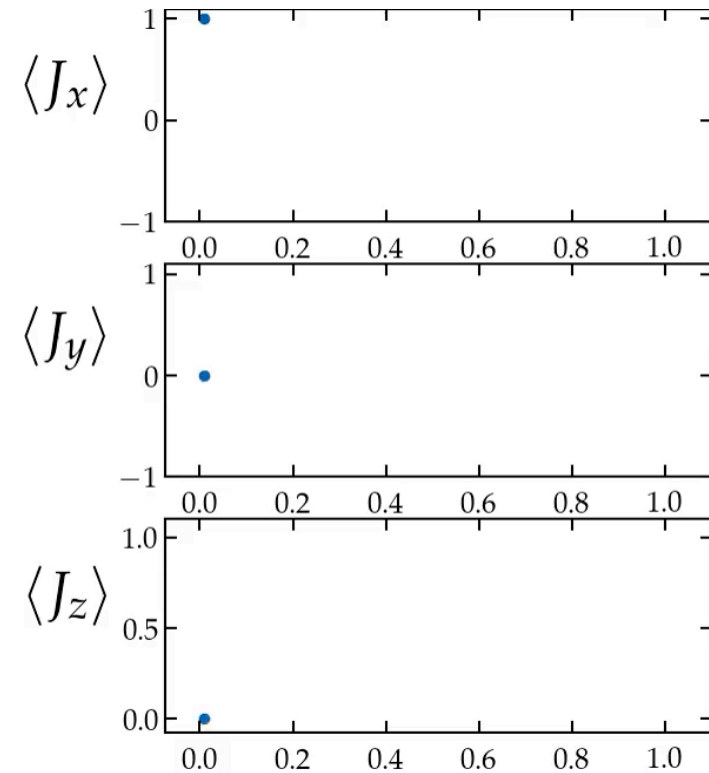
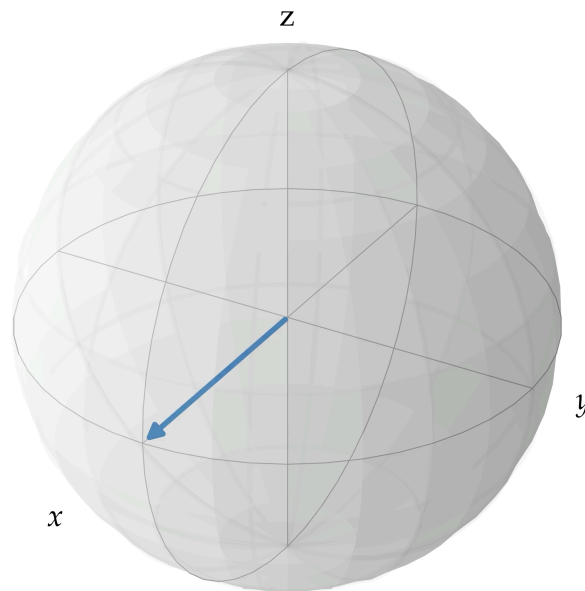
Starting from the
coherent state



Dissipative dynamics

Total **neutrino-induced collective** effects $\gamma_{\pm}$
 $\gamma_+/\gamma_- = 0.9$

Starting from the
coherent state



- Transverse coherence is only lost because system moves away from the equatorial plane
- Ratio between +/- sets decoherence timescale

$$\gamma_{\text{net}} \propto 1 - \gamma_+/\gamma_-$$

Dissipative channels

$$\frac{d\rho_S}{dt} = -i[H_{\text{int}}^I, \rho_S] + \sum_k \gamma_k \mathcal{D}_{\mathcal{O}_k}[\rho_S(t)] \quad \mathcal{D}_{\mathcal{O}}[\rho_S(t)] = \mathcal{O}\rho_S\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O}, \rho_S\}$$

Process	Operator	Rate
---------	----------	------

**Collective
effects**
 N^2

Neutrino emission	$\mathcal{D}_- = J_- \rho J_+ - \frac{1}{2}\{J_+ J_-, \rho\}$	γ_-
Neutrino absorption	$\mathcal{D}_+ = J_+ \rho J_- - \frac{1}{2}\{J_- J_+, \rho\}$	γ_+

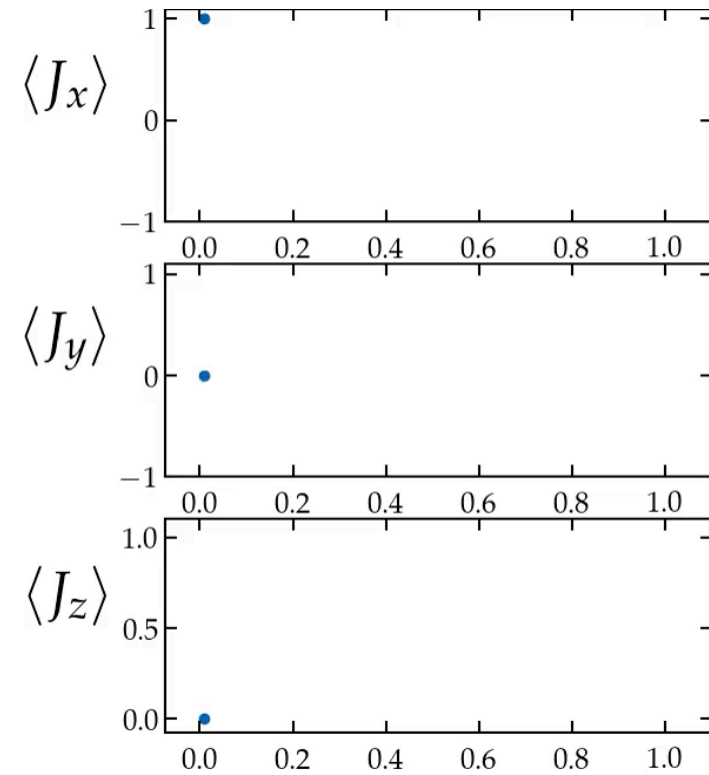
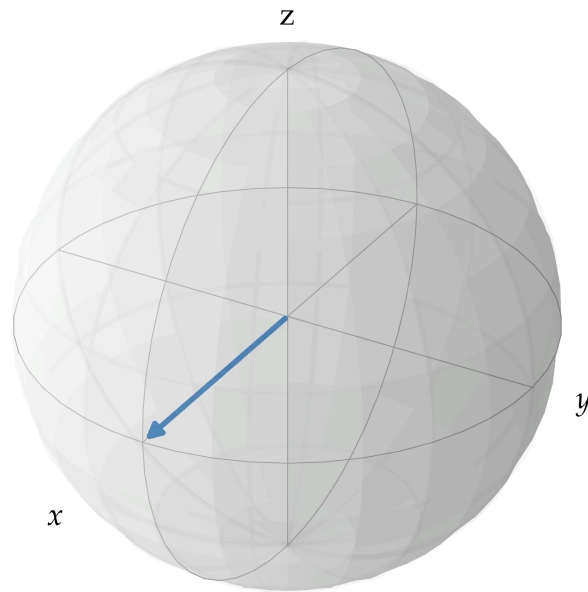
**Local
effects**
 N

Local emission	$\mathcal{D}_-^{\text{loc}} = \sum_\alpha (J_-^\alpha \rho J_+^\alpha - \frac{1}{2}\{J_+^\alpha J_-^\alpha, \rho\})$	γ_-^{loc}
Local absorption	$\mathcal{D}_+^{\text{loc}} = \sum_\alpha (J_+^\alpha \rho J_-^\alpha - \frac{1}{2}\{J_-^\alpha J_+^\alpha, \rho\})$	γ_+^{loc}
Local dephasing	$\mathcal{D}_\phi^{\text{loc}} = \sum_\alpha (J_z^\alpha \rho J_z^\alpha - \frac{1}{2}\{(J_z^\alpha)^2, \rho\})$	γ_ϕ^{loc}

Dissipative dynamics

Effects of **local dephasing** $\gamma_{\phi}^{\text{loc}}$

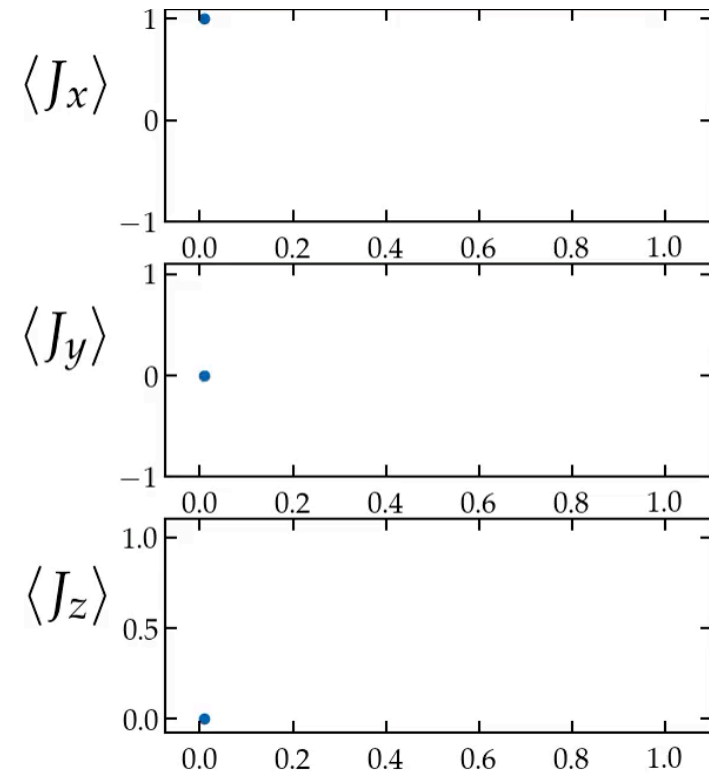
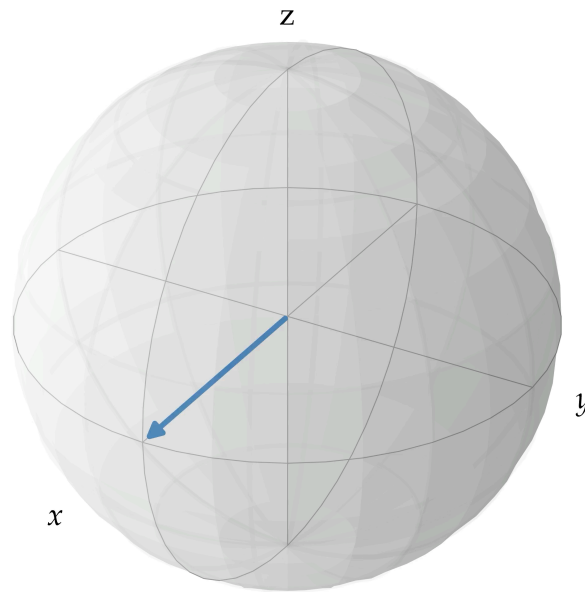
Starting from the
coherent state



Dissipative dynamics

Effects of **local dephasing** $\gamma_{\phi}^{\text{loc}}$

Starting from the
coherent state



- Transverse coherence is lost because of incoherent effects
- No energy evolution

Dissipative channels

$$\frac{d\rho_S}{dt} = -i[H_{\text{int}}^I, \rho_S] + \sum_k \gamma_k \mathcal{D}_{\mathcal{O}_k}[\rho_S(t)] \quad \mathcal{D}_{\mathcal{O}}[\rho_S(t)] = \mathcal{O}\rho_S\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O}, \rho_S\}$$

	Process	Operator	Rate
--	---------	----------	------

Collective
effects
 N^2

Neutrino emission	$\mathcal{D}_- = J_- \rho J_+ - \frac{1}{2}\{J_+ J_-, \rho\}$	γ_-
Neutrino absorption	$\mathcal{D}_+ = J_+ \rho J_- - \frac{1}{2}\{J_- J_+, \rho\}$	γ_+

Local
effects
 N

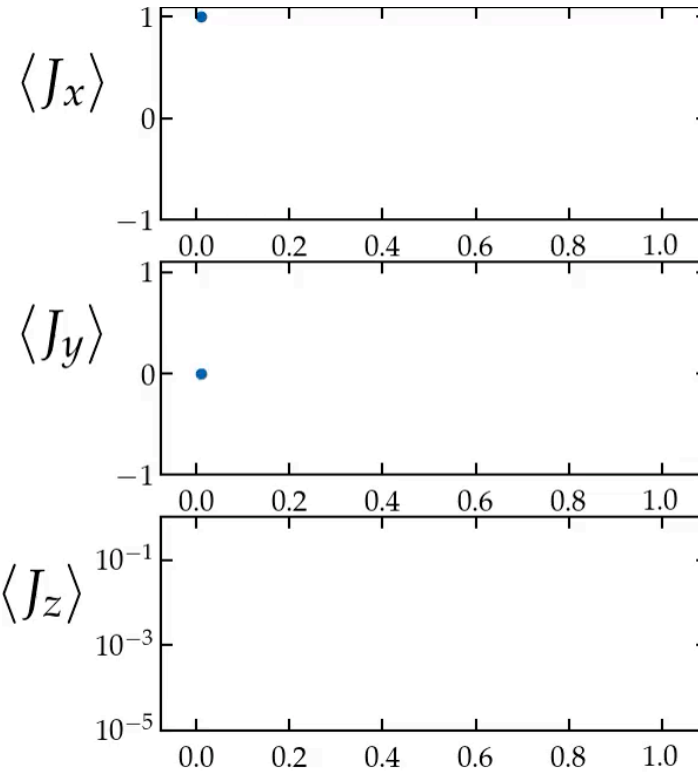
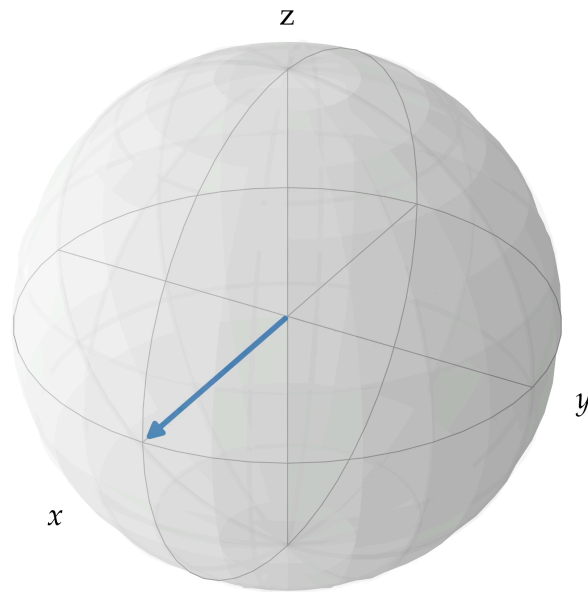
Local emission	$\mathcal{D}_-^{\text{loc}} = \sum_\alpha (J_-^\alpha \rho J_+^\alpha - \frac{1}{2}\{J_+^\alpha J_-^\alpha, \rho\})$	γ_-^{loc}
Local absorption	$\mathcal{D}_+^{\text{loc}} = \sum_\alpha (J_+^\alpha \rho J_-^\alpha - \frac{1}{2}\{J_-^\alpha J_+^\alpha, \rho\})$	γ_+^{loc}
Local dephasing	$\mathcal{D}_\phi^{\text{loc}} = \sum_\alpha (J_z^\alpha \rho J_z^\alpha - \frac{1}{2}\{(J_z^\alpha)^2, \rho\})$	γ_ϕ^{loc}

Dissipative dynamics

Effects of **local** interactions (dephasing + relaxation)

$$\gamma_{\phi}^{\text{loc}}, \gamma_{\pm}^{\text{loc}}$$

Starting from the
coherent state

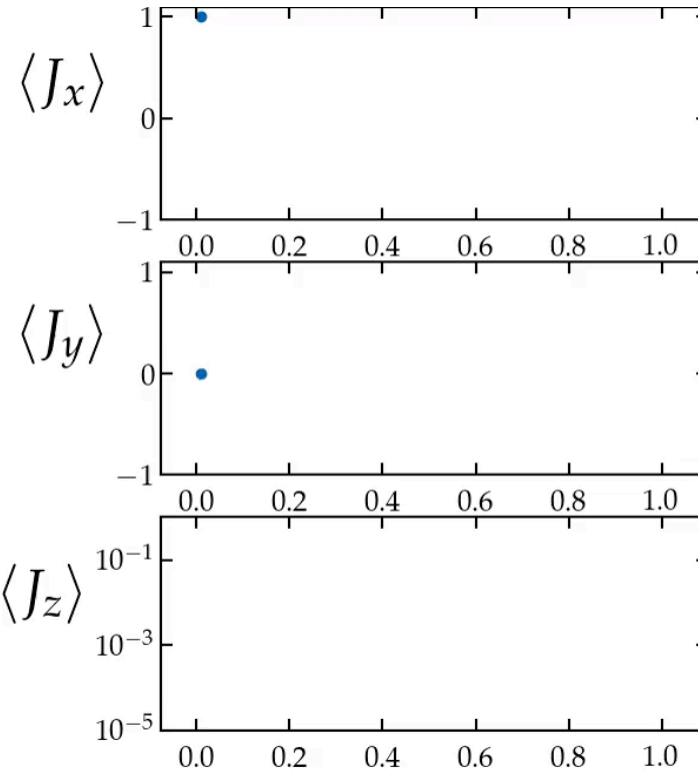
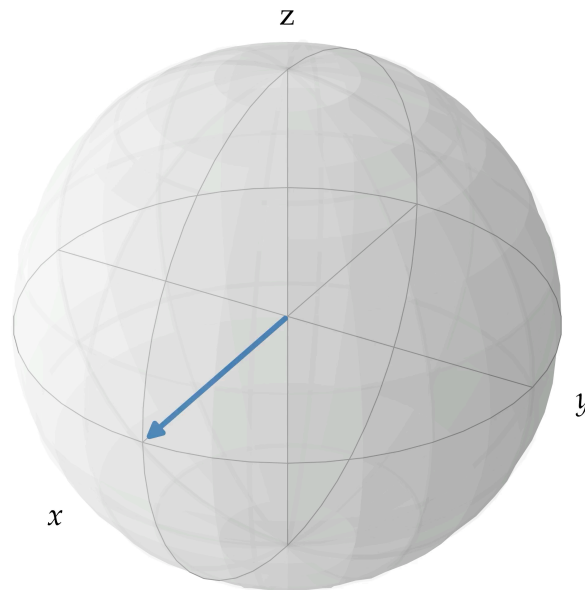


Dissipative dynamics

Effects of **local** interactions (dephasing + relaxation)

$$\gamma_{\phi}^{\text{loc}}, \gamma_{\pm}^{\text{loc}}$$

Starting from the
coherent state



- Transverse coherence is lost because of incoherent effects (both dephasing and relaxation)
- Energy evolution to the *thermal* steady-state

Relaxation and dephasing

Longitudinal relaxation: T_1

- Recovery of magnetisation along the magnetic field
- Driven by energy exchange with the lattice (spin-lattice relaxation)
- In liquid-state NMR, e.g. $T_1 \sim 1000 \text{ s}$

$$\langle \mathcal{J}_z(t) \rangle = \langle \mathcal{J}_z \rangle^{\text{eq}} - [\langle \mathcal{J}_z \rangle^{\text{eq}} - \langle \mathcal{J}_z \rangle(0)] e^{-t(\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}})}$$

$$\langle \mathcal{J}_z \rangle^{\text{eq}} = \frac{N}{2} \frac{\gamma_+^{\text{loc}} - \gamma_-^{\text{loc}}}{\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}}}$$

$$T_1^{-1} = \gamma_+^{\text{loc}} + \gamma_-^{\text{loc}}$$

Relaxation and dephasing

Longitudinal relaxation: T_1

- Recovery of magnetisation along the magnetic field
- Driven by energy exchange with the lattice (spin-lattice relaxation)
- In liquid-state NMR, e.g. $T_1 \sim 1000 \text{ s}$

$$\langle \mathcal{J}_z(t) \rangle = \langle \mathcal{J}_z \rangle^{\text{eq}} - [\langle \mathcal{J}_z \rangle^{\text{eq}} - \langle \mathcal{J}_z \rangle(0)] e^{-t(\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}})}$$

$$\langle \mathcal{J}_z \rangle^{\text{eq}} = \frac{N}{2} \frac{\gamma_+^{\text{loc}} - \gamma_-^{\text{loc}}}{\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}}}$$

$$T_1^{-1} = \gamma_+^{\text{loc}} + \gamma_-^{\text{loc}}$$

Transverse relaxation: T_2

- Loss of coherence in the xy -plane due to spin-spin interactions
- Typically shorter than in T_1 solid-state
- Experimentally limited by local field inhomogeneities (T_2^*)

$$\langle \mathcal{J}_x(t) \rangle = \frac{N}{2} e^{-t(\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}} + \gamma_\phi^{\text{loc}})/2}$$

$$T_2^{-1} = \frac{1}{2}(\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}} + \gamma_\phi^{\text{loc}})$$

$$T_2 \lesssim 1000 \text{ s}$$

Relaxation and dephasing

Longitudinal relaxation: T_1

- Recovery of magnetisation along the magnetic field
- Driven by energy exchange with the lattice (spin-lattice relaxation)
- In liquid-state NMR, e.g. $T_1 \sim 1000 \text{ s}$

$$\langle \mathcal{J}_z(t) \rangle = \langle \mathcal{J}_z \rangle^{\text{eq}} - [\langle \mathcal{J}_z \rangle^{\text{eq}} - \langle \mathcal{J}_z \rangle(0)] e^{-t(\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}})}$$

$$\langle \mathcal{J}_z \rangle^{\text{eq}} = \frac{N}{2} \frac{\gamma_+^{\text{loc}} - \gamma_-^{\text{loc}}}{\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}}}$$

$$T_1^{-1} = \gamma_+^{\text{loc}} + \gamma_-^{\text{loc}}$$

Amplitude damping channel (ADC)

Transverse relaxation: T_2

- Loss of coherence in the xy -plane due to spin-spin interactions
- Typically shorter than in T_1 solid-state
- Experimentally limited by local field inhomogeneities (T_2^*)

$$\langle \mathcal{J}_x(t) \rangle = \frac{N}{2} e^{-t(\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}} + \gamma_\phi^{\text{loc}})/2}$$

$$T_2^{-1} = \frac{1}{2}(\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}} + \gamma_\phi^{\text{loc}})$$

$$T_2 \lesssim 1000 \text{ s}$$

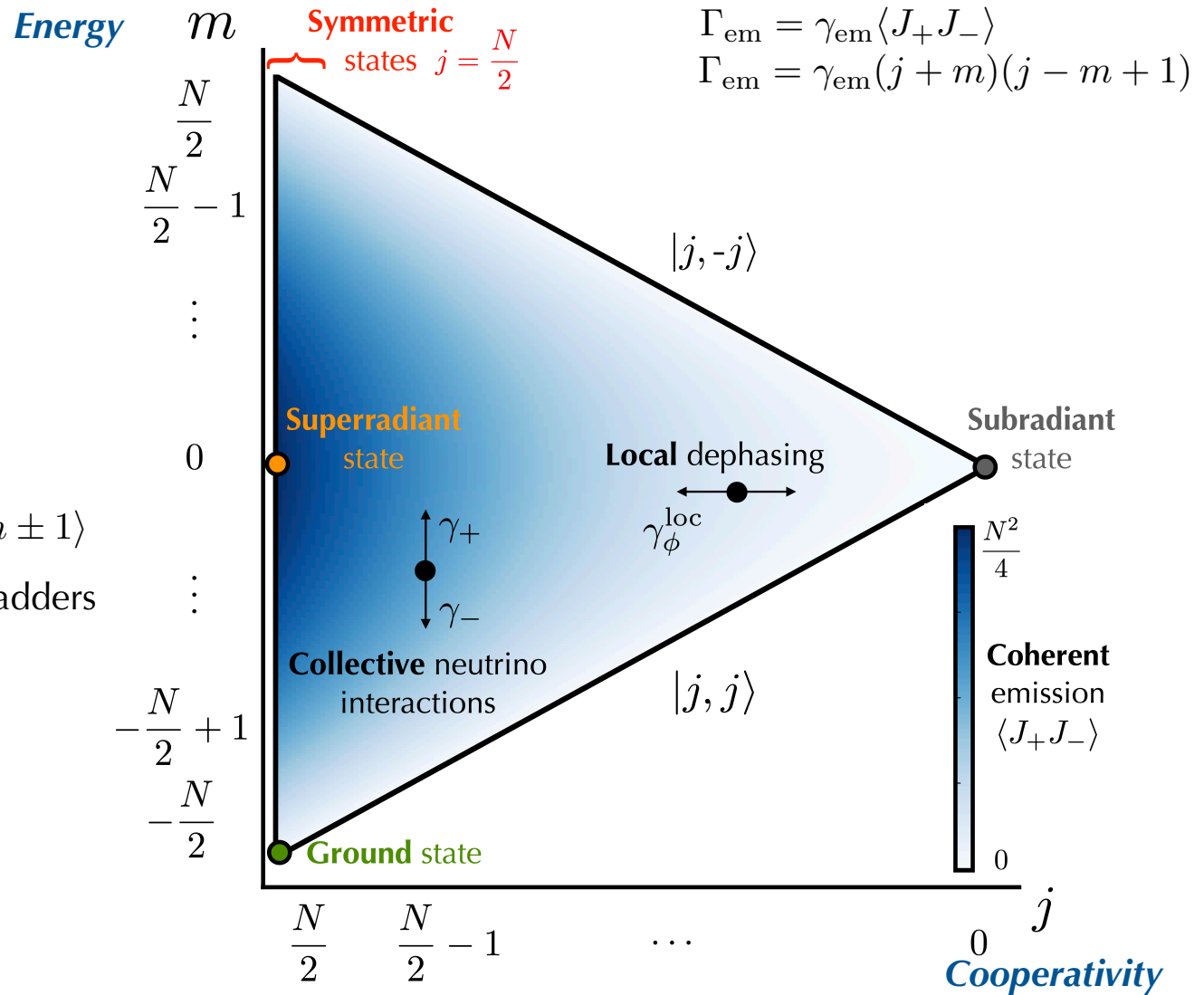
Phase damping channel (PDC)

Permutational invariance and Dicke basis

One can consider all of the two-level systems as **identical** and identically prepared at an initial time: permutational invariant subspace of the full Hilbert space

Coherent interaction: $|N/2, m\rangle \rightarrow |N/2, m \pm 1\rangle$

Local interactions connect different energy ladders



Solving the master equation

Full method

Hilbert space dimension grows as 2^N with the number of spins N
Liouvillian space grows as 4^N

With permutational symmetry up to N^3 entries in the density matrix

$$\begin{aligned}\frac{d\rho_S}{dt} &= \gamma_- \mathcal{D}_{\mathcal{J}_-}[\rho_S] + \gamma_+ \mathcal{D}_{\mathcal{J}_+}[\rho_S] \\ &\quad + \gamma_-^{\text{loc}} \sum_{\alpha} \mathcal{D}_{J_-^{\alpha}}[\rho_S] + \gamma_+^{\text{loc}} \sum_{\alpha} \mathcal{D}_{J_+^{\alpha}}[\rho_S] \\ &\quad + \gamma_{\phi}^{\text{loc}} \sum_{\alpha} \mathcal{D}_{J_z^{\alpha}}[\rho_S] \\ &\equiv \mathcal{L}\{\rho_S(t)\}\end{aligned}$$

Solving the master equation

Full method

Hilbert space dimension grows as 2^N with the number of spins N
 Liouvillian space grows as 4^N

With permutational symmetry up to N^3 entries in the density matrix

2nd order method

$$\begin{aligned}\frac{d}{dt}\langle \mathcal{J}_z \rangle &= \sum_{\pm} \pm \gamma_{\pm} \left[\langle \mathcal{J}^2 \rangle - \langle \mathcal{J}_z^2 \rangle \mp \langle \mathcal{J}_z \rangle \right] \\ &\quad + \sum_{\pm} \pm \gamma_{\pm}^{\text{loc}} \left[\mp \langle \mathcal{J}_z \rangle + \frac{N}{2} \right], \\ \frac{d}{dt}\langle \mathcal{J}^2 \rangle &= - \sum_{\pm} \gamma_{\pm}^{\text{loc}} \left[\langle \mathcal{J}^2 \rangle + \langle \mathcal{J}_z^2 \rangle \mp (N-1)\langle \mathcal{J}_z \rangle - N \right] \\ &\quad - \gamma_{\phi}^{\text{loc}} \left[\langle \mathcal{J}^2 \rangle - \langle \mathcal{J}_z^2 \rangle - \frac{N}{2} \right], \\ \frac{d}{dt}\langle \mathcal{J}_z^2 \rangle &= \sum_{\pm} \gamma_{\pm} \left[\langle \mathcal{J}^2 \rangle - 3\langle \mathcal{J}_z^2 \rangle \mp \langle \mathcal{J}_z \rangle \right. \\ &\quad \left. \mp 2\langle \mathcal{J}_z \rangle \langle \mathcal{J}_z^2 \rangle \pm 2\langle \mathcal{J}_z \rangle \langle \mathcal{J}^2 \rangle \right] \\ &\quad - \sum_{\pm} \gamma_{\pm}^{\text{loc}} \left[2\langle \mathcal{J}_z^2 \rangle \mp (N-1)\langle \mathcal{J}_z \rangle - \frac{N}{2} \right]\end{aligned}$$

$$\begin{aligned}\frac{d\rho_S}{dt} &= \gamma_- \mathcal{D}_{\mathcal{J}_-}[\rho_S] + \gamma_+ \mathcal{D}_{\mathcal{J}_+}[\rho_S] \\ &\quad + \gamma_-^{\text{loc}} \sum_{\alpha} \mathcal{D}_{J_-^{\alpha}}[\rho_S] + \gamma_+^{\text{loc}} \sum_{\alpha} \mathcal{D}_{J_+^{\alpha}}[\rho_S] \\ &\quad + \gamma_{\phi}^{\text{loc}} \sum_{\alpha} \mathcal{D}_{J_z^{\alpha}}[\rho_S] \\ &\equiv \mathcal{L}\{\rho_S(t)\}\end{aligned}$$

$$(\mathcal{J}_z, \mathcal{J}_z^2, \mathcal{J}^2)$$

Semiclassical approximation: higher moments of the operators are factorised. Truncate the hierarchy and close the system

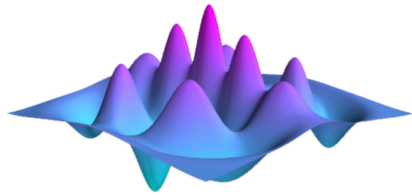
$$\begin{aligned}\langle \mathcal{J}_z^3 \rangle &\approx \langle \mathcal{J}_z \rangle \langle \mathcal{J}_z^2 \rangle \\ \langle \mathcal{J}_z \mathcal{J}^2 \rangle &\approx \langle \mathcal{J}_z \rangle \langle \mathcal{J}^2 \rangle\end{aligned}$$

Effective mean-field theory that improves for large N

Cons: gives access to limited information about the system's state (in our case only up to second moments)

Numerical methods

$$\begin{aligned}\frac{d\rho_S}{dt} &= \gamma_- \mathcal{D}_{\mathcal{J}_-}[\rho_S] + \gamma_+ \mathcal{D}_{\mathcal{J}_+}[\rho_S] \\ &\quad + \gamma_-^{\text{loc}} \sum_{\alpha} \mathcal{D}_{J_-^{\alpha}}[\rho_S] + \gamma_+^{\text{loc}} \sum_{\alpha} \mathcal{D}_{J_+^{\alpha}}[\rho_S] \\ &\quad + \gamma_{\phi}^{\text{loc}} \sum_{\alpha} \mathcal{D}_{J_z^{\alpha}}[\rho_S] \\ &\equiv \mathcal{L}\{\rho_S(t)\}\end{aligned}$$



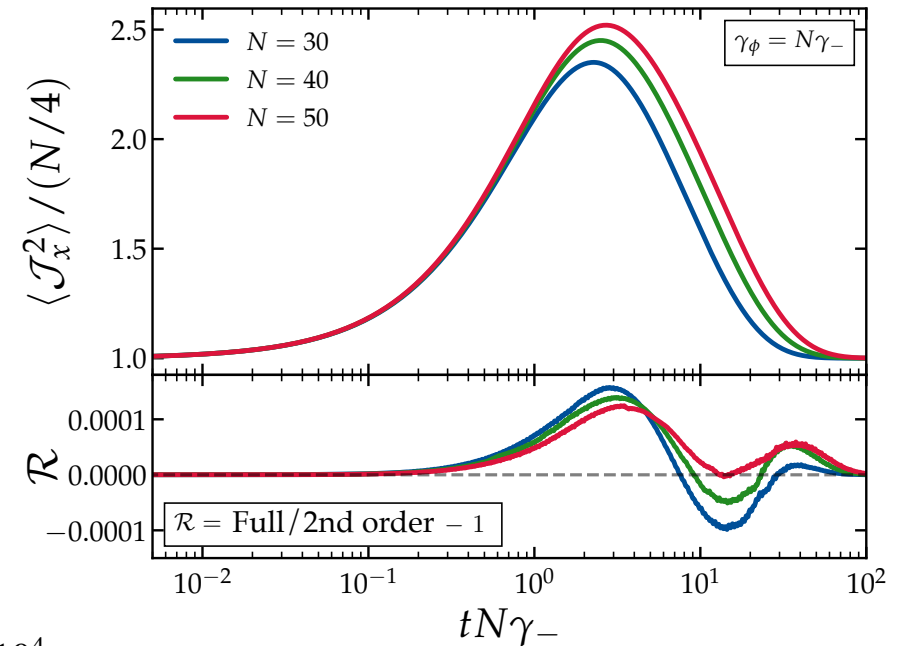
qutip.org

QuTiP

Quantum Toolbox in Python

Get Started with QuTiP v5

Try QuTiP from your Browser!



- Full density matrix solutions with collective dissipation for $N \lesssim 10^4$

github.com/gpiero/nu

- Full density matrix solutions with collective dissipation and local dissipative channels for $N \lesssim 70$

github.com/qutip/qutip

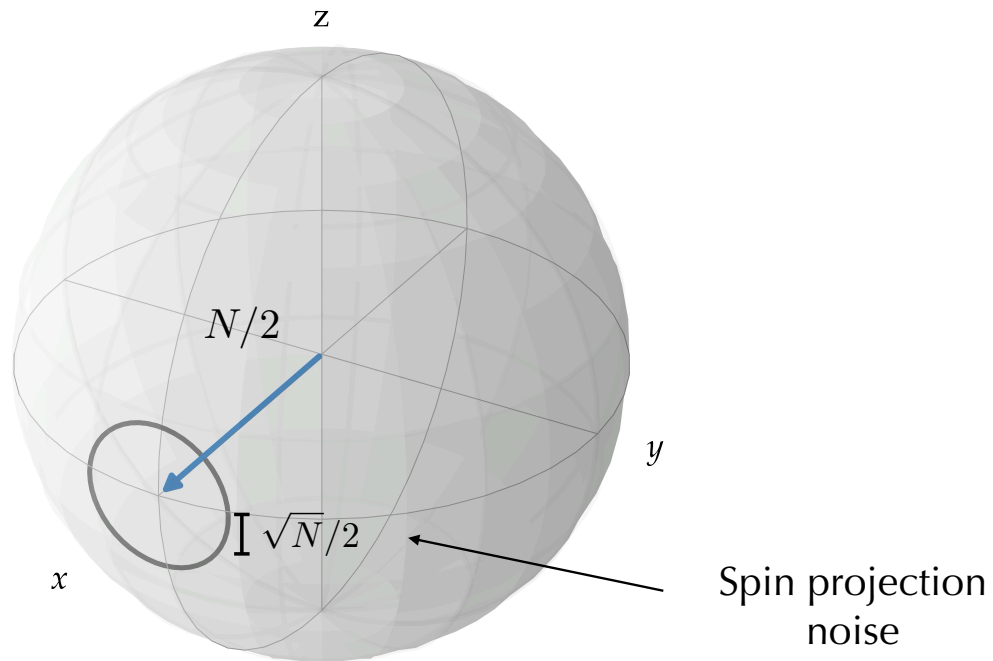
- Fast second-order approximate solutions for arbitrary N

github.com/gpiero/pyOpenNu

Drift observable

First-order observable

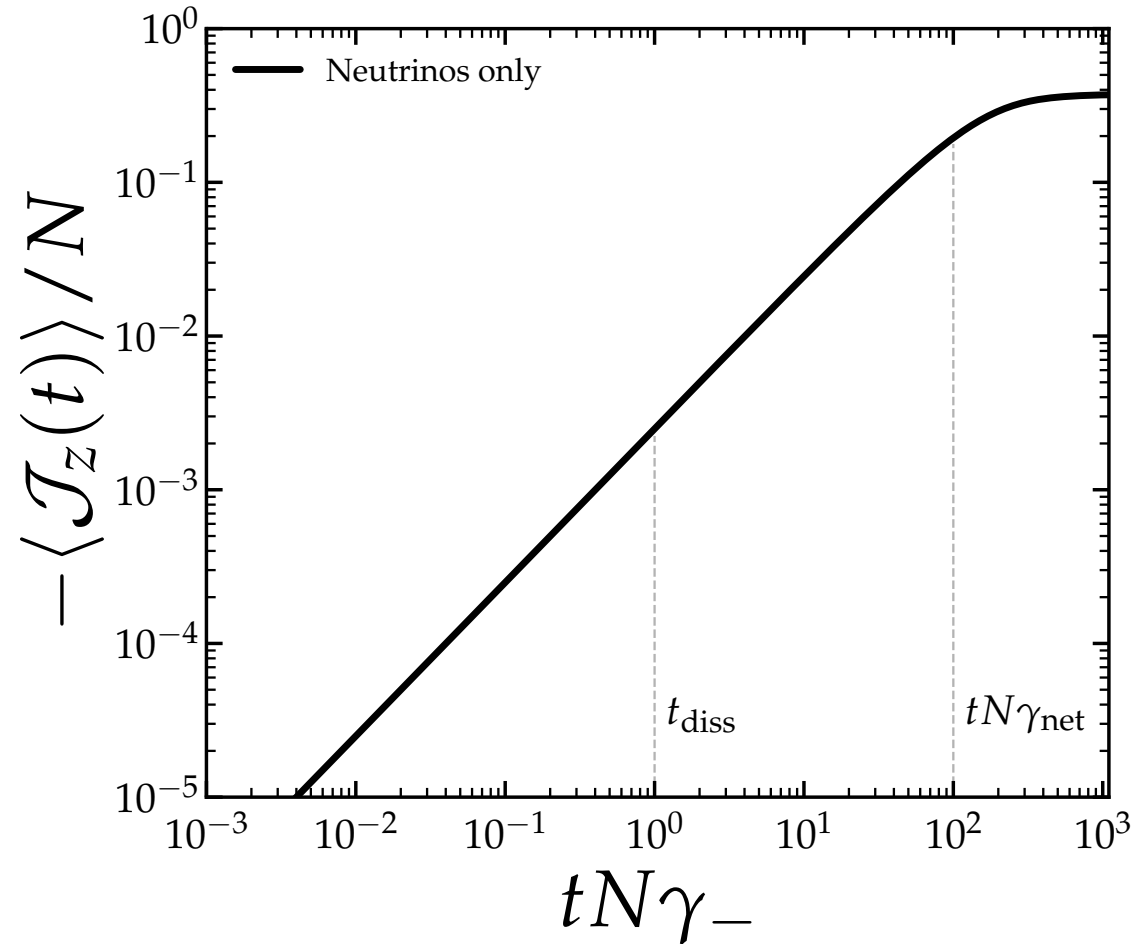
$$\langle \mathcal{J}_z \rangle \sim \gamma_{\text{net}} N^2, \quad \sigma_{\mathcal{J}_z} \sim \sqrt{N}, \quad \text{SNR} \sim \gamma_{\text{net}} N^{3/2}$$



Ideal for CνB searches

Energy evolution: including local effects

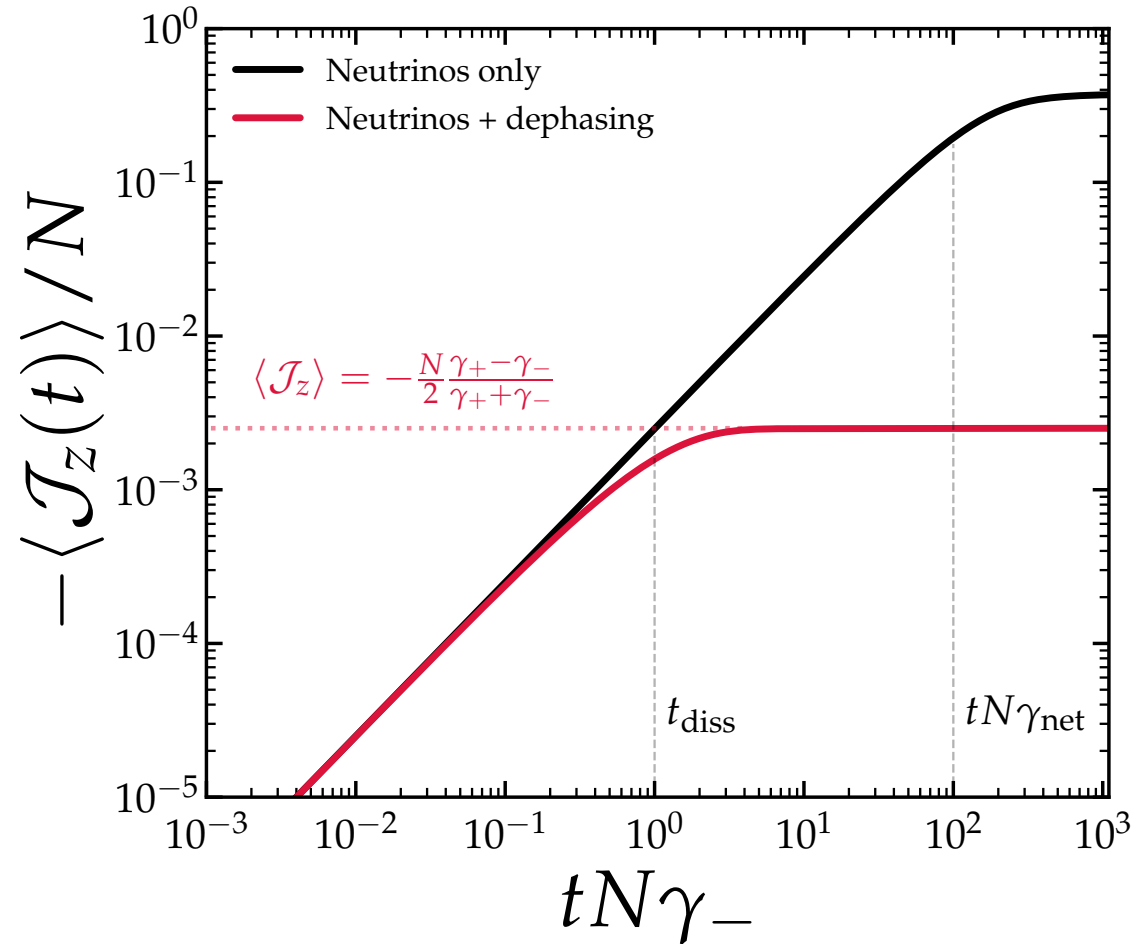
1. Neutrinos-only with $\delta_\nu \gg 1$: steady state reached after the dissipative timescale



Energy evolution: including local effects

1. Neutrinos-only with $\delta_\nu \gg 1$: steady state reached after the dissipative timescale
2. Local dephasing: coherent effects suppressed

Neutrino steady-state reached at the dissipative timescale



Energy evolution: including local effects

1. Neutrinos-only with $\delta_\nu \gg 1$: steady state reached after the dissipative timescale

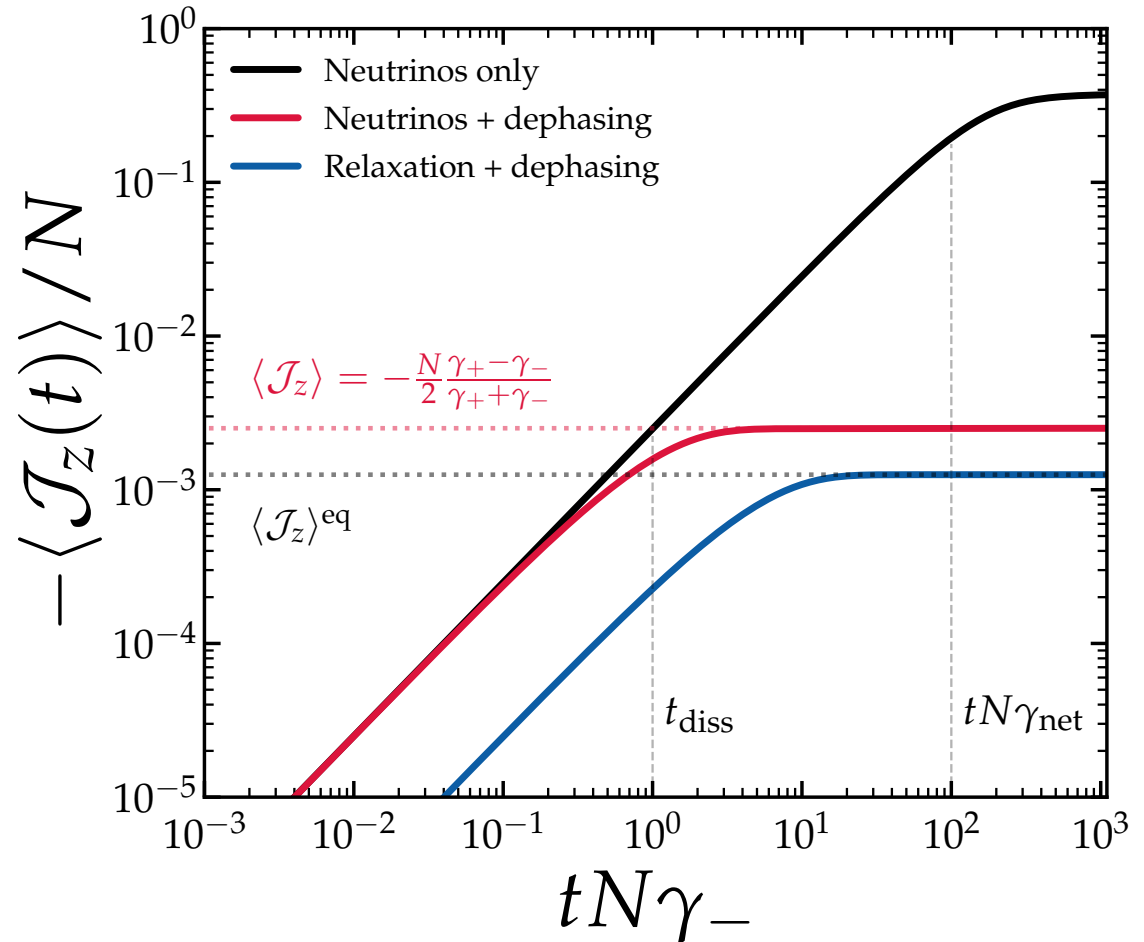
2. Local dephasing: coherent effects suppressed

Neutrino steady-state reached at the dissipative timescale

3. All local effects: relaxation adds dissipation in the energy

Thermal steady-state reached at the dissipative timescale

$$\langle \mathcal{J}_z \rangle^{\text{eq}} = \frac{N}{2} \frac{\gamma_+^{\text{loc}} - \gamma_-^{\text{loc}}}{\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}}}$$



Energy evolution: including local effects

1. Neutrinos-only with $\delta_\nu \gg 1$: steady state reached after the dissipative timescale

2. Local dephasing: coherent effects suppressed

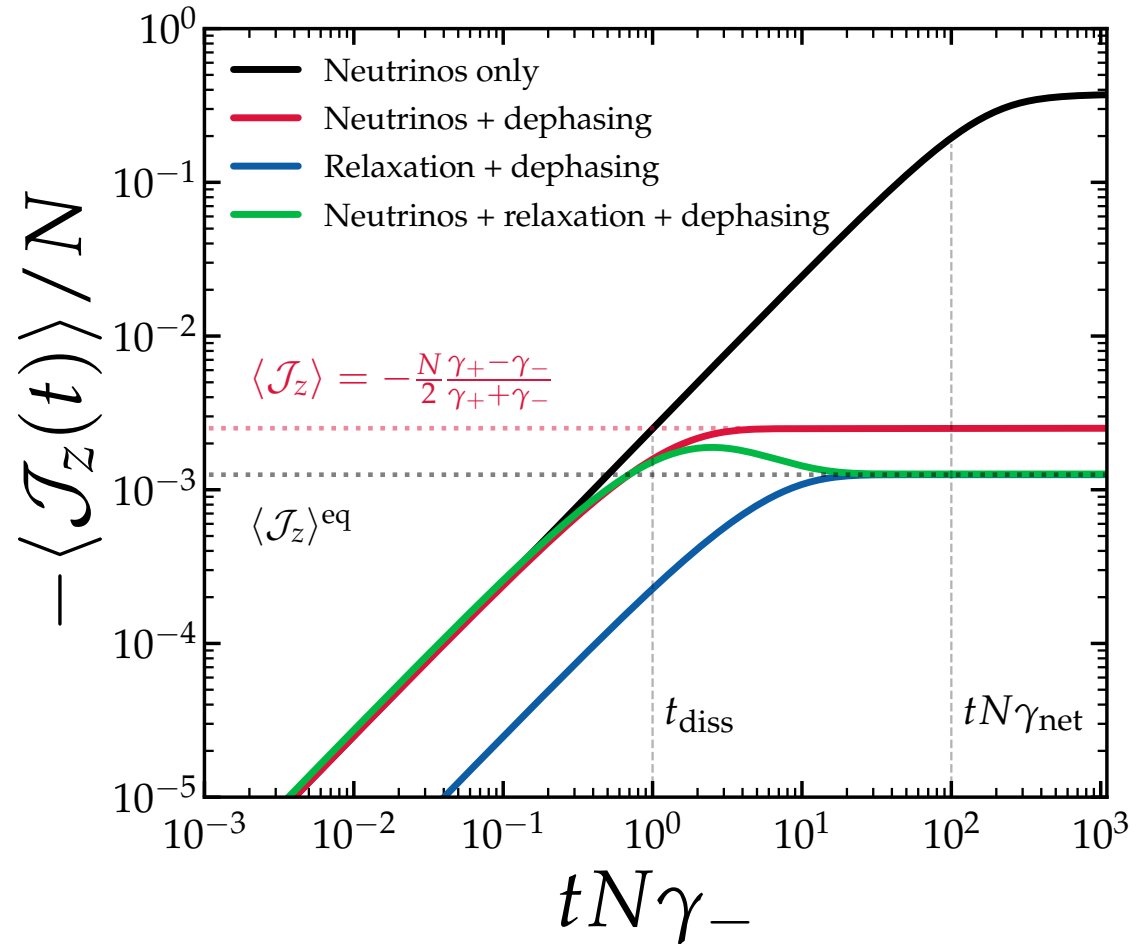
Neutrino steady-state reached at the dissipative timescale

3. All local effects: relaxation adds dissipation in the energy

Thermal steady-state reached at the dissipative timescale

$$\langle \mathcal{J}_z \rangle^{\text{eq}} = \frac{N}{2} \frac{\gamma_+^{\text{loc}} - \gamma_-^{\text{loc}}}{\gamma_+^{\text{loc}} + \gamma_-^{\text{loc}}}$$

4. Including all effects



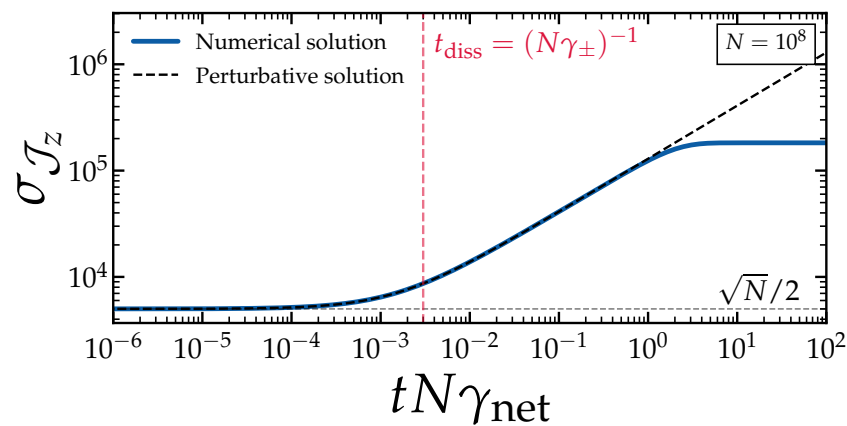
Diffusion observables

Second-order observable

$$\langle \mathcal{J}_z^2 \rangle_{|P\rangle} \approx \frac{N}{4} + \gamma_{\text{tot}} N(N+1) t,$$

$$\langle \mathcal{J}_x^2 \rangle_{|G\rangle} \approx \frac{N}{4} + 2\gamma_+ N(N+1) t,$$

$$\begin{aligned} \sigma_{\mathcal{J}_z}^2 &\equiv \langle \mathcal{J}_z^2 \rangle_{|P\rangle} - \langle \mathcal{J}_z \rangle_{|P\rangle}^2 \\ &\approx \frac{N}{4} + \gamma_{\text{tot}} N(N+1)t + \mathcal{O}(t^2) \end{aligned}$$



Diffusion observables

Second-order observable

$$\langle \mathcal{J}_z^2 \rangle_{|P\rangle} \approx \frac{N}{4} + \gamma_{\text{tot}} N(N+1) t,$$

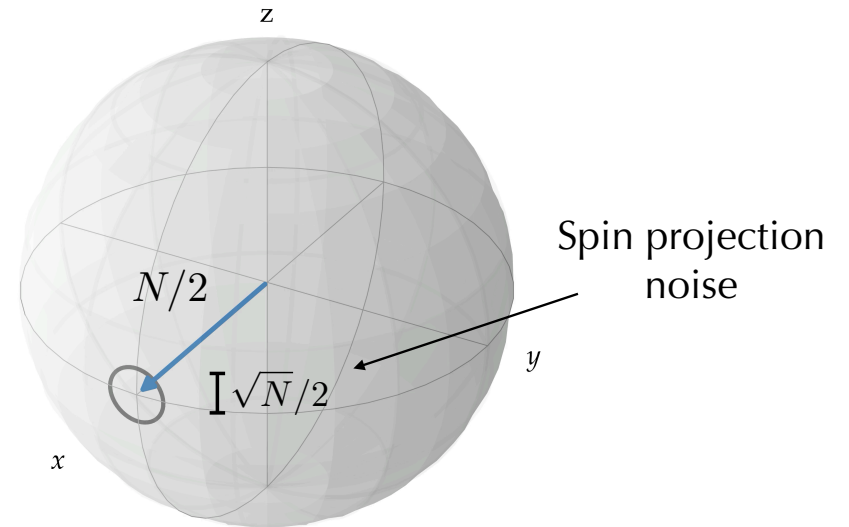
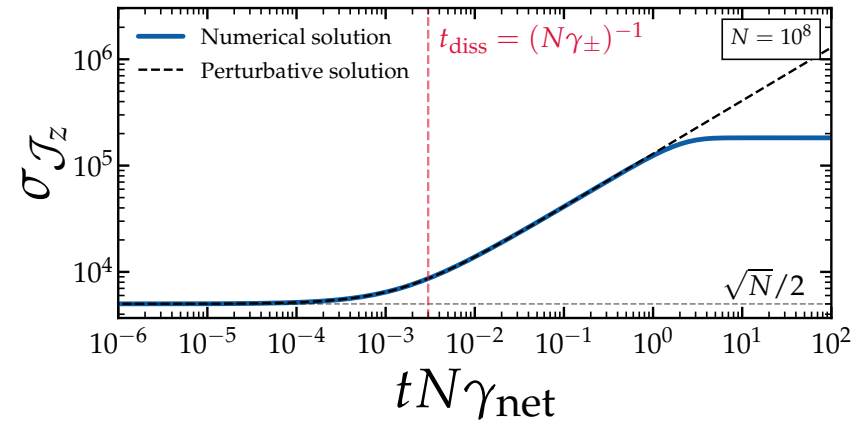
$$\langle \mathcal{J}_x^2 \rangle_{|G\rangle} \approx \frac{N}{4} + 2\gamma_+ N(N+1) t,$$

$$\sigma_{\mathcal{J}_z}^2 \equiv \langle \mathcal{J}_z^2 \rangle_{|P\rangle} - \langle \mathcal{J}_z \rangle_{|P\rangle}^2$$

$$\approx \frac{N}{4} + \gamma_{\text{tot}} N(N+1)t + \mathcal{O}(t^2)$$

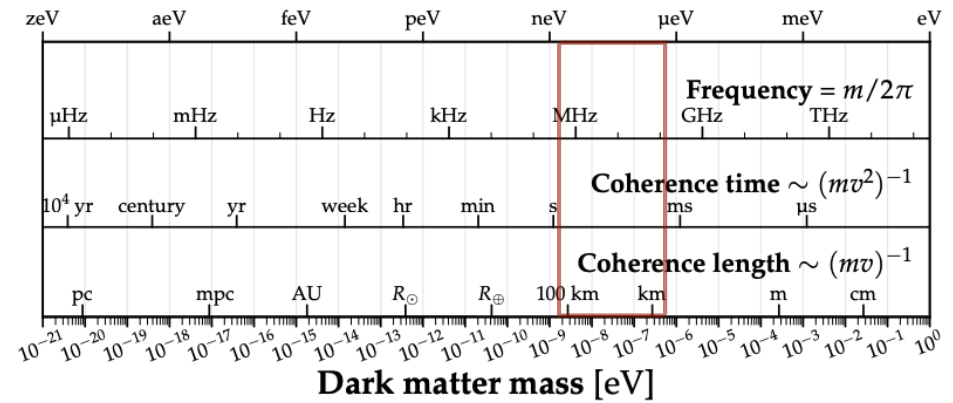
$$\langle \mathcal{J}_z \rangle \sim \gamma_{\text{tot}} N^2, \quad \sigma_{\mathcal{J}_z^2} \sim N, \quad \text{SNR} \sim \gamma_{\text{tot}} N$$

Ideal for axion DM searches



Non-Markovian case: axion DM

$$\tau_{\text{coh}} = \frac{2\pi}{m_a} \frac{1}{v\delta v} \simeq 2.8 \text{ s} \left(\frac{10^{-9} \text{ eV}}{m_a} \right)$$



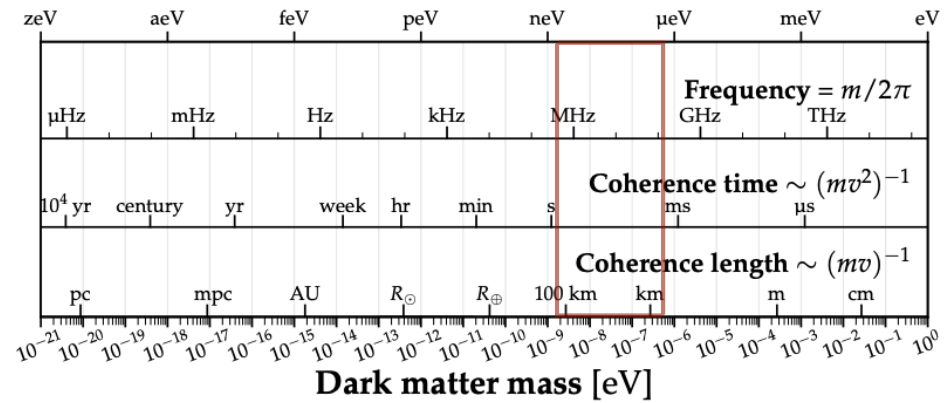
Non-Markovian case: axion DM

$$\tau_{\text{coh}} = \frac{2\pi}{m_a} \frac{1}{v\delta v} \simeq 2.8 \text{ s} \left(\frac{10^{-9} \text{ eV}}{m_a} \right)$$

For large τ limit, correlations decay

For $\tau \leq \tau_{\text{coh}}$ the axion field exhibits correlations

↓
Non-Markovian behaviour



Non-Markovian case: axion DM

$$\tau_{\text{coh}} = \frac{2\pi}{m_a} \frac{1}{v\delta v} \simeq 2.8 \text{ s} \left(\frac{10^{-9} \text{ eV}}{m_a} \right)$$

For large τ limit, correlations decay

For $\tau \leq \tau_{\text{coh}}$ the axion field exhibits correlations

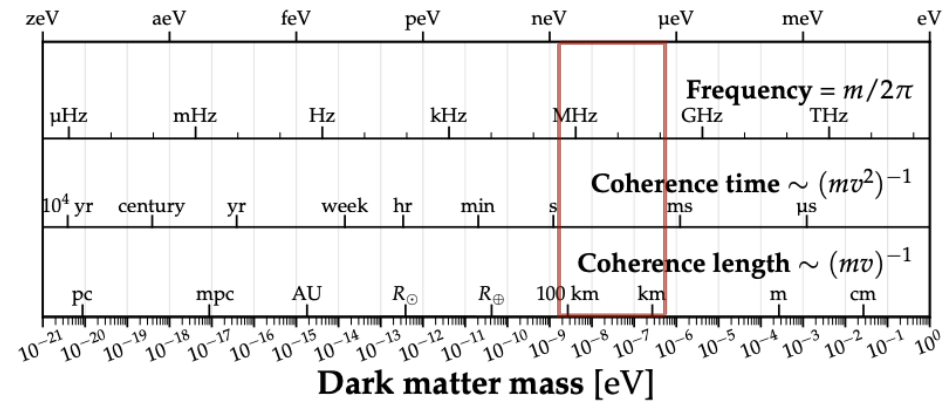


Non-Markovian behaviour

$$p(t) \propto N^2 \Omega^2 \tau_c t \quad \text{Short coherence times}$$

$$p(t) \propto N^2 \Omega^2 t^2 \quad \text{Long coherence times}$$

$$\Omega^2 = 2\rho_{\text{DM}} v^2 / f_a^2 \quad \text{Rabi frequency}$$



In the non-Markovian regime the system is influenced by a coherent drive

If the axion is on resonance, phase and direction are measured and Lindblad prediction fails

Non-Markovian case: axion DM

$$\tau_{\text{coh}} = \frac{2\pi}{m_a} \frac{1}{v\delta v} \simeq 2.8 \text{ s} \left(\frac{10^{-9} \text{ eV}}{m_a} \right)$$

For large τ limit, correlations decay

For $\tau \leq \tau_{\text{coh}}$ the axion field exhibits correlations



Non-Markovian behaviour

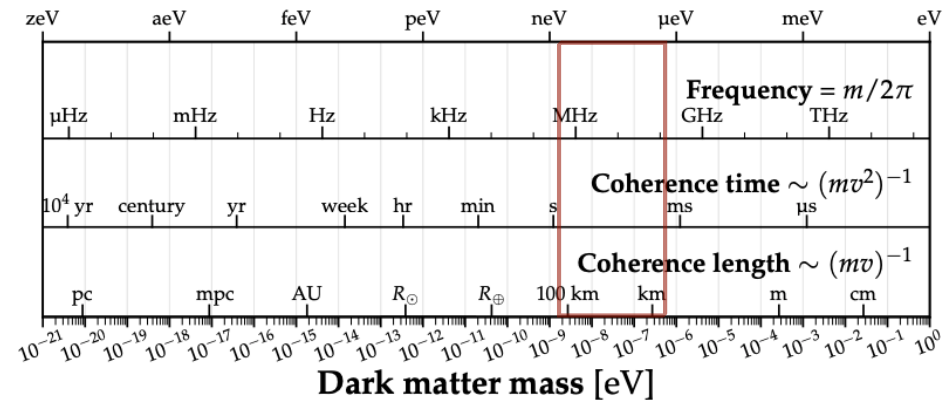
$$p(t) \propto N^2 \Omega^2 \tau_c t \quad \text{Short coherence times}$$

$$p(t) \propto N^2 \Omega^2 t^2 \quad \text{Long coherence times}$$

$$\Omega^2 = 2\rho_{\text{DM}} v^2 / f_a^2 \quad \text{Rabi frequency}$$

Pre-discovery: SNR does not care about Markovian vs non-Markovian

Post-discovery: Lindblad or coherent drive



In the non-Markovian regime the system is influenced by a coherent drive

If the axion is on resonance, phase and direction are measured and Lindblad prediction fails

Outline

1. Basics of Nuclear Magnetic Resonance (NMR)

2. Cosmic relics

- Cosmic neutrino background ($C\nu B$)
- Axion dark matter
- "Superradiant" interactions

3. Open Quantum System framework

- Lindblad master equation
- Numerical solutions
- Comments on Non-Markovian regime

4. Recent results

- Forecast for $C\nu B$ using CASPEr experiment
- Recent squeezing protocols

Experimental modeling

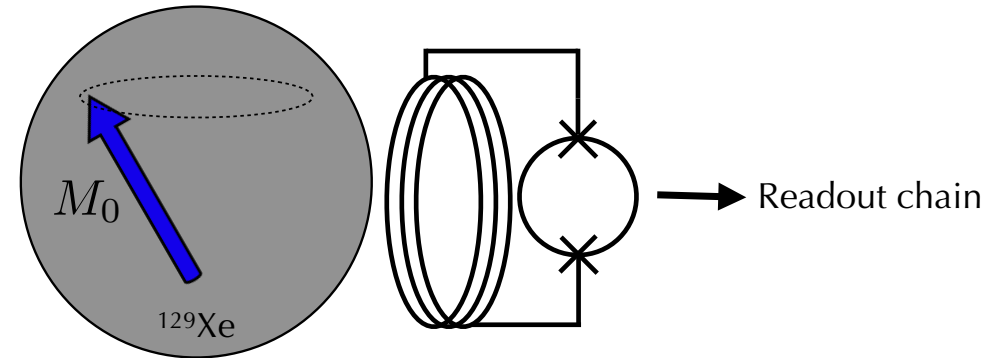
Idealised NMR experiment using polarised ^{129}Xe nuclei, as envisioned in future stages of the CASPER

$$V_f = \epsilon \Phi_{\text{pick}}$$

$$n_{\text{Xe}} \simeq 1.35 \times 10^{22} \text{ cm}^{-3}$$

$$T_2 \sim 1000 \text{ s}$$

$$T_1 \sim T_2$$



Experimental modeling

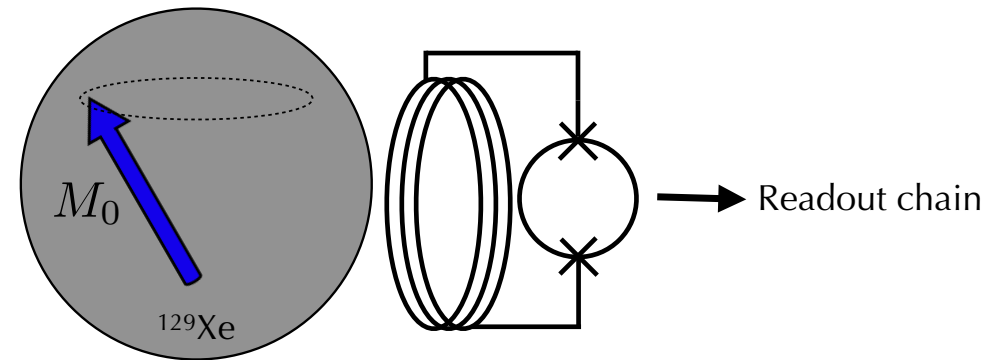
Idealised NMR experiment using polarised ^{129}Xe nuclei, as envisioned in future stages of the CASPER

$$V_f = \epsilon \Phi_{\text{pick}}$$

$$n_{\text{Xe}} \simeq 1.35 \times 10^{22} \text{ cm}^{-3}$$

$$T_2 \sim 1000 \text{ s}$$

$$T_1 \sim T_2$$



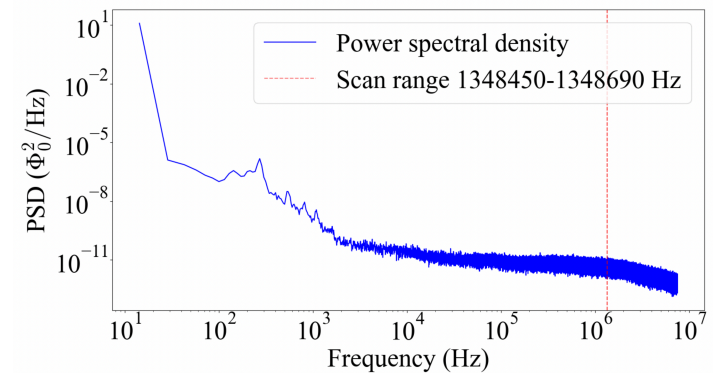
Noise sources

- Spin projection noise (shot-noise)
- Instrumental noise
 - SQUID noise
 - Electronic noise (e.g. amplifier noise)
 - Mechanical vibrations

$$\sigma_{\text{SQUID}}^2 = \frac{N}{4} \mathcal{F}_{\text{SQUID}}$$

$$\mathcal{F}_{\text{SQUID}} \approx \frac{\mathcal{P}_{\text{SQUID}}}{\mathcal{P}_{\text{SPN}}}$$

Walter et al. (CASPER) 2504.16044



Statistical analysis

$$\chi^2(\delta_\nu) = N_{\text{shots}} \sum_{ij}^{n_t} d_i [C^{-1}]_{ij} d_j$$

$$d_i \equiv \underbrace{\langle \mathcal{J}_z(t_i; \delta_\nu) \rangle_{|P\rangle}}_{\text{fiducial}} - \underbrace{\langle \hat{\mathcal{J}}_z(t_i) \rangle_{|P\rangle}}_{\text{theoretical}}$$

- Fiducial model does not contain the neutrino signal
- The theoretical prediction includes neutrino effects tuned by the overdensity parameter δ_ν

Statistical analysis

$$\chi^2(\delta_\nu) = N_{\text{shots}} \sum_{ij}^{n_t} d_i [C^{-1}]_{ij} d_j$$

$$d_i \equiv \langle \mathcal{J}_z(t_i; \delta_\nu) \rangle_{|P\rangle} - \langle \hat{\mathcal{J}}_z(t_i) \rangle_{|P\rangle}$$

$$C_{ij} = C_{ij}^{\text{th}} + \sigma_{\text{SQUID}}^2 \delta_{ij}$$

$$C_{ij}^{\text{th}} = \begin{cases} \sigma_{\mathcal{J}_z}^2(t_i; \delta_\nu), & i = j, \\ \sigma_{\mathcal{J}_z}^2(t_i; \delta_\nu) e^{-(\gamma_-^{\text{loc}} + \gamma_+^{\text{loc}})|t_j - t_i|}, & i < j, \\ \sigma_{\mathcal{J}_z}^2(t_j; \delta_\nu) e^{-(\gamma_-^{\text{loc}} + \gamma_+^{\text{loc}})|t_i - t_j|}, & i > j, \end{cases}$$

- Fiducial model does not contain the neutrino signal
- The theoretical prediction includes neutrino effects tuned by the overdensity parameter δ_ν

Statistical analysis

$$\chi^2(\delta_\nu) = N_{\text{shots}} \sum_{ij}^{n_t} d_i [C^{-1}]_{ij} d_j$$

$$d_i \equiv \langle \mathcal{J}_z(t_i; \delta_\nu) \rangle_{|P\rangle} - \langle \hat{\mathcal{J}}_z(t_i) \rangle_{|P\rangle}$$

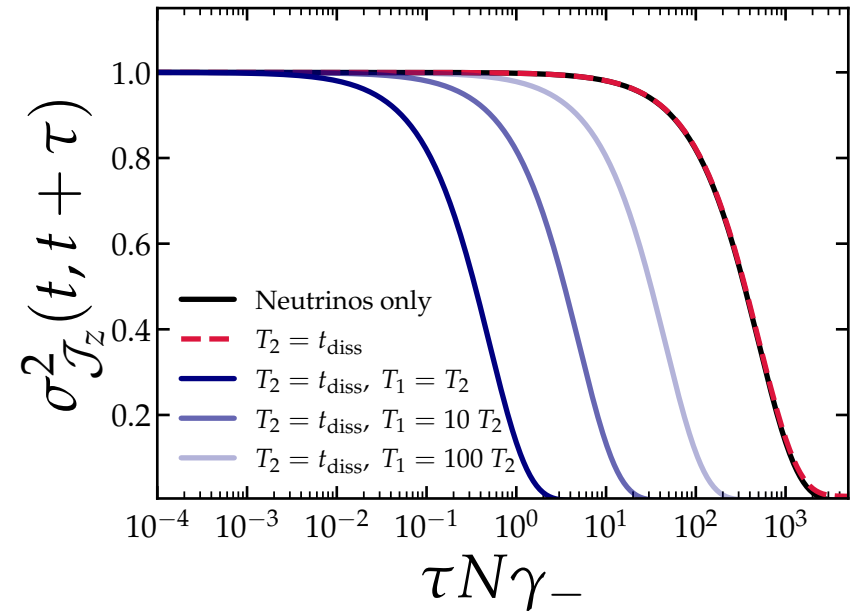
$$C_{ij} = C_{ij}^{\text{th}} + \sigma_{\text{SQUID}}^2 \delta_{ij}$$

$$C_{ij}^{\text{th}} = \begin{cases} \sigma_{\mathcal{J}_z}^2(t_i; \delta_\nu), & i = j, \\ \sigma_{\mathcal{J}_z}^2(t_i; \delta_\nu) e^{-(\gamma_-^{\text{loc}} + \gamma_+^{\text{loc}})|t_j - t_i|}, & i < j, \\ \sigma_{\mathcal{J}_z}^2(t_j; \delta_\nu) e^{-(\gamma_-^{\text{loc}} + \gamma_+^{\text{loc}})|t_i - t_j|}, & i > j, \end{cases}$$

- Fiducial model does not contain the neutrino signal
- The theoretical prediction includes neutrino effects tuned by the overdensity parameter δ_ν

Two-time correlation functions

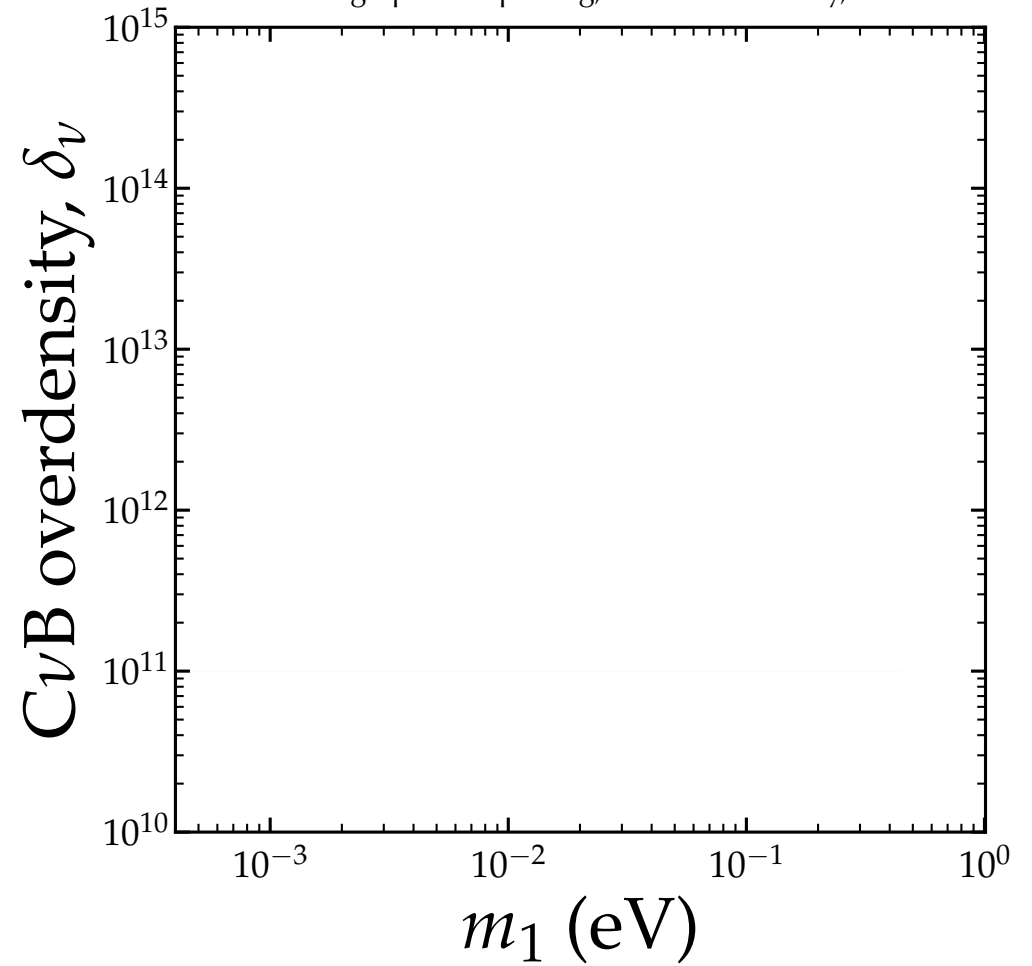
$$\begin{aligned} \frac{d}{d\tau} \sigma_{\mathcal{J}_z}^2 = & -\gamma_{\text{net}} \left[\langle \mathcal{J}_z(t) \mathcal{J}_z^2(t+\tau) \rangle - \langle \mathcal{J}_z(t) \rangle \langle \mathcal{J}_z^2(t+\tau) \rangle \right. \\ & \left. - \langle \mathcal{J}_z(t) \mathcal{J}_z^2(t+\tau) \rangle + \langle \mathcal{J}_z(t) \rangle \langle \mathcal{J}_z^2(t+\tau) \rangle \right] \\ & - (\gamma_{\text{tot}} + \gamma_+^{\text{loc}} + \gamma_-^{\text{loc}}) \sigma_{\mathcal{J}_z}^2(t, t+\tau) \end{aligned}$$



Forecast constraints

$$\omega_0^* = \min\left(\gamma B_{\max}, \frac{p_\nu}{m_\nu R}\right)$$

Assuming optimal splitting, normal hierarchy, ^{129}Xe



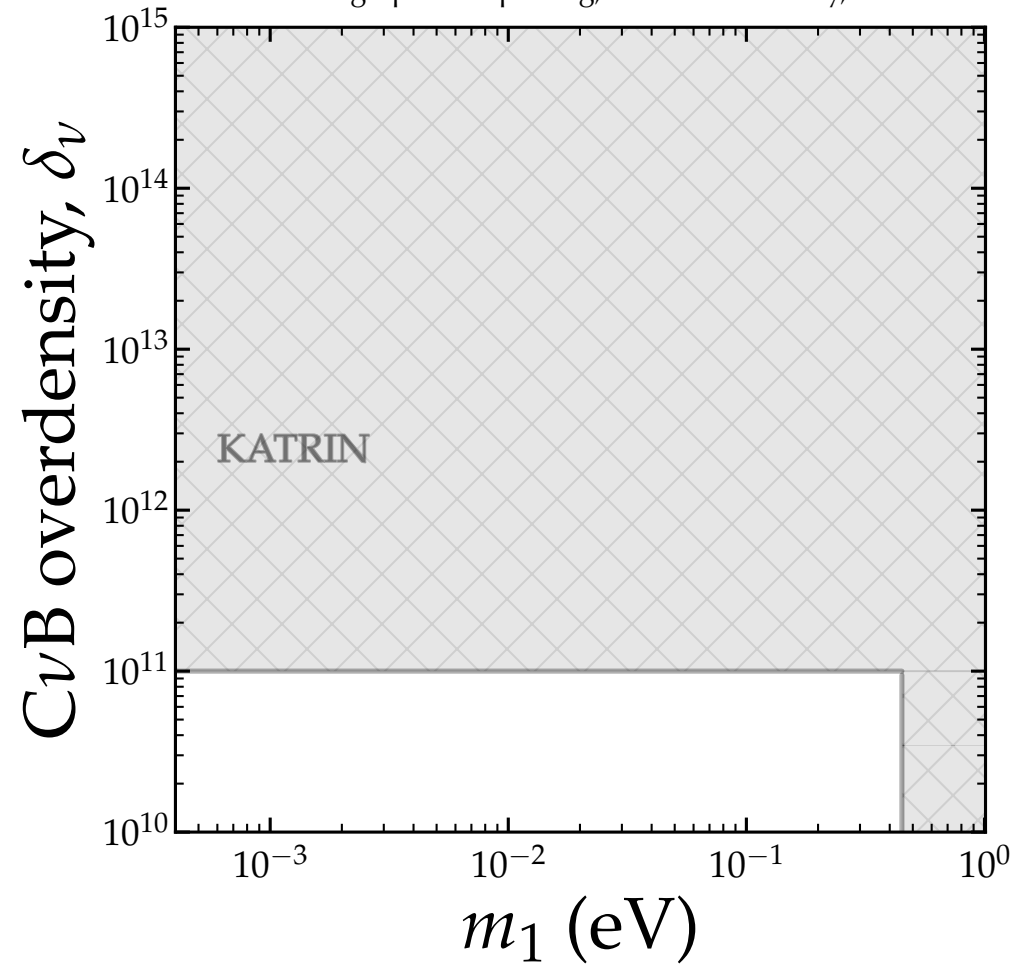
Forecast constraints

$$\omega_0^* = \min\left(\gamma B_{\max}, \frac{p_\nu}{m_\nu R}\right)$$

Assuming optimal splitting, normal hierarchy, ^{129}Xe

KATRIN mass and overdensity bounds

$$m_1 < 0.45 \text{ eV} \quad \delta_\nu \lesssim 10^{11}$$



Forecast constraints

KATRIN mass and overdensity bounds

$$m_1 < 0.45 \text{ eV} \quad \delta_\nu \lesssim 10^{11}$$

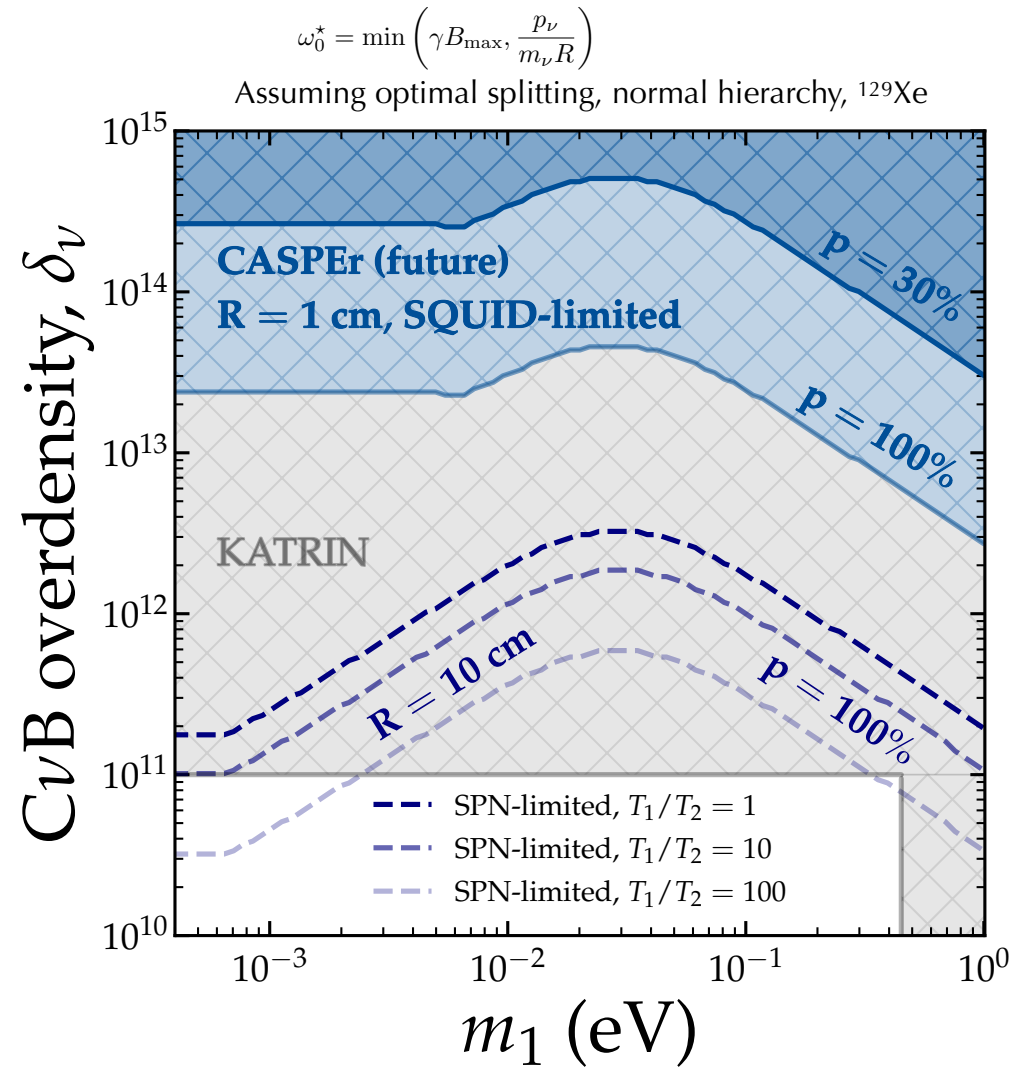
Future CASPER constraint using Xenon

$$\delta_\nu^{\text{CASPER}} \lesssim 5.3 \times 10^{13} \left(\frac{0.5}{p} \right)^2$$

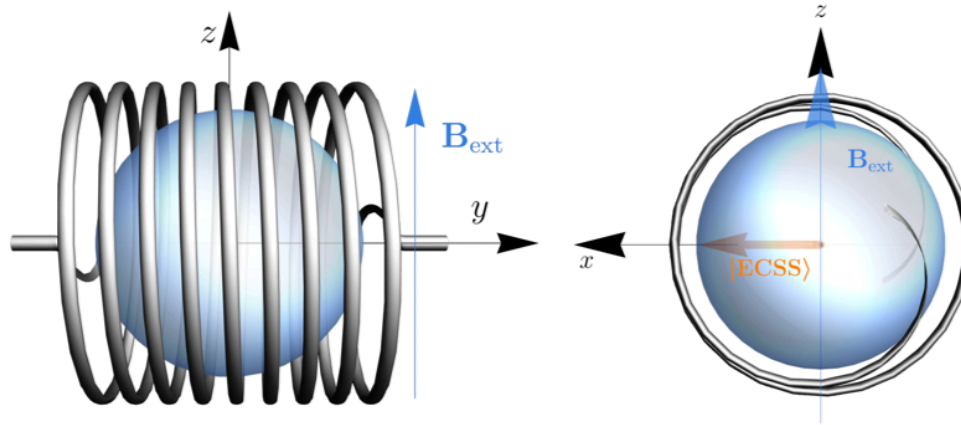
Larger sample and low instrumental noise

$$\delta_\nu^* \lesssim 1.7 \times 10^{12}, \quad 90\% \text{ C.L.}$$

Worse by factor $\mathcal{O}(10)$ compared to
perturbative estimate [2408.04021](#)

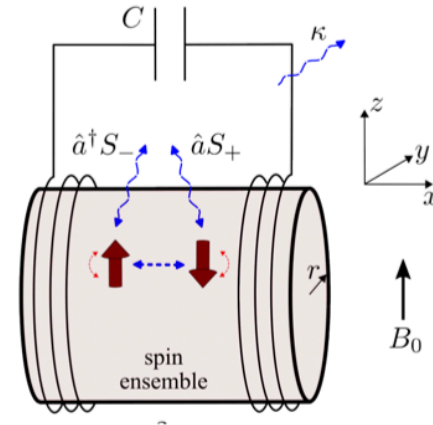


Squeezing protocols



yz plane squeezing

Galanis, Hosten, Arvanitaki, Dimopoulos, 2508.20520

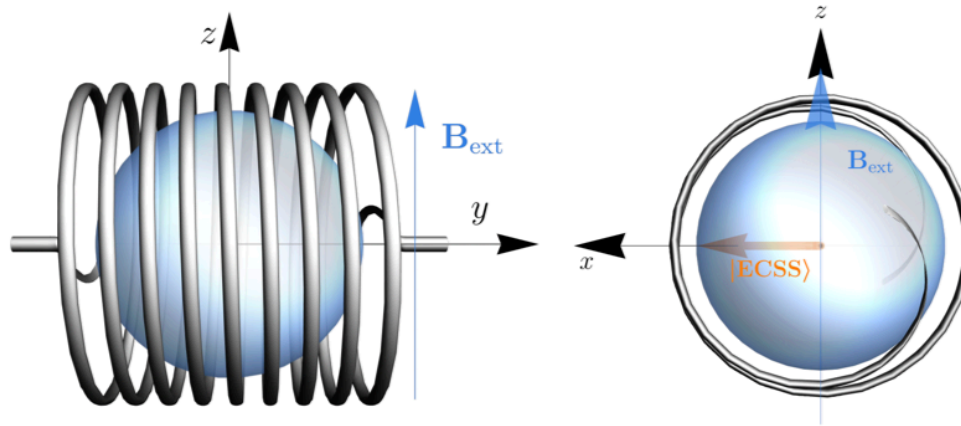


xy plane squeezing

Boyers, Goldstein, Sushkov, 2502.14103

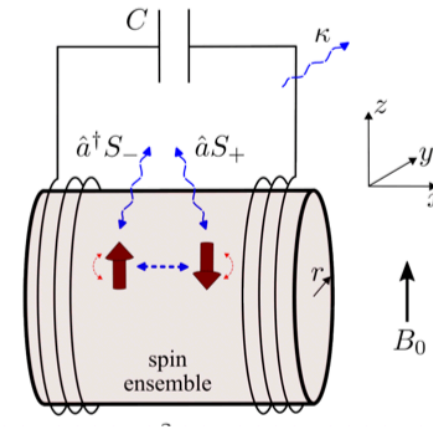
- Collection of spins inside LC circuit coil
- System very far off-resonance $\omega_{\text{LC}} \gg \omega_0$
- Effective spin-spin interaction induces non-linear squeezing Hamiltonian

Squeezing protocols



yz plane squeezing

Galanis, Hosten, Arvanitaki, Dimopoulos, 2508.20520



xy plane squeezing

Boyers, Goldstein, Sushkov, 2502.14103

- Collection of spins inside LC circuit coil
- System very far off-resonance $\omega_{LC} \gg \omega_0$
- Effective spin-spin interaction induces non-linear squeezing Hamiltonian

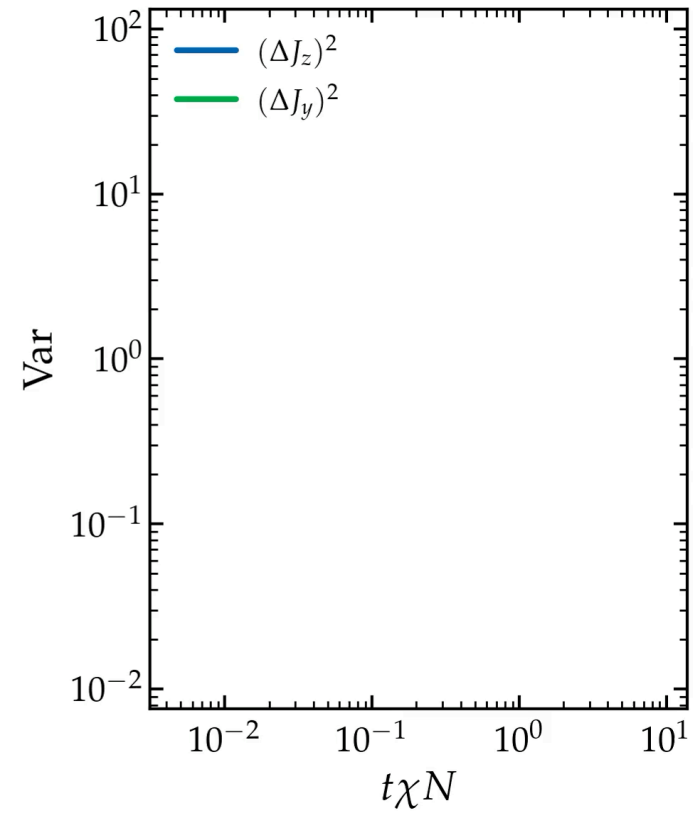
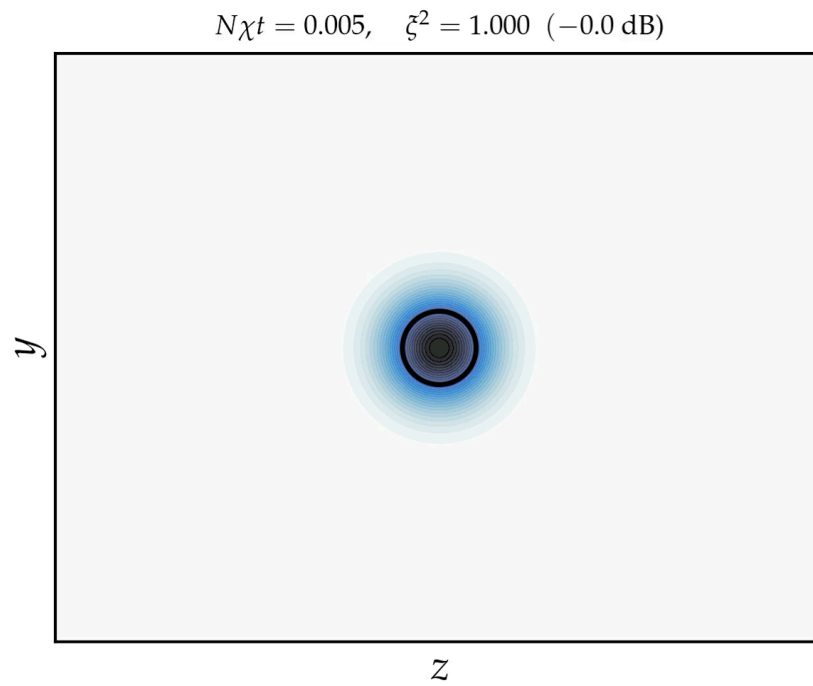
$$H = \chi J_z^2$$

$\uparrow$ Very weak coupling

$\nwarrow$ Need very large N !

One-axis twisting (OAT)
Hamiltonian

Squeezing evolution

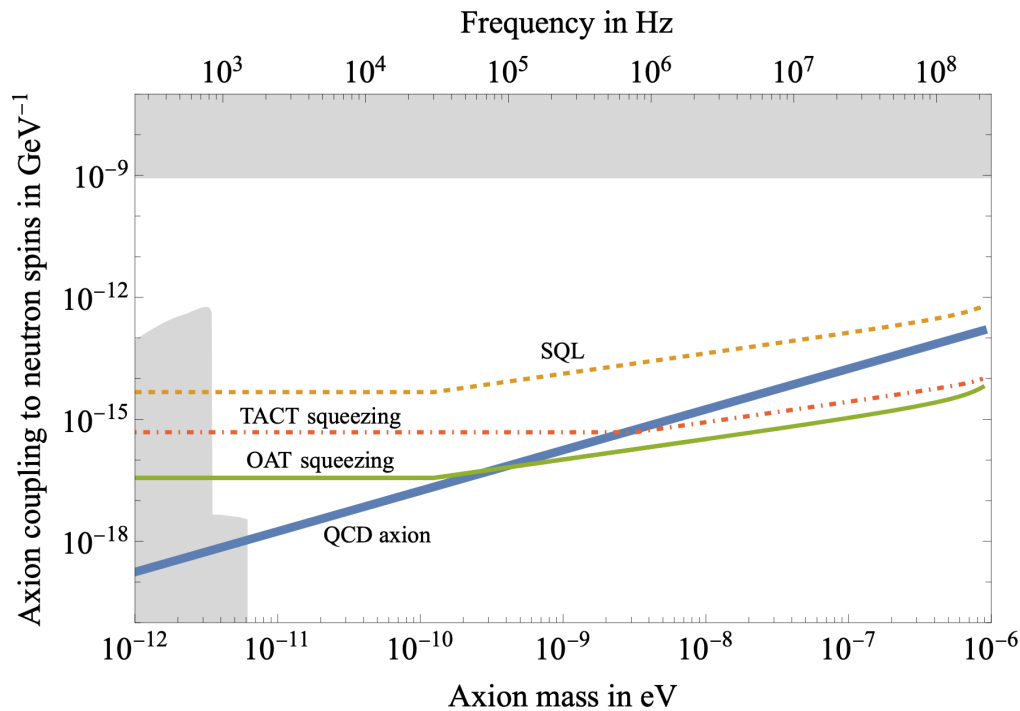


Axion DM and $C\nu B$ reach

2508.20520

Axion DM

Observe anti-squeezing induced by axion DM dissipation



$C\nu B$

Observe drift in energy with reduced variance and improved SNR

Promising potential of table-top detectors

$$\delta_\nu \simeq 10^{10} \left(\frac{0.1 \text{ eV}}{m_\nu} \right) \left(\frac{10^{26}}{N} \right) \left(\frac{R}{10 \text{ cm}} \right)^{3/2}$$

$$\delta_\nu \lesssim 10^{11} \quad \text{KATRIN}$$

Summary

- While can have enhanced rates in inelastic scatterings, $C\nu B$ is still out of reach
- Protocols for dark matter (e.g. axions) can be viable and accelerate search for dark matter
- Identified experimental challenges to probe weakly interacting relics
- New framework for future works